The Measurement of EDM and Anomalous magnetic moment $(g-2)$ for τ

Yongcheng Wu

Nanjing Normal University

Xin Chen, Y. Wu; JHEP10(2019)089, 1803.00501
Xin Chen, Y. Wu; Eur.Phys.J. C77(2017)697, 1703.04855
Xin Chen, Y. Wu; Phys.Lett.B790(2019)332, 1708.02882

Outline

- Introduction and Current Situation
- $e^+e^- \to \tau^+\tau^-$ Event Reconstruction
    - Matrix Element method and Optimal Observable
        - Expected sensitivities
- Constraints on BSM
- Summary

Introduction

- Electric Dipole Moment
  - Current Strongest CPV test
- Anomalous Magnetic Moment $(g-2)$
  - Most precisely measured observable
  - Muon g-2 anomaly
- $\Gamma^\mu(q^2) = -ieQ_f\left\{\gamma^\mu F_1(q^2) + \frac{\sigma^{\mu\nu}q_\nu}{2m_f}[i\,F_2(q^2) + F_3(q^2)\gamma_5]\right\}$
    - Low momentum transfer $F_1(0) = 1, F_2(0) = a_f = \frac{g-2}{2}, F_3(0) = d_f\frac{2m_f}{e\,Q_f}$
- BSM Contribution to EDM/g-2:
  - $\frac{\delta d_\ell}{\delta d_e} \sim \frac{m_\ell}{m_e}$
  - $\delta a_\ell \sim \frac{m_\ell^2}{\Lambda^2}$

$\frac{m_\tau}{m_e} \sim 3460$

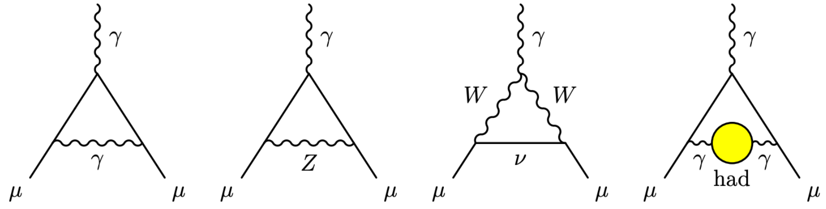
$\left(\frac{m_\tau}{m_\mu}\right)^2 \sim 280$

Mod.Phys.Lett.A22(2007)159
Phys.Lett.B255(1991)611
Phys.Lett.B395(1997)369
Rev.Mod.Phys.87(2015)531

Current Sensitivities

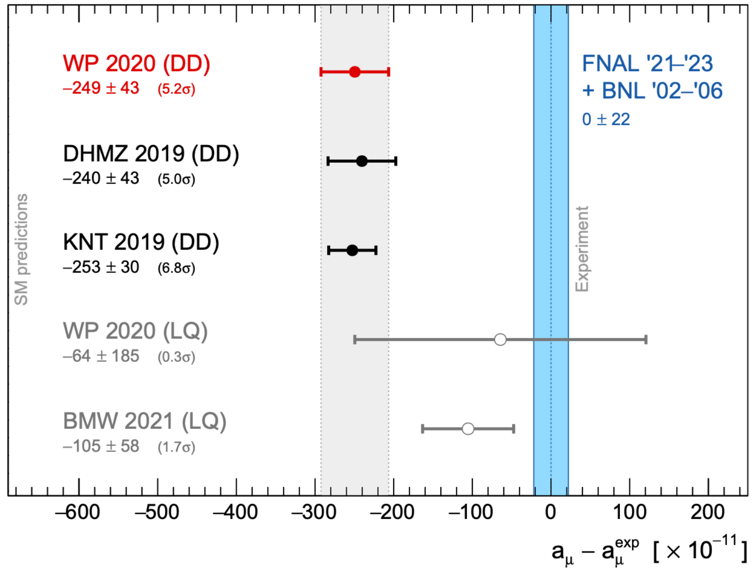
Lepton	EDM $d_\ell\ (e\cdot cm)$	$a_\ell = (g-2)/2$ PDG2023
e	$< 1.1\times10^{-29}$	$(1159.65218062^{\pm0.00000012})\times10^{-6}$
μ	$< 1.8\times10^{-19}$	$(1165.92059 \pm 0.00022)\times10^{-6}$ Combine BNL and FNAL
τ	$[-1.85, 0.61]\times10^{-17}$	$[-0.052, 0.013]$

- EDM
  - High loop corrections
  - $d_e \lesssim 10^{-38} e\cdot cm$
  - $d_\ell \sim \frac{m_\ell}{m_e}$
- Electron $(g-2)_e$: **Atoms 7 (2019) 28**
  - $a_e^{SM} = a_e^{QED} + a_e^{EW} + a_e^{Had} = (1159.65218161^{\pm0.00000023})\times10^{-6}$



- Muon $(g-2)_\mu$: **PDG2023**
  - Theoretical Prediction:
    - $a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Had} = (1165.91810 \pm 0.00001 \pm 0.00040 \pm 0.00018)\times10^{-6}$
  - The difference:
    - $\Delta a_\mu = a_\mu^{\exp} - a_\mu^{SM} = (249 \pm 22 \pm 43)\times10^{-11}$
  - Fermilab/J-PARC **J-PARC: 1901.030**
- Tau $(g-2)_\tau$:
  - $a_\tau^{SM} = 0.00117721(5)$

Mod.Phys.Lett.A22(2007)159; hep-ph/0701260

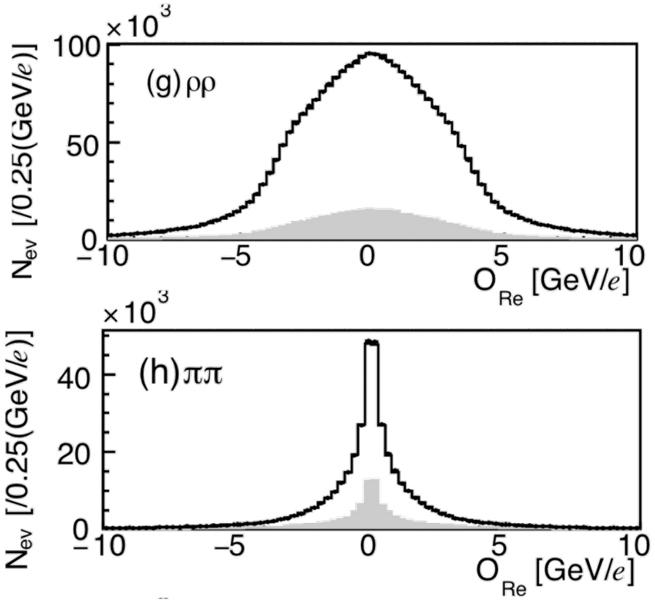


Tau Measurement

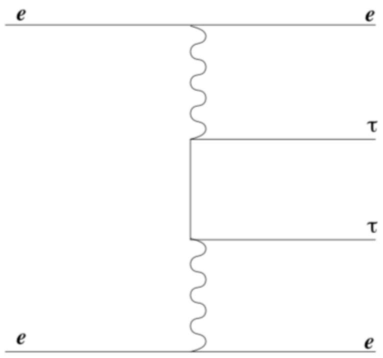
- EDM: Belle JHEP 04 (2022) 110 Phys.Lett.B 551 (2003) 16
  - $e^+e^- \to \gamma^* \to \tau^+\tau^-$
  - 8 different final states:
    - $(e\nu\bar{\nu})(\mu\nu\bar{\nu}), (e\nu\bar{\nu})(\pi\nu), (\mu\nu\bar{\nu})(\pi\nu), (e\nu\bar{\nu})(\rho\nu), (\mu\nu\bar{\nu})(\rho\nu), (\pi\nu)(\rho\nu), (\rho\nu)(\rho\bar{\nu}), (\pi\nu)(\pi\bar{\nu})$
  - Matrix Element

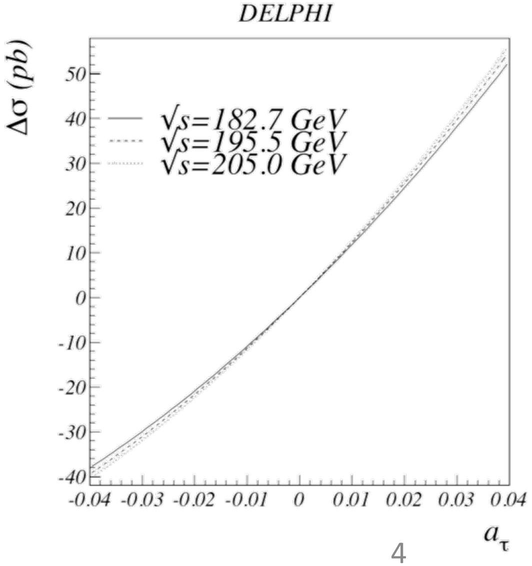
$$\mathcal{M}^2_{\rm prod} = \mathcal{M}^2_{\rm SM} + Re(d_\tau)\mathcal{M}^2_{Re} + Im(d_\tau)\mathcal{M}^2_{Im} + |d_\tau|^2\mathcal{M}^2_{d^2},$$

  - Optimal Observable
    - Average over undetectable configuration
      - Ambiguities of the tau direction



- (g-2): DELPHI EPJC 35 (2004) 159
  - $e^+e^- \to e^+e^-\tau^+\tau^-$
  - Measuring the total cross-section

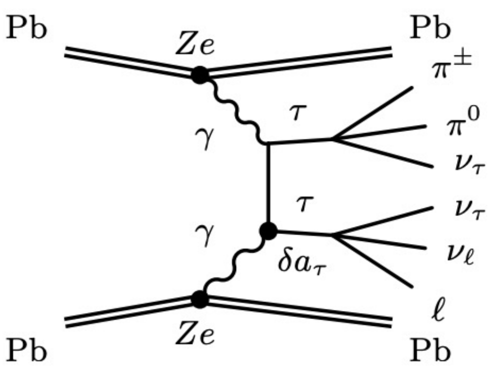


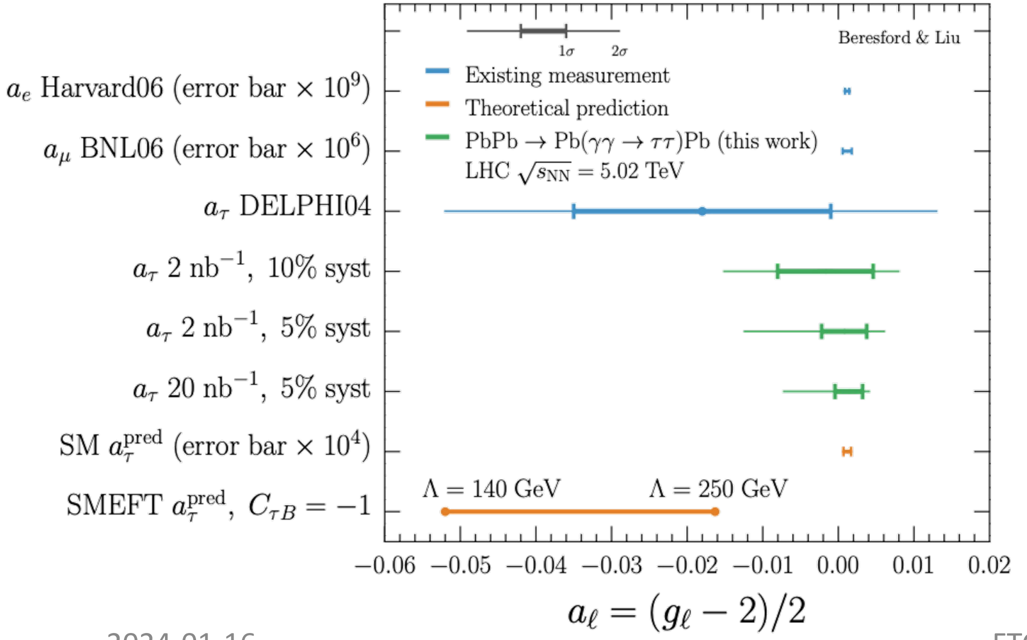


Current Studies

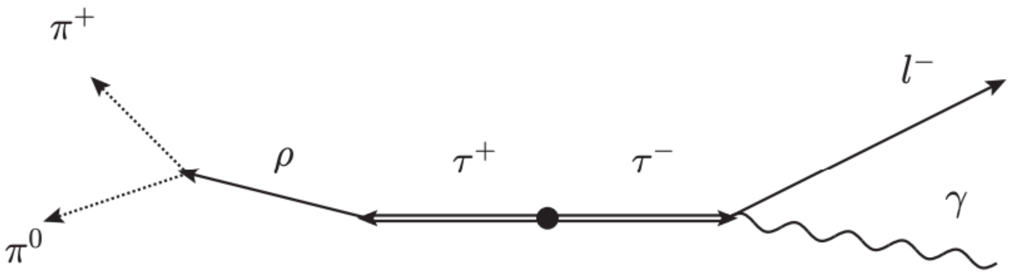
- Photon Collision/Drell-Yan
  - $\gamma\gamma \to \tau^+\tau^-(\gamma)$

JHEP 11 (2010) 060
Phys. Rev. D 98 (2018) 015017
J. Phys. G 46 (2019) 065003
1903.04135
Phys. Rev. D 102 (2020) 113008
Phys.Lett.B 809 (2020) 135682





- Radiative Decay JHEP 03 (2016) 140
  - $\tau^\pm \to \ell^\pm \nu_\tau \nu_\ell \gamma$



	Belle (ρ)	Belle II (ρ)	Belle (full)	Belle II (full)	DELPHI [2]	Belle [52]
$\tilde{a}_\tau$	0.16	0.023	0.085	0.012	0.017	—
$(m_\tau/e)\,\tilde{d}_\tau$	0.15	0.021	0.080	0.011	—	0.0015

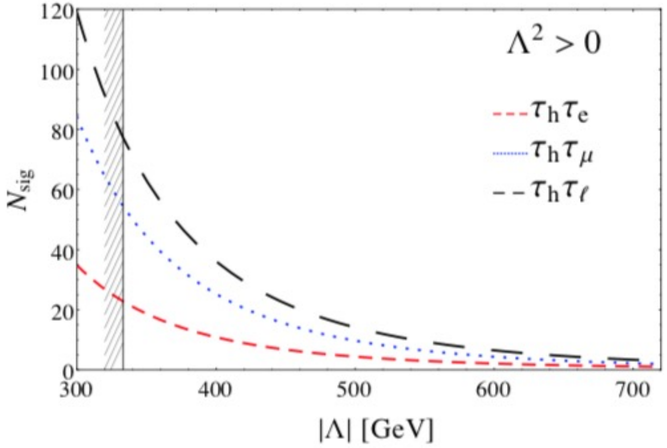
Current Studies

- $H \rightarrow \tau^+ \tau^- \gamma$ JHEP 12 (2016) 111, LHEP 2 (2019) 2, 5
  - SMEFT Dim-6:

$$c_1\ \bar{\tau}_R \sigma^{\mu\nu} B_{\mu\nu}\ H^\dagger L_3 + c_2\ \bar{\tau}_R \sigma^{\mu\nu}\ H^\dagger W_{\mu\nu} L_3 + h.c.$$

  - g-2 related to Higgs coupling

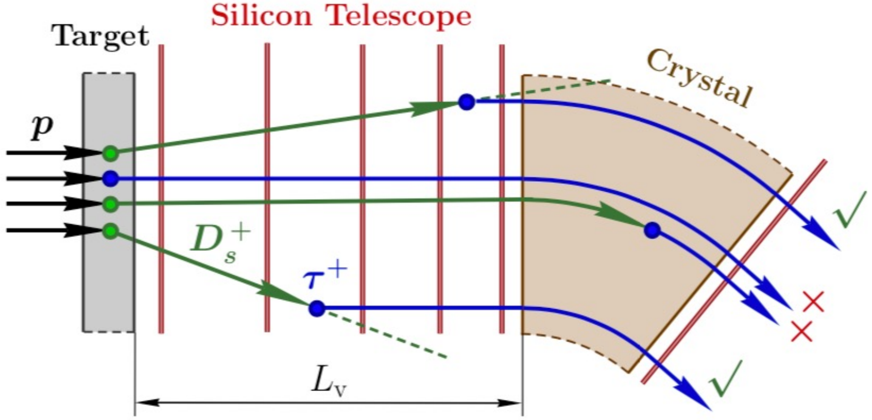
$$\frac{1}{\Lambda^2}\left\{v\ \bar{\tau}\sigma^{\mu\nu}\ \tau F_{\mu\nu} + h\ \bar{\tau}\sigma^{\mu\nu}\ \tau F_{\mu\nu}\right\}$$

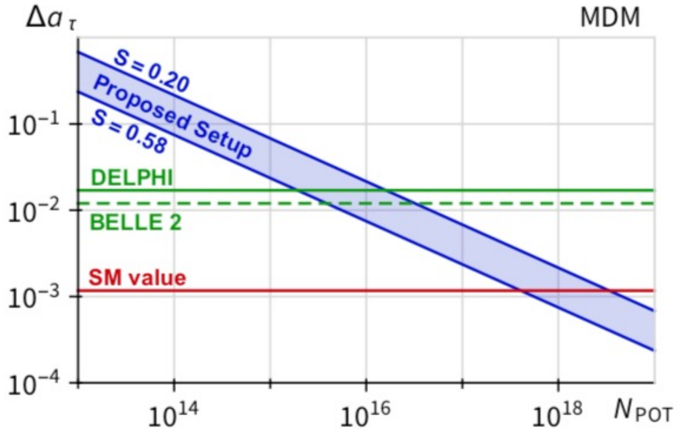


$$-0.0144 < a_\tau^\gamma < 0.0106. \quad (95\%\ \text{CL}),$$ (at the LHC)

- Bent Crystal JHEP 03 (2019) 156, Phys. Rev. Lett 123 (2019) 011801

$$pp \rightarrow D_s^+ X, \quad D_s^+ \rightarrow \tau^+ \nu_\tau, \quad \tau^+ \rightarrow \pi^+ \pi^+ \pi^- \bar{\nu}_\tau.$$

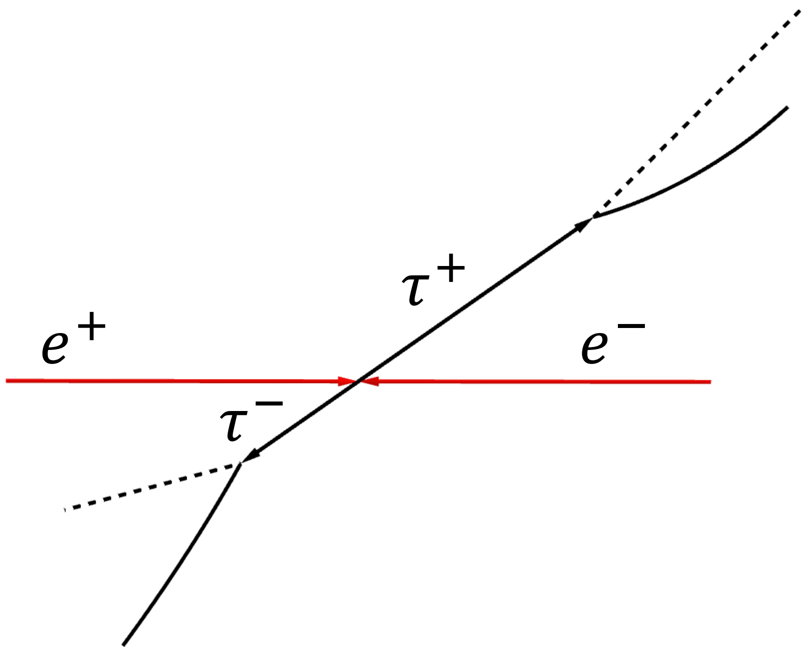




Measurement at e^+e^- machine

- Process:
  - $e^+e^- \to \gamma^* \to \tau^+\tau^-$
    - At STCF, about 3.5 billion pair of $\tau^+\tau^-$ per year
- $\tau^\pm$ Decay Channels:
  - $\tau^\pm \to \pi^\pm\nu$ (~10.8%)
  - $\tau^\pm \to \pi^\pm\pi^0\nu\ (\rho^\pm)$ (~25.4%)
  - $\tau^\pm \to \pi^\pm\pi^\pm\pi^\mp\nu\ (a^\pm)$ (~9.3%)
  - $\tau^\pm \to \ell^\pm\nu_\ell\nu$ (~17% × 2)

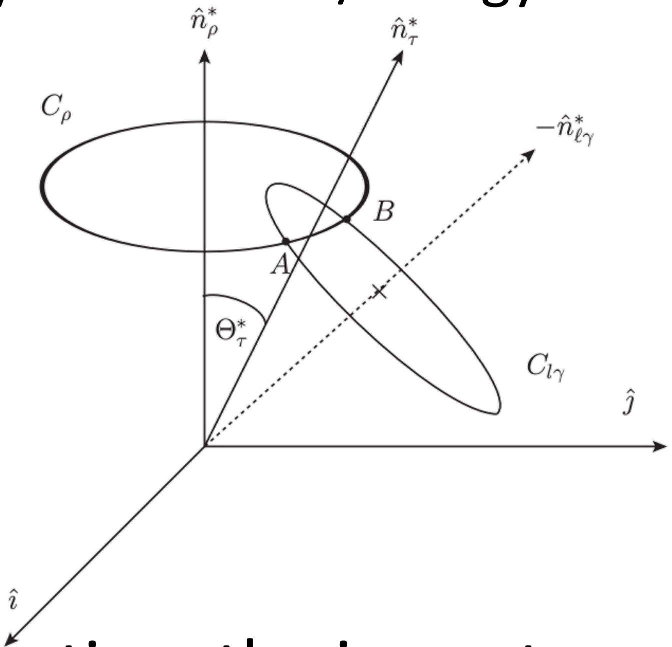
6 Combination

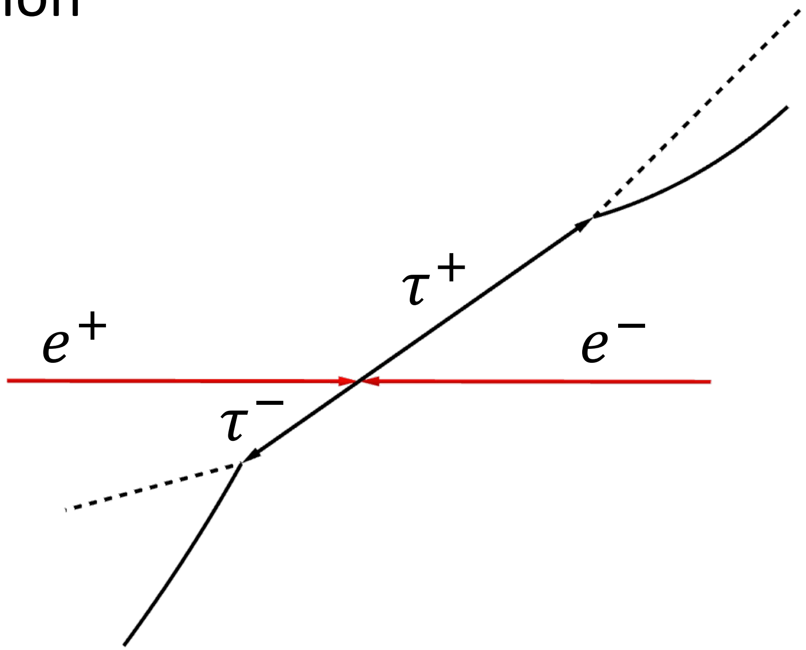


- Target:
  - Reconstruct the neutrino ⟷ Reconstruct the tau lepton

Tau Reconstruction

- The two-fold ambiguity
  - Using only momentum/Energy information

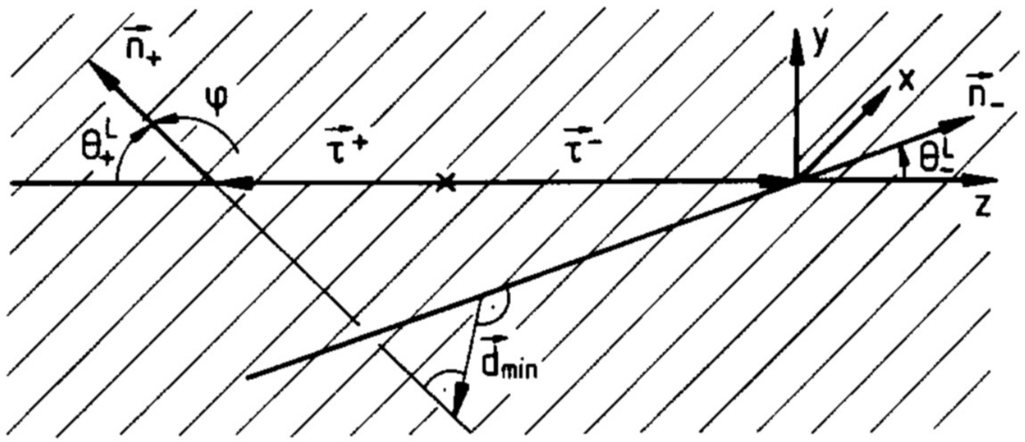




- Extra information: the impact parameter
  - $\tau^{\pm}$ life time:
    - $\tau = (290.3 \pm 0.5)\times 10^{-15} s$
    - $c\tau = 87.03 \mu m$

Impact Parameters

- The ambiguity can be resolved by using the impact parameters



Phys. Lett. B 313 (1993) 458

- Advancement in silicon trackers
  - Much better resolutions

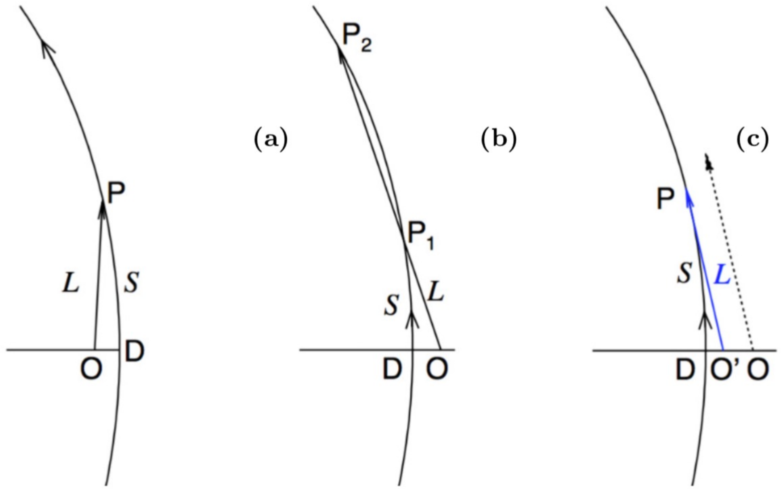
$$\sigma = a \oplus b/(p_T \sin^{1/2}\theta)$$

In mm	a	b/GeV
d_0	0.015	0.007
Belle z_0	0.020	0.010

Belle II TDR - 1011.0352

Impact parameter in reconstruction

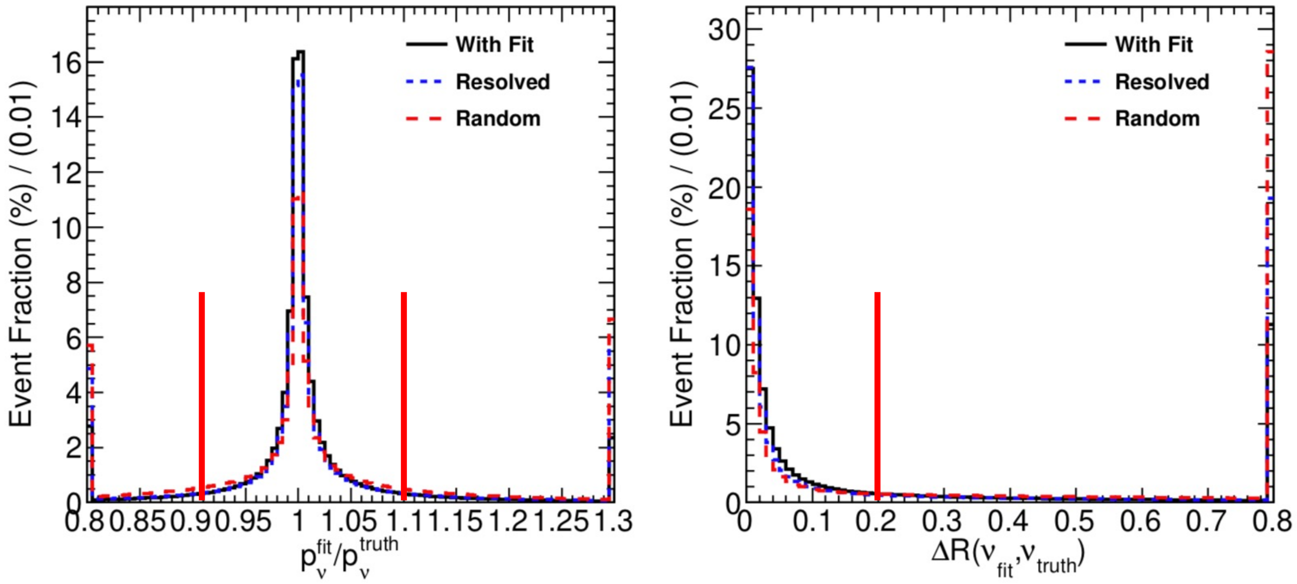
- In transverse plane:



- Along z-axis:
  - $z_0^{fit} = L \sinh \eta_\tau - S \sinh \eta_{track}$
- Contribution to χ^2:
  - $\chi^2_{Imp,i} = \left(\frac{d_0^{fit} - d_0}{\sigma_d}\right)^2 + \left(\frac{z_0^{fit} - z_0}{\sigma_z}\right)^2$

Performance

- The tau/neutrino momentum can be reconstructed by minimizing χ^2 event by event
- Compared with two other cases:
  - Random: Choose one of the two solutions randomly (PLB 313, 458)
  - Resolved: no fitting, but using the impact parameter to resolve the two-fold ambiguity



- Fractions of good reconstructed neutrinos: 42% → 59% → 65%

Sensitivities: Matrix Element/Optimal Observable

- With all momentum determined:

$$|\mathcal{M}|^2_{d_\tau} \propto M_0^d - M_1^d \frac{c_\tau^{NP}}{\Lambda} + M_2^d \left(\frac{c_\tau^{NP}}{\Lambda}\right)^2,$$

$$|\mathcal{M}|^2_{a_\tau} \propto M_0^a + M_1^a \frac{a_\tau^{NP}}{2m_\tau} + M_2^a \left(\frac{a_\tau^{NP}}{2m_\tau}\right)^2,$$

$$\mathcal{L}_{d_\ell} \supset -\frac{i}{2} d_\ell^{NP} \bar{\ell} \sigma_{\mu\nu} \gamma_5 \ell F^{\mu\nu} = \frac{i}{2} \frac{\sqrt{2}e}{v} \left(\frac{v}{\Lambda}\right)^2 c_\ell^{NP} \bar{\ell} \sigma_{\mu\nu} \gamma_5 \ell F^{\mu\nu},$$

$$\mathcal{L}_{a_\ell} \supset \frac{e}{4m_\ell} a_\ell^{NP} \bar{\ell} \sigma_{\mu\nu} \ell F^{\mu\nu}.$$

$$J_\pm^\mu(\tau^\pm \to \pi^\pm \nu) = p_{\pi^\pm}^\mu,$$

$$J_\pm^\mu(\tau^\pm \to \pi^\pm \pi^0 \nu) = p_{\pi^\pm}^\mu - p_{\pi^0}^\mu,$$

$$J_\pm^\mu(\tau^\pm \to \pi_1^\pm \pi_2^\pm \pi_3^\mp \nu) = F^{13}(q_1^\mu - q_3^\mu - G^{13} Q^\mu) + (1 \leftrightarrow 2),$$

TauDecay: EPJC 73 (2013) 2489

- Construct Optimal Observable:

$$\mathcal{O}\mathcal{O}^{(i)} \equiv \frac{(M_1^i/\mathrm{GeV})}{M_0^i},$$

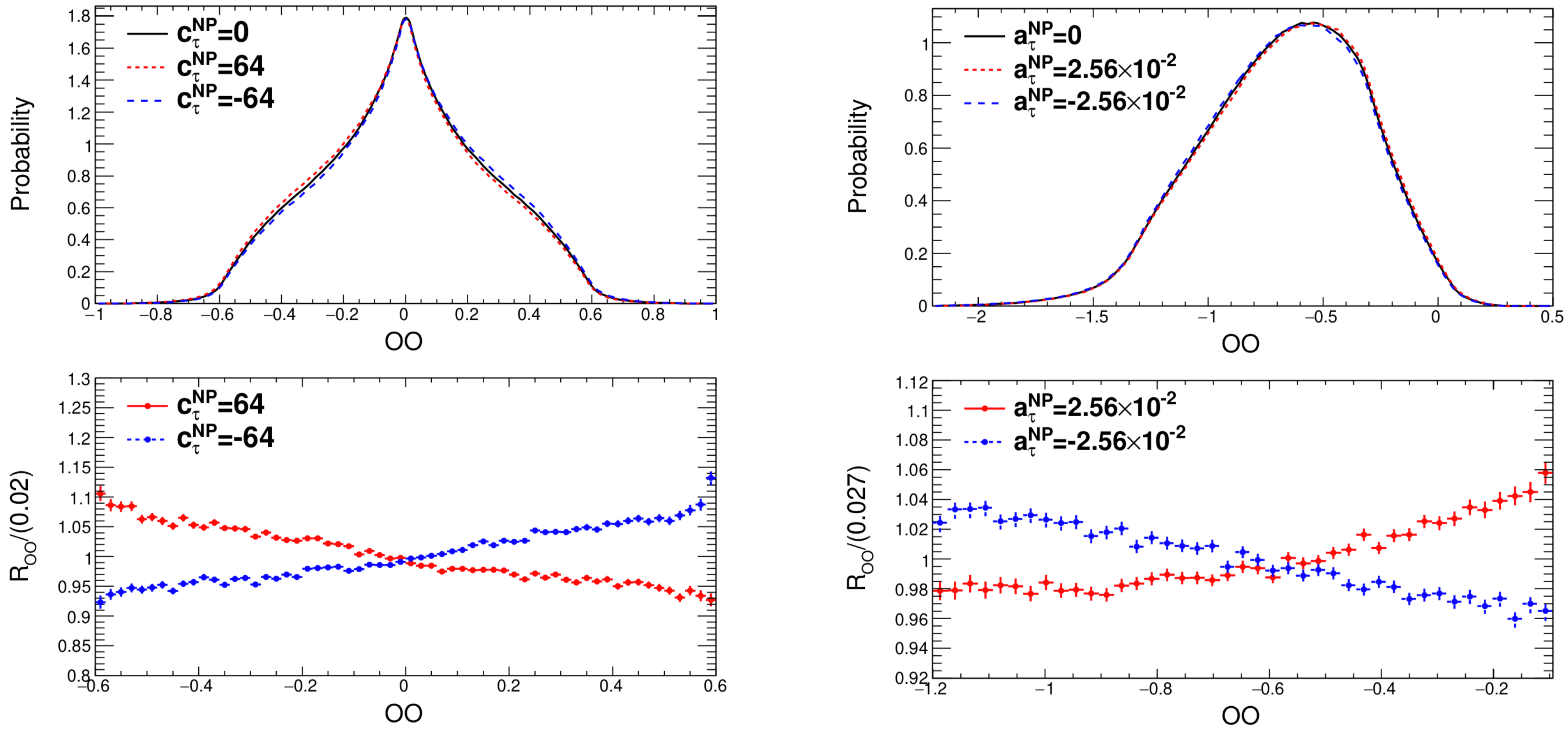
  - Ignore high order terms

Phys. Rev. D 45 (1992) 2405

Phys. Lett. B 306 (1993) 411

Z. Phys. C 62 (1994) 397

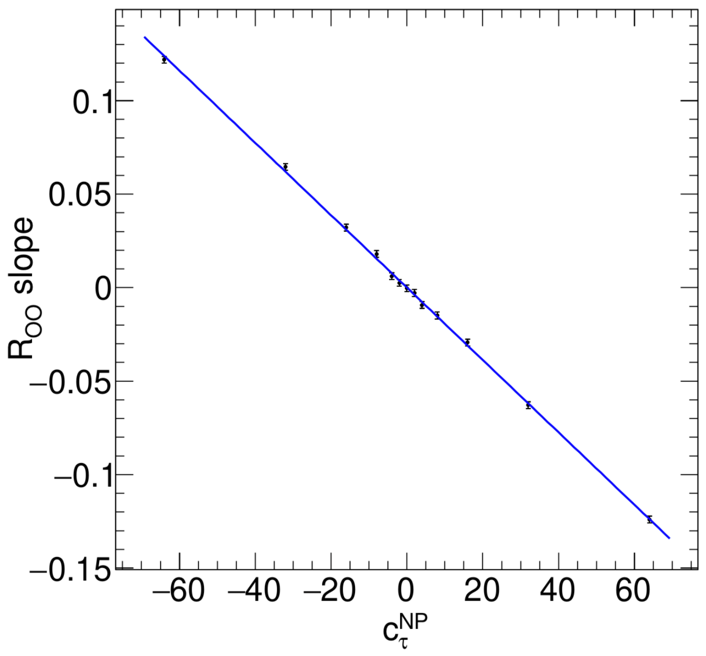
Sensitivities: OO distributions

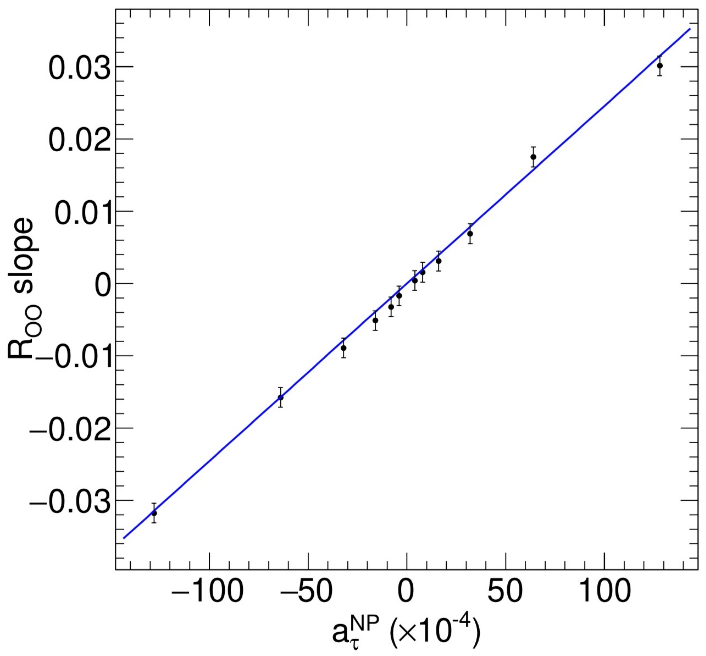


At 50 fb^{-1}

- For small c_τ and a_τ, the ratio of the distribution of different value can be parameterized as
    - $R_{OO} = 1 + b(OO - x_0)$

Sensitivities





$\mathcal{L}$	1 ab^{-1}	10 ab^{-1}	50 ab^{-1}
$\lvert d_\tau^{NP}\rvert$ (e·cm)	1.44×10^{-18}	4.56×10^{-19}	2.04×10^{-19}
$\lvert a_\tau^{NP}\rvert$	1.24×10^{-4}	3.92×10^{-5}	1.75×10^{-5}

Comparison with Belle is also performed, factor of 4 better

$a_\tau^{SM} = 0.00117721(5)$

Tau g-2 in 2HDM + Singlet (NMSSM)

- Model
  - Two Doublets + Singlet

Higgs Basis

$$\begin{pmatrix} \hat{\Phi}_1 \\ \hat{\Phi}_2 \\ \hat{S} \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ S \end{pmatrix}$$

$$\hat{\Phi}_1 = \begin{pmatrix} G^+ \\ \frac{v+\phi_1+iG^0}{\sqrt{2}} \end{pmatrix}, \quad \hat{\Phi}_2 = \begin{pmatrix} H^+ \\ \frac{\phi_2+i\phi_3}{\sqrt{2}} \end{pmatrix}, \quad \hat{S} = \frac{1}{\sqrt{2}}(\omega + \phi_4 + i\phi_5)$$

- Mixings to Mass Eigenstates h_j: $\phi_i = R_{ij} h_j$
- Couplings for Mass Eigenstates:
  - Gauge Couplings: $g_W^{h_i} = \frac{2m_W^2}{v} R_{1i}, \quad g_Z^{h_i} = \frac{m_Z^2}{v} R_{1i}$
  - Yukawa Couplings:
    - $y_d^{h_i} = \frac{m_d}{v}\left(R_{1i} + \xi_d(R_{2i} + iR_{3i})\right)$
    - $y_u^{h_i} = \frac{m_u}{v}\left(R_{1i} + \xi_u(R_{2i} - iR_{3i})\right)$
    - $y_\ell^{h_i} = \frac{m_\ell}{v}\left(R_{1i} + \xi_\ell(R_{2i} + iR_{3i})\right)$

Consider Type-II Yukawa Couplings:

- $\xi_{d,\ell} = -\tan\beta$
- $\xi_u = \cot\beta$

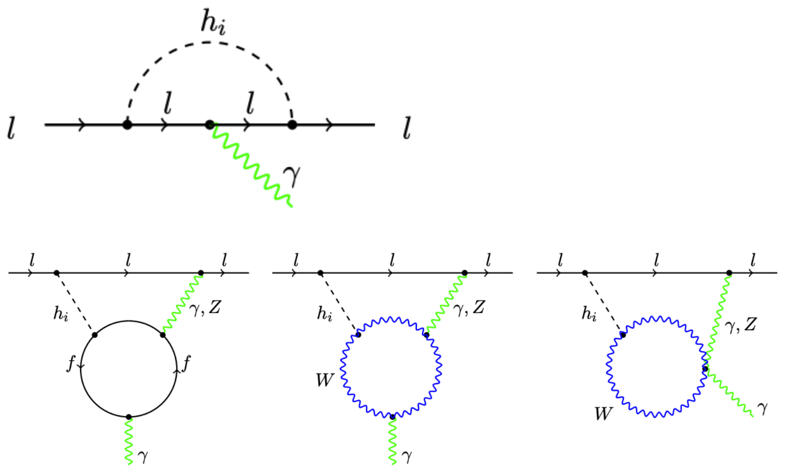
Tau g-2 in 2HDM + Singlet (NMSSM)

- Model
  - Two Doublets + Singlet

Higgs Basis

$$\hat{\Phi}_1 = \begin{pmatrix} G^+ \\ \frac{v+\phi_1+iG^0}{\sqrt{2}} \end{pmatrix}, \quad \hat{\Phi}_2 = \begin{pmatrix} H^+ \\ \frac{\phi_2+i\phi_3}{\sqrt{2}} \end{pmatrix}, \quad \hat{S} = \frac{1}{\sqrt{2}}(\omega + \phi_4 + i\phi_5)$$

- Contribution to g-2:



$$a_\ell^{1loop} = -\frac{m_\ell^2}{8\pi^2}\sum_{i=1}^{n}\int_0^1 dx\int_0^x dy \frac{y(y-1)\left|y_\ell^{h_i}\right|^2 + (y-1)Re\left(\left(y_\ell^{h_i}\right)^2\right)}{m_\ell^2[y(y-x)+(1-y)] + m_{h_i}^2 y}$$

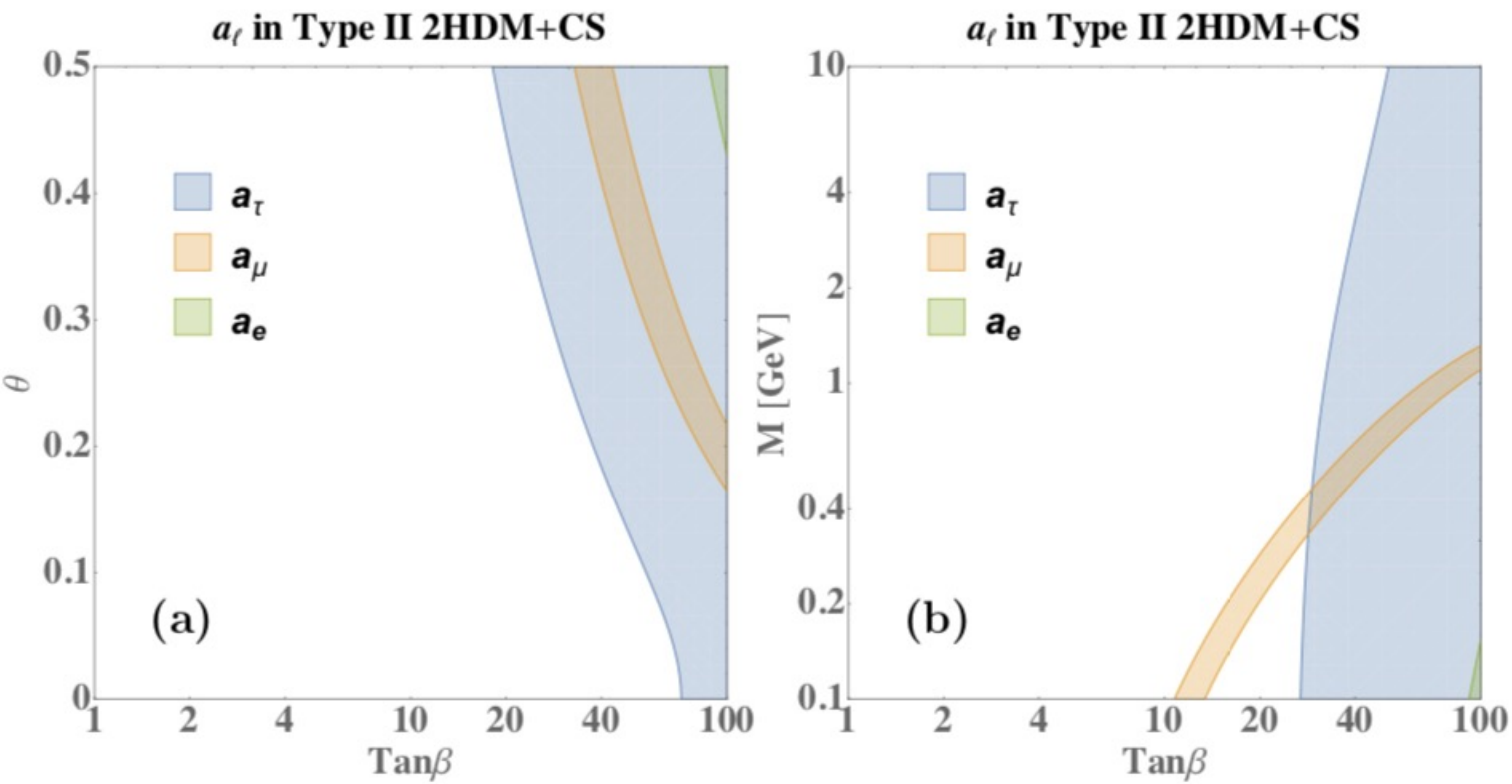
$$a_\ell^{2loop} = \sum_{f,i}\frac{2\alpha G_F v^2 m_\ell}{3\sqrt{2}\pi^3 m_f}\left[Re\left(y_\ell^{h_i}\right)Re\left(y_f^{h_i}\right)f(z_{fh_i}) - Im\left(y_\ell^{h_i}\right)Im\left(y_f^{h_i}\right)g(z_{fh_i})\right]$$

$$a_{\ell,W}^{2loop} = -\left(\frac{\alpha G_F v m_\ell}{4\sqrt{2}\pi^3}\right)\sum_i \frac{g_W^{h_i}}{2m_W^2/v} Re\left(y_\ell^{h_i}\right)\left[3f(z_{Wh_i}) + \frac{23}{4}g(z_{Wh_i}) + \frac{3}{4}h(z_{Wh_i}) + \frac{f(z_{Wh_i}) - g(z_{Wh_i})}{2z_{Wh_i}}\right]$$

JHEP 09 (2018) 059

Tau g-2 in 2HDM + Singlet (NMSSM)

- Compare with muon and electron

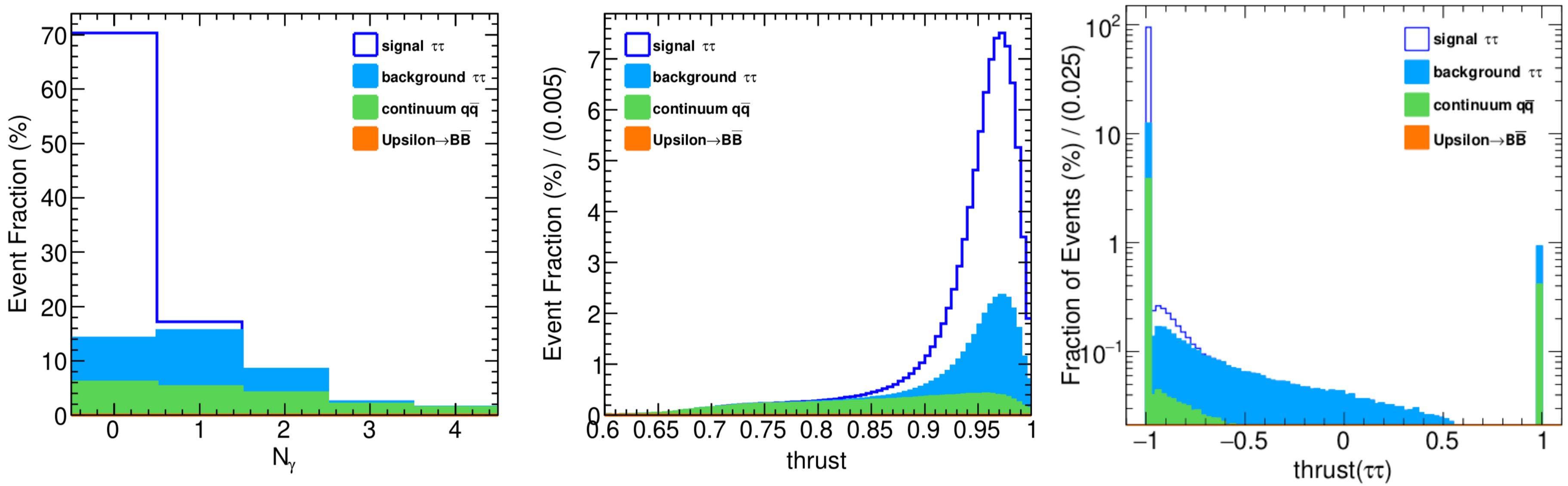


Summary

- EDM/Anomaly Magnetic Moment are sensitive to BSM
- Room of improvement for tau lepton measurement
  - Maybe more sensitive to BSM – Larger mass
  - Reconstruction of the neutrinos
    - Full matrix element – Optimal Observable
- In some case, tau lepton measurement can be more sensitive than muon/electron measurements

Backups

Event Selections



$$\text{thrust} = \max_{\mathbf{n}_{\text{thr}}} \frac{\sum_i |\mathbf{n}_{\text{thr}} \cdot \mathbf{p}_i|}{\sum_i |\mathbf{p}_i|}, \quad \text{thrust}_N(\tau_i) = \frac{\sum_j \mathbf{n}_{\text{thr}} \cdot \mathbf{p}_{ij}}{\sum_j \left|\mathbf{n}_{\text{thr}} \cdot \mathbf{p}_{ij}^{+}\right| + \sum_j \left|\mathbf{n}_{\text{thr}} \cdot \mathbf{p}_{ij}^{-}\right|},$$

Event Counts

Table 1. The effective cross sections for different processes after the selection cuts, in different $\tau^+\tau^-$ decay modes.

Mode	Signal $\tau^+\tau^-$ (pb)	Background $\tau^+\tau^-$ (pb)	Continuum (pb)	Upsilon (fb)
$a_1 + a_1$	3.09	0.00	0.22	0.37
$a_1 + \rho$	16.14	0.39	0.73	1.16
$a_1 + \pi$	9.30	0.70	0.42	0.59
$\pi + \pi$	7.42	2.50	0.51	0.68
$\pi + \rho$	24.13	3.16	0.98	1.01
$\rho + \rho$	20.96	1.20	0.73	1.19
Total	81.04	7.95	3.58	4.99