V_{us} from Tau decays

Emilie Passemar
passemar@ific.uv.es
IFIC Valencia/Indiana University

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Outline

1. What is the Cabbibo angle anomaly?
2. Why this anomaly?
3. Prospects with studying Tau physics
4. New Physics Interpretations
5. Conclusion and Outlook

1. What is the Cabbibo angle anomaly?

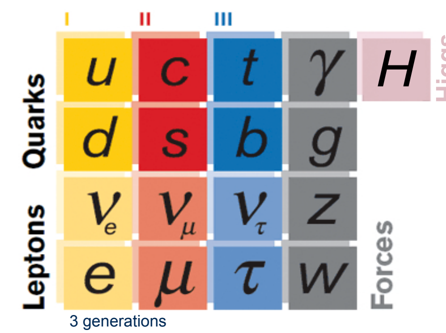
1.1 Test of the Standard Model: V_{us} and CKM unitarity

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
  - Fundamental parameter of the Standard Model

    Description of the weak interactions:

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_{\alpha}^{+} \left(\bar{D}_L V_{CKM} \gamma^{\alpha} U_L + \bar{e}_L \gamma^{\alpha} \nu_{e_L} + \bar{\mu}_L \gamma^{\alpha} \nu_{\mu_L} + \bar{\tau}_L \gamma^{\alpha} \nu_{\tau_L} \right) + \mathrm{h.c.}$$

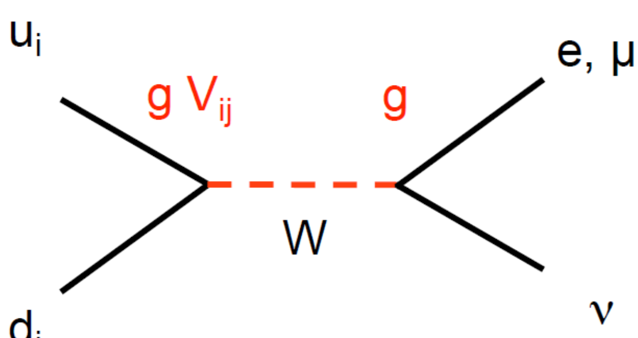
Unitary matrix



  - Check unitarity of the first row of the CKM matrix:

Cabibbo Universality: $\boxed{|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} \stackrel{?}{=} 1}$

Negligible ~2x10^{-5}
(B decays)



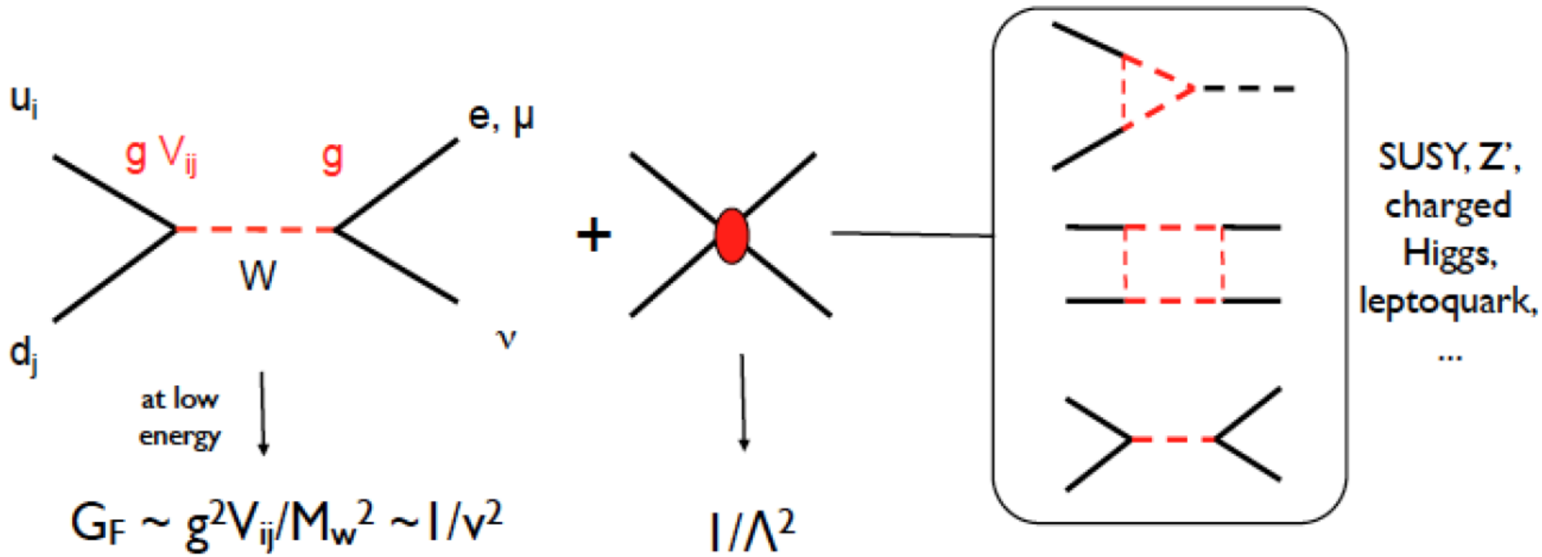
$$|V_{ud}| = \cos\theta_C \quad \text{and} \quad |V_{us}| = \sin\theta_C$$

In the SM: W exchange ⇨ V – A structure only

1.2 Constraining New Physics

- BSM: sensitive to tree-level and loop effects of a large class of models

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}}$$



BSM effects : $\boxed{\Delta \sim \dfrac{c_n}{g^2}\dfrac{M_W^2}{\Lambda^2} \leq 10^{-2} - 10^{-3} \longleftrightarrow \Lambda \sim 1\text{-}10 \text{ TeV}}$

1.2 Constraining New Physics

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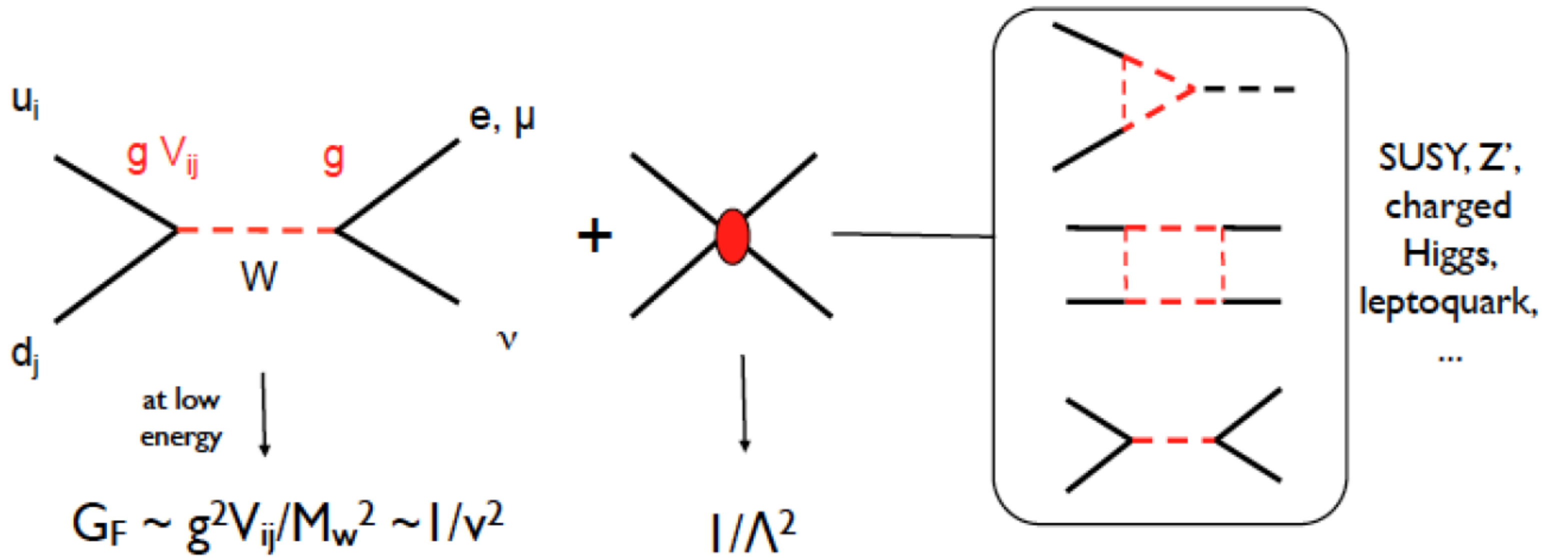
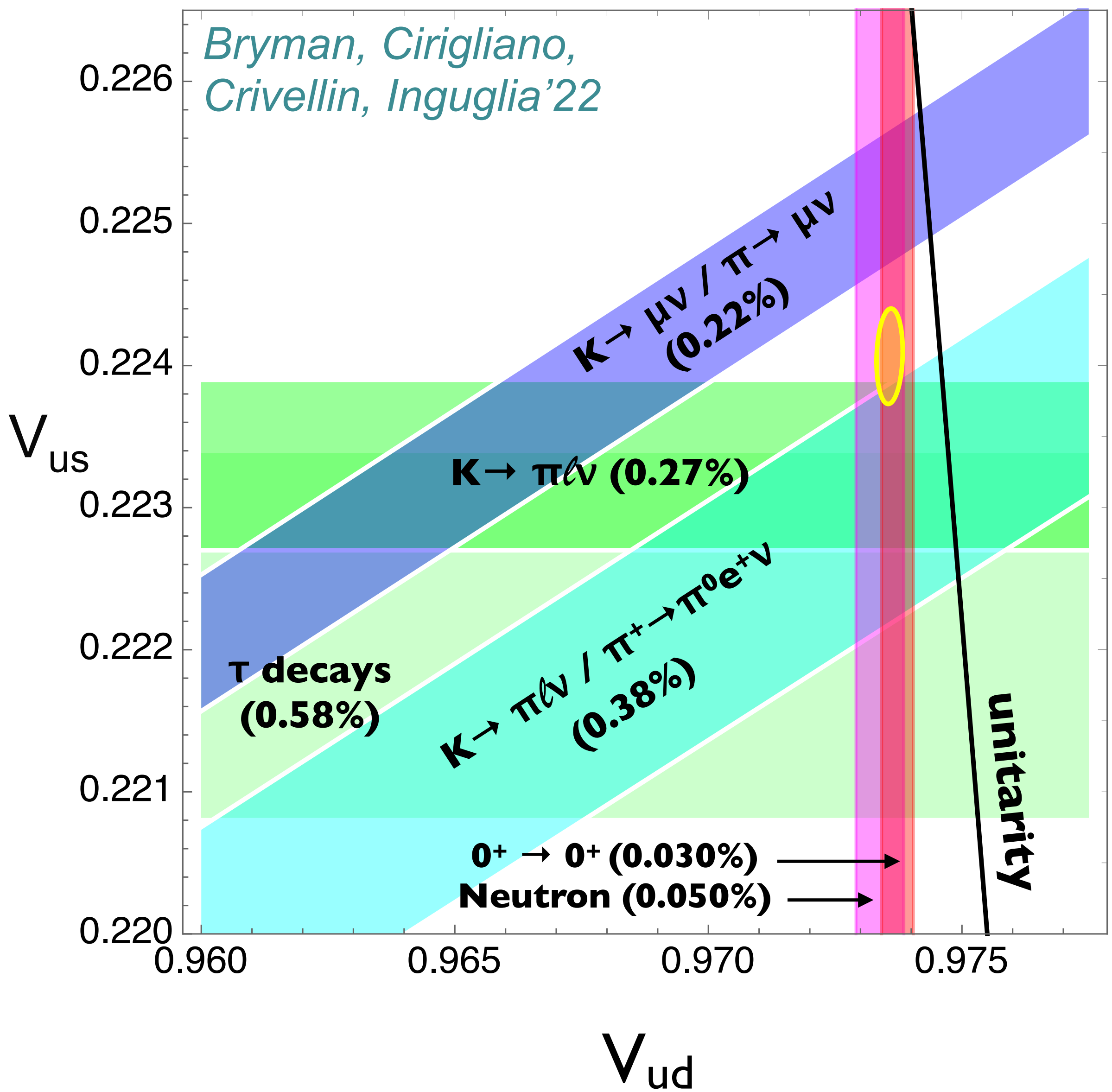


Grossman, E.P., Schacht'20

- Look for new physics by comparing the extraction of V_{us} from different processes: helicity suppressed $K_{\mu 2}$, helicity allowed K_{l3}, hadronic τ decays

1.2 Cabibbo angle anomaly

Moulson & E.P.@CKM2021



$|V_{ud}| = 0.97373(31)$

$|V_{us}| = 0.2231(6)$

$|V_{us}|/|V_{ud}| = 0.2311(5)$

Fit results, no constraint

$V_{ud} = 0.97365(30)$

$V_{us} = 0.22414(37)$

$\chi^2/\text{ndf} = 6.6/1\ (1.0\%)$

$\Delta_{CKM} = -0.0018(6)$

-2.7σ

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1 + \Delta_{CKM}$$

Negligible ~2×10^{-5}
(B decays)

Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$

$$\Gamma_k = (G_F^{(\mu)})^2 \times |V_{ij}|^2 \times |M_{\rm had}|^2 \times (1 + \delta_{RC}) \times F_{\rm kin}$$

Channel-dependent effective CKM element

Hadronic matrix element

Radiative corrections

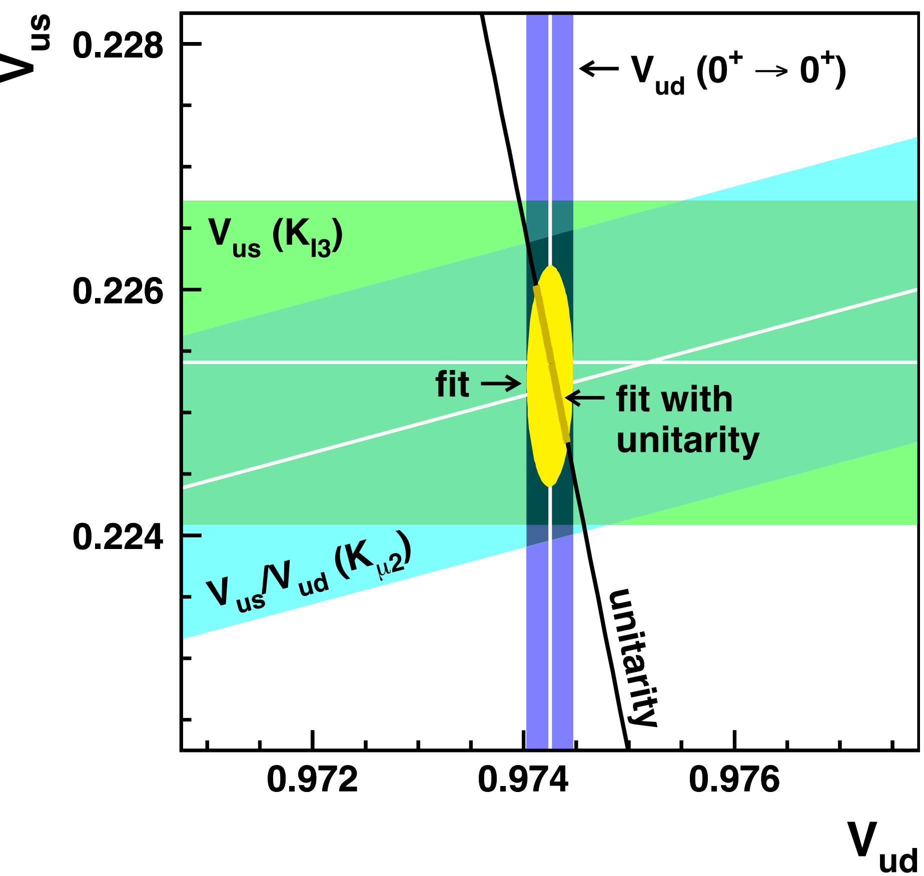
Recent progress on 1) Hadronic matrix elements from lattice QCD
2) Radiative corrections from dispersive methods + Lattice QCD

Seng, Gorchtein, Patel, Ramsey-Musolf'18,'19

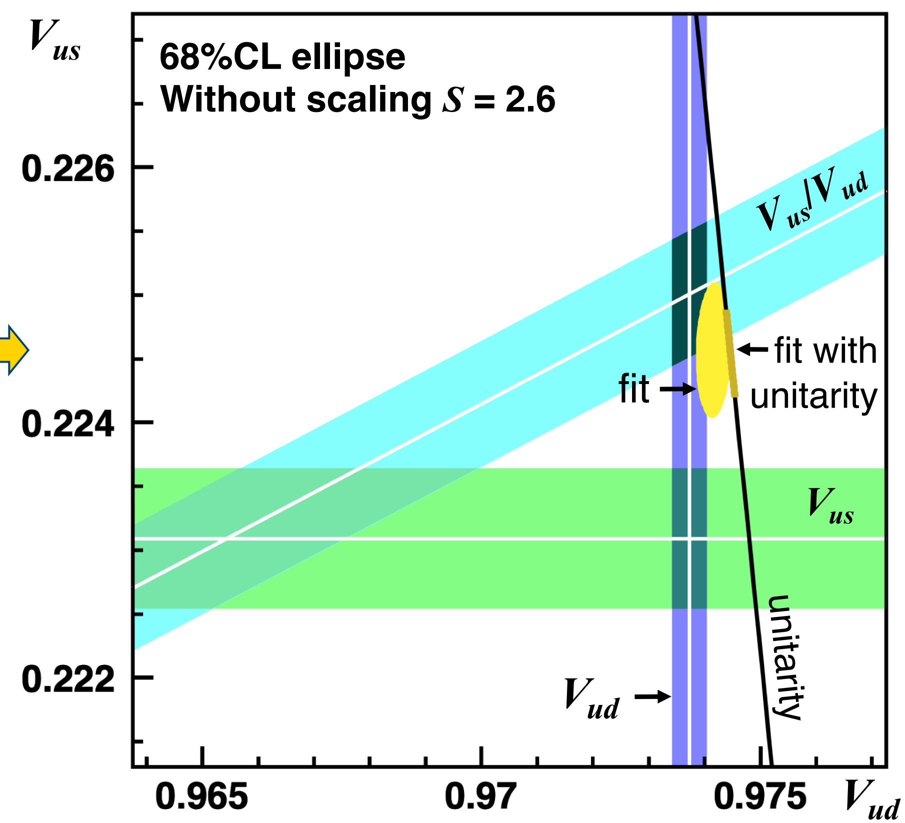
2. Why this anomaly?

2.1 Changes on V_{us} and V_{ud} since 2011

Flavianet Kaon WG: *Antonelli et al'11*



Moulson & E.P.@CKM2021



2.1 Changes on V_{us} and V_{ud} since 2011

- Almost no change on the experimental side since 2011

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Radiative corrections

- Changes in *theoretical* inputs:
  - Impressive progress on hadronic matrix element computations from *lattice QCD* for V_{us} and V_{us}/V_{ud} extraction from Kaon decays

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Channel-dependent effective CKM element

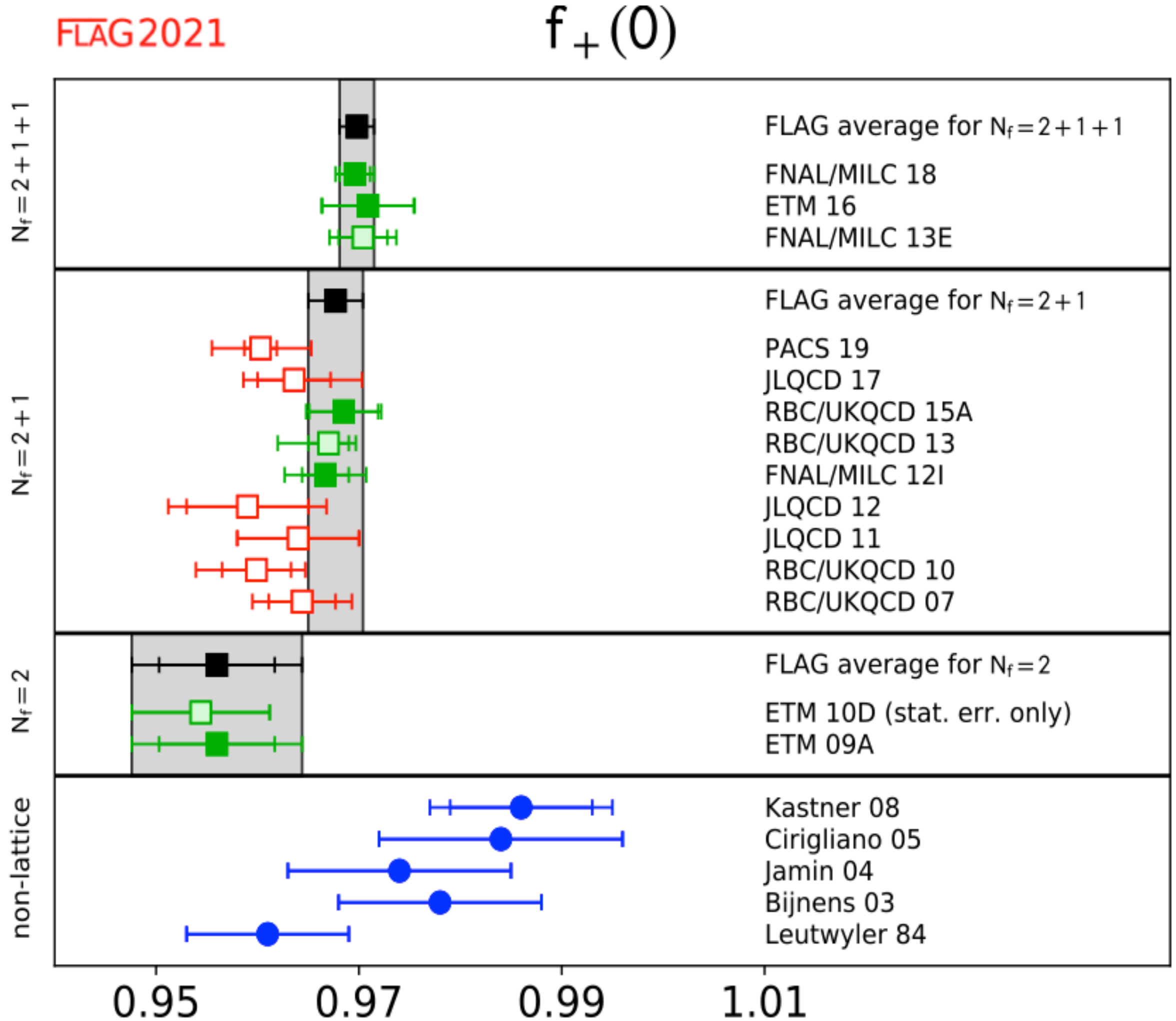
Hadronic matrix element

Radiative corrections

- Changes in *theoretical* inputs:
  - Impressive progress on hadronic matrix element computations from lattice QCD for V_{us} and V_{us}/V_{ud} extraction from Kaon decays
  - Radiative corrections from dispersive methods for V_{ud} extraction

2.2 $f_+(0)$ from lattice QCD

- Recent progress on Lattice QCD for determining $f_+(0)$



$$f_+(0)^{FLAG21}_{N_f=2+1+1} = 0.9698(17)$$

0.18% uncertainty

to be compared to

$$f_+(0)^{FLAG16}_{N_f=2+1+1} = 0.9704(32)$$

$$f_+(0)^{2010}_{N_f=2+1} = 0.959(50)$$

Uncertainty divided by ~2 w/ 2016 and by 25 w/ 2011!

Lattice uncertainties at the same level as exp.

-3.2σ away from unitarity!

2011: $V_{us} = 0.2254(5)_{exp}(11)_{lat}$ → $V_{us} = 0.2231(4)_{exp}(4)_{lat}$

V_{us}/V_{ud} from K_{l2}/π_{l2}

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\mathrm{EM}}-\frac{1}{2}\delta_{SU(2)}\right)$$

- Recent progress on radiative corrections computed on lattice:

  Di Carlo et al.'19

- Main input hadronic input: f_K/f_π

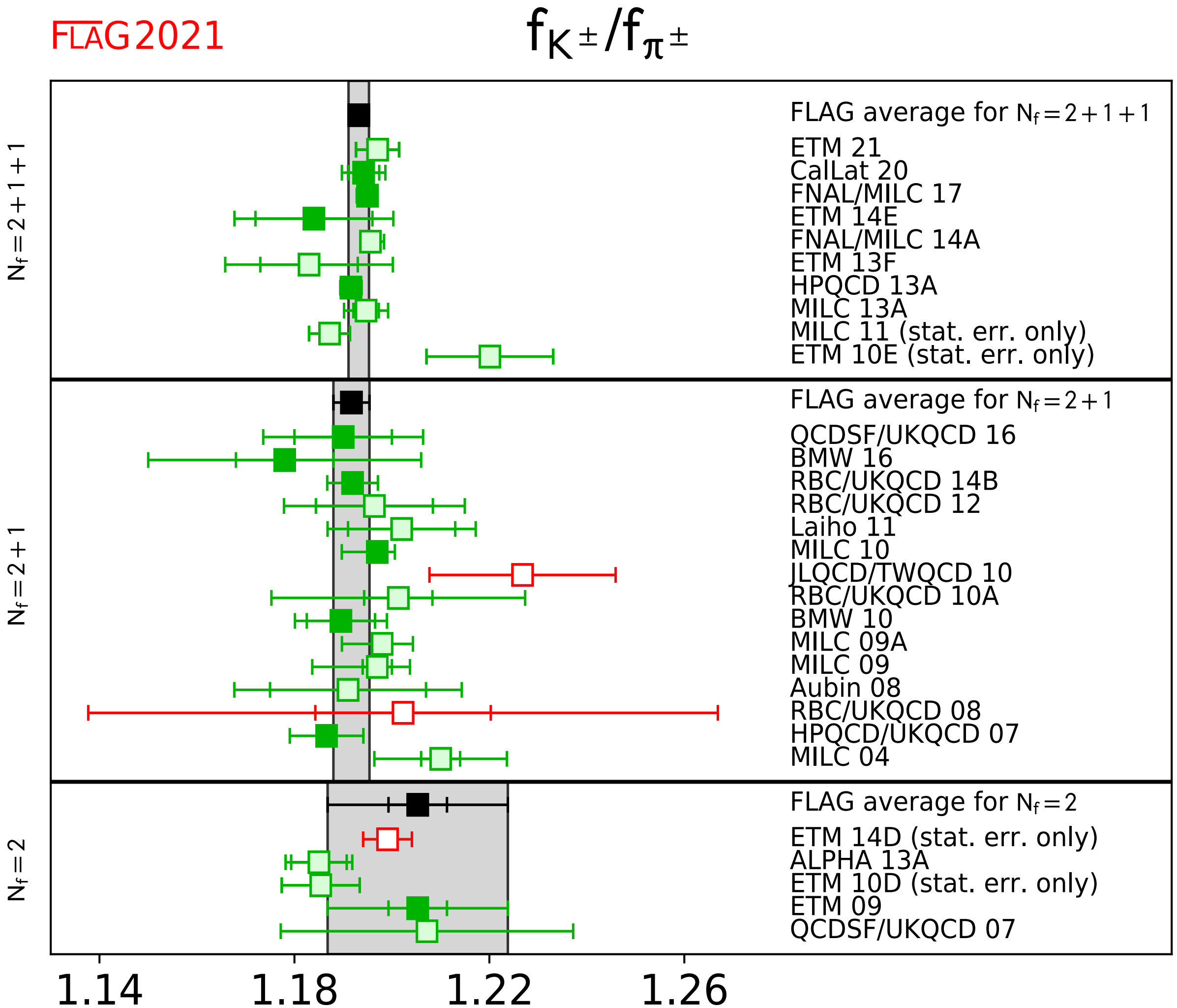
- In 2011: $V_{us}/V_{ud} = 0.2312(4)_{\text{exp}}(12)_{\text{lat}}$

- In 2021: $V_{us}/V_{ud} = 0.2311(3)_{\text{exp}}(4)_{\text{lat}}$ the lattice error is reducing by a factor of 3 compared to 2011! It is now of the same order as the experimental uncertainty.

  -1.8σ away from unitarity

2.2 f_K/f_π from lattice QCD

Progress since 2018: ➡ new results from *ETM'21* and *CalLat'20*



Now Lattice collaborations include SU(2) IB corr.
For N_f=2+1+1, FLAG2021

$$f_{K^+}/f_{\pi^+} = 1.1932(21)$$

0.18% uncertainty

Results have been stable over the years

For average substract IB corr.

$$f_K/f_\pi = 1.1967(18)$$

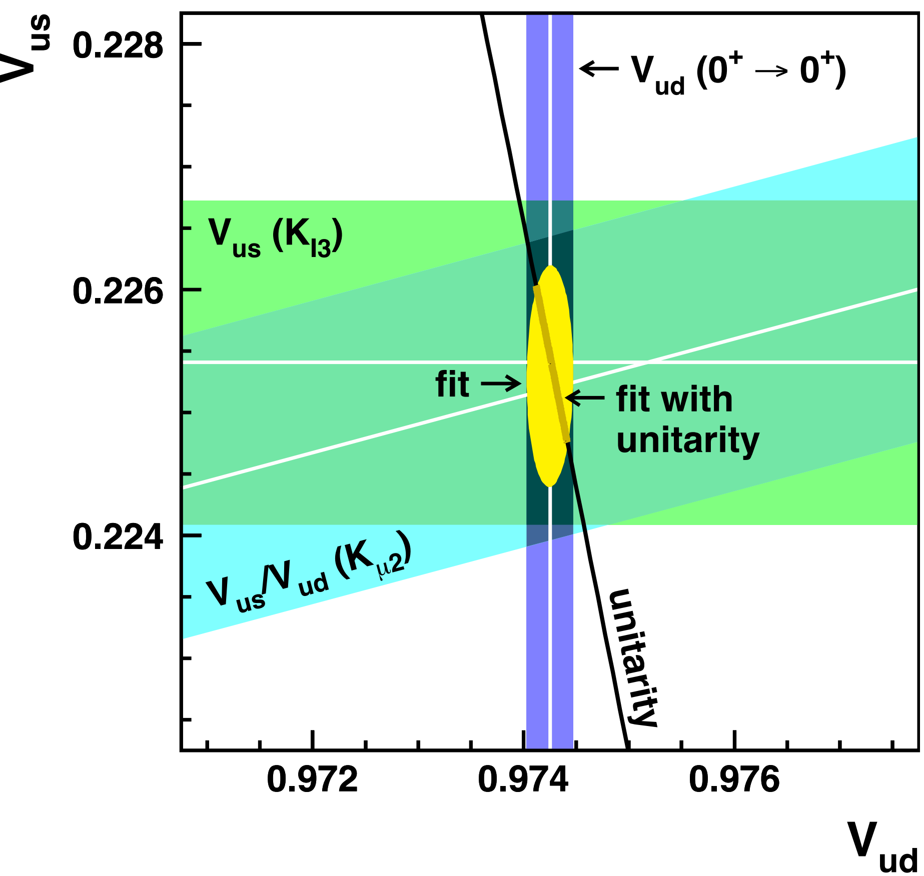
In 2011: $f_K/f_\pi = 1.193(6)$

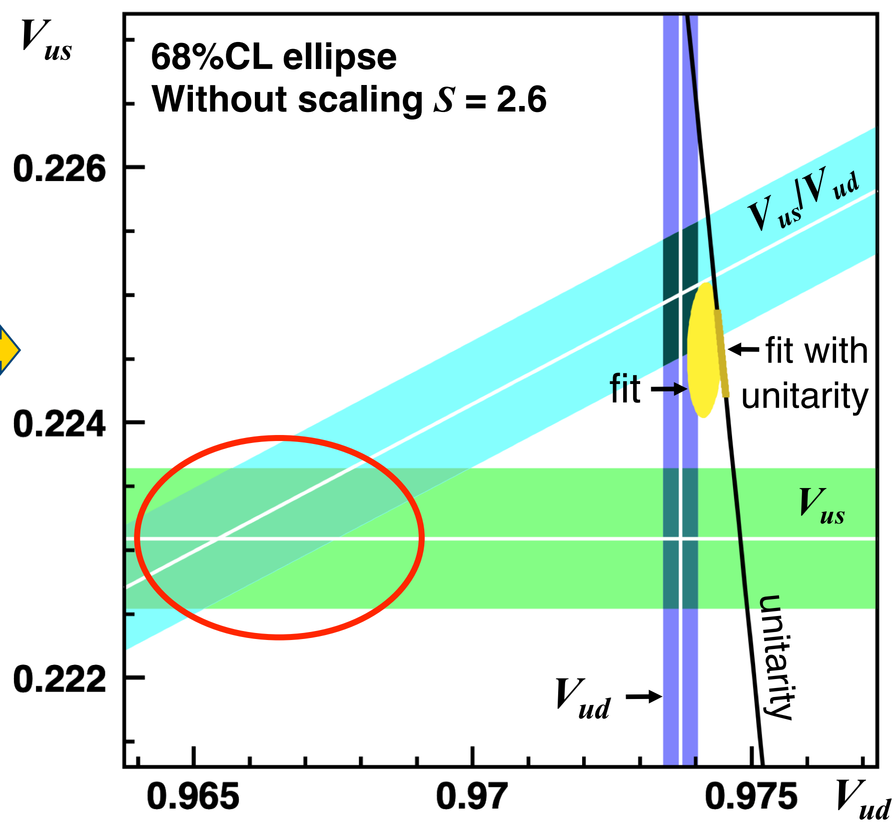
➡ $V_{us}/V_{ud} = 0.\ 23108(29)_{exp}(42)_{lat}$

Changes on V_{us} and V_{ud} since 2011

Flavianet Kaon WG: *Antonelli et al'11*

Moulson & E.P.@CKM2021



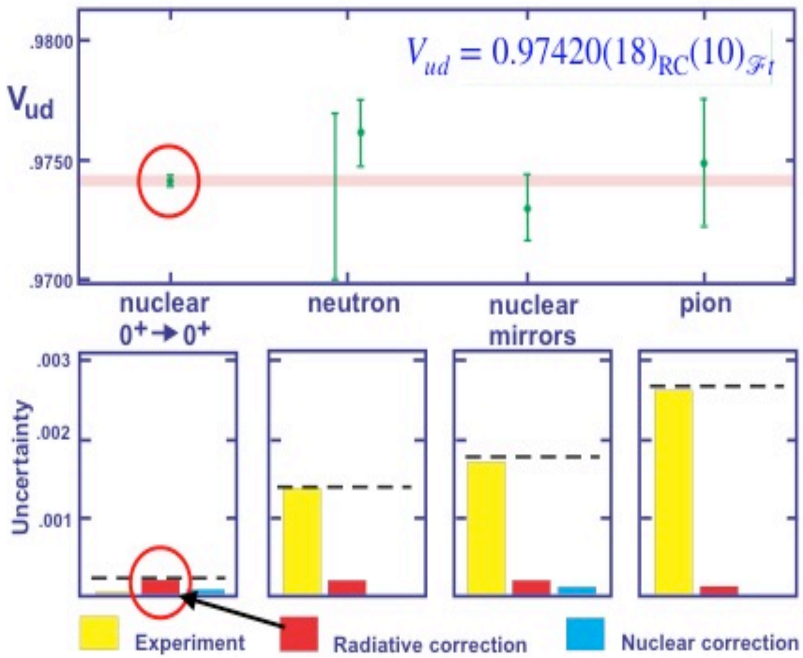


2.3 $|V_{ud}|$ from $0^+ \to 0^+$ superallowed β decays

See Talk by Misha Gorshteyn @CKM2021

PDG 2018:

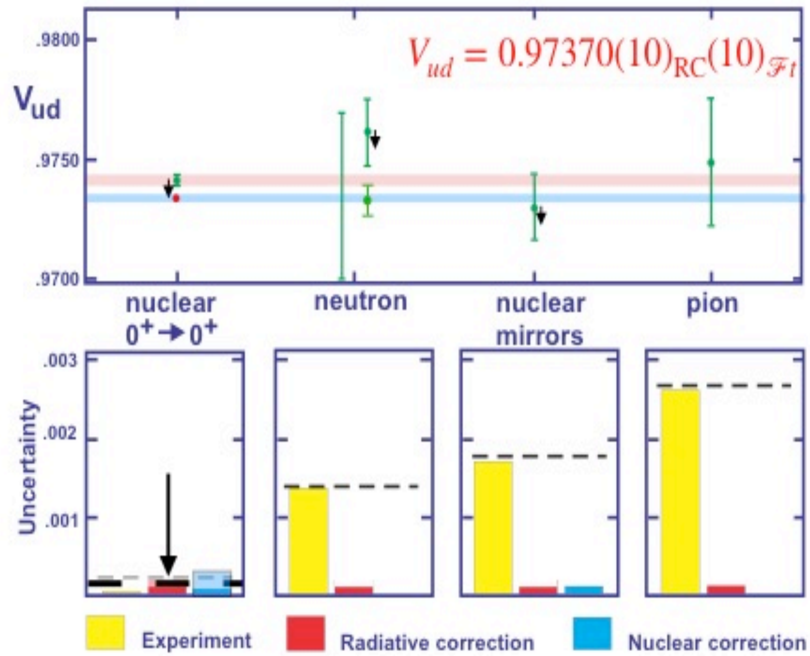
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(4)_{V_{ud}}(2)_{V_{us}}$$



PDG 2020:

Figure adapted from J. Hardy

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(3)_{V_{ud}}(4)_{V_{us}}$$



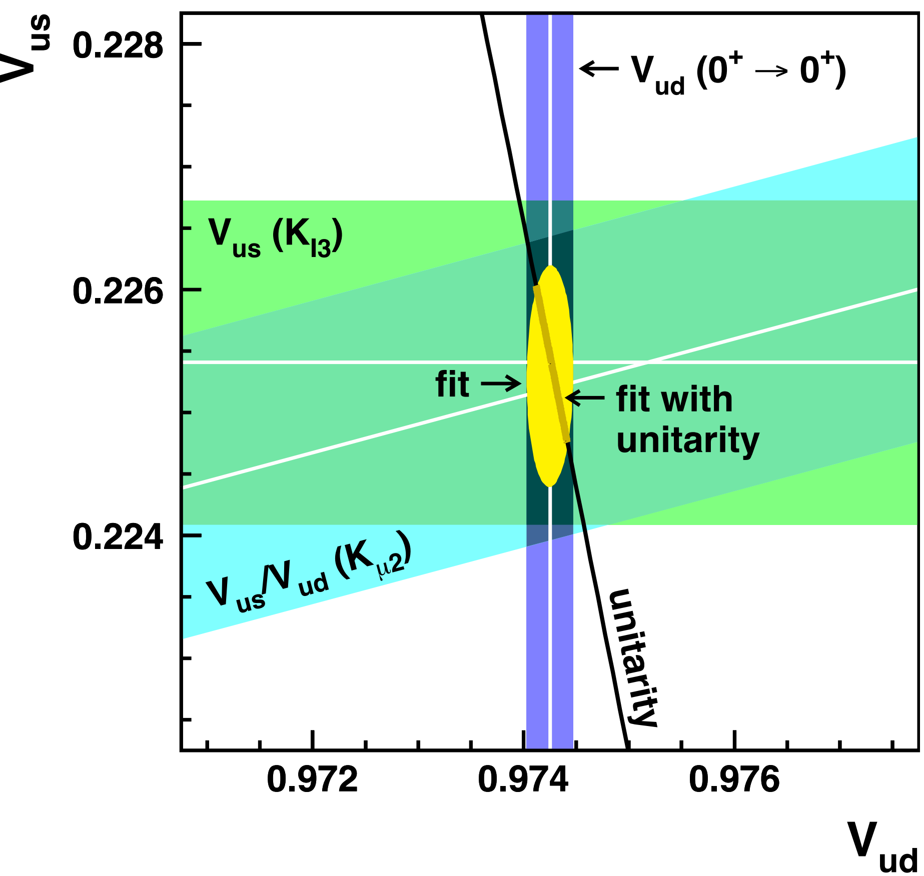
Recent improvement on the theoretical RCs +Nuclear Structure Corrections

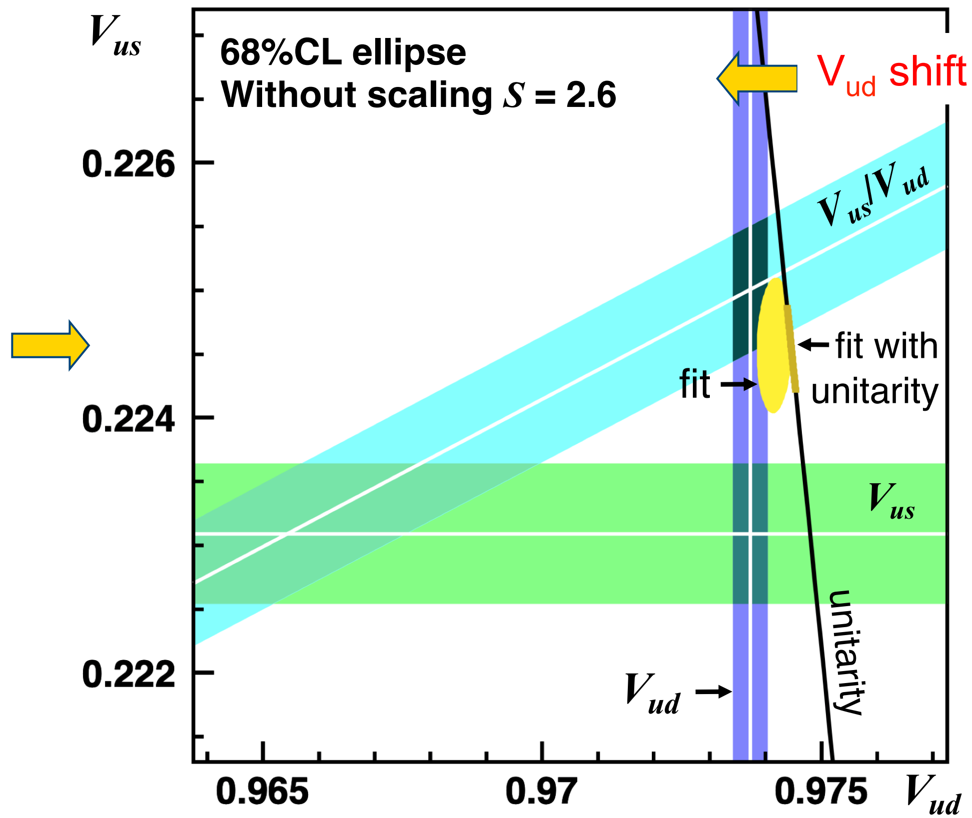
⇨ Use of a data driven dispersive approach *Seng et al.'18'19, Gorshteyn'18*

Changes on V_{us} and V_{ud} since 2011

Flavianet Kaon WG: *Antonelli et al'11*

Moulson & E.P.@CKM2021



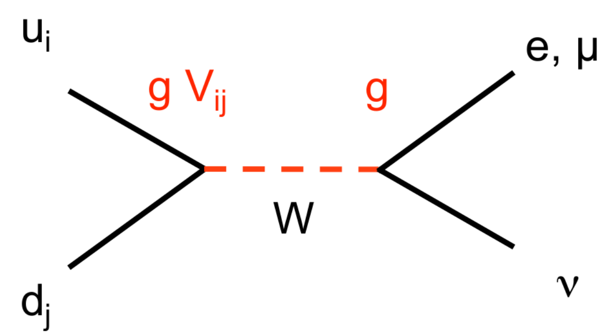


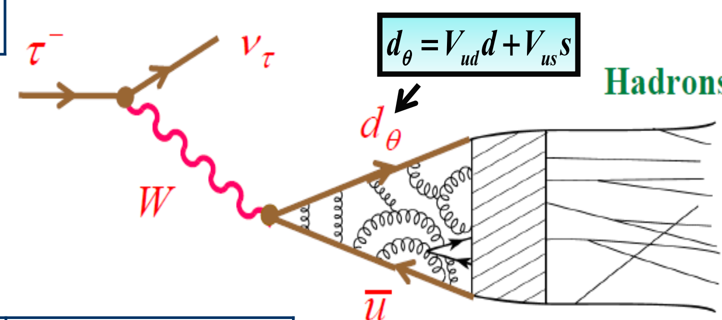
3. Prospects with studying Tau physics

3.1 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$





- From τ decays (crossed channel)

V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

3.2 Exclusive decays for V_{us}

- From τ decays (crossed channel)

V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

$$\boxed{\frac{\Gamma(\tau \to K\nu[\gamma])}{\Gamma(\tau \to \pi\nu[\gamma])} = \frac{\left(1-m_{K^\pm}^2/m_\tau^2\right)}{\left(1-m_{\pi^\pm}^2/m_\tau^2\right)} \frac{f_K^2}{f_\pi^2} \frac{|V_{us}|^2}{|V_{ud}|^2}\left(1+\delta_{\mathrm{LD}}\right)}$$

- Main input hadronic input: f_K/f_π as for Kaon physics

From Tau physics: $V_{us}/V_{ud} = 0.\,2289(18)_{exp}(4)_{lat}$ *HFLAV'23* *-2.1σ* away from unitarity

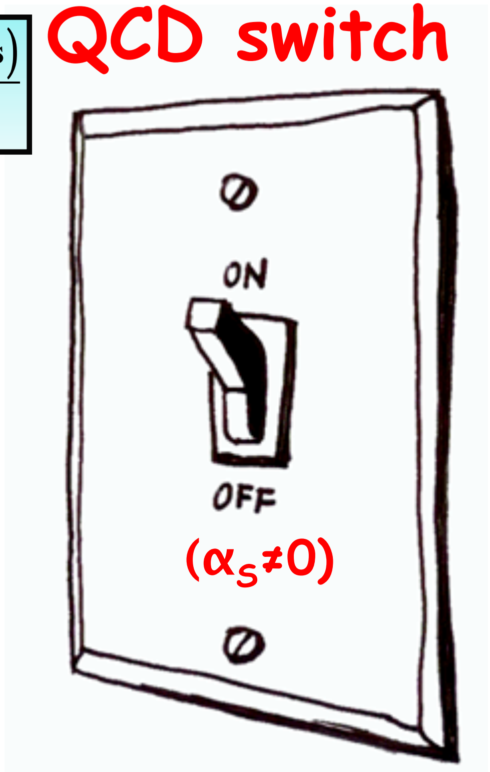
to be compared to $V_{us}/V_{ud} = 0.\,2311(3)_{exp}(4)_{lat}$ ⇨ Need important exp. improvement !

3.3 Inclusive determination of V_{us}

- With QCD on: $$\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)$$

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)}$$

QCD switch



- Use OPE: $$R_\tau^{NS}(m_\tau^2) = N_C\, S_{EW} |V_{ud}|^2 \left(1 + \delta_P + \delta_{NP}^{ud}\right)$$

$$R_\tau^{S}(m_\tau^2) = N_C\, S_{EW} |V_{us}|^2 \left(1 + \delta_P + \delta_{NP}^{us}\right)$$

- $$\delta R_\tau \equiv \frac{R_{\tau,NS}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}$$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0238(33)$$ *Gamiz et al'07, Maltman'11*

$$|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

HFLAV'23

$R_{\tau,S} = 0.1615(28)$

$R_{\tau,NS} = 3.4650(84)$

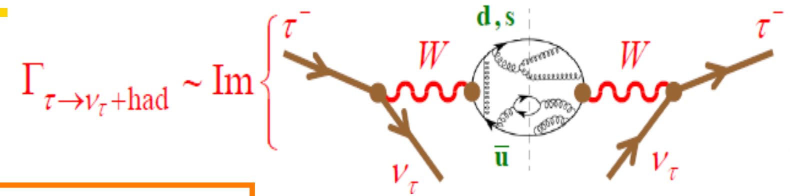
$|V_{ud}| = 0.97373(31)$

$\Rightarrow$ $$|V_{us}| = 0.2184 \pm 0.0018_{\text{exp}} \pm 0.0010_{\text{th}}$$

-3.7σ away from unitarity!

A. Lusiani@Tau'23

Calculation of the QCD corrections



- Calculation of R_τ:

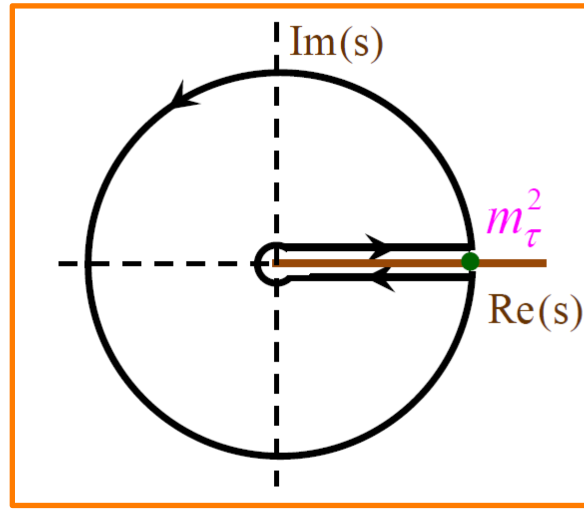
$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s+i\varepsilon) + \mathrm{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

Braaten, Narison, Pich'92

- Analyticity: Π is analytic in the entire complex plane except for s real positive

⇨ Cauchy Theorem

$$R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right]$$



- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} C^{(J)}(s,\mu)\, \langle O_D(\mu)\rangle$$

Wilson coefficients

Operators

μ: separation scale between short and long distances

Operator Product Expansion

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} \mathrm{C}^{(J)}(s,\mu)\, \langle O_D(\mu) \rangle$$

Wilson coefficients

Operators

μ separation scale between short and long distances

- D=0: Perturbative contributions
- D=2: Quark mass corrections
- D=4: Non perturbative physics operators, $\left\langle \frac{\alpha_s}{\pi} GG \right\rangle$, $\left\langle m_j \bar{q}_i q_i \right\rangle$
- D=6: 4 quarks operators, $\left\langle \bar{q}_i \Gamma_1 q_j \bar{q}_j \Gamma_2 q_i \right\rangle$
- D≥8: Neglected terms, supposed to be small…

$\Rightarrow$ $$R_{\tau,V}(s_0) = \frac{3}{2} \left|V^{ud}\right|^2 S_{EW} \left(1 + \delta^{(0)} + \sum_{D=2,4..} \delta^{(D)}_{ud,V} \right)$$ similar for $R_{\tau,A}(s_0)$ and $R_{\tau,S}(s_0)$

Calculation of $\delta R_{\tau,\mathrm{theo}}$

$$\boxed{\delta R_\tau \equiv \frac{R_{\tau,V+A}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}} \approx N_C S_{EW} \sum_{D\geq 2} \left[\delta_{ud}^{(D)} - \delta_{us}^{(D)} \right]$$

- $\delta_{ij}^{(2)}$: quark mass corrections, *neglected* for R_τ^{NS} $(\propto m_u, m_d)$ but not for R_τ^S $(\propto m_s)$

  known up to $O(\alpha_S^3)$ for both J=L and J=L+T

  Chetyrkin, Gorishry, Kataev, Larin, Sugurladze; Baikov, Chetyrkin,Kuehn
  Becchi, Narison, de Rafael; Bernreuther, Wetzel

- $\delta_{ij}^{(4)}$: fully included , e.g m_j^4 / m_τ^4 $\langle m_j \bar{q}_i q_i \rangle / m_\tau^4$
- $\delta_{ij}^{(6)}$: estimated (VSA) to be of order or smaller than errors on D=4
- D ≥ 8 : Neglected terms, expected to be small…

$$\Rightarrow \boxed{\delta R_\tau \approx 24 \frac{m_s^2(m_\tau^2)}{m_\tau^2} \Delta(\alpha_S)}$$

Results

$$|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,V+A}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

- $\delta R_{\tau,theo}$ determined from OPE (L+T) + phenomenology

$$\Rightarrow \quad \delta R_{\tau,th} = \underbrace{(0.1544 \pm 0.0037)}_{J=0} + (9.3 \pm 3.4)\, m_s^2 + (0.0034 \pm 0.0028)$$

Gamiz, Jamin, Pich, Prades, Schwab'07, Maltman'11

Input : m_s $\Rightarrow$ $m_s\left(2\text{ GeV}, \overline{\text{MS}}\right) = 93.9 \pm 1.1$ N_f=2+1+1 lattice average

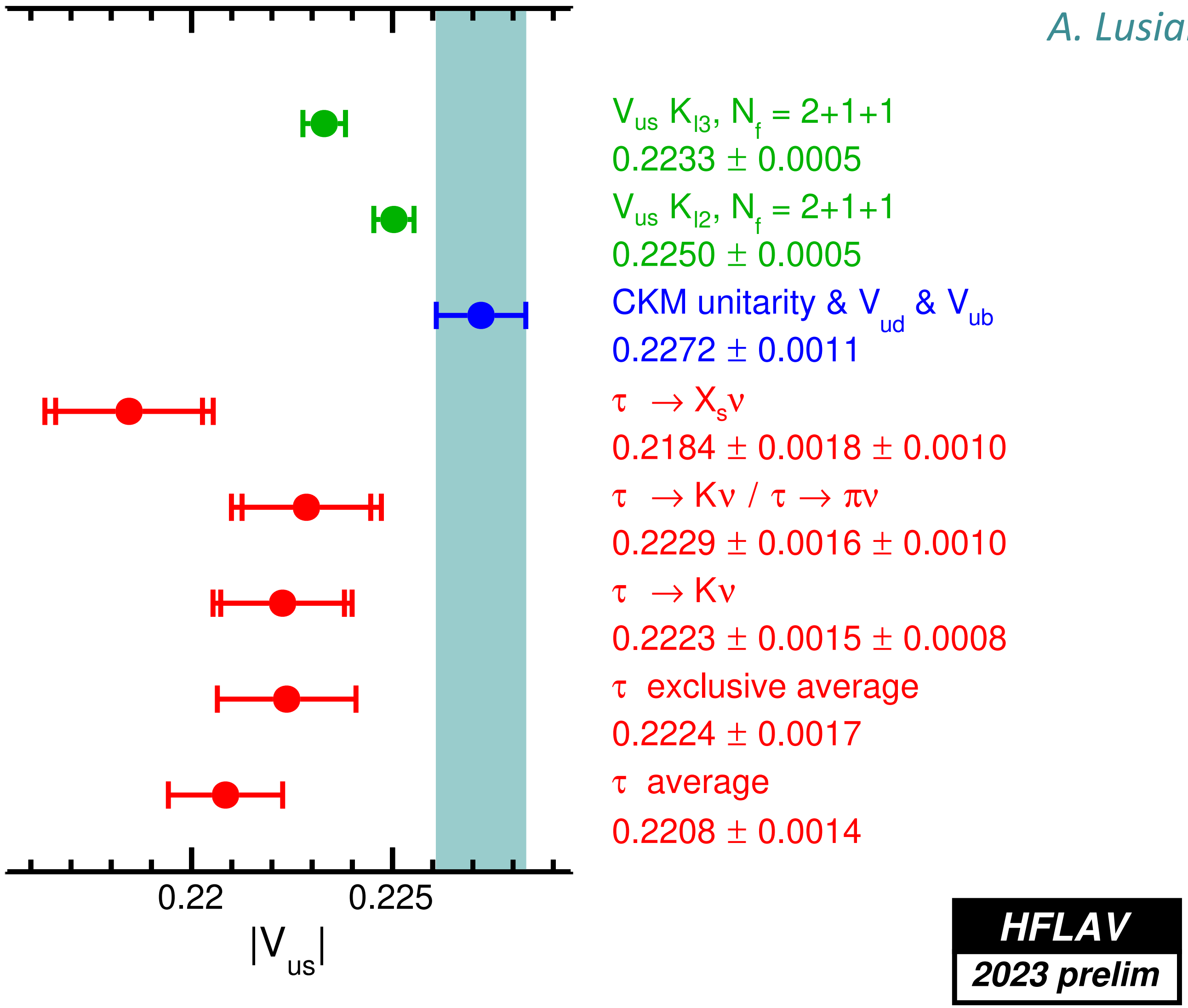
FLAG'21

- Tau data : $R_{\tau,S} = 0.1615(28)$ and $R_{\tau,NS} = 3.4650(84)$ *HFLAV'23*

- V_{ud} : $|V_{ud}| = 0.97373(31)$ *Towner & Hardy'08*

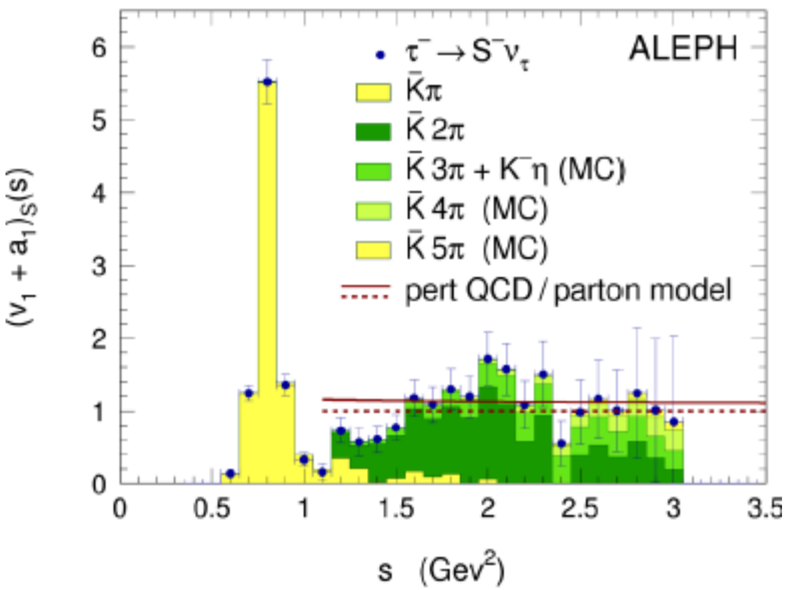
3.4 V_{us}: summary

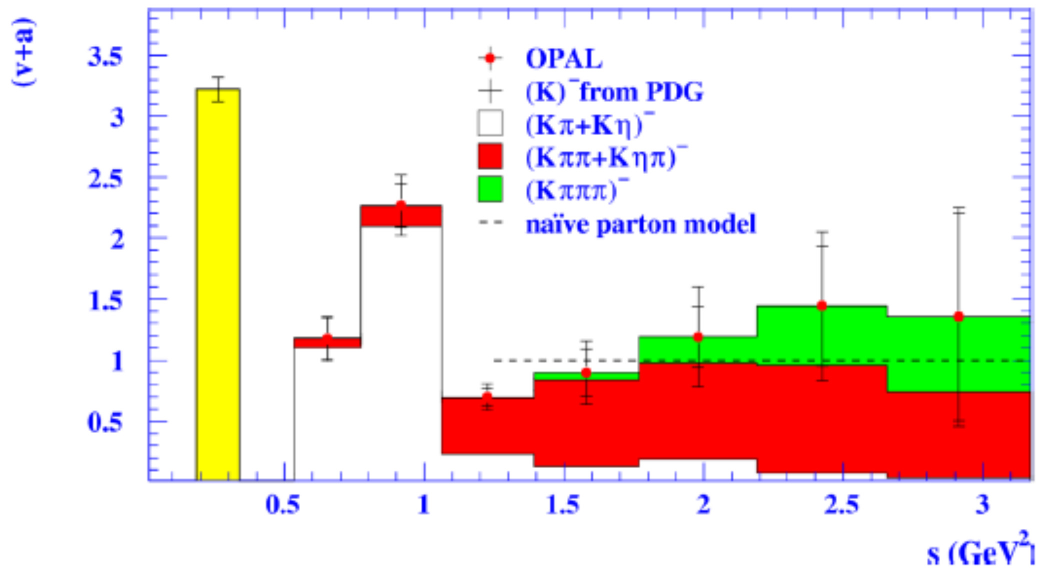
A. Lusiani@Tau'23



3.5 Prospects : τ strange Spectral functions

- Experimental measurements of the strange spectral functions not very precise





⇨ New measurements are needed !

- Before B-factories
- With B-factories new measurements :

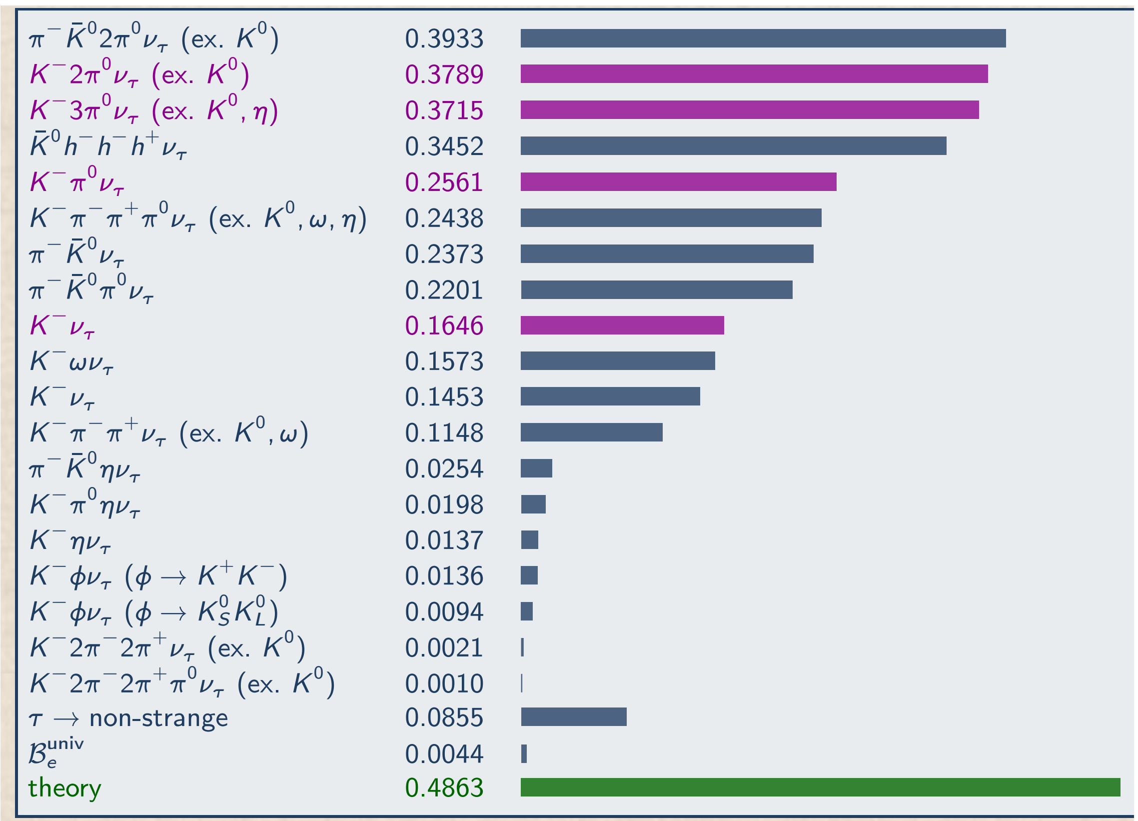
Smaller $\tau \to$ K branching ratios ⇨ smaller $R_{\tau,S}$ ⇨ smaller $\boxed{V_{us}}$

$$\boxed{R_\tau^S\big|_{\text{old}} = 0.1686(47)} \Rightarrow \boxed{R_\tau^S\big|_{\text{new}} = 0.1615(28)}$$

$$\boxed{|V_{us}|_{\text{old}} = 0.2214 \pm 0.0031_{\text{exp}} \pm 0.0010_{\text{th}}} \Rightarrow \boxed{|V_{us}|_{\text{new}} = 0.2176 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}}$$

3.5 Prospects : τ strange BRs

- Very interesting quantity to extract V_{us}: QCD part completely independent from form factors or decay constants ⇒ Use OPE
- Experimentally very challending since all Brs need to be measured



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Belle II
50 ab^{-1}
and ~4.6 x 10^{10} τ pairs
STCF?

4. New Physics Interpretations

4.1 Right-handed currents

Bernard, Oertel, E.P., Stern'08

$$\mathcal{L}_{\mathrm{W}} = \frac{e(1-\xi^2\rho_L)}{\sqrt{2}s}\left\{\bar{\mathrm{N}}_L V_{\mathrm{MNS}}\gamma^\mu L_L + (1+\delta)\bar{\mathrm{U}}_L V_L \gamma^\mu D_L + \epsilon \bar{\mathrm{U}}_R V_R \gamma^\mu D_R\right\} W_\mu^+ + \mathrm{h.c}\ .$$

- See also *Antonelli et al.'09*

  Alioli, Cirigliano, Dekens, de Vries, Mereghetti'17

  T. Kitahara@HC2NP 2019

4.1 Right-handed Currents

$$V_{us}^{K_{l3}} = \left|\sin\theta_C + \varepsilon_s\right| \,, \longleftarrow \boxed{\text{Vector s quark}}$$

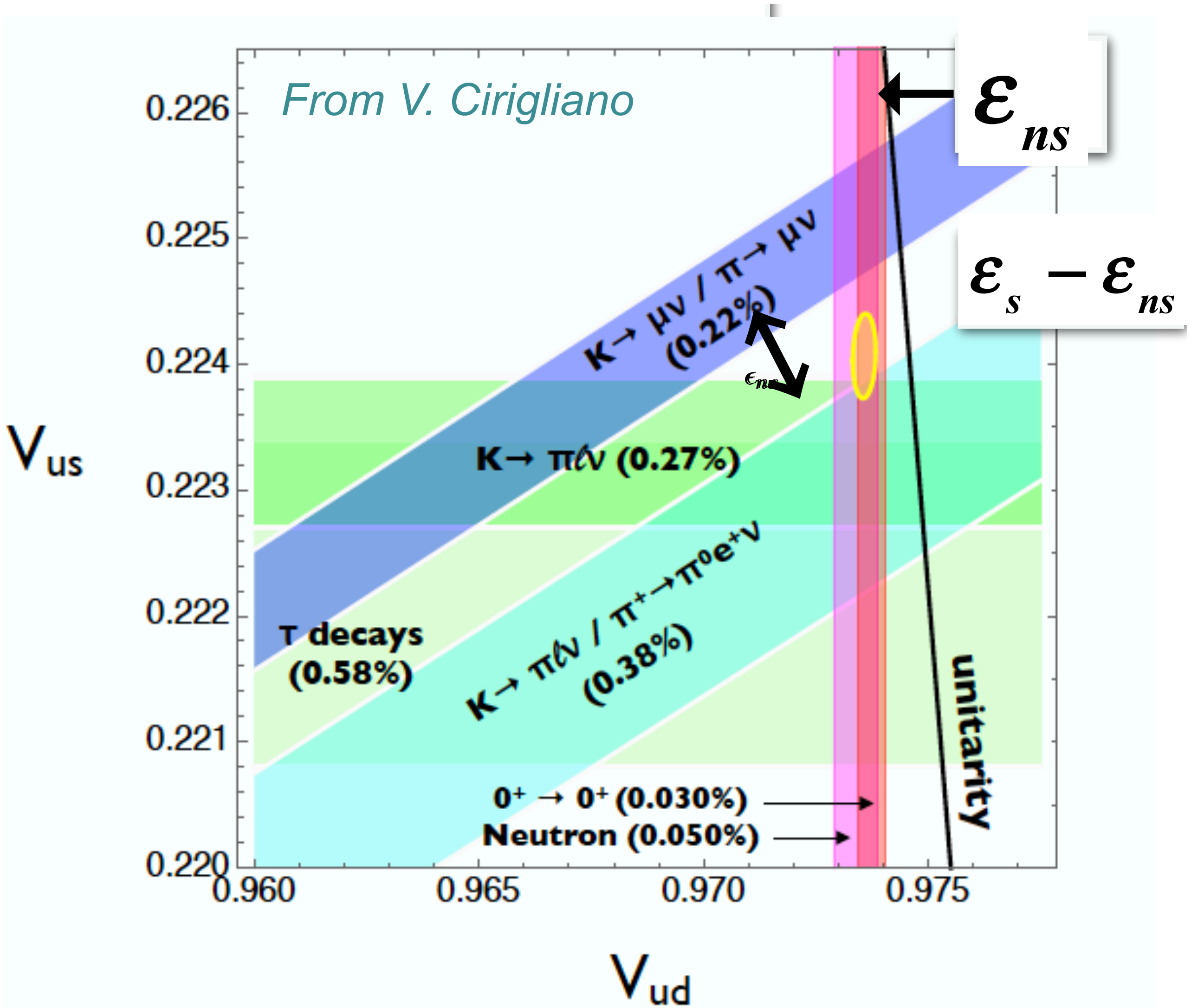
$$\left(\frac{V_{us}}{V_{ud}}\right)^{K_{l2}} = \left|\frac{\sin\theta_C - \varepsilon_s}{\cos\theta_C - \varepsilon_{ns}}\right| \,, \longleftarrow \boxed{\text{Axial}}$$

$$V_{ud}^{\beta} = \left|\cos\theta_C + \varepsilon_{ns}\right| \,. \longleftarrow \boxed{\text{Vector no s quark}}$$

$$\left(\frac{V_{us}}{V_{ud}}\right)^{K_{\ell 3}} = \left|\frac{\sin\,\theta_{\rm C} + \epsilon_{\rm s}}{\cos\,\theta_{\rm C} + \epsilon_{\rm ns}}\right| \longleftarrow \boxed{\text{Vector}}$$

- The SM is obtained in the limit $\varepsilon_s = \varepsilon_{ns} = 0$.
- Perfect fit to data $\boxed{\chi^2_{\rm min,RH} = 0}$
- Not obvious how to define CKM unitarity test in this case

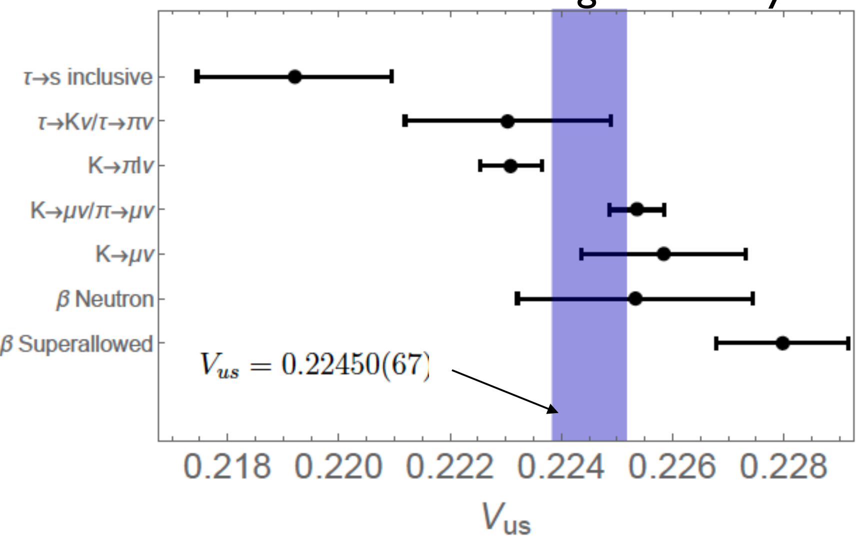
4.1 Right-handed Currents



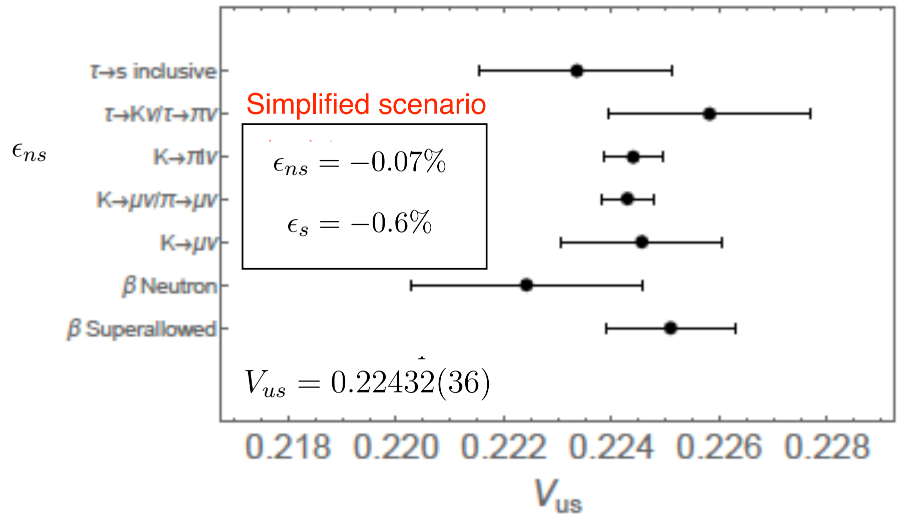
4.1 Right-handed Currents

- Global fit to CC processes involving light quarks and all lepton families
- SM hypothesis ($\varepsilon_s = \varepsilon_{ns} = 0$) disfavored (p-value 0.3%)

SM limit: Cabibbo angle anomaly



Anomaly removed by turning on the ε_R couplings



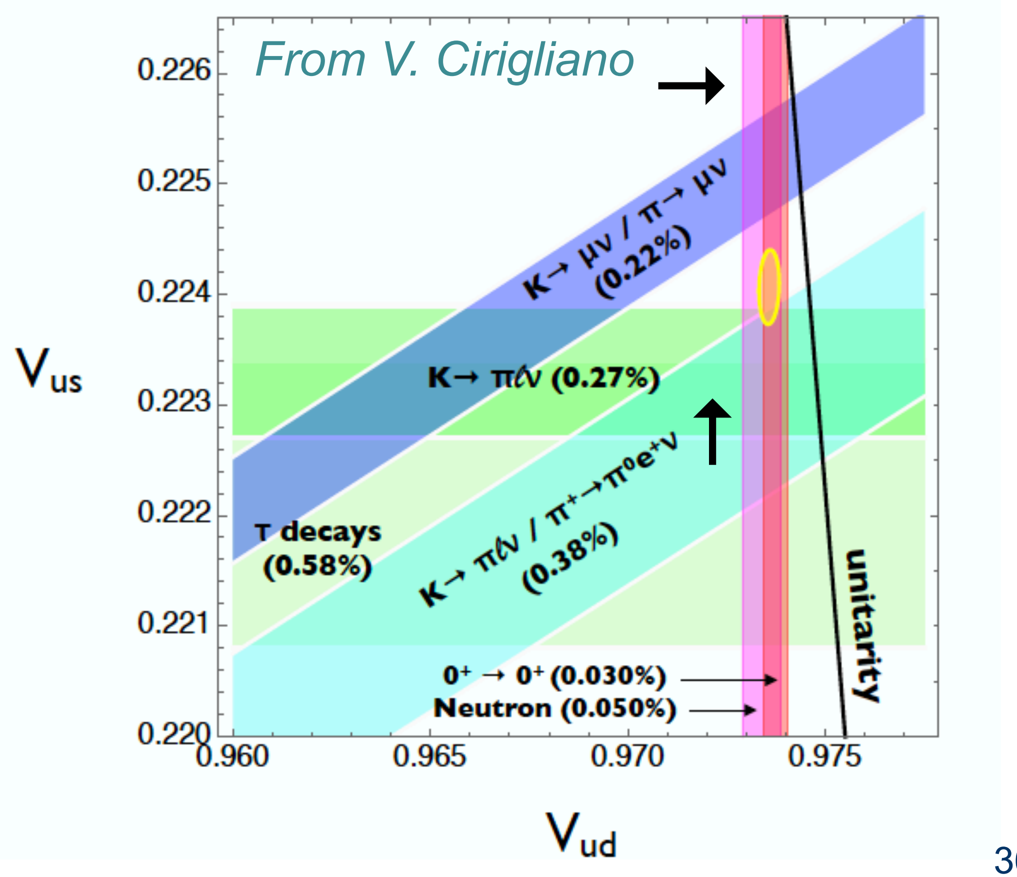
Cirigliano, Diaz-Calderon, Falkowski, Gonzalez-Alonso, Rodriguez-Sanchez'21

4.2 Other New Physics Models

- 4th quark b' *Belfatto, Beradze, Berezhiani'19*
- Gauge horizontal family symmetry
- Turn on only vertex corrections to leptons *Crivellin & Hoferichter'21*

⇒ Shift the location of the $V_{ud,us}$ bands but do not solve the tension between ratios

And many more….



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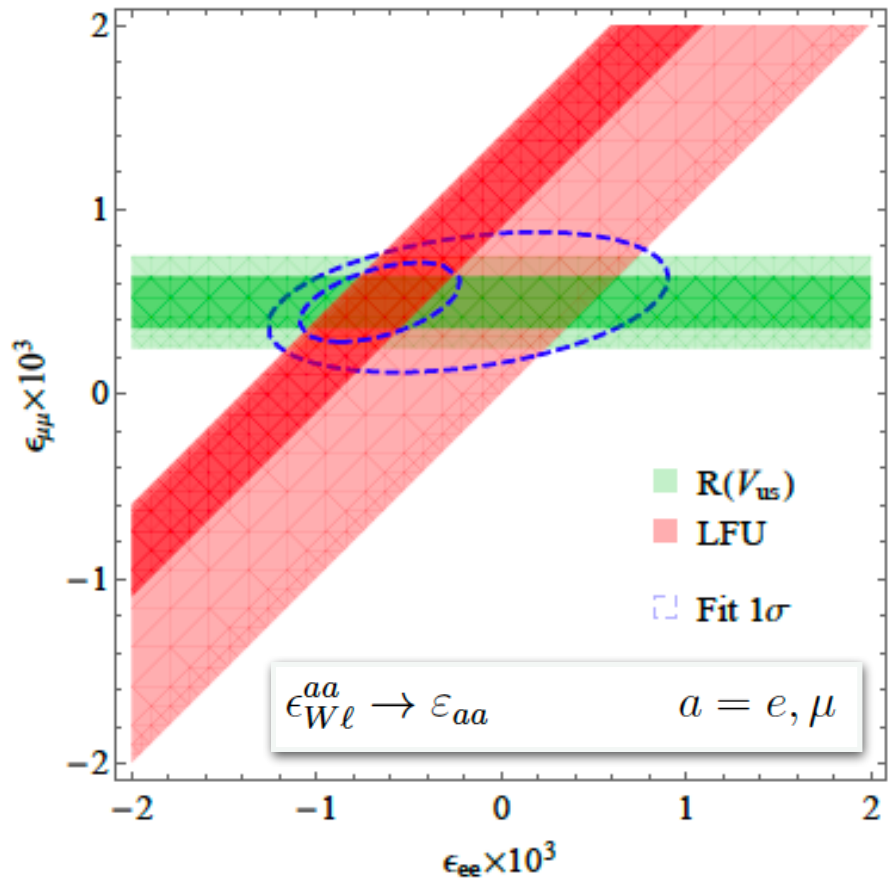


Connection with π→eν/ π→μν

$$r_\pi = 1 + 2\left(\epsilon^{ee}_{W\ell} - \epsilon^{\mu\mu}_{W\ell}\right)$$

(and other LFU probes)

And many more….

5. Conclusion and Outlook

Conclusion and Outlook

- Recent precision determinations of V_{us} and V_{ud} enable unprecedented tests of the SM and constraints on possible NP models
- Tensions in unitarity of 1st row of CKM matrix have reappeared!
- We need to work hard to understand where they come from:
  - On experimental side:
    For V_{us} we have a unique opportunity measuring hadronic tau decays from *Belle II* and *STCFs?* especially for inclusive Tau decays
  - From hyperon decays: Ex: $\boldsymbol{\Lambda \rightarrow p e \nu_e}$?
  - On theory side:
    Calculate very precisely radiative corrections, isospin breaking effects and matrix elements
    Be sure the uncertainties are under control
  - If these tensions are confirmed ⇨ what do they tell us?
- Interesting time ahead of us!

6. Back-up

3.2 Theoretical Prospects for V_{us}

- Lattice Progress on hadronic matrix elements: decay constants, FFs

- Full QCD**+**QED decay rate on the lattice,for **Leptonic decays of kaons and pions** ⇨ Inclusion of EM and IB corrections :
  - Perturbative treatment of QED on lattice established
  - Formalism for $\boldsymbol{K_{l2}}$ worked out

- Application of the method for **semileptonic Kaon** ($\mathbf{K_{l3}}$) **and Baryon decays**

⇨ **Aim: Per mille level within 10 years**

3.2 Exclusive decays for V_{us}

On Kaon side

Cirigliano et al'22

- *NA62* could measure several BRs: $K_{\mu3}/K_{\mu2}$, $K \to 3\pi$, $K_{\mu2}/K \to \pi\pi$
- Note that the high precision measurement of BR($K_{\mu2}$) (0.3%) comes only from a single experiment: KLOE. It would be good to have another measurement at the same level of accuracy

- *LHCb* : could measure BR($K_S \to \pi\mu\nu$) at the < 1% level?
  $K_S \to \pi\mu\nu$ measured by KLOE-II but not competitive
  τ_S known to 0.04% (vs 0.41% for τ_L, 0.12% for $\tau_\pm$)

- V_{us} from Tau decays at *Belle II*:

  Belle II with 50 ab^{-1} and ~4.6 x 10^{10} τ pairs will improve V_{us} extraction from τ decays
  Inclusive measurement is an opportunity to have a complete independent extraction of V_{us} ⇒ not easy as you have to measure many channels

⇒ $$\boxed{|V_{us}| = 0.2184 \pm 0.0018_{exp} \pm 0.0011_{th}}$$

To be competitive theory error will have to be improved as well

HFLAV'21

V_{us} from Hyperon decays

V_{us} can be measured from Hyperon decays:

- **$\Lambda \to p e \nu_e$** Possible measurement at *BESIII, Super τ-Charm factory?*

- Possibilities at *LHCb*?

Talk by Dettori@FPCP20

Channel	$\mathcal{R}$	ϵ_L	ϵ_D	$\sigma_L(\mathrm{MeV}/c^2)$	$\sigma_D(\mathrm{MeV}/c^2)$
$K^0_S \to \mu^+\mu^-$	1	1.0 (1.0)	1.8 (1.8)	~ 3.0	~ 8.0
$K^0_S \to \pi^+\pi^-$	1	1.1 (0.30)	1.9 (0.91)	~ 2.5	~ 7.0
$K^0_S \to \pi^0\mu^+\mu^-$	1	0.93 (0.93)	1.5 (1.5)	~ 35	~ 45
$K^0_S \to \gamma\mu^+\mu^-$	1	0.85 (0.85)	1.4 (1.4)	~ 60	~ 60
$K^0_S \to \mu^+\mu^-\mu^+\mu^-$	1	0.37 (0.37)	1.1 (1.1)	~ 1.0	~ 6.0
$K^0_L \to \mu^+\mu^-$	~ 1	2.7 (2.7) $\times 10^{-3}$	0.014 (0.014)	~ 3.0	~ 7.0
$K^+ \to \pi^+\pi^+\pi^-$	~ 2	9.0 (0.75) $\times 10^{-3}$	41 (8.6) $\times 10^{-3}$	~ 1.0	~ 4.0
$K^+ \to \pi^+\mu^+\mu^-$	~ 2	6.3 (2.3) $\times 10^{-3}$	0.030 (0.014)	~ 1.5	~ 4.5
$\Sigma^+ \to p\mu^+\mu^-$	~ 0.13	0.28 (0.28)	0.64 (0.64)	~ 1.0	~ 3.0
$\Lambda \to p\pi^-$	~ 0.45	0.41 (0.075)	1.3 (0.39)	~ 1.5	~ 5.0
$\Lambda \to p\mu^-\bar{\nu}_\mu$	~ 0.45	0.32 (0.31)	0.88 (0.86)	–	–
$\Xi^- \to \Lambda\mu^-\bar{\nu}_\mu$	~ 0.04	39 (5.7) $\times 10^{-3}$	0.27 (0.09)	–	–
$\Xi^- \to \Sigma^0\mu^-\bar{\nu}_\mu$	~ 0.03	24 (4.9) $\times 10^{-3}$	0.21 (0.068)	–	–
$\Xi^- \to p\pi^-\pi^-$	~ 0.03	0.41(0.05)	0.94 (0.20)	~ 3.0	~ 9.0
$\Xi^0 \to p\pi^-$	~ 0.03	1.0 (0.48)	2.0 (1.3)	~ 5.0	~ 10
$\Omega^- \to \Lambda\pi^-$	~ 0.001	95 (6.7) $\times 10^{-3}$	0.32 (0.10)	~ 7.0	~ 20

R = ratio of production

ϵ = ratio of efficiencies

- To be able to extract V_{us} one needs to compute form factors precisely

  ⇨ Lattice effort from *RBC/UKQCD*

3.3 Prospects for $|V_{ud}|$

See Talk by Misha Gorshteyn @CKM2021

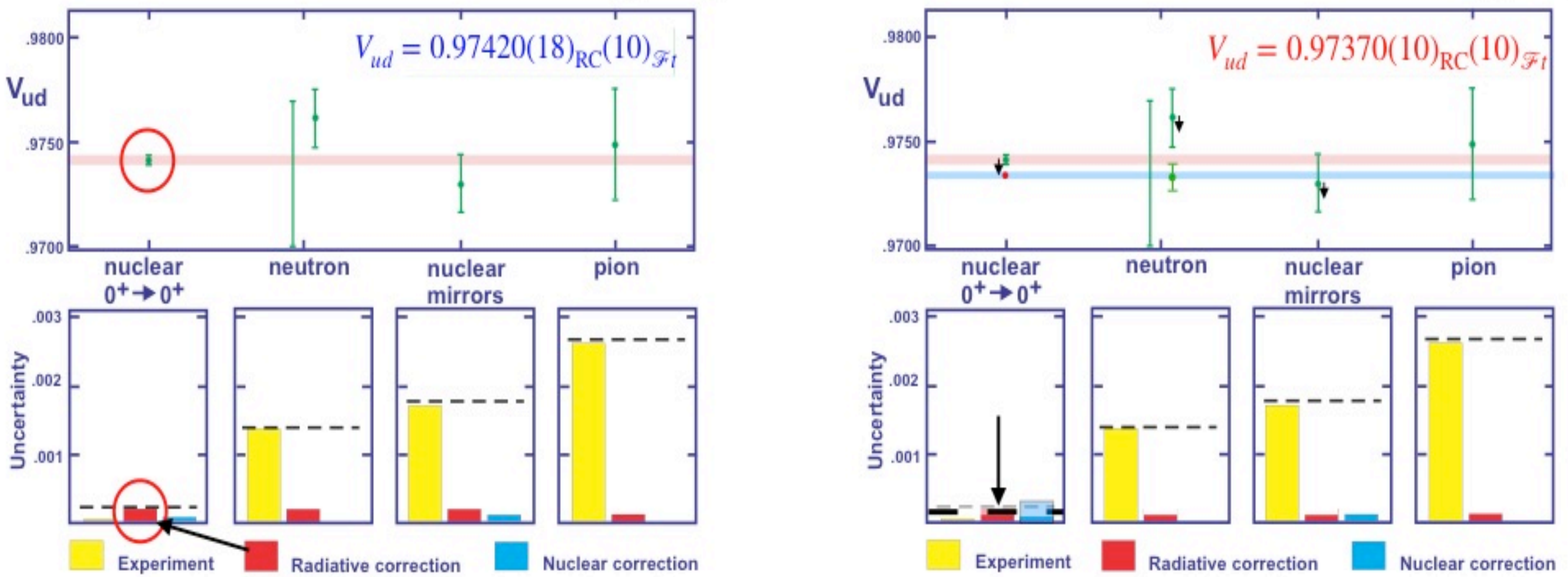
PDG 2018:

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9994(4)_{V_{ud}}(2)_{V_{us}}$$

PDG 2020:

Figure adapted from J. Hardy

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9985(3)_{V_{ud}}(4)_{V_{us}}$$



- From neutron decays : very impressive progress recently
- From pion β decay $\pi^+ \to \pi^0 e^+ \nu$: PIONEER experiment

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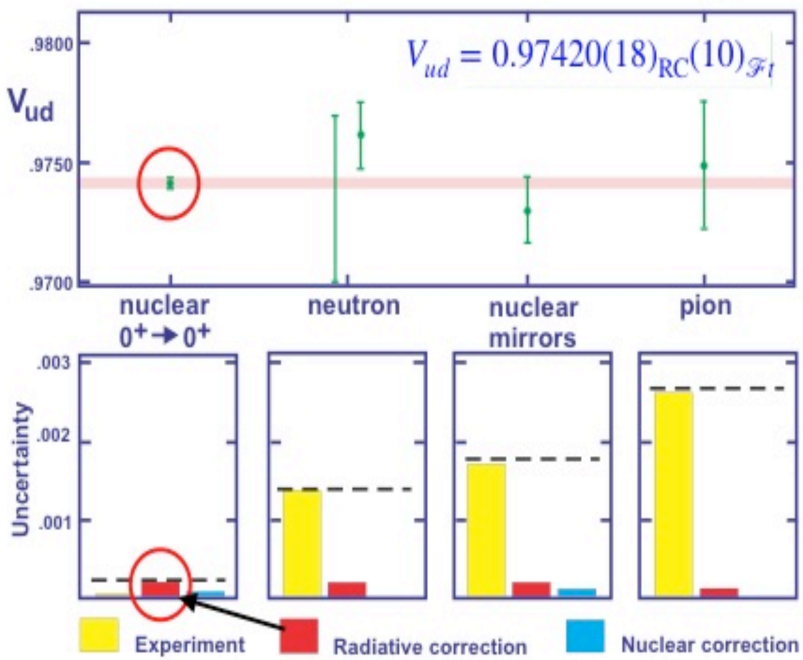

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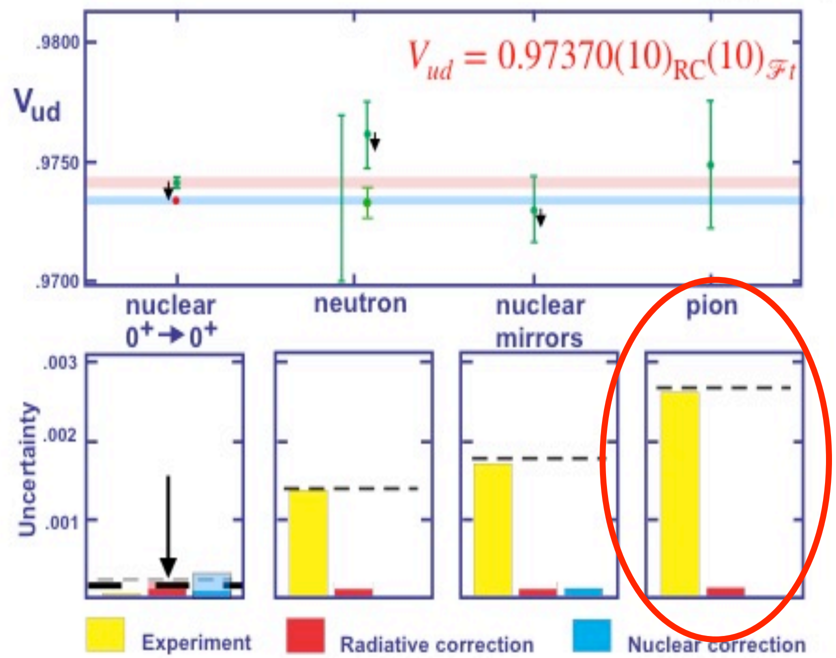


- From neutron decays
- From pion β decay $\pi^+\to\pi^0e^+\nu$: PIONEER experiment
  ⇨ (Phase-I) approved at PSI, physics starting in ~2029

$|V_{ud}|$ from pion β decay: $\pi^+ \to \pi^0 e^+ \nu$

- Theoretically cleanest method to extract V_{ud} : corrections computed in SU(2) ChPT

  Sirlin'78, Cirigliano et al.'03, Passera et al'11

- Present result: *PIBETA* Experiment (2004) → **Uncertainty: 0.64%**

$$B(\pi^+ \to \pi^0 e^+ \nu) = (1.036 \pm 0.004_{\rm stat} \pm 0.004_{\rm syst} \pm 0.003_{\pi e2})\ \mathrm{x}10^{-8} \left(\pm 0.6\%\right)$$

⇨ $\boxed{|V_{ud}| = \mathbf{0.9739(28)_{exp}(1)_{th}}}$ to be compared to $|V_{ud}| = \mathbf{0.97373(31)}$

- Reduction of the theory error thanks to a new lattice calculation for RC *Feng et al'20*

- Next generation experiment PIONEER Phase II and III measurement at 0.02% level ⇨ will be competitive with current $0^+ \to 0^+$ extraction

- Would be completely independent check! No nuclear correction and different RCs compared to neutron decay

  Czarnecki, Marciano, Sirlin'20

- **Opportunity to extract V_{us}/V_{ud}** from $\dfrac{B(K \to \pi l \nu)}{B(\pi^+ \to \pi^0 e^+ \nu)}$ EW Rad. Corr. cancel

Improve precision on $B(\pi^+ \to \pi^0 e^+ \nu)$ by x3 ⇨ $V_{us}/V_{ud} < \pm 0.2\%$

Pion decays and LFU tests

- Lepton Flavor Universality test in $R^{theory}_{e/\mu} = \dfrac{\Gamma(\pi \to e\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))}$
    - Early insight into the **V−A** structure of weak interactions
    - Exceptional precision of the SM prediction using ChPT

$$R_{e/\mu}(\mathrm{SM}) = 1.23524(015) \times 10^{-4}$$

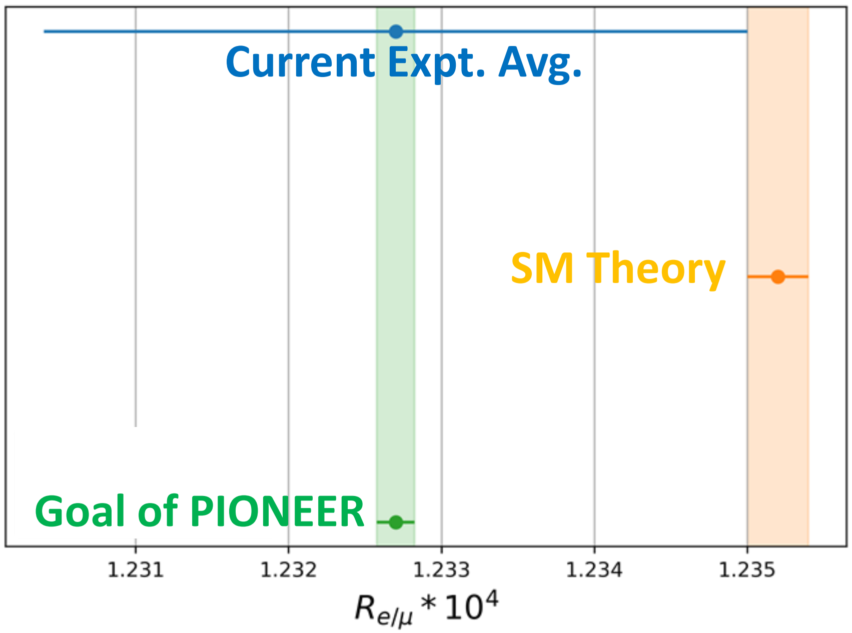
Cirigliano & Rosell'07

- World average (mainly *PIENU* at TRIUMF):

$$R_{e/\mu}(\mathrm{Exp}) = 1.23270(230) \times 10^{-4}$$

15 times worse than theory!

(dominated by PIENU expt.)



$$\frac{g_e}{g_\mu} = 0.9990 \pm 0.0009 \quad (\pm 0.09\%)$$

Goal of PIONEER: reduce unc. by a factor of 10 ! ⇒ by far most precise test of LFU

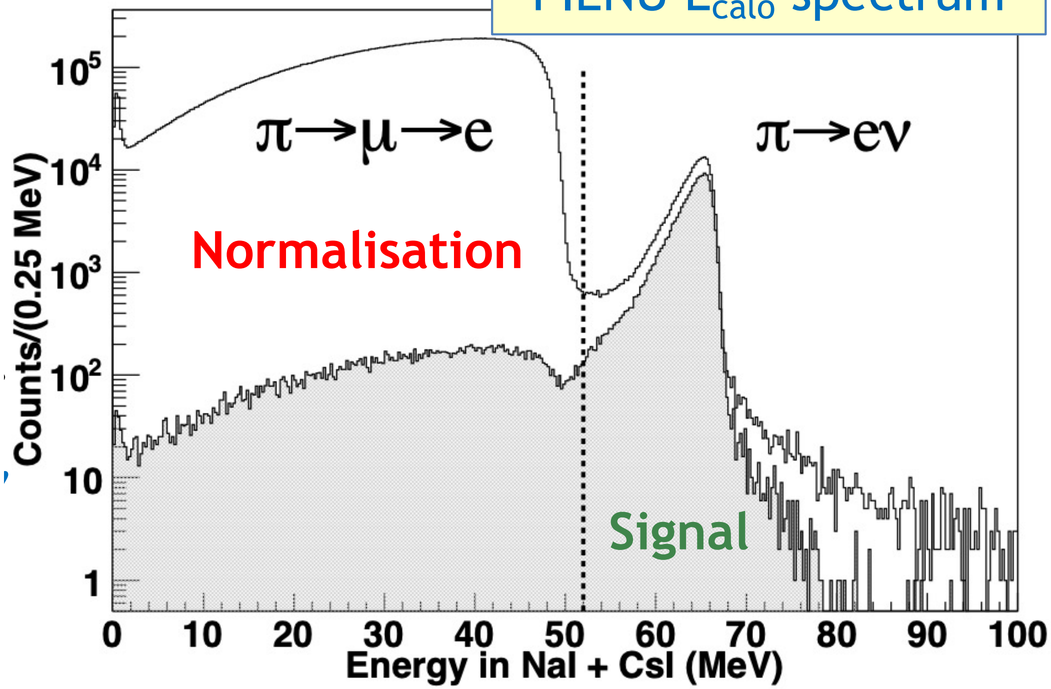
PIONEER (Phase-I)

PIONEER (Phase-I) approved at PSI, physics starting in ~2029

- Goal: matching the SM precision on $R_{e/\mu}$
  ⇨ Test of New Physics at *1 PeV scale*
- Stopped π^+ at high rate (300 kHz), focus on reduction of systematics.
- Detectors: highly-segmented LGAD active target, positron tracker, LXe calorimeter
- Collection of 2×10^8 $\pi^+ \to e^+\nu_e$ events in three years.
- Key point: control of the $\pi^+ \to e^+\nu_e$ signal tail in the calorimeter to a 10^{-4} precision

PIONEER Phase II,III:
V_{ud} from $\pi^+ \to \pi^0 e^+\nu_e$ decays to a *0.02%* level

PIENU E_{calo} spectrum



1.1 Test of the Standard Model: V_{us} and CKM unitarity

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
  - Fundamental parameter of the Standard Model

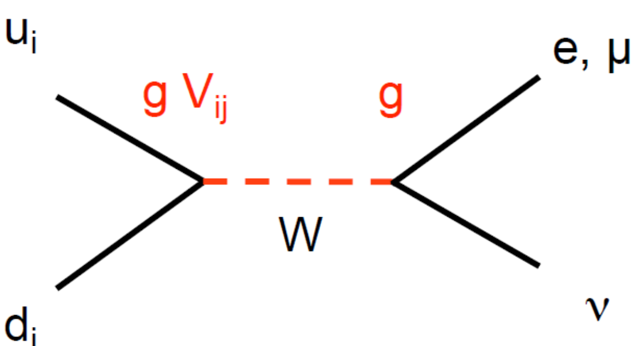
    Description of the weak interactions:

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L V_{CKM} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{e_L} + \bar{\mu}_L \gamma^\alpha \nu_{\mu_L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau_L} \right) + \text{h.c.}$$

Gauge coupling

  - Universality: Is G_F from μ decay equals to G_F from π, K, nuclear β decay?

$$G_\mu^2 = (g_\mu g_e)^2 / M_W^4 \quad \stackrel{?}{=} \quad G_{\rm CKM}^2 = (g_q g_\ell)^2 \left(|V_{ud}|^2 + |V_{us}|^2\right) / M_W^4$$



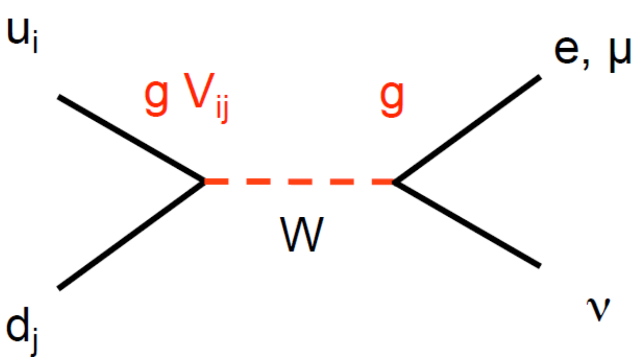
1.2 Constraining New Physics

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
  - Fundamental parameter of the Standard Model

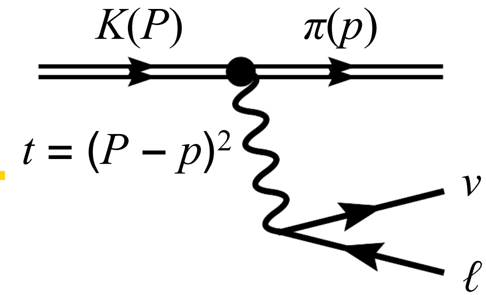
    Description of the weak interactions :

$$\boxed{\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L V_{CKM} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{eL} + \bar{\mu}_L \gamma^\alpha \nu_{\mu L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau L} \right) + \text{h.c.}}$$

- Look for *new physics*
  - In the Standard Model : W exchange ⇨ only V-A structure



2.2 V_{us} from K_{l3} ($K \to \pi l \nu_l$)



- Master formula for $K \to \pi l \nu_l$: $K = \{K^+, K^0\}$, $l=\{e,\mu\}$

$$\boxed{\Gamma\left(K \to \pi l \nu[\gamma]\right) = Br(K_{l3})/\tau = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K |V_{us}|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl}\left(1+2\Delta_{\mathrm{EM}}^{Kl}+2\Delta_{\mathrm{SU(2)}}^{K\pi}\right)}$$

Average and work by Flavianet Kaon WG *Antonelli et al'11* and then by *M. Moulson*, see e.g. *Moulson.@CKM2021*

Theoretically

- Update on long-distance EM corrections for K_{e3} *Seng et al.'21*
- Improvement on Isospin breaking evaluation due to more precise dominant input: quark mass ratio from $\eta \to 3\pi$ *Colangelo et al.'18*
- Progress from lattice QCD on the $K \to \pi$ FF

$$\boxed{\left\langle \pi^-(p)\right| \bar{s}\gamma_\mu u \left|K^0(P)\right\rangle = f_+^{K^0\pi^-}(0)\left[(P+p)_\mu \bar{f}_+^{K^0\pi^-}(t) + (P-p)_\mu \bar{f}_-^{K^0\pi^-}(t)\right]}$$

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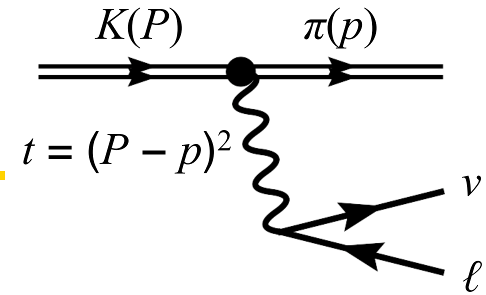

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2.3 V_{us}/V_{ud} from K_{l2}/π_{l2}

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\mathrm{EM}}-\frac{1}{2}\delta_{SU(2)}\right)$$

- Recent progress on radiative corrections computed on lattice:

First lattice calculation of EM corrections to P_{l2} decays

- Ensembles from ETM
- $N_f = 2+1+1$ Twisted-mass Wilson fermions

Giusti et al.'18

$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0122(16)}$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld '11:

$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0112(21)}$

Update, extended description, and systematics of Giusti et al.

$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0126(14)}$

Di Carlo et al.'19

2.1 V_{us} from K_{l3}

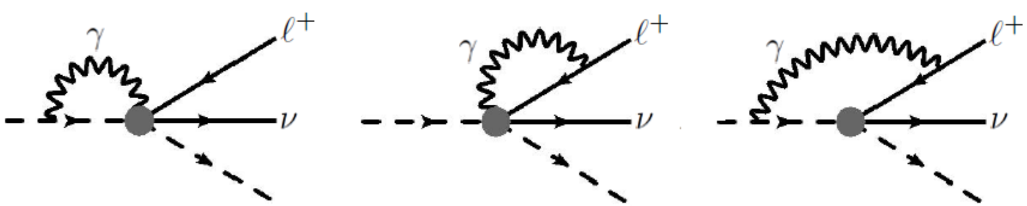
Matthew Moulson, Chien Yeah Seng

Progress since 2018:

- First experimental measurement of BR of $K_S \to \pi\mu\nu$
  BR($K_S \to \pi\mu\nu$) = (4.56 ± 0.20) ×10^{-4}

KLOE-2
PLB 804 (2020)

- Theoretically update on long-distance EM corrections:



Up to now computation at fixed order e^2p^2 + model estimate for the LECs

Cirigliano et al. '08

New calculation of complete EW RC using hybrid current algebra and ChPT (Sirlin's representation) with resummation of largest terms to all chiral orders

- Reduced uncertainties at O(e^2p^4)
- Lattice evaluation of QCD contributions to γW box diagrams

Seng et al.'21

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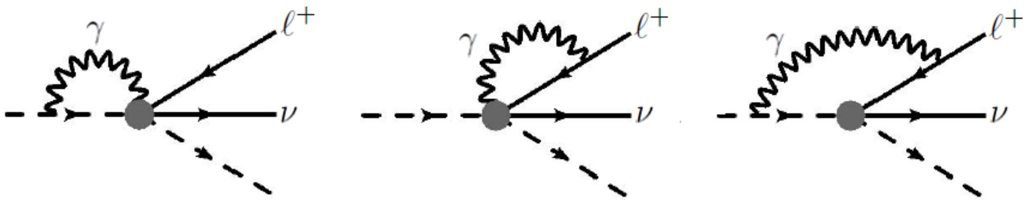


Only K_{e3} at present
For $K_{\mu 3}$ modes continue to use **Cirigliano et al. '08**

	Cirigliano et al. '08	Seng et al. '21
$\Delta^{EM}(K^0{}_{e3})$ [%]	0.50 ± 0.11	**0.580 ± 0.016**
$\Delta^{EM}(K^+{}_{e3})$ [%]	0.05 ± 0.13	**0.105 ± 0.024**
ρ	+0.081	**–0.039**

Progress since 2018:

- Theoretical progress on isospin breaking correction

Strong isospin breaking

Quark mass differences, η-π^0 mixing in $K^+\pi^0$ channel

$$\Delta^{SU(2)} \equiv \frac{f_+(0)^{K^+\pi^0}}{f_+(0)^{K^0\pi^-}} - 1$$

$$= \frac{3}{4}\frac{1}{Q^2}\left[\frac{\overline{M}_K^2}{\overline{M}_\pi^2} + \frac{\chi_{p^4}}{2}\left(1 + \frac{m_s}{\hat{m}}\right)\right] \qquad Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

$\chi_p{}^4 = 0.252$
NLO in strong interaction
O(e^2p^2) term $\varepsilon_{EM}{}^{(4)} \sim 10^{-6}$

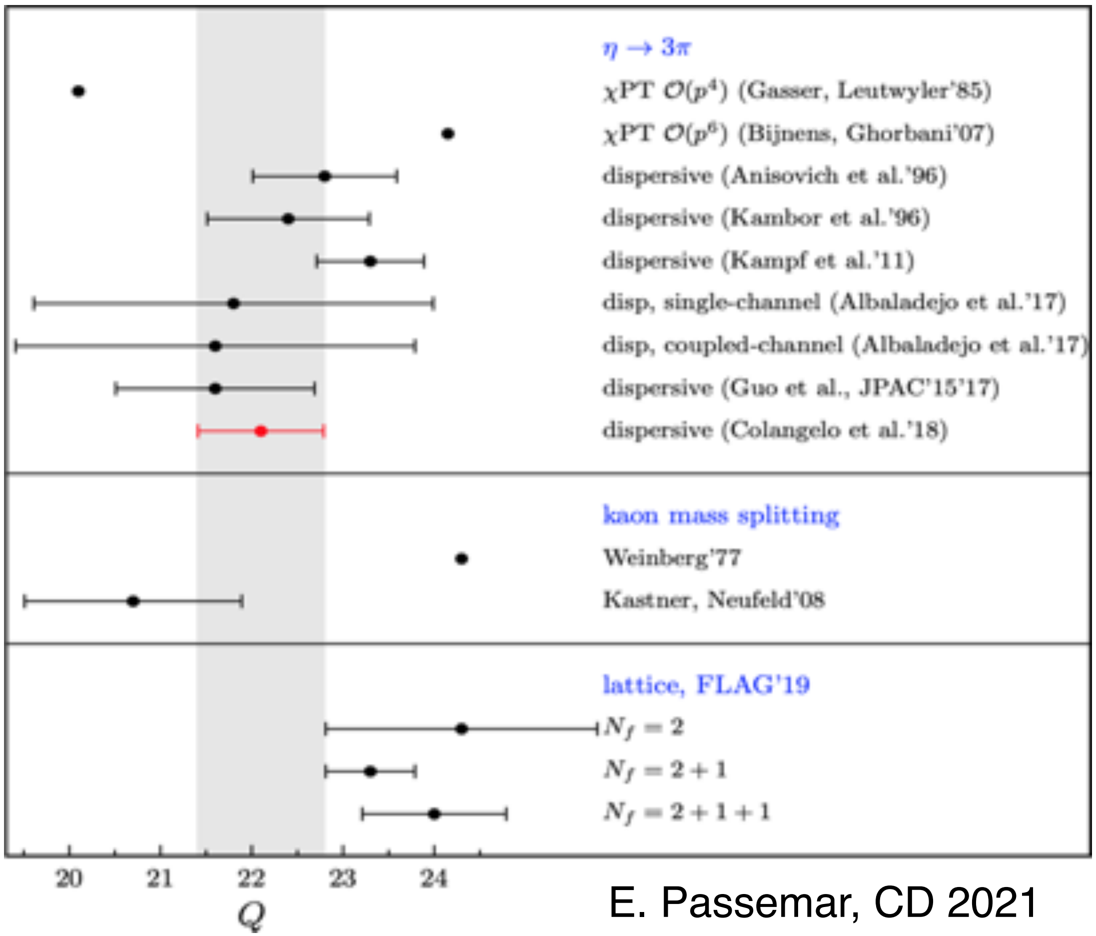
Cirigliano et al., ’02; Gasser & Leutwyler, ’85

$=$ **+2.61(17)%** **Calculated using:**

Q = 22.1(7)	Colangelo et al. ’18, avg. from $\eta \to 3\pi$
$m_s/\hat{m}$ = 27.23(10)	FLAG ’20, N_f = 2+1+1 avg.
M_K = 494.2(3)	Isospin-limit meson masses from FLAG ’17
M_π = 134.8(3)	

Test by evaluating V_{us} from $K^\pm$ and K^0 data with **no** corrections:
Equality of V_{us} values would require **$\Delta^{SU(2)}$ = 2.86(34)%**

Previous to recent results for Q, uncertainty on $\Delta^{SU(2)}$ was leading contributor to uncertainty on V_{us} from $K^{\pm}$ decays



E. Passemar, CD 2021

Reference value of Q from dispersion relation analyses of $\eta \to 3\pi$ Dalitz plots

Colangelo et al., '18

$Q = 22.1 \pm 0.7$

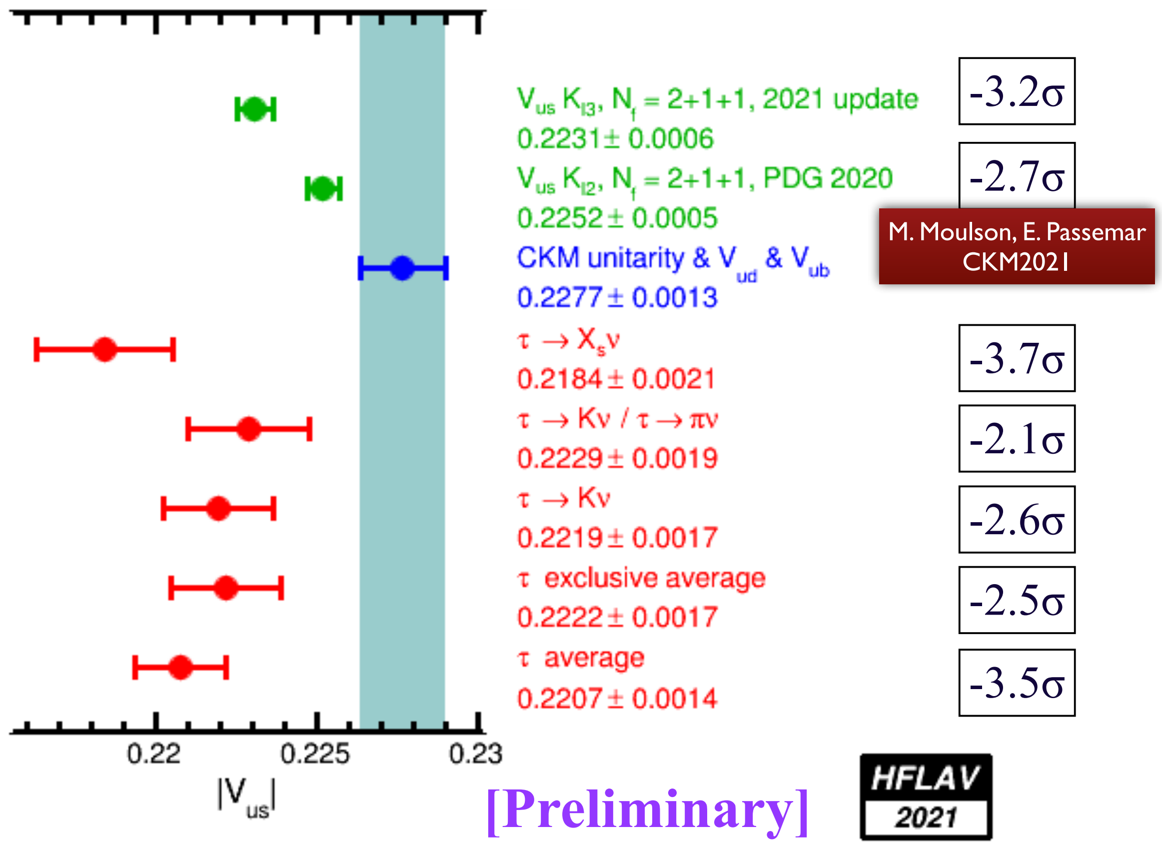
Lattice results for Q somewhat higher than analytical results

But, lattice results have finite correction to LO expectation:

$$Q_M^2 \equiv \frac{\hat{M}_K^2}{\hat{M}_\pi^2} \frac{\hat{M}_K^2 - \hat{M}_\pi^2}{\hat{M}_{K^0}^2 - \hat{M}_{K^+}^2}$$

Low-energy theorem: Q has no correction at NLO

V_{us} from Tau decays



- Belle II with 50 ab^{-1} and ~4.6 x 10^{10} τ pairs will improve V_{us} extraction
- Inclusive measurement is an opportunity to have a complete independent measurement of V_{us} ⇒ not easy as you have to measure many channels

V_{us} from Tau decays

e 13: HFLAV 2021 τ branching fractions to strange final states.

Branching fraction	HFLAV 2021 fit (%)
$K^-\nu_\tau$	0.6957 ± 0.0096
$K^-\pi^0\nu_\tau$	0.4322 ± 0.0148
$K^-2\pi^0\nu_\tau$ (ex.K^0)	0.0634 ± 0.0219
$K^-3\pi^0\nu_\tau$ (ex.K^0,η)	0.0465 ± 0.0213
$\pi^-\overline{K}^0\nu_\tau$	0.8375 ± 0.0139
$\pi^-\overline{K}^0\pi^0\nu_\tau$	0.3810 ± 0.0129
$\pi^-\overline{K}^0 2\pi^0\nu_\tau$ (ex.K^0)	0.0234 ± 0.0231
$\overline{K}^0 h^- h^- h^+\nu_\tau$	0.0222 ± 0.0202
$K^-\eta\nu_\tau$	0.0155 ± 0.0008
$K^-\pi^0\eta\nu_\tau$	0.0048 ± 0.0012
$\pi^-\overline{K}^0\eta\nu_\tau$	0.0094 ± 0.0015
$K^-\omega\nu_\tau$	0.0410 ± 0.0092
$K^-\phi(K^+K^-)\nu_\tau$	0.0022 ± 0.0008
$K^-\phi(K_S^0K_L^0)\nu_\tau$	0.0015 ± 0.0006
$K^-\pi^-\pi^+\nu_\tau$ (ex.K^0,ω)	0.2924 ± 0.0068
$K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex.K^0,ω,η)	0.0387 ± 0.0142
$K^-2\pi^-2\pi^+\nu_\tau$ (ex.K^0)	0.0001 ± 0.0001
$K^-2\pi^-2\pi^+\pi^0\nu_\tau$ (ex.K^0)	0.0001 ± 0.0001
$X_s^-\nu_\tau$	2.9076 ± 0.0478

HFLAV'21

$$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C}$$

parton model prediction

$$\boxed{\delta R_\tau \equiv \frac{R_{\tau,NS}}{\left|V_{ud}\right|^2} - \frac{R_{\tau,S}}{\left|V_{us}\right|^2}}$$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0242(32)$$

Gamiz et al'07, Maltman'11

$$\boxed{\left|V_{us}\right|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{\left|V_{ud}\right|^2} - \delta R_{\tau,th}}}$$

2.9σ away from unitarity!

$$\Rightarrow \quad \boxed{\left|V_{us}\right| = 0.2184 \pm 0.0018_{\text{exp}} \pm 0.0011_{\text{th}}}$$

$|V_{ud}|$ from Neutrons

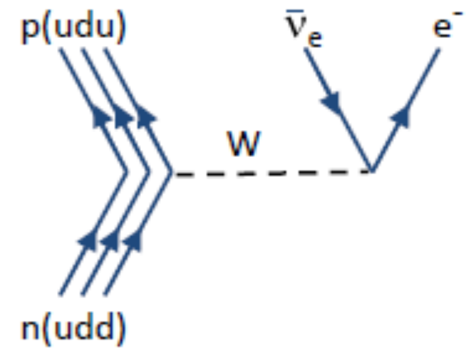
- Master Formula:

$$\boxed{\left|V_{ud}\right|^2 = \frac{5024.7s}{\tau_n\left(1+3\lambda^2\right)\left(1+\Delta_R\right)}}$$

Lifetime

$\lambda = g_A/g_V$

$$n \to p + e^- + \overline{\nu}_e$$



- Needs $\delta\lambda/\lambda \approx 3 \times 10^{-4}$ and $\delta\tau_n \approx 0.3$ s to compete with $0^+ \to 0^+$ transitions.
- Theoretically, the radiative corrections are under control (same as for $0^+ \to 0^+$)
- Recent progress :
  - New Perkeo III result: *PERKEO III* result improves world-average of beta asymmetry by factor 5! Half of it is due to the reduction of the scale factor

$$\boxed{A = -0.11958(21),\ S = 1.2 \quad \lambda_A = -1.2757(5)}$$

  - Tension with *aSPECT* result: $\boxed{\lambda_{\text{avg}} = -1.2754(13),\ S = 2.7}$

$|V_{ud}|$ from Neutrons

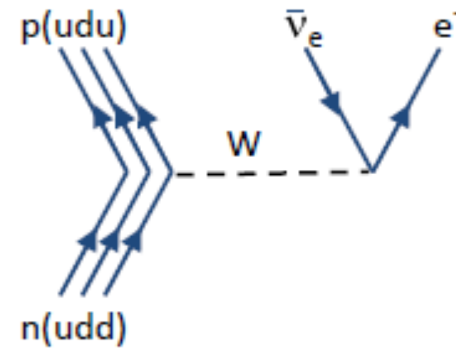
- Master Formula:

$$n \rightarrow p + e^{-} + \overline{\nu}_e$$

$$\boxed{|V_{ud}|^2 = \frac{5024.7s}{\tau_n(1+3\lambda^2)(1+\Delta_R)}}$$

Lifetime

$$\lambda = g_A / g_V$$



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- Theoretically, the radiative corrections are under control (same as for $0^+ \rightarrow 0^+$)
- Recent progress :
  - New Perkeo III result: *PERKEO III* result improves world-average of beta asymmetry by factor 5! Half of it is due to the reduction of the scale factor

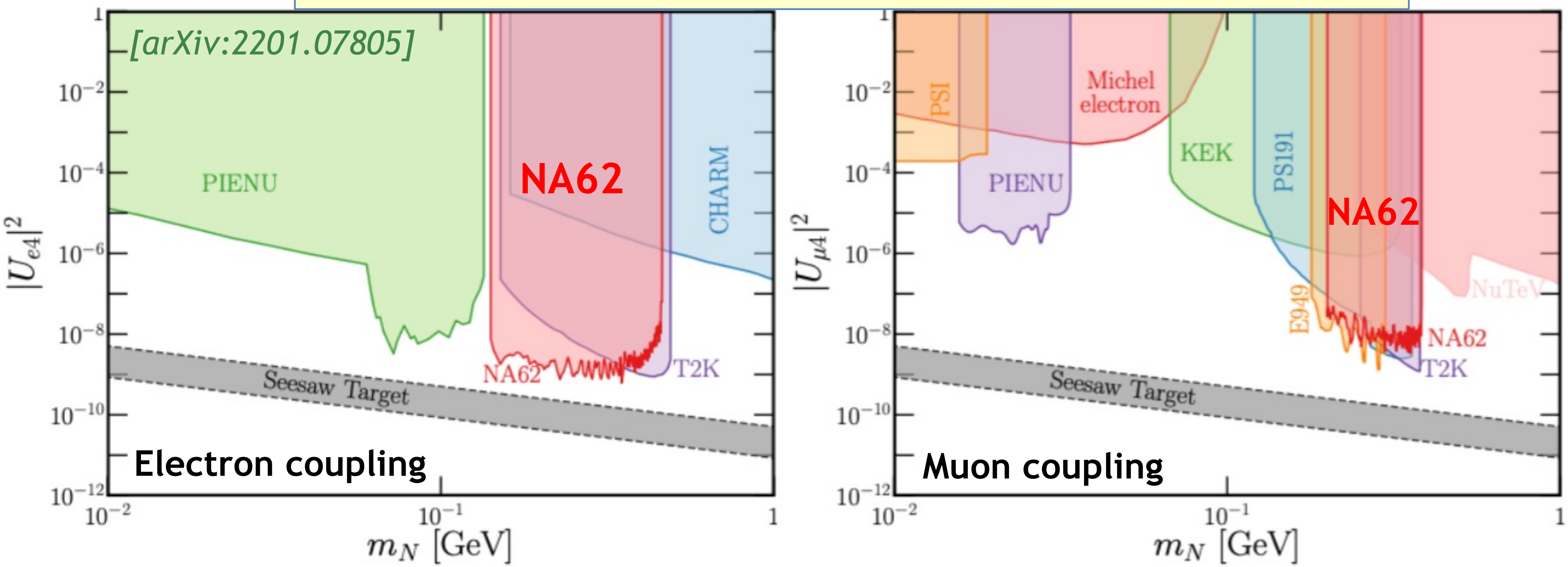
    ⇨ $\boxed{A = -0.11958(21),\ S = 1.2 \quad \lambda_A = -1.2757(5)}$

  - New result for Lifetime from *UCNτ* $\tau_n = 877.75 \pm 0.28^{+0.22}_{-0.16}$ s

    ⇨ improvement by a factor of 2.25 compared to previous result

3.3 Example: Constraints on Heavy Neutral Leptons

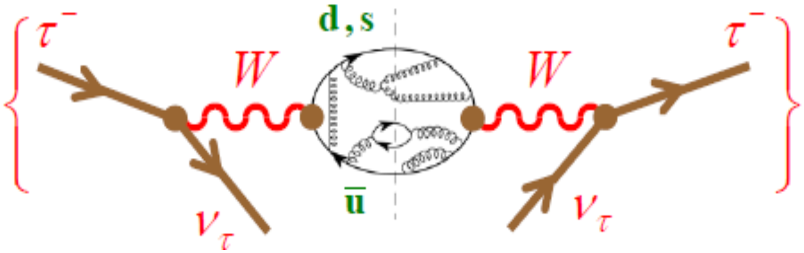
$|U_{\ell 4}|^2$ limits vs m_{HNL} from production & decay searches



- Strongest **$|U_{e4}|^2$** limits below **400 MeV**: **$K^+, \pi^+ \to e^+N$** from *NA62* & *PIENU*.
- Also important limits on **$|U_{\mu 4}|^2$** from *E949*, *NA62* and *PIENU*.
- *NA62/E949* limits are complementary to HNL *decay* searches at T2K.
- Next-generation **K^+** and p**+** experiments (*NA62^{++}*, *PIONEER*) to improve by up to factor **10**, reaching the seesaw bound.

Inclusive τ-decays

Braaten, Narison, Pich'92

$\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}$ 

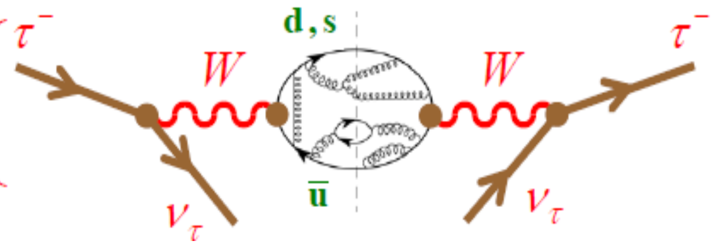
- Quantity of interest : $$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \mathbf{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)}}$$

3.2 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \text{Im}\left\{\right.$$



$$\left.\right\}$$

$$\Rightarrow \quad \boxed{R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s+i\varepsilon) + \text{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]}$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2\left(\Pi^{(J)}_{ud,V}(s) + \Pi^{(J)}_{ud,A}(s)\right) + |V_{us}|^2\left(\Pi^{(J)}_{us,V}(s) + \Pi^{(J)}_{us,A}(s)\right)$$

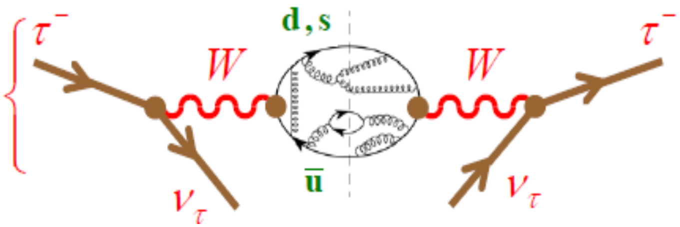
J

$$\Pi^{\mu\nu}_{ij,V/A}(q) = \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi^{(1)}_{ij,V/A}(q^2) + q^\mu q^\nu\,\Pi^{(0)}_{ij,V/A}(q^2)$$

3.2 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}\Big\{$$  $$\Big\}$$

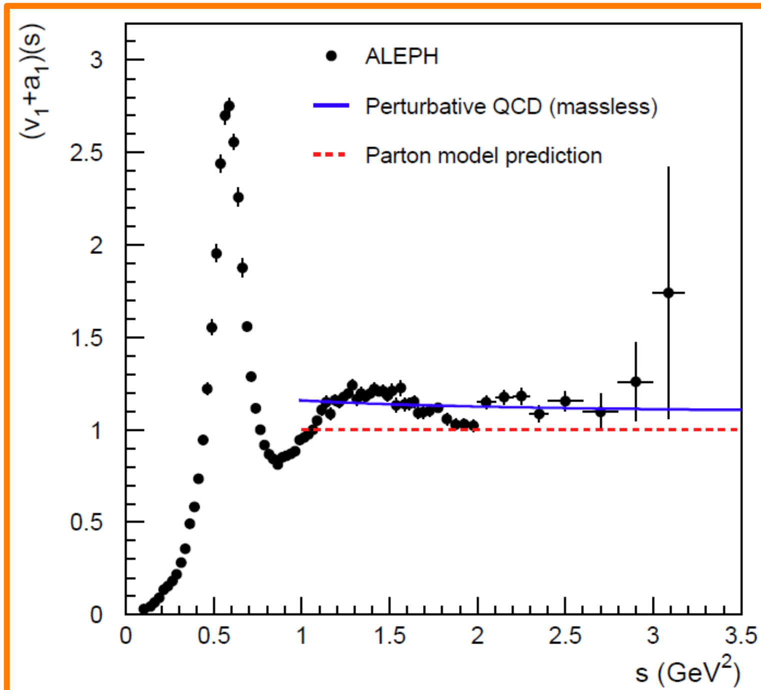
$$\Rightarrow \boxed{R_\tau(m_\tau^2) = 12\pi S_{EW}\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s+i\varepsilon)+\mathrm{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]}$$

- Spectral functions:

$$\mathrm{Im}\Pi^{(1)}_{\bar{u}d,V/A}(s) = \frac{1}{2\pi}v_1/a_1(s)$$

- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system

$\Rightarrow$ mix of non-perturbative and perturbative effects



Measurements

- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \textbf{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} = ?$$

- Decomposition as a function of observed and separated final states:

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

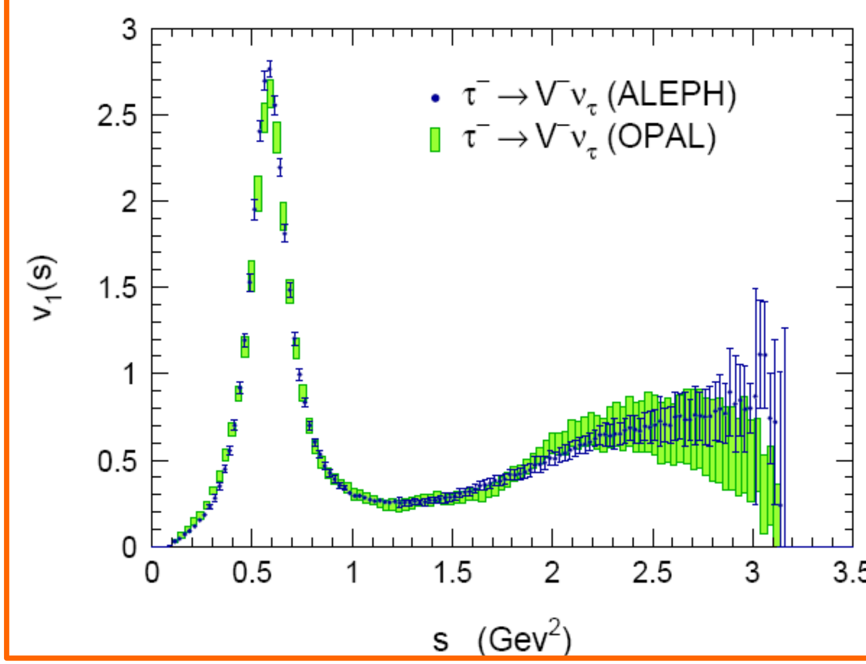
$R_{\tau,V} \Rightarrow \boxed{\tau^- \to \nu_\tau + h_{v,s=0}}$

(even number of pions)

$R_{\tau,A} \Rightarrow \tau^- \to \nu_\tau + h_{A,s=0}$

(odd number of pions)

$R_{\tau,S} \Rightarrow \tau^- \to \nu_\tau + h_{V+A,s=1}$



Measurements

- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \mathbf{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} = ?$$

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(odd number of pions)

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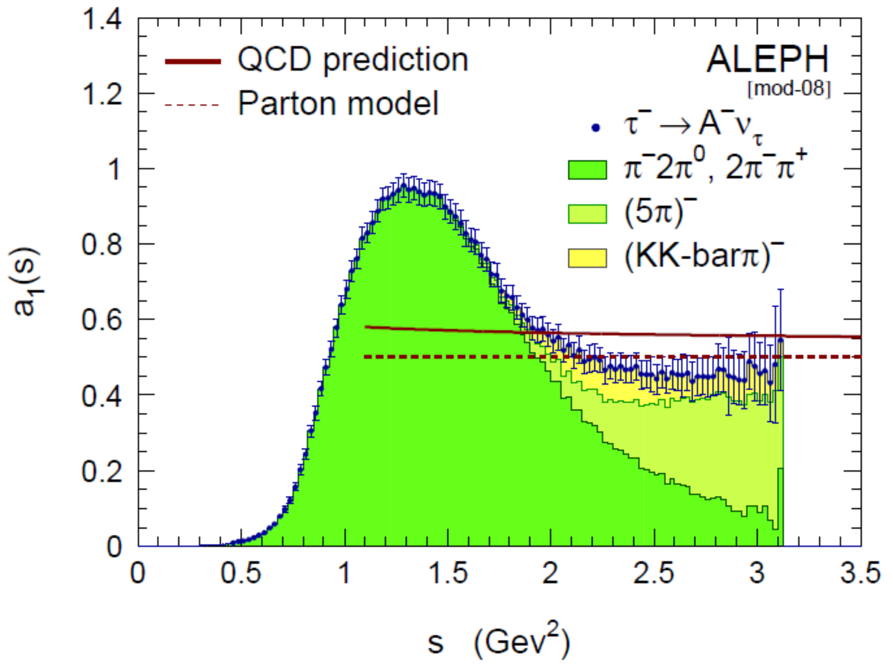

Measurements

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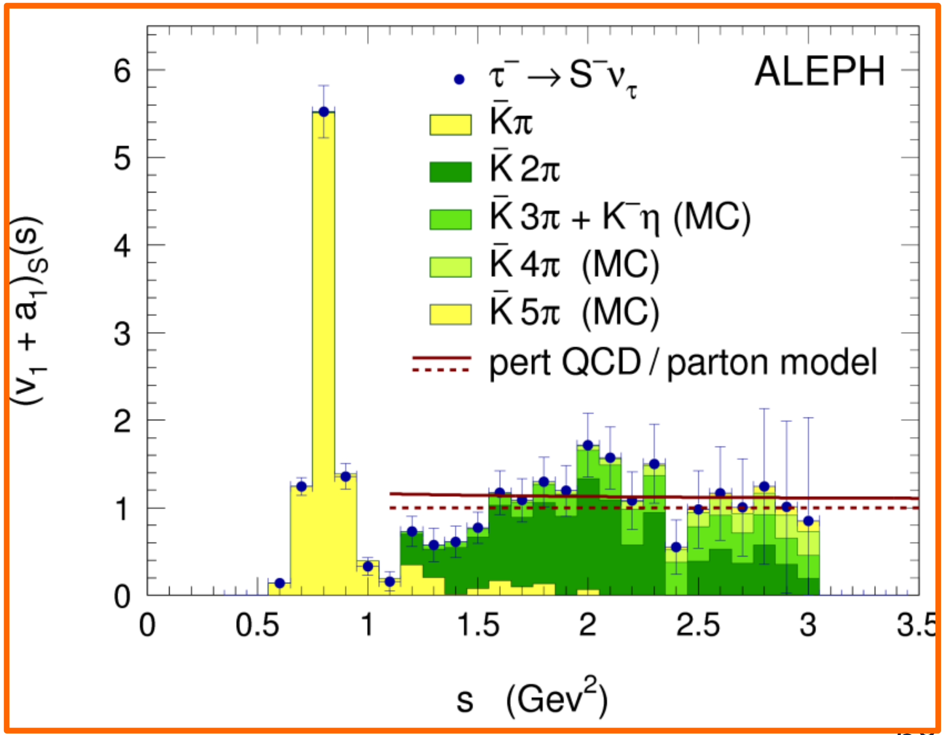
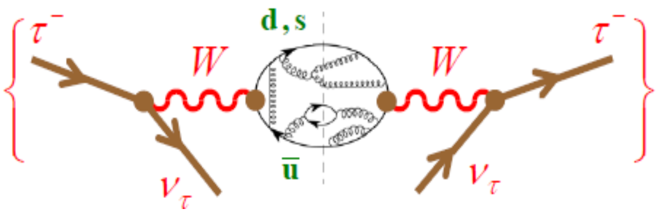


3.2 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

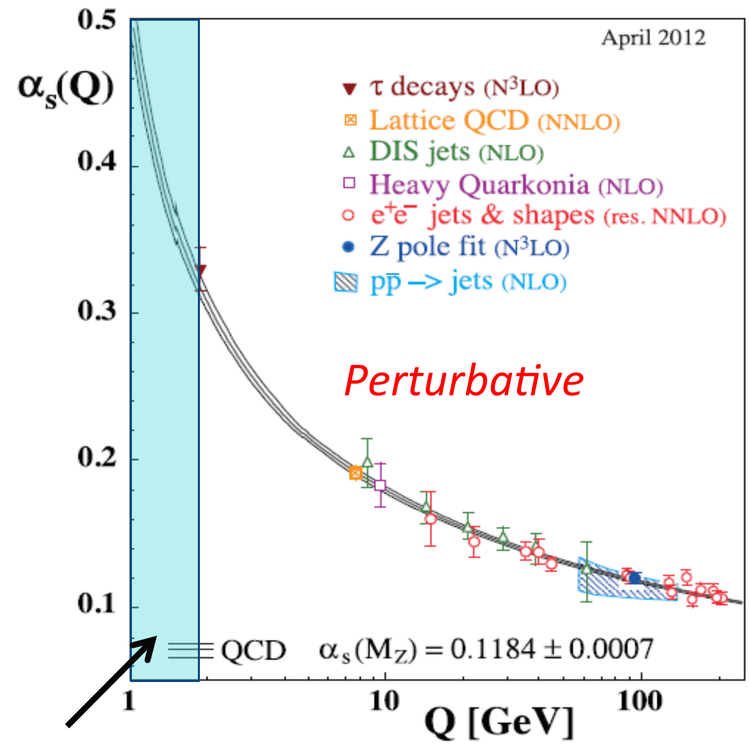
$$\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}\left\{ \right\}$$



$$\boxed{R_\tau(m_\tau^2) = 12\pi S_{EW}\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s+i\varepsilon)+\mathrm{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]}$$

- We are in the *non-perturbative* region: we do not know how to compute!

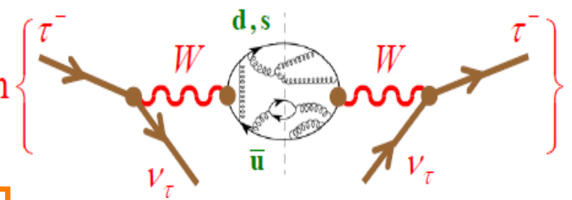
- Trick: use the analytical properties of Π!



Non-Perturbative

3.2 Calculation of the QCD corrections

- Calculation of R_τ:

$\Gamma_{\tau \to \nu_\tau + \text{had}} \sim \text{Im}$ 

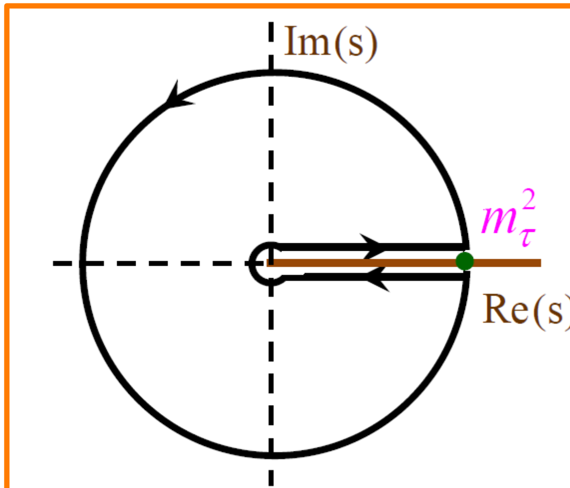
$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s+i\varepsilon) + \text{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

Braaten, Narison, Pich'92

- Analyticity: Π is analytic in the entire complex plane except for s real positive

⇨ Cauchy Theorem

$$R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right]$$



- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O = D} C^{(J)}(s,\mu)\, \langle O_D(\mu) \rangle$$

Wilson coefficients

Operators

μ: separation scale between short and long distances

3.3 Operator Product Expansion

$$\boxed{\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} \mathrm{C}^{(J)}(s,\mu)\ \langle O_D(\mu)\rangle}$$

Wilson coefficients

Operators

μ separation scale between short and long distances

- D=0: Perturbative contributions
- D=2: Quark mass corrections
- D=4: Non perturbative physics operators, $\left\langle \frac{\alpha_s}{\pi} GG \right\rangle$, $\left\langle m_j \bar{q}_i q_i \right\rangle$
- D=6: 4 quarks operators, $\left\langle \bar{q}_i \Gamma_1 q_j \bar{q}_j \Gamma_2 q_i \right\rangle$
- D≥8: Neglected terms, supposed to be small…

$\Rightarrow$ $\boxed{R_{\tau,V}(s_0) = \frac{3}{2}\left|V^{ud}\right|^2 S_{EW}\left(1+\delta^{(0)} + \sum_{D=2,4..} \delta^{(D)}_{ud,V}\right)}$ similar for $R_{\tau,A}(s_0)$ and $R_{\tau,S}(s_0)$

Perturbative Part

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ Marciano &Sirlin'88, Braaten & Li'90, Erler'04

- Perturbative part (D=0):

$$a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$$

$$\boxed{\delta_P = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + \ldots \approx 20\%}$$

Baikov, Chetyrkin, Kühn'08

Non-perturbative part

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

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Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS}\ \left(\propto m_u, m_d\right)$ but not for $R_\tau^{S}\ \left(\propto m_s\right)$

- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  ⇨ Use of weighted distributions

Ex: In the non-strange sector: $\boxed{\delta_{NP}^{NS} = -0.0064(13)}$ Davier et al.'14

Non-Perturbative part

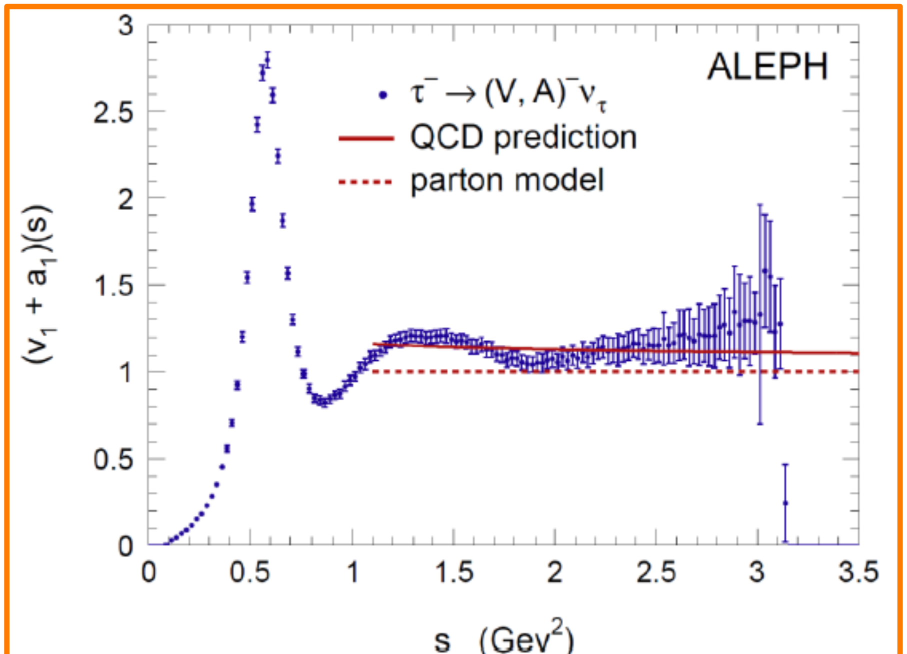
Le Diberder&Pich'92

- D ≥ 4: Non perturbative part, not known, *fitted from the data*

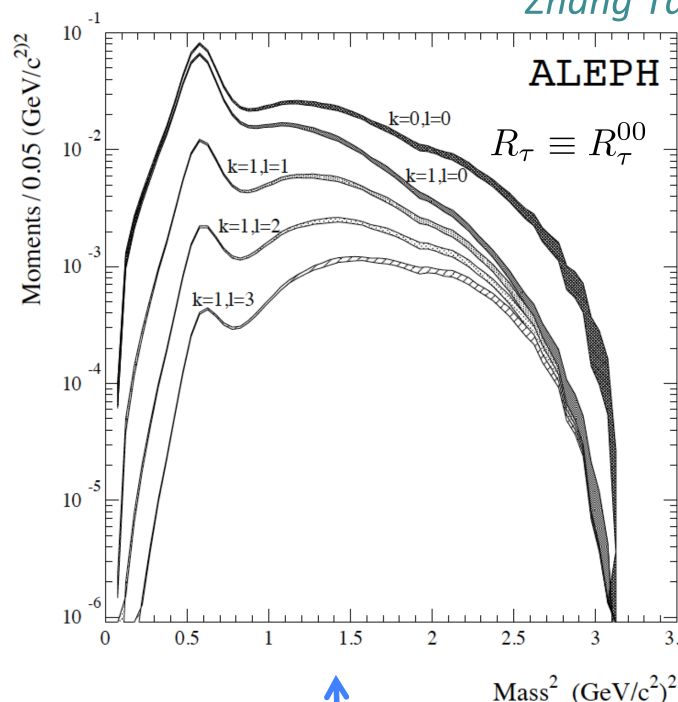
  ⇨ Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

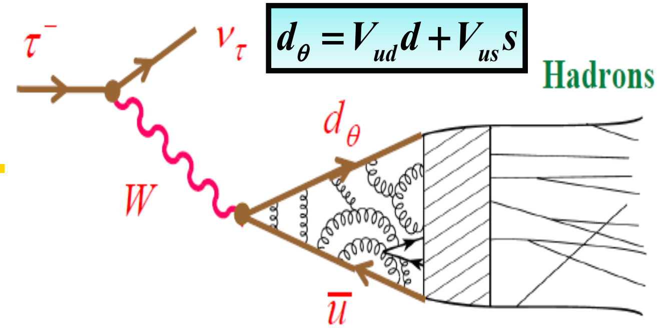
$$R_{\tau,U}^{k\ell}(s_0) = \int_0^{s_0} ds \left(1-\frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$



Zhang'Tau14



3.4 Determination of α_s



- $$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C}$$

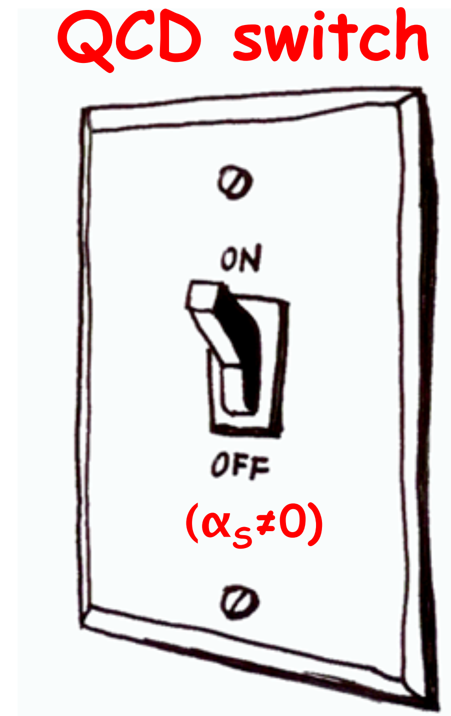
- Use OPE: $$\boxed{R_\tau^{NS}\left(m_\tau^2\right) = N_C\, S_{EW} \left|V_{ud}\right|^2 \left(1 + \delta_P + \delta_{NP}^{ud}\right)}$$

$$\boxed{R_\tau^{S}\left(m_\tau^2\right) = N_C\, S_{EW} \left|V_{us}\right|^2 \left(1 + \delta_P + \delta_{NP}^{us}\right)}$$

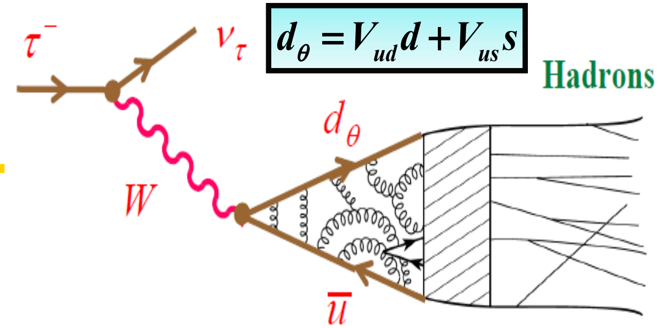
- Extraction of the strong coupling constant :

$$R_\tau^{NS} = \left|V_{ud}\right|^2 N_C + O\left(\alpha_s\right) \Longrightarrow \alpha_s$$

measured calculated



3.3 Determination of V_{us}



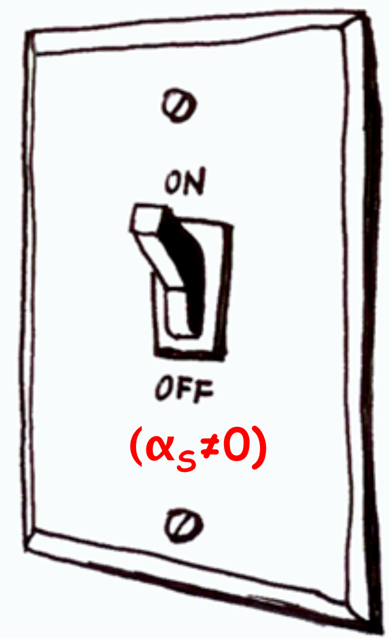
- $$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \approx N_C$$

- Use OPE: $$R_\tau^{NS}(m_\tau^2) = N_C\, S_{EW} |V_{ud}|^2 \left(1 + \delta_P + \delta_{NP}^{ud}\right)$$

$$R_\tau^{S}(m_\tau^2) = N_C\, S_{EW} |V_{us}|^2 \left(1 + \delta_P + \delta_{NP}^{us}\right)$$

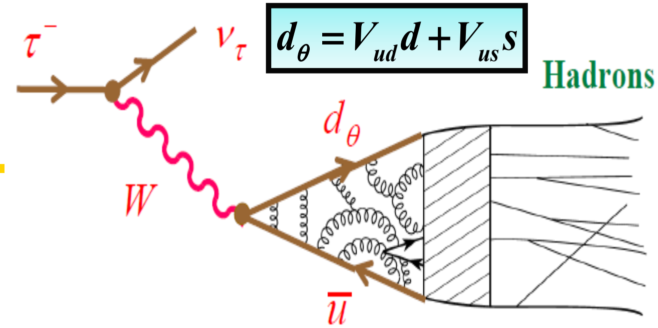
- From the measurement of R_τ^S $\Rightarrow$ $|V_{us}|$

QCD switch



($\alpha_s \neq 0$)

3.3 Determination of V_{us}



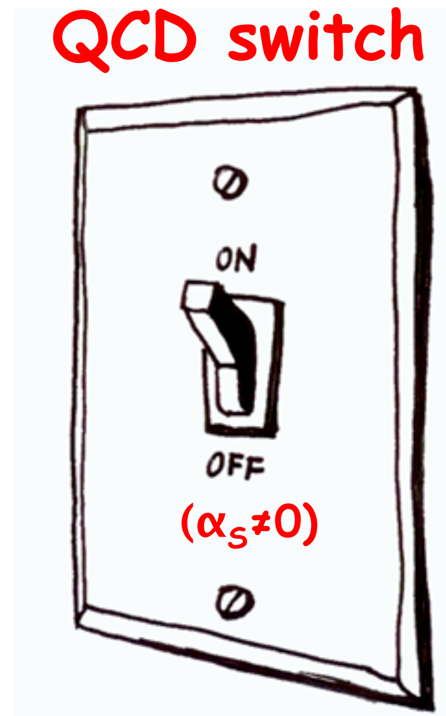
- $$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C}$$

- Use OPE: $$\boxed{R_\tau^{NS}\left(m_\tau^2\right) = N_C\, S_{EW} \left|V_{ud}\right|^2 \left(1 + \delta_P + \delta_{NP}^{ud}\right)}$$

$$\boxed{R_\tau^{S}\left(m_\tau^2\right) = N_C\, S_{EW} \left|V_{us}\right|^2 \left(1 + \delta_P + \delta_{NP}^{us}\right)}$$

- From the measurement of R_τ^S ⇨ $\left|V_{us}\right|$

- Use instead : $$\boxed{\delta R_\tau \equiv \frac{R_{\tau,NS}}{\left|V_{ud}\right|^2} - \frac{R_{\tau,S}}{\left|V_{us}\right|^2}}$$

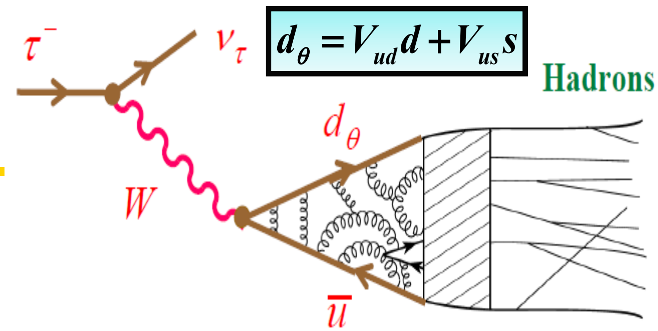


⇨ *SU(3) breaking* quantity: the flavour independent piece:

$\delta_P \sim 20\%$ cancels!

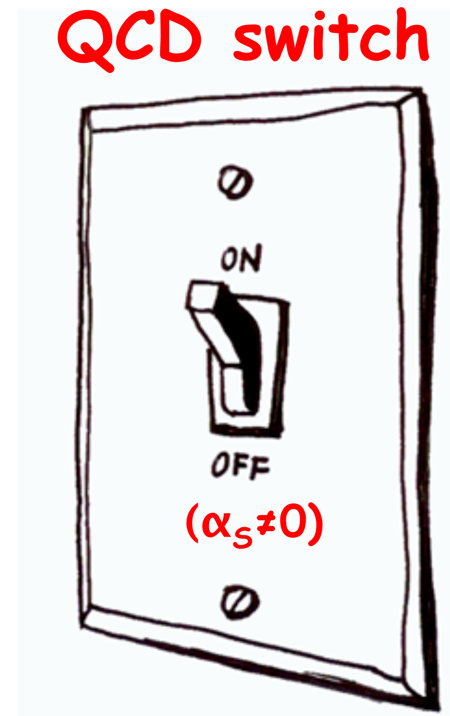
3.3 Determination of V_{us}

- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C$$



- Use OPE: $$R_\tau^{NS}\left(m_\tau^2\right) = N_C\, S_{EW} \left|V_{ud}\right|^2 \left(1 + \delta_P + \delta_{NP}^{ud}\right)$$

$$R_\tau^{S}\left(m_\tau^2\right) = N_C\, S_{EW} \left|V_{us}\right|^2 \left(1 + \delta_P + \delta_{NP}^{us}\right)$$



$$\delta R_\tau \equiv \frac{R_{\tau,NS}}{\left|V_{ud}\right|^2} - \frac{R_{\tau,S}}{\left|V_{us}\right|^2}$$

Calculation of $\delta R_{\tau,\mathrm{theo}}$

$$\boxed{\delta R_\tau \equiv \frac{R_{\tau,V+A}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}} \approx N_C S_{EW} \sum_{D\geq 2} \left[\delta_{ud}^{(D)} - \delta_{us}^{(D)}\right]$$

- $\delta_{ij}^{(2)}$: quark mass corrections, *neglected* for R_τ^{NS} $(\propto m_u, m_d)$ but not for R_τ^S $(\propto m_s)$

  known up to $O(\alpha_S^3)$ for both J=L and J=L+T

  Chetyrkin, Gorishry, Kataev, Larin, Sugurladze; Baikov, Chetyrkin,Kuehn
  Becchi, Narison, de Rafael; Bernreuther, Wetzel

- $\delta_{ij}^{(4)}$: fully included , e.g m_j^4 / m_τ^4 $\langle m_j \bar{q}_i q_i \rangle / m_\tau^4$

- $\delta_{ij}^{(6)}$: estimated (VSA) to be of order or smaller than errors on D=4

- D ≥ 8 : Neglected terms, expected to be small…

$$\Rightarrow \quad \boxed{\delta R_\tau \approx 24 \frac{m_s^2(m_\tau^2)}{m_\tau^2} \Delta(\alpha_S)}$$

Results

$$|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,V+A}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

- $\delta R_{\tau,theo}$ determined from OPE (L+T) + phenomenology

$$\Rightarrow \quad \delta R_{\tau,th} = \underbrace{(0.1544 \pm 0.0037)}_{J=0} + (9.3 \pm 3.4)\, m_s^2 + (0.0034 \pm 0.0028)$$

Gamiz, Jamin, Pich, Prades, Schwab'07, Maltman'11

Input : m_s $\Rightarrow$ $m_s\left(2\ \mathrm{GeV}, \overline{\mathrm{MS}}\right) = 93.9 \pm 1.1$ N_f=2+1+1 lattice average

FLAG'16

- Tau data : $R_{\tau,S} = 0.1646(23)$ and $R_{\tau,NS} = 3.4721(77)$ *HFLAV'16 + BaBar@ICHEP18*
- V_{ud} : $|V_{ud}| = 0.97425(22)$ *Towner & Hardy'08*