Physics of near-threshold resonances

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Outline

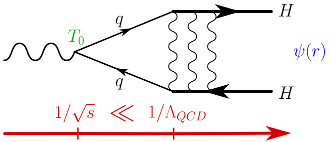
1. Motivation.
2. Our approach to the description of near-threshold resonances.
3. Application of our approach to various processes.
4. Summary.

Motivation

- The cross sections of many processes with hadronic pairs production demonstrate significant enhancement near the thresholds ($e^+e^- \to p\bar{p}, n\bar{n}, \Lambda\bar{\Lambda}, \Lambda_c\bar{\Lambda}_c, B\bar{B}, B^*\bar{B}, B^*\bar{B}^*$, some J/ψ decays, etc.).
- Strong energy dependence of the cross sections is observed in these processes.
- Standard description of resonances based on the Breit-Wigner parametrization is not valid near the threshold. Other approaches are necessary.
- A nontrivial energy dependence of the cross sections is also observed in other processes at energies close to the thresholds for the production of hadronic pairs ($e^+e^- \to 6\pi, K^+K^-\pi^+\pi^-$ and others).
- We would like to have a unified interpretation of near-threshold resonances in various processes.

Final-state interaction

Near-threshold resonances in hadroproduction cross sections appear as the result of interaction between produced hadrons.



- $q\bar{q}$ pair is produced at small distances $r \sim 1/\sqrt{s}$.
- Hadronization takes place at larger distances $r \sim 1/\Lambda_{QCD}$. Hadronic system is described by a wave function $\psi(\boldsymbol{r})$, which characteristic size is much larger than $1/\Lambda_{QCD}$.
- Near the threshold, the velocity of hadrons is small, so they interact for quite a long time.
- The interaction between slow hadrons significantly changes the wave function of a pair.
- Thanks to the separation of scales, the amplitude is factorized:

$$T = T_0 \cdot \psi(r \lesssim 1/\Lambda_{QCD}) \approx T_0 \cdot \psi(0)\,, \quad \text{where } T_0 \approx \text{const}$$

- The function $\psi(0)$ has a strong energy dependence near the threshold.

Interaction of hadrons

The exact equation for the wave function of the hadronic system depends on their quantum numbers:
spin of particles, isospin, angular momentum, total angular momentum.

For the production of spinless particles with angular momentum l, total kinetic energy E and mass M the radial Schrödinger equation has form

$$\left(p_r^2 + M\,V(r) + \frac{l(l+1)}{r^2} - k^2\right)\psi(r) = 0\,, \qquad k = \sqrt{ME}\,.$$

- We find the solution with asymptotic behavior

$$\psi(r\to\infty) = \tfrac{1}{2ikr}\left(S\,\chi_l^+ - \chi_l^-\right), \qquad \chi_l^\pm = \exp\left[\pm i\left(kr - \pi l/2\right)\right]$$

- Pair production cross section: $\boxed{\sigma = \dfrac{\pi k \alpha^2}{2M^3}\left|g\,\psi^{(l)}(0)\right|^2}$

  $g \approx$ const — the amplitude of pair production at small distances,
  $\psi^{(l)}(0) = \frac{d^l}{dr^l}\psi(r)\Big|_{r=0}$

Properties of hadroproduction cross section

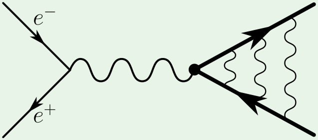
- The scattering cross section ($\sigma_{\mathrm{sc}} = \frac{\pi}{k^2}\left(2l+1\right)\left|S-1\right|^2$) and the cross section of pair production are determined by the behavior of wave function in different regions.
- In the resonance case the energy dependence of the cross sections is determined by a small number of parameters, such as scattering length and effective range of interaction.
- One can use different parametrizations of the potential that reproduce these parameters.
- We consider the potentials we used to be phenomenological effective potentials rather than real ones.

For $l = 0$ and the rectangular potential well: $V(r) = \begin{cases} -U & r \leqslant R \\ 0 & r > R \end{cases}$

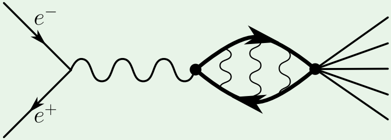
$$\sigma = \frac{\pi k \alpha^2 g^2}{2M^3} \frac{q^2}{q^2 \cos^2\left(qR\right) + k^2 \sin^2\left(qR\right)}, \qquad q = \sqrt{M\left(E+U\right)}$$

Elastic and inelastic processes

- Elastic processes — production of real hadronic pair with low relative velocity



- Inelastic processes — production of virtual hadronic pair in the intermediate state with subsequent annihilation into other particles



- In order to describe inelastic processes we use optical potentials that contain imaginary part: $V(r) = U(r) - i\,W(r)$.
- The total cross section is expressed via the Green's function of the Schrödinger equation at $r = r' = 0$:

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{inel}} = \frac{\pi\alpha^2 g^2}{2M^3}\,\mathrm{Im}\,\mathcal{D}(0,0|E)$$

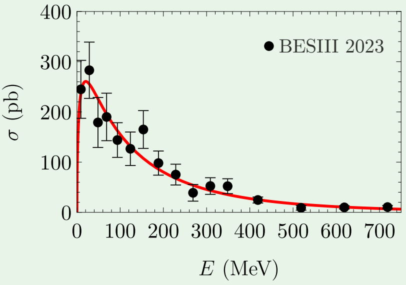
The process $e^+e^- \to \Lambda\bar{\Lambda}$

- Near the threshold, $\Lambda\bar{\Lambda}$ pair is produced mainly with quantum numbers $l = 0$, $S = 1$, $I = 0$.
- The simple model described before can be applied as is: $R = 0.45\,\mathrm{fm}$, $U = 584\,\mathrm{MeV}$.

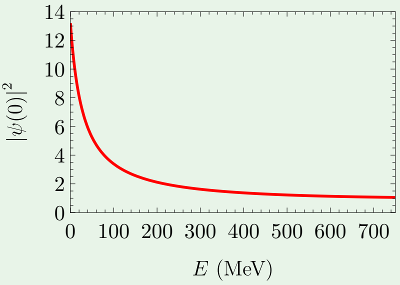
$$\sigma = \frac{2\pi k\alpha^2}{M\,s}\,F_D^2(s)\,g^2\,|\psi(0)|^2\,,$$

$$F_D(s) = \left(1 - \tfrac{s}{1\,\mathrm{GeV}^2}\right)^{-2}, \qquad s = (2M + E)^2$$

Cross section



Enhancement factor



Tensor interaction in the process $e^+e^- \to \Lambda_c\bar{\Lambda}_c$

The interaction potentials of hadrons contain the contribution of the tensor operator $S_{12} = 3\,(\boldsymbol{\sigma}_1\boldsymbol{n})\,(\boldsymbol{\sigma}_2\boldsymbol{n}) - (\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)$.

- Conserves the total spin S of particles and total angular momentum J.
- Changes the angular momentum l by 2. As a result, the states with $S = 1$ and $l = J \pm 1$ are mixed. Coupled-channels Schrödinger equation should be considered.

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- The $\Lambda_c\bar{\Lambda}_c$ pair has the quantum numbers $S = 1$, $J = 1$, $l = 0, 2$.
- The cross section is expressed via the components of two independent solutions of coupled-channels Schrödinger equation:

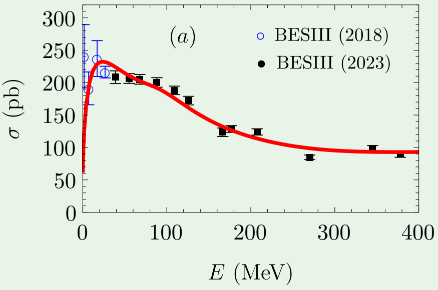
$\psi_1(r \to \infty)$	$\psi_2(r \to \infty)$
$u_1 \to \frac{1}{2ikr}\left(S_{11}\chi_0^+ - \chi_0^-\right)$	$u_2 \to \frac{1}{2ikr}\,S_{21}\chi_0^+$
$w_1 \to \frac{1}{2ikr}\,S_{12}\chi_2^+$	$w_2 \to \frac{1}{2ikr}\left(S_{22}\chi_2^+ - \chi_2^-\right)$

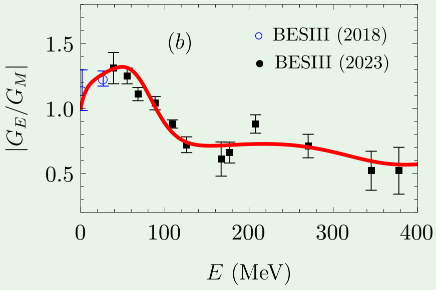
Tensor interaction in the process $e^+e^- \to \Lambda_c\bar{\Lambda}_c$

$$\sigma = \frac{\pi k \alpha^2 g^2}{2M^3}\left(|u_1(0)|^2 + |u_2(0)|^2\right), \qquad \frac{G_E}{G_M} = \frac{u_1(0) - \sqrt{2}\,u_2(0)}{u_1(0) + \frac{1}{\sqrt{2}}u_2(0)}$$

The functions $u_1(0)$ and $u_2(0)$ have nontrivial energy dependence.

The ratio G_E/G_M depends on energy and differs from unity due to tensor interaction.





$p\bar{p}$ and $n\bar{n}$ pairs production in e^+e^- annihilation

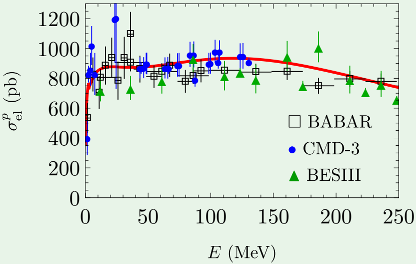
Aspects to consider:

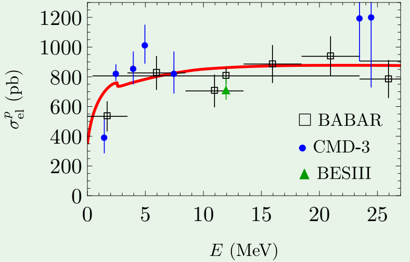
- Nontrivial dependence of G_E/G_M on energy means that the tensor interaction plays an important role.
- Nucleon-antinucleon pairs are produced with isospins $I = 0$ and $I = 1$, which are mixed because of violation of isotopic invariance.
- The proton-neutron mass difference and the Coulomb interaction are important at low energies.
- The cross section of $p\bar{p}$ and $n\bar{n}$ annihilation into mesons is large, therefore the imaginary part of interaction potential is important.
- The predictions based on nucleon-antinucleon interaction potential should reproduce a lot of experimental data (partial-wave scattering cross sections, cross sections of pair production, electromagnetic form factors, etc.).

All these can be taken into account within our approach.

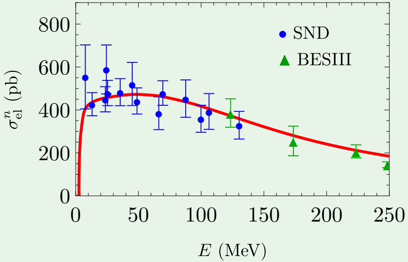
$p\bar{p}$ and $n\bar{n}$ pairs production in e^+e^- annihilation

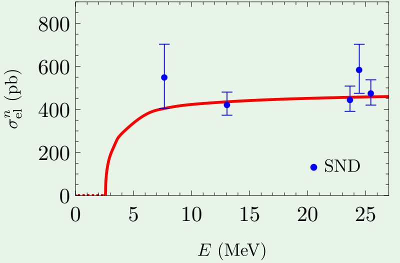
$p\bar{p}$ cross section





$n\bar{n}$ cross section

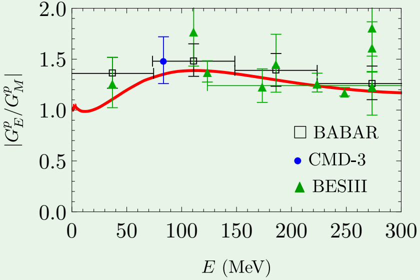




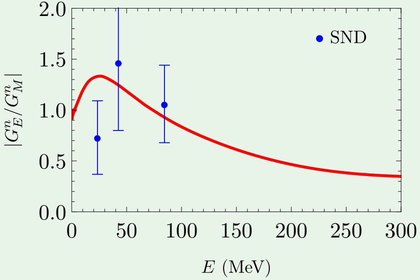
Form factors of p and n in the time-like region

Energy dependence of the ratio of electromagnetic form factors G_E/G_M is the consequence of tensor interaction.

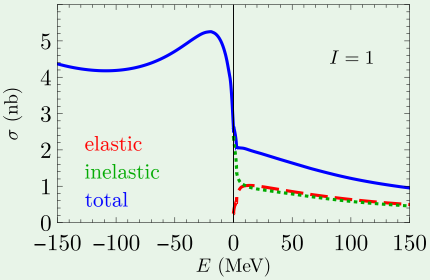
Proton form factors



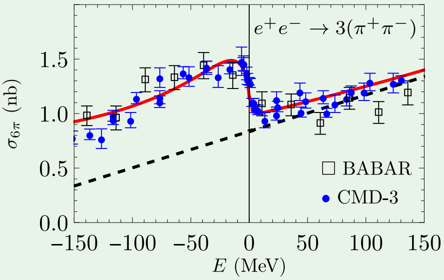
Neutron form factors

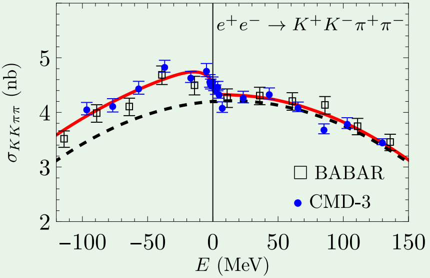


Contribution of $N\bar{N}$ interaction to inelastic processes



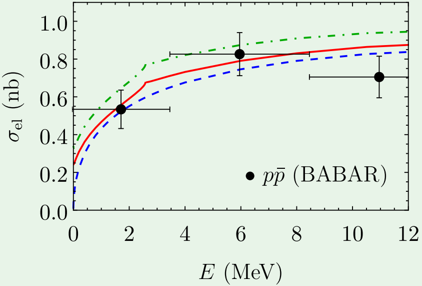
- Significant drop of the inelastic cross section is predicted in the state with $I = 1$.
- This results in strong energy dependence of cross sections of some processes near the $N\bar{N}$ production threshold.



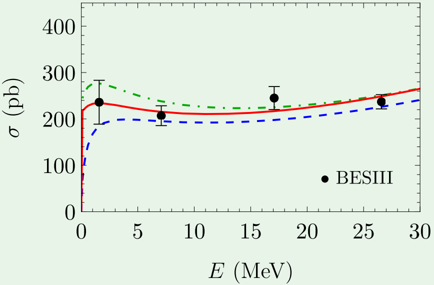


Effect of the Coulomb interaction near the threshold

$p\bar{p}$ cross section



$\Lambda_c\bar{\Lambda}_c$ cross section



- blue – cross section without the Coulomb interaction
- green – the same multiplied by the Sommerfeld factor $C = \frac{2\pi\eta}{1-e^{-2\pi\eta}}$
- red – cross section with proper account for the Coulomb interaction within our approach

Multiplication of pair production cross section by the Sommerfeld factor is an incorrect way of account for the Coulomb interaction.

$B\bar{B}$, $B^*\bar{B}$ and $B^*\bar{B}^*$ production in e^+e^- annihilation

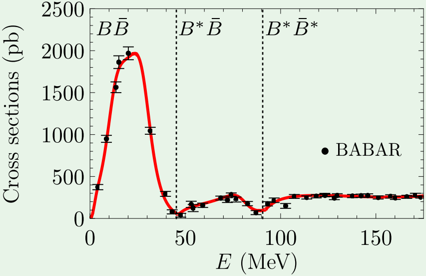
Aspects to consider:

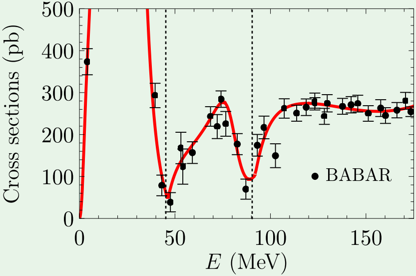
- The masses are close : $M_B = 5280\,\mathrm{MeV}$, $M_{B^*} = 5326\,\mathrm{MeV}$.
- Meson pairs are produced with quantum numbers $I = 0$, $l = 1$, $J^{PC} = 1^{--}$. Transitions between different states are possible.
- Pairs of charged or neutral mesons can be produced.

For simplicity, we use rectangular potential wells to describe the interaction in each final state, as well as the mixing between states. The potential is a matrix $V_{ij}(r) = U_{ij} \cdot \theta(R_{ij} - r)$.

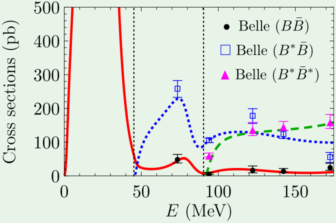
Cross sections of $B\bar{B}$, $B^*\bar{B}$ and $B^*\bar{B}^*$ production

Sum of elastic cross sections





Exclusive cross sections



- Sharp dips appear because of transitions between channels.
- The peak at $E \approx 75\,\mathrm{MeV}$ in $B^*\bar{B}$ channel could be due to mixing with bound state in $B^*\bar{B}^*$ channel.

Summary

- We developed an approach for the description of nontrivial energy dependence of pair production cross sections near the thresholds.
- Our approach allows one to easily consider various effects: the Coulomb interaction, different masses of particles, tensor interaction, coupling between channels.
- The approach allows one to obtain the contribution of interaction in the intermediate state to the cross sections of inelastic processes.
- Within this approach, good agreement with the experimental data was obtained for many processes.

Summary

- We developed an approach for the description of nontrivial energy dependence of pair production cross sections near the thresholds.
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THANK YOU FOR ATTENTION!

Backup slides

The scattering theory

- The scattering cross section: $\sigma_{\text{sc}} = \dfrac{\pi}{k^2}(2l+1)\left|S-1\right|^2$, $S = e^{2i\delta_l}$, where δ_l is the scattering phase.

The scattering cross section and the cross section of pair production are determined by the behavior of wave function in different regions.

Low-energy scattering

The scattering length $a = \lim\limits_{k\to 0}\left(-\dfrac{\delta_0}{k}\right)$

- For resonance scattering $|a| \gg R$ (the range of interaction)
  - Loosely bound state: $a > 0$, binding energy $\varepsilon = -1/Ma^2$
  - Virtual state: $a < 0$, energy $\varepsilon = 1/Ma^2$

In both cases $|\varepsilon| \ll U$, $\sigma_{\text{sc}} = 4\pi a^2 \gg 4\pi R^2$

Example: rectangular well for $l = 0$

Consider potential $V(r) = \begin{cases} -U & r \leqslant R\,, \\ 0 & r > R\,, \end{cases}$ where $U \gg E$

$$\boxed{\sigma = \frac{\pi k \alpha^2 g^2}{2M^3} \frac{q^2}{q^2 \cos^2(qR) + k^2 \sin^2(qR)}}\,, \quad q = \sqrt{M(E+U)} \gg k = \sqrt{ME}$$

- Resonant pair production takes place for

$$q_0 R = \pi\left(n + \tfrac{1}{2}\right) + \varphi\,, \qquad q_0 = \sqrt{MU}, \quad |\varphi| \ll 1, \quad n \; - \; \text{integer}$$

- Then the cross section of pair production is

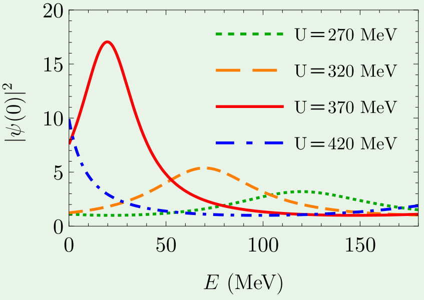
$$\sigma \approx \frac{\pi \alpha^2 g^2 \sqrt{M}}{2M^3} \frac{\gamma U \sqrt{E}}{(E - E_0)^2 + \gamma E} \qquad \longleftarrow \text{one-channel Flatté formula}$$

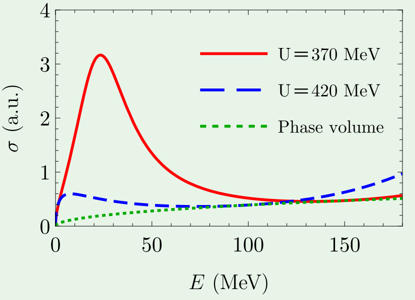
$$\gamma = \frac{4U}{q_0^2 R^2} = 4\,|\varepsilon|\,\frac{a^2}{R^2}\,, \quad E_0 = -\frac{2U\varphi}{q_0 R} \approx -2\,|\varepsilon|\,\frac{a}{R}\,, \quad |\varepsilon| \ll |E_0| \ll U$$

Cross section has maximum at $E \approx |\varepsilon| = U\varphi^2$, but not E_0!

Example: rectangular well for $l = 0$

The energy dependence of enhancement factor $|\psi(0)|^2$ (left) and cross section of pair production (right) for $M = 5279\,\mathrm{MeV}$, $R = 1.5\,\mathrm{fm}$.





A bound state appears at $U = 396\,\mathrm{MeV}$.

- $U = 370\,\mathrm{MeV}$ — a virtual state above the threshold.
- $U = 420\,\mathrm{MeV}$ — a loosely bound state below the threshold.

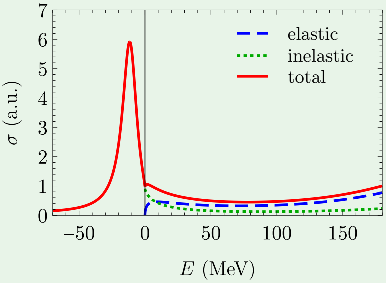
Virtual pair production

- We use optical potentials that contain imaginary part to take inelastic processes into account.
- The total cross section is expressed via the Green's function of the Schrödinger equation at $r = r' = 0$:

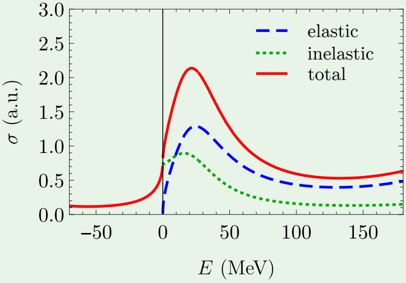
$$\sigma_{\mathrm{tot}} = \sigma_{\mathrm{el}} + \sigma_{\mathrm{inel}} = \frac{\pi\alpha^2 g^2}{2M^3}\,\mathrm{Im}\,\mathcal{D}(0,0|E)$$

For $l = 0$ we have: $\mathrm{Im}\,\mathcal{D}(0,0|E) = \mathrm{Im}\left[q\,\frac{q\sin(qR)+ik\cos(qR)}{q\cos(qR)-ik\sin(qR)}\right]$

Loosely bound state



Virtual state



	$\widetilde{U}_S^0$	$\widetilde{U}_D^0$	$\widetilde{U}_T^0$	$\widetilde{U}_S^1$	$\widetilde{U}_D^1$	$\widetilde{U}_T^1$
U_i (MeV)	-196	80.8	-2.2	-36.3	401.6	15.2
W_i (MeV)	167.3	225.4	-2	-16.4	217.2	1.5
a_i (fm)	0.701	1.185	2.704	1.294	0.739	1.289
g_i	$g_p = 14.1$			$g_n = 3.6 - 1.1i$		

$$\sigma_{\text{el}}^p = \frac{4\pi k_p \alpha^2}{q^3} F_D^2(q) \left[\left| g_p u_1^p(0) + g_n u_1^n(0) \right|^2 + \left| g_p u_2^p(0) + g_n u_2^n(0) \right|^2 \right],$$
$$\sigma_{\text{el}}^n = \frac{4\pi k_n \alpha^2}{q^3} F_D^2(q) \left[\left| g_p u_3^p(0) + g_n u_3^n(0) \right|^2 + \left| g_p u_4^p(0) + g_n u_4^n(0) \right|^2 \right].$$

$B\bar{B}$, $B^*\bar{B}$ and $B^*\bar{B}^*$ interaction

	V_{XX}	V_{YY}	V_{ZZ}	V_{XY}	V_{XZ}	V_{YZ}
U_{ij} (MeV)	-613.1	-360.6	-586.7	26.7	20	78.6
a_{ij} (fm)	1.361	1.804	1.809	0.953	2.819	2.209
g_i (fm)	$g_X = 0.118$		$g_Y = -0.004 + 0.217\,i$		$g_Z = -0.6 + 0.193\,i$	

$$\sigma_X = \frac{2\pi\beta_X\alpha^2}{s}\left|g_X u_{1R}^{(1)}(0) + g_Y v_{1R}^{(1)}(0) + g_Z w_{1R}^{(1)}(0)\right|^2,$$
$$\sigma_Y = \frac{2\pi\beta_Y\alpha^2}{s}\left|g_X u_{2R}^{(1)}(0) + g_Y v_{2R}^{(1)}(0) + g_Z w_{2R}^{(1)}(0)\right|^2,$$
$$\sigma_Z = \frac{2\pi\beta_Z\alpha^2}{s}\left|g_X u_{3R}^{(1)}(0) + g_Y v_{3R}^{(1)}(0) + g_Z w_{3R}^{(1)}(0)\right|^2.$$

Related publications

- A. I. Milstein, S. G. Salnikov. *Fine structure of the cross sections of e^+e^- annihilation near the thresholds of $p\bar{p}$ and $n\bar{n}$ production*. Nucl. Phys. A. 977, 60 (2018).
- A. I. Milstein, S. G. Salnikov. *Coulomb effects in the decays $\Upsilon(4S) \to B\bar{B}$*. Phys. Rev. D 104, 014007 (2021).
- A. I. Milstein, S. G. Salnikov. *Invariant-mass spectrum of $\Lambda\bar{\Lambda}$ pair in the process $e^+e^- \to \phi\Lambda\bar{\Lambda}$*. Phys. Rev. D. 105, L031501 (2022).
- A. I. Milstein, S. G. Salnikov. *Final-state interaction in the process $e^+e^- \to \Lambda_c\bar{\Lambda}_c$*. Phys. Rev. D. 105, 074002 (2022).
- A. E. Bondar, A. I. Milstein, R. V. Mizuk, S. G. Salnikov. *Effects of isospin violation in the $e^+e^- \to B^{(*)}\bar{B}^{(*)}$ cross sections*. J. High Energ. Phys. 2022, 170 (2022).
- A. I. Milstein, S. G. Salnikov. *$N\bar{N}$ production in e^+e^- annihilation near the threshold revisited*. Phys. Rev. D. 106, 074012 (2022).
- A. I. Milstein, S. G. Salnikov. *Natural explanation of recent results on $e^+e^- \to \Lambda\bar{\Lambda}$*. JETP Letters 117, 905 (2023).
- A. I. Milstein, S. G. Salnikov. *Near-threshold resonances in $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ process*. Phys. Rev. D. 108, L071505 (2023).
- A. E. Bondar, A. I. Milstein, S. G. Salnikov. *Coupled channels and production of near-threshold $B^{(*)}\bar{B}^{(*)}$ resonances in e^+e^- annihilation*. Nucl. Phys. A. 1041, 122764 (2024).