CPT tests with neutral kaons

Antonio Di Domenico
Dipartimento di Fisica, Sapienza Università di Roma
and INFN sezione di Roma, Italy

Testing CPT: introduction

The three discrete symmetries of QM, C (charge conjugation: q → -q), P (parity: x → -x), and T (time reversal: t → -t) are known to be violated in nature both singly and in pairs. Only CPT appears to be an exact symmetry of nature.

CPT theorem holds for any QFT formulated on flat space-time which assumes: (1) Lorentz invariance (2) Locality (3) Unitarity (i.e. conservation of probability).

Extension of CPT theorem to a theory of quantum gravity far from obvious.
(e.g. CPT violation appears in several QG models)
huge effort in the last decades to study and shed light on QG phenomenology
$\Rightarrow$ Phenomenological CPTV parameters to be constrained by experiments

Consequences of CPT symmetry: equality of masses, lifetimes, |q| and |μ| of a particle and its anti-particle.

Neutral meson systems offer unique possibilities to test CPT invariance; e.g. taking as figure of merit the fractional difference between the masses of a particle and its anti-particle:

neutral K system	neutral B system	proton- anti-proton
$\left|m_{K^0} - m_{\overline{K}^0}\right| / m_K < 10^{-18}$	$\left|m_{B^0} - m_{\overline{B}^0}\right| / m_B < 10^{-14}$	$\left|m_p - m_{\bar{p}}\right| / m_p < 10^{-9}$

Other interesting CPT tests: e.g. the study of anti-hydrogen atoms, etc..

Testing CPT: introduction

The three discrete symmetries of QM, C (charge conjugation: q → -q), P (parity: x → -x), and T (time reversal: t → -t) are known to be violated in nature both singly and in pairs. Only CPT appears to be an exact symmetry of nature.

CPT theorem holds for any QFT formulated on flat space-time which assumes: (1) Lorentz invariance (2) Locality (3) Unitarity (i.e. conservation of probability).

Extension of CPT theorem to a theory of quantum gravity far from obvious. (e.g. CPT violation appears in several QG models)
huge effort in the last decades to study and shed light on QG phenomenology
⇒ Phenomenological CPTV parameters to be constrained by experiments

Consequences of CPT symmetry: equality of masses, lifetimes, |q| and |μ| of a particle and its anti-particle.

Neutral meson systems offer unique possibilities to test CPT invariance; e.g. taking as figure of merit the fractional difference between the masses of a particle and its anti-particle:

neutral K system	neutral B system	proton- anti-proton
$\left|m_{K^0} - m_{\overline{K}^0}\right| / m_K < 10^{-18}$	$\left|m_{B^0} - m_{\overline{B}^0}\right| / m_B < 10^{-14}$	$\left|m_p - m_{\bar{p}}\right| / m_p < 10^{-9}$

Other interesting CPT tests: e.g. the study of anti-hydrogen atoms, etc..

The neutral kaon two-level oscillating system in a nutshell

$$|K_{S,L}\rangle \propto \left[\left(1+\varepsilon_{S,L}\right)|K^0\rangle \pm \left(1-\varepsilon_{S,L}\right)|\bar{K}^0\rangle\right]$$

CP violation:

$$\varepsilon_{S,L} = \varepsilon \pm \delta$$

T violation:

$$\varepsilon = \frac{H_{12}-H_{21}}{2\left(\lambda_S-\lambda_L\right)} = \frac{-i\Im M_{12} - \Im\Gamma_{12}/2}{\Delta m + i\Delta\Gamma/2}$$

CPT violation:

$$\delta = \frac{H_{11}-H_{22}}{2\left(\lambda_S-\lambda_L\right)} = \frac{1}{2}\frac{\left(m_{\bar{K}^0}-m_{K^0}\right)-\left(i/2\right)\left(\Gamma_{\bar{K}^0}-\Gamma_{K^0}\right)}{\Delta m + i\Delta\Gamma/2}$$

- $\delta \neq 0$ implies CPT violation
- $\varepsilon \neq 0$ implies T violation
- $\varepsilon \neq 0$ or $\delta \neq 0$ implies CP violation

(with a phase convention $\Im\Gamma_{12} = 0$)

$$\Delta m = m_L - m_S \quad , \quad \Delta\Gamma = \Gamma_S - \Gamma_L$$

$$\Delta m = 3.5 \times 10^{-15} \text{ GeV}$$

$$\Delta\Gamma \approx \Gamma_S \approx 2\Delta m = 7 \times 10^{-15} \text{ GeV}$$

The neutral kaon two-level oscillating system in a nutshell

$$|K_{S,L}\rangle \propto \left[\left(1+\varepsilon_{S,L}\right)|K^0\rangle \pm \left(1-\varepsilon_{S,L}\right)|\bar{K}^0\rangle\right]$$

CP violation:

$$\varepsilon_{S,L} = \varepsilon \pm \delta$$

T violation:

$$\varepsilon = \frac{H_{12}-H_{21}}{2\left(\lambda_S-\lambda_L\right)} = \frac{-i\Im M_{12} - \Im\Gamma_{12}/2}{\Delta m + i\Delta\Gamma/2}$$

CPT violation:

$$\delta = \frac{H_{11}-H_{22}}{2\left(\lambda_S-\lambda_L\right)} = \frac{1}{2}\frac{\left(m_{\bar{K}^0}-m_{K^0}\right)-(i/2)\left(\Gamma_{\bar{K}^0}-\Gamma_{K^0}\right)}{\Delta m + i\Delta\Gamma/2}$$

huge amplification factor!!

- $\delta \neq 0$ implies CPT violation
- $\varepsilon \neq 0$ implies T violation
- $\varepsilon \neq 0$ or $\delta \neq 0$ implies CP violation

(with a phase convention $\Im\Gamma_{12}=0$)

$$\Delta m = m_L - m_S \quad , \quad \Delta\Gamma = \Gamma_S - \Gamma_L$$

$$\Delta m = 3.5\times 10^{-15}\ \text{GeV}$$

$$\Delta\Gamma \approx \Gamma_S \approx 2\Delta m = 7\times 10^{-15}\ \text{GeV}$$

neutral kaons vs other oscillating meson systems

	$\langle m \rangle$ (GeV)	Δm (GeV)	$\langle \Gamma \rangle$ (GeV)	$\Delta\Gamma/2$ (GeV)
K^0	0.5	$3\text{x}10^{-15}$	$3\text{x}10^{-15}$	$3\text{x}10^{-15}$
D^0	1.9	$6\text{x}10^{-15}$	$2\text{x}10^{-12}$	$1\text{x}10^{-14}$
$B^0{}_d$	5.3	$3\text{x}10^{-13}$	$4\text{x}10^{-13}$	$O(10^{-15})$ (SM prediction)
$B^0{}_s$	5.4	$1\text{x}10^{-11}$	$4\text{x}10^{-13}$	$3\text{x}10^{-14}$

$K_{S,L}$ semileptonic charge asymmetries

K_S and K_L semileptonic charge asymmetry

T CPT viol. in mixing

$$A_{S,L} = \frac{\Gamma\left(K_{S,L} \to \pi^- e^+ \nu\right) - \Gamma\left(K_{S,L} \to \pi^+ e^- \overline{\nu}\right)}{\Gamma\left(K_{S,L} \to \pi^- e^+ \nu\right) + \Gamma\left(K_{S,L} \to \pi^+ e^- \overline{\nu}\right)} = 2\Re\varepsilon \pm 2\Re\delta - 2\Re y \pm 2\Re x_-$$

CPTV in $\Delta S=\Delta Q$ $\Delta S\neq\Delta Q$ decays

$A_{S,L}\neq 0$ signals CP violation

$A_S\neq A_L$ signals CPT violation ($\Delta A=A_S-A_L\neq 0$

A_L=(3.322 ± 0.058 ± 0.047) × 10^{-3}

KTEV PRL88,181601(2002)

A_S = (-3.8 ± 5.0 ± 2.6) × 10^{-3}

KLOE-2 JHEP 09 (2018) 21

$K_{S,L}$ semileptonic charge asymmetries

K_S and K_L semileptonic charge asymmetry

T CPT viol. in mixing

$$A_{S,L} = \frac{\Gamma\left(K_{S,L} \to \pi^- e^+ \nu\right) - \Gamma\left(K_{S,L} \to \pi^+ e^- \bar{\nu}\right)}{\Gamma\left(K_{S,L} \to \pi^- e^+ \nu\right) + \Gamma\left(K_{S,L} \to \pi^+ e^- \bar{\nu}\right)} = 2\Re\varepsilon \pm 2\Re\delta - 2\Re y \pm 2\Re x_-$$

CPTV in ΔS=ΔQ ΔS≠ΔQ decays

$A_{S,L} \neq 0$ signals CP violation

$A_S \neq A_L$ signals CPT violation ($\Delta A = A_S - A_L \neq 0$)

$$A_L = (3.322 \pm 0.058 \pm 0.047) \times 10^{-3}$$

KTEV PRL88,181601(2002)

$$A_S = (-3.8 \pm 5.0 \pm 2.6) \times 10^{-3}$$

KLOE-2 JHEP 09 (2018) 21

CPT test

$$\Delta A/4 = (A_S - A_L)/4 = \Re\delta + \Re x_- = (-1.8 \pm 1.4) \times 10^{-3}$$

$$A_S - A_L = 4\left(\Re\delta + \Re x_-\right) \longrightarrow \Re x_- = (-2.0 \pm 1.4) \times 10^{-3}$$

CPT & $\Delta S = \Delta Q$ viol. JHEP 09 (2018) 21

$$A_S + A_L = 4\left(\Re\varepsilon - \Re y\right) \longrightarrow \Re y = (1.7 \pm 1.4) \times 10^{-3}$$

CPT viol.

input from other experiments

K_S tagging at KLOE

Initial entangled state from $\phi \to K^0\bar{K}^0$ decay:

$$|i\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle\right]$$

For times $t_1 >> \tau_S$ (or $t_2 >> \tau_S$):

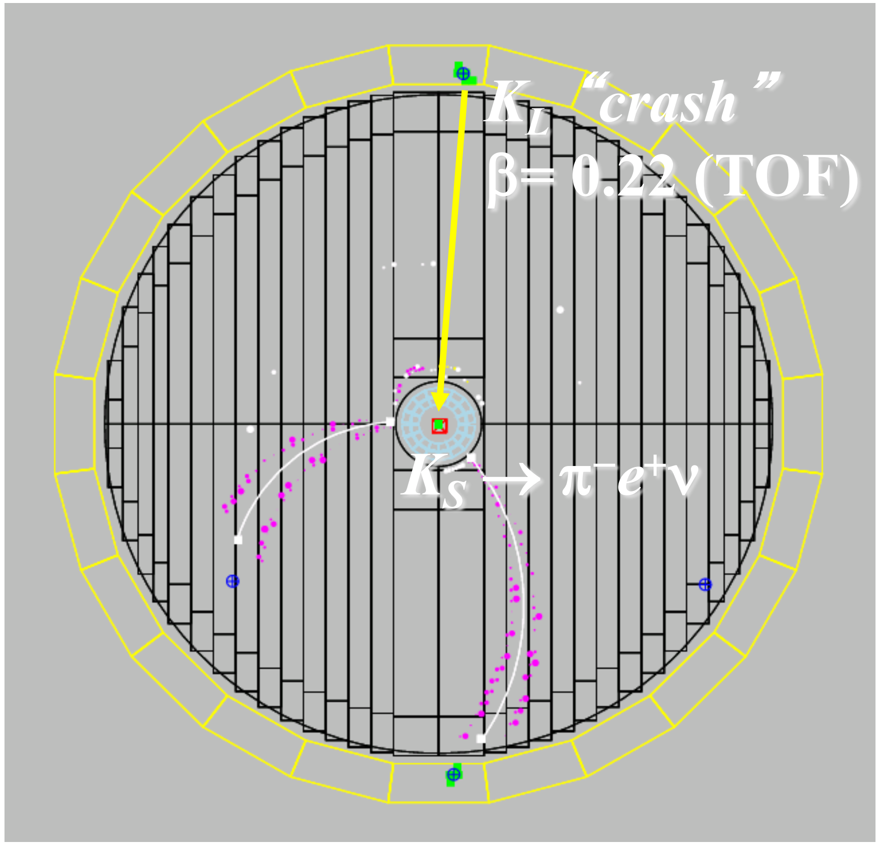
$$I(f_1,t_1;f_2,t_2) = C_{12}\{|\eta_1|^2 e^{-\Gamma_L t_1 - \Gamma_S t_2} + |\eta_2|^2 e^{-\Gamma_S t_1 - \Gamma_L t_2}$$
$$- 2|\eta_1||\eta_2| e^{-(\Gamma_S+\Gamma_L)(t_1+t_2)/2}\cos[\Delta m(t_2-t_1)+\phi_1-\phi_2]\}$$

=> the state behaves like an incoherent mixture of states:

$$|K_S(t_1)\rangle|K_L(t_2)\rangle \quad \text{or} \quad |K_L(t_1)\rangle|K_S(t_2)\rangle$$

the selection of a pure K_S beam is possible exploiting entanglement

unique at a ϕ-factory, not possible at fixed target experiments

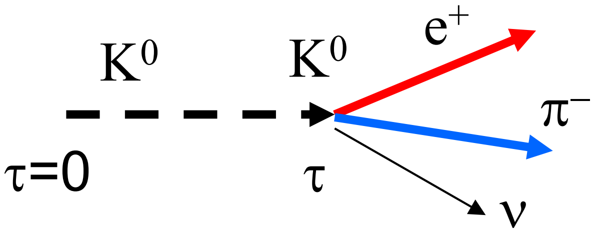


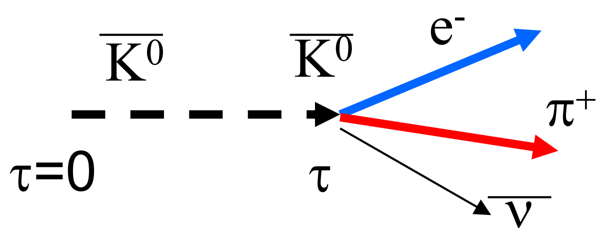
K_S tagged by K_L interaction in EmC
Efficiency ~ 30% (largely geometrical)
K_S angular resolution: ~ 1° (0.3° in ϕ)
K_S momentum resolution: ~ 2 MeV

A recent study on this quantum effect:
J. Bernabeu, A.D.D. "Can future observation of the living partner post-tag the past decayed state in entangled neutral K mesons?" PRD 105 116004(2022)

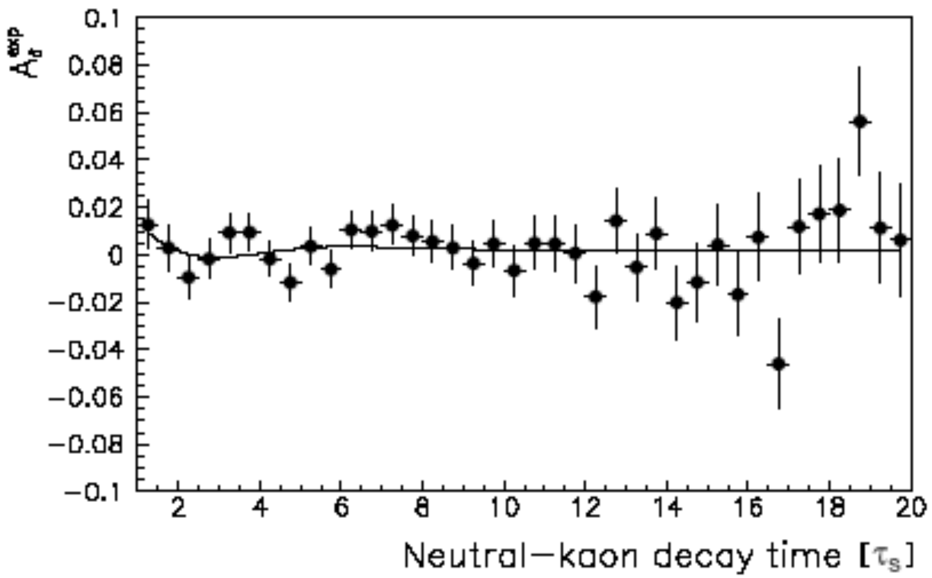
CPT test at CPLEAR

Test of **CPT** in the time evolution of neutral kaons using the semileptonic Asymmetry, i.e. comparing **"survival" probabilities**: $K^0 \to K^0$ vs $\bar{K}^0 \to \bar{K}^0$





$$\begin{cases} A_\delta(\tau) = \dfrac{\bar{R}_+(\tau) - \alpha R_-(\tau)}{\bar{R}_+(\tau) + \alpha R_-(\tau)} + \dfrac{\bar{R}_-(\tau) - \alpha R_+(\tau)}{\bar{R}_-(\tau) + \alpha R_+(\tau)} \\ R_{+(-)}(\tau) = R\left(K^0{}_{t=0} \to (e^{+(-)}\pi^{-(+)}\nu)_{t=\tau}\right) \\ \bar{R}_{-(+)}(\tau) = R\left(\bar{K}^0{}_{t=0} \to (e^{-(+)}\pi^{+(-)}\nu)_{t=\tau}\right) \\ \alpha = 1 + 4\Re\varepsilon_L \end{cases}$$



$$A_\delta(\tau >> \tau_S) = 8\Re\delta$$

$$\Re\delta = (0.30 \pm 0.33 \pm 0.06) \times 10^{-3}$$

CPLEAR PLB444 (1998) 52

"Standard" CPT test

J. Bell

J. Steinberger

using the unitarity constraint
(**Bell-Steinberger** relation)

$$2\Im\delta = \Im\left[\langle K_L | K_S \rangle\right] = \Im\left[\frac{\sum_f \langle f|T|K_S\rangle \langle f|T|K_L\rangle^*}{i\left(\lambda_S - \lambda_L^*\right)}\right]$$

All observables quantities (BRs, lifetimes etc..)
updated inputs from KLOE, NA48, KTEV, CPLEAR, etc.

$$\text{Im}\,\delta = (-0.3 \pm 1.4) \times 10^{-5}$$

PDG fit (2023)

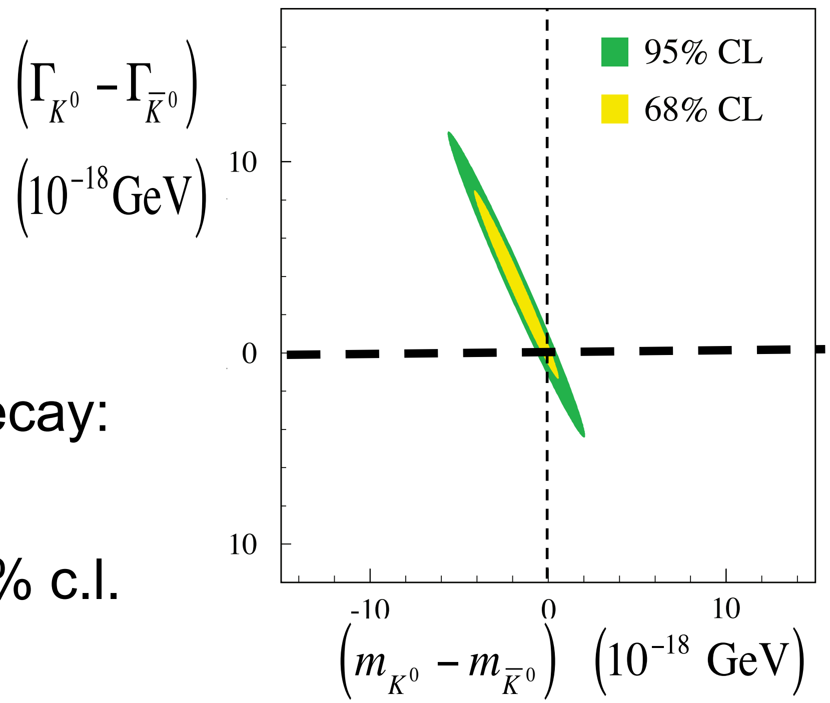
$$\delta = \frac{1}{2}\frac{\left(m_{\bar{K}^0} - m_{K^0}\right) - (i/2)\left(\Gamma_{\bar{K}^0} - \Gamma_{K^0}\right)}{\Delta m + i\Delta\Gamma/2}$$

Combining Reδ and Imδ results →

Assuming $\left(\Gamma_{\bar{K}^0} - \Gamma_{K^0}\right) = 0$, i.e. no CPT viol. in decay:

$$\left|m_{\bar{K}^0} - m_{K^0}\right| < 4.0 \times 10^{-19} \quad \text{GeV}$$

at 95% c.l.



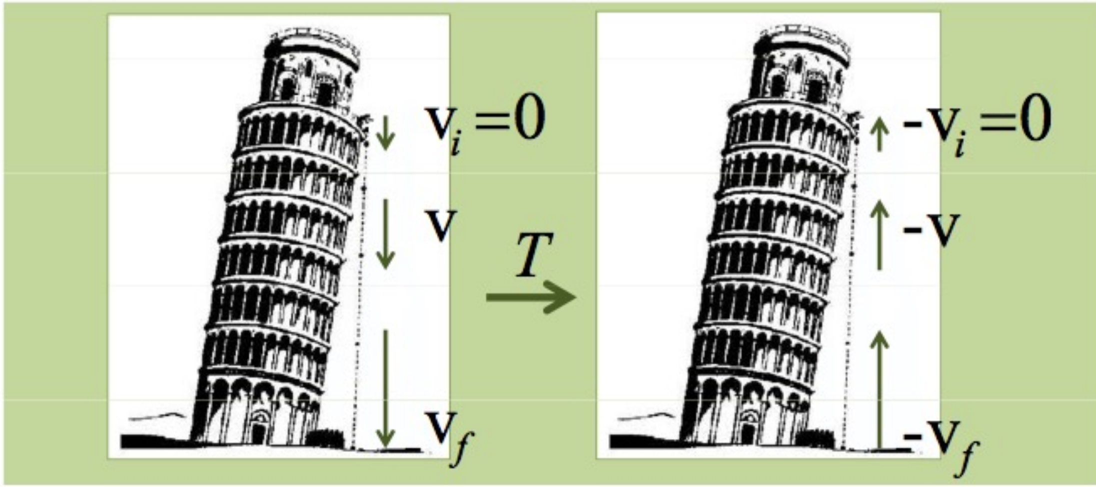
Direct CPT test in transitions

- Is it possible to test the CPT symmetry directly in transition processes between kaon states, rather than comparing masses, lifetimes, or other intrinsic properties of particle and anti-particle states?
- CPT violating effects may not appear at first order in diagonal mass terms (survival probabilities) while they can manifest at first order in transitions (non-diagonal terms).
- Clean formulation required. Possible spurious effects induced by CP violation in the decay and/or a violation of the $\Delta S = \Delta Q$ rule have to be well under control.
- In standard WWA the test is related to Reδ, a genuine CPT violating effect independent of $\Delta\Gamma$, i.e. not requiring the decay as an essential ingredient.

Probing CPT: J. Bernabeu, A.D.D., P. Villanueva, JHEP 10 (2015) 139
Time-reversal violation: J. Bernabeu, A.D.D., P. Villanueva, NPB 868 (2013) 102

Time Reversal

- The transformation of a system corresponding to the inversion of events in time, or reversed dynamics, with the formal substitution $\Delta t \rightarrow -\Delta t$, is usually called '**time reversal**', but a more appropriate name would actually be **motion reversal**.



- Exchange of in ↔ out states and reversal of all momenta and spins tests time reversal, i.e. the symmetry of the responsible dynamics for the observed process under time reversal (transformation implemented in QM by an antiunitary operator)
- Similarly for CPT tests: the exchange of in ↔ out states etc.. is required.

Direct test of CPT symmetry in neutral kaon transitions

CPT symmetry test

Reference		$\mathcal{CPT}$-conjugate	
Transition	Decay products	Transition	Decay products
$K^0 \to K_+$	$(\ell^-, \pi\pi)$	$K_+ \to \bar{K}^0$	$(3\pi^0, \ell^-)$
$K^0 \to K_-$	$(\ell^-, 3\pi^0)$	$K_- \to \bar{K}^0$	$(\pi\pi, \ell^-)$
$\bar{K}^0 \to K_+$	$(\ell^+, \pi\pi)$	$K_+ \to K^0$	$(3\pi^0, \ell^+)$
$\bar{K}^0 \to K_-$	$(\ell^+, 3\pi^0)$	$K_- \to K^0$	$(\pi\pi, \ell^+)$

One can define the following ratios of probabilities:

$$R_{1,\mathcal{CPT}}(\Delta t) = P\left[K_+(0) \to \bar{K}^0(\Delta t)\right] / P\left[K^0(0) \to K_+(\Delta t)\right]$$
$$R_{2,\mathcal{CPT}}(\Delta t) = P\left[K^0(0) \to K_-(\Delta t)\right] / P\left[K_-(0) \to \bar{K}^0(\Delta t)\right]$$
$$R_{3,\mathcal{CPT}}(\Delta t) = P\left[K_+(0) \to K^0(\Delta t)\right] / P\left[\bar{K}^0(0) \to K_+(\Delta t)\right]$$
$$R_{4,\mathcal{CPT}}(\Delta t) = P\left[\bar{K}^0(0) \to K_-(\Delta t)\right] / P\left[K_-(0) \to K^0(\Delta t)\right]$$

Any deviation from $R_{i,CPT}=1$ constitutes a violation of CPT-symmetry

J. Bernabeu, A.D.D., P. Villanueva, JHEP 10 (2015) 139

Direct test of T symmetry in neutral kaon transitions

T symmetry test

Reference		T-conjugate	
Transition	Final state	Transition	Final state
$\bar{K}^0 \to K_-$	$(\ell^+, \pi^0\pi^0\pi^0)$	$K_- \to \bar{K}^0$	$(\pi^0\pi^0\pi^0, \ell^-)$
$K_+ \to K^0$	$(\pi^0\pi^0\pi^0, \ell^+)$	$K^0 \to K_+$	$(\ell^-, \pi\pi)$
$\bar{K}^0 \to K_+$	$(\ell^+, \pi\pi)$	$K_+ \to \bar{K}^0$	$(\pi^0\pi^0\pi^0, \ell^-)$
$K_- \to K^0$	$(\pi\pi, \ell^+)$	$K^0 \to K_-$	$(\ell^-, \pi\pi)$

One can define the following ratios of probabilities:

$$
\begin{aligned}
R_1(\Delta t) &= P\left[K^0(0) \to K_+(\Delta t)\right] / P\left[K_+(0) \to K^0(\Delta t)\right] \\
R_2(\Delta t) &= P\left[K^0(0) \to K_-(\Delta t)\right] / P\left[K_-(0) \to K^0(\Delta t)\right] \\
R_3(\Delta t) &= P\left[\bar{K}^0(0) \to K_+(\Delta t)\right] / P\left[K_+(0) \to \bar{K}^0(\Delta t)\right] \\
R_4(\Delta t) &= P\left[\bar{K}^0(0) \to K_-(\Delta t)\right] / P\left[K_-(0) \to \bar{K}^0(\Delta t)\right] .
\end{aligned}
$$

Any deviation from R_i=1 constitutes a violation of T-symmetry

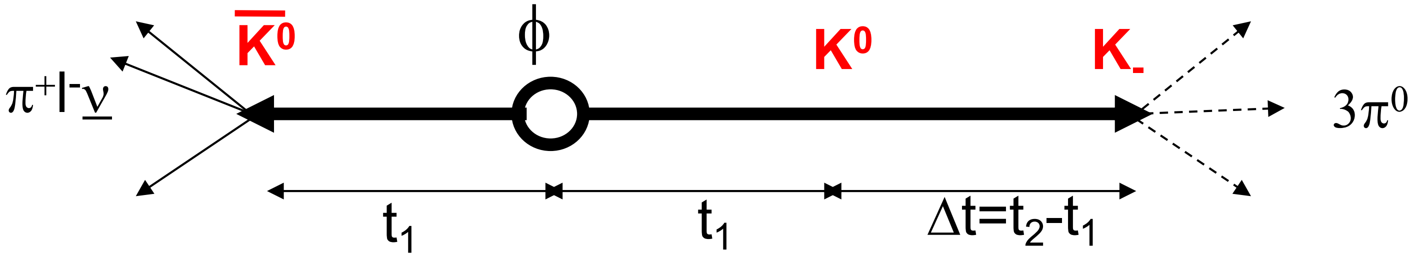
J. Bernabeu, A.D.D., P. Villanueva: NPB 868 (2013) 102

Entanglement in neutral kaon pairs

- EPR correlations at a ϕ-factory can be exploited to study transitions involving orthogonal "CP states" K_+ and K_-

$$
\begin{aligned}
|i\rangle &= \frac{1}{\sqrt{2}}\left[\left|K^0(\vec{p})\right\rangle\left|\overline{K}^0(-\vec{p})\right\rangle - \left|\overline{K}^0(\vec{p})\right\rangle\left|K^0(-\vec{p})\right\rangle\right] \\
&= \frac{1}{\sqrt{2}}\left[\left|K_+(\vec{p})\right\rangle\left|K_-(-\vec{p})\right\rangle - \left|K_-(\vec{p})\right\rangle\left|K_+(-\vec{p})\right\rangle\right]
\end{aligned}
$$

- decay as filtering measurement
- entanglement -> preparation of state

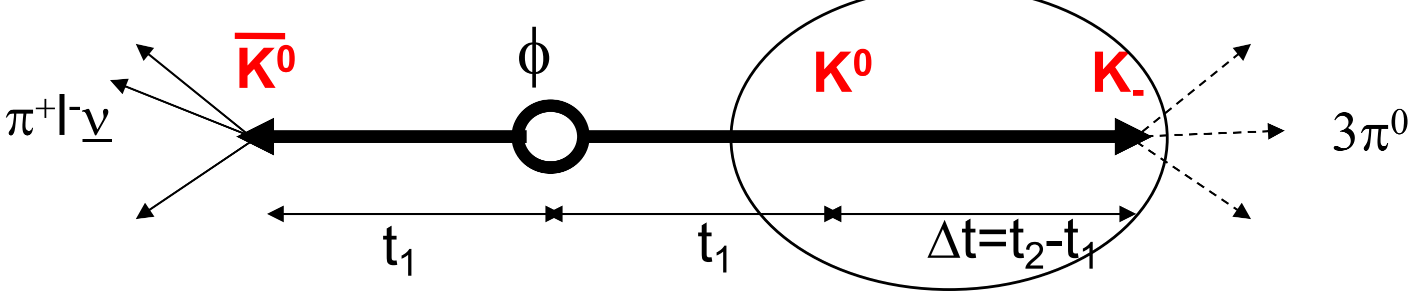


Entanglement in neutral kaon pairs

- EPR correlations at a ϕ-factory can be exploited to study transitions involving orthogonal "CP states" K_+ and K_-

$$|i\rangle = \frac{1}{\sqrt{2}}\left[|K^0(\vec{p})\rangle|\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(\vec{p})\rangle|K^0(-\vec{p})\rangle\right]$$
$$= \frac{1}{\sqrt{2}}\left[|K_+(\vec{p})\rangle|K_-(-\vec{p})\rangle - |K_-(\vec{p})\rangle|K_+(-\vec{p})\rangle\right]$$

- decay as filtering measurement
- entanglement -> preparation of state



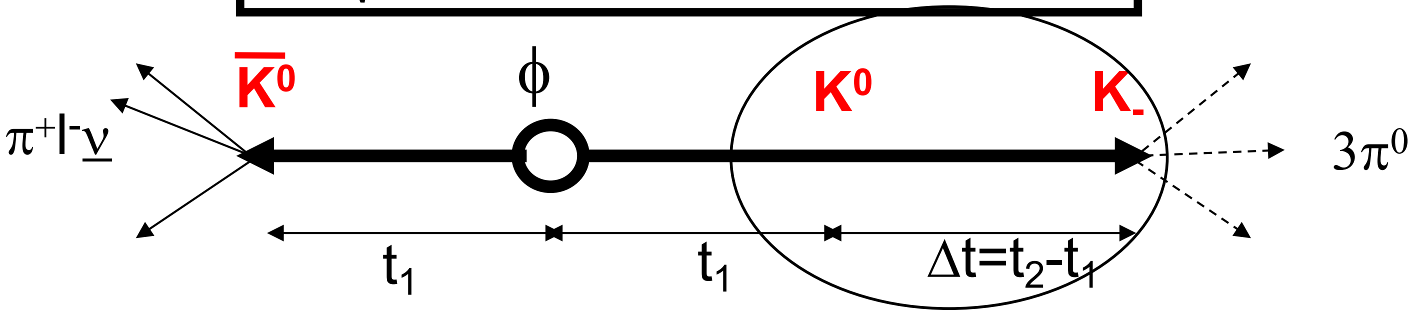
$$K^0 \rightarrow K_-$$ reference process

Entanglement in neutral kaon pairs

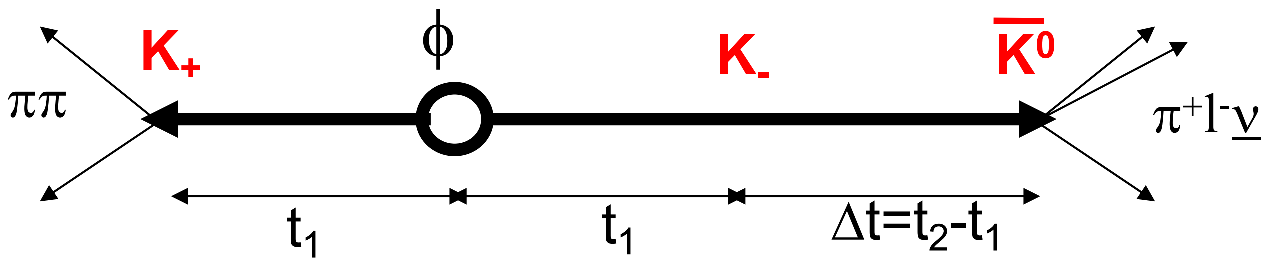
- EPR correlations at a ϕ-factory can be exploited to study transitions involving orthogonal "CP states" K_+ and K_-

$$|i\rangle = \frac{1}{\sqrt{2}}\left[\left|K^0(\vec{p})\right\rangle\left|\bar{K}^0(-\vec{p})\right\rangle - \left|\bar{K}^0(\vec{p})\right\rangle\left|K^0(-\vec{p})\right\rangle\right]$$
$$= \frac{1}{\sqrt{2}}\left[\left|K_+(\vec{p})\right\rangle\left|K_-(-\vec{p})\right\rangle - \left|K_-(\vec{p})\right\rangle\left|K_+(-\vec{p})\right\rangle\right]$$

- decay as filtering measurement
- entanglement -> preparation of state



$K^0 \rightarrow K_-$ reference process

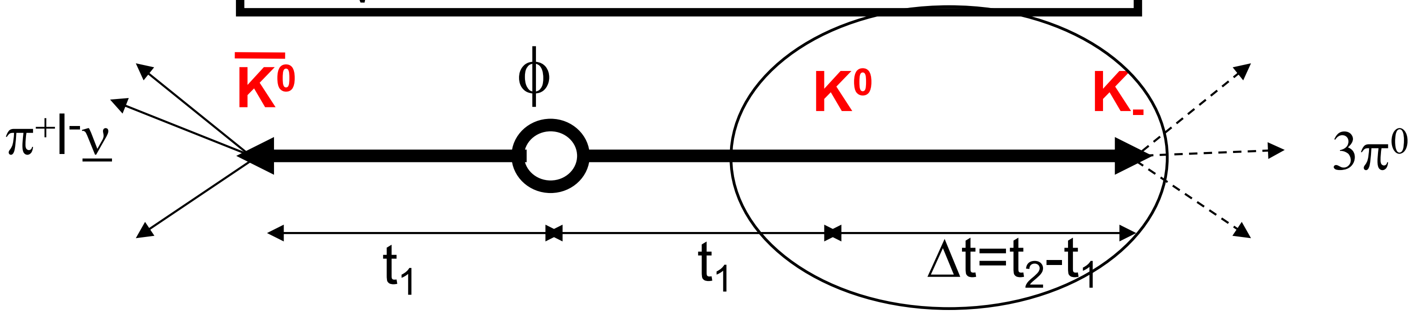


Entanglement in neutral kaon pairs

- EPR correlations at a ϕ-factory can be exploited to study transitions involving orthogonal "CP states" K_+ and K_-

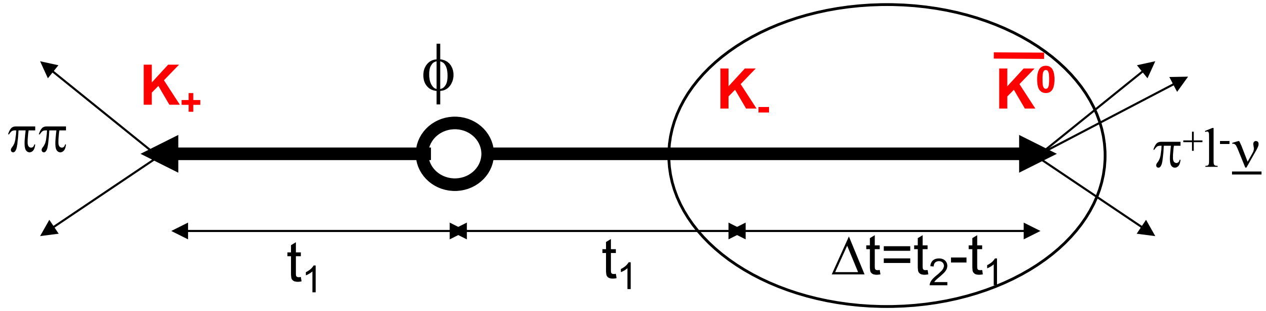
$$|i\rangle = \frac{1}{\sqrt{2}}\left[|K^0(\vec{p})\rangle|\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(\vec{p})\rangle|K^0(-\vec{p})\rangle\right]$$
$$= \frac{1}{\sqrt{2}}\left[|K_+(\vec{p})\rangle|K_-(-\vec{p})\rangle - |K_-(\vec{p})\rangle|K_+(-\vec{p})\rangle\right]$$

- decay as filtering measurement
- entanglement -> preparation of state



$K^0 \rightarrow K_-$ reference process

$K_- \rightarrow \bar{K}^0$ CPT-conjugated process

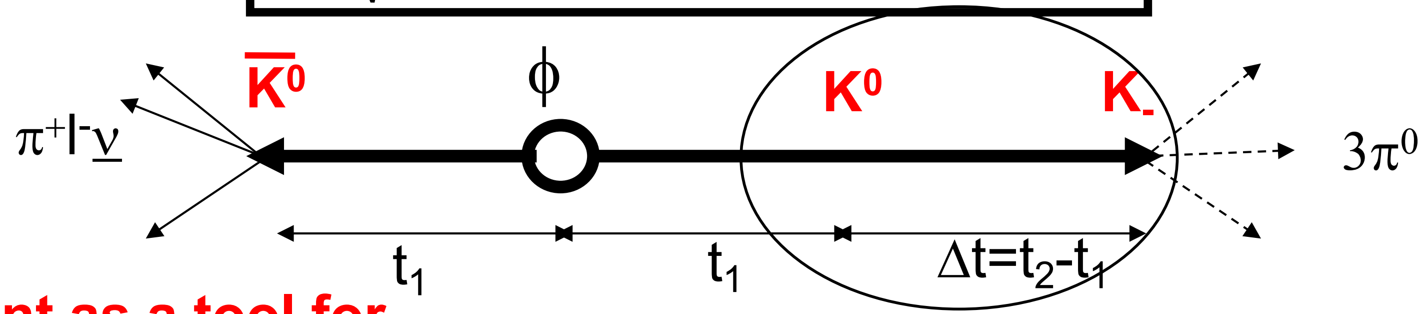


Entanglement in neutral kaon pairs

- EPR correlations at a ϕ-factory can be exploited to study transitions involving orthogonal "CP states" K_+ and K_-

$$|i\rangle = \frac{1}{\sqrt{2}}\left[|K^0(\vec{p})\rangle|\overline{K}^0(-\vec{p})\rangle - |\overline{K}^0(\vec{p})\rangle|K^0(-\vec{p})\rangle\right]$$
$$= \frac{1}{\sqrt{2}}\left[|K_+(\vec{p})\rangle|K_-(-\vec{p})\rangle - |K_-(\vec{p})\rangle|K_+(-\vec{p})\rangle\right]$$

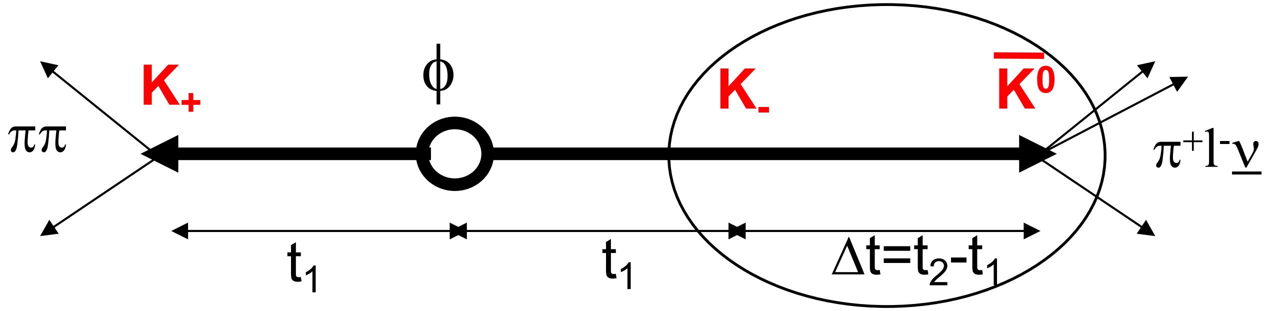
- decay as filtering measurement
- entanglement -> preparation of state



Entanglement as a tool for in <-> out state inversion

$K^0 \rightarrow K_-$ reference process

$K_- \rightarrow \overline{K}^0$ CPT-conjugated process

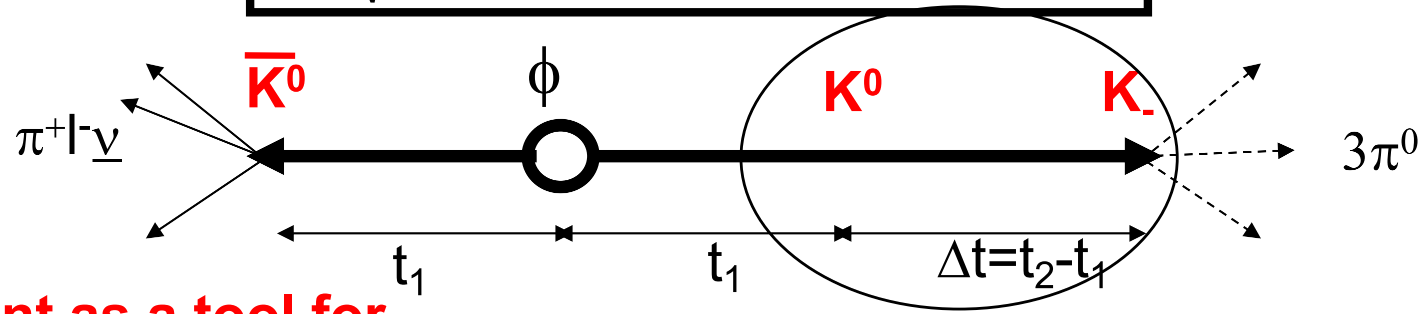


Entanglement in neutral kaon pairs

- EPR correlations at a ϕ-factory can be exploited to study transitions involving orthogonal “CP states” K_+ and K_-

$$|i\rangle = \frac{1}{\sqrt{2}}\left[|K^0(\vec{p})\rangle|\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(\vec{p})\rangle|K^0(-\vec{p})\rangle\right]$$
$$= \frac{1}{\sqrt{2}}\left[|K_+(\vec{p})\rangle|K_-(-\vec{p})\rangle - |K_-(\vec{p})\rangle|K_+(-\vec{p})\rangle\right]$$

- decay as filtering measurement
- entanglement -> preparation of state



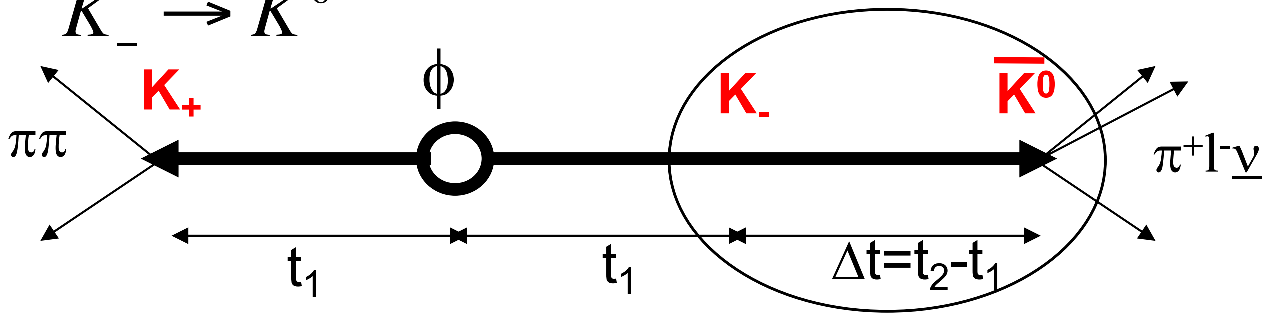
Entanglement as a tool for in <-> out state inversion

$K^0 \to K_-$ reference process

$K_- \to \bar{K}^0$ CPT-conjugated process

Note: CP and T conjugated process

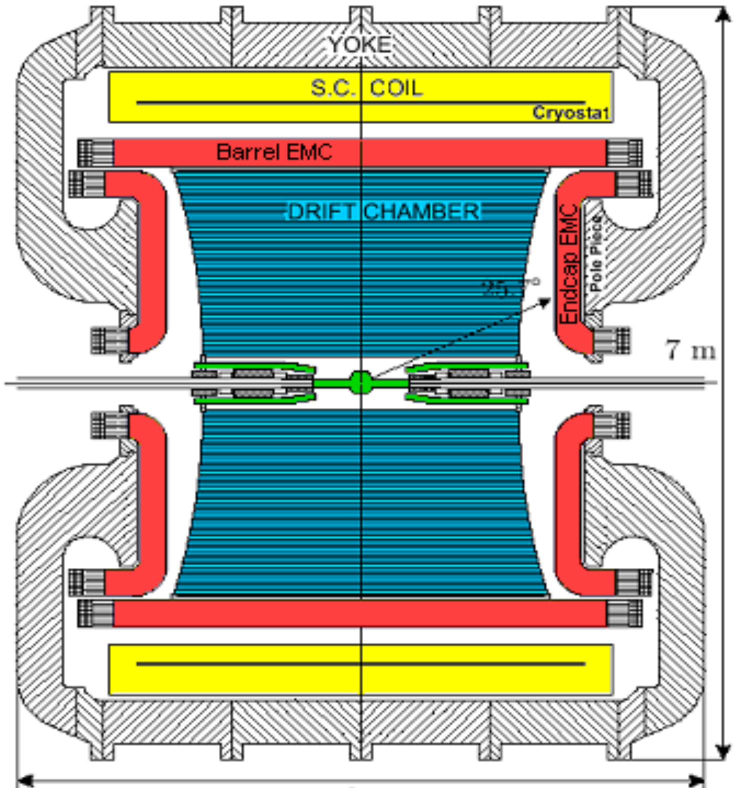
$\bar{K}^0 \to K_-$ $\quad K_- \to K^0$



KLOE and KLOE-2 at the Frascati ϕ-factory DAΦNE

KLOE detector

Superconducting coil B = 0.52 T



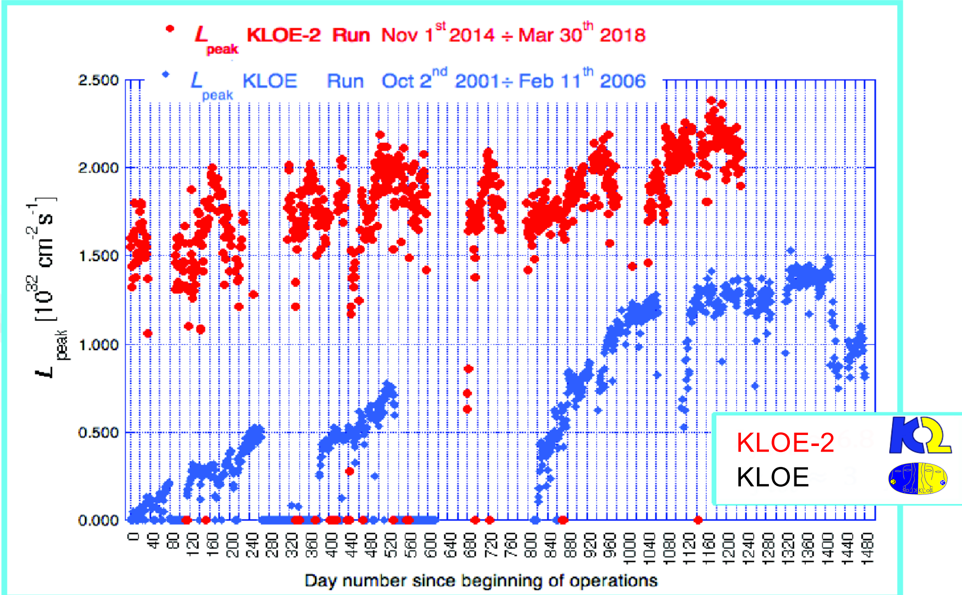
Lead/scintillating fiber calorimeter $\sigma_E/E \cong 5.7\% /\sqrt{E(GeV)}$
$\sigma_t \cong 54$ ps $/\sqrt{E(GeV)} \oplus 50$ ps

drift chamber; 4 m diameter × 3.3 m length
90% He - 10% isobutane gas mixture

$\sigma(p_\perp)/p_\perp \simeq 0.4$ % $\sigma_{xy} \simeq 150$ μm $\sigma_z \simeq 2$ mm

DAΦNE collider

KLOE-2: $L_{int} \sim 5.5$ fb^{-1}
KLOE: $L_{int} \sim 2.5$ fb^{-1}



KLOE + KLOE-2 data sample:
- ~ 8 fb^{-1} ⇒ 2.4 × 10^{10} ϕ's produced
- ~ 8 x10^9 $K_S K_L$ pairs
- ~ 3 x10^8 η's
- ⇒ the largest sample ever collected at the ϕ(1020) peak in e^+e^- collisions

T, CP, CPT tests in neutral kaon transitions at KLOE

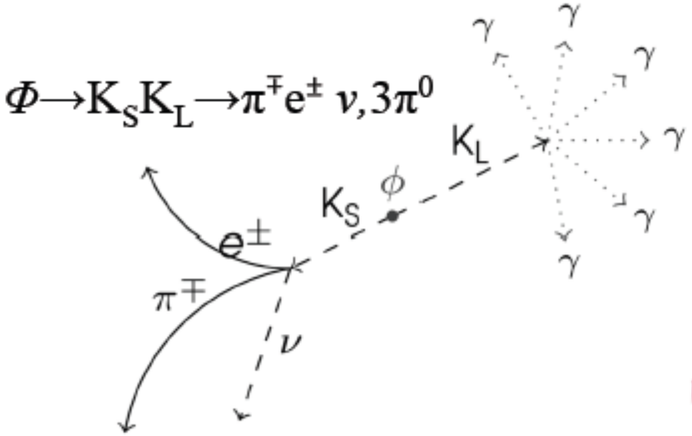
CPT | **T** | **CP** | **observables**

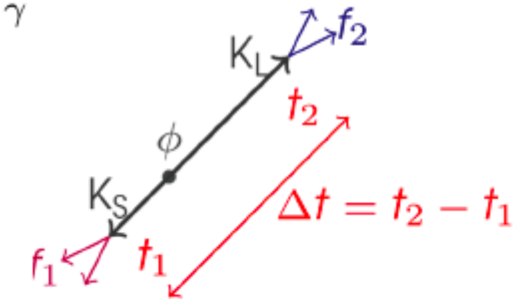
$$R_{2,\mathcal{CPT}}^{\rm exp}(\Delta t) \equiv \frac{I(\ell^-,3\pi^0;\Delta t)}{I(\pi\pi,\ell^-;\Delta t)} \qquad R_{2,\mathcal{T}}^{\rm exp}(\Delta t) \equiv \frac{I(\ell^-,3\pi^0;\Delta t)}{I(\pi\pi,\ell^+;\Delta t)} \qquad R_{2,\mathcal{CP}}^{\rm exp}(\Delta t) \equiv \frac{I(\ell^-,3\pi^0;\Delta t)}{I(\ell^+,3\pi^0;\Delta t)}$$

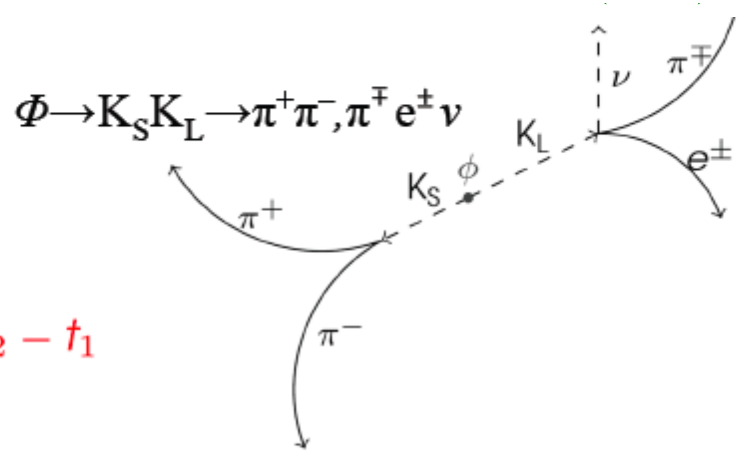
$$R_{4,\mathcal{CPT}}^{\rm exp}(\Delta t) \equiv \frac{I(\ell^+,3\pi^0;\Delta t)}{I(\pi\pi,\ell^+;\Delta t)} \qquad R_{4,\mathcal{T}}^{\rm exp}(\Delta t) \equiv \frac{I(\ell^+,3\pi^0;\Delta t)}{I(\pi\pi,\ell^-;\Delta t)} \qquad R_{4,\mathcal{CP}}^{\rm exp}(\Delta t) \equiv \frac{I(\pi\pi,\ell^+;\Delta t)}{I(\pi\pi,\ell^-;\Delta t)}$$

$$\mathcal{DR}_{\mathcal{CPT}}(\Delta t \gg \tau_S) \equiv \frac{R_{2,\mathcal{CPT}}^{\rm exp}(\Delta t \gg \tau_S)}{R_{4,\mathcal{CPT}}^{\rm exp}(\Delta t \gg \tau_S)} \qquad \mathcal{DR}_{\mathcal{T},\mathcal{CP}}(\Delta t \gg \tau_S) \equiv \frac{R_{2,\mathcal{T}}^{\rm exp}(\Delta t \gg \tau_S)}{R_{4,\mathcal{T}}^{\rm exp}(\Delta t \gg \tau_S)} \equiv \frac{R_{2,\mathcal{CP}}^{\rm exp}(\Delta t \gg \tau_S)}{R_{4,\mathcal{CP}}^{\rm exp}(\Delta t \gg \tau_S)}$$

Corresponding to study the following processes at KLOE:



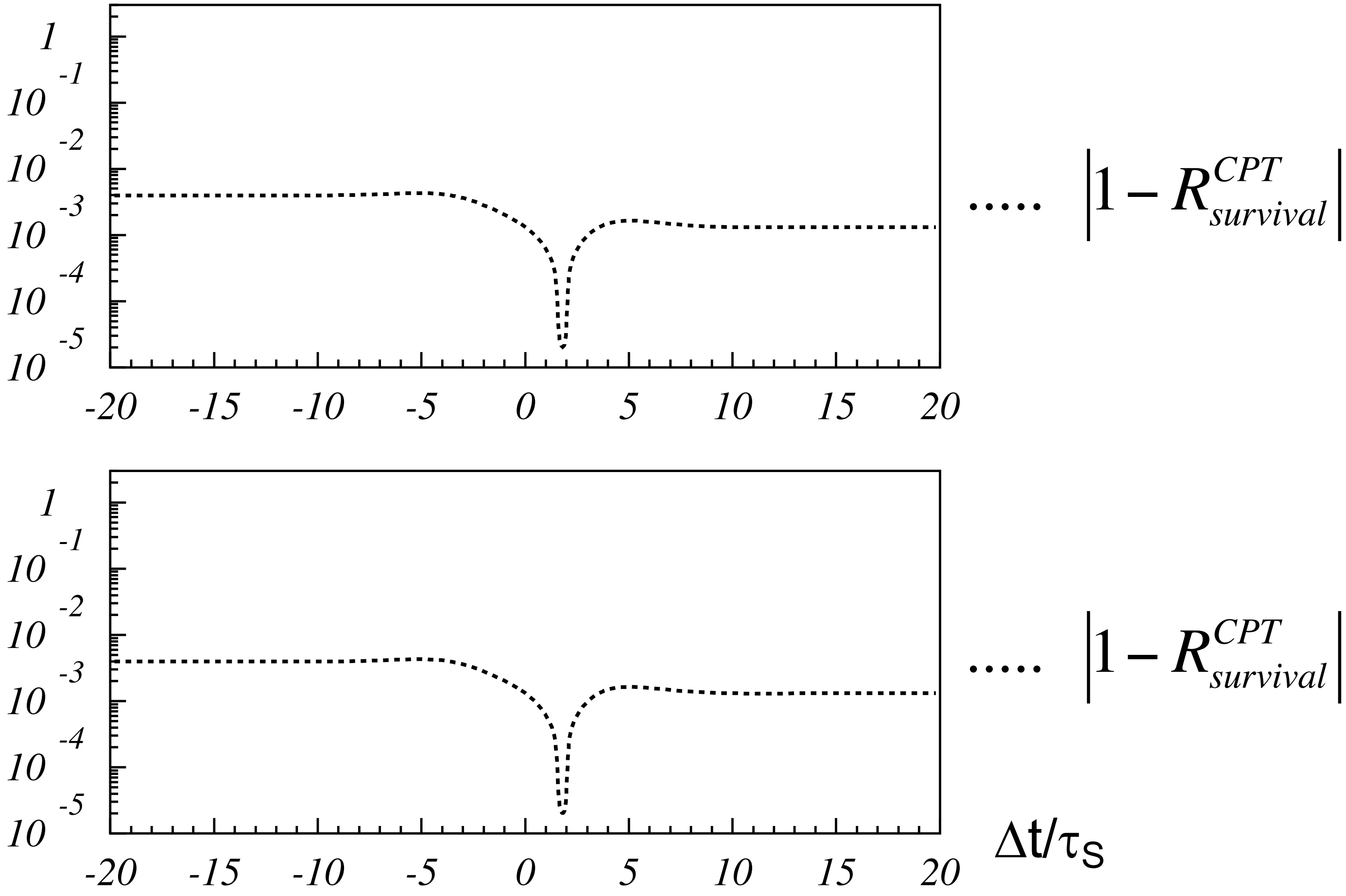




Direct test of CPT symmetry in neutral kaon transitions

Survival probabilities vs Transitions:
comparison of sensitivity to “standard” CPT violation parameter δ

(for visualization purposes, plots with $\mathrm{Re}(\delta)=3.3\ 10^{-4}$ $\mathrm{Im}(\delta)=1.4\ 10^{-5}$ = present limits)

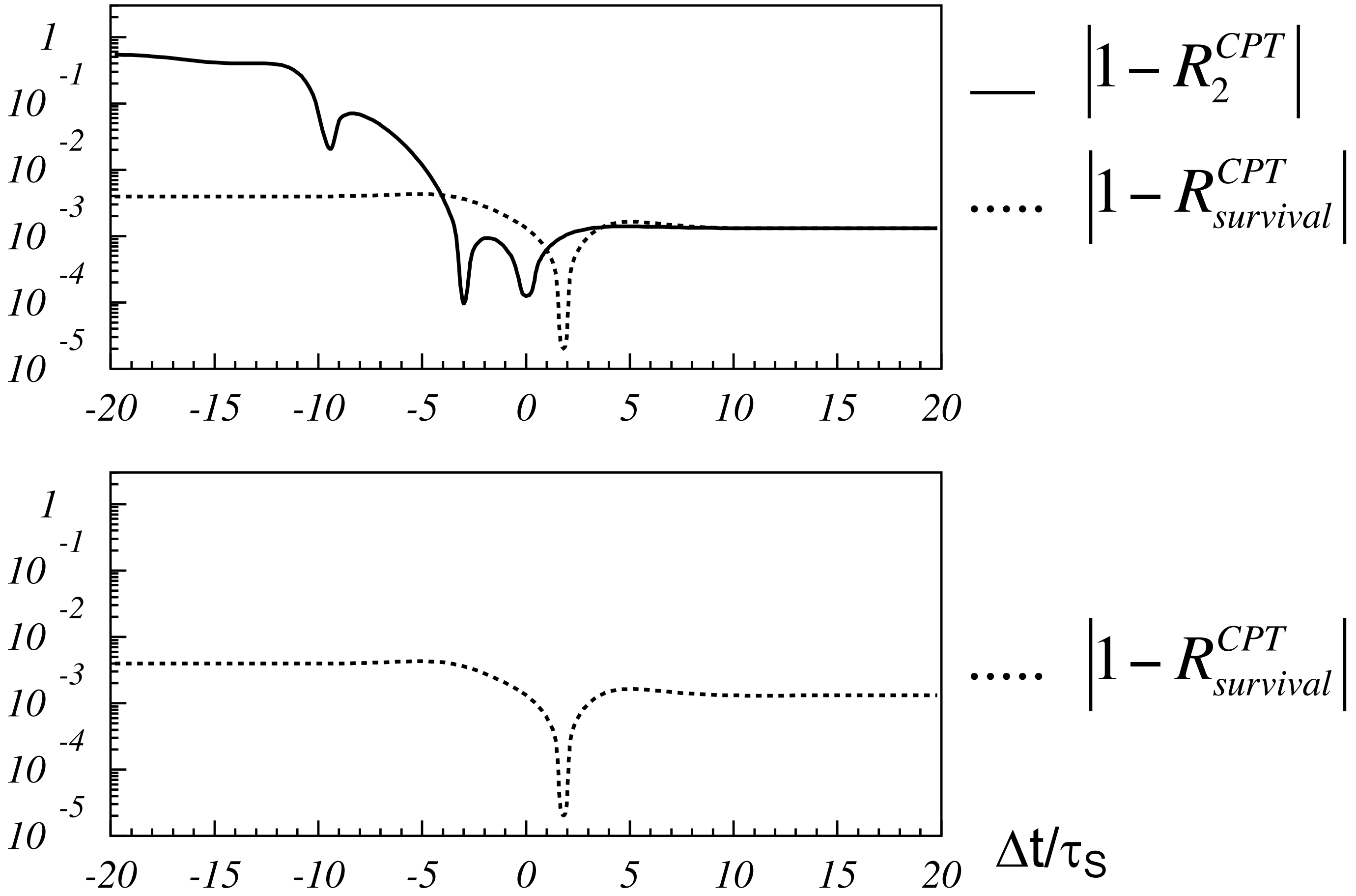


Direct test of CPT symmetry in neutral kaon transitions

Survival probabilities vs Transitions:

comparison of sensitivity to "standard" CPT violation parameter δ

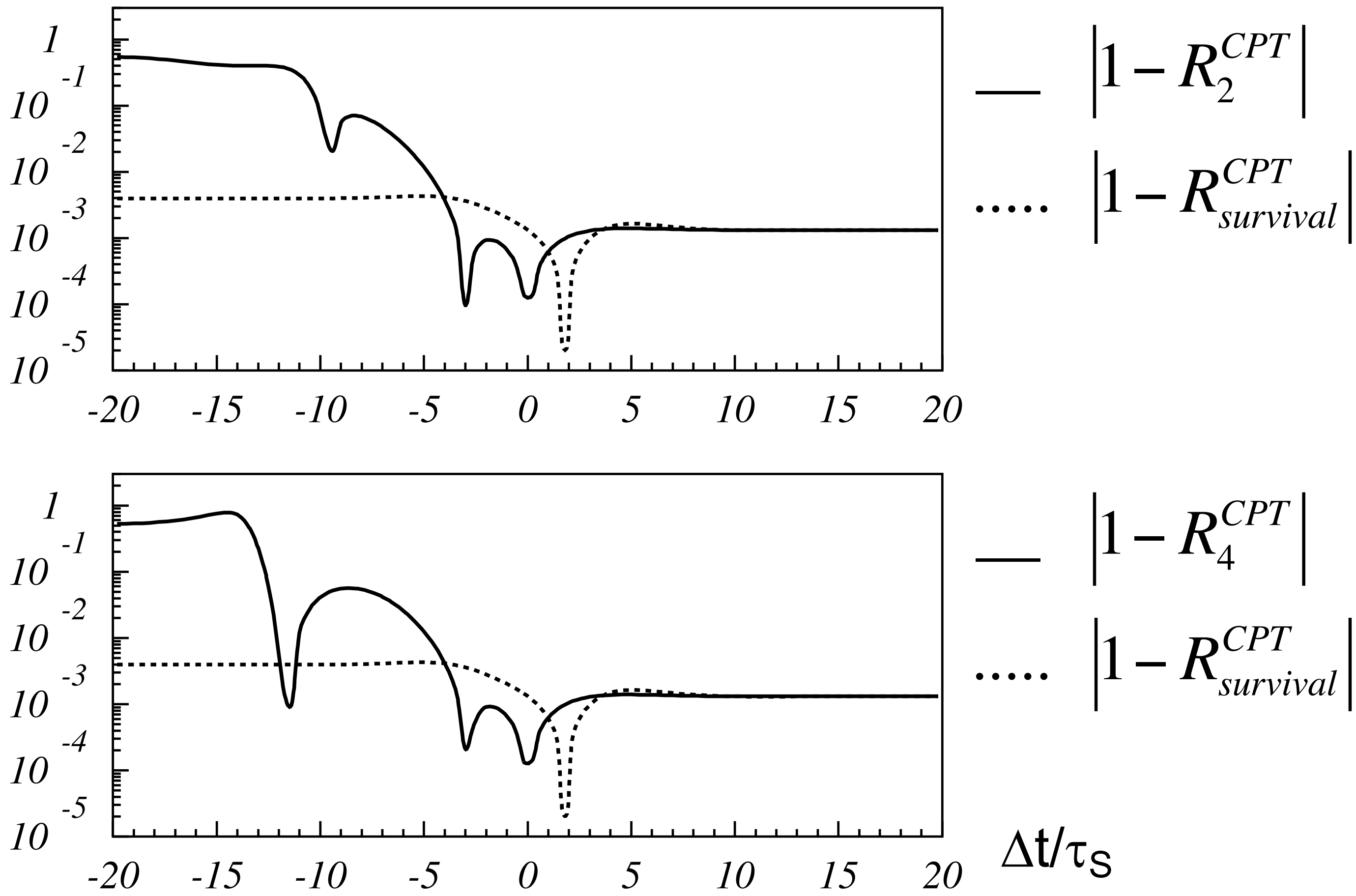
(for visualization purposes, plots with $Re(\delta)=3.3\ 10^{-4}$ $Im(\delta)=1.4\ 10^{-5}$ = present limits)



Direct test of CPT symmetry in neutral kaon transitions

Survival probabilities vs Transitions:
comparison of sensitivity to "standard" CPT violation parameter δ

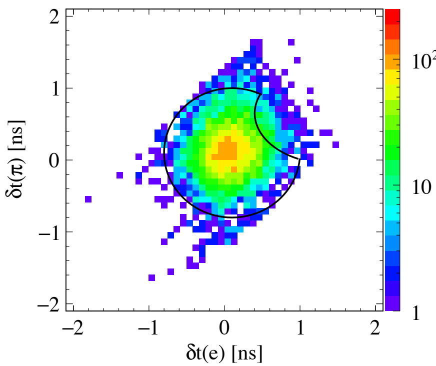
(for visualization purposes, plots with $Re(\delta)=3.3\ 10^{-4}$ $Im(\delta)=1.4\ 10^{-5}$ = present limits)



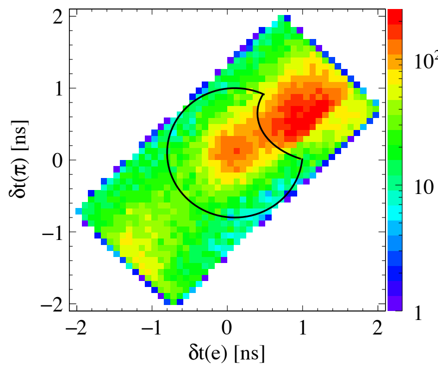
T, CP, CPT tests in neutral kaon transitions at KLOE

- Analysed data $L=1.7\ fb^{-1}$
- Four processes studied:
$\phi \to K_S K_L \to \pi e^{\pm} \nu\ 3\pi^0$ and $\pi^+\pi^-\ \pi e^{\pm}\nu$ in the asympotic regime: $\Delta t \gg \tau_S$
- Time of flight technique to identify semileptonic decays

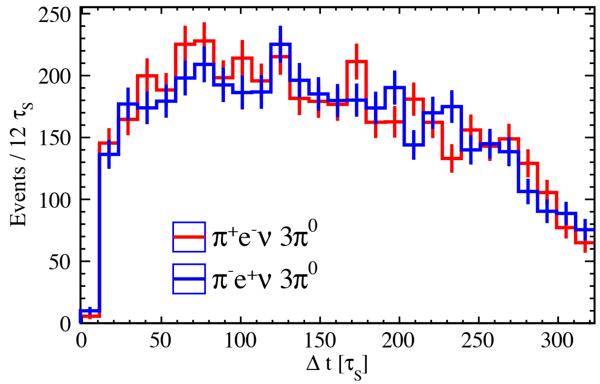
MC K_S signal

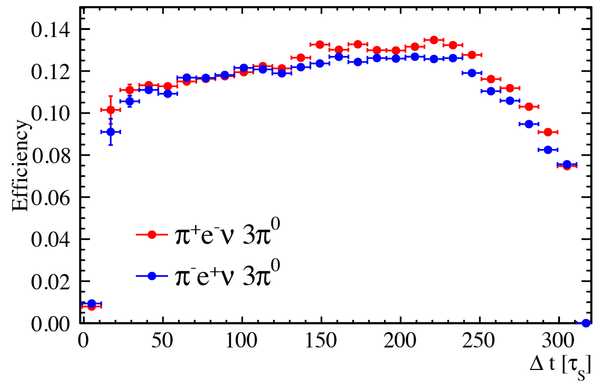


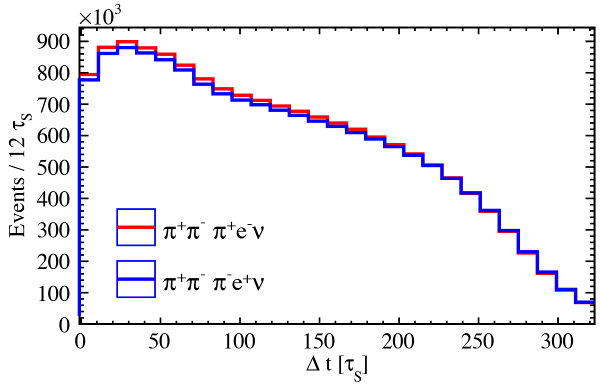
Data

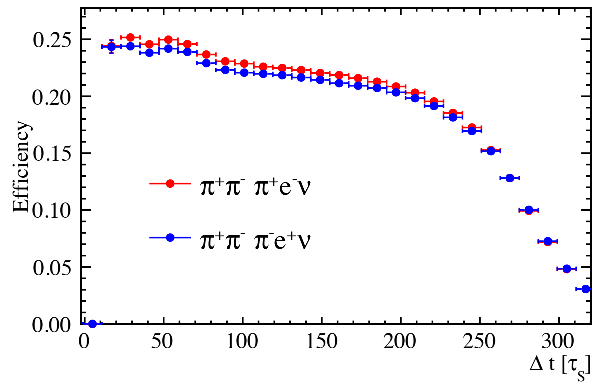


Measured double kaon decay intensities









- residual background subtraction for $\pi e^{\pm}\nu\ 3\pi^0$ channel
- MC selection efficiencies corrected from data with 4 independent control samples

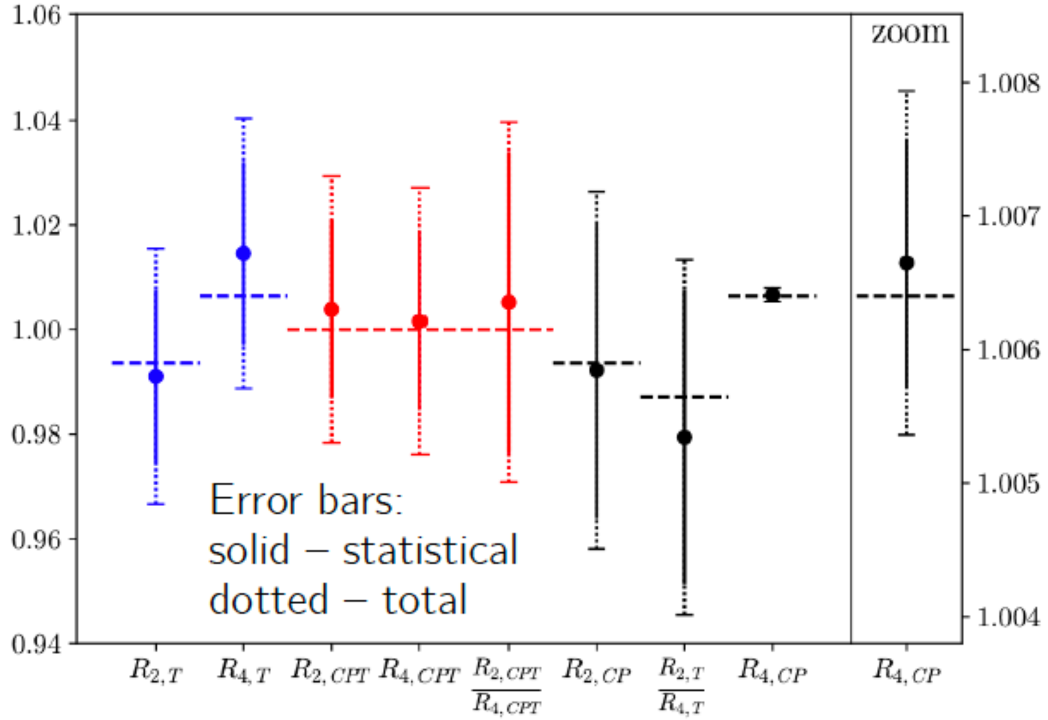
$K_S K_L \to \pi e^{\pm} \nu\ 3\pi^0$ → $K_S \to \pi e \nu$ Klcrash ; $K_S K_L \to \pi^+\pi^-\ 3\pi^0$

$K_S K_L \to \pi^+\pi^-\ \pi e^{\pm}\nu$ → $K_S \to \pi^+\pi^-$ Klcrash ; $K_S K_L \to \pi^0\pi^0\ \pi e \nu$

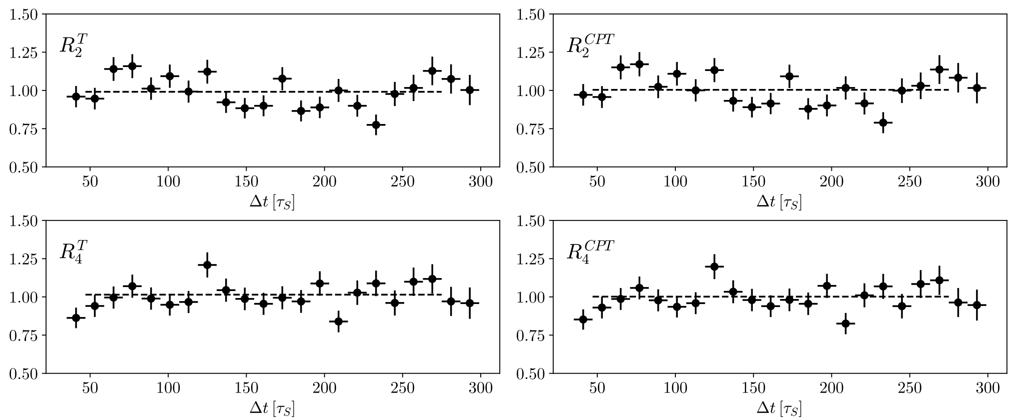
T, CP, CPT tests in neutral kaon transitions at KLOE

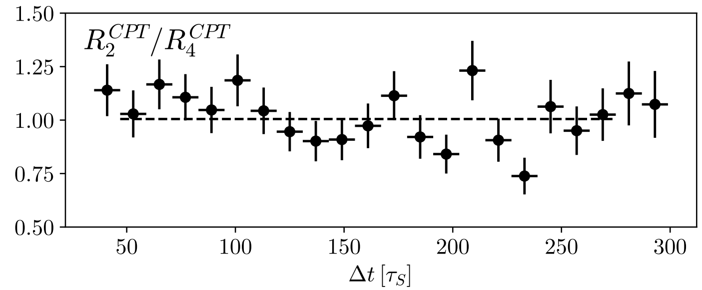
horizontal dashed lines denote expected values:
CPT invariance and TV extrapolated from observed CPV (PDG)

KLOE-2 result
PLB 845 (2023) 138164



$$R_{2,\mathrm{T}} = 0.991 \pm 0.017_{stat} \pm 0.014_{syst} \pm 0.012_{D} \ ,$$
$$R_{4,\mathrm{T}} = 1.015 \pm 0.018_{stat} \pm 0.015_{syst} \pm 0.012_{D} \ ,$$
$$R_{2,\mathrm{CPT}} = 1.004 \pm 0.017_{stat} \pm 0.014_{syst} \pm 0.012_{D} \ ,$$
$$R_{4,\mathrm{CPT}} = 1.002 \pm 0.017_{stat} \pm 0.015_{syst} \pm 0.012_{D} \ ,$$
$$R_{2,\mathrm{CP}} = 0.992 \pm 0.028_{stat} \pm 0.019_{syst} \ ,$$
$$R_{4,\mathrm{CP}} = 1.00665 \pm 0.00093_{stat} \pm 0.00089_{syst} \ ,$$
$$DR_{\mathrm{T,CP}} = R_{2,\mathrm{T}}/R_{4,\mathrm{T}} = 0.979 \pm 0.028_{stat} \pm 0.019_{syst} \ ,$$
$$DR_{\mathrm{CPT}} = R_{2,\mathrm{CPT}}/R_{4,\mathrm{CPT}} = 1.005 \pm 0.029_{stat} \pm 0.019_{syst} \ .$$





First T and CPT test in kaon transitions

$\phi \to K_S K_L \to \pi^+\pi^- \, \pi^+\pi^-$: test of quantum coherence

$$|i\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle|\overline{K}^0\rangle - |\overline{K}^0\rangle|K^0\rangle \right]$$

The EPR correlation suggested a simple test of quantum coherence

$$I(\pi^+\pi^-,\pi^+\pi^-;\Delta t) = \tfrac{N}{2}\Big[\left|\langle \pi^+\pi^-,\pi^+\pi^- | K^0\overline{K}^0(\Delta t)\rangle\right|^2 + \left|\langle \pi^+\pi^-,\pi^+\pi^- | \overline{K}^0 K^0(\Delta t)\rangle\right|^2$$

$$-2\Re\left(\langle \pi^+\pi^-,\pi^+\pi^- | K^0\overline{K}^0(\Delta t)\rangle \langle \pi^+\pi^-,\pi^+\pi^- | \overline{K}^0 K^0(\Delta t)\rangle^* \right)\Big]$$

$\phi \to K_S K_L \to \pi^+\pi^- \, \pi^+\pi^-$: test of quantum coherence

$$|i\rangle = \frac{1}{\sqrt{2}}\left[\left|K^0\right\rangle\left|\overline{K}^0\right\rangle - \left|\overline{K}^0\right\rangle\left|K^0\right\rangle\right]$$

The EPR correlation suggested a simple test of quantum coherence

$$I\left(\pi^+\pi^-,\pi^+\pi^-;\Delta t\right) = \frac{N}{2}\left[\left|\left\langle \pi^+\pi^-,\pi^+\pi^- \middle| K^0\overline{K}^0(\Delta t)\right\rangle\right|^2 + \left|\left\langle \pi^+\pi^-,\pi^+\pi^- \middle| \overline{K}^0K^0(\Delta t)\right\rangle\right|^2\right.$$
$$\left. -\left(1-\zeta_{0\bar{0}}\right)\cdot 2\Re\left(\left\langle \pi^+\pi^-,\pi^+\pi^- \middle| K^0\overline{K}^0(\Delta t)\right\rangle\left\langle \pi^+\pi^-,\pi^+\pi^- \middle| \overline{K}^0K^0(\Delta t)\right\rangle^*\right)\right]$$

$\phi \to K_S K_L \to \pi^+\pi^- \pi^+\pi^-$: test of quantum coherence

$$|i\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle|\overline{K}^0\rangle - |\overline{K}^0\rangle|K^0\rangle\right]$$

The EPR correlation suggested a simple test of quantum coherence

$$I(\pi^+\pi^-,\pi^+\pi^-;\Delta t) = \frac{N}{2}\left[\left|\langle\pi^+\pi^-,\pi^+\pi^-|K^0\overline{K}^0(\Delta t)\rangle\right|^2 + \left|\langle\pi^+\pi^-,\pi^+\pi^-|\overline{K}^0K^0(\Delta t)\rangle\right|^2\right.$$

$$\left. -(1-\zeta_{0\bar{0}})\cdot 2\Re\left(\langle\pi^+\pi^-,\pi^+\pi^-|K^0\overline{K}^0(\Delta t)\rangle\langle\pi^+\pi^-,\pi^+\pi^-|\overline{K}^0K^0(\Delta t)\rangle^*\right)\right]$$

Decoherence parameter:

$\zeta_{0\bar{0}} = 0 \quad \rightarrow \quad$ QM

$\zeta_{0\bar{0}} = 1 \quad \rightarrow \quad$ total decoherence

(also known as Furry's hypothesis or spontaneous factorization)
W.Furry, PR 49 (1936) 393

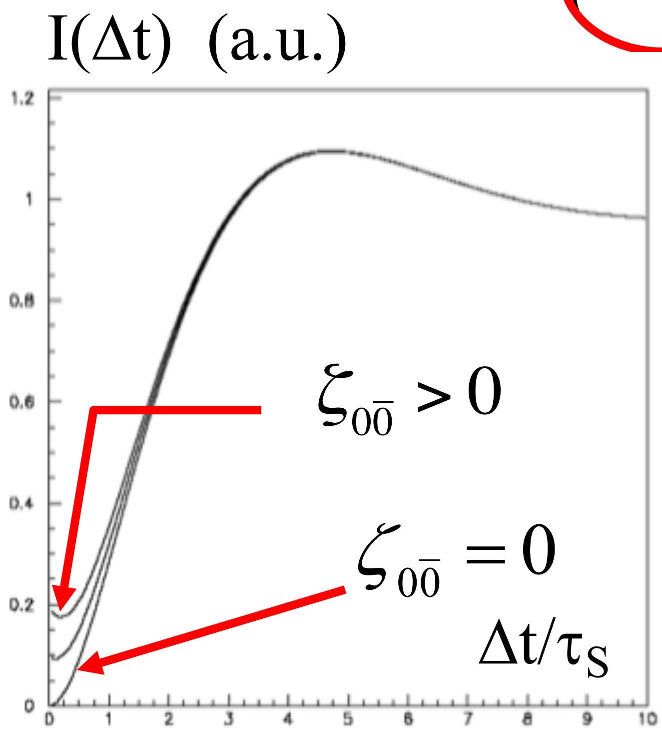
Bertlmann, Grimus, Hiesmayr PR D60 (1999) 114032
Bertlmann, Durstberger, Hiesmayr PRA 68 012111 (2003)

$\phi \rightarrow K_S K_L \rightarrow \pi^+\pi^- \; \pi^+\pi^-$: test of quantum coherence

$$|i\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle|\overline{K}^0\rangle - |\overline{K}^0\rangle|K^0\rangle\right]$$

The EPR correlation suggested a simple test of quantum coherence

$$I(\pi^+\pi^-,\pi^+\pi^-;\Delta t) = \frac{N}{2}\left[\left|\langle\pi^+\pi^-,\pi^+\pi^-|K^0\overline{K}^0(\Delta t)\rangle\right|^2 + \left|\langle\pi^+\pi^-,\pi^+\pi^-|\overline{K}^0K^0(\Delta t)\rangle\right|^2\right.$$
$$\left. -\left(1-\zeta_{0\bar{0}}\right)\cdot 2\Re\left(\langle\pi^+\pi^-,\pi^+\pi^-|K^0\overline{K}^0(\Delta t)\rangle\langle\pi^+\pi^-,\pi^+\pi^-|\overline{K}^0K^0(\Delta t)\rangle^*\right)\right]$$



Decoherence parameter:

$\zeta_{0\bar{0}} = 0 \quad \rightarrow \quad$ QM

$\zeta_{0\bar{0}} = 1 \quad \rightarrow \quad$ total decoherence

(also known as Furry's hypothesis or spontaneous factorization)
W.Furry, PR 49 (1936) 393

Bertlmann, Grimus, Hiesmayr PR D60 (1999) 114032
Bertlmann, Durstberger, Hiesmayr PRA 68 012111 (2003)

$\phi \to K_S K_L \to \pi^+\pi^- \, \pi^+\pi^-$: test of quantum coherence

KLOE-2 JHEP 04 (2022) 059

$$\zeta_{0\bar{0}} = (-0.5 \pm 8.0_{stat} \pm 3.7_{syst}) \times 10^{-7}$$

CP violating process:

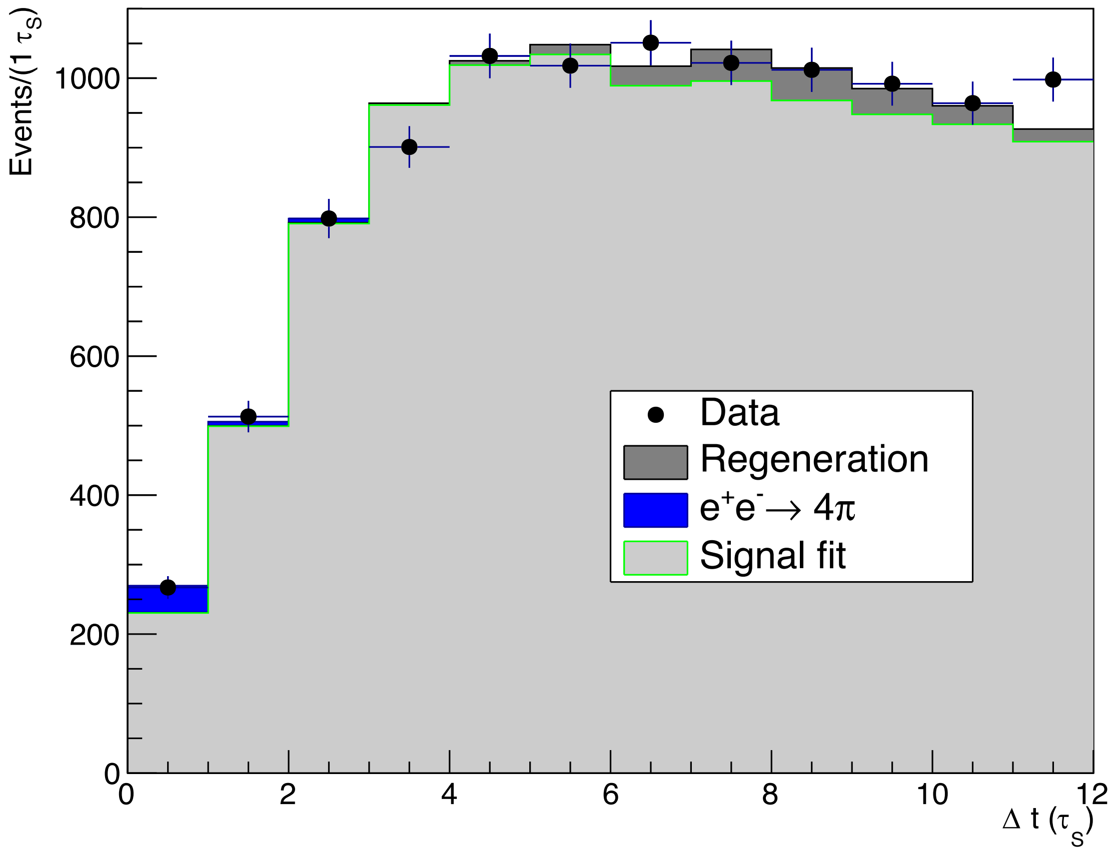
terms $\zeta_{00}/|\eta_{+-}|^2$ with $|\eta_{+-}|^2 \sim |\varepsilon|^2 \sim 10^{-6}$

=> high sensitivity to ζ_{00} ;

CP violation in kaon mixing acts as amplification mechanism

In the B-meson system, BELLE coll. (PRL 99 (2007) 131802) obtains:

$$\zeta^{B}_{0\bar{0}} = 0.029 \pm 0.057$$



Possible decoherence due quantum gravity effects (apparent loss of unitarity) implying also **CPT** violation => modified Liouville – von Neumann equation for the density matrix of the kaon system depends on a CPTV parameter γ [J. Ellis et al. PRD53 (1996) 3846]

In this scenario γ can be at most:

$$O(m_K^2/M_{PLANCK}) = 2 \times 10^{-20}\ GeV$$

KLOE-2 result

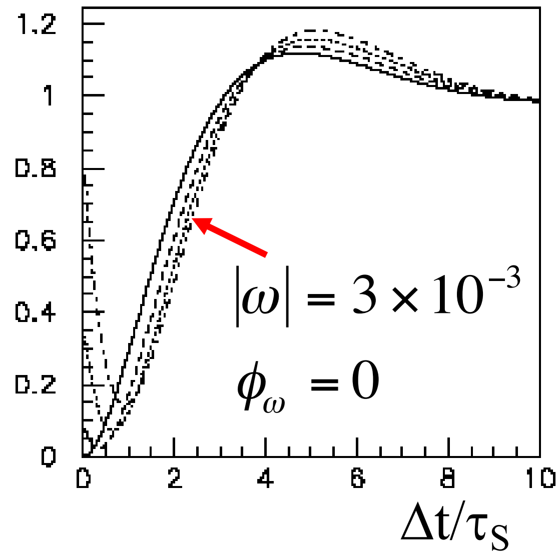
$$\gamma = (1.3 \pm 9.4_{stat} \pm 4.2_{syst}) \times 10^{-22}\ \text{GeV}$$

$\phi \rightarrow K_S K_L \rightarrow \pi^+\pi^- \pi^+\pi^-$: CPT violation in entangled K states

In presence of decoherence and **CPT violation** induced by quantum gravity (CPT operator "ill-defined") the definition of the particle-antiparticle states could be modified. This in turn could induce a breakdown of the correlations imposed by Bose statistics (EPR correlations) to the kaon state:

[Bernabeu, et al. PRL 92 (2004) 131601, NPB744 (2006) 180].

$$|i\rangle \propto \left(|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle\right) + \omega\left(|K^0\rangle|\bar{K}^0\rangle + |\bar{K}^0\rangle|K^0\rangle\right)$$



at most one expects:

$$|\omega|^2 = O\left(\frac{E^2/M_{PLANCK}}{\Delta\Gamma}\right) \approx 10^{-5} \Rightarrow |\omega| \sim 10^{-3}$$

In some microscopic models of space-time foam arising from non-critical string theory [Bernabeu, Mavromatos, Sarkar PRD 74 (2006) 045014] : $|\omega| \sim 10^{-4} \div 10^{-5}$

The maximum sensitivity to ω is expected for $f_1=f_2=\pi^+\pi^-$ (terms: $|\omega|/|\eta_{+-}|$)
All **CPTV** effects induced by QG $(\alpha,\beta,\gamma,\omega)$ could be simultaneously disentangled.

$\phi \rightarrow K_S K_L \rightarrow \pi^+\pi^- \pi^+\pi^-$: CPT violation in entangled K states

In presence of decoherence and **CPT violation** induced by quantum gravity (CPT operator "ill-defined") the definition of the particle-antiparticle states could be modified. This in turn could induce a breakdown of the correlations imposed by Bose statistics (EPR correlations) to the kaon state:

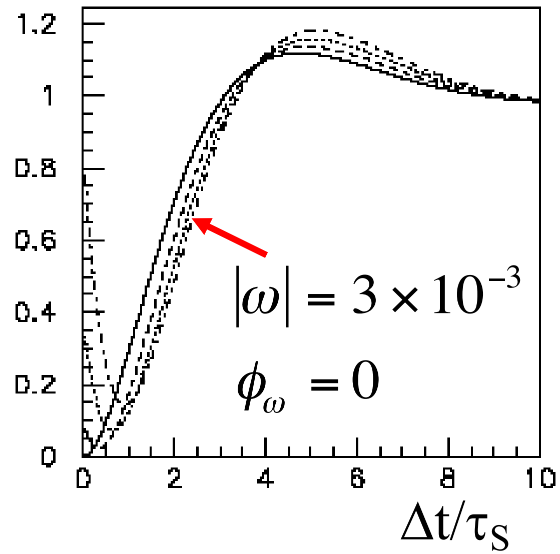
[Bernabeu, et al. PRL 92 (2004) 131601, NPB744 (2006) 180].

$$|i\rangle \propto \left(|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle\right) + \omega\left(|K^0\rangle|\bar{K}^0\rangle + |\bar{K}^0\rangle|K^0\rangle\right)$$

$$\propto \left(|K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle\right) + \omega\left(|K_S\rangle|K_S\rangle - |K_L\rangle|K_L\rangle\right)$$

at most one expects:

$$|\omega|^2 = O\left(\frac{E^2/M_{PLANCK}}{\Delta\Gamma}\right) \approx 10^{-5} \Rightarrow |\omega| \sim 10^{-3}$$



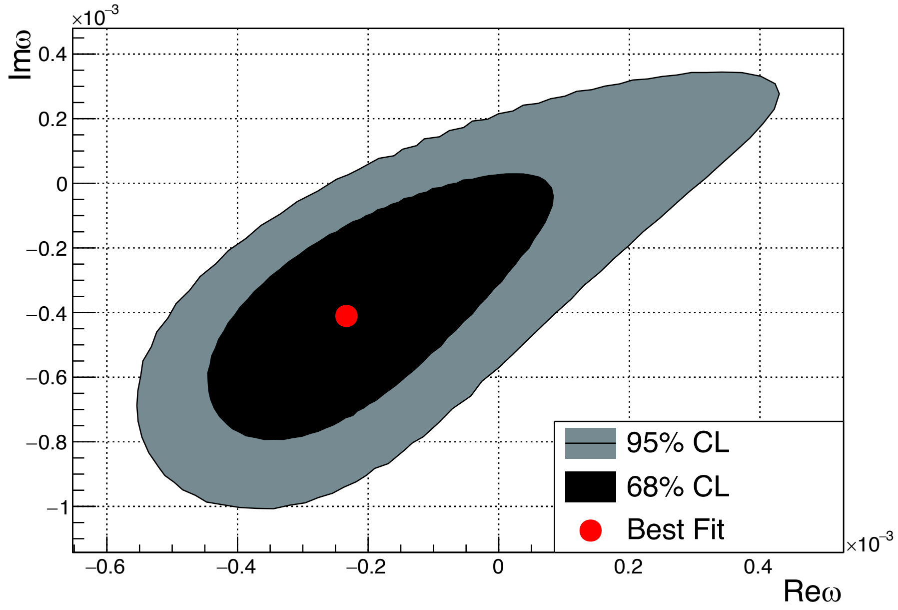
In some microscopic models of space-time foam arising from non-critical string theory [Bernabeu, Mavromatos, Sarkar PRD 74 (2006) 045014] : $|\omega| \sim 10^{-4} \div 10^{-5}$

The maximum sensitivity to ω is expected for $f_1=f_2=\pi^+\pi^-$ (terms: $|\omega|/|\eta_{+-}|$)
All **CPTV** effects induced by QG $(\alpha,\beta,\gamma,\omega)$ could be simultaneously disentangled.

$\phi \to K_S K_L \to \pi^+\pi^- \pi^+\pi^-$: CPT violation in entangled K states

The fit with $I(\pi^+\pi^-,\pi^+\pi^-;\Delta t,\omega)$ yields (1.7 fb-1):

$$\Re\omega = \left(-2.3^{+1.9}_{-1.5}{}_{stat} \pm 0.6_{syst}\right)\times 10^{-4}$$
$$\Im\omega = \left(-4.1^{+2.8}_{-2.6}{}_{stat} \pm 0.9_{syst}\right)\times 10^{-4}$$
$$|\omega| = (4.7 \pm 2.9_{stat} \pm 1.0_{syst})\times 10^{-4}$$
$$\phi_\omega = -2.1 \pm 0.2_{stat} \pm 0.1_{syst} \text{ rad}$$

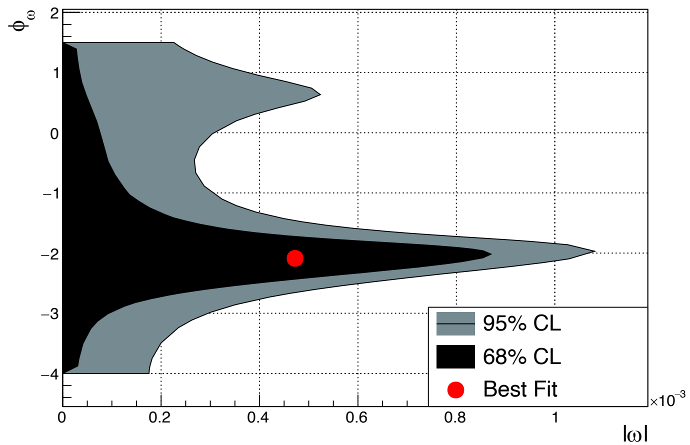


from $|\omega|^2 = \dfrac{BR(\phi \to K_S K_S, K_L K_L)}{BR(\phi \to K_S K_L)}$

$$BR(\phi \to K_S K_S, K_L K_L) < 2.4\times 10^{-7}$$
at 90% C.L.

KLOE-2 JHEP 04 (2022) 059

Experimental disproof of prediction:
$BR(\phi \to K_S K_S) \sim 3.6\times 10^{-6}$
PRL 96, 192001(2006)



In the B system:
$-0.0084 \leq \Re\omega \leq 0.0100$ at 95% C.L.
Alvarez, Bernabeu, Nebot JHEP 11 (2006) 087 (see also Bernabeu et al, EPJC (2017) 77:865)

Conclusions

- The neutral kaon system is a jewel donated to us by Nature.
- It is an excellent laboratory for the study of CPT symmetry and the basic principles of Quantum Mechanics.
- These studies are widely expanded with entangled neutral kaon pairs at a ϕ-factory.

- Several parameters related to possible:
  - CPT violation (within QM)
  - CPT violation and decoherence

  have been measured, in same cases with a precision reaching the interesting Planck's scale region;
- Direct test of CPT in transitions has been performed at KLOE-2
- All results so far are consistent with no CPT violation

- Other interesting tests (not discussed here) related to CPT violation and Lorentz symmetry breaking

- The deep connection of entanglement with discrete symmetries of neutral kaons is still surprising and continues to provide interesting results (e.g. the "from future to past" paradox, post-tagging effect in entangled kaons [PRD 105 116004(2022)]) with new results expected from the analysis of the whole dataset of KLOE+KLOE-2 of ~8 fb^{-1}.

spare slides

Definition of states

We need two orthogonal bases:

1) $|K^0\rangle$ and $|\bar{K}^0\rangle$ assuming $\Delta S=\Delta Q$ rule identified by their $\pi l \nu$ decay (l^+ or l^-)

2) $|K_+\rangle$ and $|K_-\rangle$ (* not to be confused with charged kaons K^+ and K^-)

$|K_+\rangle$ is the state filtered by the decay into $\pi\pi$, a pure CP=+1 state
$|K_-\rangle$ is the state filtered by the decay into $3\pi^0$, a pure CP=-1 state
Their orthogonal states correspond to the states that cannot decay into $\pi\pi$ or $3\pi^0$:

$$|\widetilde{K}_-\rangle \equiv \widetilde{N}_- \left[|K_L\rangle - \eta_{\pi\pi}|K_S\rangle\right]$$
$$|\widetilde{K}_+\rangle \equiv \widetilde{N}_+ \left[|K_S\rangle - \eta_{3\pi^0}|K_L\rangle\right]$$

$$\eta_{\pi\pi} = \frac{\langle\pi\pi|T|K_L\rangle}{\langle\pi\pi|T|K_S\rangle}$$
$$\eta_{3\pi^0} = \frac{\langle 3\pi^0|T|K_S\rangle}{\langle 3\pi^0|T|K_L\rangle}$$

The orthogonal bases are: $\{K_+, \widetilde{K}_-\}$ $\{\widetilde{K}_+, K_-\}$

Even though the decay products are orthogonal, the filtered $|K_+\rangle$ and $|K_-\rangle$ states can still be non-orthogonal.
Condition of orthogonality:

$$\eta_{\pi\pi} + \eta^{\star}_{3\pi^0} = \epsilon_L + \epsilon^{\star}_S$$

Neglecting direct CP violation ε' →

$$|K_+\rangle \equiv |\widetilde{K}_+\rangle$$
$$|K_-\rangle \equiv |\widetilde{K}_-\rangle$$

Definition of states

We need two orthogonal bases:

1) $|K^0\rangle$ and $|\bar{K}^0\rangle$ assuming ΔS=ΔQ rule identified by their πlν decay (l⁺ or l⁻)

2) $|K_+\rangle$ and $|K_-\rangle$ (* not to be confused with charged kaons K⁺ and K⁻)

$|K_+\rangle$ is the state filtered by the decay into ππ, a pure CP=+1 state
$|K_-\rangle$ is the state filtered by the decay into $3\pi^0$, a pure CP=-1 state
Their orthogonal states correspond to the states that cannot decay into ππ or $3\pi^0$:

$$|\widetilde{K}_-\rangle \equiv \widetilde{N}_- \left[|K_L\rangle - \eta_{\pi\pi}|K_S\rangle\right]$$
$$|\widetilde{K}_+\rangle \equiv \widetilde{N}_+ \left[|K_S\rangle - \eta_{3\pi^0}|K_L\rangle\right]$$

$$\eta_{\pi\pi} = \frac{\langle\pi\pi|T|K_L\rangle}{\langle\pi\pi|T|K_S\rangle}$$
$$\eta_{3\pi^0} = \frac{\langle 3\pi^0|T|K_S\rangle}{\langle 3\pi^0|T|K_L\rangle}$$

The orthogonal bases are: $\{K_+, \widetilde{K}_-\}$ $\{\widetilde{K}_+, K_-\}$

Even though the decay products are orthogonal, the filtered $|K_+\rangle$ and $|K_-\rangle$ states can still be non-orthogonal.
Condition of orthogonality:

$$\eta_{\pi\pi} + \eta^\star_{3\pi^0} = \epsilon_L + \epsilon^\star_S$$

Neglecting direct CP violation ε' →

$$|K_+\rangle \equiv |\widetilde{K}_+\rangle$$
$$|K_-\rangle \equiv |\widetilde{K}_-\rangle$$

Direct test of symmetries with neutral kaons

Reference	T-conjugate	CP-conjugate	CPT-conjugate
$K^0 \to K^0$	$K^0 \to K^0$	$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$
$K^0 \to \bar{K}^0$	$\bar{K}^0 \to K^0$	$\bar{K}^0 \to K^0$	$K^0 \to \bar{K}^0$
$K^0 \to K_+$	$K_+ \to K^0$	$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$
$K^0 \to K_-$	$K_- \to K^0$	$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$
$\bar{K}^0 \to K^0$	$K^0 \to \bar{K}^0$	$K^0 \to \bar{K}^0$	$\bar{K}^0 \to K^0$
$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$	$K^0 \to K^0$	$K^0 \to K^0$
$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$	$K^0 \to K_+$	$K_+ \to K^0$
$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$	$K^0 \to K_-$	$K_- \to K^0$
$K_+ \to K^0$	$K^0 \to K_+$	$K_+ \to \bar{K}^0$	$\bar{K}^0 \to K_+$
$K_+ \to \bar{K}^0$	$\bar{K}^0 \to K_+$	$K_+ \to K^0$	$K^0 \to K_+$
$K_+ \to K_+$	$K_+ \to K_+$	$K_+ \to K_+$	$K_+ \to K_+$
$K_+ \to K_-$	$K_- \to K_+$	$K_+ \to K_-$	$K_- \to K_+$
$K_- \to K^0$	$K^0 \to K_-$	$K_- \to \bar{K}^0$	$\bar{K}^0 \to K_-$
$K_- \to \bar{K}^0$	$\bar{K}^0 \to K_-$	$K_- \to K^0$	$K^0 \to K_-$
$K_- \to K_+$	$K_+ \to K_-$	$K_- \to K_+$	$K_+ \to K_-$
$K_- \to K_-$	$K_- \to K_-$	$K_- \to K_-$	$K_- \to K_-$

Direct test of symmetries with neutral kaons

Conjugate= reference

Reference	T-conjugate	CP-conjugate	CPT-conjugate
$K^0 \to K^0$	~~$K^0 \to K^0$~~	$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$
$K^0 \to \bar{K}^0$	$\bar{K}^0 \to K^0$	$\bar{K}^0 \to K^0$	~~$K^0 \to \bar{K}^0$~~
$K^0 \to K_+$	$K_+ \to K^0$	$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$
$K^0 \to K_-$	$K_- \to K^0$	$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$
$\bar{K}^0 \to K^0$	$K^0 \to \bar{K}^0$	$K^0 \to \bar{K}^0$	~~$\bar{K}^0 \to K^0$~~
$\bar{K}^0 \to \bar{K}^0$	~~$\bar{K}^0 \to \bar{K}^0$~~	$K^0 \to K^0$	$K^0 \to K^0$
$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$	$K^0 \to K_+$	$K_+ \to K^0$
$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$	$K^0 \to K_-$	$K_- \to K^0$
$K_+ \to K^0$	$K^0 \to K_+$	$K_+ \to \bar{K}^0$	$\bar{K}^0 \to K_+$
$K_+ \to \bar{K}^0$	$\bar{K}^0 \to K_+$	$K_+ \to K^0$	$K^0 \to K_+$
$K_+ \to K_+$	~~$K_+ \to K_+$~~	~~$K_+ \to K_+$~~	~~$K_+ \to K_+$~~
$K_+ \to K_-$	$K_- \to K_+$	~~$K_+ \to K_-$~~	$K_- \to K_+$
$K_- \to K^0$	$K^0 \to K_-$	$K_- \to \bar{K}^0$	$\bar{K}^0 \to K_-$
$K_- \to \bar{K}^0$	$\bar{K}^0 \to K_-$	$K_- \to K^0$	$K^0 \to K_-$
$K_- \to K_+$	$K_+ \to K_-$	~~$K_- \to K_+$~~	$K_+ \to K_-$
$K_- \to K_-$	~~$K_- \to K_-$~~	~~$K_- \to K_-$~~	~~$K_- \to K_-$~~

Direct test of symmetries with neutral kaons

Conjugate= reference (red bar)

already in the table with conjugate as reference (black bar)

Reference	T-conjugate	CP-conjugate	CPT-conjugate
$K^0 \to K^0$	~~$K^0 \to K^0$~~	$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$
$K^0 \to \bar{K}^0$	$\bar{K}^0 \to K^0$	$\bar{K}^0 \to K^0$	~~$K^0 \to \bar{K}^0$~~
$K^0 \to K_+$	$K_+ \to K^0$	$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$
$K^0 \to K_-$	$K_- \to K^0$	$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$
$\bar{K}^0 \to K^0$	~~$K^0 \to \bar{K}^0$~~	~~$K^0 \to \bar{K}^0$~~	~~$\bar{K}^0 \to K^0$~~
$\bar{K}^0 \to \bar{K}^0$	~~$\bar{K}^0 \to \bar{K}^0$~~	~~$K^0 \to K^0$~~	~~$K^0 \to K^0$~~
$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$	~~$K^0 \to K_+$~~	$K_+ \to K^0$
$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$	~~$K^0 \to K_-$~~	$K_- \to K^0$
$K_+ \to K^0$	~~$K^0 \to K_+$~~	$K_+ \to \bar{K}^0$	~~$\bar{K}^0 \to K_+$~~
$K_+ \to \bar{K}^0$	~~$\bar{K}^0 \to K_+$~~	~~$K_+ \to K^0$~~	~~$K^0 \to K_+$~~
$K_+ \to K_+$	~~$K_+ \to K_+$~~	~~$K_+ \to K_+$~~	~~$K_+ \to K_+$~~
$K_+ \to K_-$	$K_- \to K_+$	~~$K_+ \to K_-$~~	$K_- \to K_+$
$K_- \to K^0$	~~$K^0 \to K_-$~~	$K_- \to \bar{K}^0$	~~$\bar{K}^0 \to K_-$~~
$K_- \to \bar{K}^0$	~~$\bar{K}^0 \to K_-$~~	~~$K_- \to K^0$~~	~~$K^0 \to K_-$~~
$K_- \to K_+$	~~$K_+ \to K_-$~~	~~$K_- \to K_+$~~	~~$K_+ \to K_-$~~
$K_- \to K_-$	~~$K_- \to K_-$~~	~~$K_- \to K_-$~~	~~$K_- \to K_-$~~

Direct test of symmetries with neutral kaons

Conjugate= reference

already in the table with conjugate as reference

Two identical conjugates for one reference

Reference	T-conjugate	CP-conjugate	CPT-conjugate
$K^0 \to K^0$	$K^0 \to K^0$	$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$
$K^0 \to \bar{K}^0$	$\bar{K}^0 \to K^0$	$\bar{K}^0 \to K^0$	$K^0 \to \bar{K}^0$
$K^0 \to K_+$	$K_+ \to K^0$	$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$
$K^0 \to K_-$	$K_- \to K^0$	$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$
$\bar{K}^0 \to K^0$	[redacted]	[redacted]	$\bar{K}^0 \to K^0$
$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$	[redacted]	[redacted]
$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$	[redacted]	$K_+ \to K^0$
$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$	[redacted]	$K_- \to K^0$
$K_+ \to K^0$	[redacted]	$K_+ \to \bar{K}^0$	[redacted]
$K_+ \to \bar{K}^0$	[redacted]	[redacted]	[redacted]
$K_+ \to K_+$	$K_+ \to K_+$	$K_+ \to K_+$	$K_+ \to K_+$
$K_+ \to K_-$	$K_- \to K_+$	$K_+ \to K_-$	$K_- \to K_+$
$K_- \to K^0$	[redacted]	$K_- \to \bar{K}^0$	[redacted]
$K_- \to \bar{K}^0$	[redacted]	[redacted]	[redacted]
$K_- \to K_+$	[redacted]	$K_- \to K_+$	[redacted]
$K_- \to K_-$	$K_- \to K_-$	$K_- \to K_-$	$K_- \to K_-$

Direct test of symmetries with neutral kaons

Conjugate= reference (red bar)

already in the table with conjugate as reference (black bar)

Two identical conjugates for one reference (blue bar)

Reference	T-conjugate	CP-conjugate	CPT-conjugate
$K^0 \to K^0$	$K^0 \to K^0$	$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$
$K^0 \to \bar{K}^0$	$\bar{K}^0 \to K^0$	$\bar{K}^0 \to K^0$	$K^0 \to \bar{K}^0$
$K^0 \to K_+$	$K_+ \to K^0$	$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$
$K^0 \to K_-$	$K_- \to K^0$	$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$
$\bar{K}^0 \to K^0$	[redacted]	[redacted]	$\bar{K}^0 \to K^0$
$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$	[redacted]	[redacted]
$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$	[redacted]	$K_+ \to K^0$
$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$	[redacted]	$K_- \to K^0$
$K_+ \to K^0$	[redacted]	$K_+ \to \bar{K}^0$	[redacted]
$K_+ \to \bar{K}^0$	[redacted]	[redacted]	[redacted]
$K_+ \to K_+$	$K_+ \to K_+$	$K_+ \to K_+$	$K_+ \to K_+$
$K_+ \to K_-$	$K_- \to K_+$	$K_+ \to K_-$	$K_- \to K_+$
$K_- \to K^0$	[redacted]	$K_- \to \bar{K}^0$	[redacted]
$K_- \to \bar{K}^0$	[redacted]	[redacted]	[redacted]
$K_- \to K_+$	[redacted]	$K_- \to K_+$	[redacted]
$K_- \to K_-$	$K_- \to K_-$	$K_- \to K_-$	$K_- \to K_-$

4 distinct tests of T symmetry

4 distinct tests of CP symmetry

4 distinct tests of CPT symmetry