



***On possibility
to measure all
Michel
parameters at
STCF***

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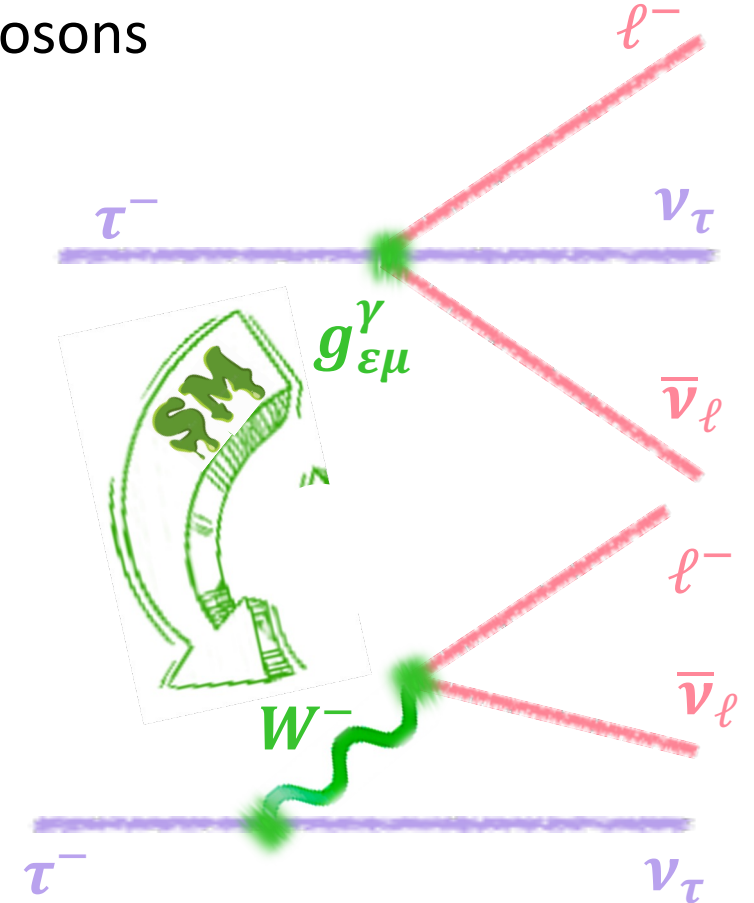
$\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau$ decays

- The most general low-energy expression for the decay matrix element for 4-fermion interaction mediated by heavy (scalar, vector and/or tensor) bosons

$$M = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \varepsilon,\mu=L,R}} g_{\varepsilon\mu}^\gamma \langle \bar{\ell}_\varepsilon | \Gamma^\gamma | (\nu_\ell)_\alpha \rangle \langle (\bar{\nu}_\tau)_\beta | \Gamma^\gamma | \tau_\mu \rangle$$

$$\Gamma^S = 1, \quad \Gamma^V = \gamma^\mu, \quad \Gamma^T = \frac{1}{\sqrt{2}} \sigma^{\mu\nu} = \frac{i}{2\sqrt{2}} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$$

- The only nonzero term in the SM: $g_{LL}^V = 1$
- Precise low-energy tests of $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau$ provide a window through which we can take a glance into the nature of the interactions at higher energies.
- Deviations can be caused by anomalous coupling of the W -boson, new gauge or charged Higgs bosons, presence of massive neutrinos...

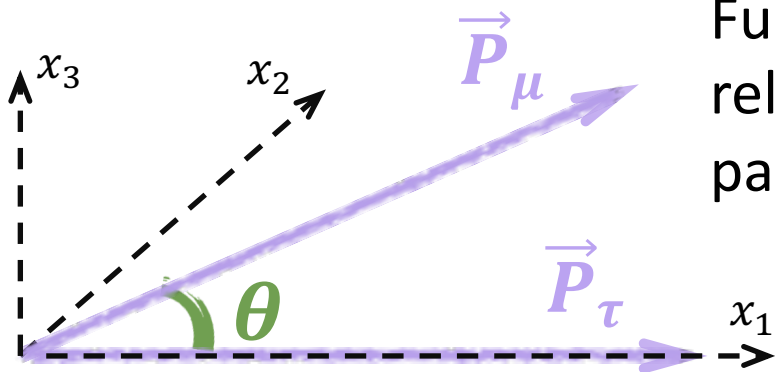


Michel parameters

- At experiment we measure some kinematical characteristics of decay products (charged daughter lepton). The distributions are more convenient to express in terms of Michel parameters, which are bilinear combinations of coupling constants.
- Differential decay width of τ lepton integrated over neutrino momenta:

$$\frac{d^2\Gamma}{dx d\cos\theta} = \frac{m_\tau}{4\pi^3} W_{\ell\tau}^4 G_F^2 \sqrt{x^2 - x_0^2} \left(F_{IS}(x) \pm F_{AS}(x) P_\tau \cos\theta + F_{T_1}(x) P_\tau \sin\theta \zeta_1 \right. \\ \left. + F_{T_2}(x) P_\tau \sin\theta \zeta_2 + (\pm F_{IP}(x) + F_{AP}(x) P_\tau \cos\theta) \zeta_3 \right)$$

$$W_{\ell\tau} = \max E_\ell = \frac{m_\tau^2 + m_\ell^2}{2m_\tau}, \quad x = \frac{E_\ell}{\max E_\ell}, \quad x_0 = \frac{m_\ell}{\max E_\ell}, \quad P_\tau = |\mathbf{P}_\tau|$$



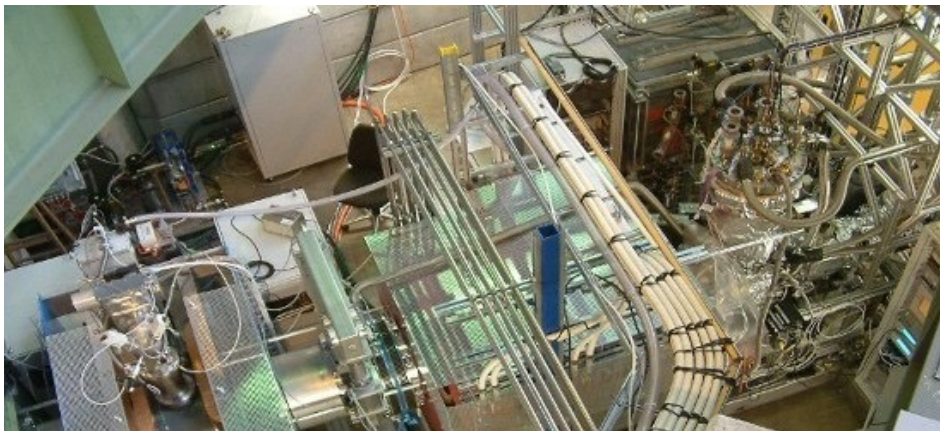
Functions $F(x)$ are related to Michel parameters:

$$\left. \begin{aligned} F_{IS}(x) &: \rho, \eta \\ F_{AS}(x) &: \xi, \xi\delta \\ F_{IP}(x) &: \xi', \xi, \xi\delta \\ F_{AP}(x) &: \xi'', \rho, \eta'' \\ F_{T_1}(x) &: \xi'', \rho, \eta, \eta'' \\ F_{T_2}(x) &: \alpha'/A, \beta'/A \end{aligned} \right\}$$

Need to measure daughter lepton polarization

Michel parameters status

MP	SM	$\mu \rightarrow e\nu_\mu\bar{\nu}_e$	$\tau \rightarrow e\nu_\tau\bar{\nu}_e$	$\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu$
ρ	0.75	0.74979 ± 0.00026	0.747 ± 0.010	0.747 ± 0.010
η	0	0.057 ± 0.034	0.013 ± 0.020	0.094 ± 0.073
$\delta(\xi\delta)$	0.75	0.75047 ± 0.00034	0.734 ± 0.028	0.778 ± 0.037
$\xi(\xi\delta/\rho)$	1	$1.0018_{-0.0007}^{+0.0016}$	0.994 ± 0.040	1.030 ± 0.059
ξ'	1	1.00 ± 0.04	2.6 ± 4.8	
ξ''	1	0.98 ± 0.04		6.2 ± 6.8
α'/A	0	-0.010 ± 0.020		
β'/A	0	0.002 ± 0.007		

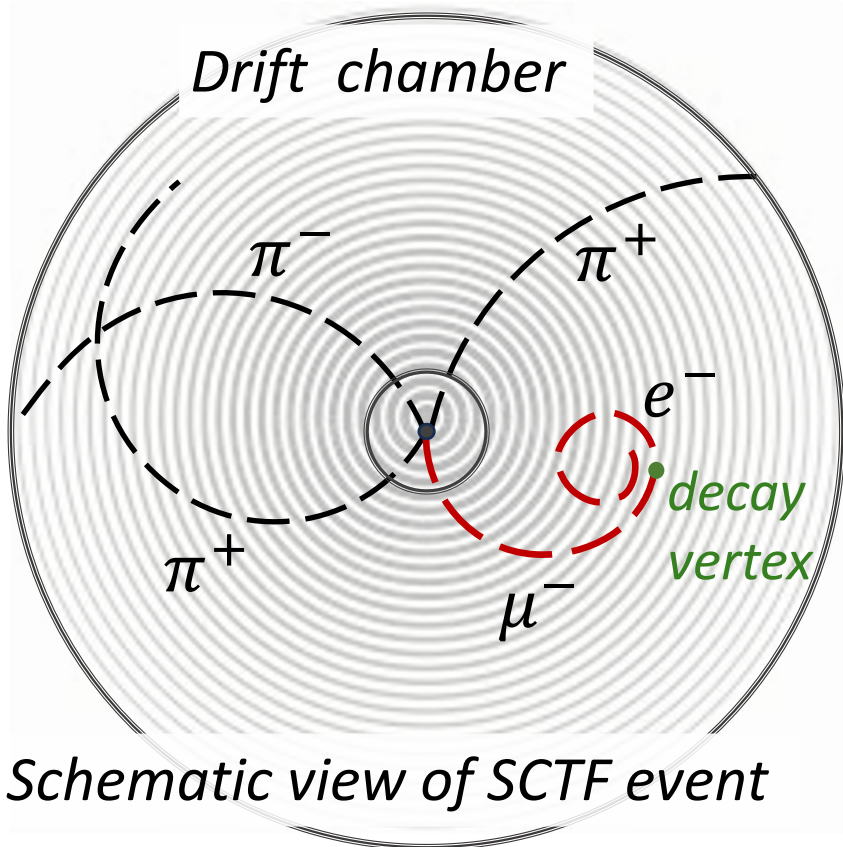


- in $\mu \rightarrow e\nu_\mu\bar{\nu}_e$ precise MP measurements at TWIST (Canada) and PSI (Switzerland)
- in $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau$ MP not related to daughter lepton polarization are measured quite precisely at LEP, CLEO, Argus.

Method of the muon polarization measurement

is based on $\mu \rightarrow e \nu_\mu \bar{\nu}_e$ decay-in-flight reconstruction in the tracker as a track kink.

- The information about muon spin can be inferred from the μ daughter (τ granddaughter) electron direction in the muon rest frame due to P-violation in the μ weak decay



- Muons from τ decays will fly hundreds of meters before decay \Rightarrow events where muon decays at ~ 1 meter from IP (DC size) will be VERY rare
- But even more critical problem: if even $\tau\bar{\tau}$ events can be selected with high purity over continuum/charm background, decays-in-flight of pions/kaons from τ is much more probable than muon decays: π -lifetime is 100 times shorter! $\tau_\pi : \tau_K : \tau_\mu \approx 1 : 0.5 : 100$
- $\frac{dE}{dx}$ in DC to separate π/μ ? No chance!

hopeless?

How many such events at STCF?

- Assuming $\mathcal{L} \sim 10 \text{ ab}^{-1}$ above $\tau\bar{\tau}$ threshold, one can collect $2 \cdot 10^{10}$ $\tau\bar{\tau}$ pairs. Huge number!
- We need a muon to decays in DC volume so that both the mother muon and the daughter electron can be reconstructed. Assume that if the decay took place in the middle third (along the radius) of the DC, there are enough hits to reconstruct both mother muon and daughter electron tracks. Thus allowed decay volume is ~ 40 cm. Assume that muon decays at the first turn in the magnetic field (otherwise track pattern recognition is too sophisticated). Thus allowed decay time is $\lesssim 10$ ns. Probability to decays in this range is $10^{-3} - 10^{-4}$ depending on muon's γ -factor.
- In spite of tiny efficiency still huge sample for analysis: \sim million of selected candidates \Rightarrow we can hope to achieve (statistical) accuracy better than 1%, if we manage to
 - handle backgrounds
 - control systematics

Backgrounds

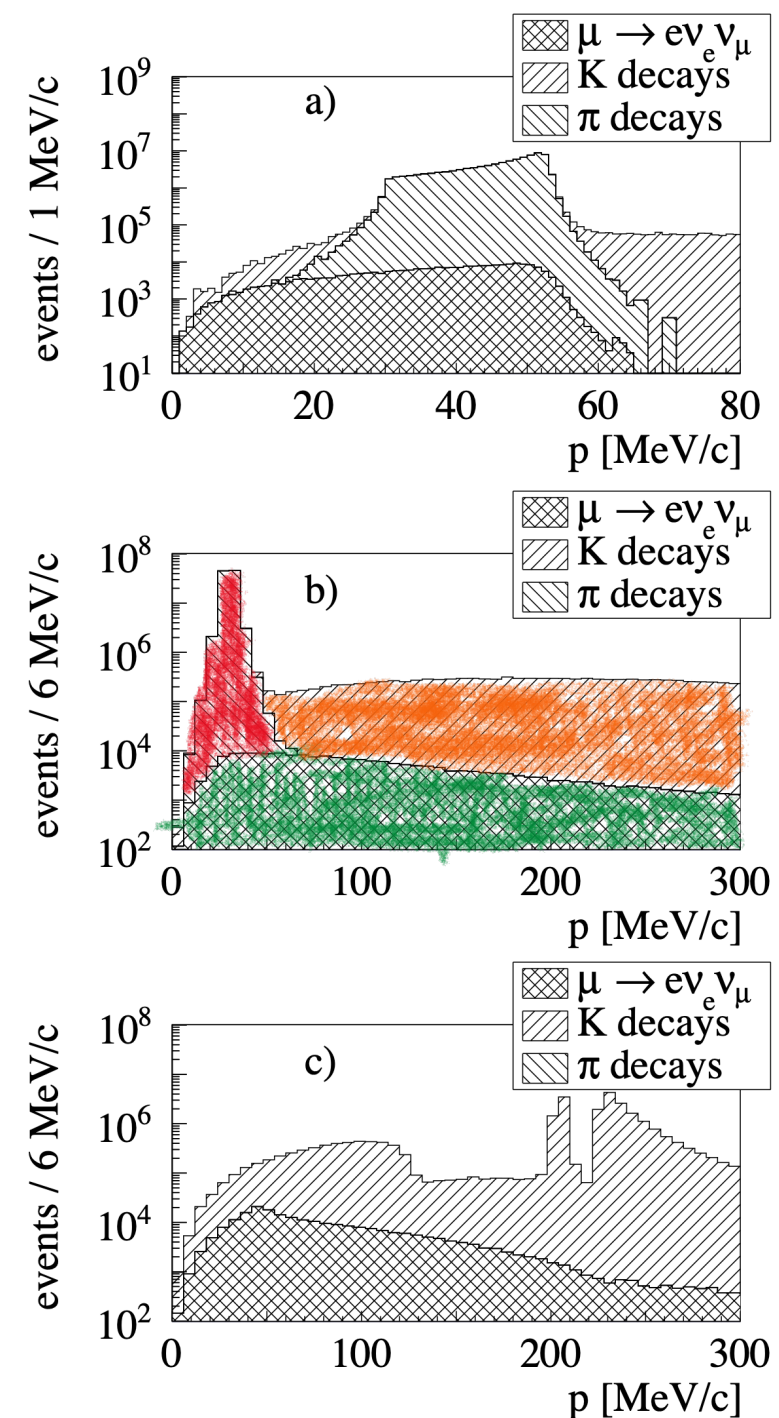
- Non $\tau\bar{\tau}$ events: BES/CLEOc/KEDR experience proves that this background can be effectively suppressed with high efficiency for all $\tau\bar{\tau}$ topologies
- Random cross of two tracks in the DC volume: tracks topology is different – both tracks from IP, they are not started/ended at the crossing point
- Real kinks from $\tau\bar{\tau}$ events:
 - Electron scattering on DC wires/gas ($\mathcal{B}(\tau \rightarrow e\bar{\nu}\nu) = \mathcal{B}(\tau \rightarrow \mu\bar{\nu}\nu)$; DC radiation length $\lesssim 0.01X_0$)
 - Hadronic scattering on DC wires/gas (*# of hadrons is ~5 times larger than muons in $\tau\bar{\tau}$ events, but nuclear length of DC is much smaller than radiation length*)
 - $\pi^\pm \rightarrow \mu^\pm\nu$ (*# of π 's is ~5 times larger than muons, $\tau_\pi:\tau_\mu \approx 1:100$*)
 - $K^\pm \rightarrow \mu^\pm\nu$ (*# of kaons is ~ # of muons, $\tau_K:\tau_\mu \approx 1:50$, $\mathcal{B}(K \rightarrow \mu\bar{\nu}) = 64\%$*)
 - $K^\pm \rightarrow \pi^\pm\pi^0$ (*... , $\mathcal{B}(K \rightarrow \pi^+\pi^0) = 21\%$*)
 - $K^\pm \rightarrow X\ell^\pm\nu_\ell, 3\pi$ (*... , $\mathcal{B}(3 - \text{body}) = 15\%$*)

Background suppression

Real kinks can not be suppressed by any requirements on vertex quality, but fortunately all such backgrounds have specific kinematics!

- Elastic scattering is characterized by 0 momentum transfer
 $|\vec{P}_{secondary}| = |\vec{P}_{primary}|$, unlike decay $|\vec{P}_{sec}| < |\vec{P}_{prim}|$
- $\pi^\pm \rightarrow \mu^\pm \nu$, $K^\pm \rightarrow \mu^\pm \nu$, $K^\pm \rightarrow \pi^\pm \pi^0$ are two-body decays, thus $|\vec{P}_{sec}|$ spectrum is monochromatic in mother particle rest frame, BUT only in proper mass (for both mother and daughter) hypotheses. Thus, the strategy is to veto monochromatic peaks in three spectra, corresponding to these three processes
- 3-body kaon decays are small, and primary kaon track can be identified with $\frac{dE}{dx}$ measurements

Our estimate: with $\frac{\sigma_p}{p} \sim 1\%$ all these processes can be vetoed (5σ) keeping signal efficiency $> 80\%$



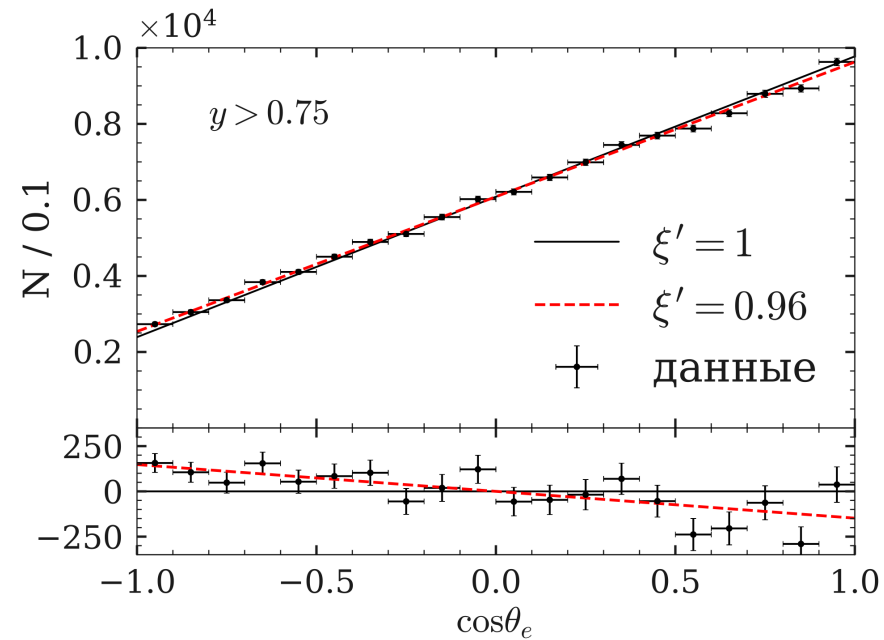
Potential application at STCF

[1] [Phys.Atom.Nuclei 84 \(2021\) 212–215](#)

[2] [J. High Energy Phys. 2022 \(10\) 035 \(2022\)](#)

- The expected number of signal events is $N \approx 5 \cdot 10^5$ at full luminosity. Assume 80% polarization of positron beam at STCF \Rightarrow achieved accuracy (assuming ρ , η , ξ , and $\xi\delta$ will be measured with accuracy $\sim 10^{-3}$):

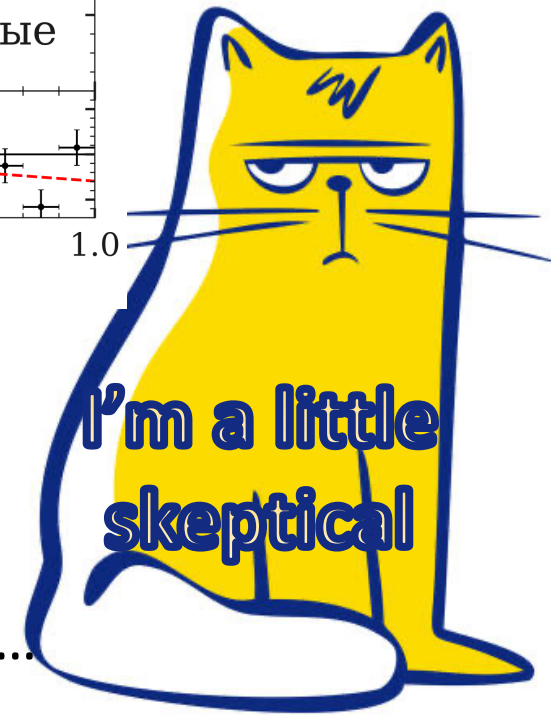
MP	SM	$\mu \rightarrow e\nu_\mu\bar{\nu}_e$	$\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu$
ξ'	1	1.00 ± 0.04	$? \pm 0.006$
ξ''	1	0.98 ± 0.04	$? \pm 0.03$
ξ'''	0	-0.010 ± 0.020	$? \pm 0.02$
α'/A	0	-0.010 ± 0.020	$? \pm 0.014$
β'/A	0	0.002 ± 0.007	$? \pm 0.007$



- New physics leading to $\xi' = 0.96 < 1$ can be discovered with 5σ

You might be skeptical... Indeed, we use just toy MC. Could our estimates be close to reality?

We would like to prove that you must be skeptical of your skepticism...



Measurement of ξ' in $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ at Belle

Belle experiment is a 4-pi spectrometer at e^+e^- asymmetric energies collider KEKB $E_{cms} \sim 10.6$ GeV

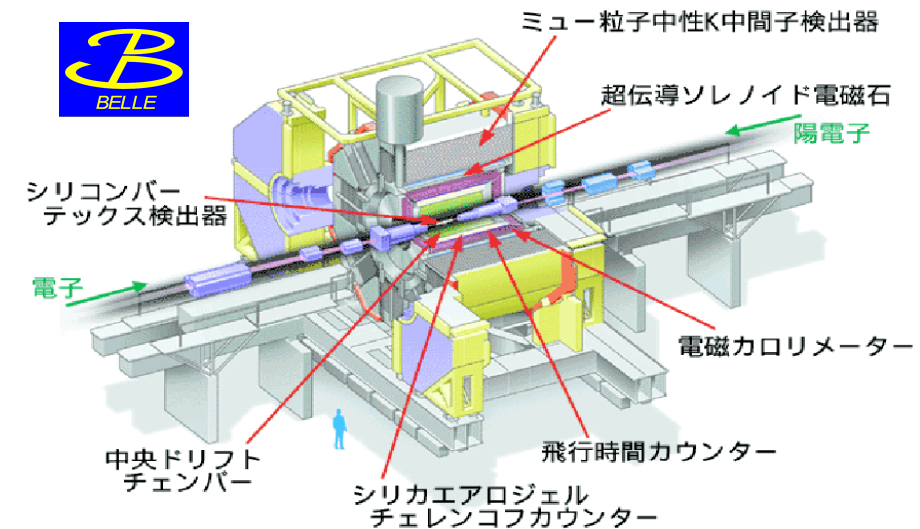
- $\mathcal{L} \sim 10^{34} / \text{cm}^2 / \text{s}$
- Integrated luminosity over 10 years of operation is 1 ab^{-1}
 $10^9 \tau\bar{\tau}$ pairs

Bad things for this task at Belle vs STCF

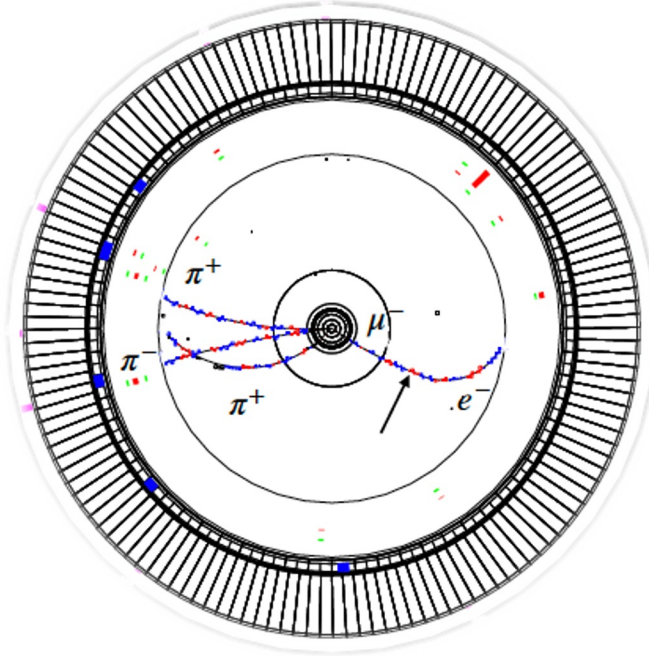
- τ -boost: 3 times larger γ -factor for muons – decay length larger, efficiency lower
- τ -boost: need to know τ -momentum – can be inferred from the second τ only approximately
- no special secondary track finding – efficiency for daughter electron is extremely low
- greedy algorithm to find primary tracks – hits from the secondary track are ascribed to the primary, which deteriorate both vertex and momentum resolution.

Good things for this task at Belle vs STCF

- τ -boost: two τ are separated topologically – slightly helpful for background suppression
- after 10 years of data taking and 20 years of analysis the detector is studied with high accuracy



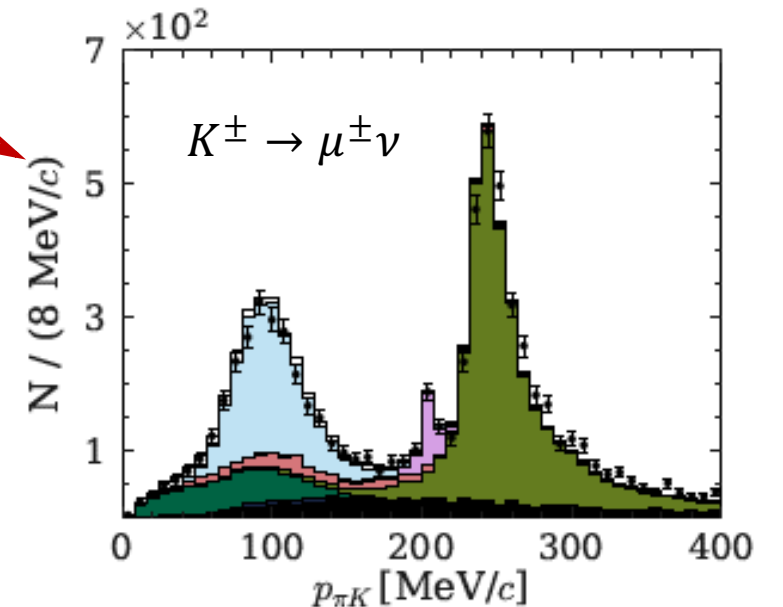
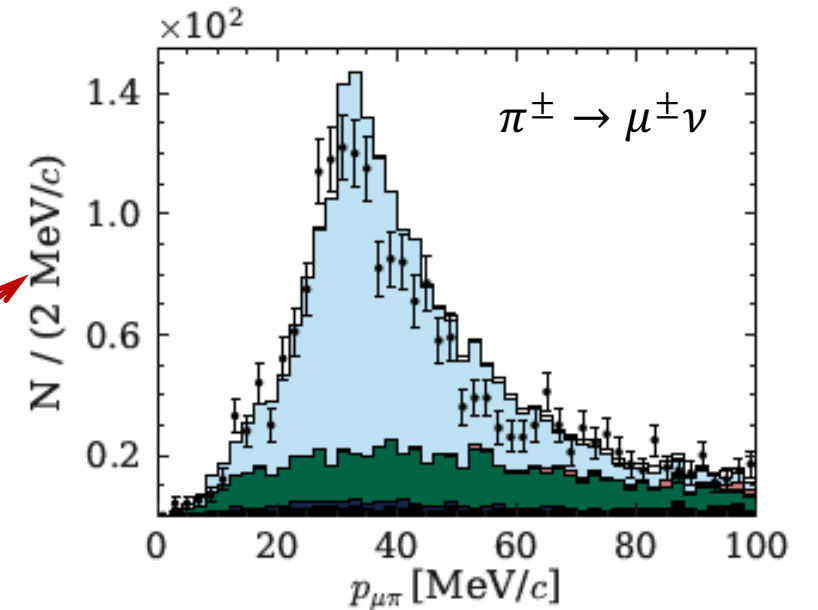
Measurement of ξ' in $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ at Belle



Signal MC event display

Daughter particle momentum in the mother rest frame for kink candidates in the selected $\tau\bar{\tau}$ sample at Belle

Backgrounds are seen as expected. While the momentum resolution is poor (much worse, than those with proper kink reconstruction), the peaks in the corresponding mass hypothesis are seen and MC well describe them.



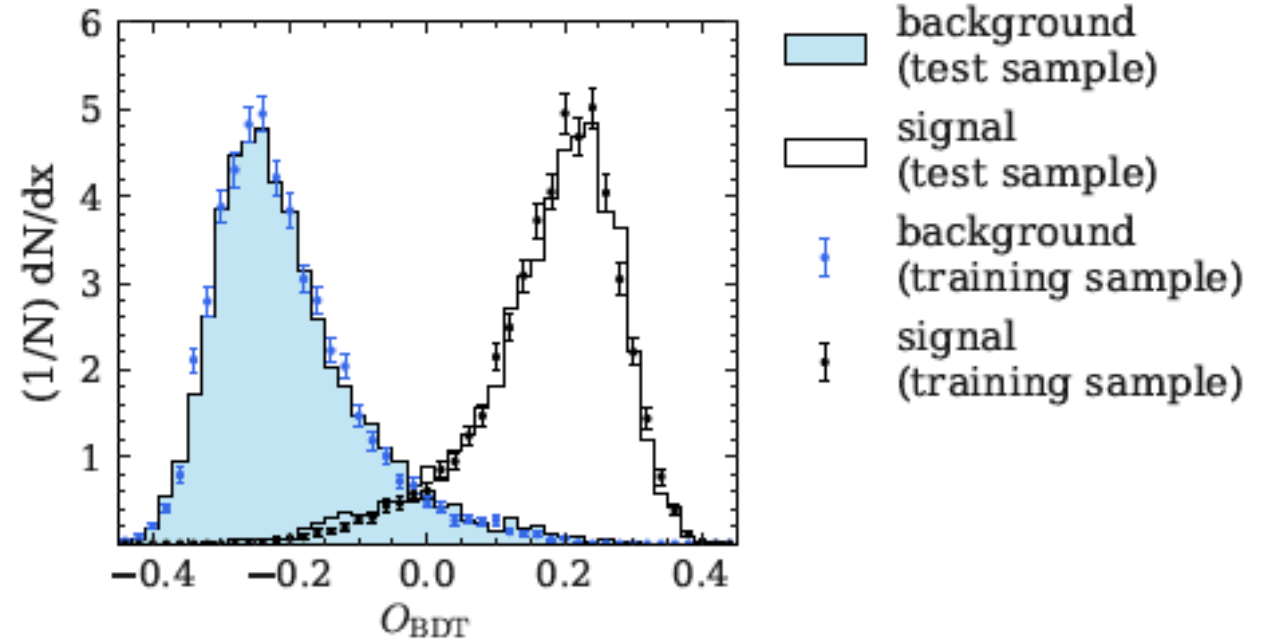
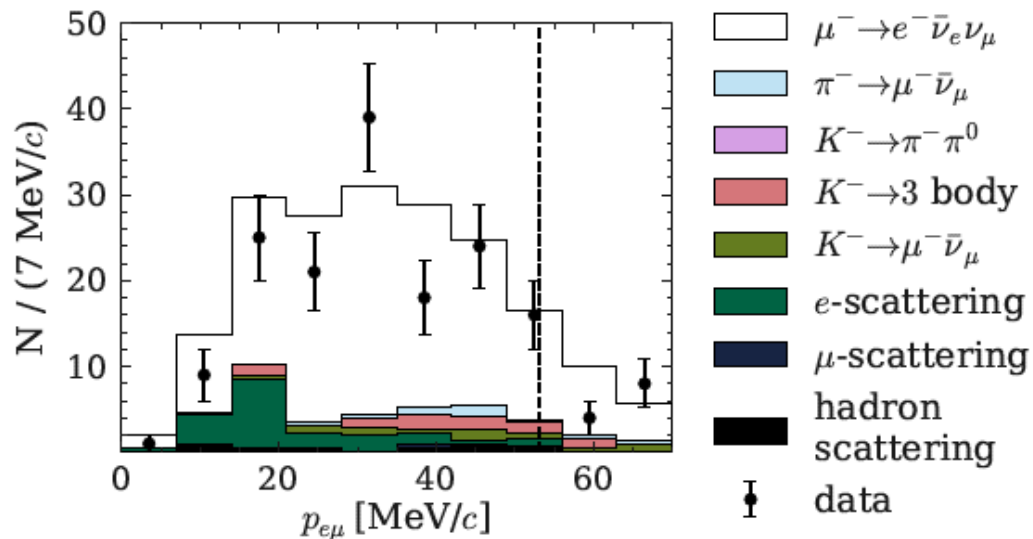
Measurement of ξ' in $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ at Belle

Use neural network to suppress backgrounds:

- BDT inputs: particle ID, and kinematics (momentum of daughter in the mother rest frame in different mass hypotheses)

Achieve:

- Signal efficiency 80%,
- All backgrounds are suppressed by a factor >50



Using BDT select 165 events in the data;
 MC expectation: 139 ± 2 signal 50 ± 5
 background events

Fit to the data

[1] [Phys.Rev.Lett. 131 \(2023\) 021801](#)

[2] [Phys.Rev.D 108 \(2023\) 012003](#)

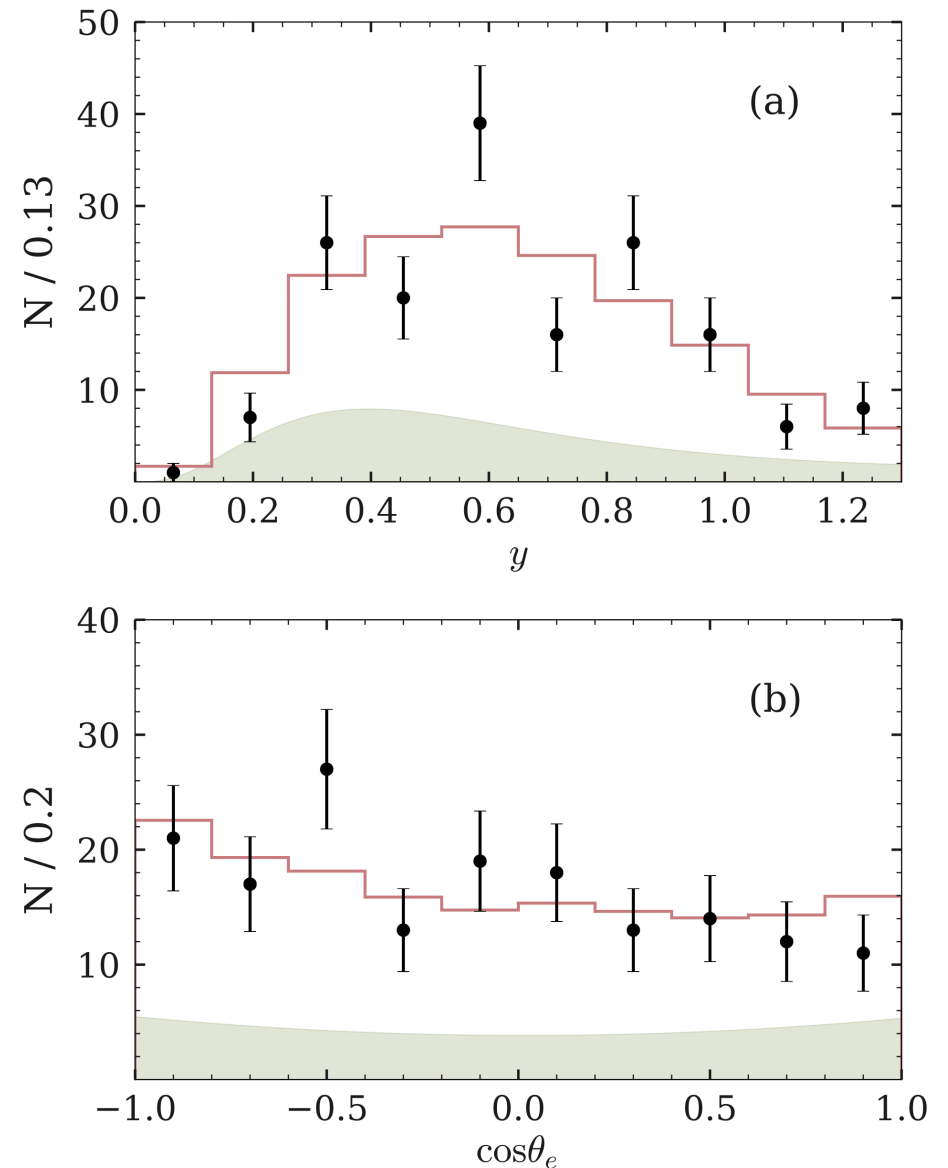
2D fit of y (reduced electron energy in muon decay) and electron emission angle with the only free parameter ξ'

$$\xi' = 0.22 \pm 0.94(\text{stat}) \pm 0.42(\text{syst})$$

The main loss in sensitivity is due to poor y resolution.

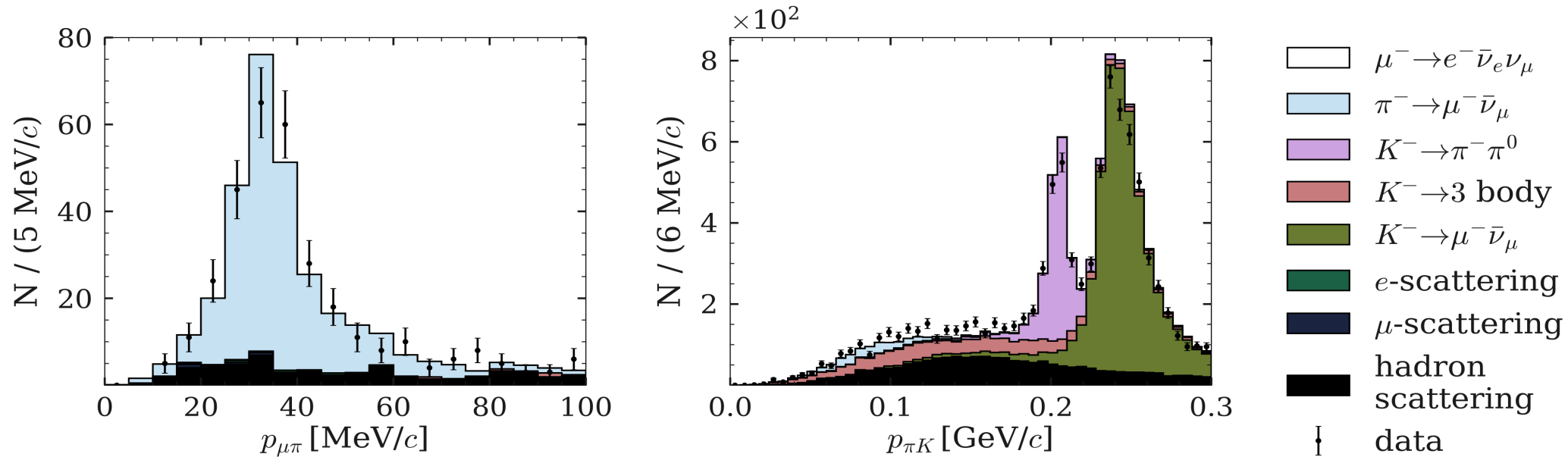
The only previous ξ' measurement was also done by Belle in radiative decays:

$$\xi' = -2.2 \pm 2.4$$



Fit projections on (a) reduced electron energy and (b) emission direction

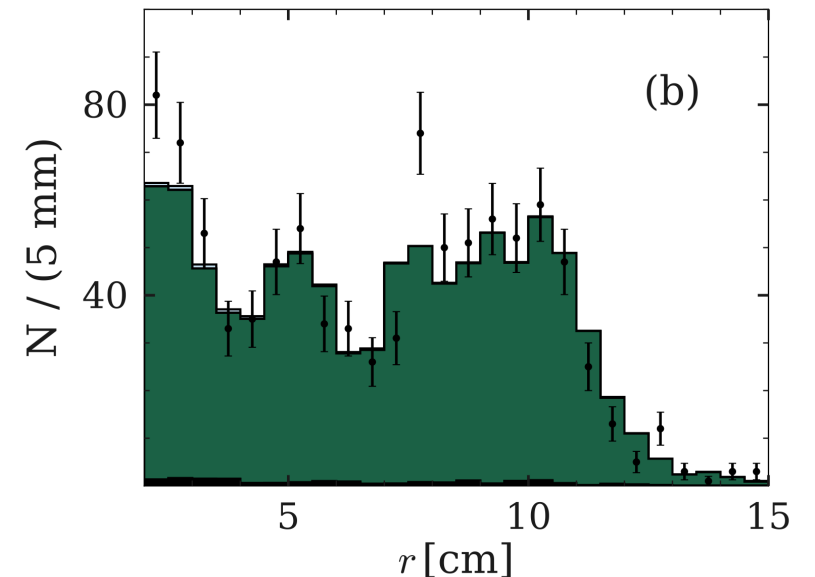
Tagged kink control samples



For all backgrounds tagged control samples are available

- Pion and kaon kinks selected from $D^0 \rightarrow K^- \pi^+$ decays
- Electron scattering selected from γ -conversion

The only missing control sample is signal muons. The good one $e^+ e^- \rightarrow \mu^+ \mu^- \gamma$ is unfortunately suppressed by trigger.



Lessons from Belle analysis for STCF

One needs:

- Big drift chamber to increase the number of reconstructed $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ decays
- Kink reconstruction algorithm (improves the sensitivity, reconstruction efficiency, and background suppression). It is also useful for hyperon (Σ^\pm, Ξ^-) reconstruction, to have absolutely clean $K^\pm \rightarrow \mu^\pm \nu$ sample
- Control sample of $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ events with $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ decays selection
- Polarized beam is crucial to obtain all the Michel parameters precisely and check the polarization in the control $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ sample.
- High decay-vertex resolution and good dE/dx separation to suppress background

Conclusion

- Method allows for the first measurement of all Michel parameters that describe daughter muon polarization in $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ decays
- STCF with polarized beam is an optimal experiment for precise measurement
- Experiment **without** polarized beam **complicates** the measurement and **degrades precision** for all discussed MPs except for ξ'
- Systematics can be controlled at least at the same level as statistics
- Uncertainties are comparable with one obtained in muon decays!
- Method can be further developed for other studies with muon polarization

Thank
you!!

謝謝