

τ Physics

Opportunities at the STCF

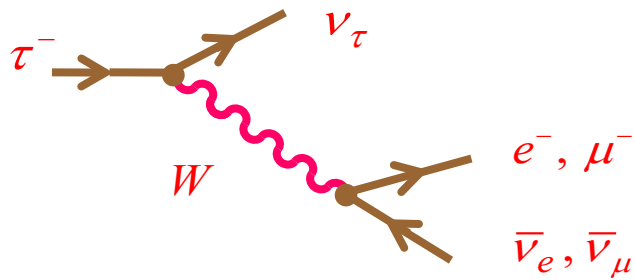
Antonio Pich

IFIC, Univ. Valencia - CSIC

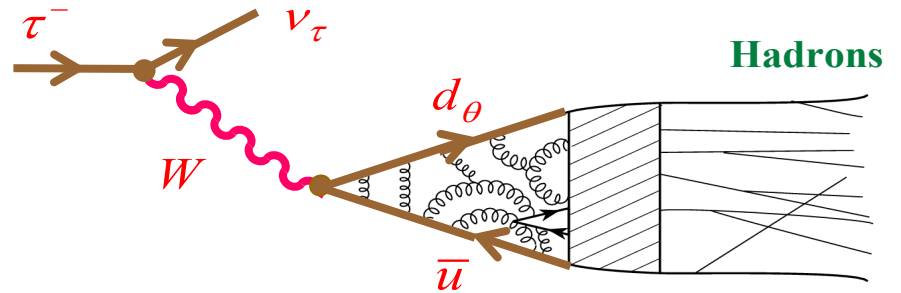


τ Physics

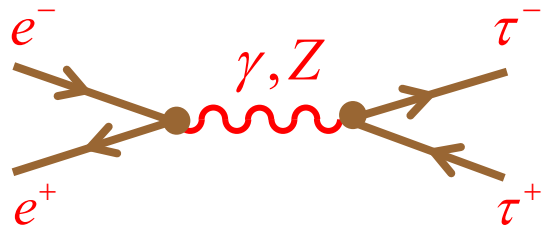
Decay



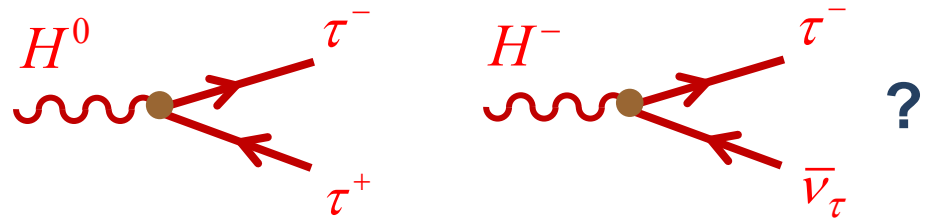
QCD



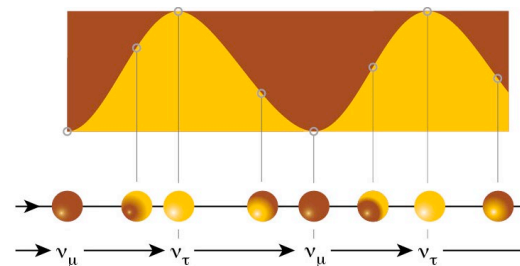
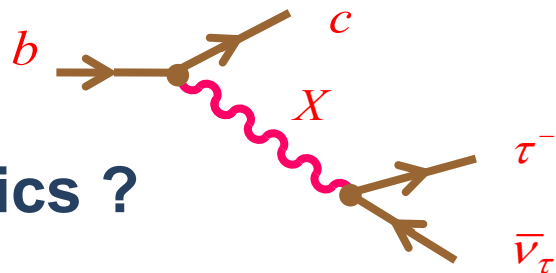
Production



Higgs Interactions



New Physics ?



Neutrinos


τ Data Samples

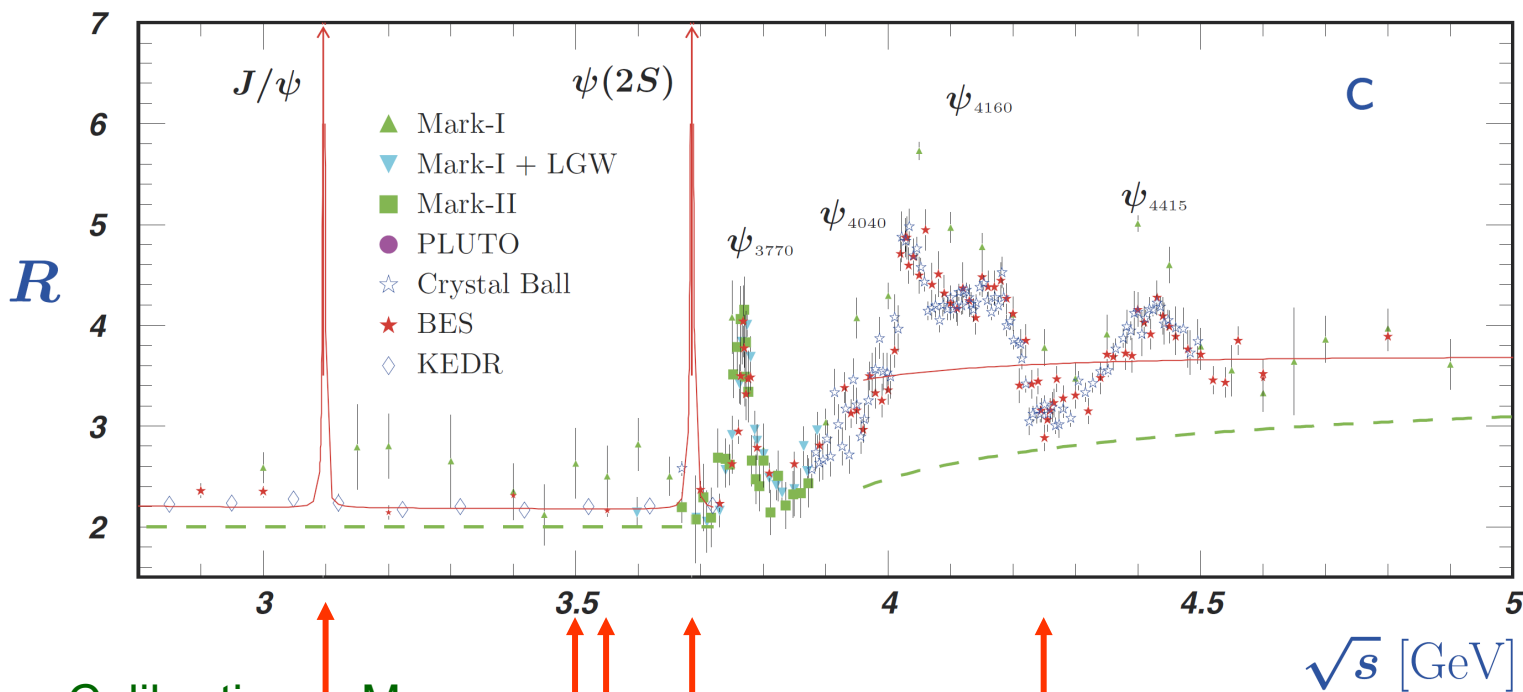
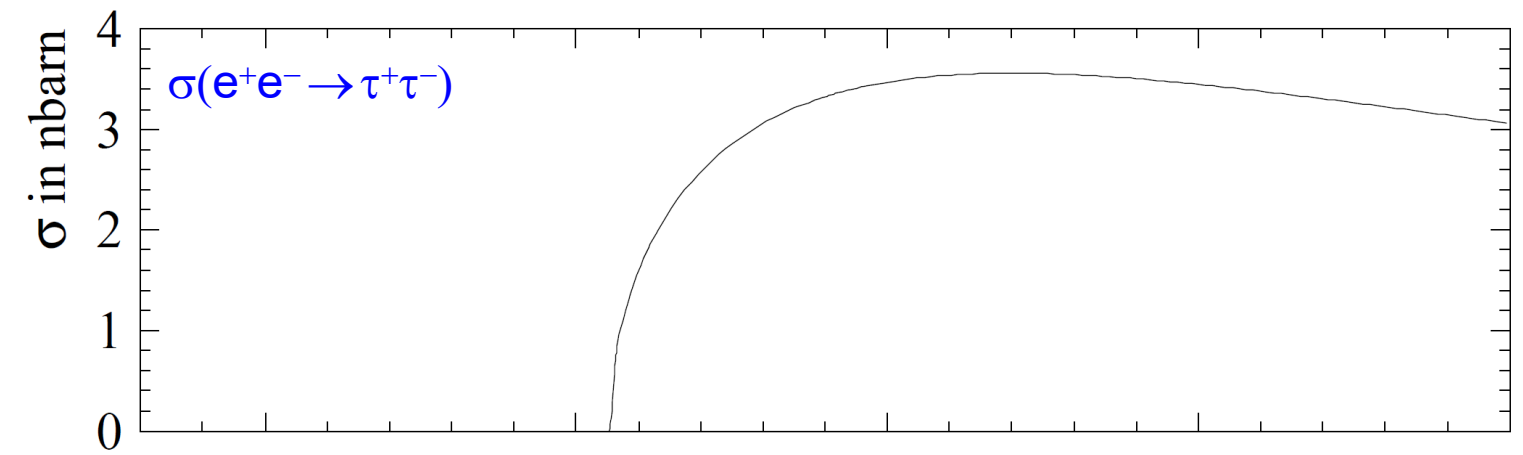
ALEPH:	$3.3 \cdot 10^5$	reconstructed τ decays	
BaBar / Belle:	$1.4 \cdot 10^9$	$\tau^+\tau^-$ pairs	
Belle-II:	$4.6 \cdot 10^{10}$	$\tau^+\tau^-$ pairs	
sτcF:	$2.1 \cdot 10^{10}$	$\tau^+\tau^-$ pairs	(10^8 near threshold)
Tera-Z:	$1.7 \cdot 10^{11}$	$\tau^+\tau^-$ pairs	

Luminosity is important. Systematics & backgrounds also!

Different experimental conditions at different energies ($\tau^+\tau^-$ threshold, ψ , Υ , Z)

Advantages of the threshold region:

- Ability to measure backgrounds (running below threshold)
- Free of heavy quark backgrounds
- Single-Tagging  Precise measurement of absolute branching fractions
- Monochromatic spectra for two-body decays (π , K)



Calibration

Measure τ backgr.

$\tau^+\tau^-$ Thr.

Highest $\sigma(\tau^+\tau^-)$ below open c

Highest $\sigma(\tau^+\tau^-)$

Ortho-ditauonium can be observed at the STCF

Hua-Sheng Shao

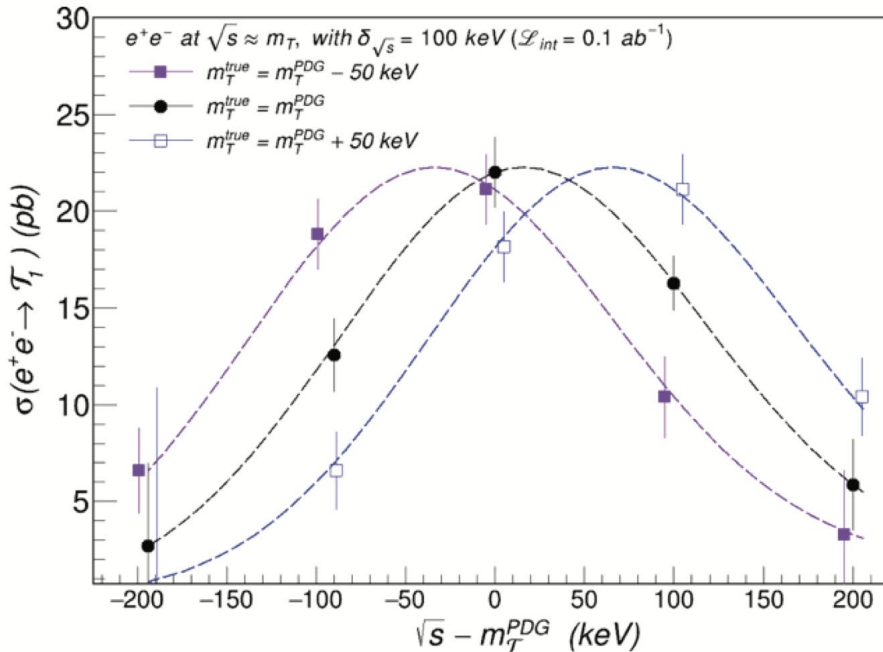
$\mathcal{T}_1 (1^3S_1, 1^{--})$

Threshold scan at e^+e^-

d'Enterria-Shao, 2302.07365

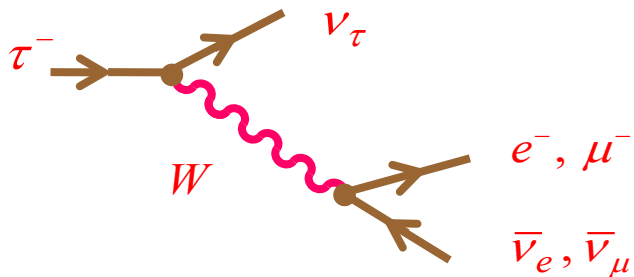
Colliding system, \sqrt{s} ($\delta_{\sqrt{s}}$ spread), \mathcal{L}_{int} , experiment	σ			N			S/\sqrt{B}
	\mathcal{T}_1	$\tau^+\tau^-$	$\mu^+\mu^-$	\mathcal{T}_1	$\mathcal{T}_1 \rightarrow \mu^+\mu^-$	$\mu^+\mu^-$	
e^+e^- at 3.5538 GeV (1.47 MeV), 5.57 pb ⁻¹ , BES III	1.9 pb	117 pb	6.88 nb	10.4	2.1	38 300	0.01 σ
e^+e^- at $\sqrt{s} \approx m_\tau$ (1.24 MeV), 140 pb ⁻¹ , BES III	2.2 pb	103 pb	6.88 nb	310	63	$9.63 \cdot 10^5$	0.06 σ
e^+e^- at $\sqrt{s} \approx m_\tau$ (1 MeV), 1 ab ⁻¹ , STCF	2.6 pb	95 pb	6.88 nb	$2.6 \cdot 10^6$	$5.3 \cdot 10^5$	$6.88 \cdot 10^9$	6.4 σ ←
e^+e^- at $\sqrt{s} \approx m_\tau$ (100 keV), 0.1 ab ⁻¹ , STCF	22 pb	46 pb	6.88 nb	$2.2 \cdot 10^6$	$4.5 \cdot 10^5$	$6.88 \cdot 10^8$	17 σ

$$m_\tau = (m_{\mathcal{T}} - E_{\text{bind}})/2 \longrightarrow \delta m_\tau = 25 \text{ keV}$$



The tau mass can be also precisely determined with monochromatization

LEPTONIC DECAYS



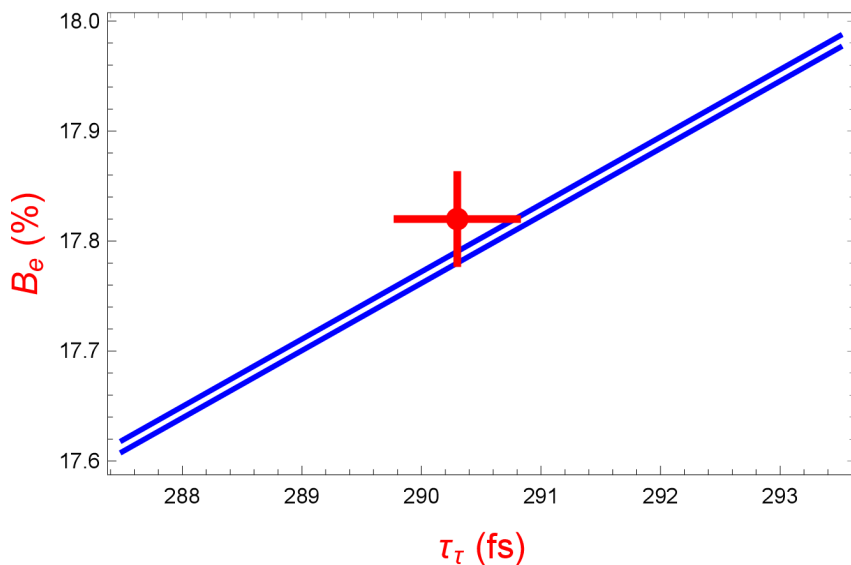
$$\Gamma(\tau \rightarrow \nu_\tau l \bar{\nu}_l) = \frac{G_F^2 m_\tau^5}{192 \pi^3} f(m_l^2/m_\tau^2) (1 + \delta_{\text{RC}})$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$



$$B_e = \frac{B_\mu}{0.972564 \pm 0.000003} = \frac{\tau_\tau}{(1632.3 \pm 0.5) \times 10^{-15} \text{ s}}$$

τ_τ (Belle), m_τ (Bes III, Belle II)



$$\left(B_\mu / B_e \right)_{\text{exp}} = 0.9762 \pm 0.0028$$

PDG 2023

Non-BF: 0.9725 ± 0.0039

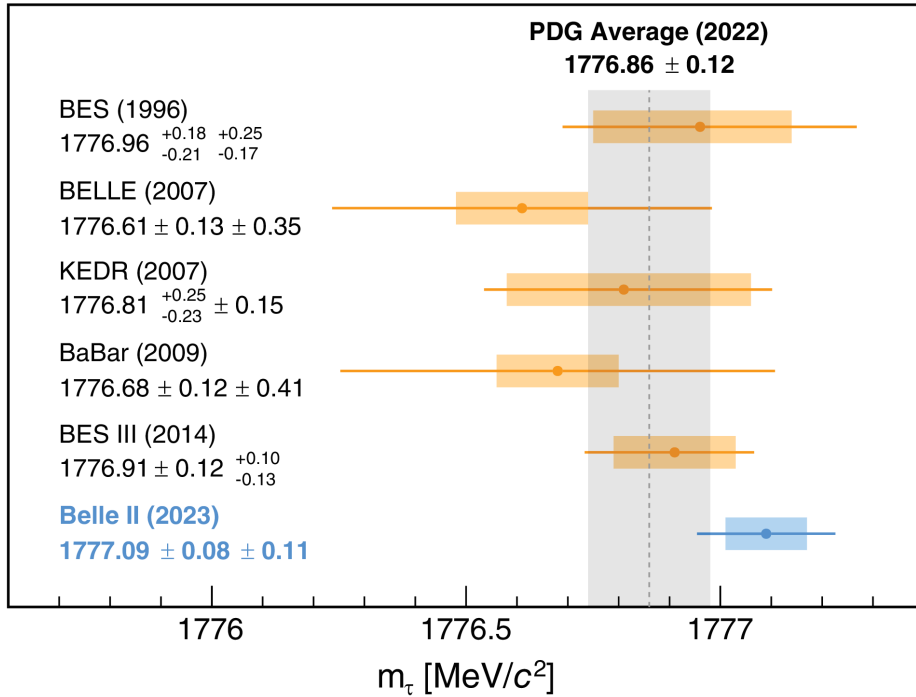
BaBar '10: 0.9796 ± 0.0039



$$B_e^{\text{univ}} = (17.812 \pm 0.022)\%$$

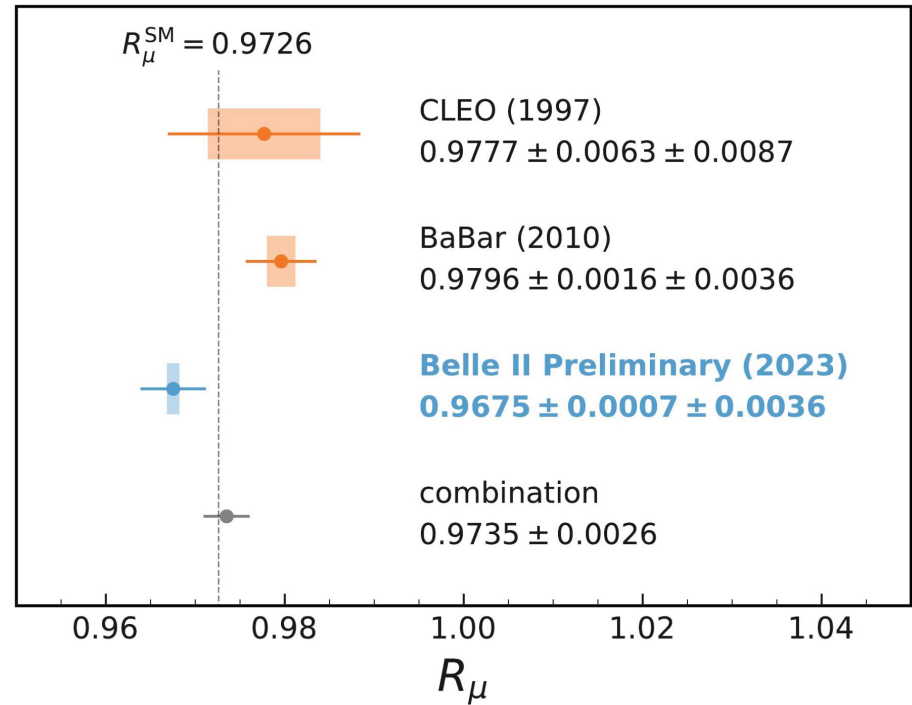
New Belle II measurements

2305.19116



P. Feichtinger, TAU2023

$$R_\mu = \frac{\mathcal{B}(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu(\gamma))}{\mathcal{B}(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma))}$$

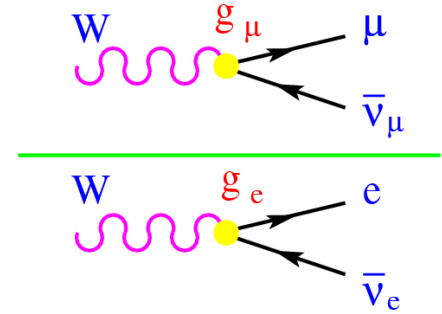
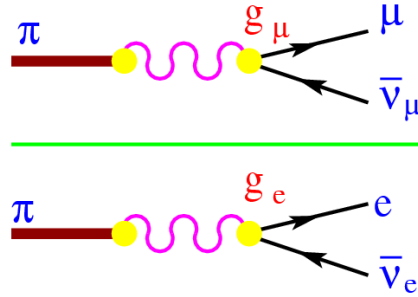
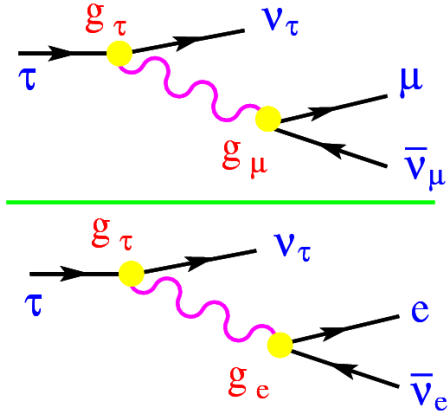


$m_\tau = (1776.96 \pm 0.09) \text{ MeV}$

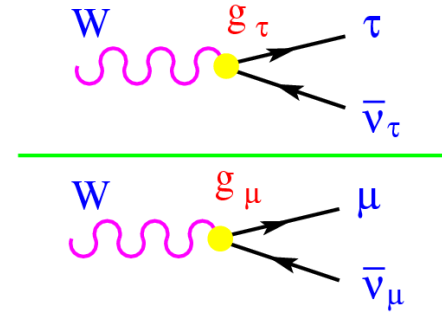
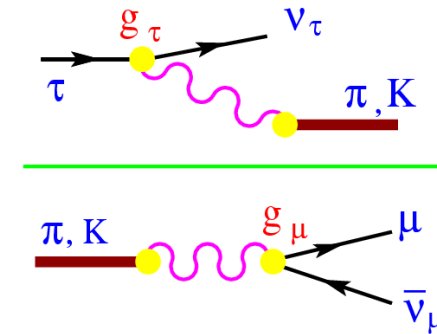
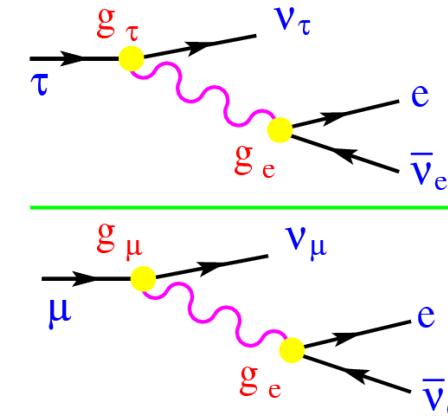
$|g_\mu / g_e| = 1.0005 \pm 0.0013$

Lepton Universality

$\frac{\sigma_{\mu}}{\sigma_{e}}$



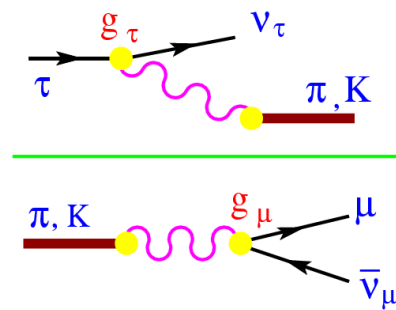
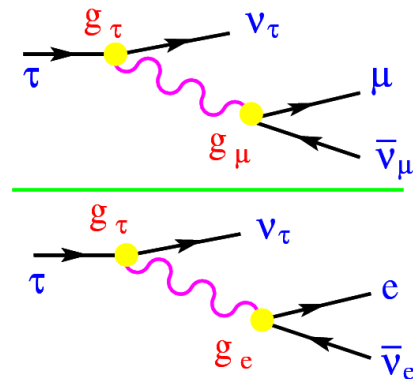
$\frac{\sigma_{\tau}}{\sigma_{\mu}}$



Lepton Universality

$$\left| g_\mu / g_e \right|$$

$B_{\tau \rightarrow \mu} / B_{\tau \rightarrow e}$	1.0017 ± 0.0016
$B_{\pi \rightarrow \mu} / B_{\pi \rightarrow e}$	1.0010 ± 0.0009
$B_{K \rightarrow \mu} / B_{K \rightarrow e}$	0.9978 ± 0.0018
$B_{K \rightarrow \pi\mu} / B_{K \rightarrow \pi e}$	1.0010 ± 0.0025
$B_{W \rightarrow \mu} / B_{W \rightarrow e}$	1.001 ± 0.003



$$\left| g_\tau / g_\mu \right|$$

$$\left| g_\tau / g_e \right|$$

$B_{\tau \rightarrow \mu} \tau_\mu / \tau_\tau$	1.0028 ± 0.0015
$B_{W \rightarrow \tau} / B_{W \rightarrow e}$	1.008 ± 0.012

$B_{\tau \rightarrow e} \tau_\mu / \tau_\tau$	1.0011 ± 0.0014
$\Gamma_{\tau \rightarrow \pi} / \Gamma_{\pi \rightarrow \mu}$	0.9964 ± 0.0038
$\Gamma_{\tau \rightarrow K} / \Gamma_{K \rightarrow \mu}$	0.986 ± 0.008
$B_{W \rightarrow \tau} / B_{W \rightarrow \mu}$	1.001 ± 0.010

Lorentz Structure: $l^- \rightarrow l'^- \bar{\nu}_{l'} \nu_l$

Effective Hamiltonian:

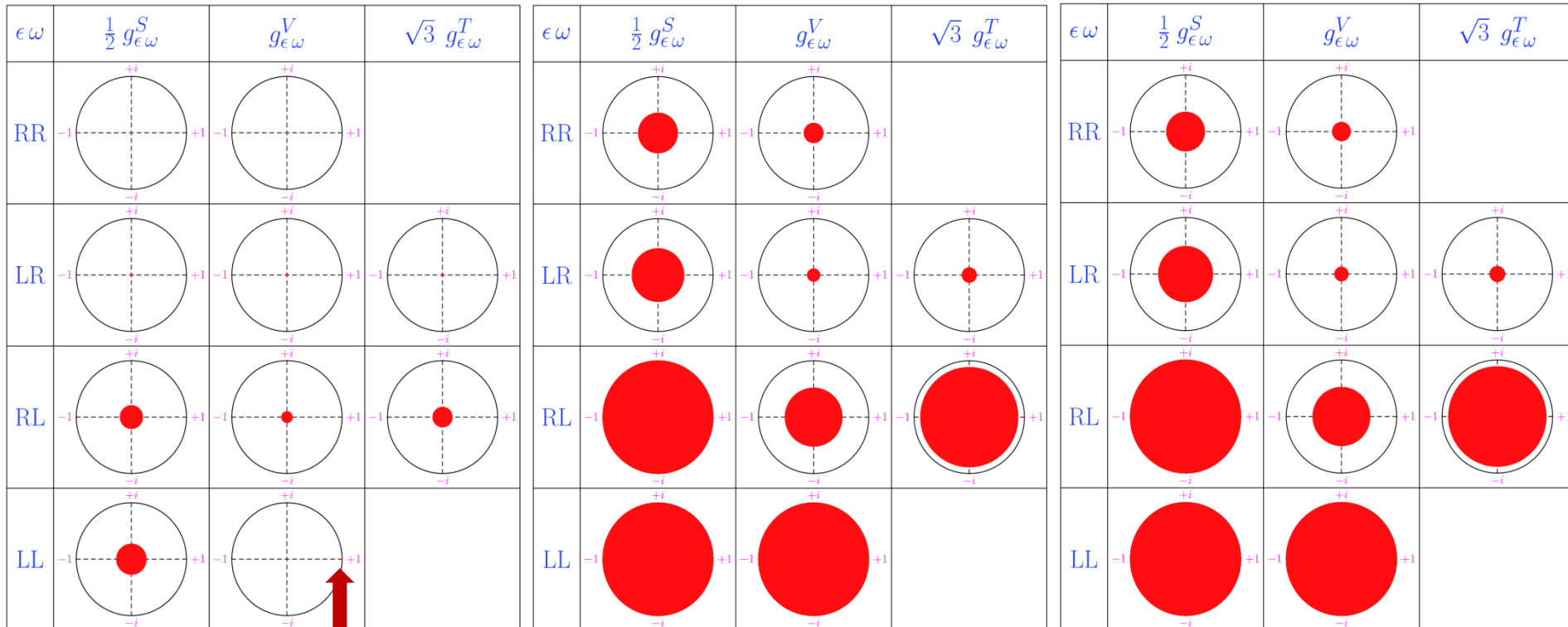
$$\mathcal{H} = 4 \frac{G_{\ell' \ell}}{\sqrt{2}} \sum_{n, \epsilon, \omega} g_{\epsilon \omega}^n [\bar{l}'_{\epsilon} \Gamma^n (\nu_{\ell'})_{\sigma}] \left[\overline{(\nu_{\ell})_{\lambda}} \Gamma_n l_{\omega} \right]$$

Normalization: $\Gamma \propto \frac{1}{4} (|g_{RR}^S|^2 + |g_{RL}^S|^2 + |g_{LR}^S|^2 + |g_{LL}^S|^2) + 3 (|g_{RR}^T|^2 + |g_{LR}^T|^2) + (|g_{RR}^V|^2 + |g_{RL}^V|^2 + |g_{LR}^V|^2 + |g_{LL}^V|^2) \equiv 1$

$\mu \rightarrow e \bar{\nu}_e \nu_{\mu}$

$\tau \rightarrow \mu \bar{\nu}_{\mu} \nu_{\tau}$

$\tau \rightarrow e \bar{\nu}_e \nu_{\tau}$



$|g_{LL}^V| > 0.960$ (90% CL)

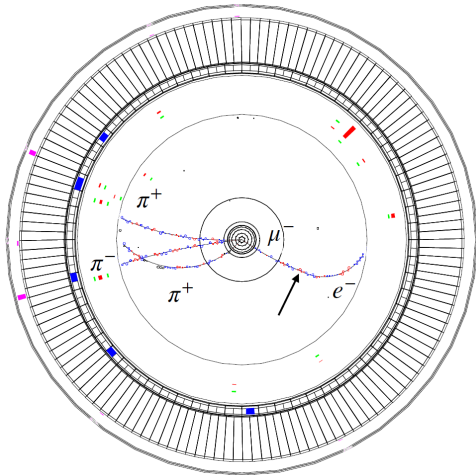
High-precision τ data needed!

μ^- Longitudinal Polarization in $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$

Probability to decay into a right-handed muon:

$$Q_{\mu_R} = Q_{RR} + Q_{RL} = \frac{1}{4} (|g_{RR}^S|^2 + |g_{RL}^S|^2) + 3|g_{RL}^T|^2 + |g_{RR}^V|^2 + |g_{RL}^V|^2 = \frac{1}{2} (1 - \xi')$$

MC $\tau^+\tau^-$



Tiny probability of muon decaying inside the detector compensated by huge statistics

$$\xi' = 0.22 \pm 0.94 \pm 0.42 \quad \text{Belle, 2303.10570}$$

➔

$$Q_{\mu_R} \leq 1.23 \quad (90\% \text{ CL})$$

Not yet constraining. Error dominated by statistics...

Estimated STCF sensitivity

with 80% positron polarization

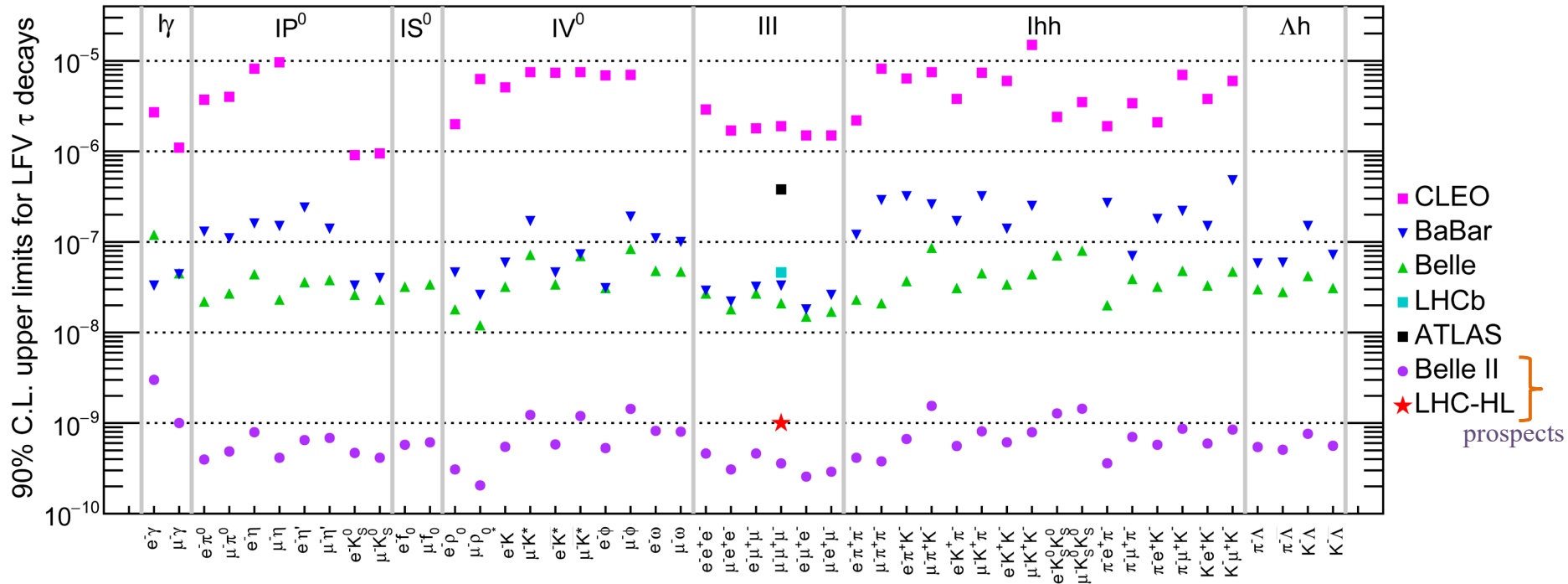
(assuming ρ , η , ξ and $\xi\delta$ are measured with 10^{-3} accuracy)

P. Pakhlov

MP	SM	$\mu \rightarrow e \nu_\mu \bar{\nu}_e$	$\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu$
ξ'	1	1.00 ± 0.04	$? \pm 0.006$
ξ''	1	0.98 ± 0.04	$? \pm 0.03$
ξ'''	0	-0.010 ± 0.020	$? \pm 0.02$
α'/A	0	-0.010 ± 0.020	$? \pm 0.014$
β'/A	0	0.002 ± 0.007	$? \pm 0.007$

Bounds on Lepton Flavour Violation

τ Decays (90% CL)



$$\text{Br}(\mu \rightarrow e \gamma) < 4.2 \times 10^{-13} \quad (\text{MEG, 90\% CL})$$

$$\text{Br}(K_L \rightarrow \mu e) < 4.7 \times 10^{-12} \quad (\text{BNL-E871, 90\% CL})$$

$$\text{Br}(B^0 \rightarrow e \mu) < 1.0 \times 10^{-9} \quad (\text{LHCb, 90\% CL})$$

$$\text{Br}(\mu \rightarrow 3e) < 1.0 \times 10^{-12} \quad (\text{SINDRUM, 90\% CL})$$

$$\text{Br}(K^+ \rightarrow \pi^+ \mu^+ e^-) < 1.3 \times 10^{-11} \quad (\text{BNL-E865, 90\% CL})$$

$$\text{Br}(D^0 \rightarrow e \mu) < 1.3 \times 10^{-8} \quad (\text{LHCb, 90\% CL})$$

$$\text{Br}(Z^0 \rightarrow e \mu) < 7.5 \times 10^{-7} \quad (\text{ATLAS, 95\% CL})$$

$$\text{Br}(Z^0 \rightarrow e \tau) < 5.0 \times 10^{-6} \quad (\text{ATLAS, 95\% CL})$$

$$\text{Br}(Z^0 \rightarrow \mu \tau) < 6.5 \times 10^{-6} \quad (\text{ATLAS, 95\% CL})$$

$$\text{Br}(H \rightarrow e \mu) < 6.1 \times 10^{-5} \quad (\text{ATLAS, 95\% CL})$$

$$\text{Br}(H \rightarrow e \tau) < 2.2 \times 10^{-3} \quad (\text{CMS, 95\% CL})$$

$$\text{Br}(H \rightarrow \mu \tau) < 1.5 \times 10^{-3} \quad (\text{CMS, 95\% CL})$$



CP Asymmetry

Xin-Qiang Li

$$A_\tau \equiv \frac{\Gamma(\tau^+ \rightarrow \pi^+ K_S \bar{\nu}_\tau) - \Gamma(\tau^- \rightarrow \pi^- K_S \nu_\tau)}{\Gamma(\tau^+ \rightarrow \pi^+ K_S \bar{\nu}_\tau) + \Gamma(\tau^- \rightarrow \pi^- K_S \nu_\tau)} = (-3.6 \pm 2.3 \pm 1.1) \cdot 10^{-3}$$

BaBar'11
($\geq 0\pi^0$)

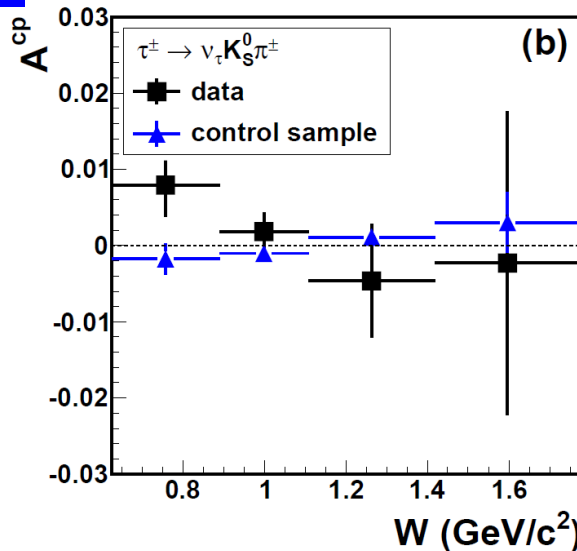
$$A_\tau^{\text{SM}}(\tau^+ \rightarrow \pi^+ K_S \bar{\nu}_\tau) = (3.6 \pm 0.1) \cdot 10^{-3}$$

Bigi-Sanda, Grossman-Nir

2.8 σ discrepancy



Belle does not see any asymmetry at the 10^{-2} level



$$A_i^{\text{CP}} \simeq \langle \cos \beta \cos \psi \rangle_i^{\tau^-} - \langle \cos \beta \cos \psi \rangle_i^{\tau^+}$$

bins (i) of $W = \sqrt{Q^2}$

$\beta = K_S$ direction in hadronic rest frame ; $\psi = \tau$ direction

**BaBar signal incompatible (with EFT)
with other sets of flavour data**

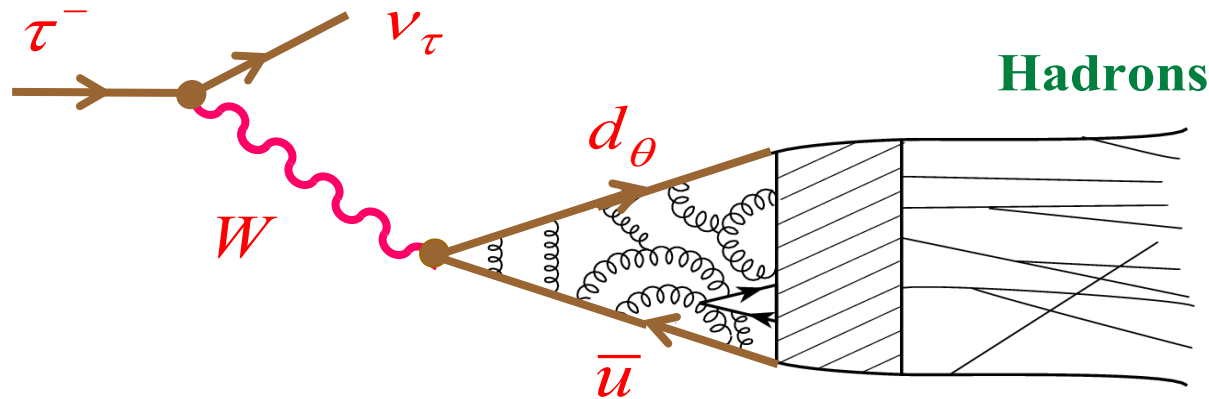
Cirigliano-Crivellin-Hoferichter, 1712.06595 ; Rendón-Roig-Toledo, 1902.08143

CP asymmetry in angular distribution sensitive to BSM scalar & tensor interactions

Measurable at Belle II & STCF?

X.-Q. Li et al

HADRONIC TAU DECAY



$$d_\theta = V_{ud} d + V_{us} s$$

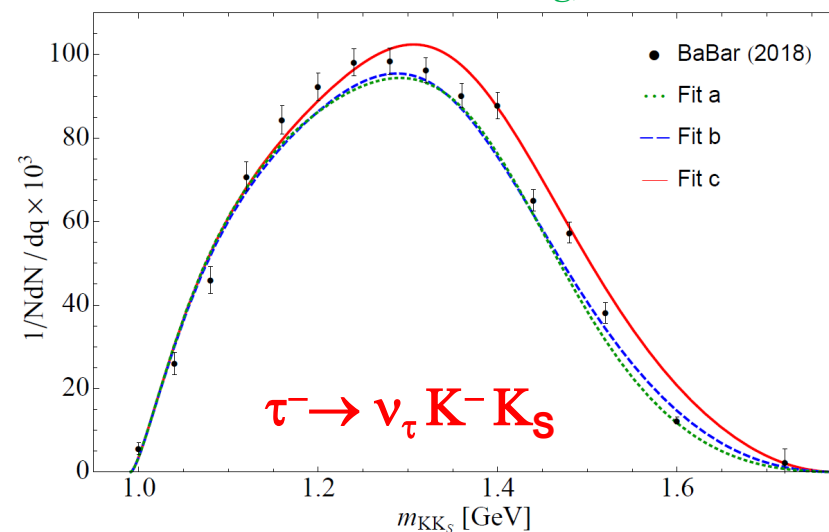
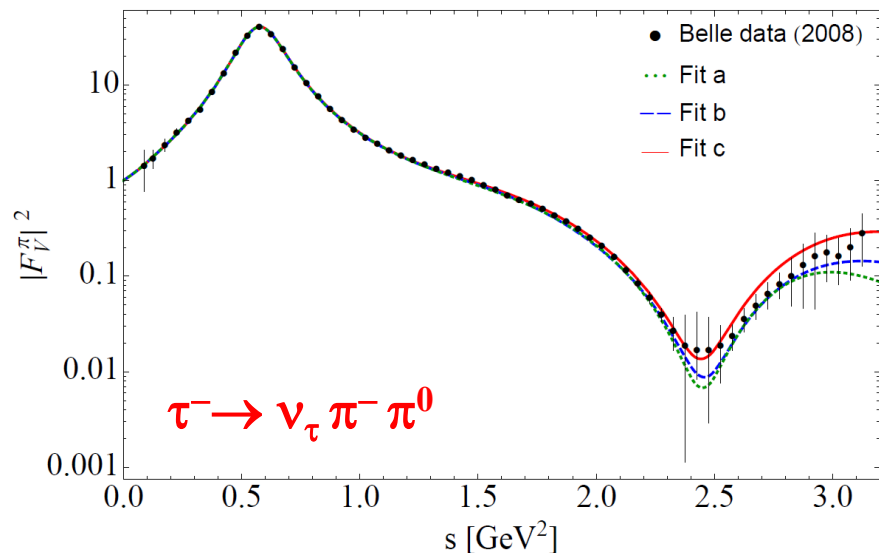
Only lepton massive enough to decay into hadrons

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{Hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \approx N_C \quad ; \quad R_\tau = \frac{1 - B_e - B_\mu}{B_e^{\text{univ}}} = 3.6381 \pm 0.0075$$

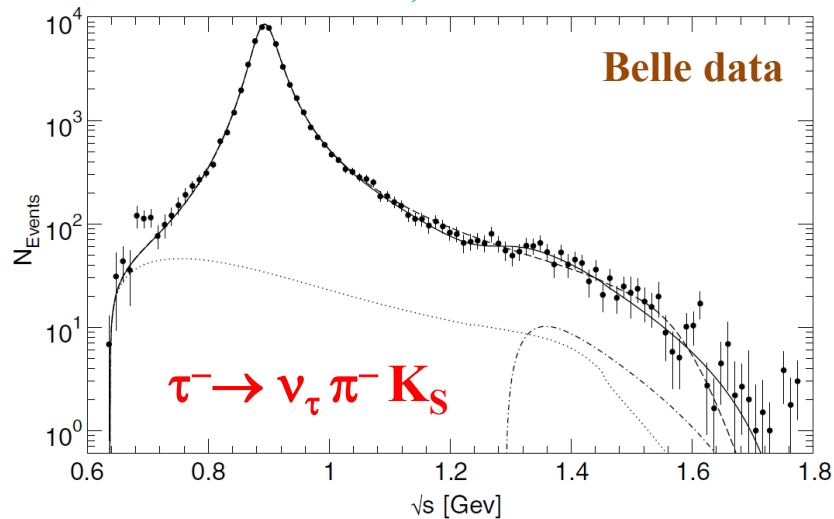
$$R_\tau = \frac{1}{B_e^{\text{univ}}} - 1.972564 = 3.6417 \pm 0.0070 \quad ; \quad R_\tau = \frac{\text{Br}(\tau^- \rightarrow \nu_\tau + \text{Hadrons})}{B_e^{\text{univ}}} = 3.6343 \pm 0.0082$$

Invariant Mass Spectra

González-Solís – Roig, 1902.02273

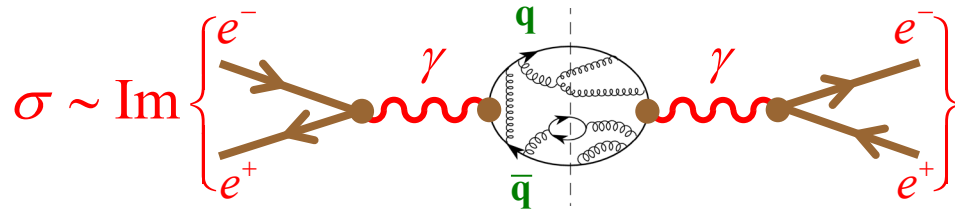


Jamin-Pich-Portolés, 0803.1786



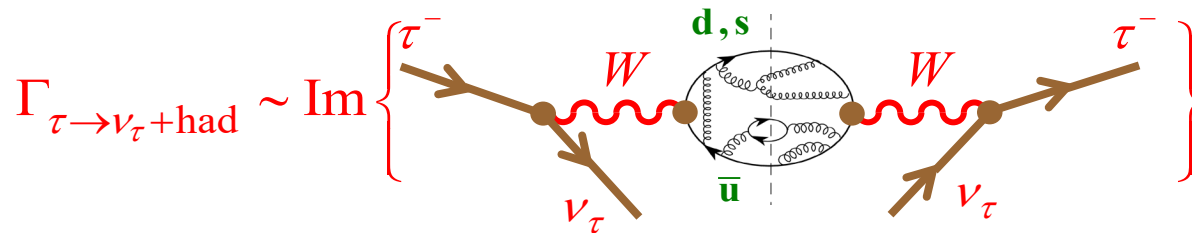
Useful tests of QCD Dynamics
 Form Factors
 Non-perturbative parameters

Resonance Chiral Theory (RχT)



$$\frac{\sigma(e^+e^- \rightarrow \text{had})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 12\pi \text{Im} \Pi_{\text{em}}(s)$$

$$\Pi_{\text{em}}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | T [J_{\text{em}}^\mu(x) J_{\text{em}}^\nu(0)] | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{\text{em}}(q^2)$$



$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{had})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im} \Pi^{(1)}(s) + \text{Im} \Pi^{(0)}(s) \right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left[\Pi_{ud,V}^{(J)}(s) + \Pi_{ud,A}^{(J)}(s) \right] + |V_{us}|^2 \left[\Pi_{us,V}^{(J)}(s) + \Pi_{us,A}^{(J)}(s) \right]$$

$$\Pi_{ij,J}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | T [J_{ij}^\mu(x) J_{ij}^\nu(0)^\dagger] | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{ij,J}^{(1)}(q^2) + q^\mu q^\nu \Pi_{ij,J}^{(0)}(q^2)$$

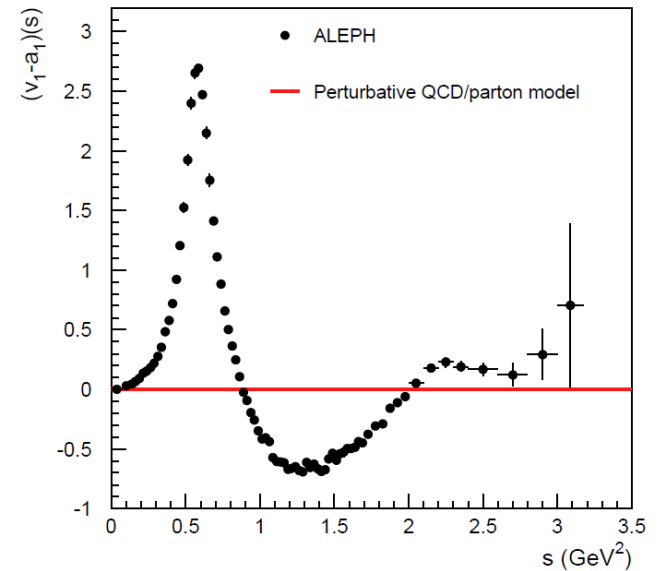
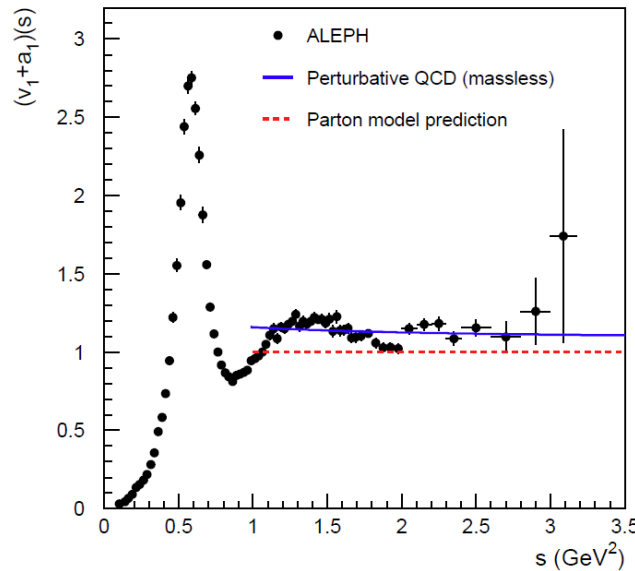
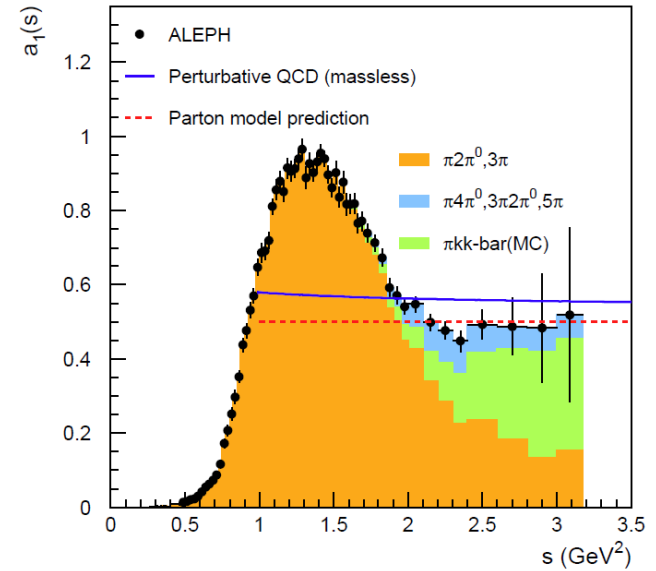
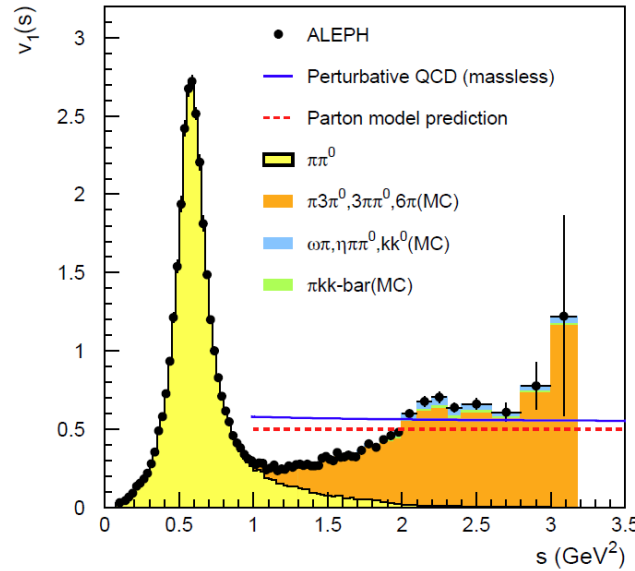
SPECTRAL FUNCTIONS

Davier et al, 1312.1501

$$v_1(s) = 2\pi \text{Im} \Pi_{ud,V}^{(0+1)}(s)$$

$$a_1(s) = 2\pi \text{Im} \Pi_{ud,A}^{(0+1)}(s)$$

**Better
data
needed**

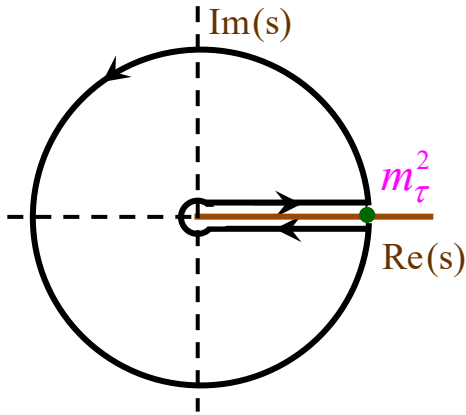


QCD Prediction of R_τ

Braaten-Narison-Pich'92

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{had})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx (1-x)^2 \left[(1+2x) \text{Im} \Pi^{(1)}(x m_\tau^2) + \text{Im} \Pi^{(0)}(x m_\tau^2) \right]$$

$$x \equiv s/m_\tau^2$$



$$R_\tau = 6\pi i \oint_{|x|=1} dx (1-x)^2 \left[(1+2x) \Pi^{(0+1)}(x m_\tau^2) - 2x \Pi^{(0)}(x m_\tau^2) \right]$$

$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s, \mu) \langle O_D(\mu) \rangle}{(-s)^{D/2}}$$

OPE

$$R_\tau = N_C S_{EW} (1 + \delta_P + \delta_{NP}) = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$$S_{EW} = 1.0201(3)$$

Marciano-Sirlin, Braaten-Li, Erler

$$\delta_{NP} = -0.0064 \pm 0.0013$$

Fitted from data (Davier et al)

$$\delta_P = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \dots \approx 20\%$$

Baikov-Chetyrkin-Kühn

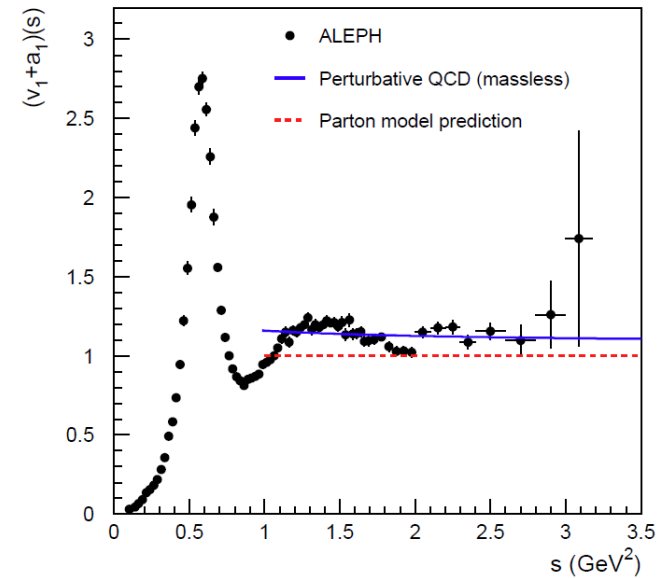
$$a_\tau \equiv \alpha_s(m_\tau) / \pi$$

Spectral Function Distribution

Moments:

$$R_{\tau}^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{m_{\tau}^2}\right)^l \frac{dR_{\tau}}{ds}$$

Sensitivity to power corrections (k,l)



The non-perturbative contribution to R_{τ} can be obtained from the invariant-mass distribution of the final hadrons

Detailed experimental analyses by ALEPH, CLEO and OPAL

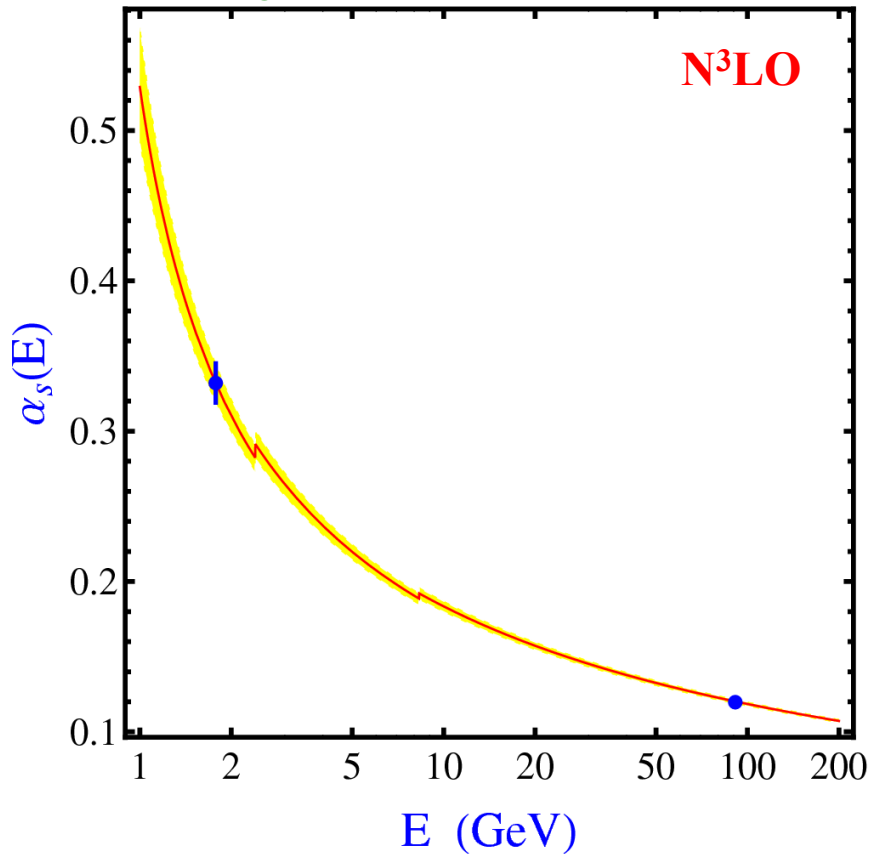
$$\delta_{\text{NP}} = -0.0064 \pm 0.0013$$

$$\alpha_s(m_{\tau}^2) = 0.332 \pm 0.005_{\text{exp}} \pm 0.011_{\text{th}}$$

Davier et al., 1312.1501
(ALEPH data)

α_s at N³LO from τ and Z

Rodríguez-Sánchez, Pich, 1605.06830



$$\alpha_s(m_\tau^2) = 0.328 \pm 0.013$$



$$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

$$\alpha_s(M_Z^2)_{Z_{\text{width}}} = 0.1199 \pm 0.0029$$

**Very precise test of
Asymptotic Freedom**

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0002 \pm 0.0015_\tau \pm 0.0029_Z$$

Improved spectral function data



Better control of non-perturbative contributions

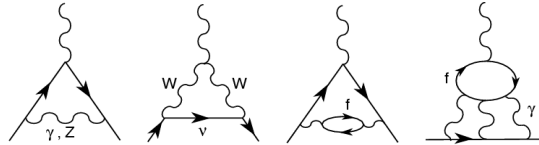
Rodríguez-Sánchez, Pich 2205.07587

Better theoretical understanding of higher-order perturbative corrections needed
(CIPT / FOPT, K5, renormalons...)

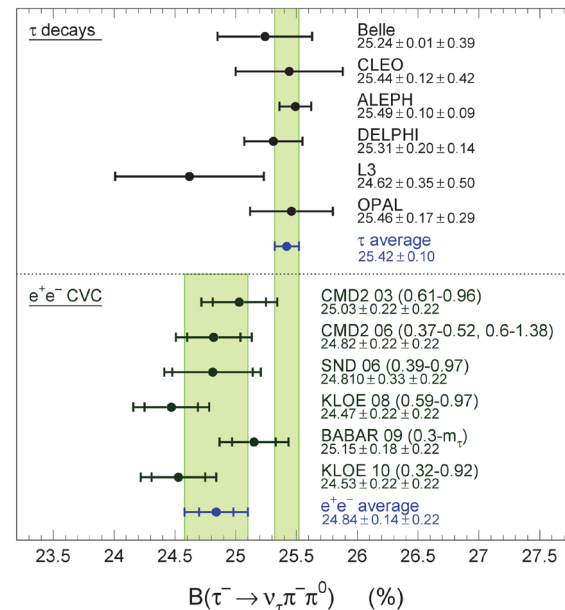
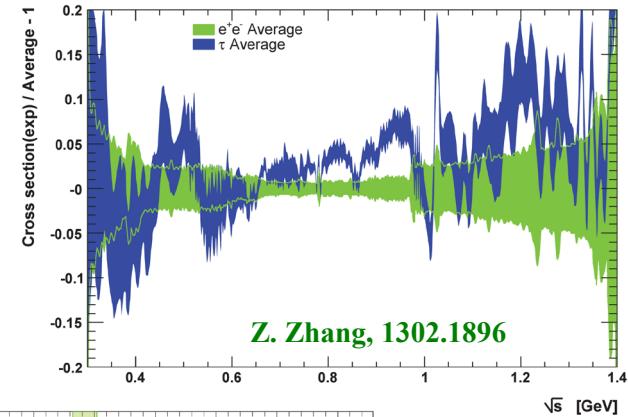
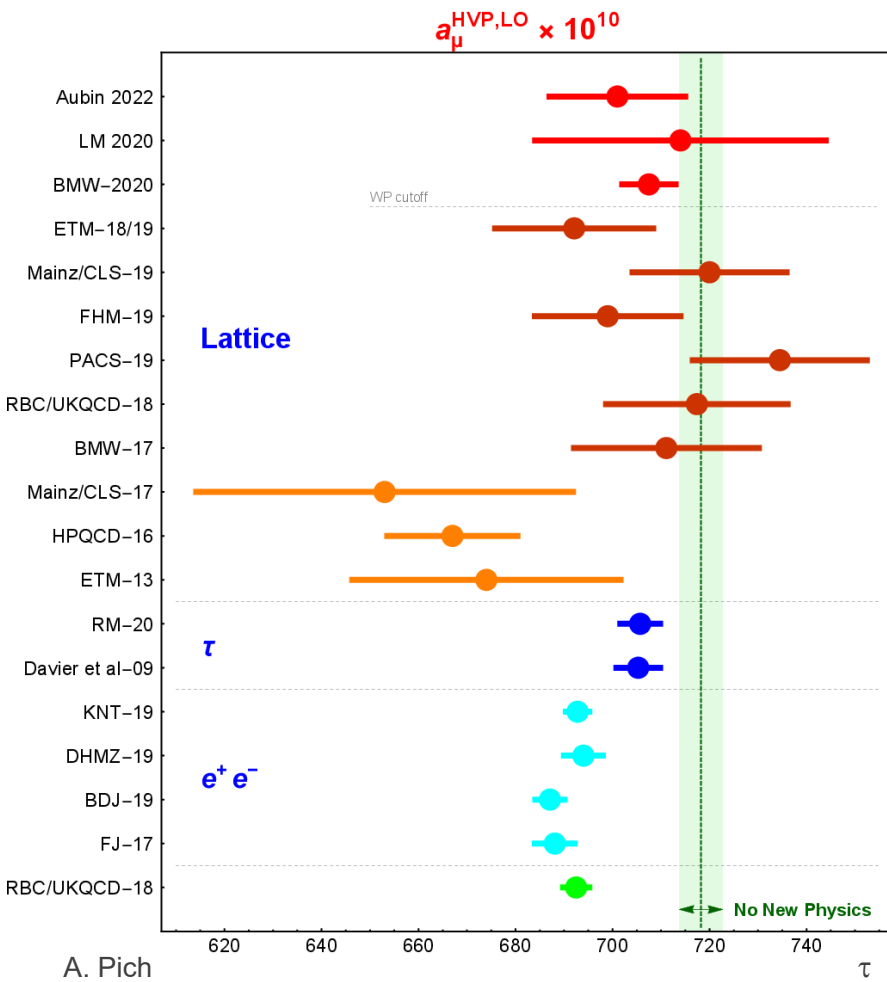
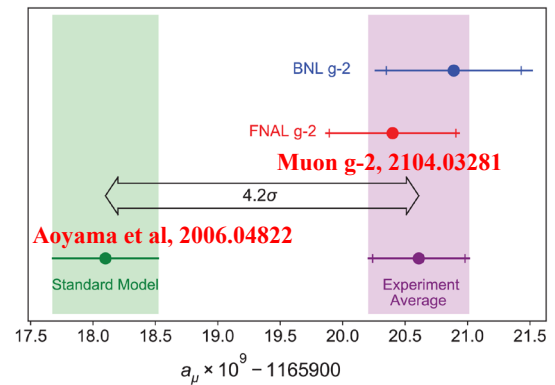
Caprini, Beneke et al, Golterman et al, Hoang et al...

μ Anomalous Magnetic Moment

$$a_{\mu}^{\text{HVP,LO}} = \frac{\alpha^2 m_{\mu}^2}{9\pi^2} \int_{s_{\text{th}}}^{\infty} \frac{ds}{s^2} \hat{K}(s) R(s)$$

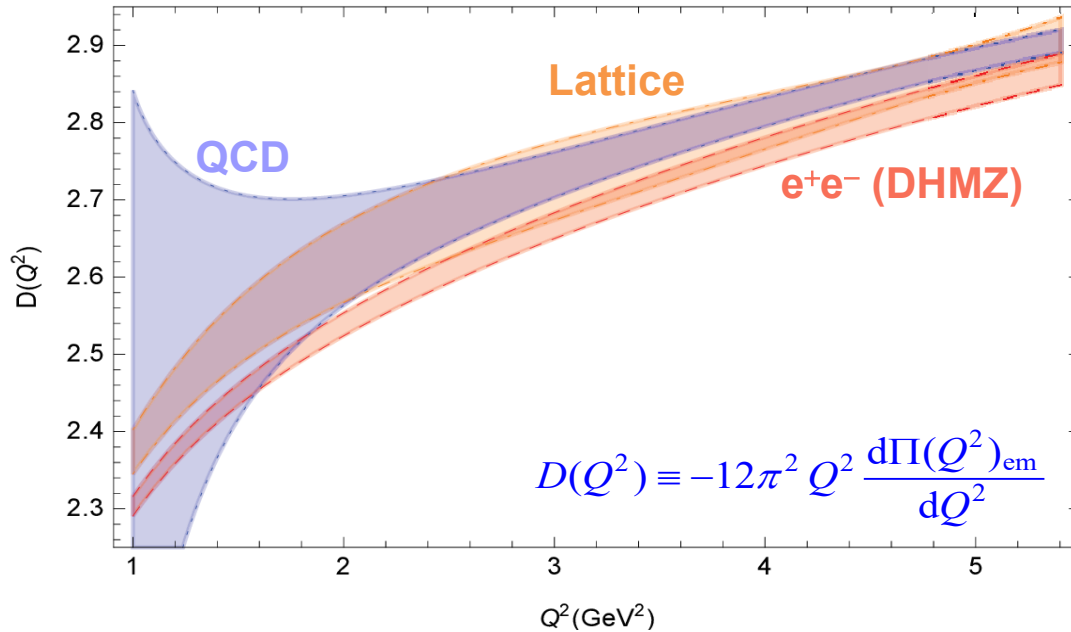


Dominated (75%) by 2π



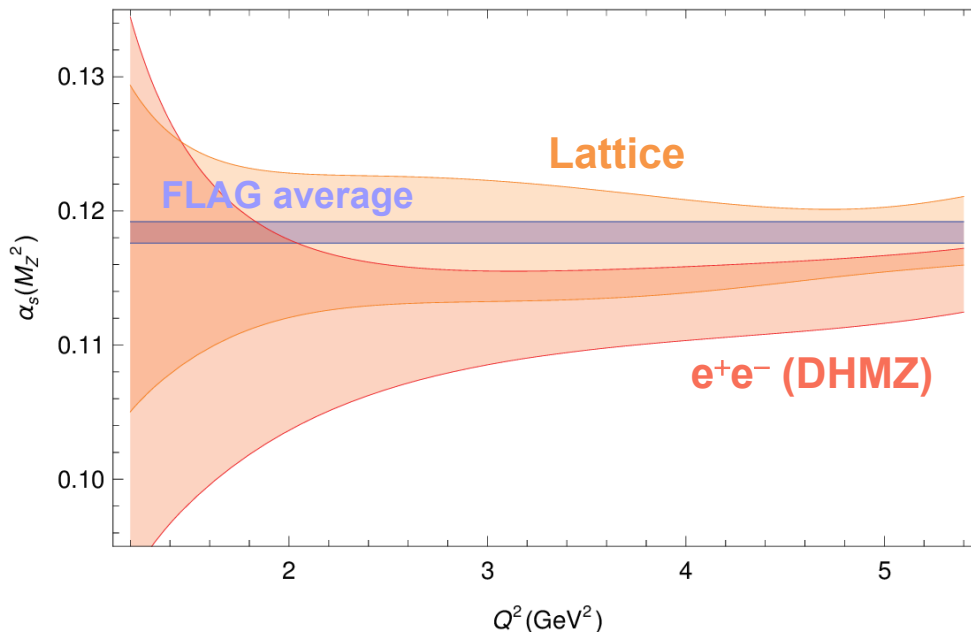
Euclidean Adler Function

$(Q^2 = -q^2)$



M. Davier, D. Díaz-Calderón, B. Malaescu,
A. Pich, A. Rodríguez-Sánchez,
Z. Zhang, 2302.01359

**2 σ discrepancy
between
e⁺e⁻ data & QCD**

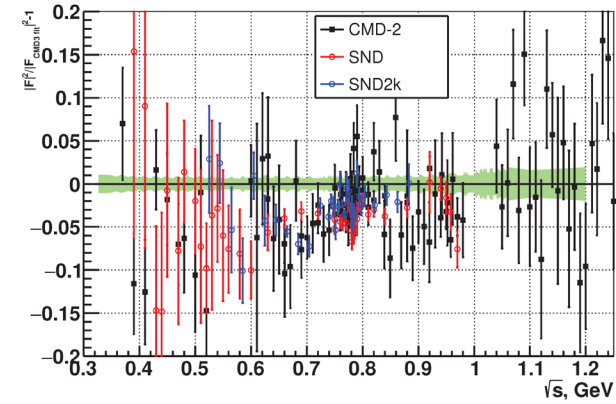
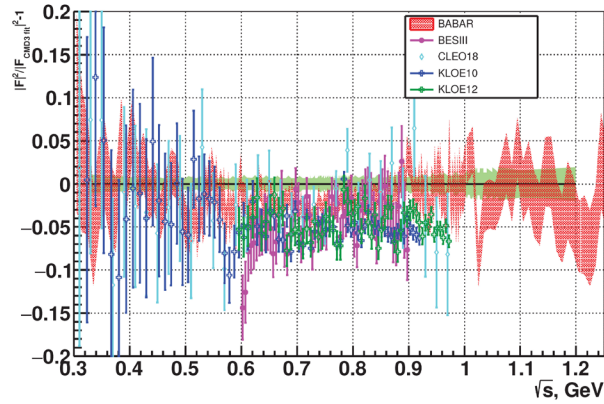
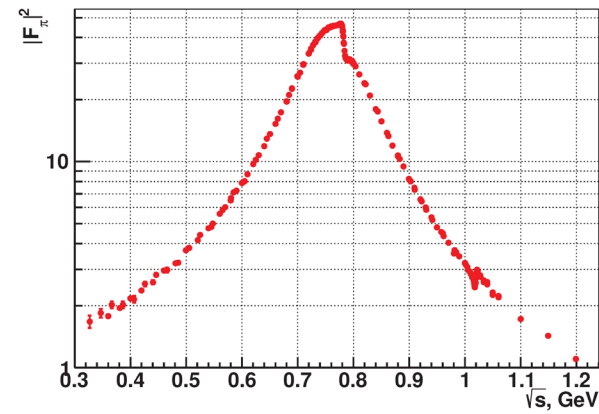


FLAG average: $\alpha_s(M_Z^2) = 0.1184 \pm 0.0008$

$$\alpha_s(M_Z^2) = \begin{cases} 0.1136 \pm 0.0025 & (e^+e^- \text{ data}) \\ 0.1179 \pm 0.0025 & (\text{Lattice}) \end{cases}$$

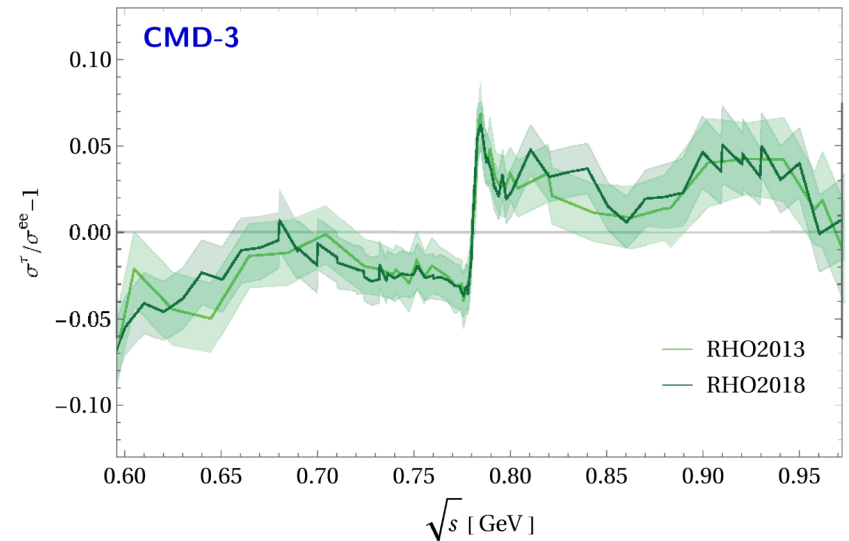
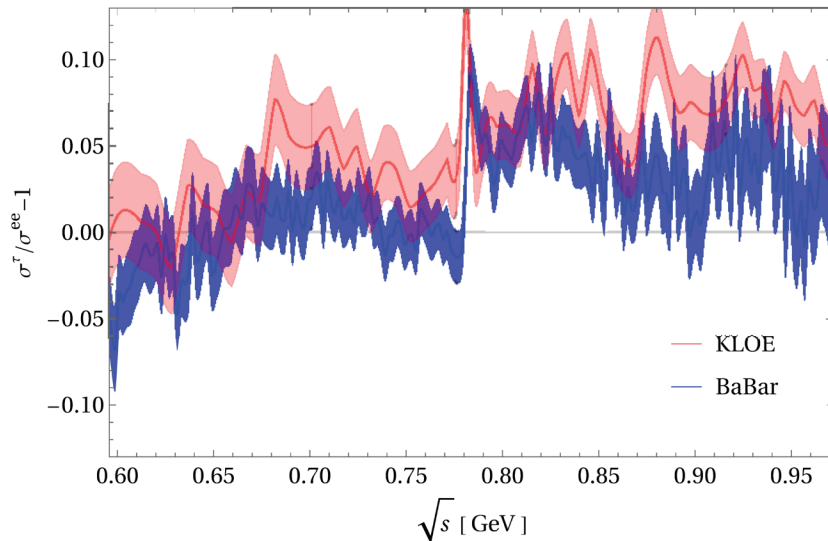
2023 CMD-3 $e^+e^- \rightarrow \pi^+\pi^-$ Data

2302.08834



$\tau \rightarrow 2\pi\nu_\tau$ & $e^+e^- \rightarrow 2\pi$ Spectral Functions

Masjuan-Miranda-Roig 2305.20005



τ Anomalous Magnetic Moment

Difficult to measure!

$$a_{\tau}^{\text{exp}} = (-0.018 \pm 0.017)$$

DELPHI

$$-0.007 < a_{\tau}^{\text{New Phys}} < 0.005$$

González-Springer, Santamaria, Vidal '00 (LEP/SLD data)

Eidelman, Passera

$$\begin{aligned}
 10^8 \cdot a_{\tau}^{\text{th}} &= 117\,324 \pm 2 && \text{QED} \\
 &+ 47.4 \pm 0.5 && \text{EW} \\
 &+ 337.5 \pm 3.7 && \text{hvp} \\
 &+ 7.6 \pm 0.2 && \text{hvp NLO} \\
 &+ 5 \pm 3 && \text{light-by-light} \\
 &= \mathbf{117\,721 \pm 5}
 \end{aligned}$$

Enhanced sensitivity to

new physics: $(m_{\tau}/m_{\mu})^2 = 283$

	Electron	Muon	Tau
$a^{\text{EW}}/a^{\text{HAD}}$	1/56	1/45	1/7
$a^{\text{EW}}/\delta a^{\text{HAD}}$	1.6	3	10

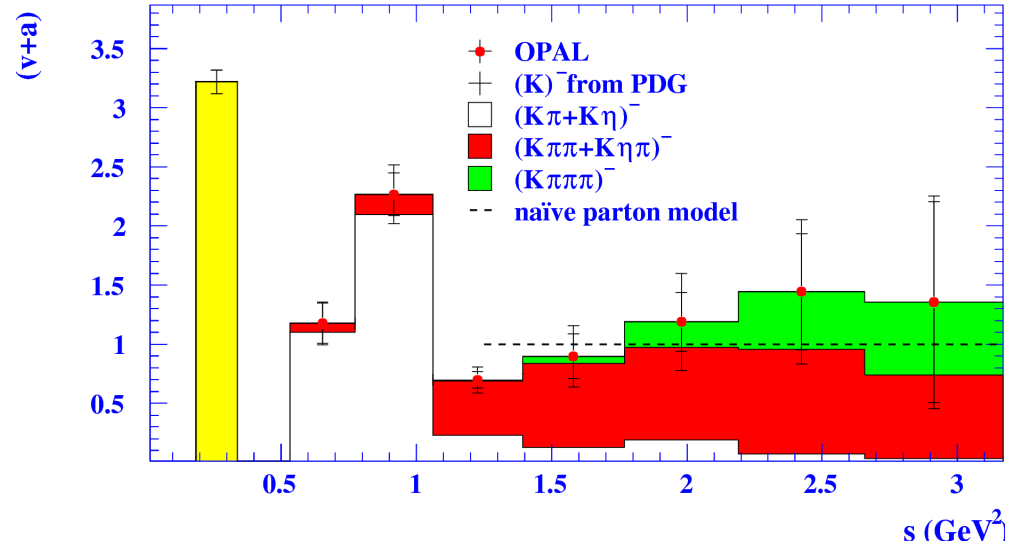
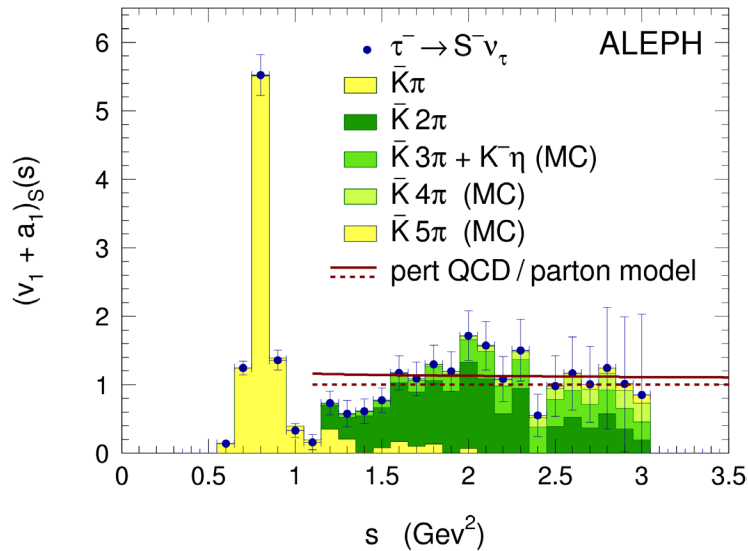
Essentially unknown. May be accessible through radiative leptonic decays (Fael et al) or with a polarized electron beam (Crivellin et al)

STCF sensitivity

Yongcheng Wu

\mathcal{L}	1 ab ⁻¹	10 ab ⁻¹	50 ab ⁻¹
$ d_{\tau}^{\text{NP}} $ (e·cm)	1.44×10^{-18}	4.56×10^{-19}	2.04×10^{-19}
$ a_{\tau}^{\text{NP}} $	1.24×10^{-4}	3.92×10^{-5}	1.75×10^{-5}

Strange Spectral Function



Very low statistics. Large experimental uncertainties

Sensitive to SU(3) breaking: m_s , V_{us}

V_{us} Determination

Gámiz-Jamin-Pich-Prades-Schwab '03

$$|V_{us}|^2 = \frac{R_{\tau,us}}{\frac{R_{\tau,ud}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

$$\delta R_{\tau,th} \approx 24 \frac{m_s^2(m_\tau^2)}{m_\tau^2} \Delta(\alpha_s)$$

$$\delta R_{\tau,th} \equiv \underbrace{0.1544(37)}_{J=0} + \underbrace{0.084(33)}_{m_s(2 \text{ GeV}) = 93.0(8.5) \text{ MeV}} = 0.238(33)$$

$$R_{\tau,uq} = \Gamma(\tau^- \rightarrow \nu_\tau \bar{u}q) / \Gamma(\tau^- \rightarrow \nu_\tau \bar{\nu}_e e^-)$$

HFLAV 2022:

$$\text{Br}(\tau^- \rightarrow \nu_\tau \bar{u}s) = (2.908 \pm 0.048)\%$$

$$\text{Br}(\tau^- \rightarrow \nu_\tau \bar{u}d) = (61.83 \pm 0.10)\%$$

$$\text{Br}(\tau^- \rightarrow \nu_\tau \bar{\nu}_e e^-)_{\text{univ}} = (17.812 \pm 0.022)\%$$

$$V_{ud} = 0.97373 \pm 0.00031$$

$$\Rightarrow |V_{us}| = 0.2184 \pm 0.0018_{\text{exp}} \pm 0.0011_{\text{th}}$$

$$\mathbf{K}_{13}: |V_{us}| = 0.2232 \pm 0.0006$$

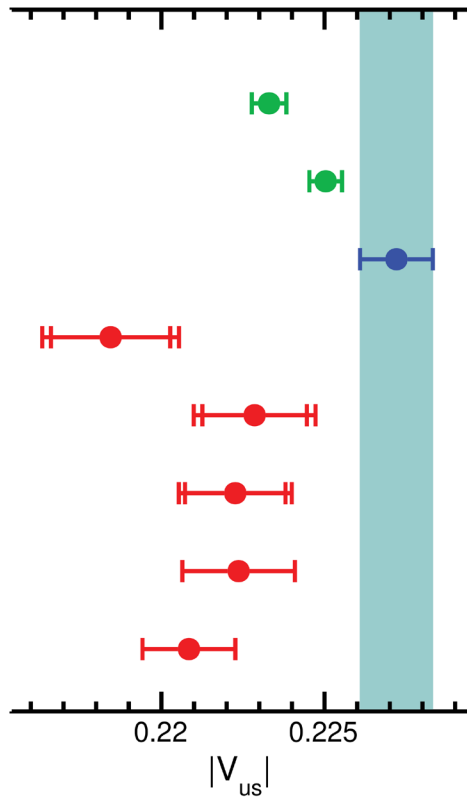
$$[f_+(0) = 0.9698 \pm 0.0017]$$

FLAG 2021

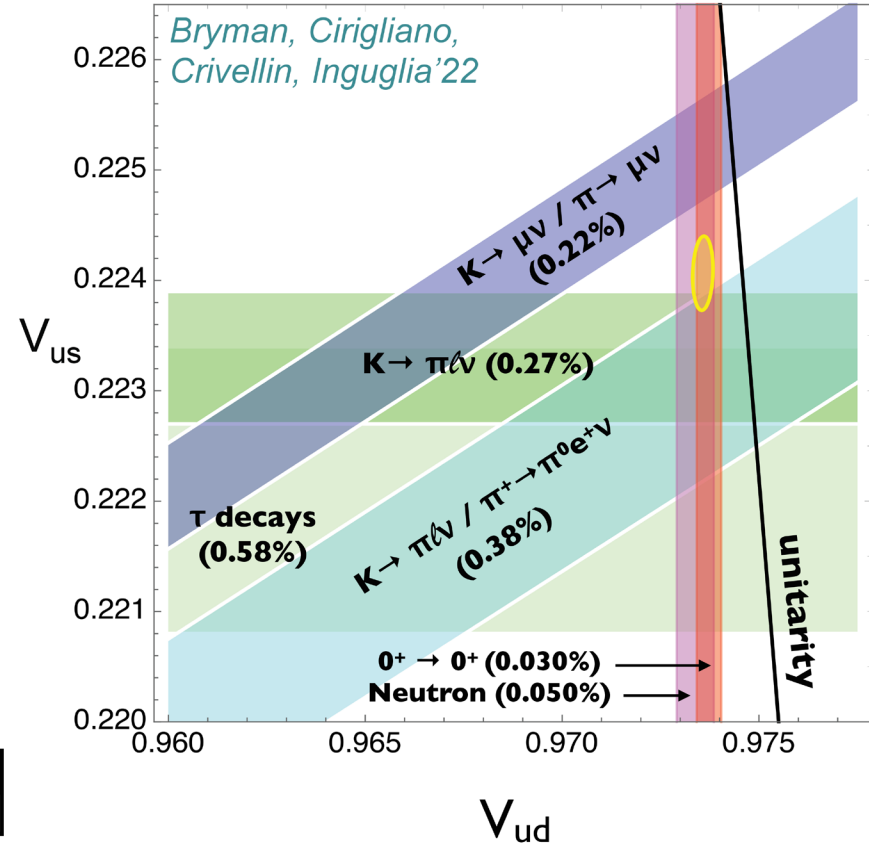
Sizeable discrepancy. Improvements needed

Cabibbo Anomaly

A. Lusiani



HFLAV
2023 prelim



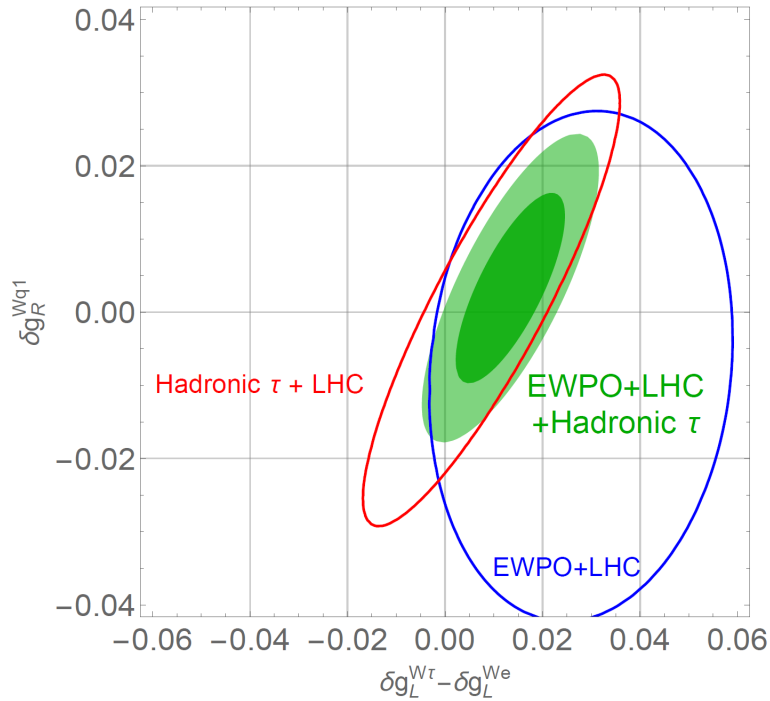
Sizeable violation of CKM unitarity

Hadronic τ Decay & New Physics

$$\mathcal{L}_{\text{eff}} = -\frac{G_F V_{ud}}{\sqrt{2}} \left[\left(1 + \epsilon_L^\tau\right) \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 - \gamma_5) d + \epsilon_R^\tau \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 + \gamma_5) d \right. \\ \left. + \bar{\tau} (1 - \gamma_5) \nu_\tau \cdot \bar{u} \left[\epsilon_S^\tau - \epsilon_P^\tau \gamma_5 \right] d + \epsilon_T^\tau \bar{\tau} \sigma_{\mu\nu} (1 - \gamma_5) \nu_\tau \cdot \bar{u} \sigma^{\mu\nu} (1 - \gamma_5) d \right] + \text{h.c.}$$

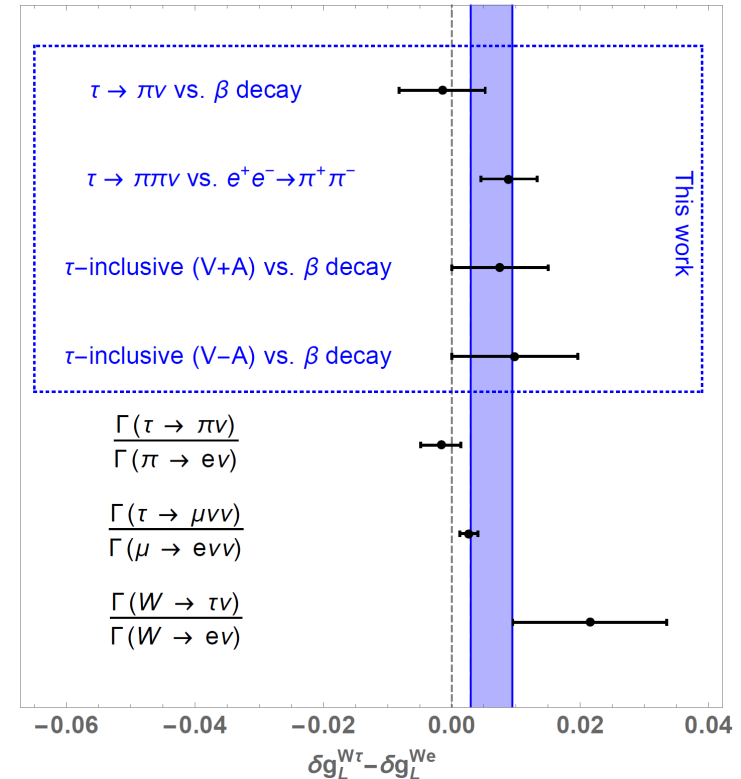
Cirigliano, Falkowski, González-Alonso, Rodríguez-Sánchez, 1809.01161

$$\epsilon_L^\tau - \epsilon_L^e = \delta g_L^{W\tau} - \delta g_L^{We} - [c_{\ell q}^{(3)}]_{\tau\tau 11} + [c_{\ell q}^{(3)}]_{ee 11} \quad \epsilon_{S,P}^\tau = -\frac{1}{2} [c_{\ell e q} \pm c_{\ell e d q}]_{\tau\tau 11}^* \\ \epsilon_R^\tau = \delta g_R^{Wq_1}, \quad \epsilon_T^\tau = -\frac{1}{2} [c_{\ell e q u}]_{\tau\tau 11}^*$$

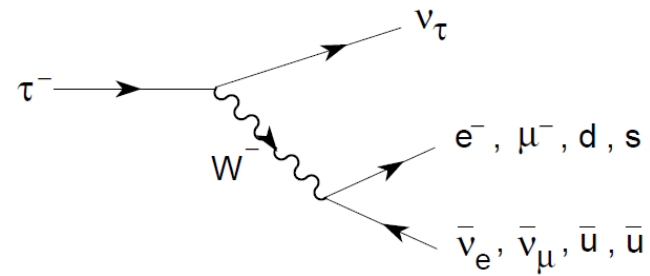


95% CL 10⁻³ units

Coefficient	ATLAS $\tau\nu$	τ decays	τ and π decays
$[c_{\ell q}^{(3)}]_{\tau\tau 11}$	[0.0, 1.6]	[-12.6, 0.2]	[-7.6, 2.1]
$[c_{\ell e q u}]_{\tau\tau 11}$	[-5.6, 5.6]	[-8.4, 4.1]	[-5.6, 2.3]
$[c_{\ell e d q}]_{\tau\tau 11}$	[-5.6, 5.6]	[-3.5, 9.0]	[-2.1, 5.8]
$[c_{\ell e q u}^{(3)}]_{\tau\tau 11}$	[-3.3, 3.3]	[-10.4, -0.2]	[-8.6, 0.7]



SUMMARY



Many interesting τ topics

- ❑ Tests of QCD and the Electroweak Theory
- ❑ Looking for Signals of New Phenomena
- ❑ Superb Tool for New Physics Searches

Better data samples needed

Lots of data will be produced @ Belle-II & STCF

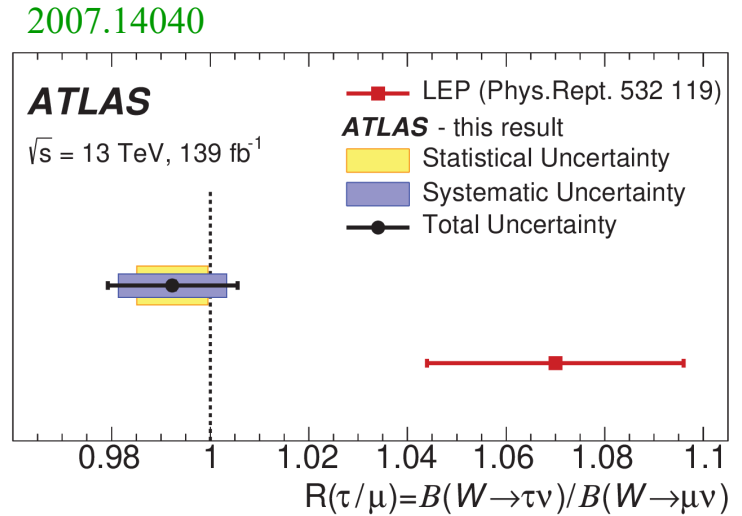
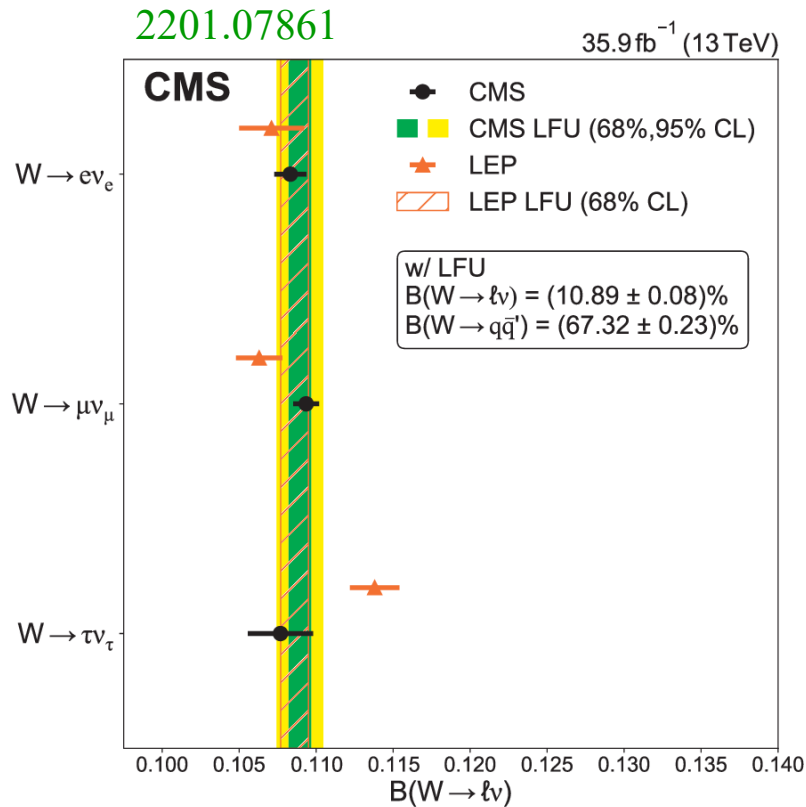
Improving systematics brings a great reward

Clean threshold environment at the STCF

Backup

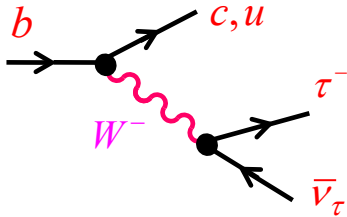


Lepton Universality in W decays

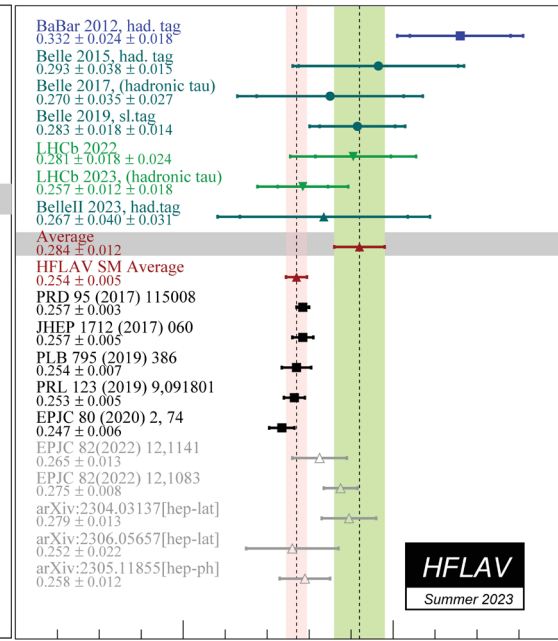
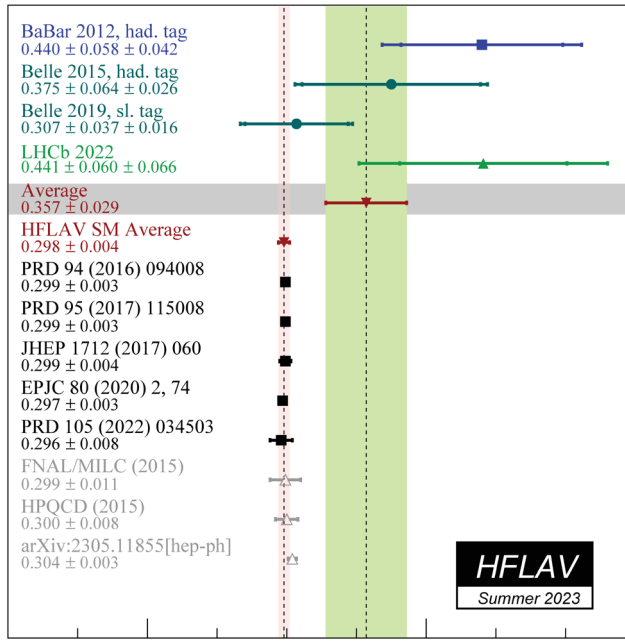


	CMS	LEP	ATLAS	LHCb	CDF	D0
$R_{\mu/e}$	1.009 ± 0.009	0.993 ± 0.019	1.003 ± 0.010	0.980 ± 0.012	0.991 ± 0.012	0.886 ± 0.121
$R_{\tau/e}$	0.994 ± 0.021	1.063 ± 0.027	—	—	—	—
$R_{\tau/\mu}$	0.985 ± 0.020	1.070 ± 0.026	0.992 ± 0.013	—	—	—
$R_{\tau/\ell}$	1.002 ± 0.019	1.066 ± 0.025	—	—	—	—

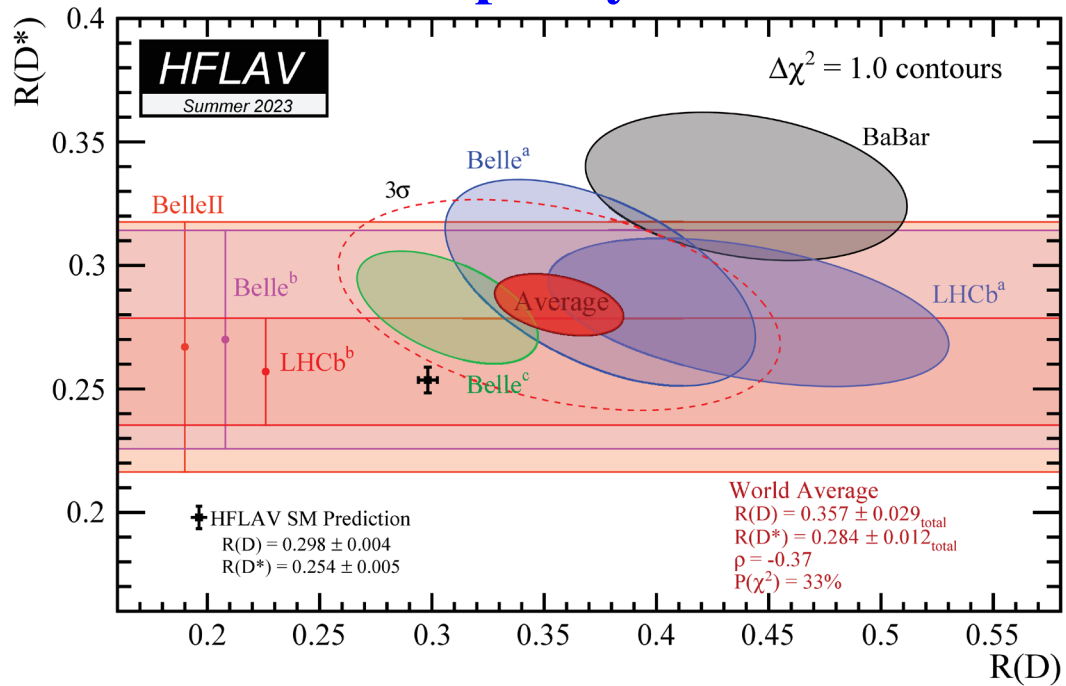
Flavour Anomaly



$$R(D^{(*)}) \equiv \frac{\text{Br}(\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau)}{\text{Br}(\bar{B} \rightarrow D^{(*)} \ell^- \bar{\nu}_\ell)}$$

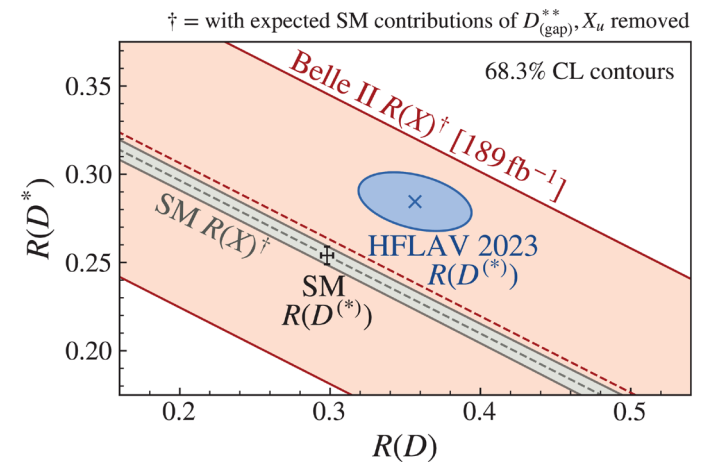


3.3 sigma discrepancy



$$R(X) \equiv \frac{\text{Br}(\bar{B} \rightarrow X \tau^- \bar{\nu}_\tau)}{\text{Br}(\bar{B} \rightarrow X \ell^- \bar{\nu}_\ell)} = 0.228 \pm 0.016 \pm 0.036$$

Belle-II 2311.07248



LORENTZ STRUCTURE

$$\mathcal{H} = 4 \frac{G_{\mu l}}{\sqrt{2}} \sum_{n, \epsilon, \omega} g_{\epsilon \omega}^n \left[\bar{l}'_{\epsilon} \Gamma^n (\nu_{l'})_{\sigma} \right] \left[\overline{(\nu_l)_{\lambda}} \Gamma_n l_{\omega} \right]$$

90% CL

Fetscher-Gerber, PDG2020

$$\mu \rightarrow e \bar{\nu}_e \nu_{\mu}$$

$ g_{RR}^S < 0.035$	$ g_{RR}^V < 0.017$	$ g_{RR}^T \equiv 0$
$ g_{LR}^S < 0.050$	$ g_{LR}^V < 0.023$	$ g_{LR}^T < 0.015$
$ g_{RL}^S < 0.420$	$ g_{RL}^V < 0.105$	$ g_{RL}^T < 0.105$
$ g_{LL}^S < 0.550$	$ g_{LL}^V > 0.960$	$ g_{LL}^T \equiv 0$
$ g_{LR}^S + 6g_{LR}^T < 0.143$	$ g_{RL}^S + 6g_{RL}^T < 0.418$	
$ g_{LR}^S + 2g_{LR}^T < 0.108$	$ g_{RL}^S + 2g_{RL}^T < 0.417$	
$ g_{LR}^S - 2g_{LR}^T < 0.070$	$ g_{RL}^S - 2g_{RL}^T < 0.418$	
$Q_{RR} + Q_{LR} < 8.2 \times 10^{-4}$		

95% CL

Stahl, PDG2020

$$\tau \rightarrow e \bar{\nu}_e \nu_{\tau}$$

$ g_{RR}^S < 0.70$	$ g_{RR}^V < 0.17$	$ g_{RR}^T \equiv 0$
$ g_{LR}^S < 0.99$	$ g_{LR}^V < 0.13$	$ g_{LR}^T < 0.082$
$ g_{RL}^S < 2.01$	$ g_{RL}^V < 0.52$	$ g_{RL}^T < 0.51$
$ g_{LL}^S < 2.01$	$ g_{LL}^V < 1.005$	$ g_{LL}^T \equiv 0$

$$\tau \rightarrow \mu \bar{\nu}_{\mu} \nu_{\tau}$$

$ g_{RR}^S < 0.72$	$ g_{RR}^V < 0.18$	$ g_{RR}^T \equiv 0$
$ g_{LR}^S < 0.95$	$ g_{LR}^V < 0.12$	$ g_{LR}^T < 0.079$
$ g_{RL}^S < 2.01$	$ g_{RL}^V < 0.52$	$ g_{RL}^T < 0.51$
$ g_{LL}^S < 2.01$	$ g_{LL}^V < 1.005$	$ g_{LL}^T \equiv 0$

$$\tau \rightarrow \pi \nu_{\tau}$$

$ g_R^V < 0.15$	$ g_L^V > 0.992$
------------------	-------------------

$$\tau \rightarrow \rho \nu_{\tau}$$

$ g_R^V < 0.10$	$ g_L^V > 0.995$
------------------	-------------------

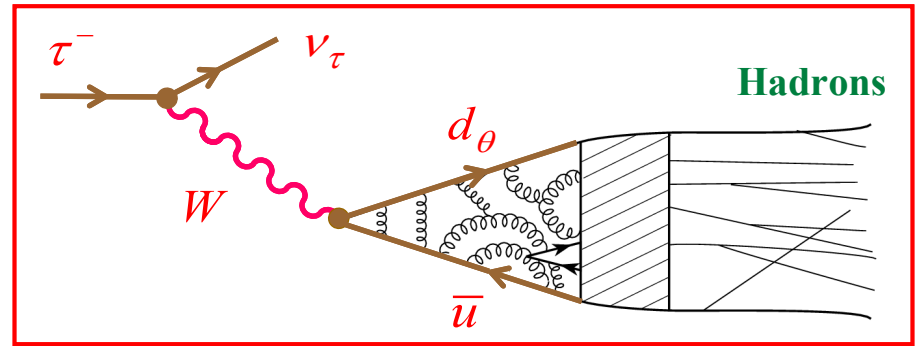
$$\tau \rightarrow a_1 \nu_{\tau}$$

$ g_R^V < 0.16$	$ g_L^V > 0.987$
------------------	-------------------

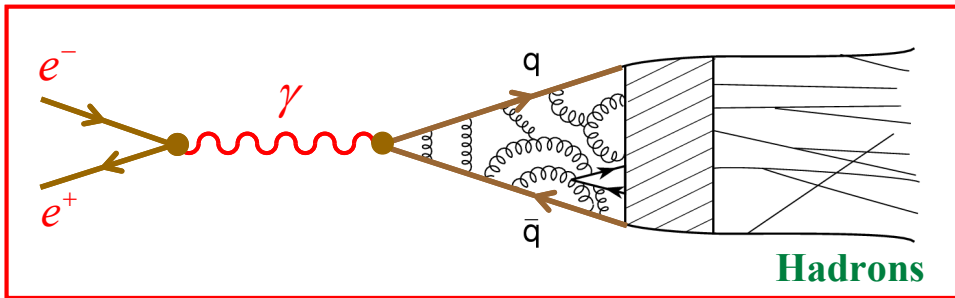
Only Lepton Massive Enough to Decay into Hadrons

$\tau^- \rightarrow \nu_\tau H^-$ probes the hadronic V-A current

$$\langle H^- | \bar{d}_\theta \gamma^\mu (1 - \gamma_5) u | 0 \rangle$$



$e^+e^- \rightarrow H^0$ probes the hadronic electromagnetic current



$$\langle H^0 | \sum_q Q_q \bar{q} \gamma^\mu q | 0 \rangle$$

Isospin:
$$\frac{\Gamma(\tau^- \rightarrow \nu_\tau V^-)}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = \frac{3 \cos^2 \theta_C}{2\pi\alpha^2} S_{EW} \int_0^1 dx (1-x)^2 (1+2x) x \sigma_{e^+e^- \rightarrow V^0}(x m_\tau^2)$$

Perturbative ($m_q=0$)

$$-s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0} K_n \left(\frac{\alpha_s(-s)}{\pi} \right)^n$$

$$K_0 = K_1 = 1, \quad K_2 = 1.63982, \quad K_3 = 6.37101, \quad K_4 = 49.07570$$

Baikov-Chetyrkin-Kühn '08

➔
$$\delta_P = \sum_{n=1} K_n A^{(n)}(\alpha_s) = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \dots$$

Le Diberder-Pich '92

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1 - 2x + 2x^3 - x^4) \left(\frac{\alpha_s(-s)}{\pi} \right)^n = a_\tau^n + \dots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau) / \pi$$

Power Corrections

$$\Pi_{\text{OPE}}^{(0+1)}(s) \approx \frac{1}{4\pi^2} \sum_{n \geq 2} \frac{C_{2n} \langle O_{2n} \rangle}{(-s)^n}$$

Braaten-Narison-Pich '92

$$C_4 \langle O_4 \rangle \approx \frac{2\pi}{3} \langle 0 | \alpha_s G^{\mu\nu} G_{\mu\nu} | 0 \rangle$$

$$\delta_{\text{NP}} \approx \frac{-1}{2\pi i} \oint_{|x|=1} dx (1 - 3x^2 + 2x^3) \sum_{n \geq 2} \frac{C_{2n} \langle O_{2n} \rangle}{(-x m_\tau^2)^n} = -3 \frac{C_6 \langle O_6 \rangle}{m_\tau^6} - 2 \frac{C_8 \langle O_8 \rangle}{m_\tau^8}$$

Suppressed by m_τ^6 [additional chiral suppression in $C_6 \langle O_6 \rangle^{V+A}$]

Exhaustive Analysis of ALEPH Data

Rodríguez-Sánchez, Pich, 1605.06830

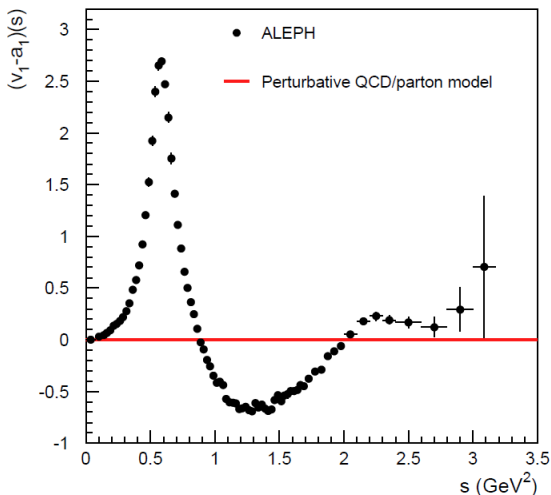
Method (V + A)	$\alpha_s(m_\tau^2)$		
	CIPT	FOPT	Average
ALEPH moments ¹	$0.339^{+0.019}_{-0.017}$	$0.319^{+0.017}_{-0.015}$	$0.329^{+0.020}_{-0.018}$
Mod. ALEPH moments ²	$0.338^{+0.014}_{-0.012}$	$0.319^{+0.013}_{-0.010}$	$0.329^{+0.016}_{-0.014}$
$A^{(2,m)}$ moments ³	$0.336^{+0.018}_{-0.016}$	$0.317^{+0.015}_{-0.013}$	$0.326^{+0.018}_{-0.016}$
s_0 dependence ⁴	0.335 ± 0.014	0.323 ± 0.012	0.329 ± 0.013
Borel transform ⁵	$0.328^{+0.014}_{-0.013}$	$0.318^{+0.015}_{-0.012}$	$0.323^{+0.015}_{-0.013}$
Combined value	0.335 ± 0.013	0.320 ± 0.012	0.328 ± 0.013



$$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

- 1) $\omega_{kl}(x) = (1 + 2x)(1 - x)^{2+k}x^l$ $(k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$
- 2) $\tilde{\omega}_{kl}(x) = (1 - x)^{2+k}x^l$ $(k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$
- 3) $\omega^{(2,m)}(x) = (1 - x)^2 \sum_{k=0}^m (k + 1)x^k = 1 - (m + 2)x^{m+1} + (m + 1)x^{m+2}$, $1 \leq m \leq 5$
- 4) $\omega^{(2,m)}(x)$ $0 \leq m \leq 2$, 1 single moment in each fit
- 5) $\omega_a^{(1,m)}(x) = (1 - x^{m+1})e^{-ax}$ $0 \leq m \leq 6$

Chiral Sum Rules



$$\Pi(s) \equiv \Pi_{VV}(s) - \Pi_{AA}(s) \quad \text{Pure non-perturbative quantity}$$

$$\lim_{s \rightarrow \infty} s^2 \Pi(s) = 0 \quad \rightarrow \quad \Pi^{\text{OPE}}(s) = -\frac{O_6}{s^3} + \frac{O_8}{s^4} - \dots$$

$$\chi\text{PT} (s \rightarrow 0): \quad \Pi(s) = \frac{2F^2}{s} - 8L_{10}^r(\mu^2) + \frac{1}{16\pi^2} \left(\frac{5}{3} - \ln \frac{-s}{\mu^2} \right) + 16C_{87}^r(\mu^2) \frac{s}{F^2} + \dots$$

$$\int_{s_{\text{th}}}^{s_0} ds \omega(s) \frac{1}{\pi} \text{Im} \Pi(s) + \frac{1}{2\pi i} \oint_{|s|=s_0} ds \omega(s) \Pi(s) = 2f_\pi^2 \omega(m_\pi^2) + \text{Res}[\omega(s)\Pi(s), s=0]$$

Statistical analysis:

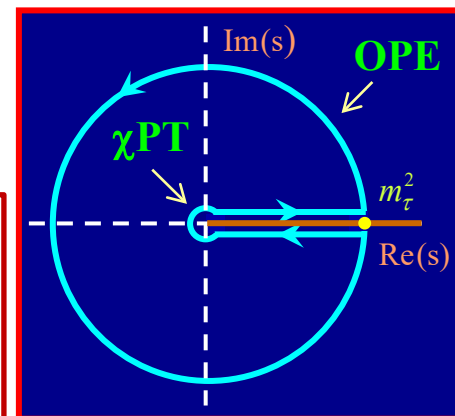
$$C_{87}^{\text{eff}} = (8.40 \pm 0.18) \cdot 10^{-3} \text{ GeV}^{-2}$$

$$L_{10}^{\text{eff}} = (-6.48 \pm 0.05) \cdot 10^{-3}$$

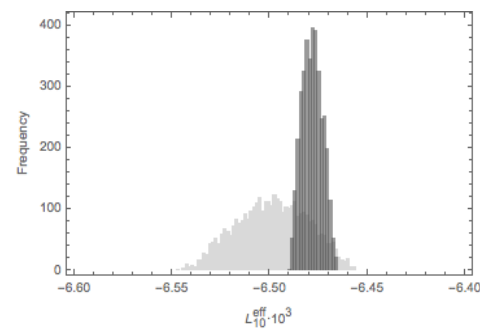
González-Pich-Rodríguez, 1602.06112

$$O_6 = (-3.6 \pm 0.7) \cdot 10^{-3} \text{ GeV}^6$$

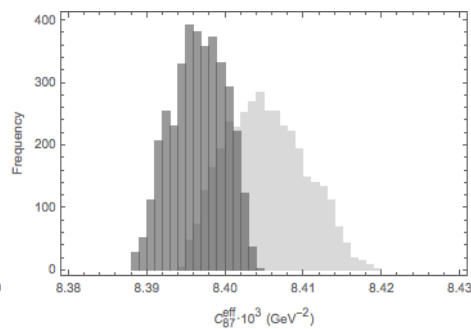
$$O_8 = (-1.0 \pm 0.4) \cdot 10^{-2} \text{ GeV}^8$$



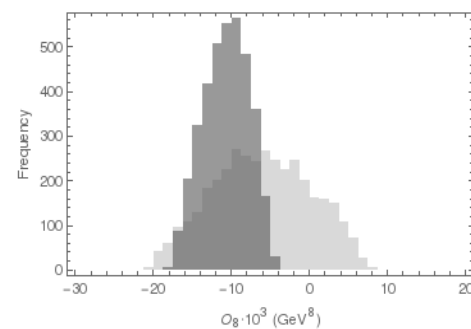
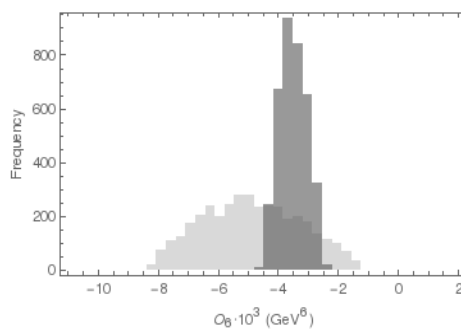
Non-pinched & pinched weights



A. Pich



τ Physics

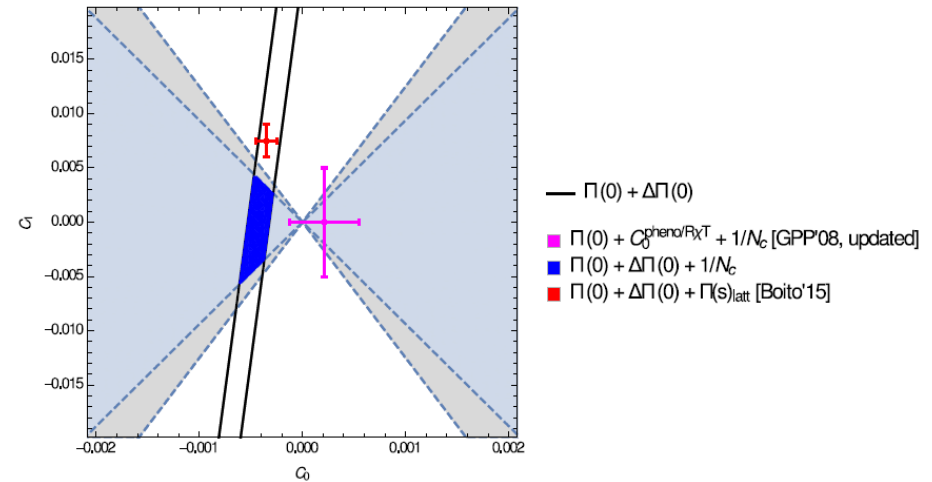


● χ PT Parameters:

$$L_{10}^{\text{eff}} = L_{10}^r - 0.00126 + \mathcal{O}(p^6)$$

$$L_{10}^{\text{eff}} = 1.53 L_{10}^r + 0.263 L_9^r - 0.00179 - \frac{1}{8} (C_0^r + C_1^r) + \mathcal{O}(p^8)$$

$$C_{87}^{\text{eff}} = C_{87}^r + 0.296 L_9^r + 0.00155 + \mathcal{O}(p^8)$$



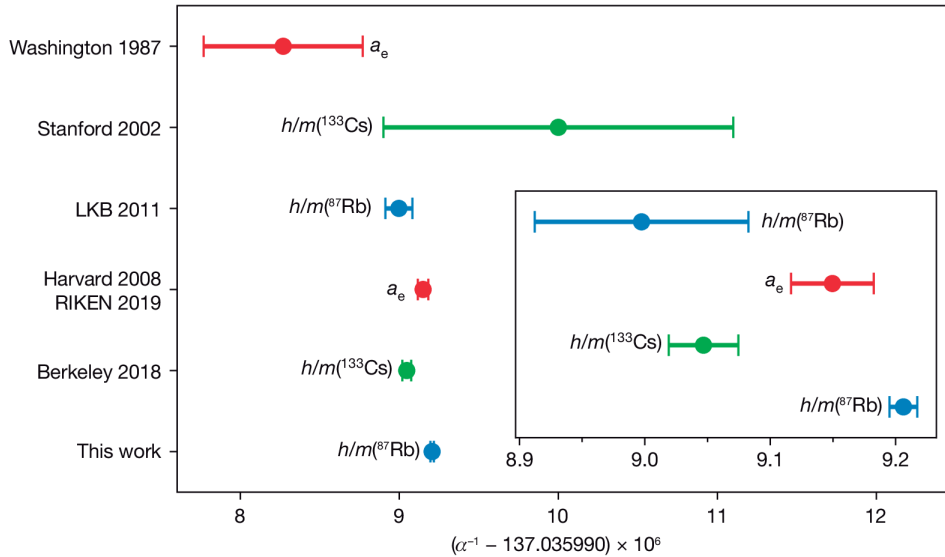
- $\mathcal{O}(p^4)$ analysis: $L_{10}^r(M_\rho) = -(5.22 \pm 0.05) \cdot 10^{-3}$
- $\mathcal{O}(p^6)$ analysis: $L_{10}^r(M_\rho) = -(4.1 \pm 0.4) \cdot 10^{-3}$
 $C_{87}^r(M_\rho) = (5.10 \pm 0.22) \cdot 10^{-3} \text{ GeV}^{-2}$

- $\varepsilon'_K / \varepsilon_K$: $\mathcal{O}_6 \Rightarrow \langle (\pi\pi)_{I=2} | Q_8 | K^0 \rangle \Rightarrow$ e.m. penguin contribution

$$(\varepsilon'_K / \varepsilon_K)_{\text{EWP}}^{I=2} = (-4.5 \pm 1.8) \cdot 10^{-4}$$

Pich-Rodríguez, 2102.09308

Electron Anomalous Magnetic Moment



Morel et al, Nature 588 (2020) 61

New measurement of α

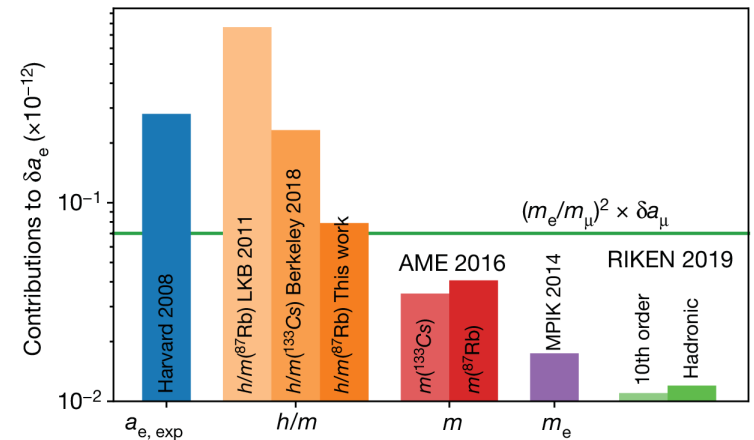
$$\alpha^{-1}(\text{Rb}) = 137.035\,999\,206\ (11)$$

8.1×10^{-11} accuracy

5.8 σ discrepancy with Cs experiment

$$\Delta a_e \equiv a_e^{\text{exp}} - a_e^{\text{SM}}$$

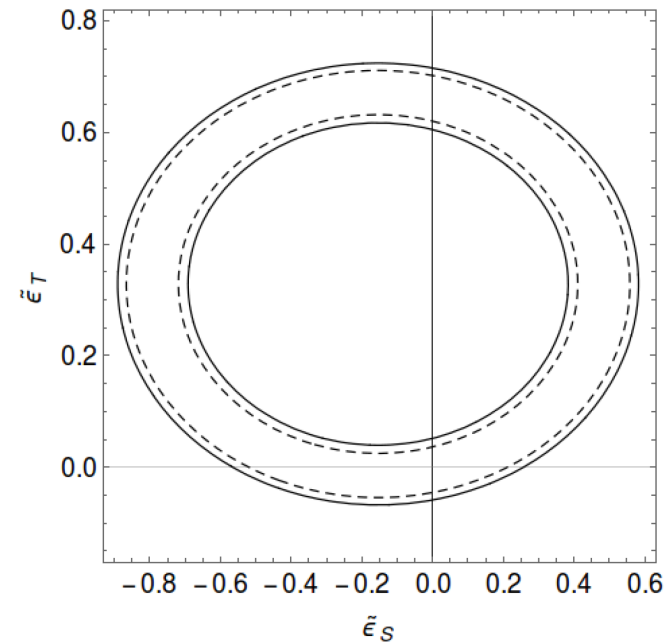
$$= \begin{cases} (-8.8 \pm 3.6) \cdot 10^{-13} & (\text{Cs}, -2.4\sigma) \\ (+4.8 \pm 3.0) \cdot 10^{-13} & (\text{Rb}, +1.6\sigma) \end{cases}$$



EFT analysis of $\tau \rightarrow \nu_\tau K \pi$

Rendón-Roig-Toledo, 1902.08143

$$\mathcal{L}_{cc} = -\frac{G_F V_{us}}{\sqrt{2}} (1 + \epsilon_L + \epsilon_R) \left[\bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\ell \cdot \bar{u} [\gamma^\mu - (1 - 2\hat{\epsilon}_R) \gamma^\mu \gamma_5] s \right. \\ \left. + \bar{\tau} (1 - \gamma_5) \nu_\ell \cdot \bar{u} [\hat{\epsilon}_s - \hat{\epsilon}_p \gamma_5] s + 2\hat{\epsilon}_T \bar{\tau} \sigma_{\mu\nu} (1 - \gamma_5) \nu_\ell \cdot \bar{u} \sigma^{\mu\nu} s \right] + \text{h.c.}$$



Best fit values	$\hat{\epsilon}_S$	$\hat{\epsilon}_T$	χ^2	χ^2 in the SM
Excluding $i = 5, 6, 7$ bins	$(1.3 \pm 0.9) \times 10^{-2}$	$(0.7 \pm 1.0) \times 10^{-2}$	[72, 73]	[74, 77]
Including $i = 5, 6, 7$ bins	$(0.9 \pm 1.0) \times 10^{-2}$	$(1.7 \pm 1.7) \times 10^{-2}$	[83, 86]	[91, 95]



$$\Lambda_{\text{NP}} \geq 2 - 5 \text{ TeV}$$

Complementary to kaon and hyperon data analyses

τ 's @ LHC

Excellent signature to probe New Physics

Difficult to identify light objects (Z, W^\pm) with only Jets
 QCD Jets orders of magnitude larger
 Must rely on leptons

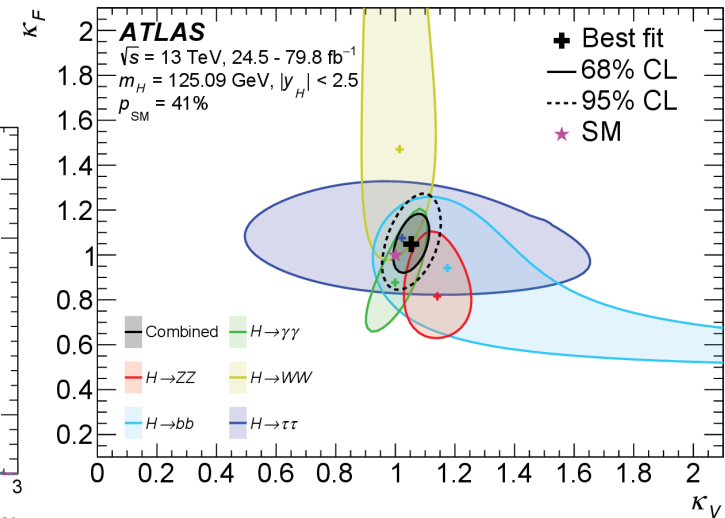
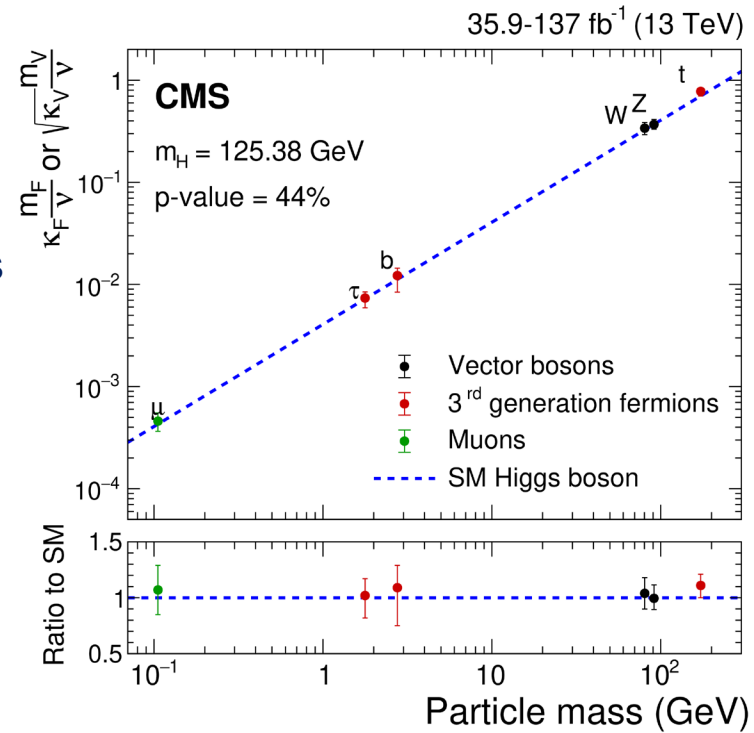
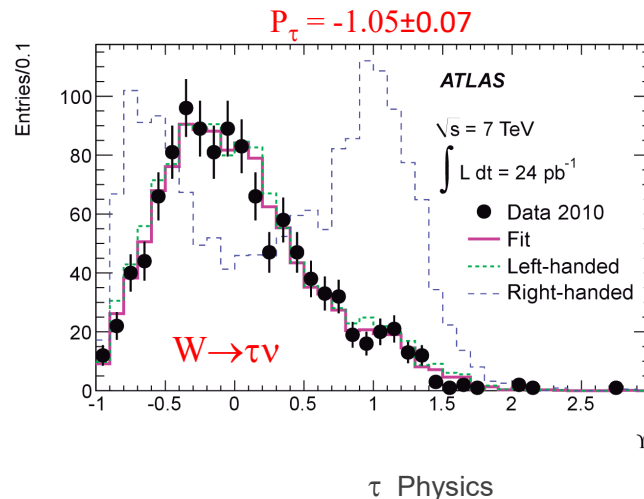
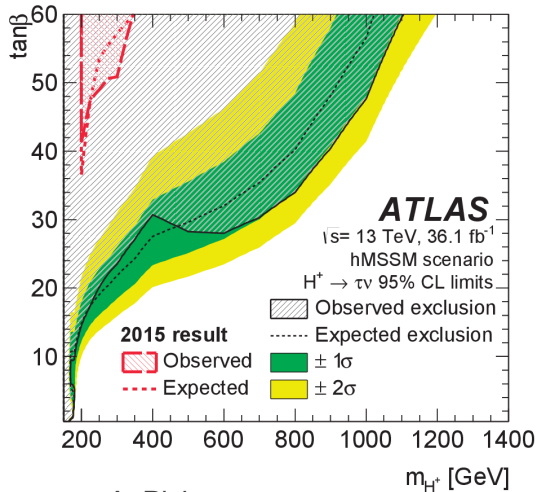
LHC produces high-momenta τ 's

Tightly collimated decay products (mini-jet like)
 Momentum reconstruction possible

Low multiplicity. Good tagging efficiency

Heaviest lepton coupling to the Higgs (4th H Br)

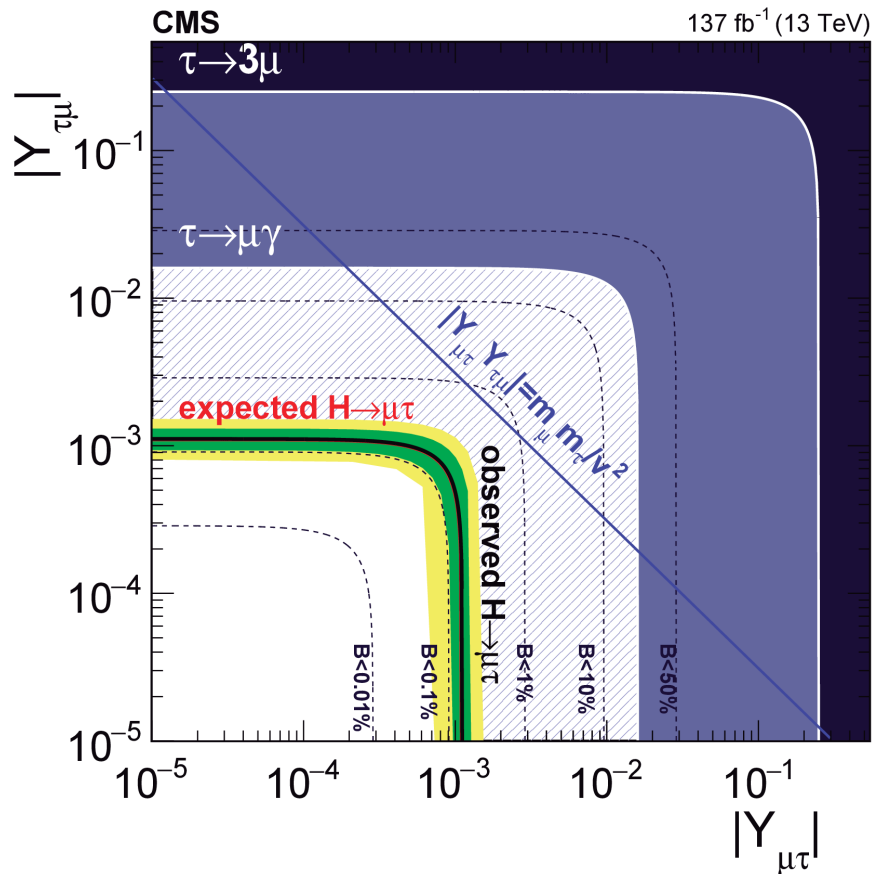
Polarization information



Flavour-Violating Higgs Couplings

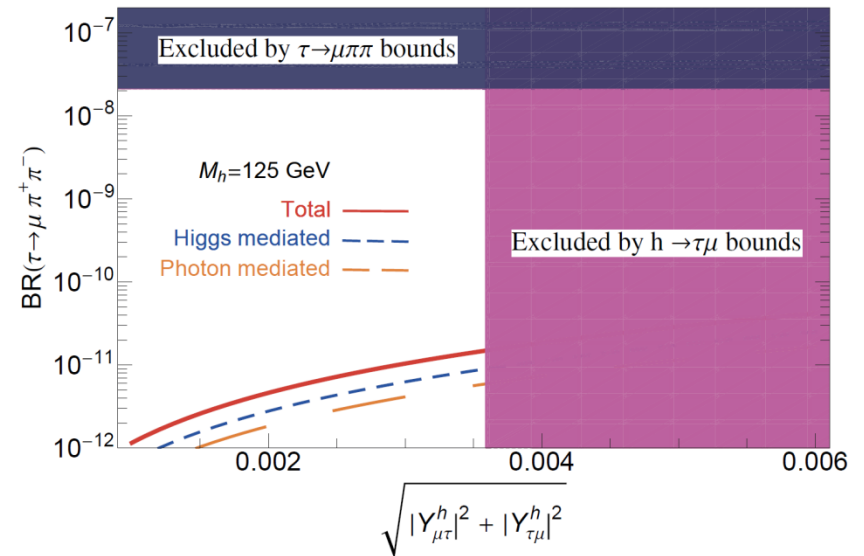
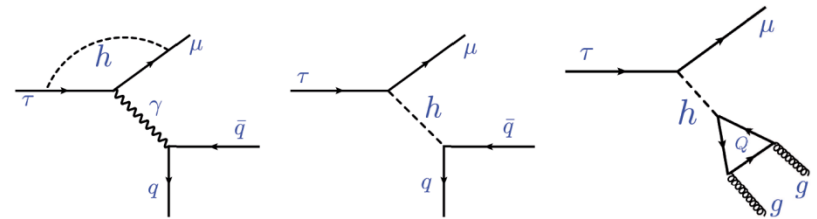
$$\mathcal{L} = -H \{ Y_{e\mu} \bar{e}_L \mu_R + Y_{e\tau} \bar{e}_L \tau_R + Y_{\mu\tau} \bar{\mu}_L \tau_R + \dots \}$$

$\text{Br}(H \rightarrow \mu\tau) < 0.15\%$ (95% CL)



$\tau \rightarrow \mu \pi^+ \pi^-$

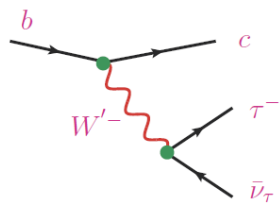
Celis et al., 1409.4439



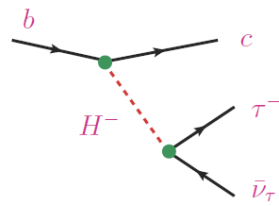
Effective Field Theory Analysis

$$\mathcal{H}_{\text{eff}}^{b \rightarrow c \ell \nu} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[(1 + C_{V_L}) \mathcal{O}_{V_L} + C_{V_R} \mathcal{O}_{V_R} + C_{S_R} \mathcal{O}_{S_R} + C_{S_L} \mathcal{O}_{S_L} + C_T \mathcal{O}_T \right] + \text{h.c.}$$

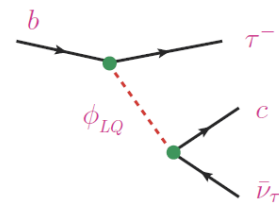
$$\mathcal{O}_{V_{L,R}} = (\bar{c} \gamma^\mu b_{L,R}) (\bar{\ell}_L \gamma_\mu \nu_{\ell L}), \quad \mathcal{O}_{S_{L,R}} = (\bar{c} b_{L,R}) (\bar{\ell}_R \nu_{\ell L}), \quad \mathcal{O}_T = (\bar{c} \sigma^{\mu\nu} b_L) (\bar{\ell}_R \sigma_{\mu\nu} \nu_{\ell L})$$



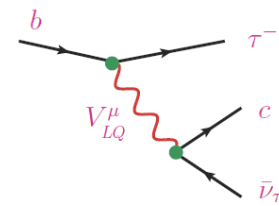
C_{V_L}



$C_{S_L}, C_{S_R} \quad (C_T)$



C_{V_L}, C_{S_L}, C_T



C_{V_L}, C_{S_R}

Many analyses (usually with single operator/mediator and partial data information)

Freytsis et al, Bardhan et al, Cai et al, Hu et al, Celis et al, Datta et al, Bhattacharya et al, Alonso et al, ...

Global fit to all data

(q^2 distributions included)

Murgui-Peñuelas-Jung-Pich, 1904.09311

$F_L^{D*}, \mathcal{B}_{10}$	Min 1	Min 2
$\chi^2/\text{d.o.f.}$	37.4/54	40.4/54
C_{LL}^V	0.09 ± 0.13 -0.12	0.34 ± 0.05 -0.07
C_{RL}^S	0.09 ± 0.12 -0.61	-1.10 ± 0.48 -0.07
C_{LL}^S	-0.14 ± 0.52 -0.07	-0.30 ± 0.11 -0.50
C_{LL}^T	0.008 ± 0.046 -0.044	0.093 ± 0.029 -0.030

$\mathcal{B}(B_c \rightarrow \tau \bar{\nu}) < 10\%$

F_L^{D*} included