TECHNISCHE UNIVERSITÄT MÜNCHEN

XYZs with Effective Field Theories

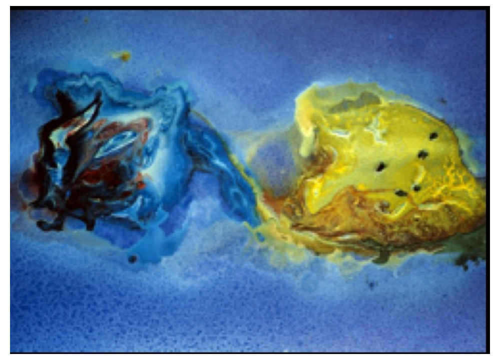
Nora Brambilla

T30f

TUMQCD LATTICE Collaboration

STRONG 2020

TUM-IAS Focus Group Physics with Effective Field Theories



Quark Confinement and the Hadron Spectrum since 1994

MCQST

MDSI

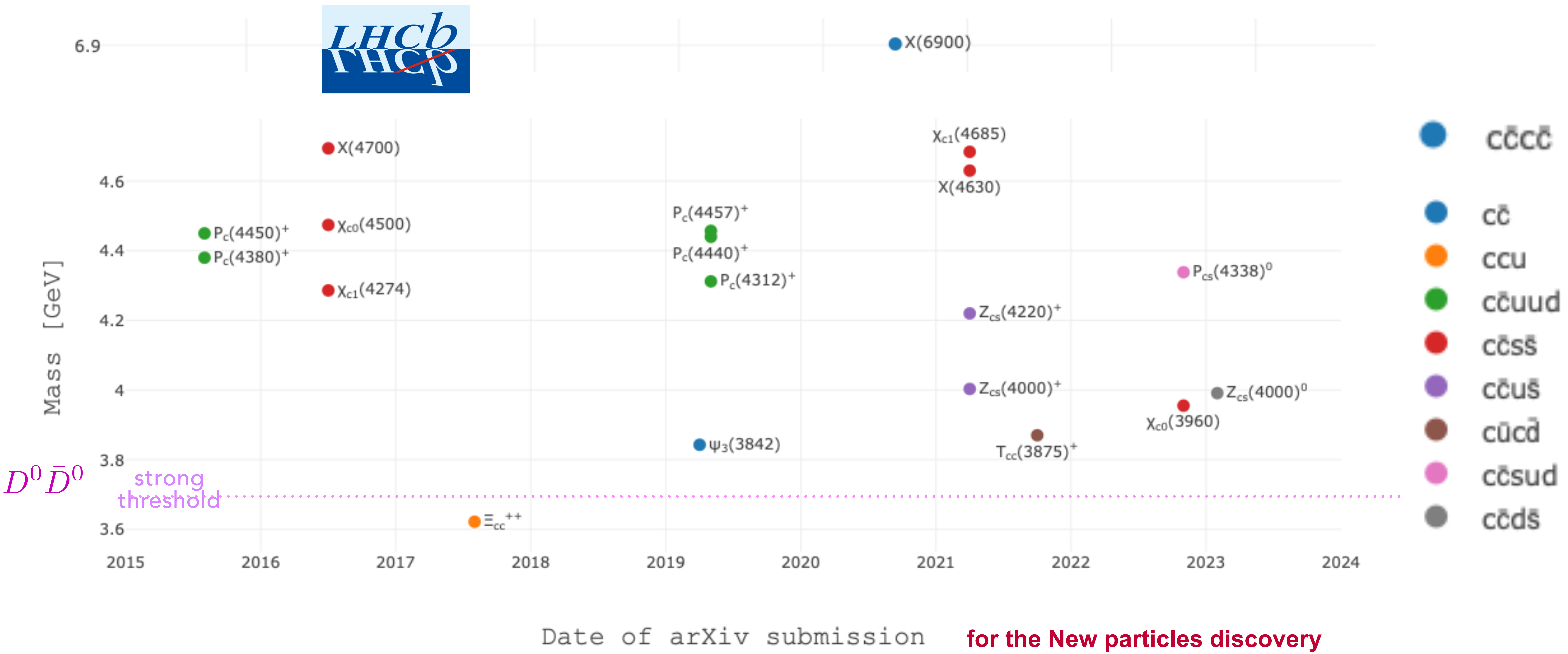
Munich Data Science Institute

ORIGINS Excellence Cluster

QWG

We are currently through a new revolution in particle physics

The present revolution: new particles discoveries beyond the Quark Model in the QQbar sector at/above the strong decay thres



for the New particles discovery

CMS BESIII Belle II ATLAS EXPERIMENT

Plots for each experiment available at our QWG https://qwg.ph.nat.tum.de/exoticshub/

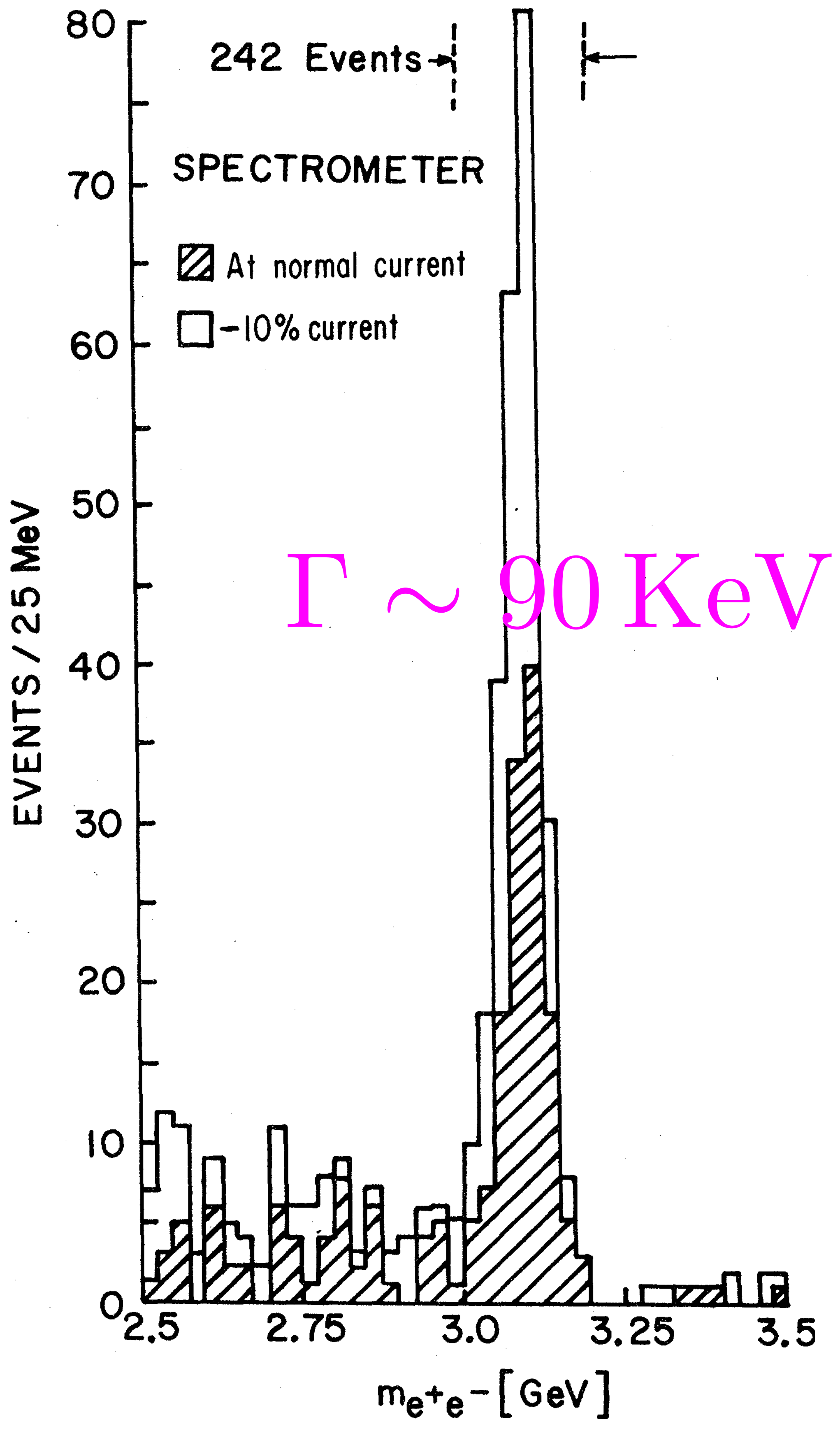
upcoming experiments: GlueX, Electron ion collider, panda

Future tau-charm Facilities STCF

Past Revolution

The discovery of the J/psi (ccbar lowest state) is at the origin of the November revolution in 1974

Samuel Ting: It was like to stumble in a village where people were living 70000 years



$\Gamma \sim 90\,\mathrm{KeV}$

- Discovery of the first quark of heavy type Q (m_c > Lambda_QCD)
- Confirmation of the quark model and QCD

Aubert et al. BNL 74

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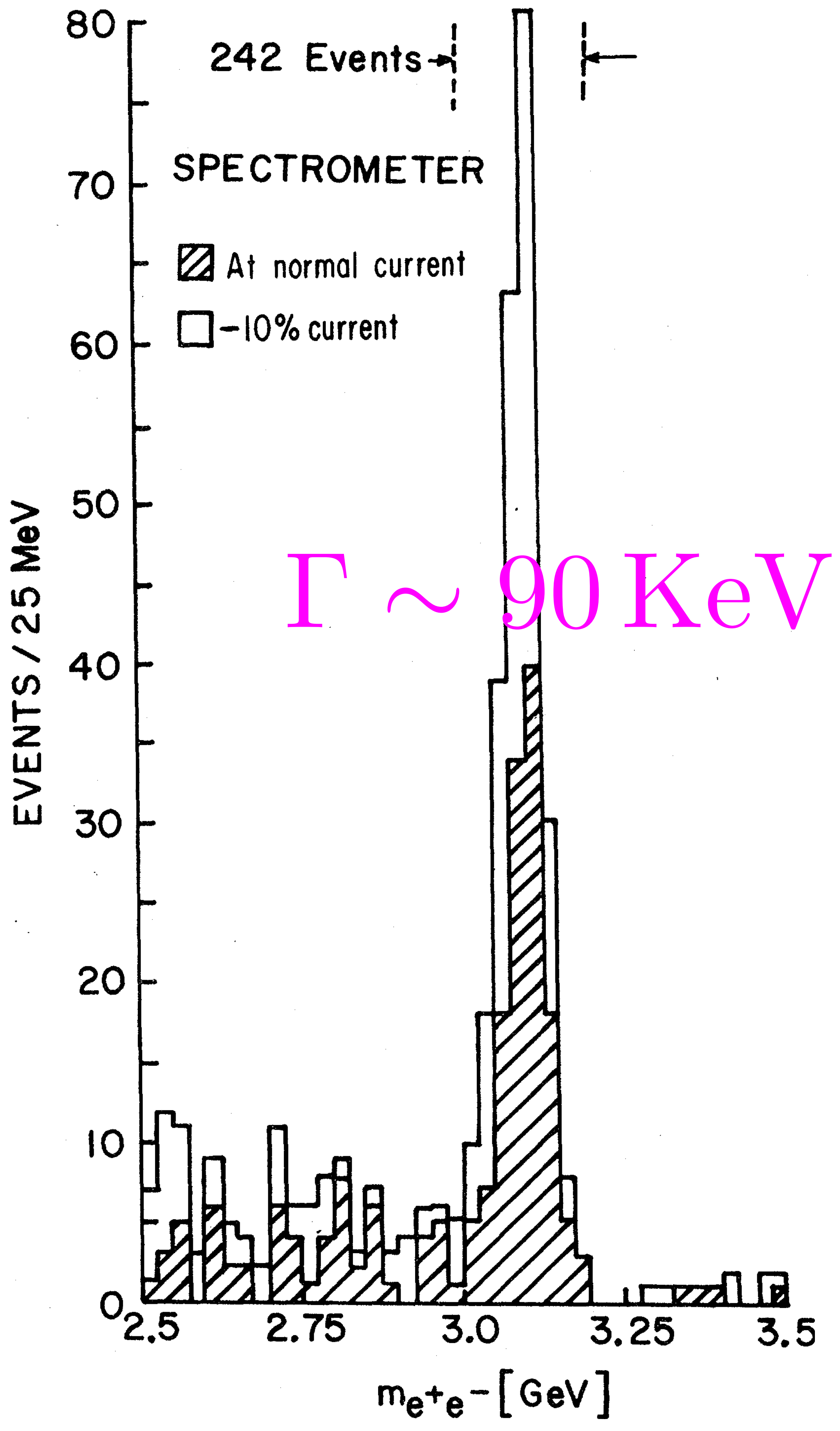

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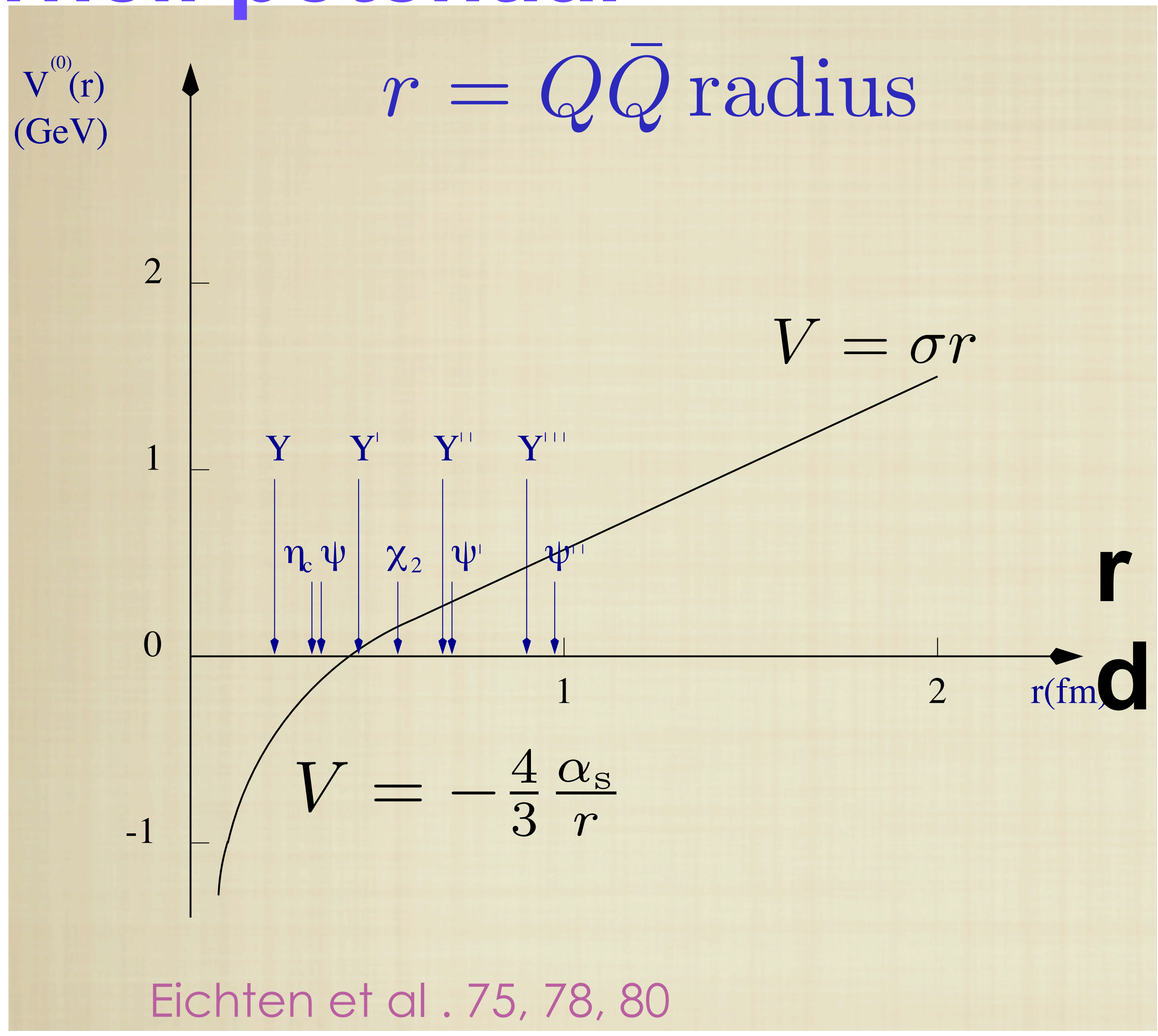
—>**narrow width and asymptotic freedom:** annihilation at large scale controlled by a small coupling constant $\alpha_s(2m_c) \ll 1$

—>**energy levels and confinement:** structure of levels controlled by a Cornell potential in a Schroedinger eq.

Cornell potential



Eichten et al . 75, 78, 80

r QQbar distance

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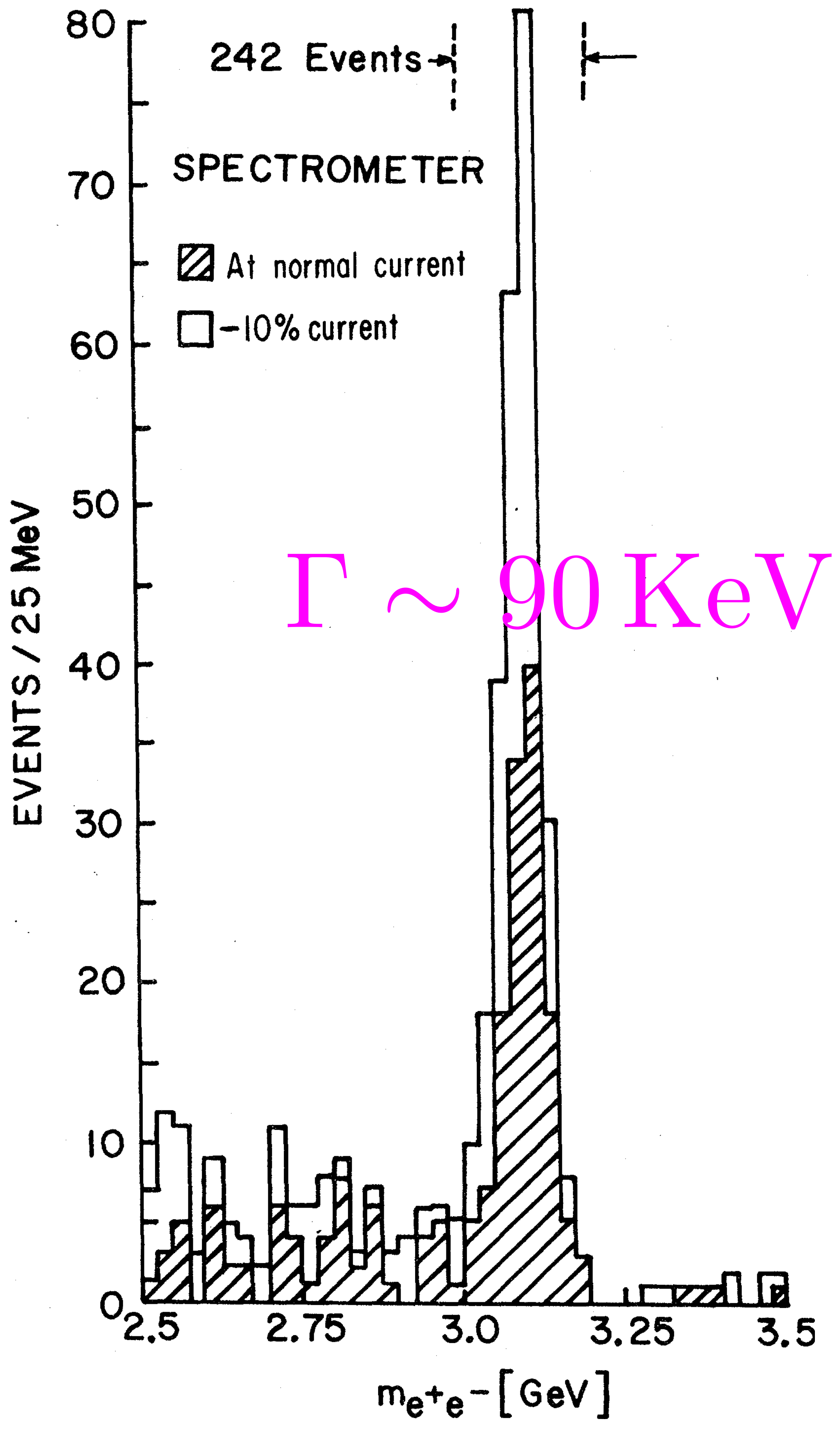

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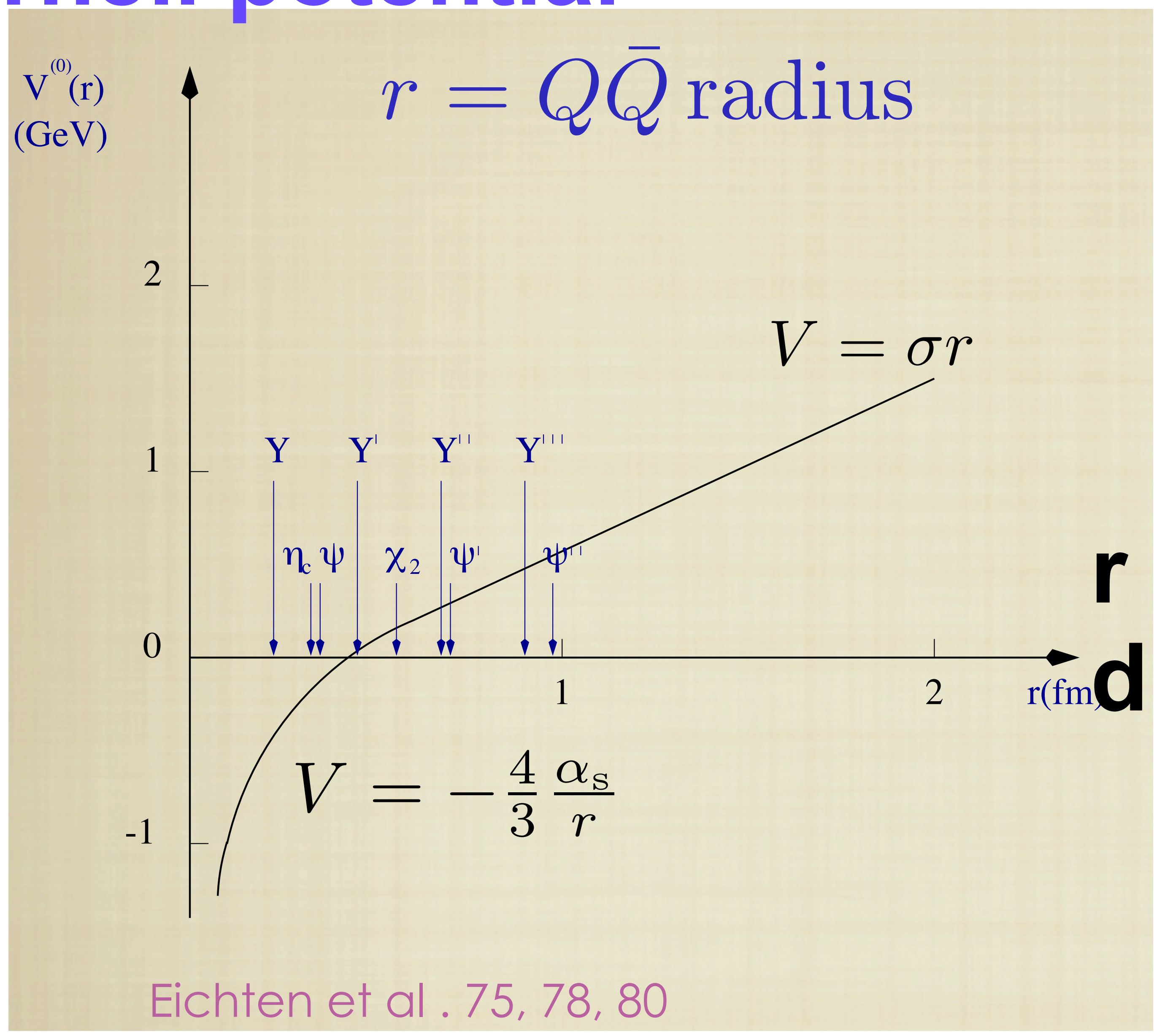
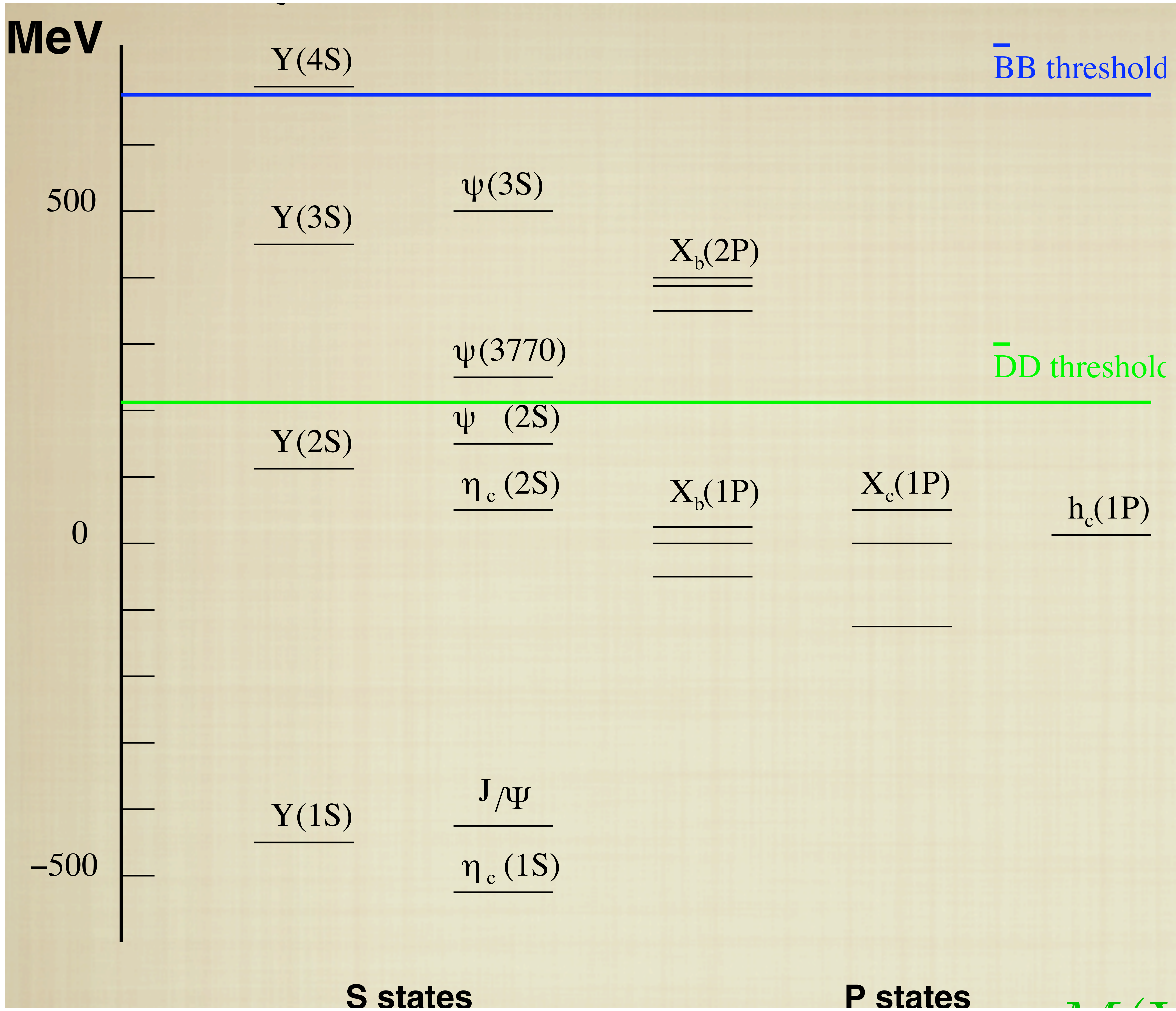


r QQbar distance

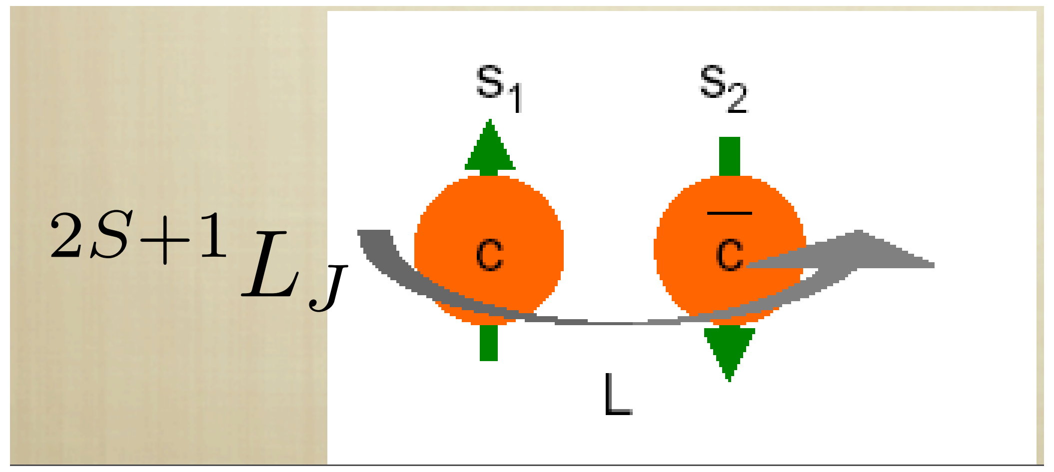
Heavy quarkonia are nonrelativistic systems:multiscale systems

Many scales: a challenge and an opportunity

Quarkonium scales



Levels normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$



$$M(Y(1S)) = 9460\,\mathrm{GeV}$$
$$M(J/\Psi) = 3097\,\mathrm{GeV}$$

The mass scale is perturbative

$$m_Q \gg \Lambda_{\mathrm{QCD}}$$

$$m_b \simeq 5\,\mathrm{GeV};\, m_c \simeq 1.5\,\mathrm{GeV}$$

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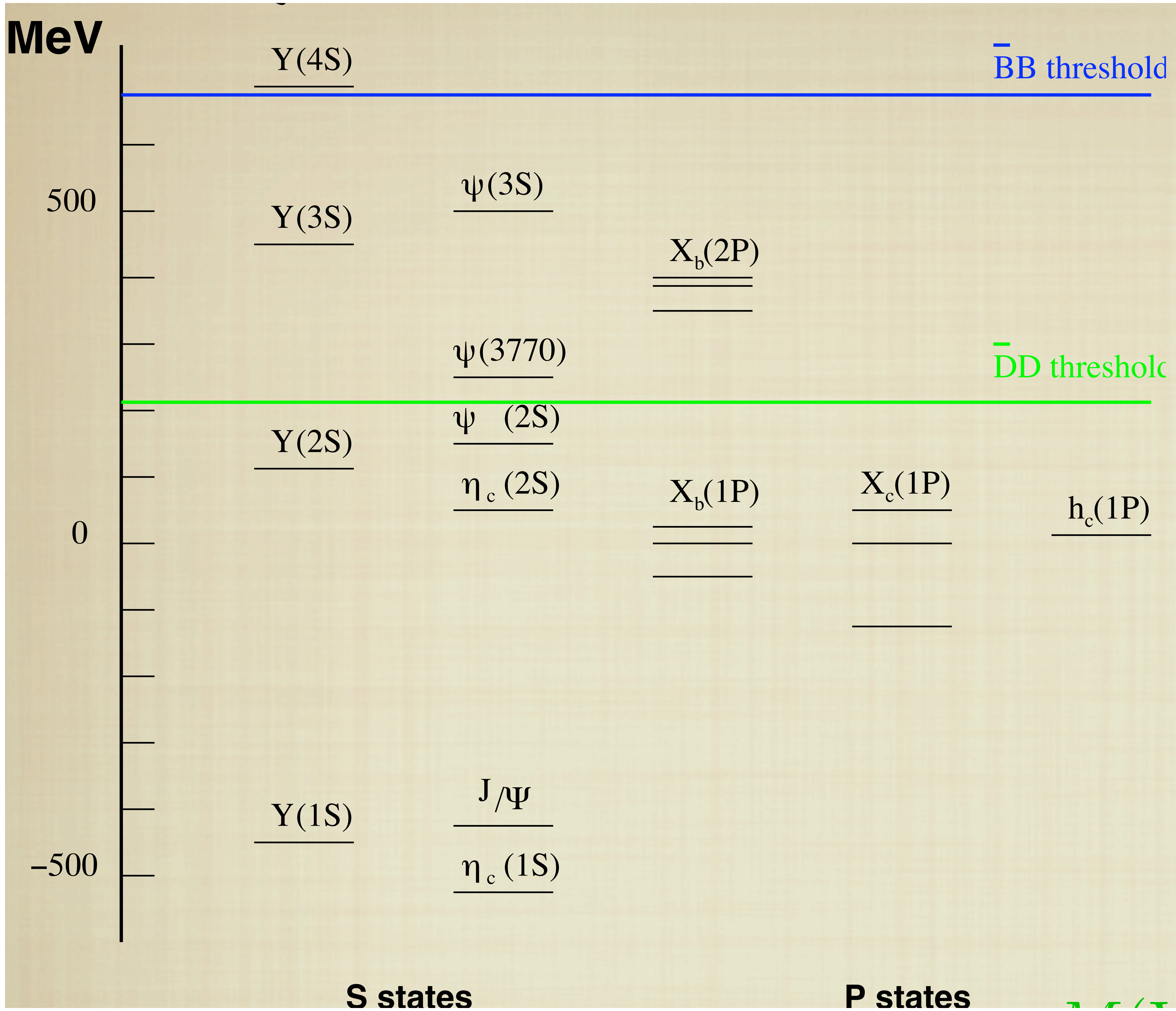

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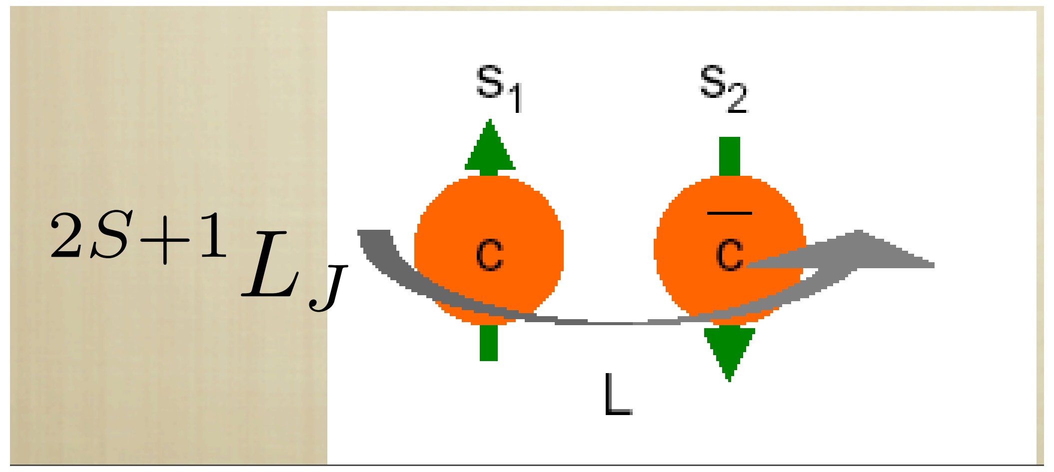

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NR bound states have at least 3 scales

$$m \gg mv \gg mv^2 \qquad v \ll 1$$

$$mv \sim r^{-1}$$

The system is nonrelativistic(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

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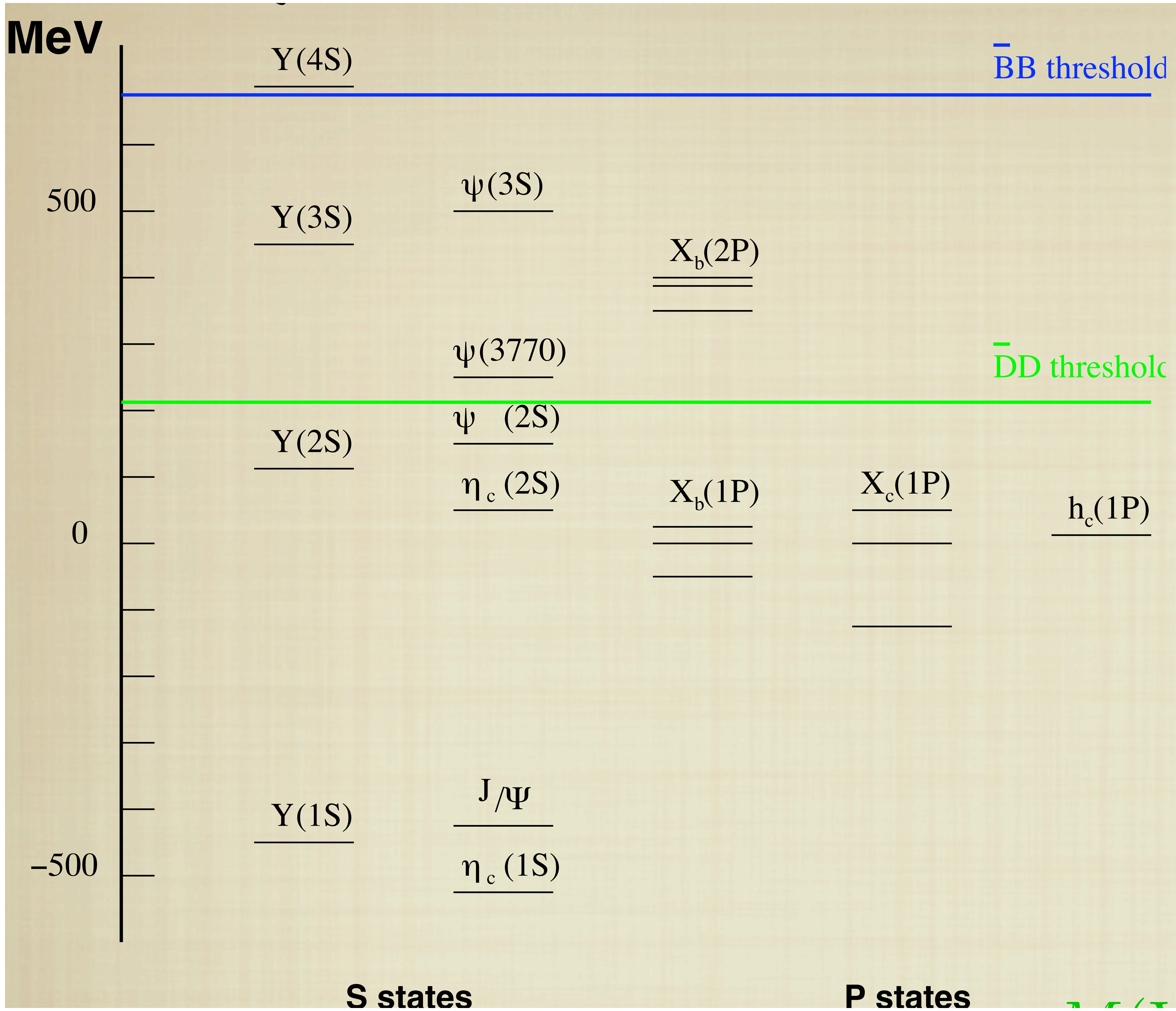

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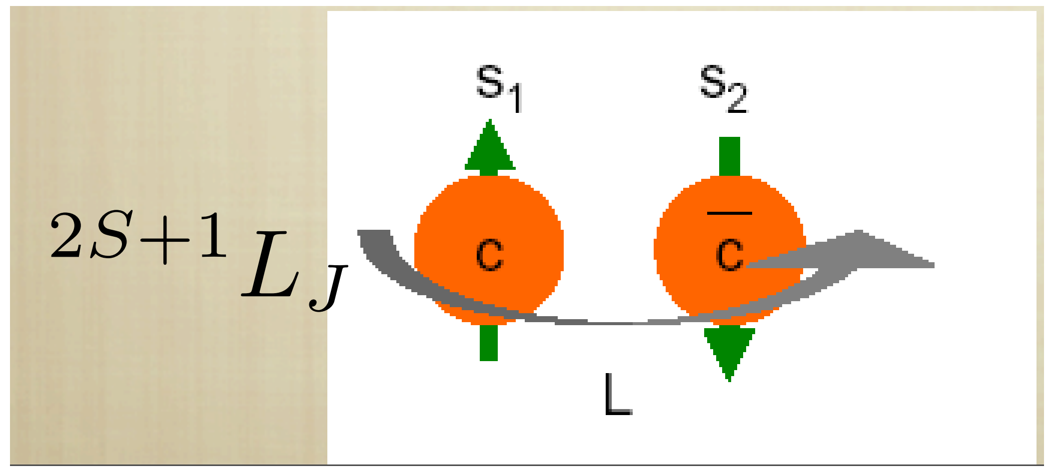

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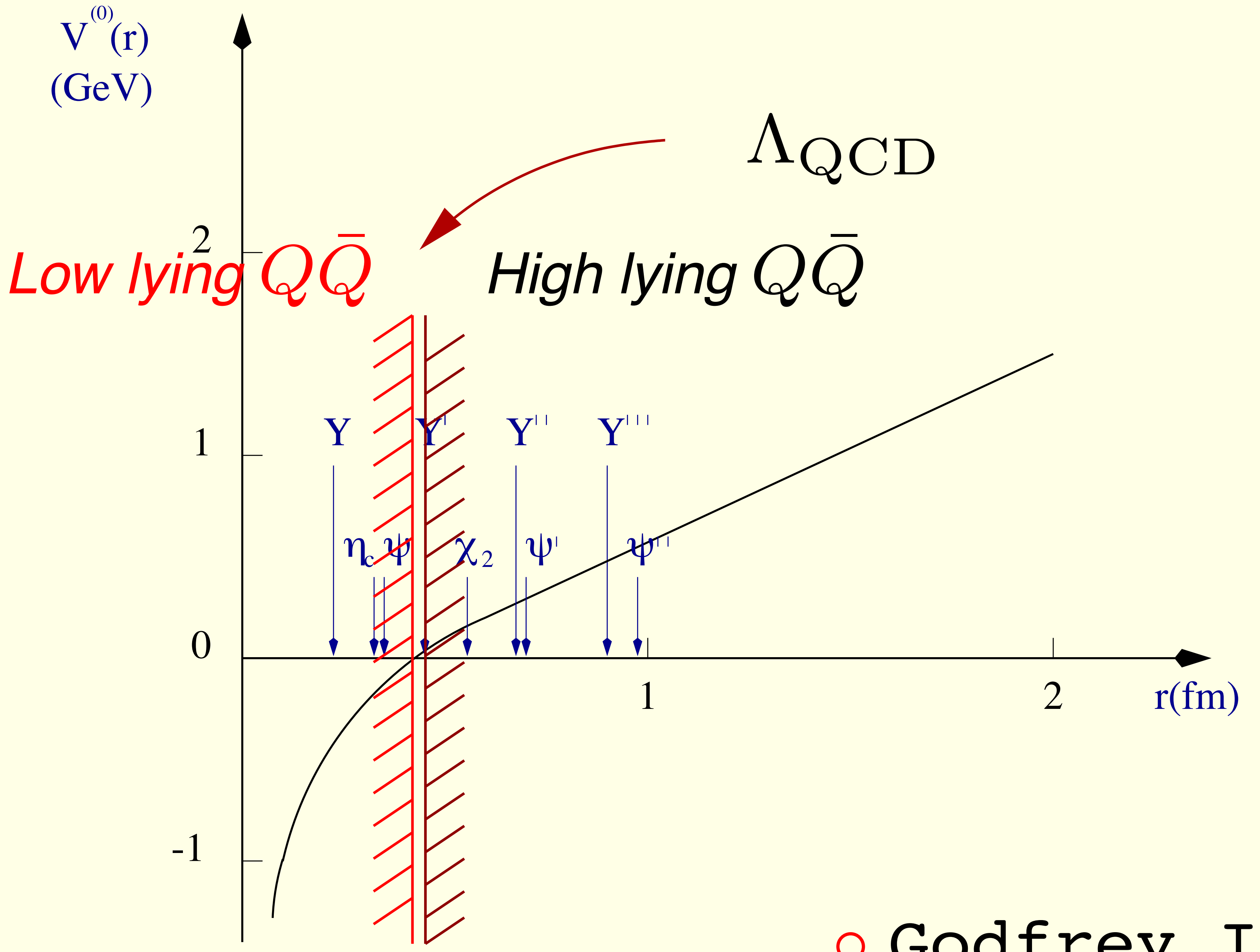
The rich structure of separated energy scales makes QQbar systems an ideal probe of strong interactions

- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.



○ Godfrey Isgur PRD 32(85)189

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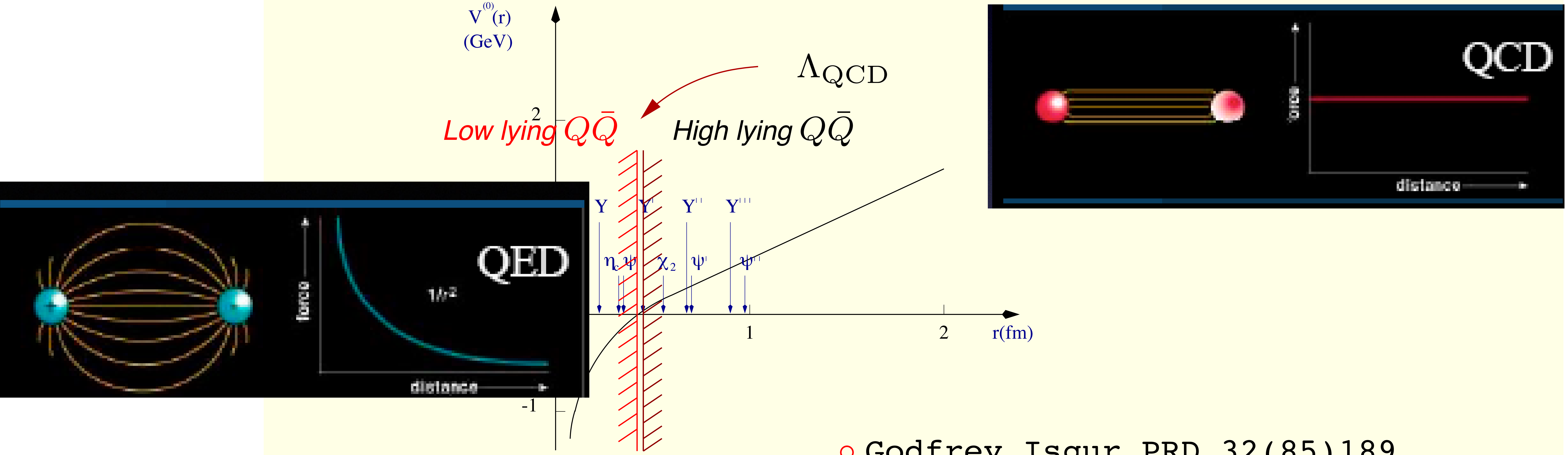

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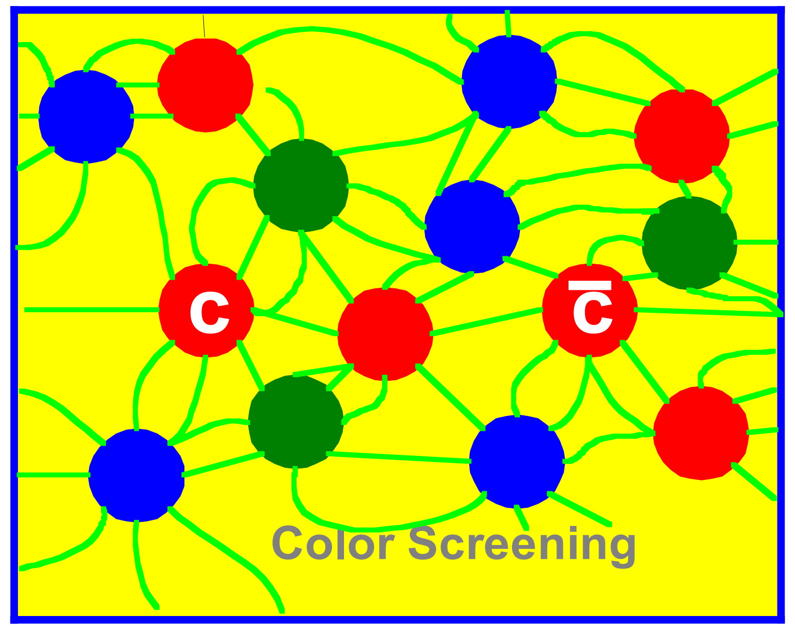
The present revolutions: nuclear matter phase diagram

Quarkonia are probe of QGP formation

Matsui Satz 1986 idea of **color screening in medium**



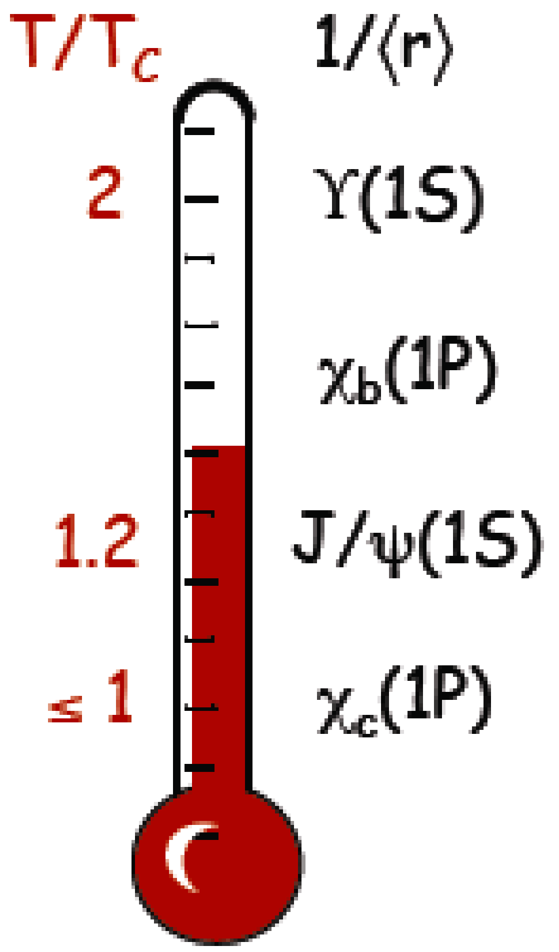
Debye charge screening

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

$$m_D \sim gT$$

$$r \sim \frac{1}{m_D}$$

Bound state dissolve



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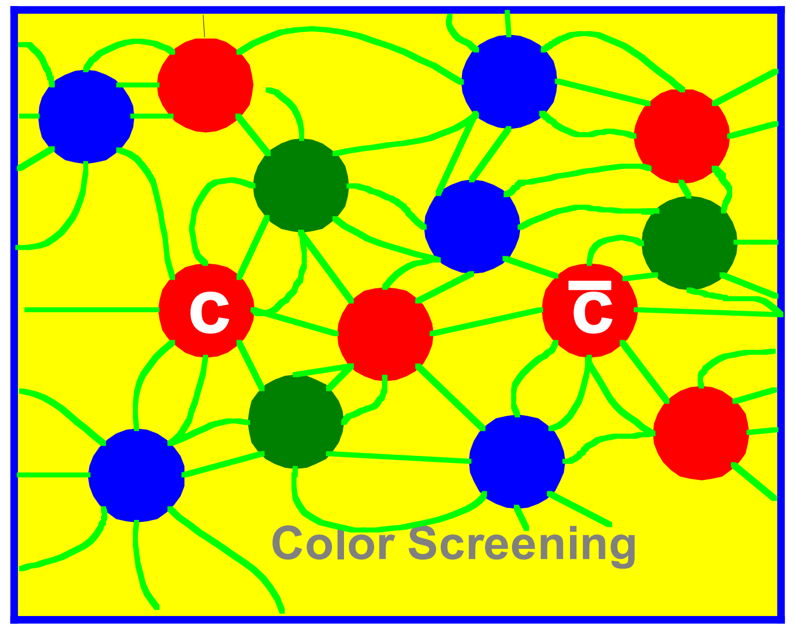

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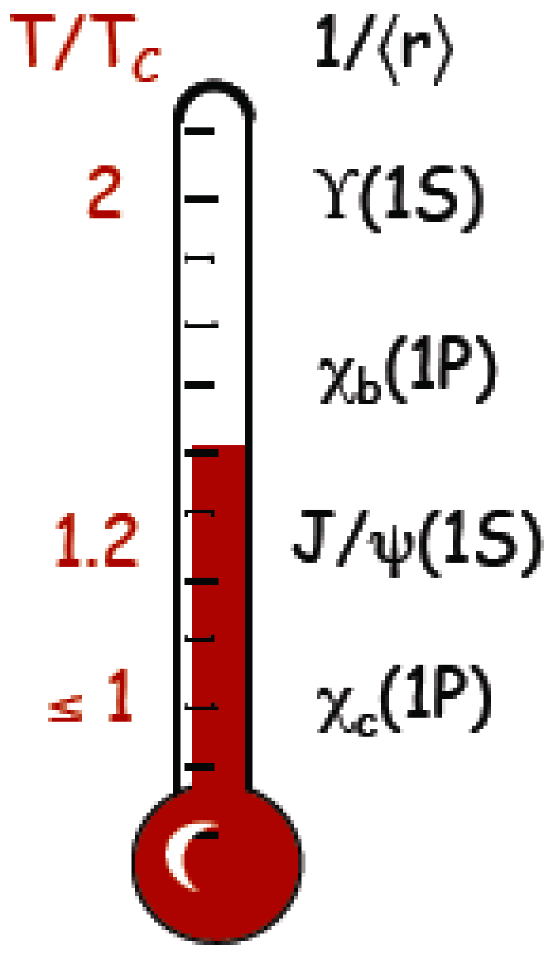

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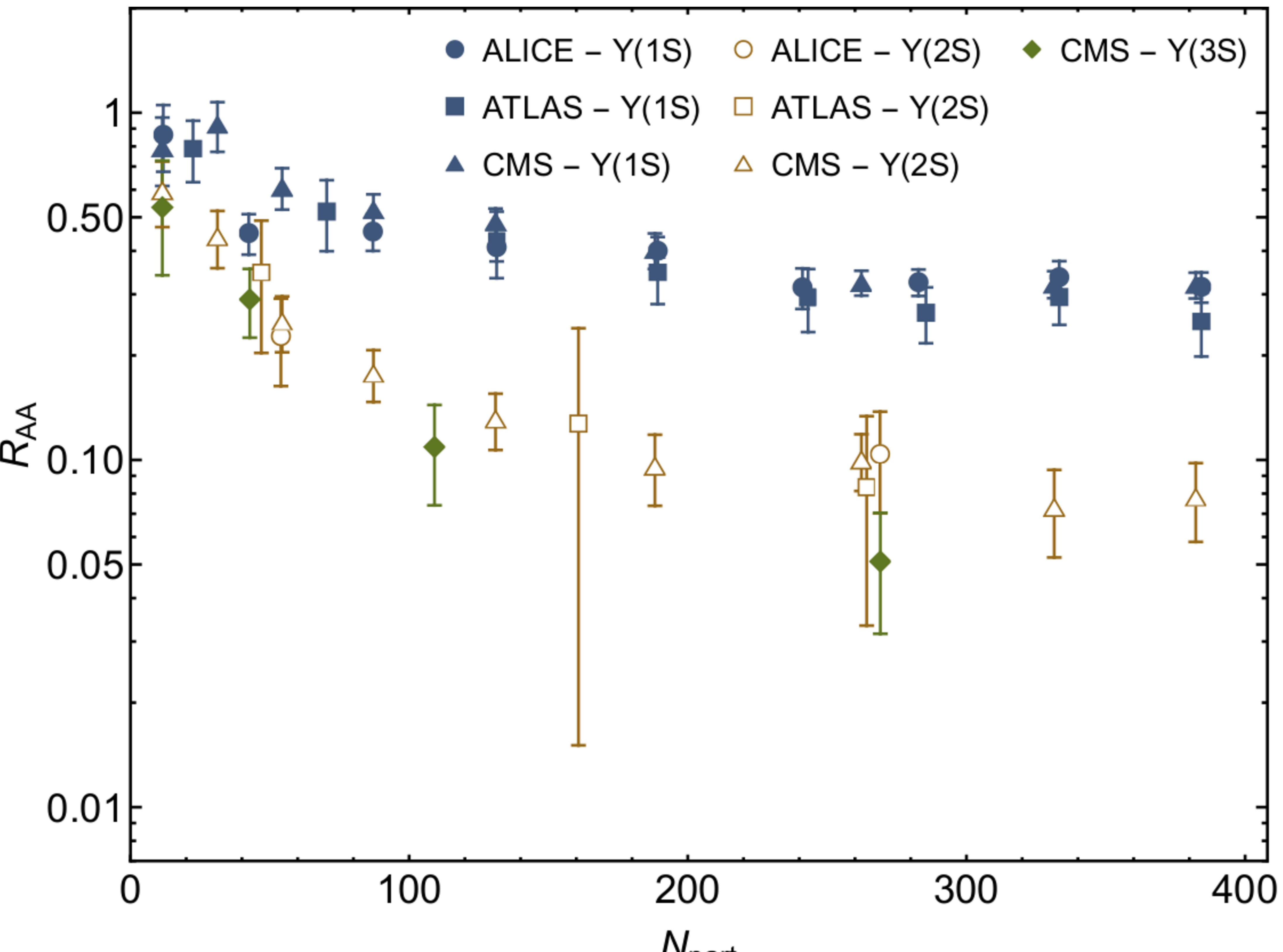
Experimental measurements:

R_{AA} is the nuclear modification factor = yield of quarkonium in PbPb / yield in pp.



CMS PLB 790 (2019) 270
ALICE PLB 822 (2021) 136579
ATLAS PRC 107 (2023) 054912

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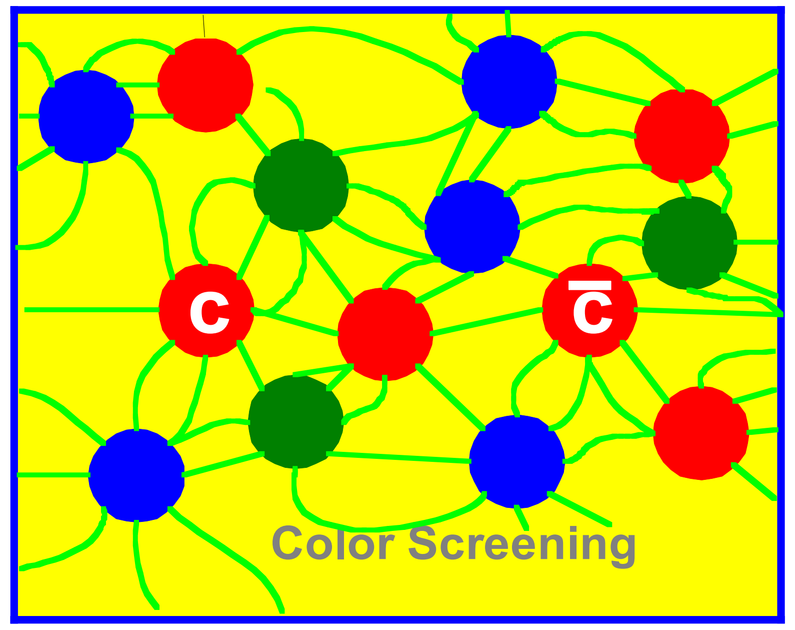

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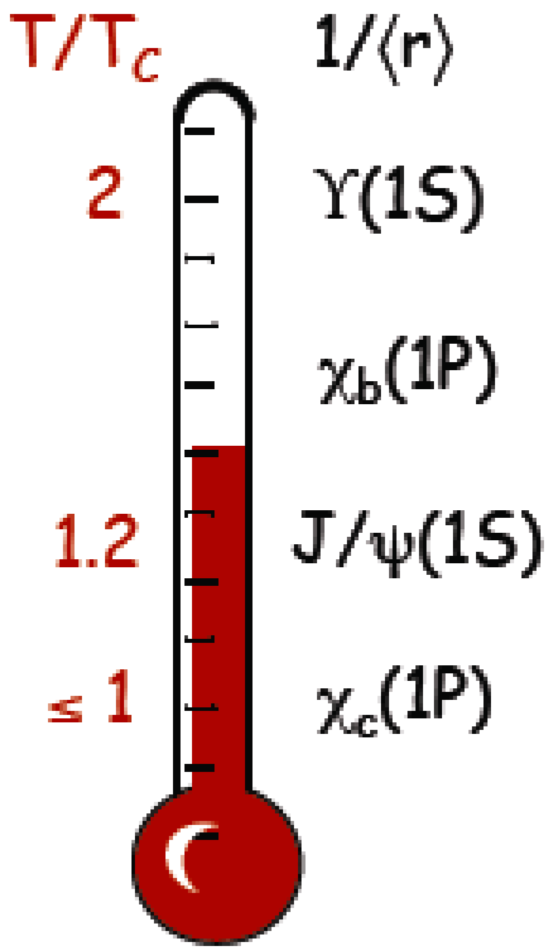

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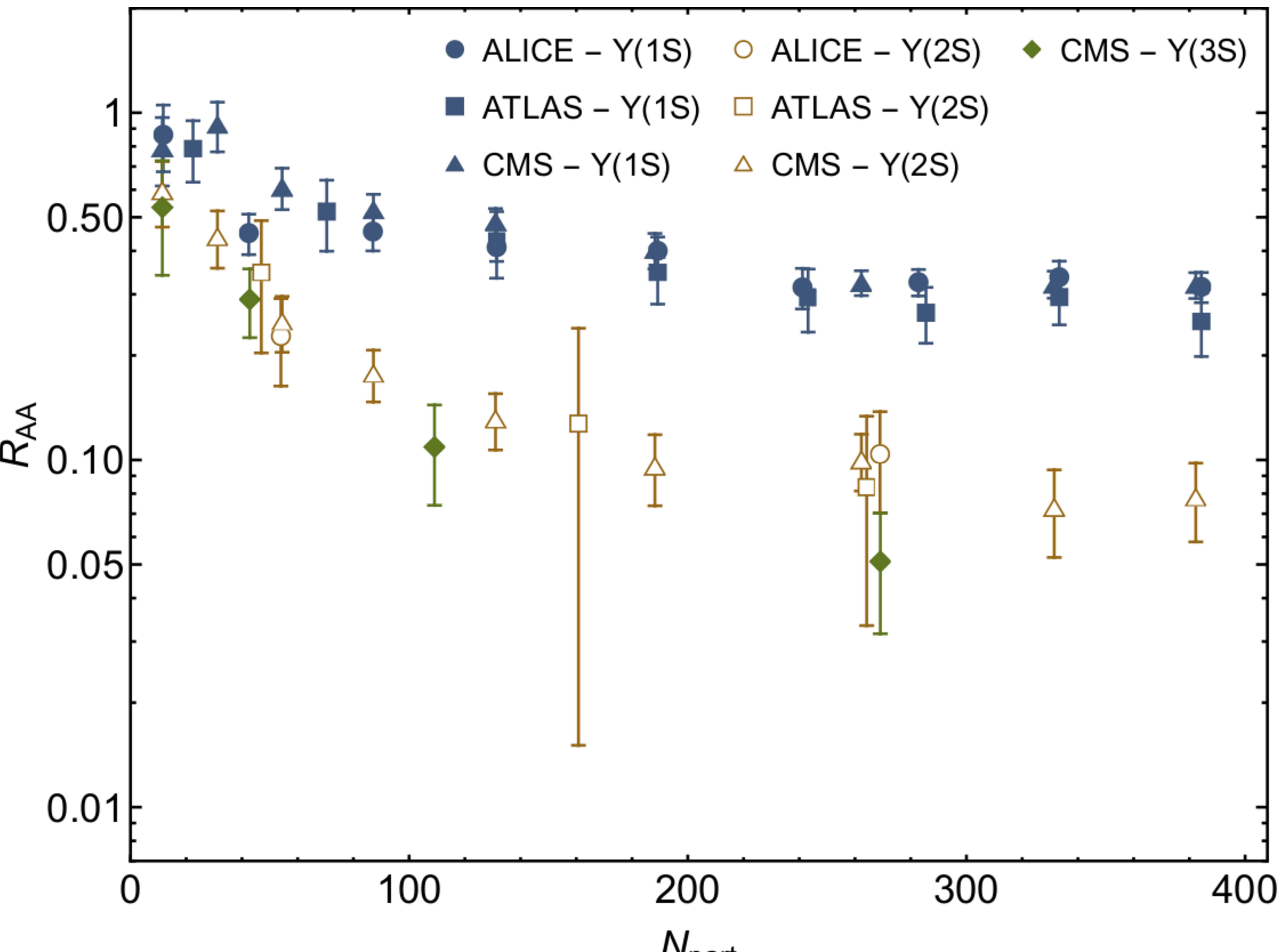

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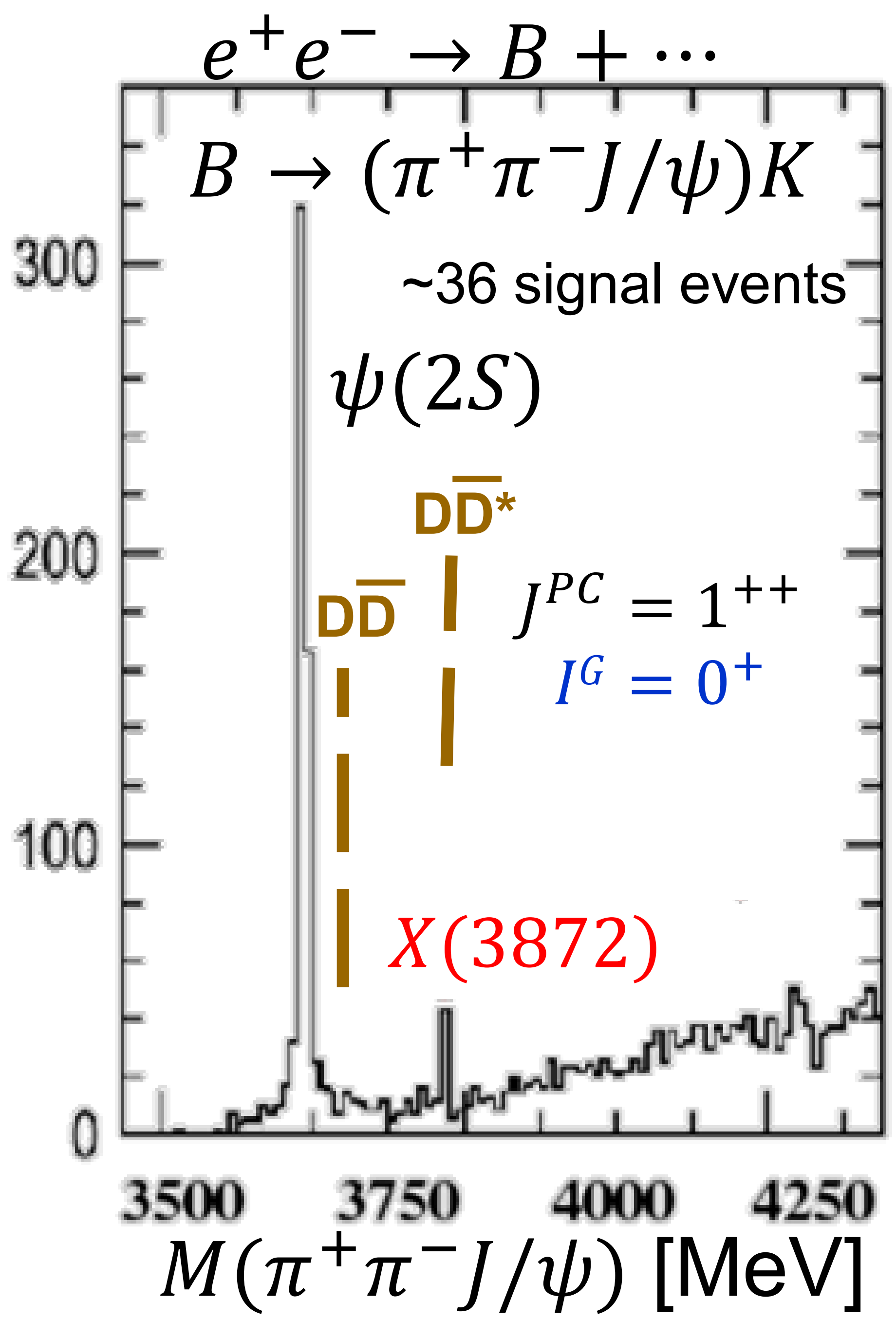
Today a new paradigm emerged **beyond screening** relating the R_AA to the **nonequilibrium evolution of the heavy pair in medium:** medium induced dissociation and color singlet/octet recombination. **Quantum phenomenon to be addresses with quantum master equations**

The present revolution: XYZ striking characteristics

Belle 2003: discovery of X(3872)



$\Gamma = 1.2 \pm 0.2\ MeV\ (PDG\ 2022)$

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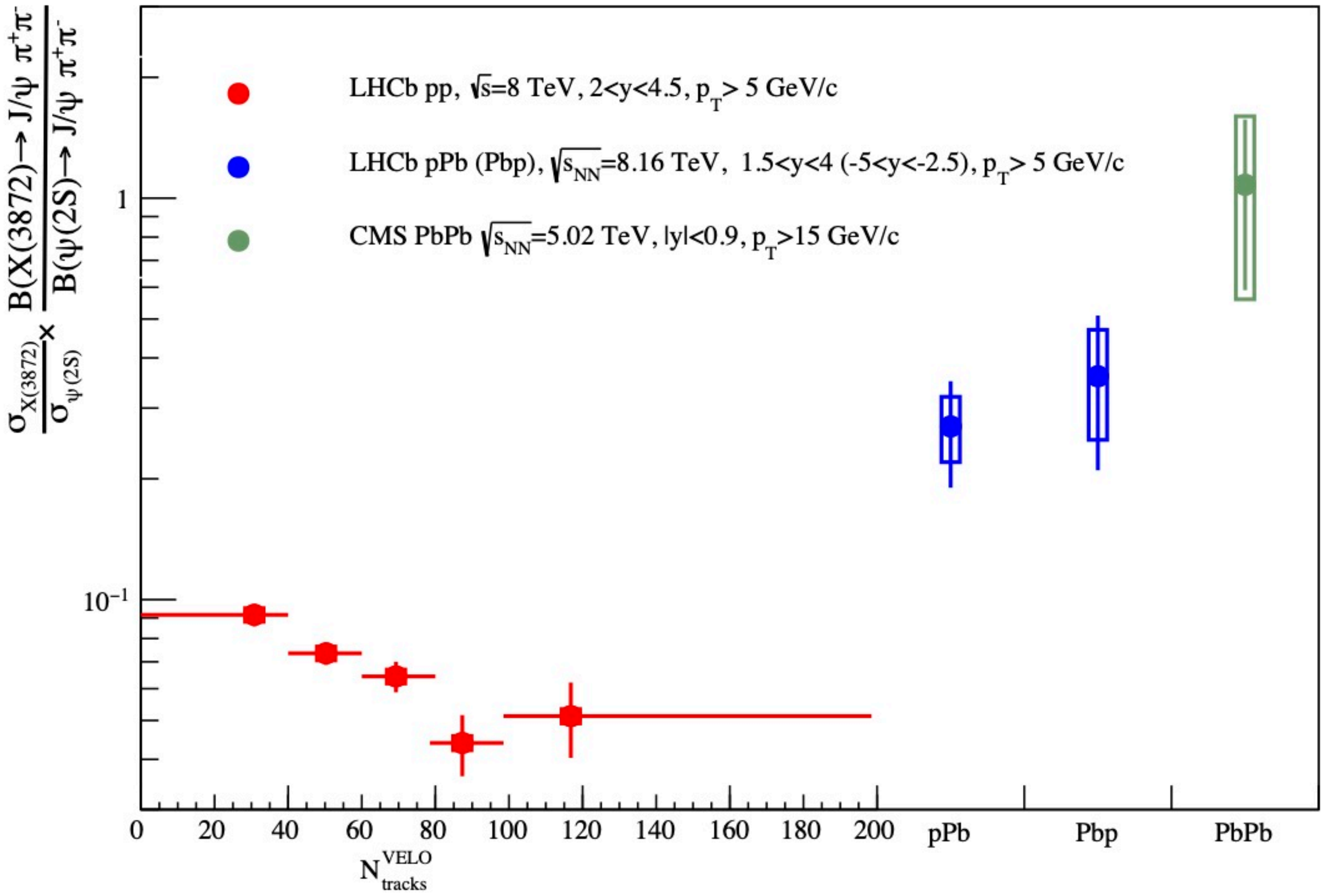
Sizeable hadronic prompt production cross sections (Tevatron, CMS, ATLAS, LHCb)

production in heavy ions where the strongly coulpled QCD medium is formed



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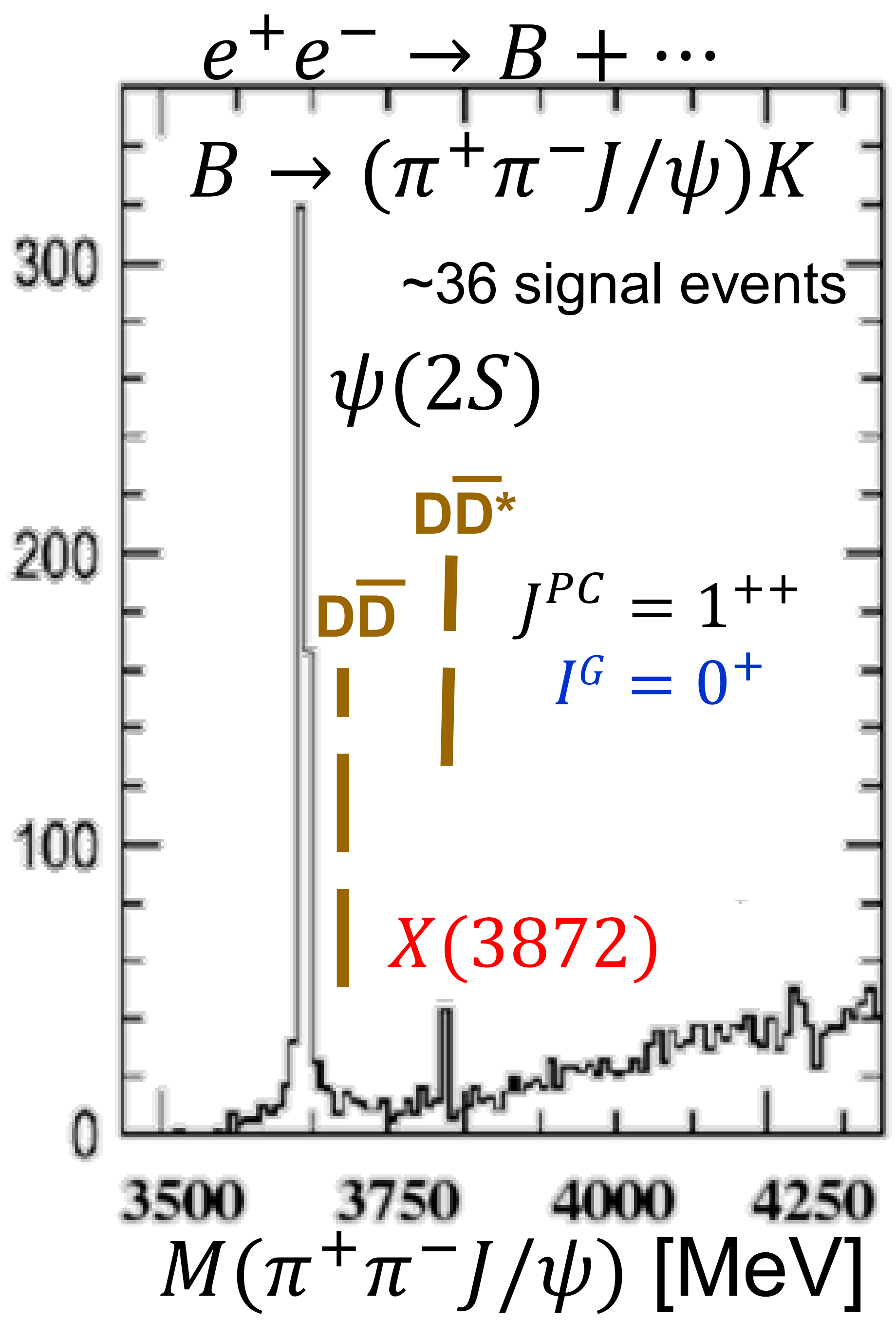

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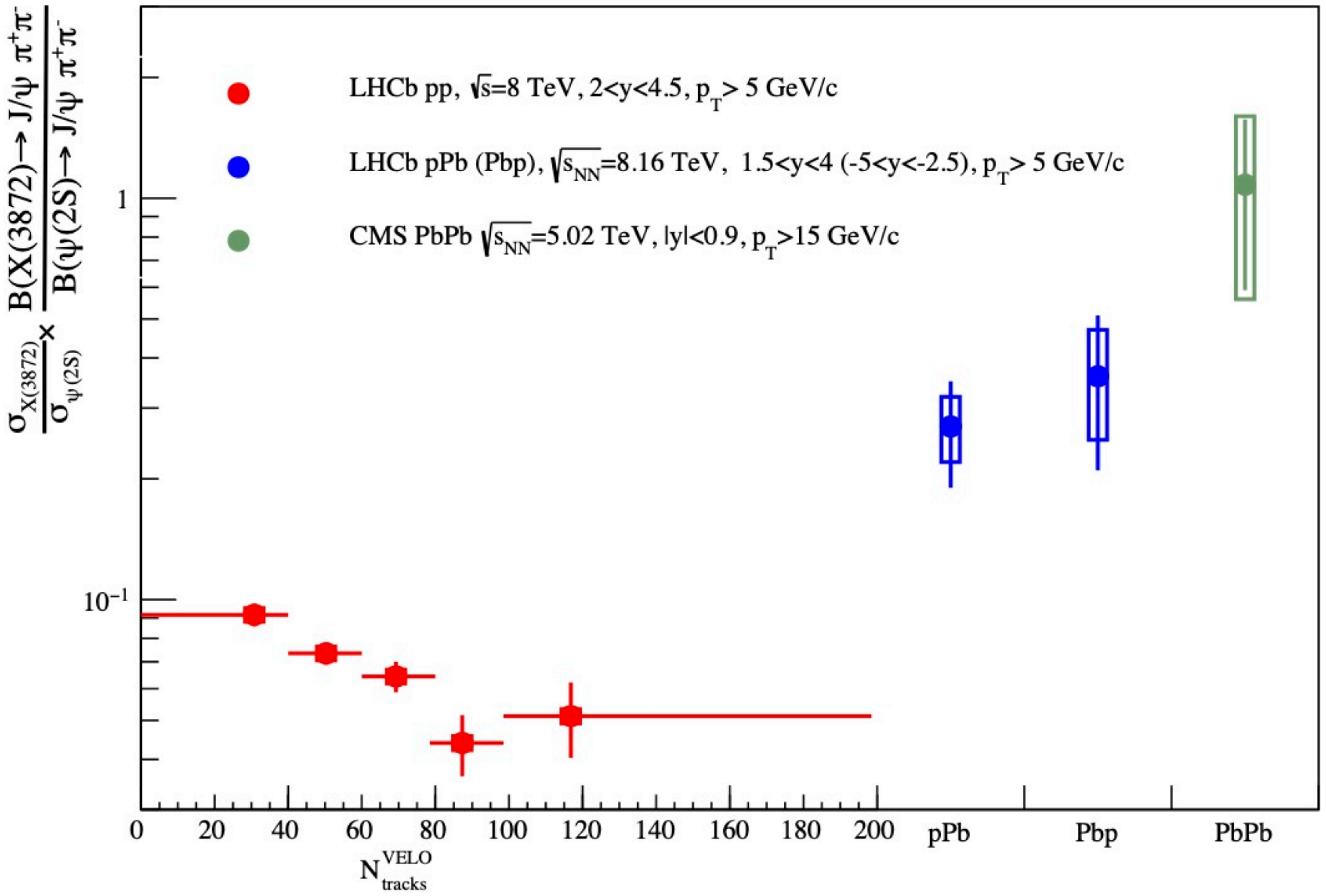

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XYZs not merely composite particles, have unique properties

Novel strongly correlated exotics systems

It is of fundamental interest to provide first principle predictions for spectra, transitions, production and medium evolution from QCD

The present revolution: XYZ a formidable theoretical challenge

Close/above threshold new degrees of freedom like glue and light quarks are nonperturbative part in the binding.

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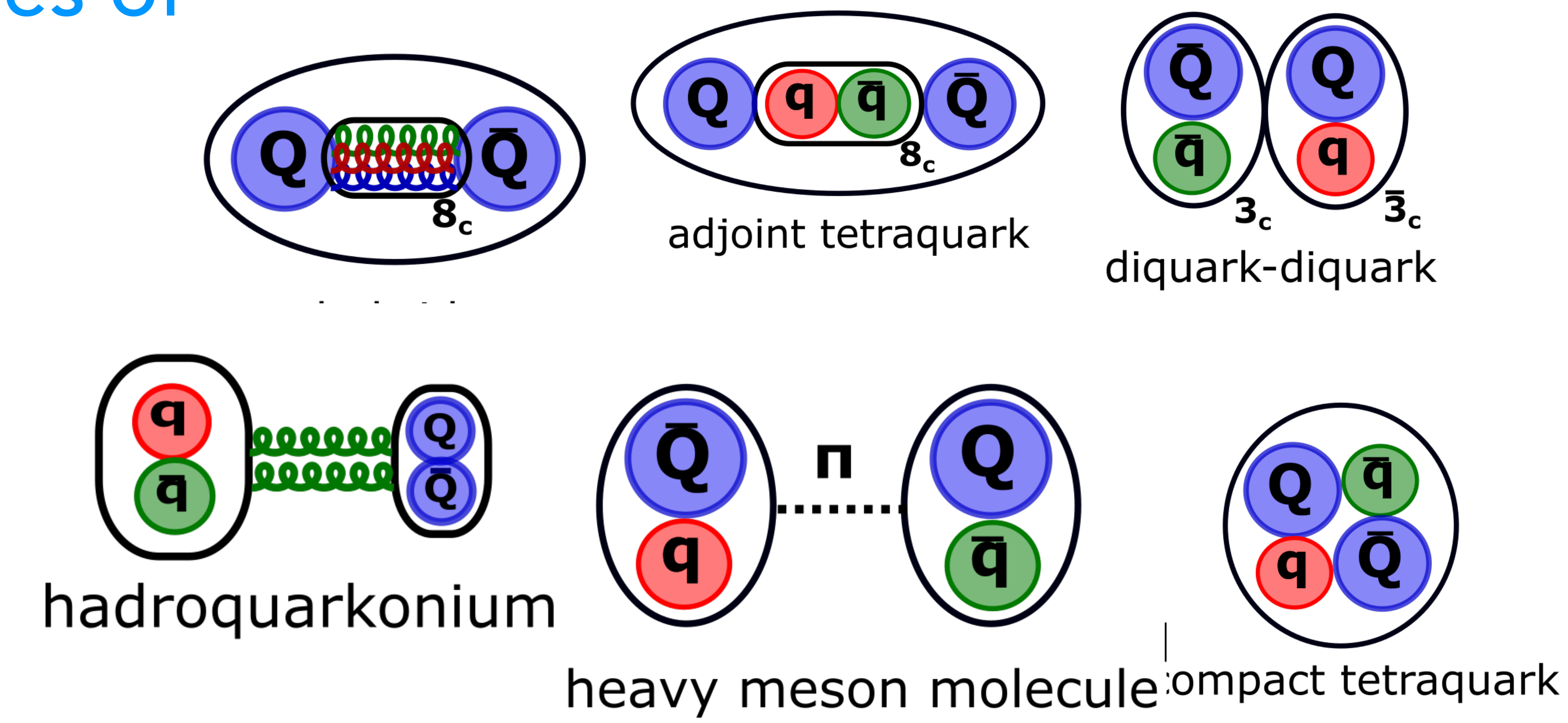
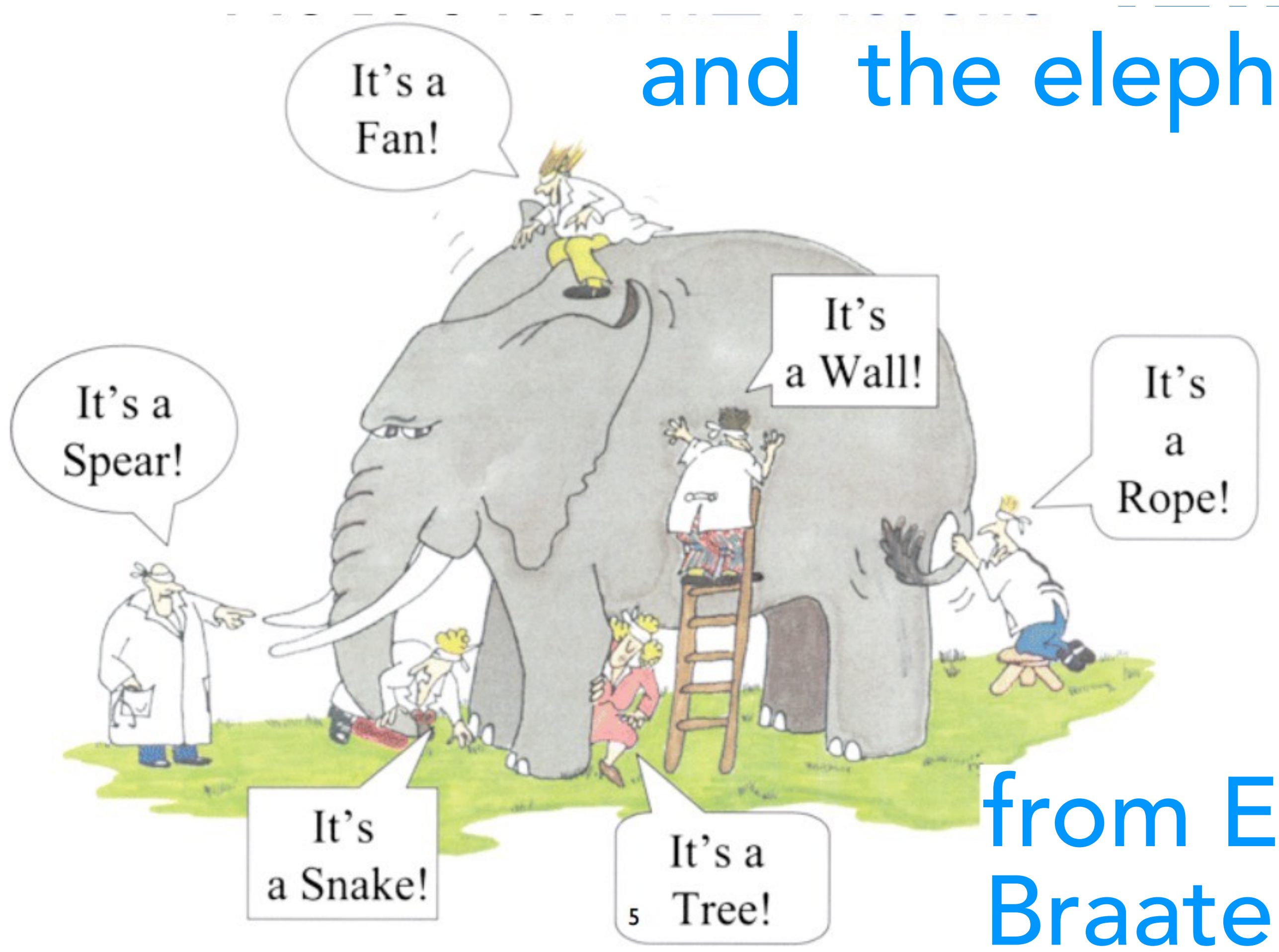
- Models assume some special degrees of freedom and a model interaction



the blind men and the elephant



from E. Braaten

- Lattice calculation of exotics masses are limited by the large number of open decay modes and they are not Immediately suited for production and in medium studies

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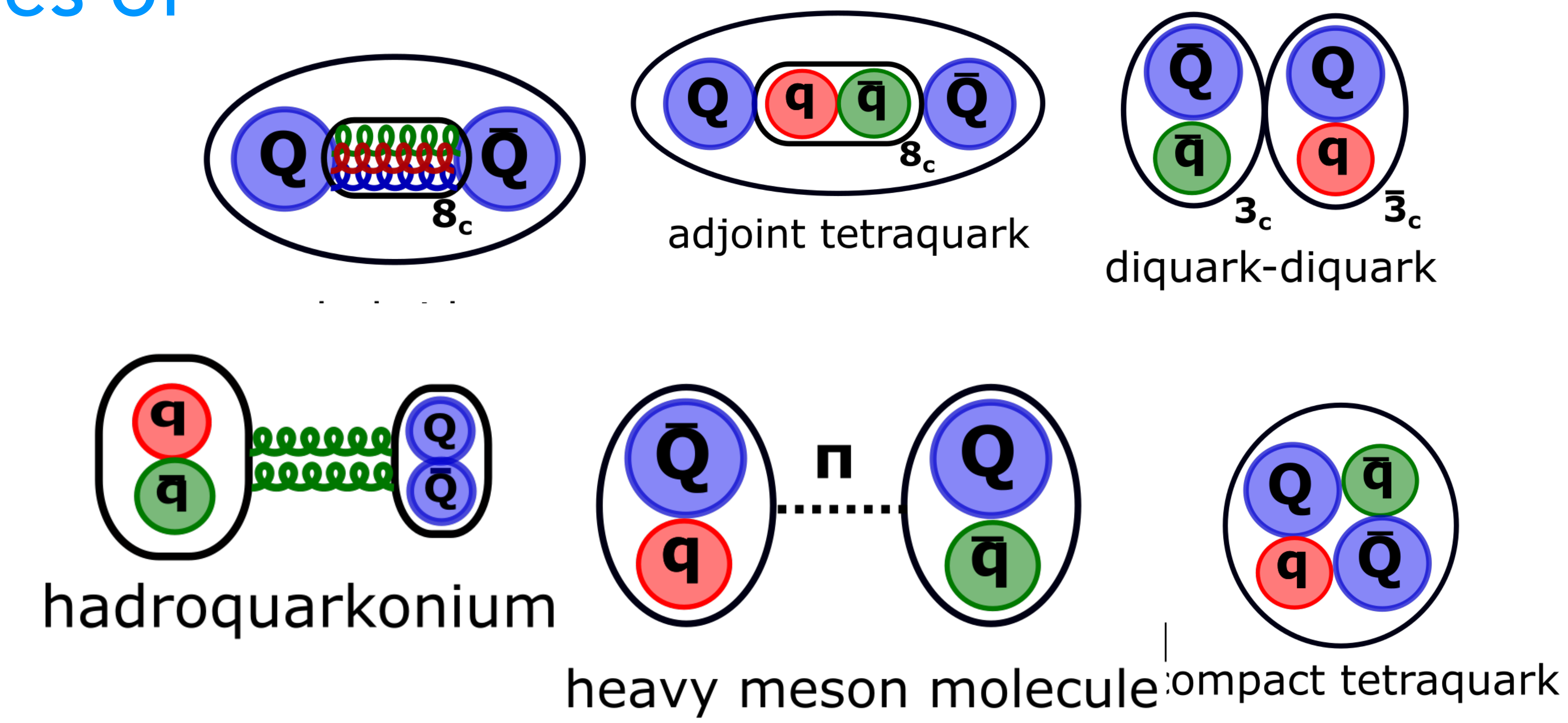

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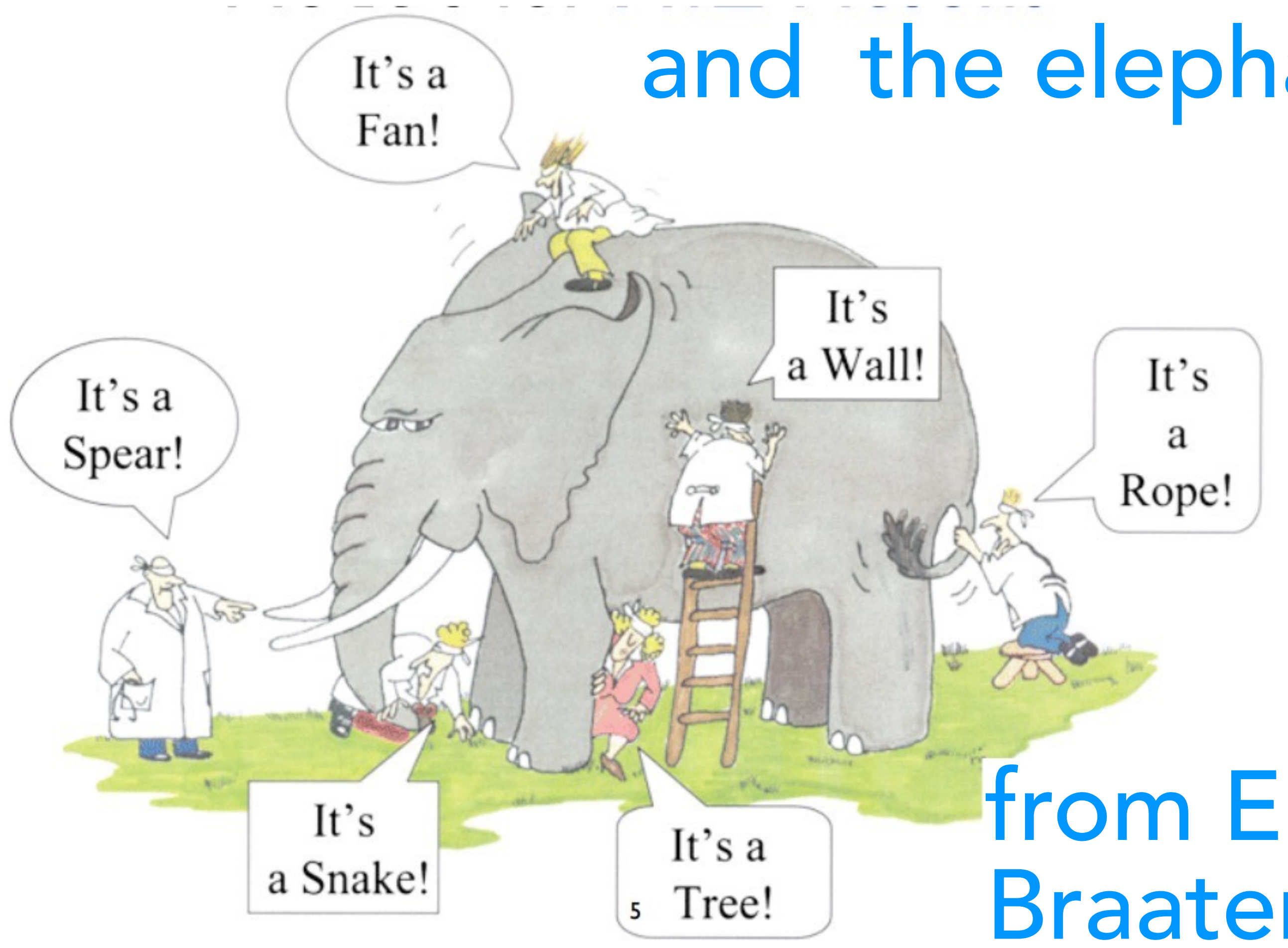

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We need a flexible approach rooted in QCD that can address all properties of XYZ spectra, production and propagation in medium : Born Oppenheimer Effective Field Theory (BOEFT)

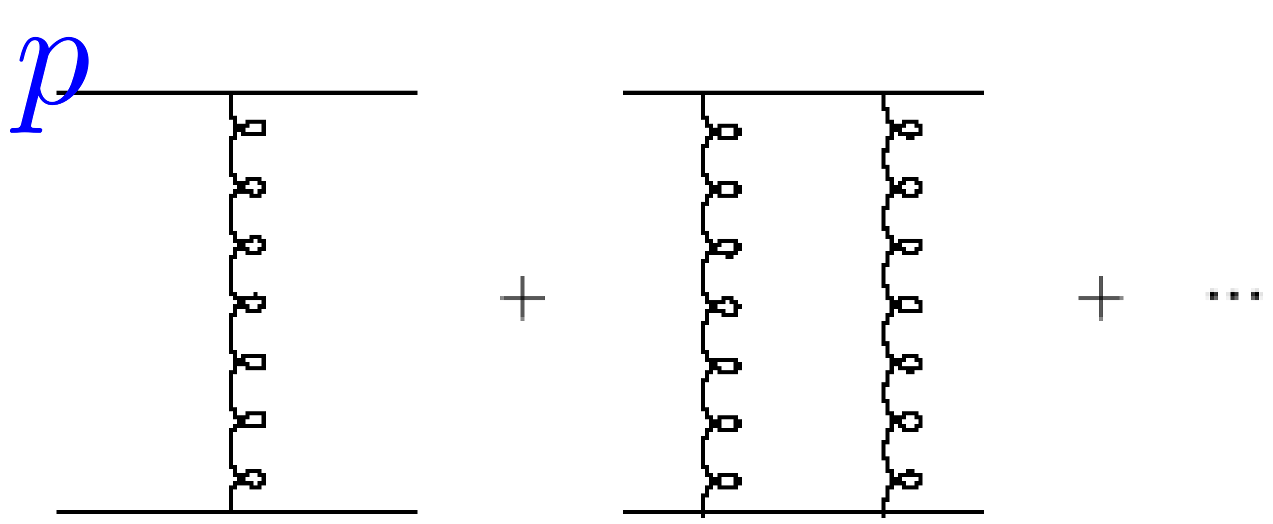
For quarkonium to become a probe of strong interactions, it should be treated in QCD :a very hard problem

Close to the bound state $\alpha_s \sim v$

Q

$p \sim m\alpha_s$

$\bar{Q}$



$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

- From $(\frac{p^2}{m} + V)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

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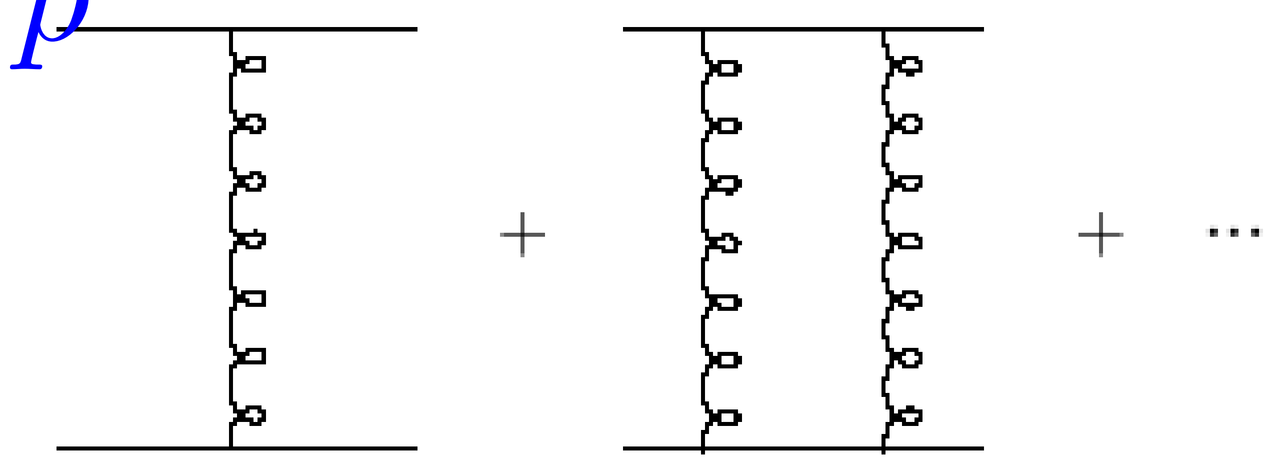

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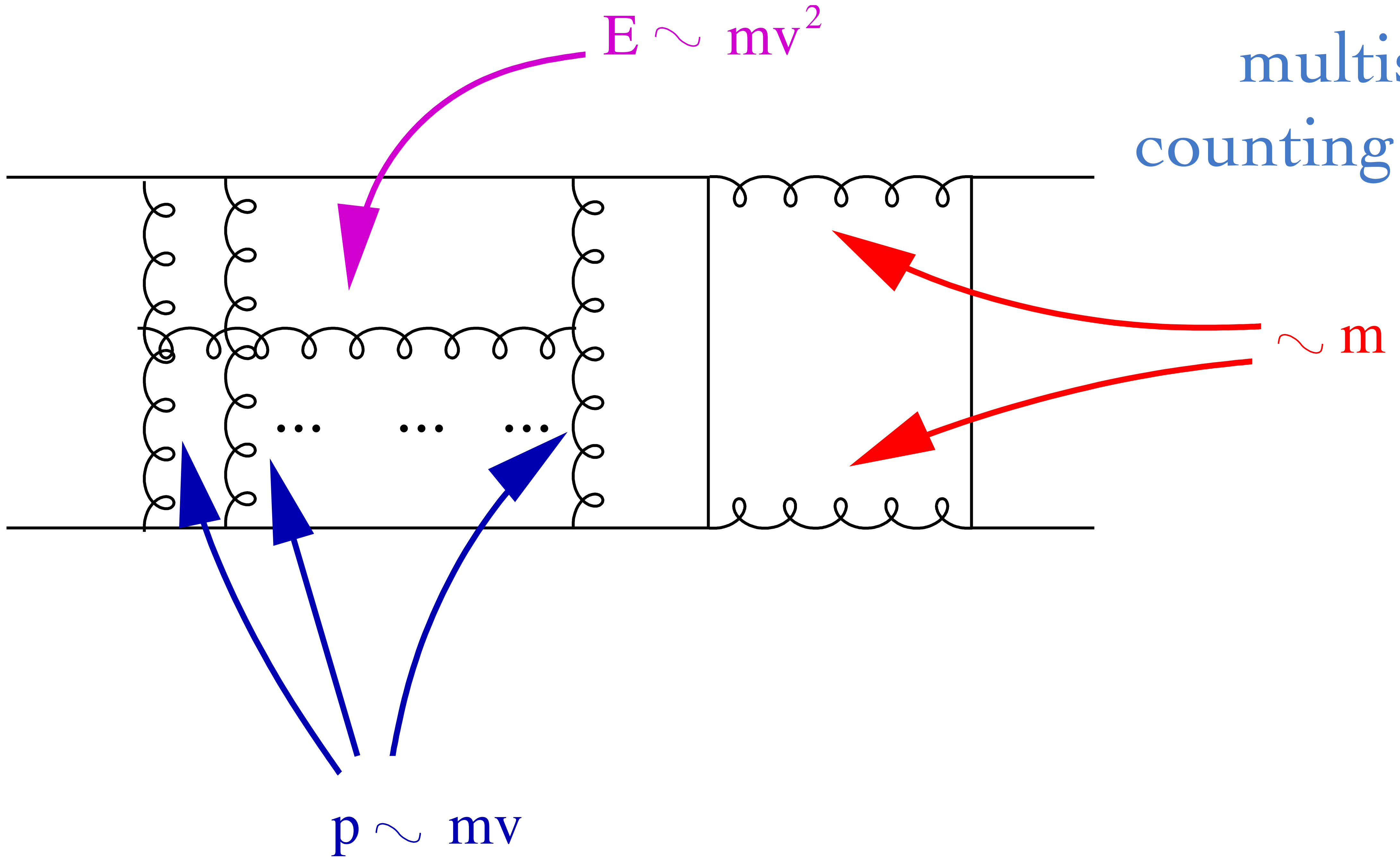

multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

Difficult also
for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

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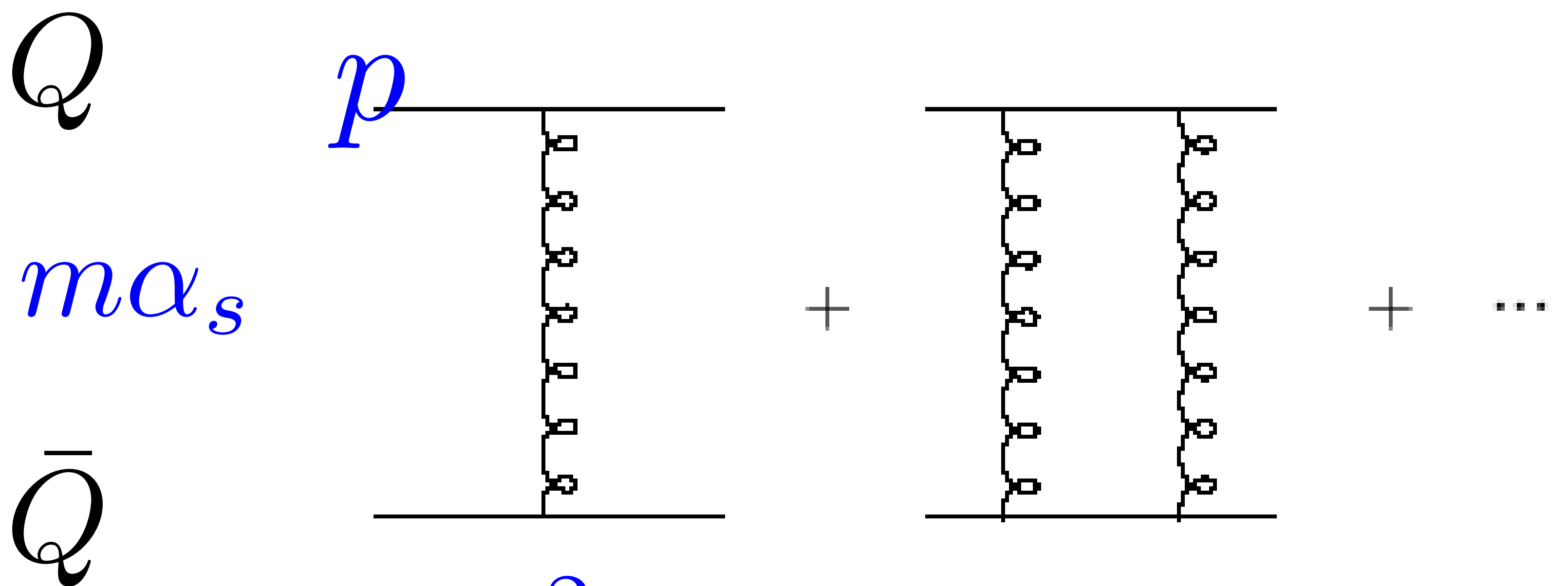

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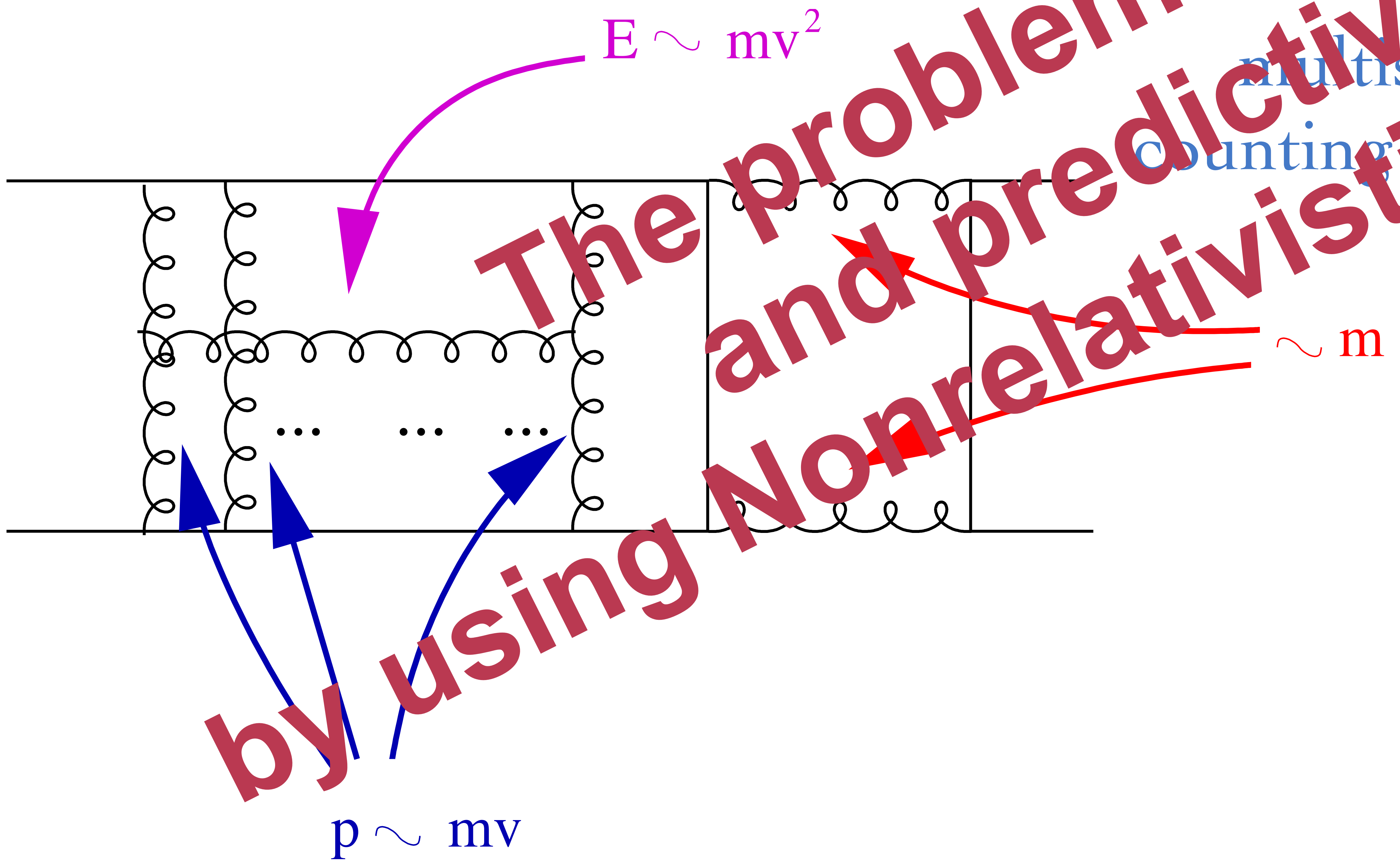

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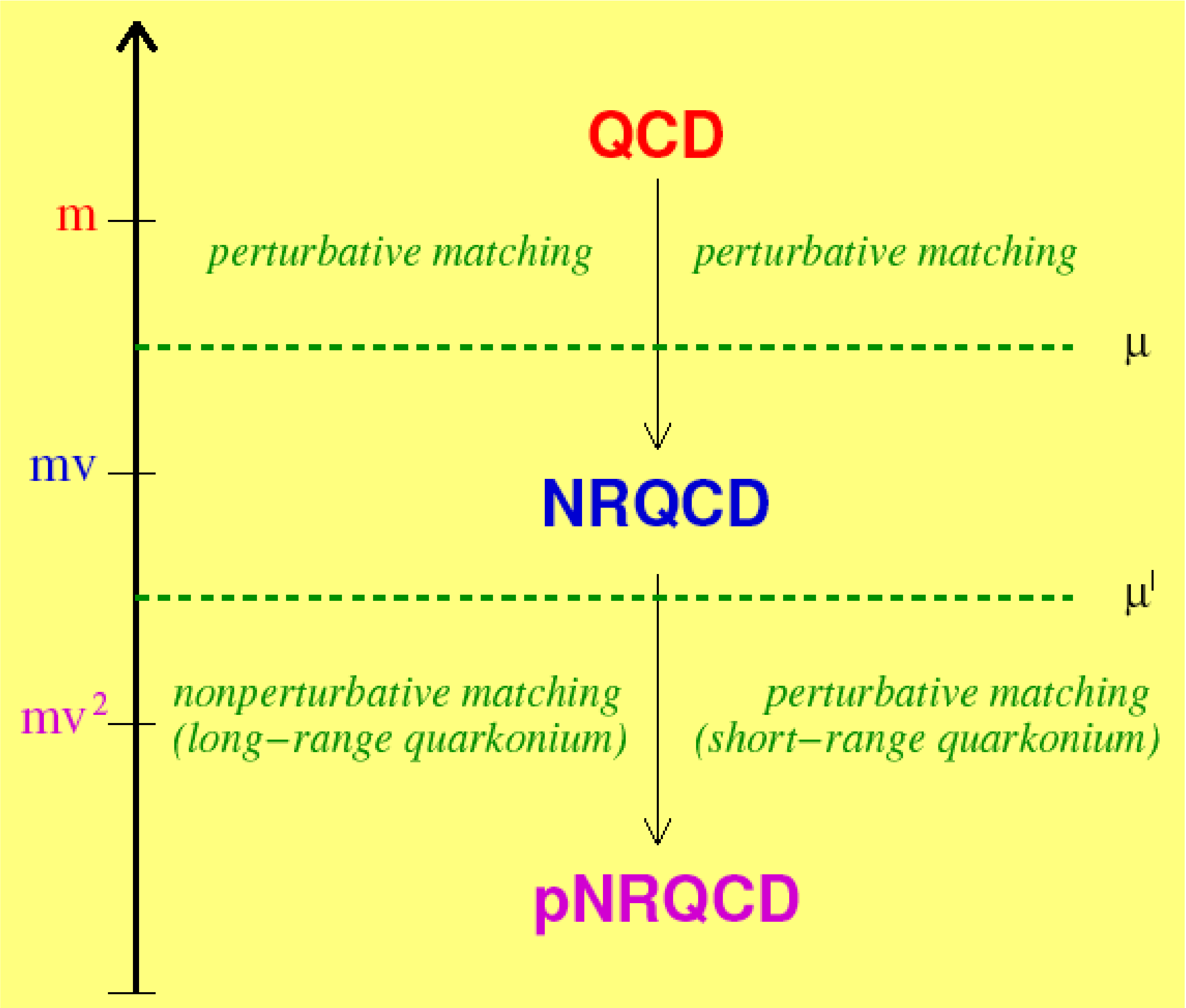
The problem is greatly simplified and predictivity is achieved by using Nonrelativistic Effective Field Theories

QQbar systems with NR EFT

Color degrees of freedom
3X3=1+8
singlet and octet QQbar



Hard

Soft
(relative
momentum)

Ultrasoft
(binding energy)

$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

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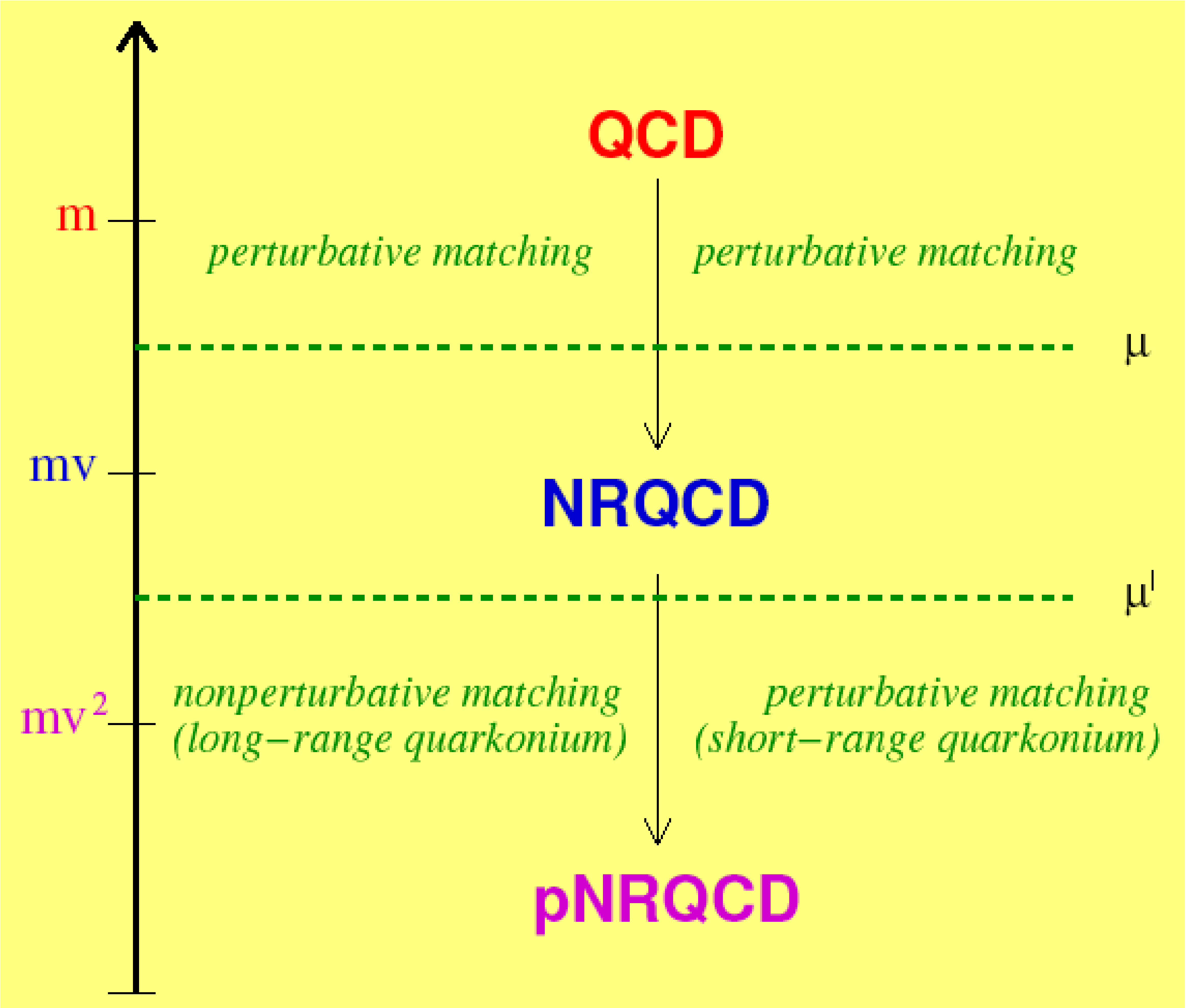

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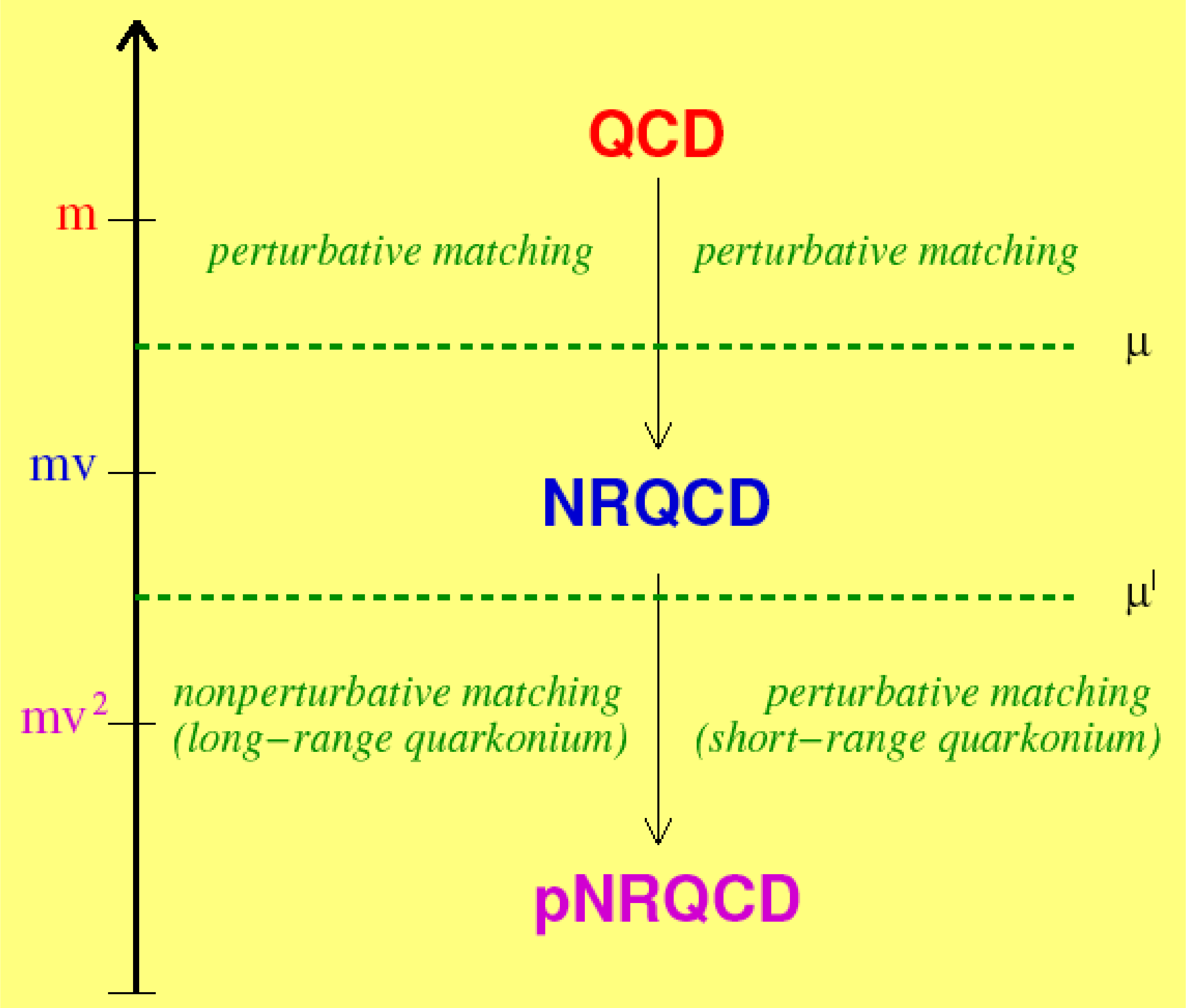

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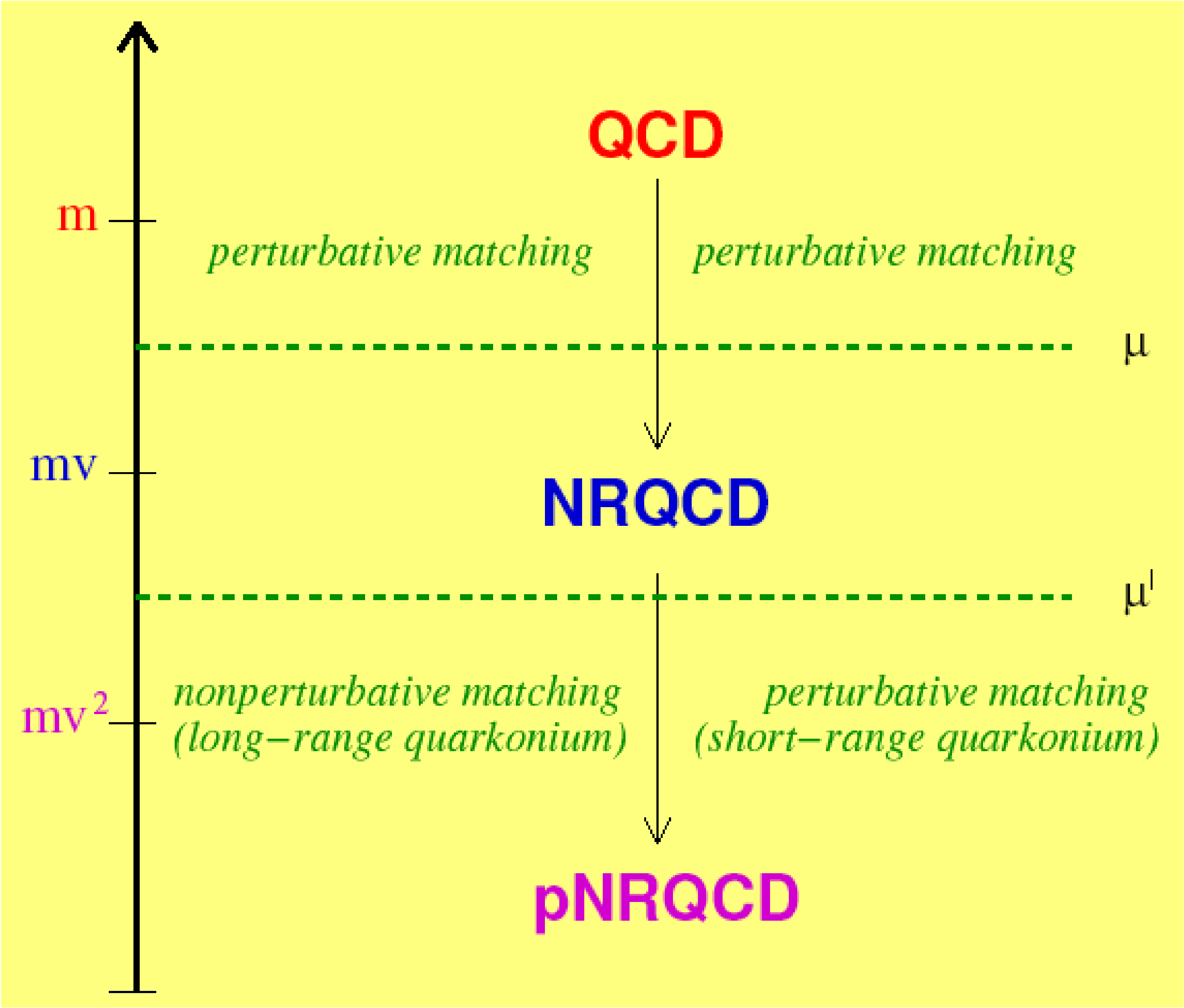
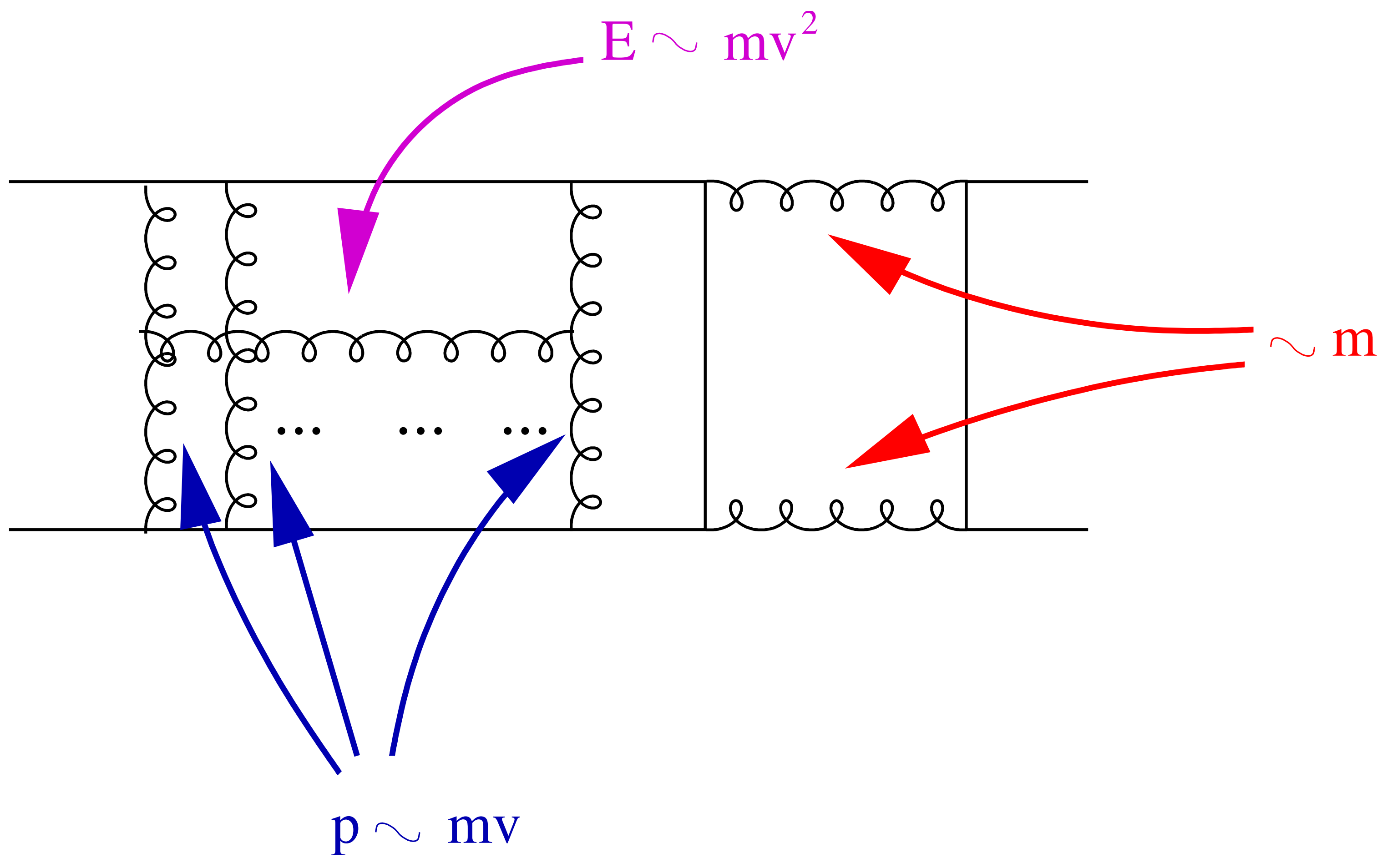
QQbar systems with NR EFT: Non Relativistic QCD (NRQCD)

Caswell, Lepage 86, Lepage Thacker 88,
Bodwin, Braaten, Lepage 95





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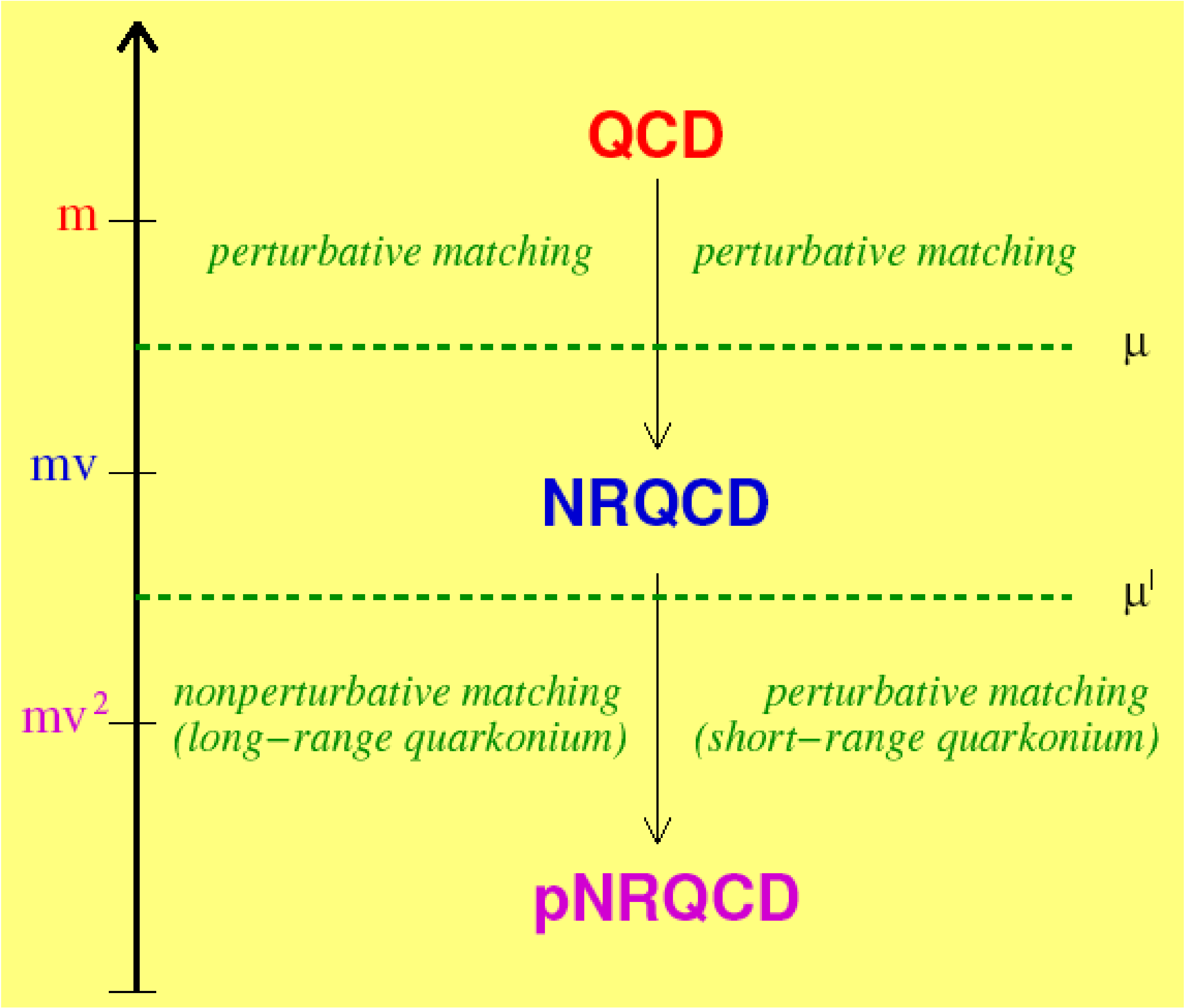
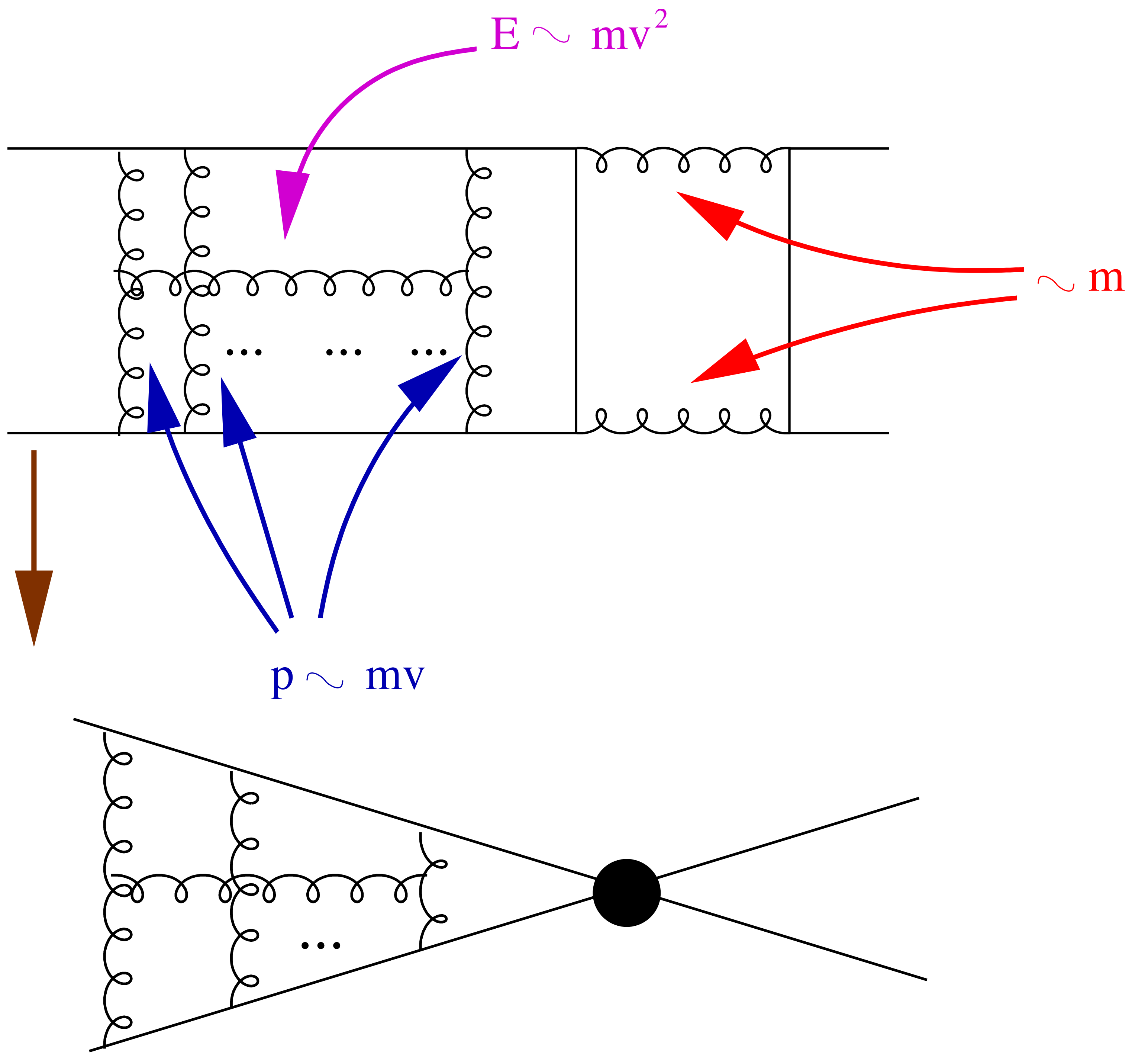
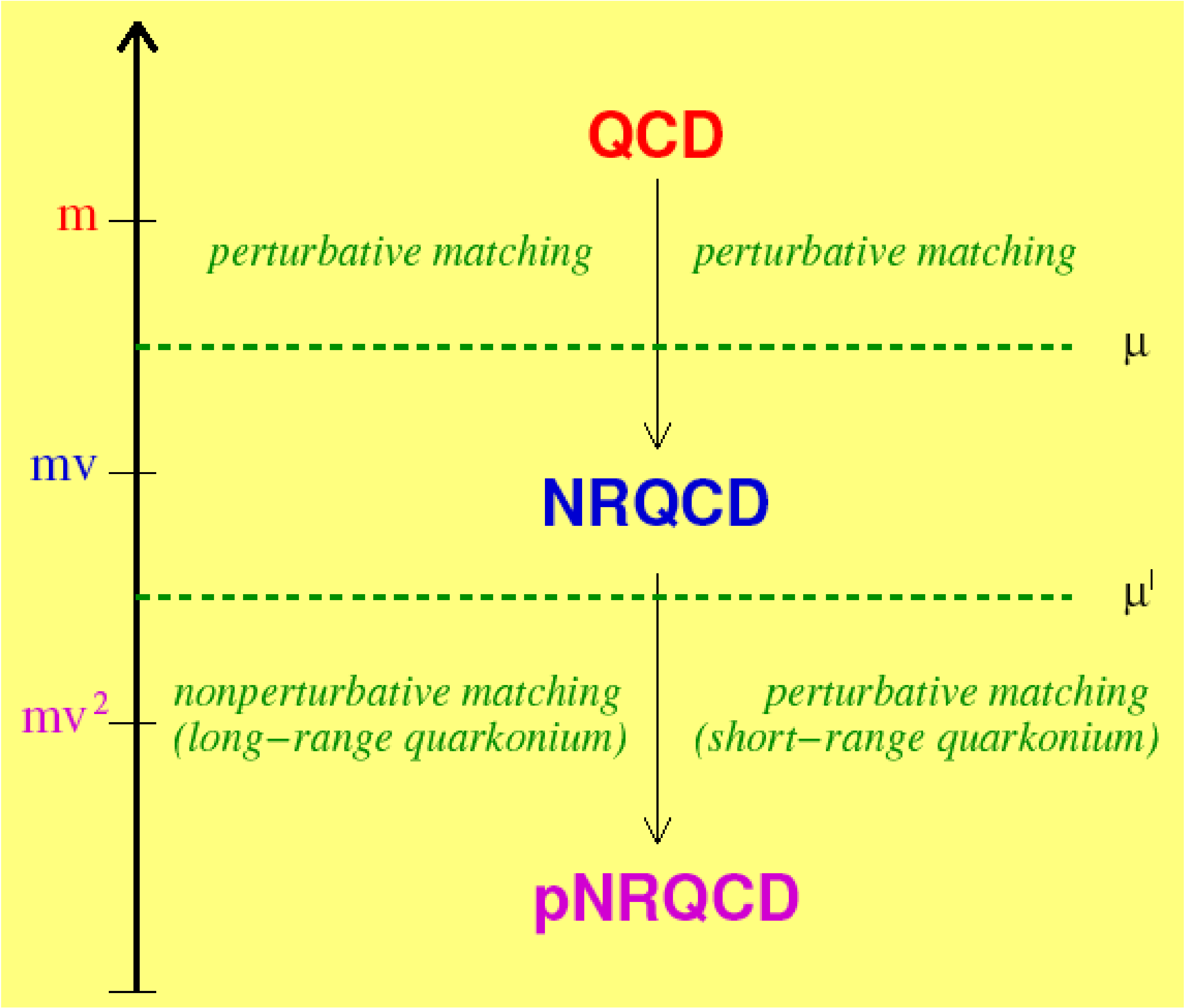
Bodwin, Braaten, Lepage 95

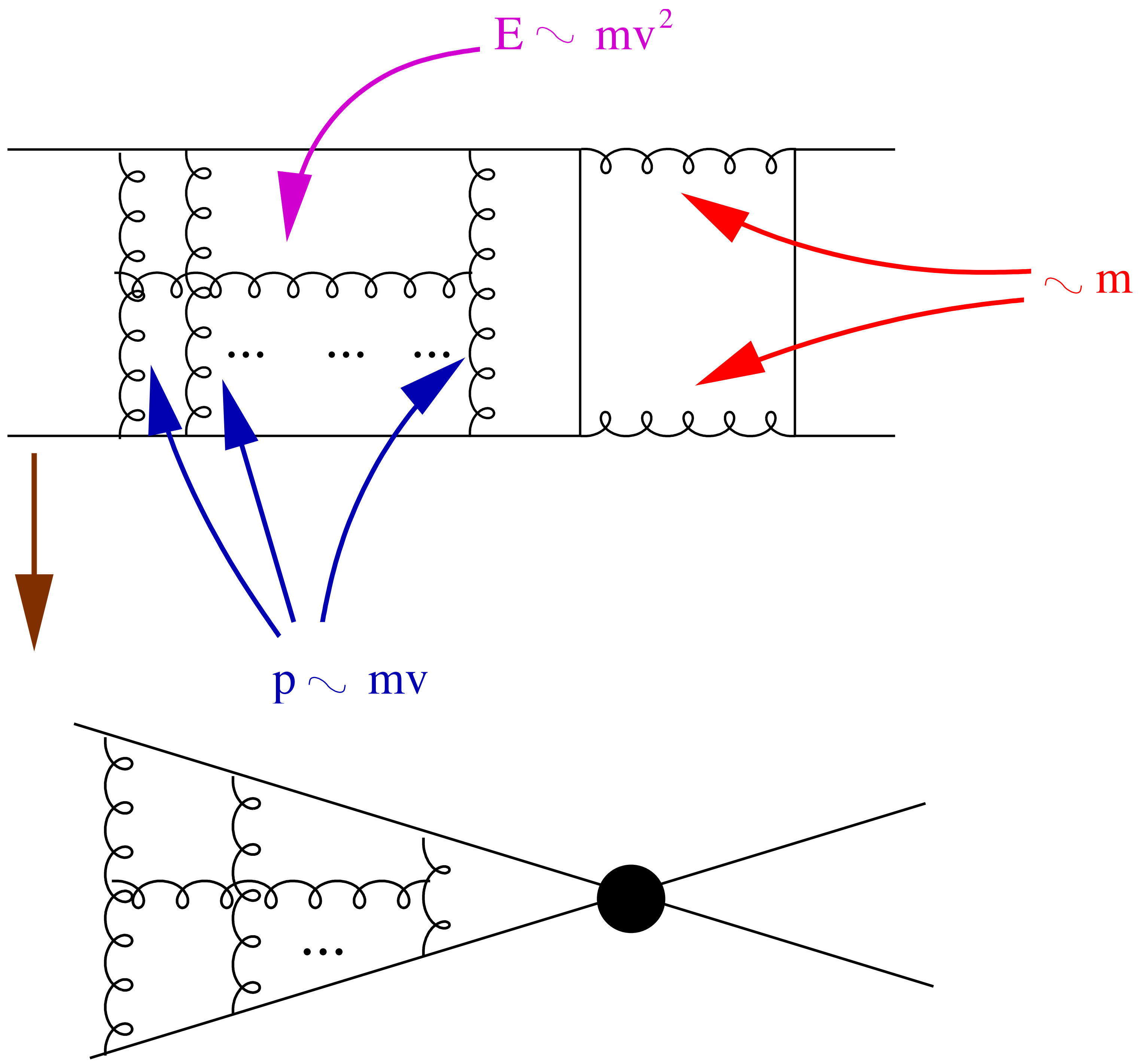




QQbar systems with NR EFT: Non Relativistic QCD (NRQCD)

Caswell, Lepage 86, Lepage Thacker 88,
Bodwin, Braaten, Lepage 95





$$\mathcal{L}_{\rm NRQCD} = \sum_n c(\alpha_{\rm s}(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

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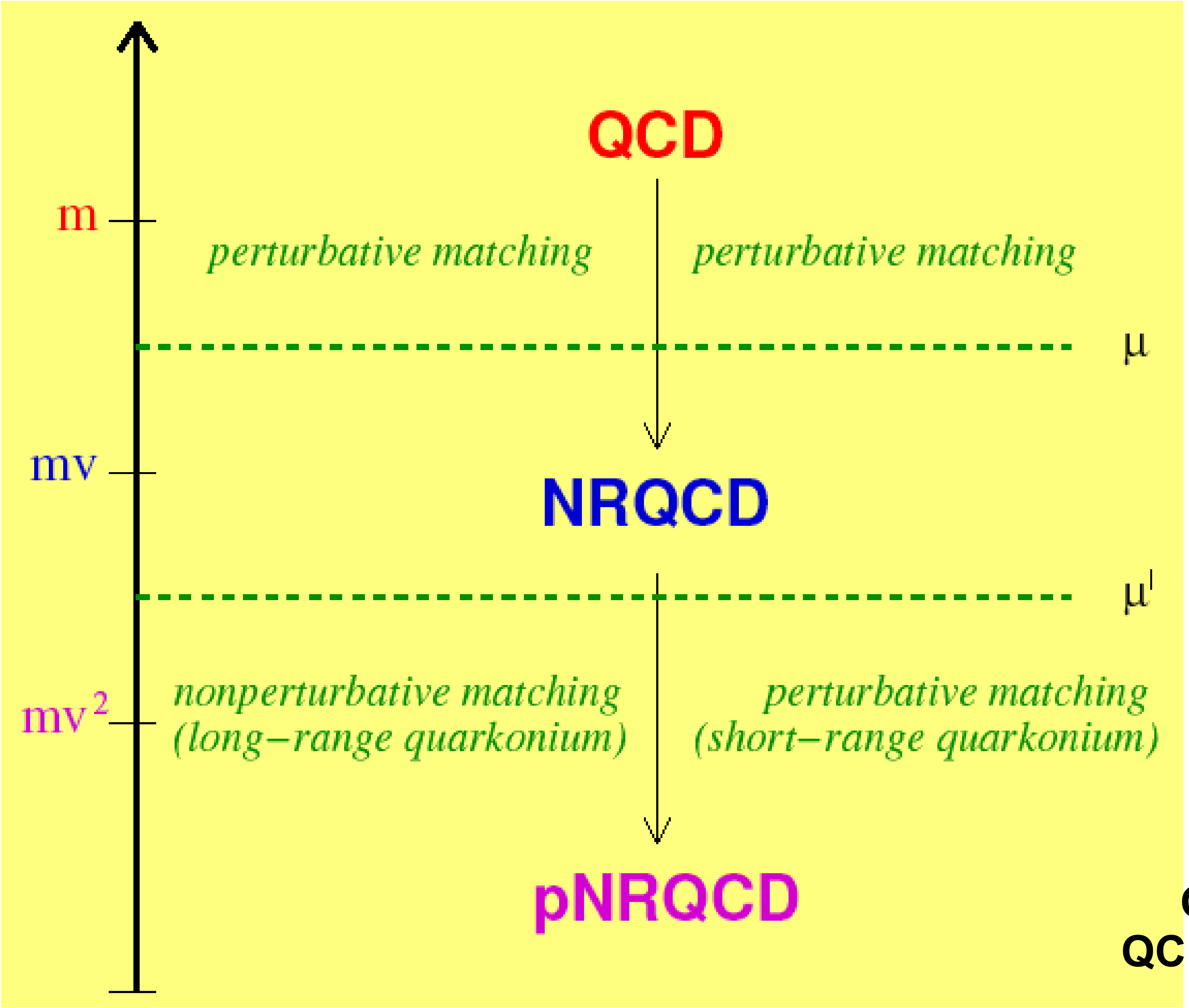
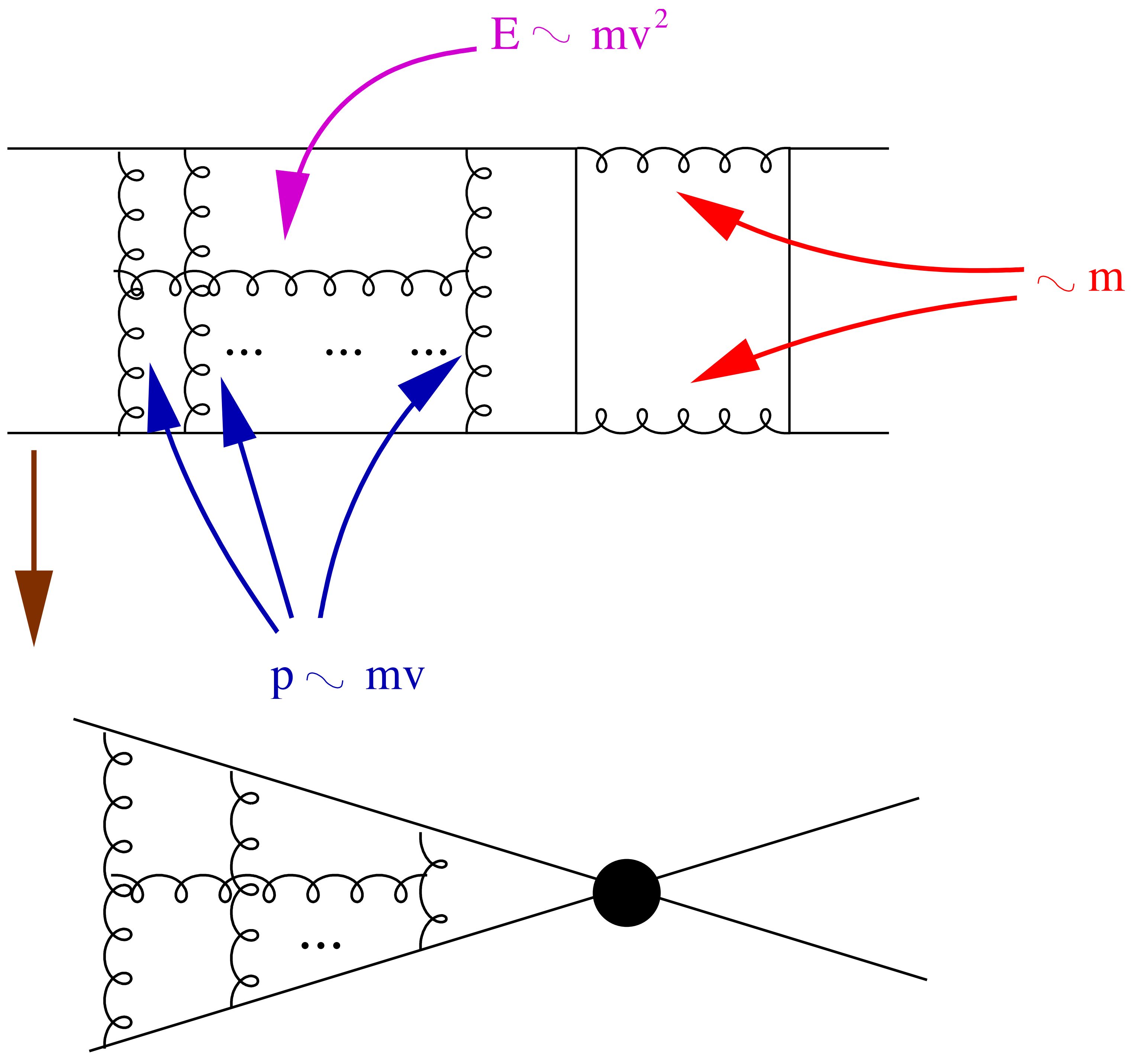


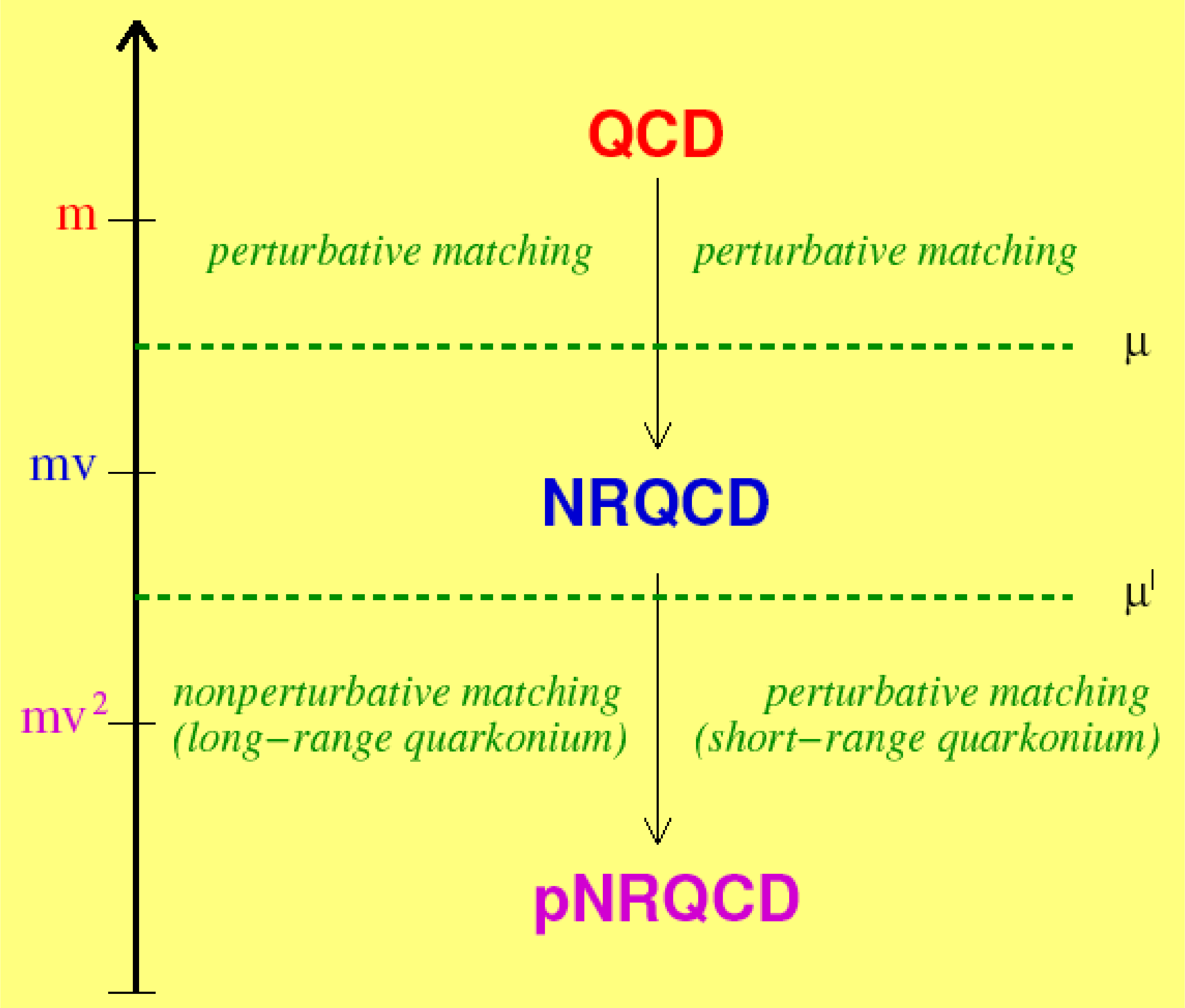


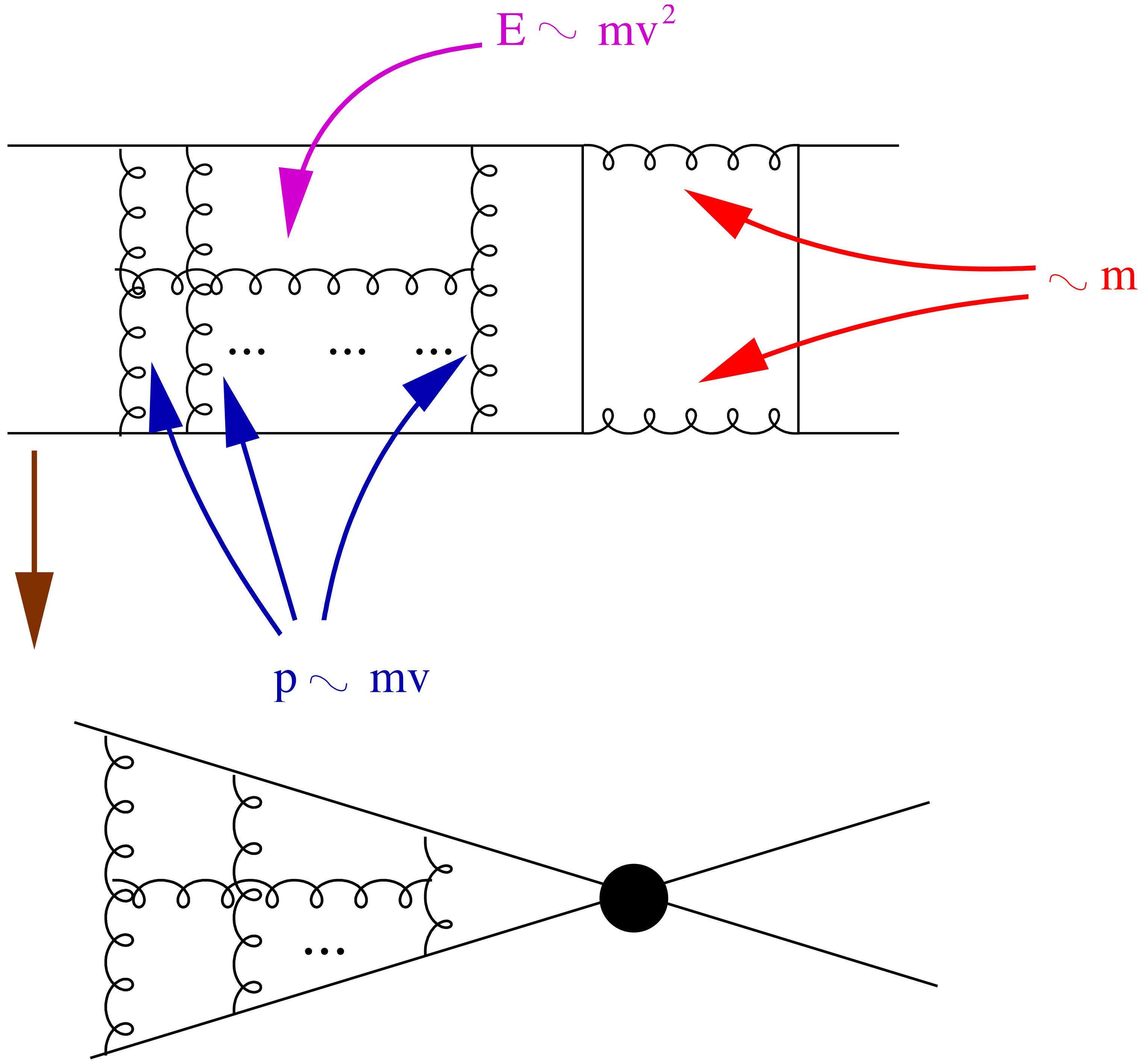
Only at the level of pNRQCD we obtain the potentials from QCD and the zero order problem is the Schroedinger equations

$$\mathcal{L}_{\mathrm{NRQCD}} = \sum_n c(\alpha_s(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

Quarkonium with NR EFT: potential Non Relativistic QCD (pNRQCD)

Pineda Soto 97, N. B., Pineda, Soto, Vairo 98





For mv >>Lambda_QCD

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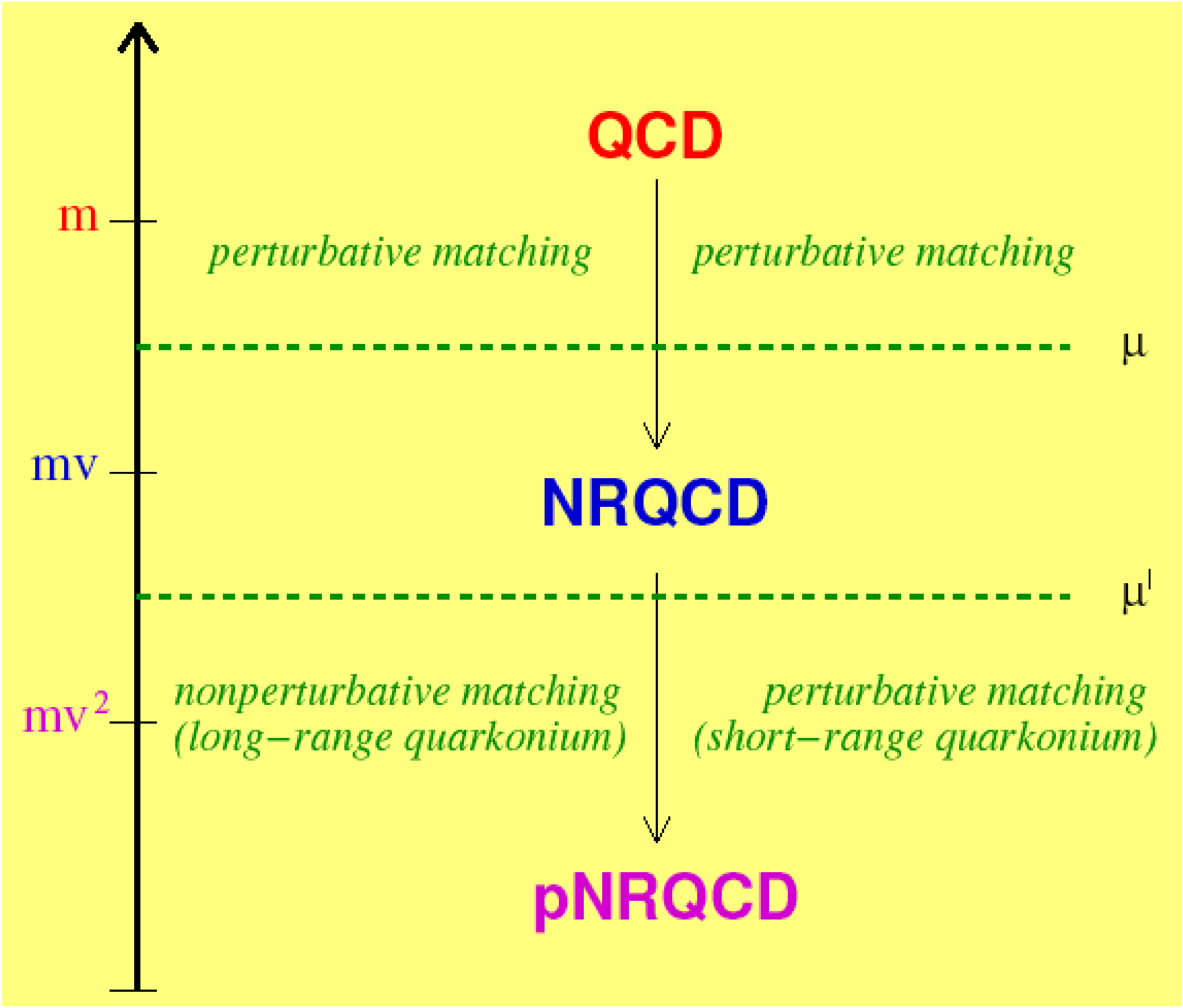
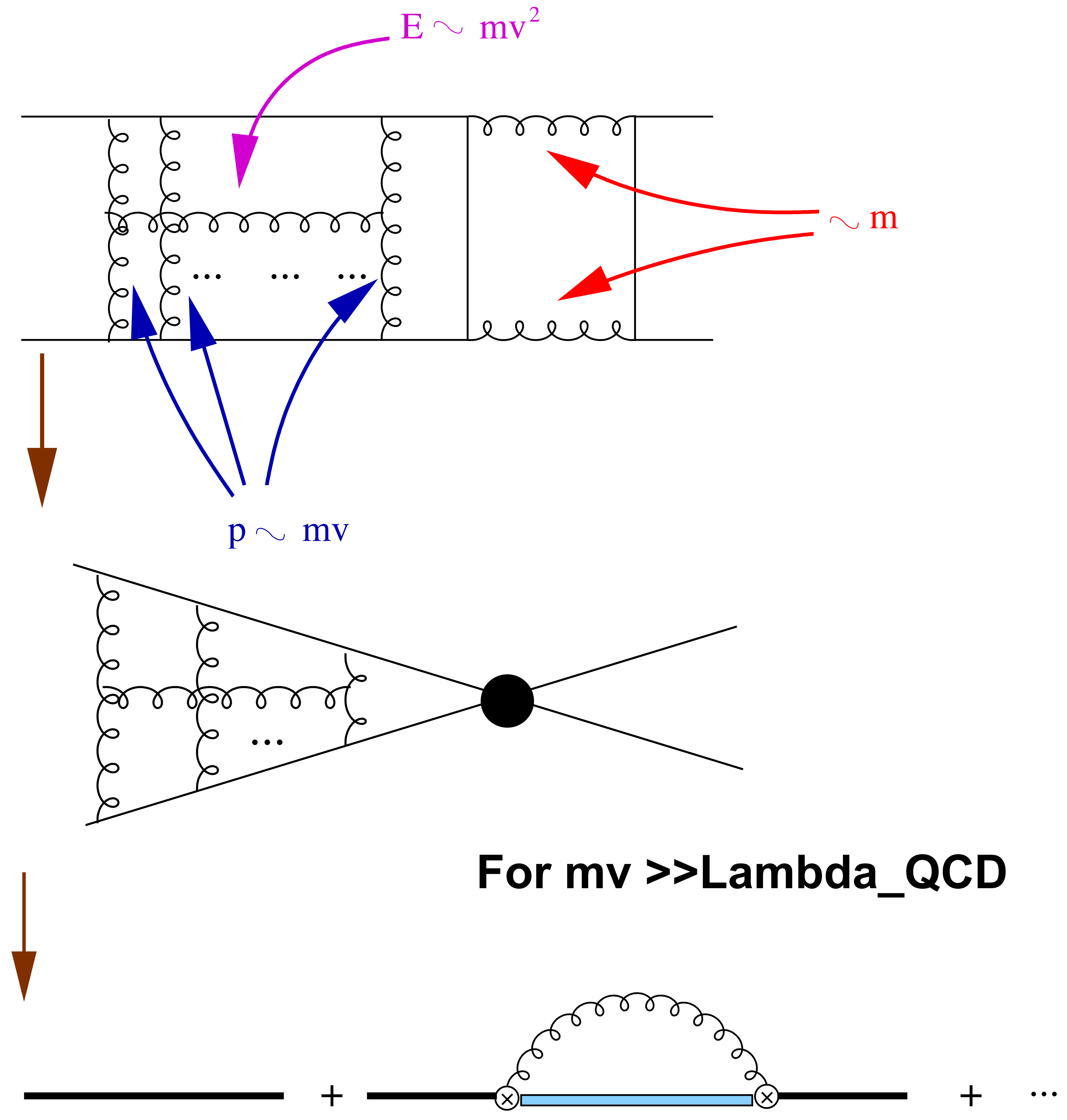
Pineda Soto 97, N. B., Pineda, Soto, Vairo 98





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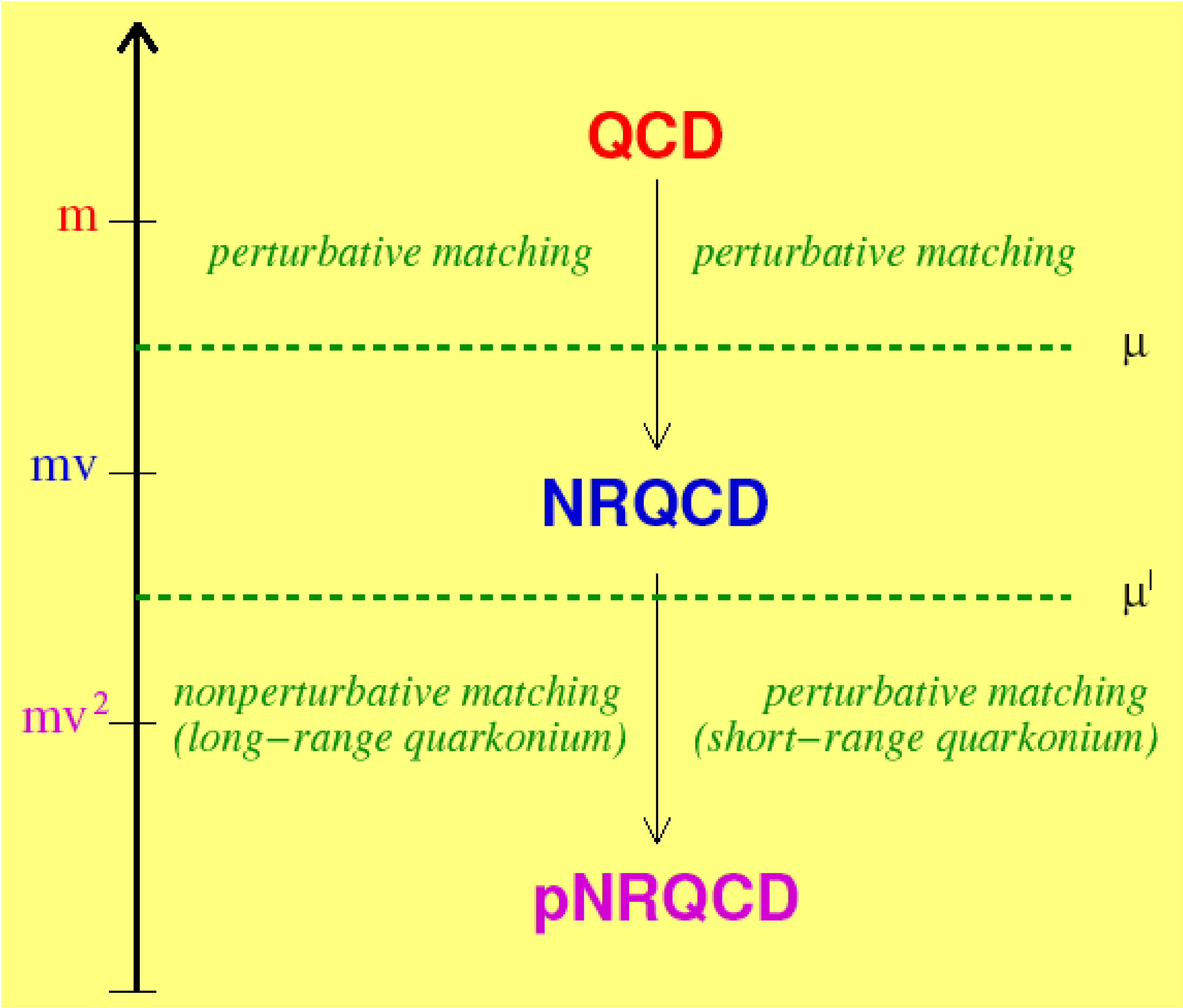

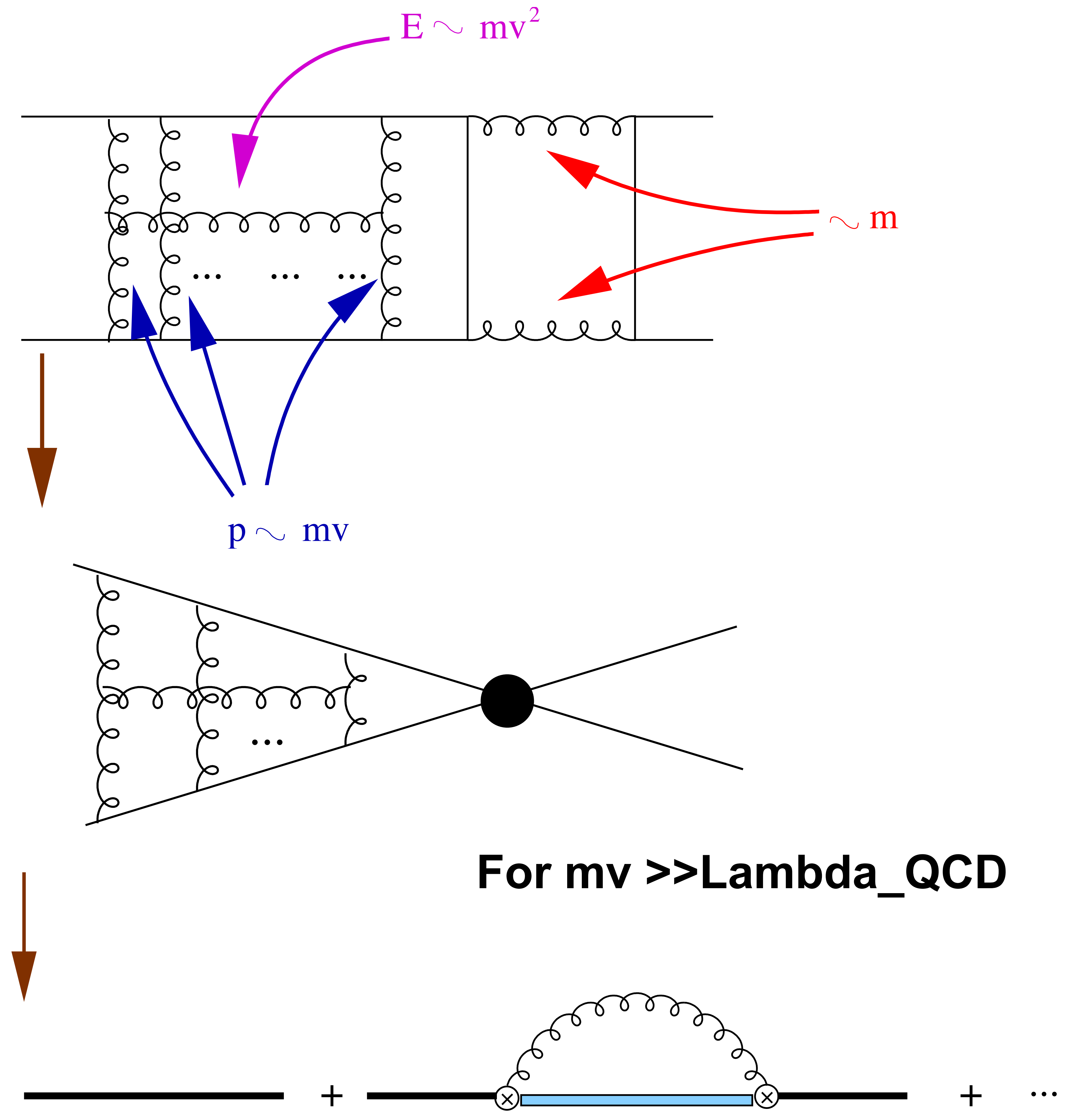

For mv >>Lambda_QCD

$$\mathcal{L}_{\rm pNRQCD} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_{\rm s}(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

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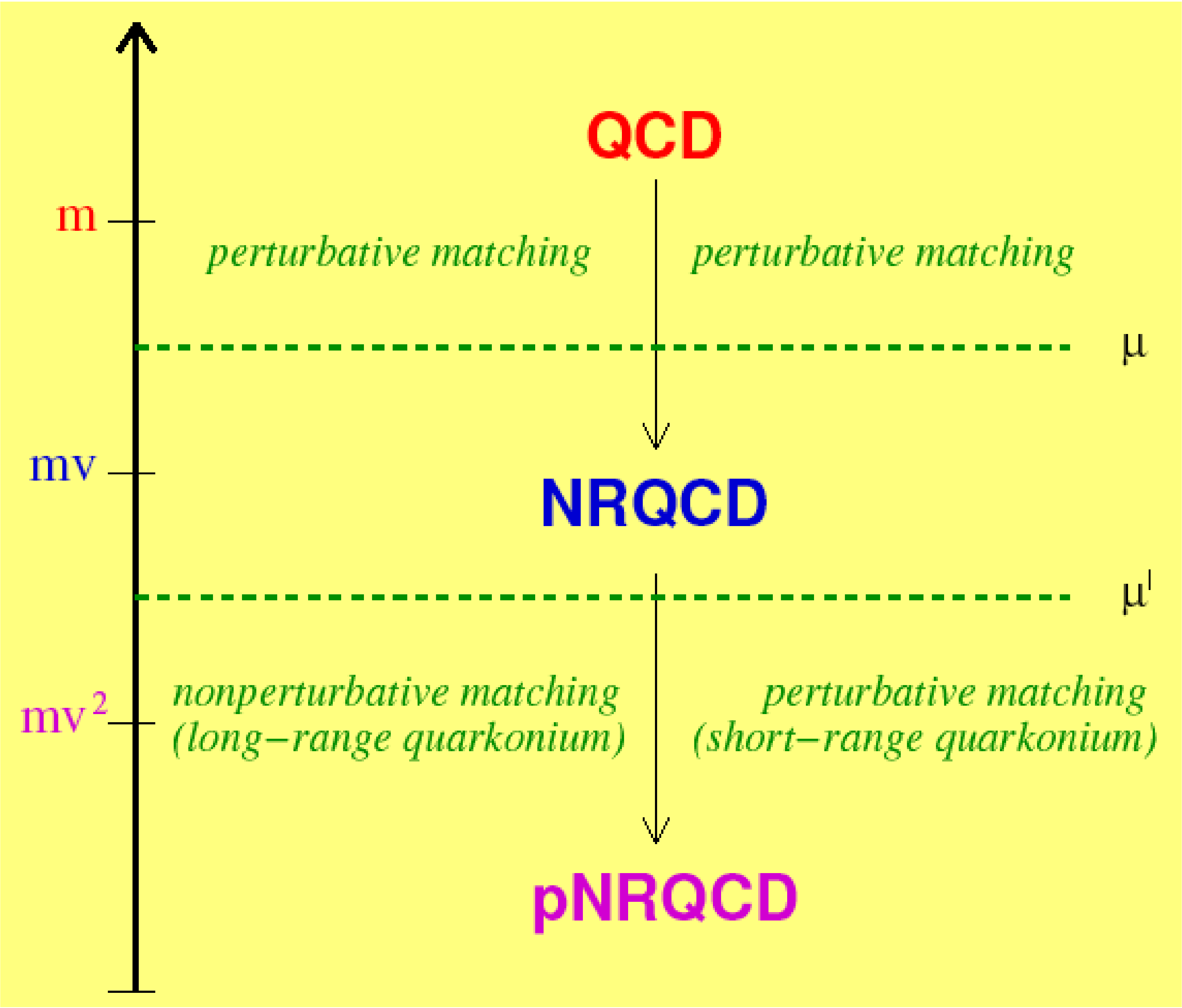

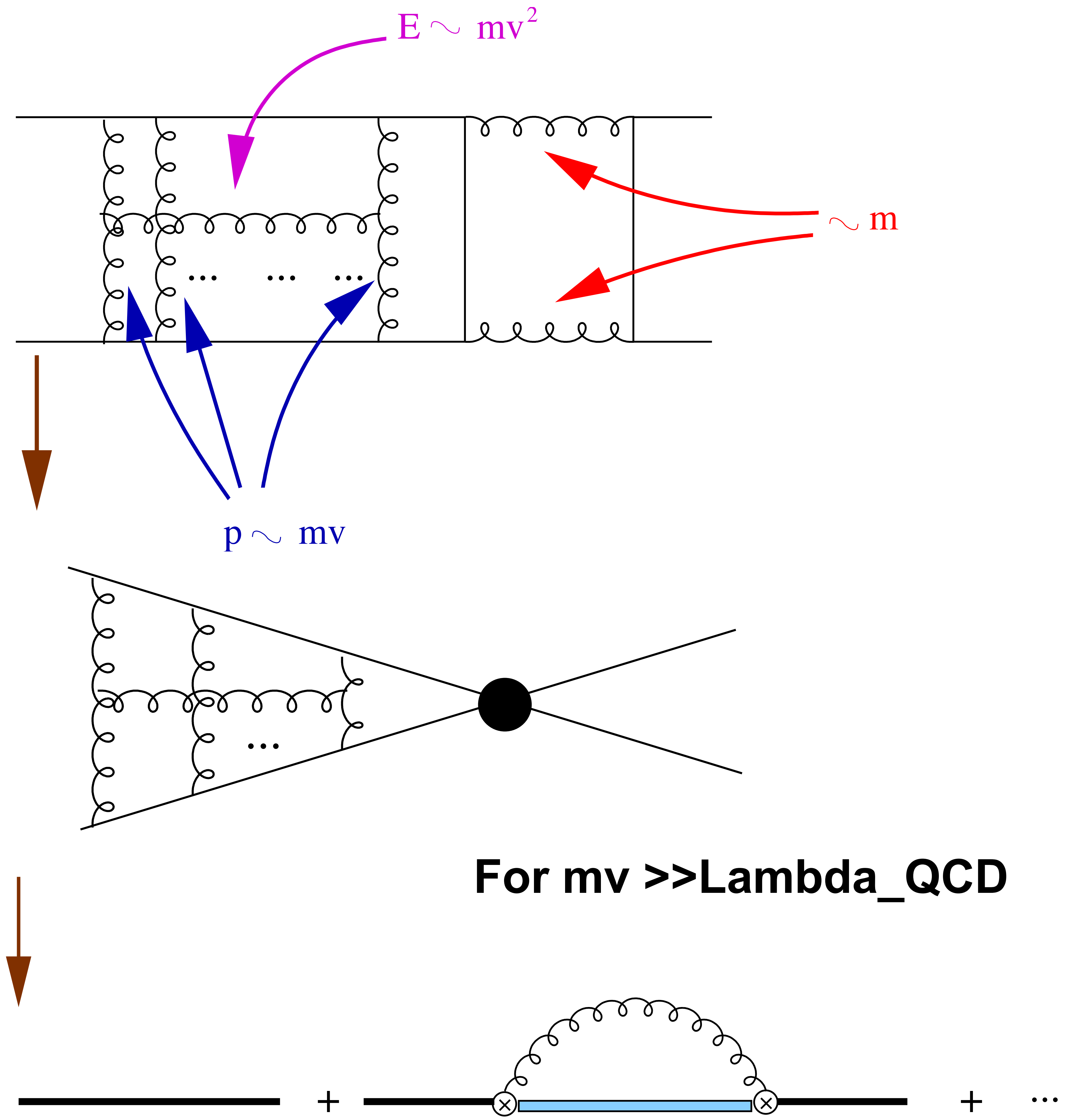

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Weakly coupled pNRQCD at the perturbative soft scale

Pineda Soto NP PS 64 (1998) 428
Brambilla Pineda Soto Vairo NPB 566 (2000) 275

- If $mv \gg \Lambda_{\mathrm{QCD}}$, the matching is perturbative

  Non-analytic behaviour in $r \to$ matching coefficients V

The gauge fields are multipole expanded:
$A(R,r,t) = A(R,t) + \mathbf{r}\cdot\nabla A(R,t) + \ldots$

$\mathbf{R}$ = center of mass
$\mathbf{r}$ = $Q\bar{Q}$ distance

$$\mathcal{L}^{\mathrm{pNRQCD}} = \int d^3r\, \mathrm{Tr}\,\{S^\dagger(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_S + \cdots)S + O^\dagger(iD_0 - \frac{\mathbf{p}^2}{m} - V_O + \cdots)O +$$

LO in r

$$+V_A(S^\dagger\mathbf{r}\cdot g\mathbf{E}O + O^\dagger\mathbf{r}\cdot g\mathbf{E}S) + \frac{V_B}{2}(O^\dagger\mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E})\} + \ldots$$

NLO in r

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

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NLO in r

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

The matching coefficients are the Coulomb potential

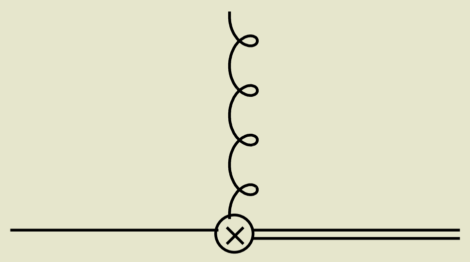
$$V_S(r) = -C_F\frac{\alpha_{\rm s}}{r} + \dots, \qquad V_o(r) = \frac{1}{2N}\frac{\alpha_{\rm s}}{r} + \dots,$$

$$V_A = 1 + \mathcal{O}(\alpha_{\rm s}^2),\ V_B = 1 + \mathcal{O}(\alpha_{\rm s}^2).$$

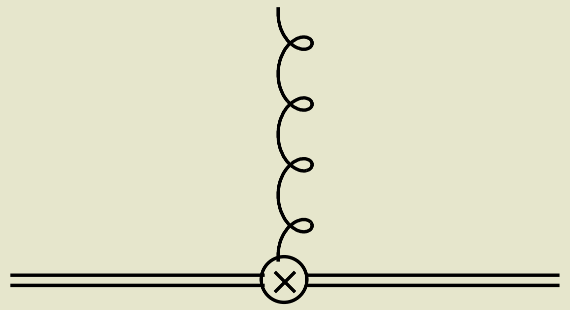
Feynman rules

$$= \theta(t)\, e^{-it(\mathbf{p}^2/m + V)}$$

$$= \theta(t)\, e^{-it(\mathbf{p}^2/m + V_o)}\left(e^{-i\int dt\, A^{\rm adj}}\right)$$



$= {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm S}$



$= {\rm O}^\dagger\{\mathbf{r}\cdot g\mathbf{E},\ {\rm O}\}$

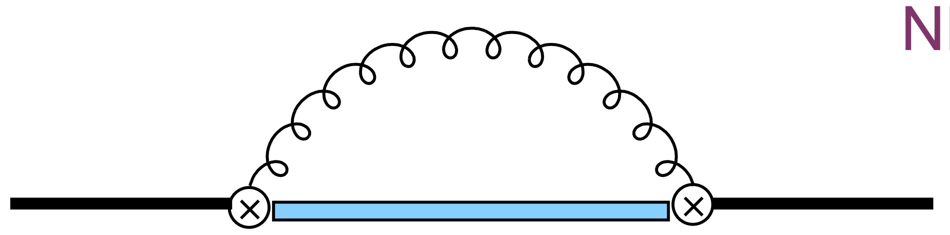
Using weakly coupled pNRQCD quarkonium becomes a tool for precision physics in QCD

$m\alpha_s^5 \ln \alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99

$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02 NNLL Pineda 02

NNNLL Peset Pineda et al 2018,2019, Kiyo Sumino 2014, Beneke, Kiyo Schuler 05,08

Energies at order m alpha^5 (NNNLO)

$$E_n = 2m + \langle n | \frac{p^2}{m} + V_s | n \rangle + \langle n |$$

$$| n \rangle$$

$$\sim e^{i\Lambda_{\rm QCD} t}$$

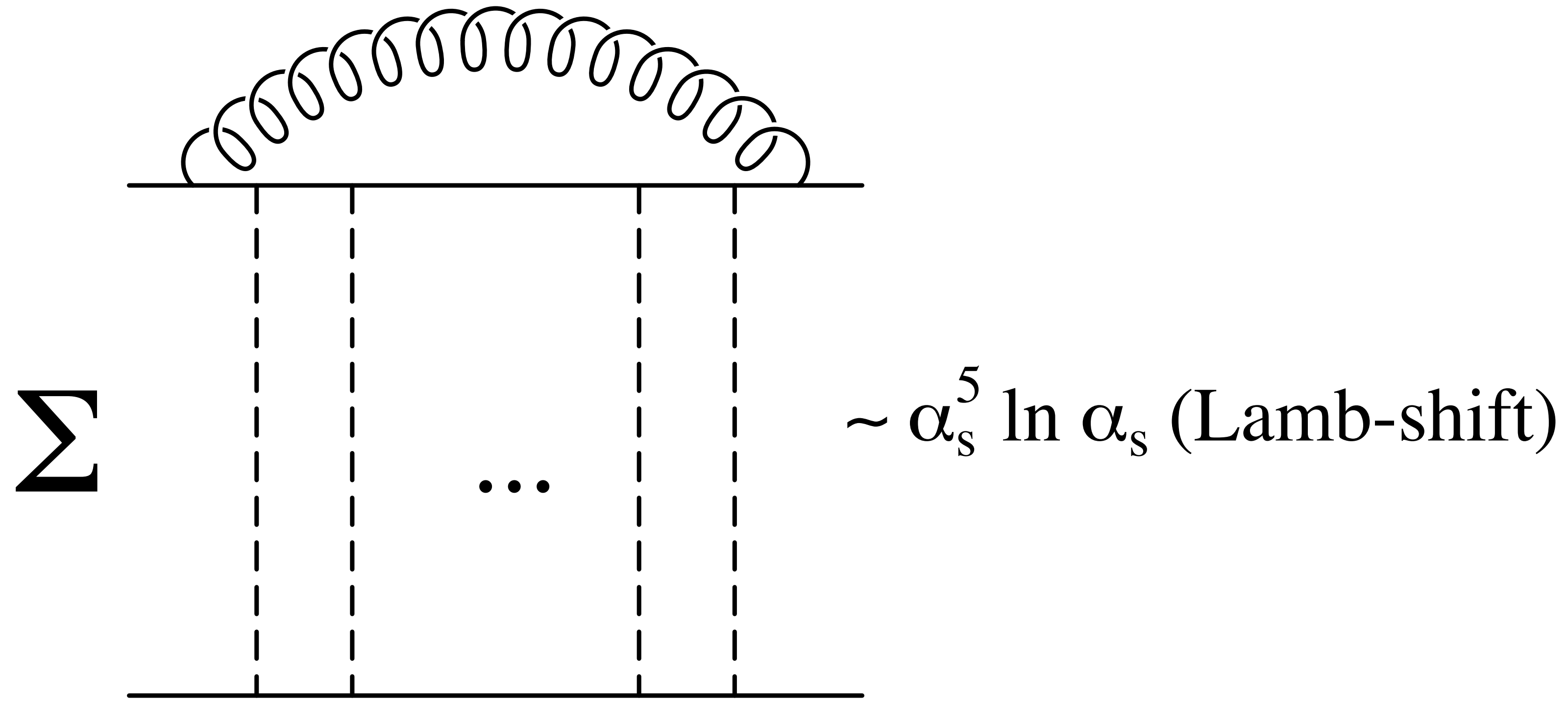
$$E_n = \langle n | H_s(\mu) | n \rangle - i \frac{g^2}{3N_c} \int_0^\infty dt \, \langle n | \mathbf{r} e^{it(E_n^{(0)} - H_o)} \mathbf{r} | n \rangle \, \langle \mathbf{E}(t) \, \mathbf{E}(0) \rangle(\mu)$$

$E_n^{(0)} - H_o \gg \Lambda_{\rm QCD} \Rightarrow \langle \mathbf{E}(t)\mathbf{E}(0)\rangle(\mu) \to \langle \mathbf{E}^2(0) \rangle$

local condensates as predicted in a paper by Misha Voloshin in 1979

—>used to extract precise (NNNLO) determination of m_c and m_b

$E_n^{(0)} - H_o \sim \Lambda_{\rm QCD} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED

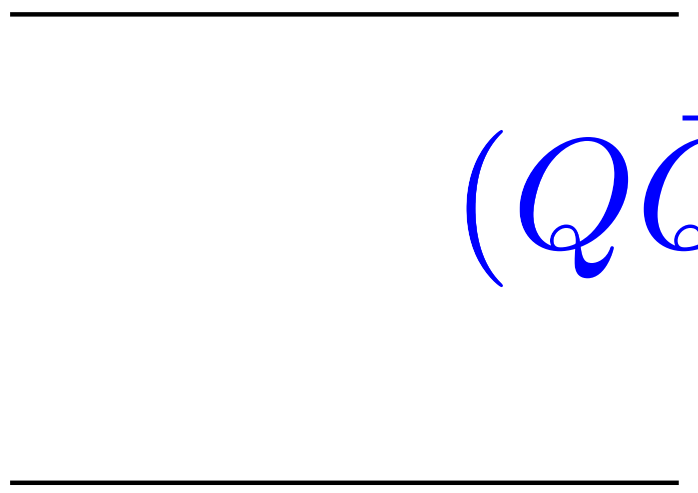


Applications of weakly coupled pNRQCD include: precise alphas extraction from the static energy,ttbar production, quarkonia spectra, decays, E1 and M1 transitions, QQq and QQQ energies, thermal masses and potentials

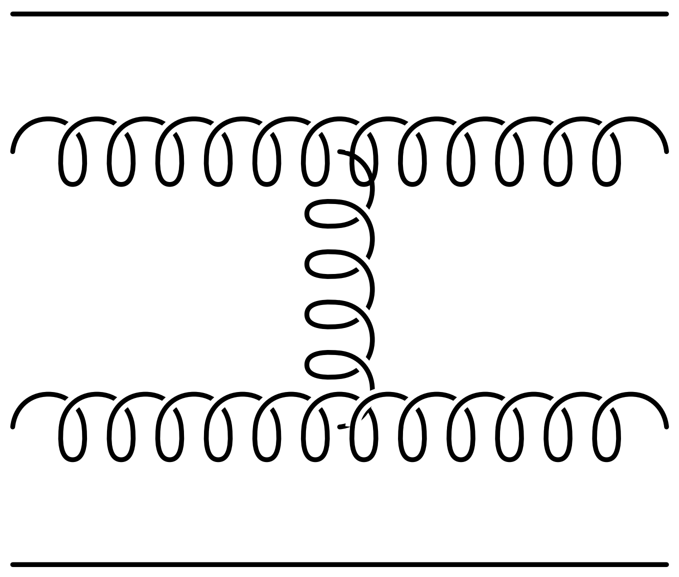
At the nonperturbative soft scale

Hitting the scale Λ_{QCD} $\quad r \sim \Lambda_{QCD}^{-1}$

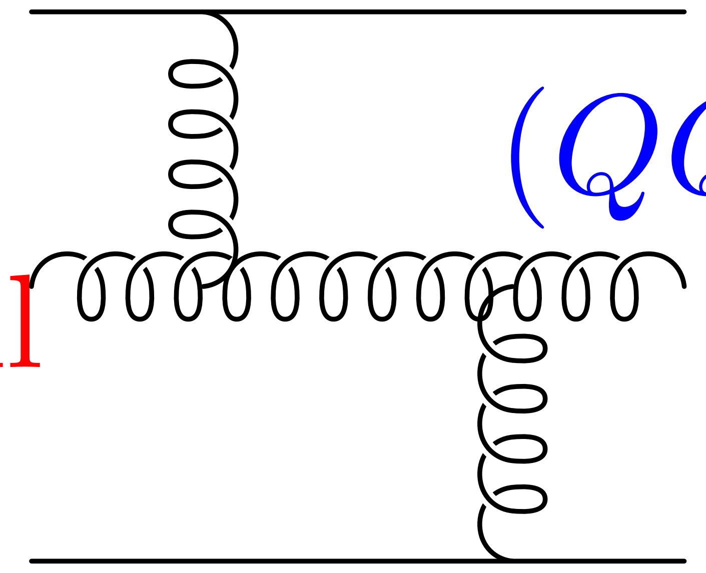
The degrees of freedoms now are



$(Q\bar{Q})_1$



$(Q\bar{Q})_1 + \mathrm{Glueball}$



$(Q\bar{Q})_8 G$

hybrid **/tetraquarks**

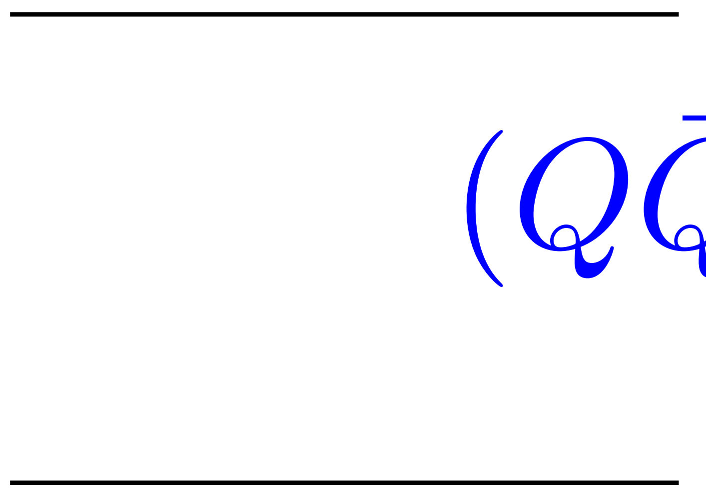
with gluons/light quarks becoming
part of the binding over the strong decay threshold

—>nonperturbative problem, use lattice

At the nonperturbative soft scale

Hitting the scale Λ_{QCD} $r \sim \Lambda_{QCD}^{-1}$

The degrees of freedoms now are



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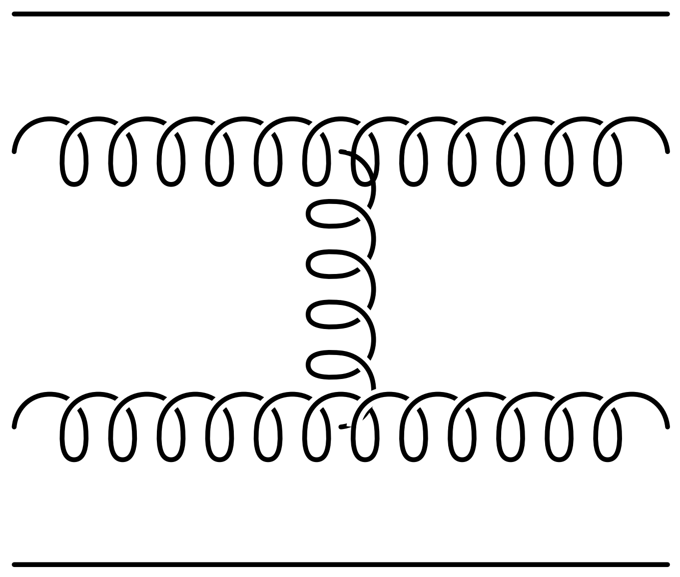

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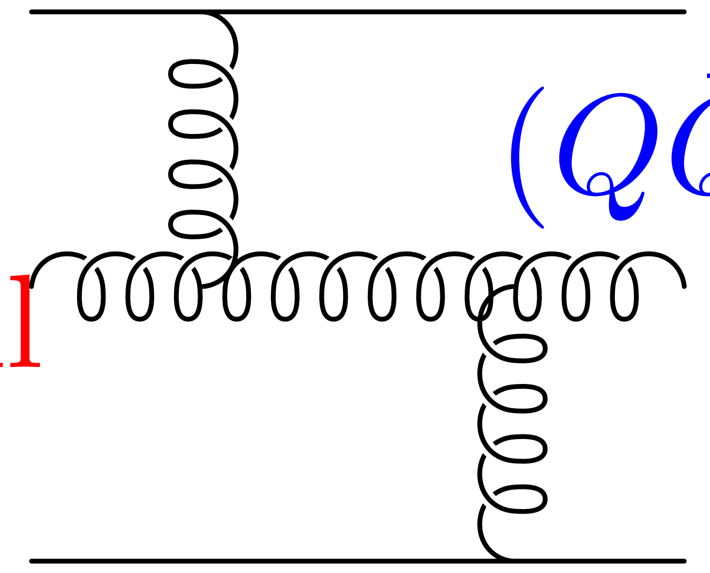

$(Q\bar{Q})_8 G$

hybrid **/tetraquarks**

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—>nonperturbative problem, use lattice

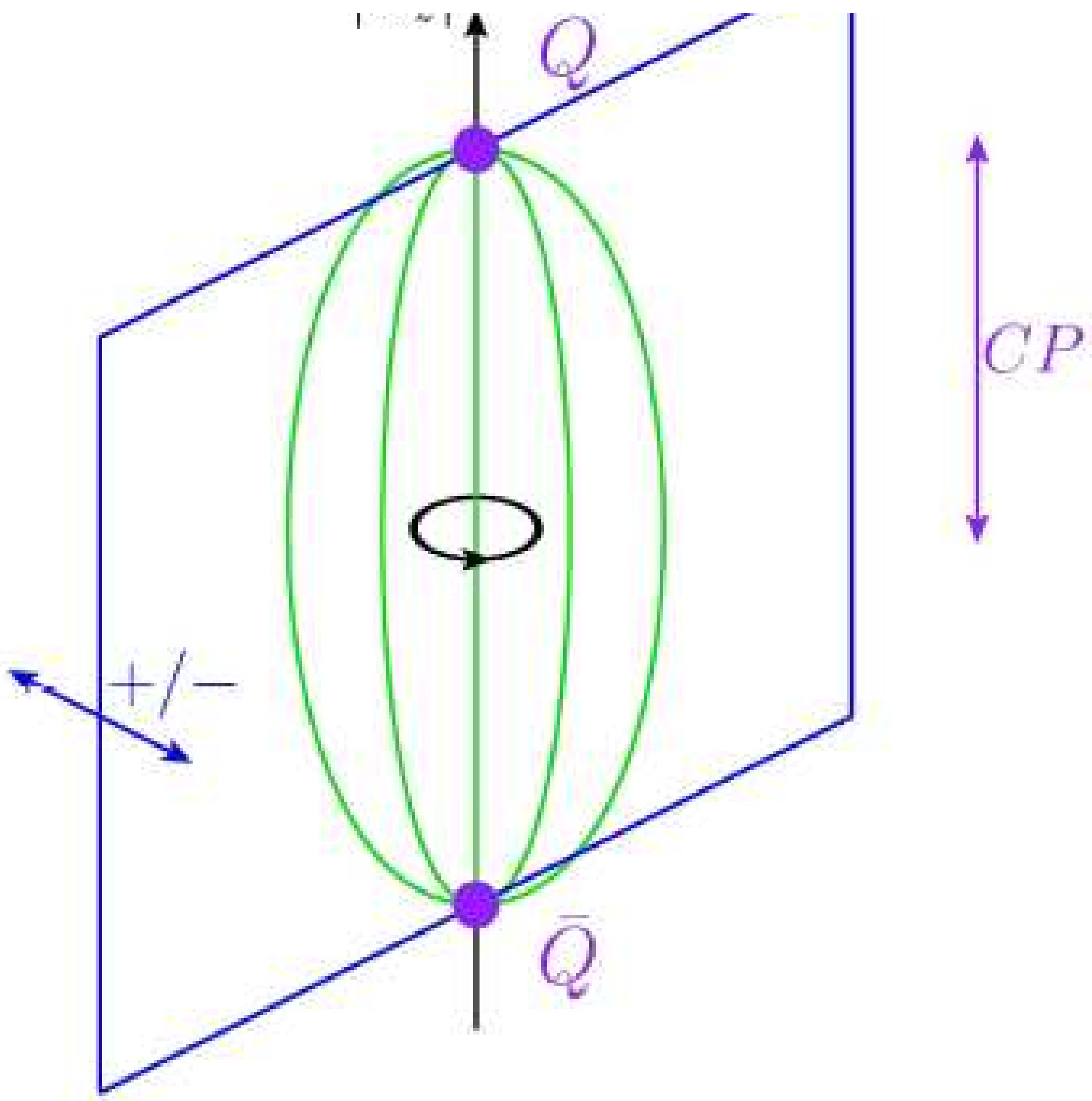
Use symmetry and scale separation: $m > \Lambda_{QCD}$ NRQCD holds

$\Lambda_{QCD} > mv^2$ fast (gluons, light quarks) and slow (heavy quarks)

like in molecular physics (fast-electrons, slow nuclei)

The spectrum of static energies can be calculated in NRQCD

Symmetry of a system with a static Q in x_1 and a Qbar in x_2



Irreducible representations of $D_{\infty h}$

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots \; (\Sigma, \Pi, \Delta, \Phi, \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g), -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)

$$\Lambda_\eta^\sigma$$

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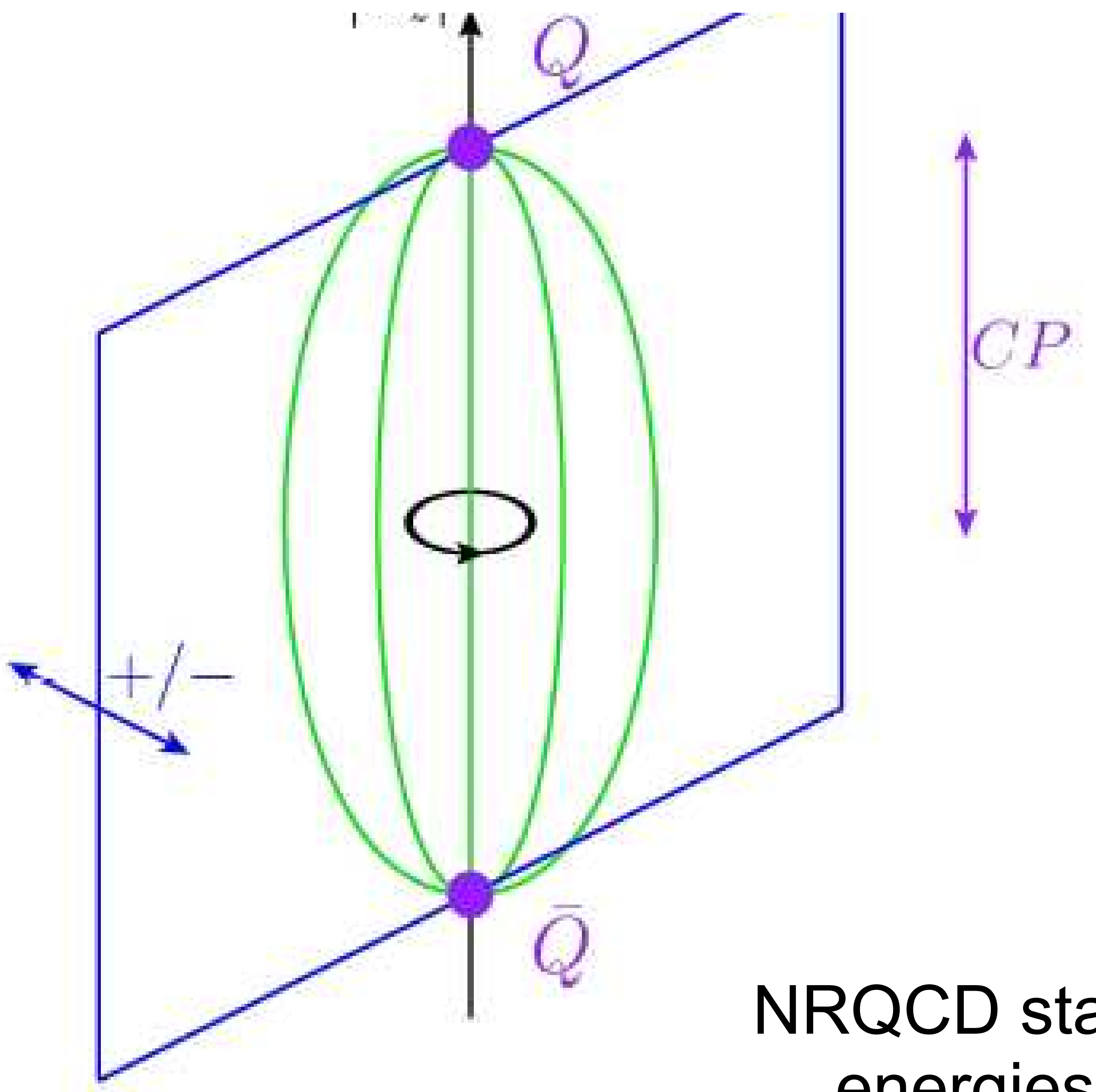

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NRQCD static energies

$$\mathcal{H}^{(0)}|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1,\mathbf{x}_2)|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|n;\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}$$

$$E_n^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle X_n,\, T/2 | X_n,\, -T/2\rangle$$

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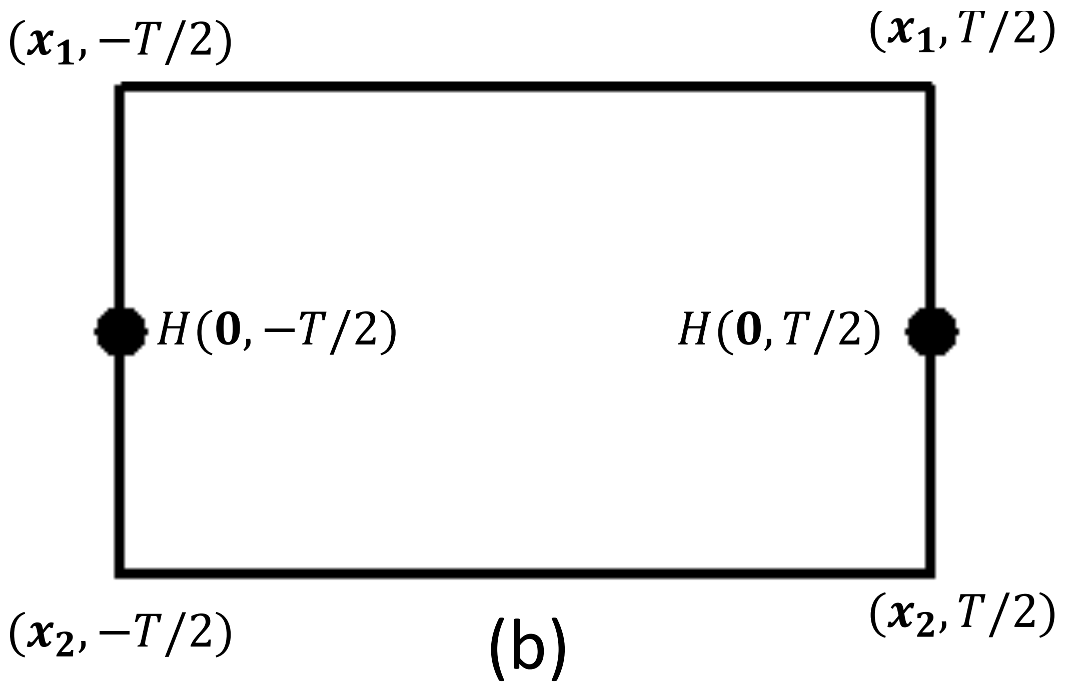
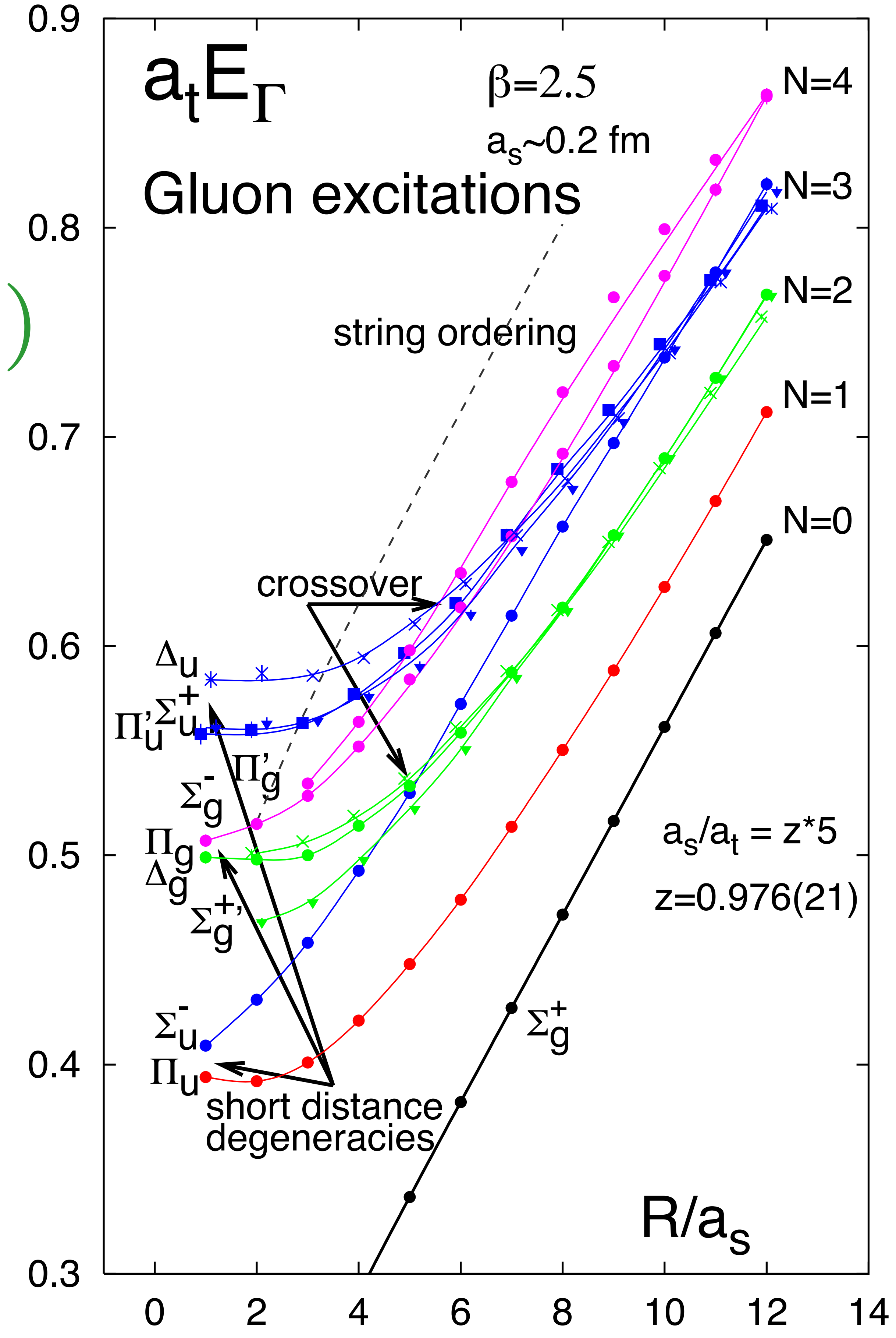


(b)

The spectrum of static energies can be calculated in NRQCD

Lattice Spectrum of NRQCD hybrid static energies E^0_n

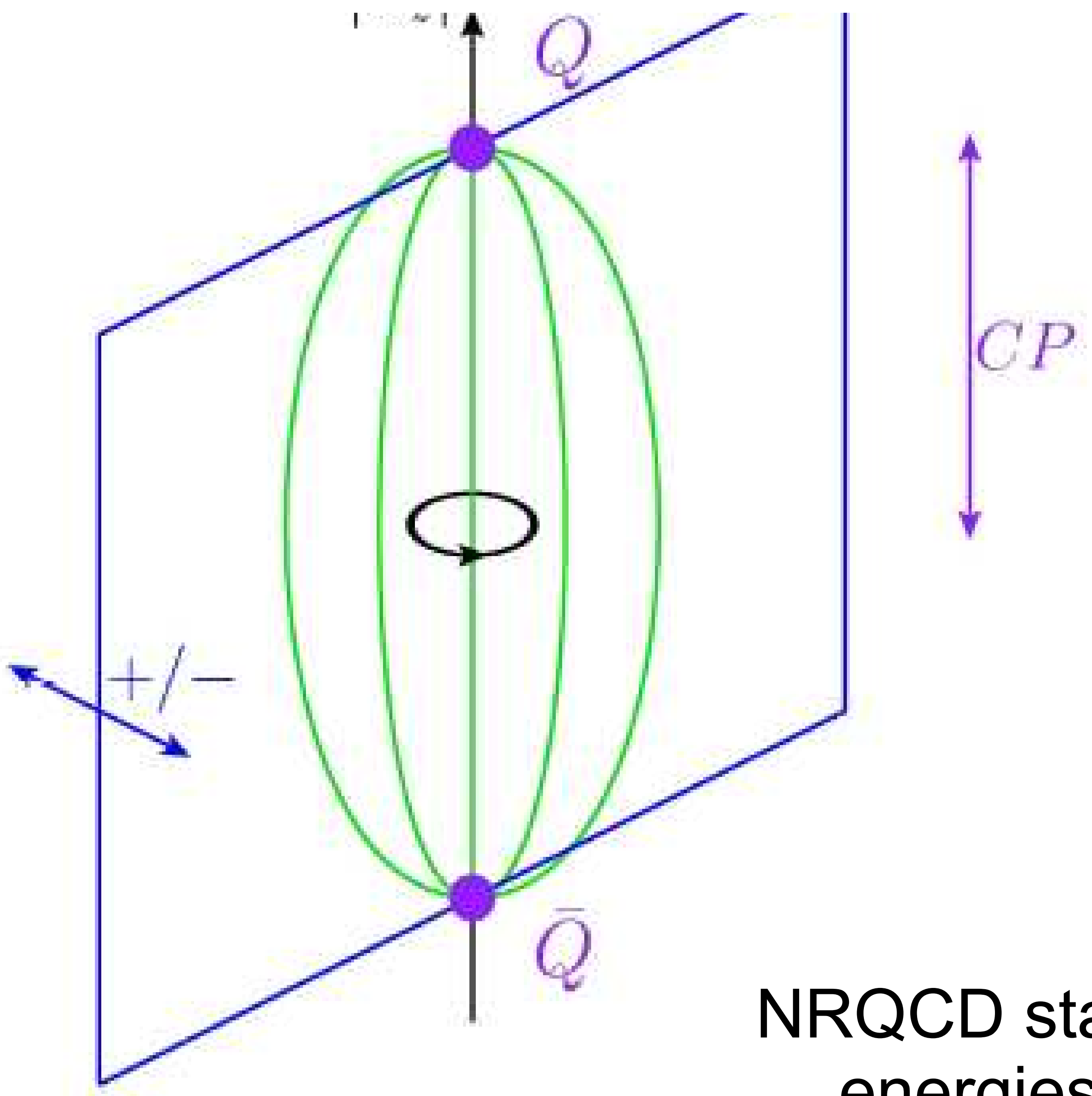
$$E_n^{(0)}$$



$$r = |\mathbf{x}_1 - \mathbf{x}_2|$$

Juge Kuti Mornigstar 98-06

Schlosser, Wagner 2111.00741, Bali Pineda 2004



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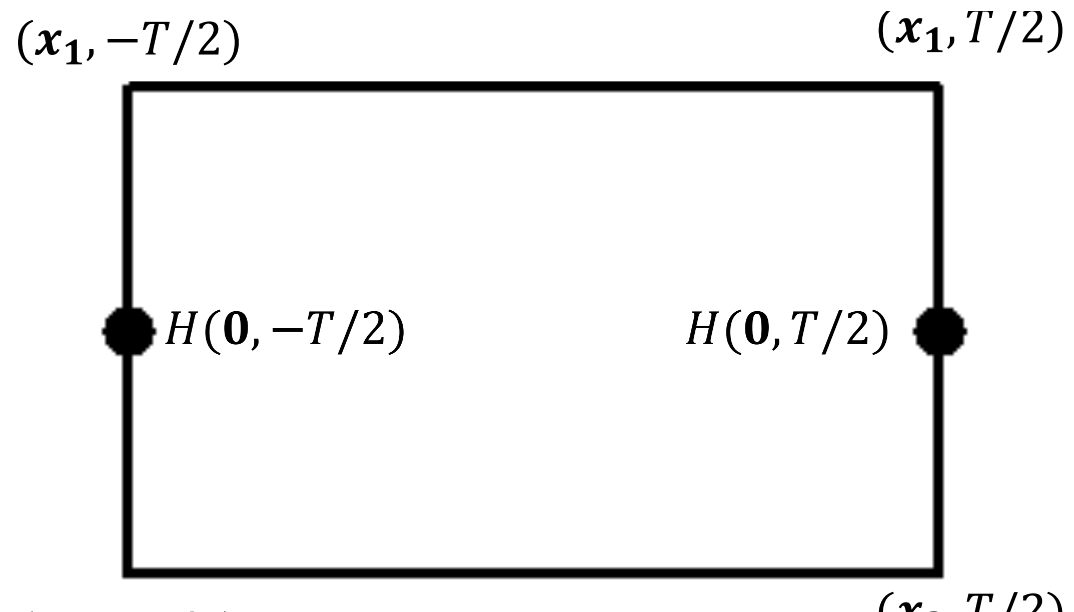


(b)

Capitani Philipsen Reisinger Riehl Wagner PRD 99 (2019) 034502

The spectrum of static energies can be calculated in NRQCD

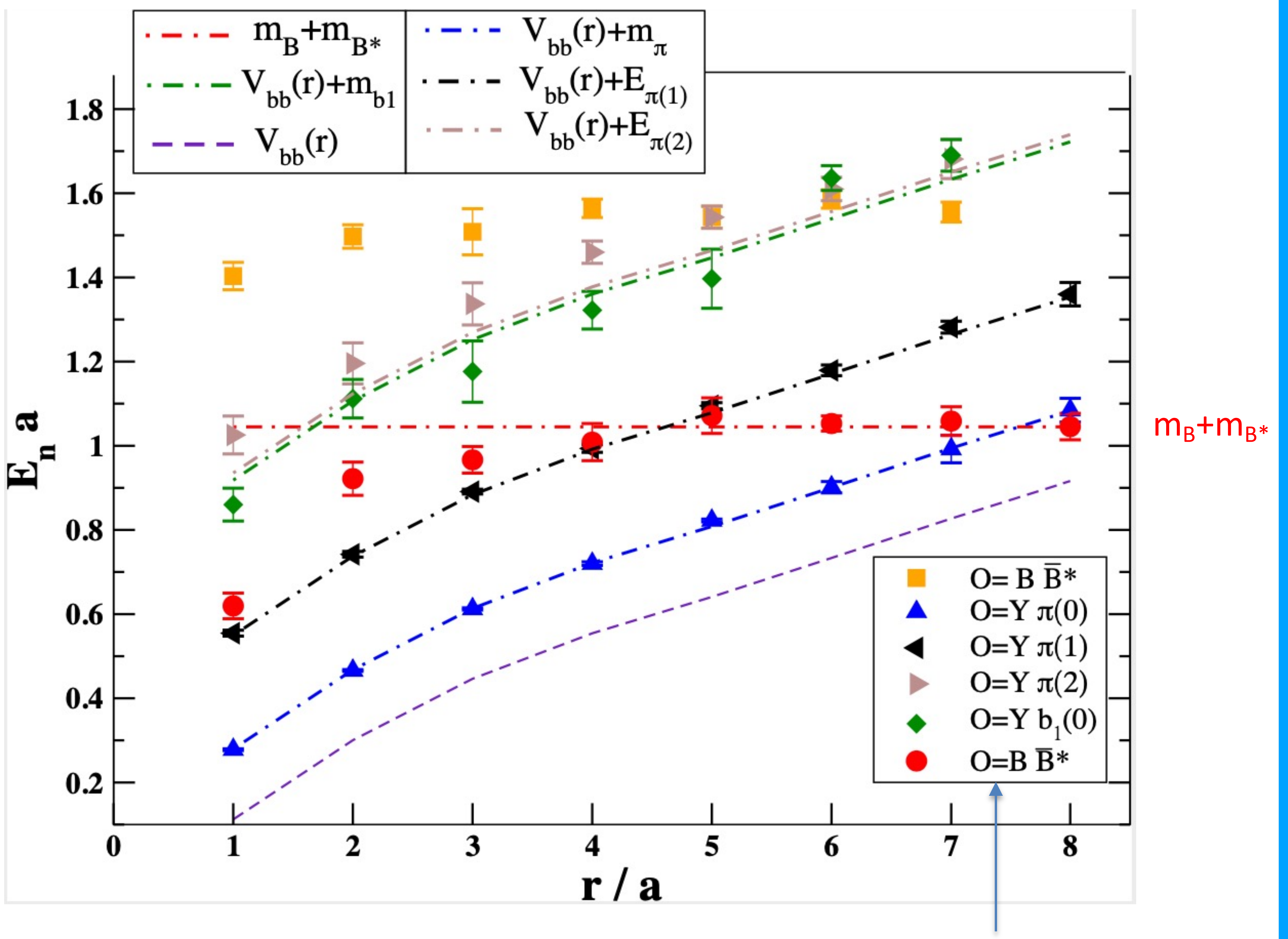
Tetraquark static energies

$\bar b b \bar d u$

Sadl, Prevlosek, 211014568

Z_b channe $\vec S_h = \vec S_b + \vec S_{\bar b}$

Eigen-energies $E_n(r)$: channel S_h=1, CP=-1, ε=-1



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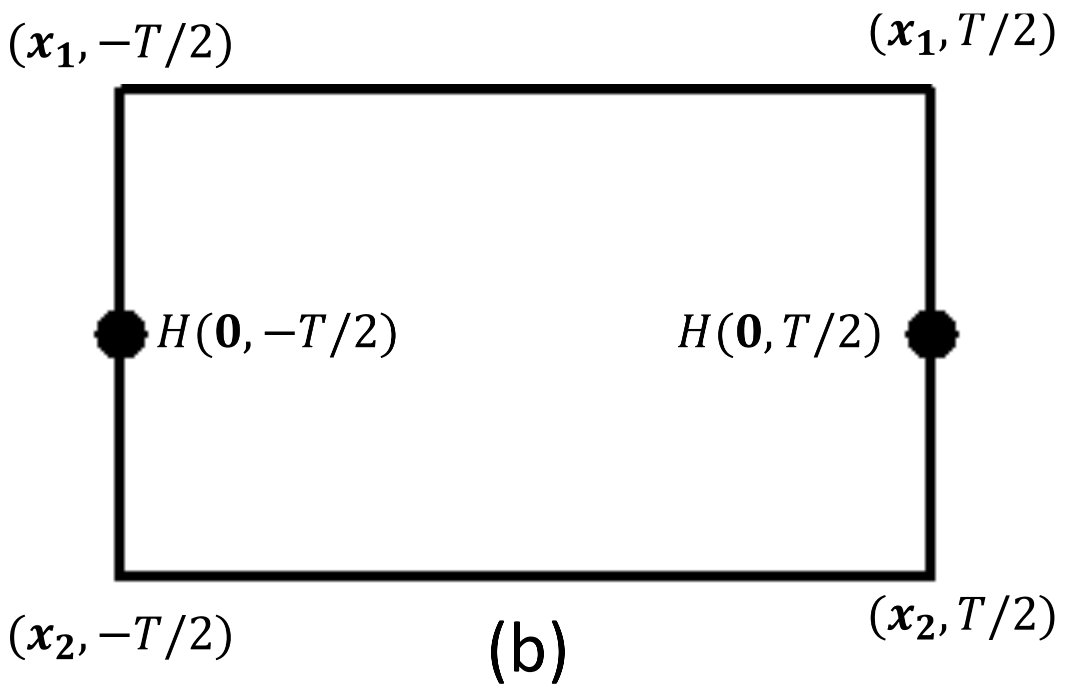
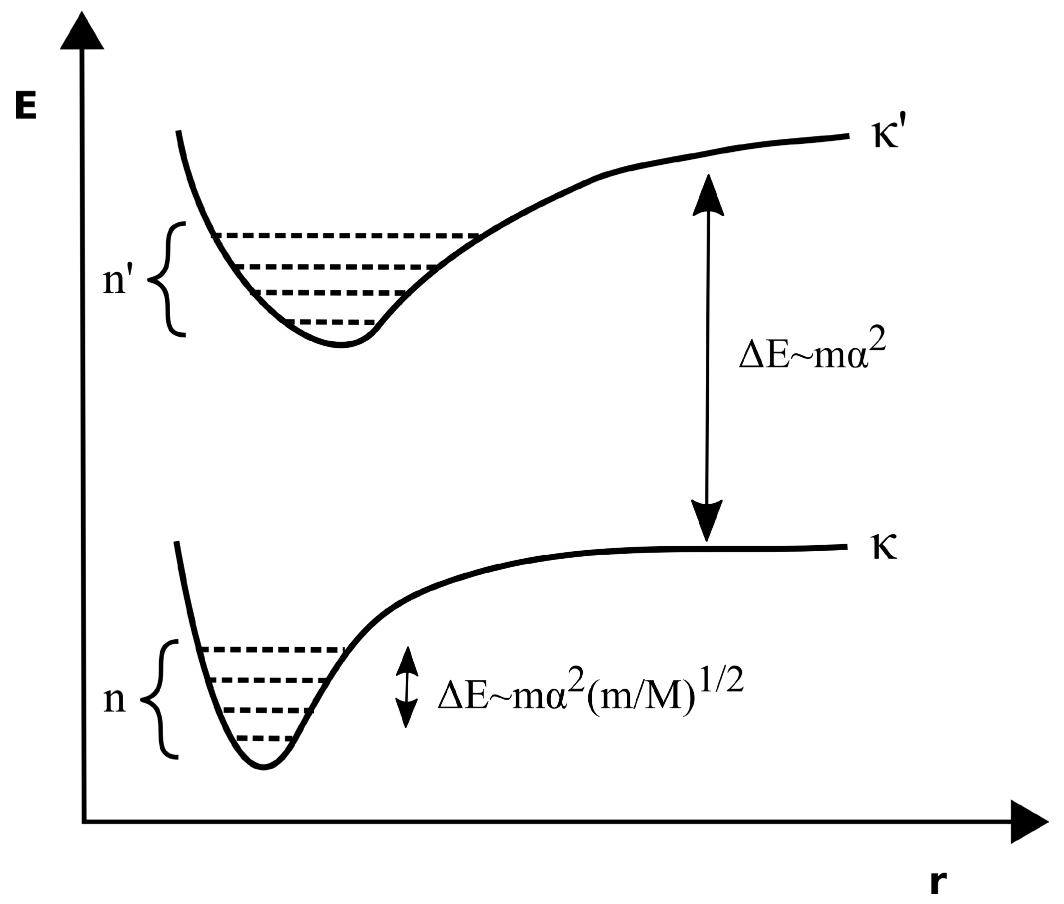


(b)

Notice: in presence of light quark in the binding one adds ispospin quantum numbers and measure tetraquark static energies

QED

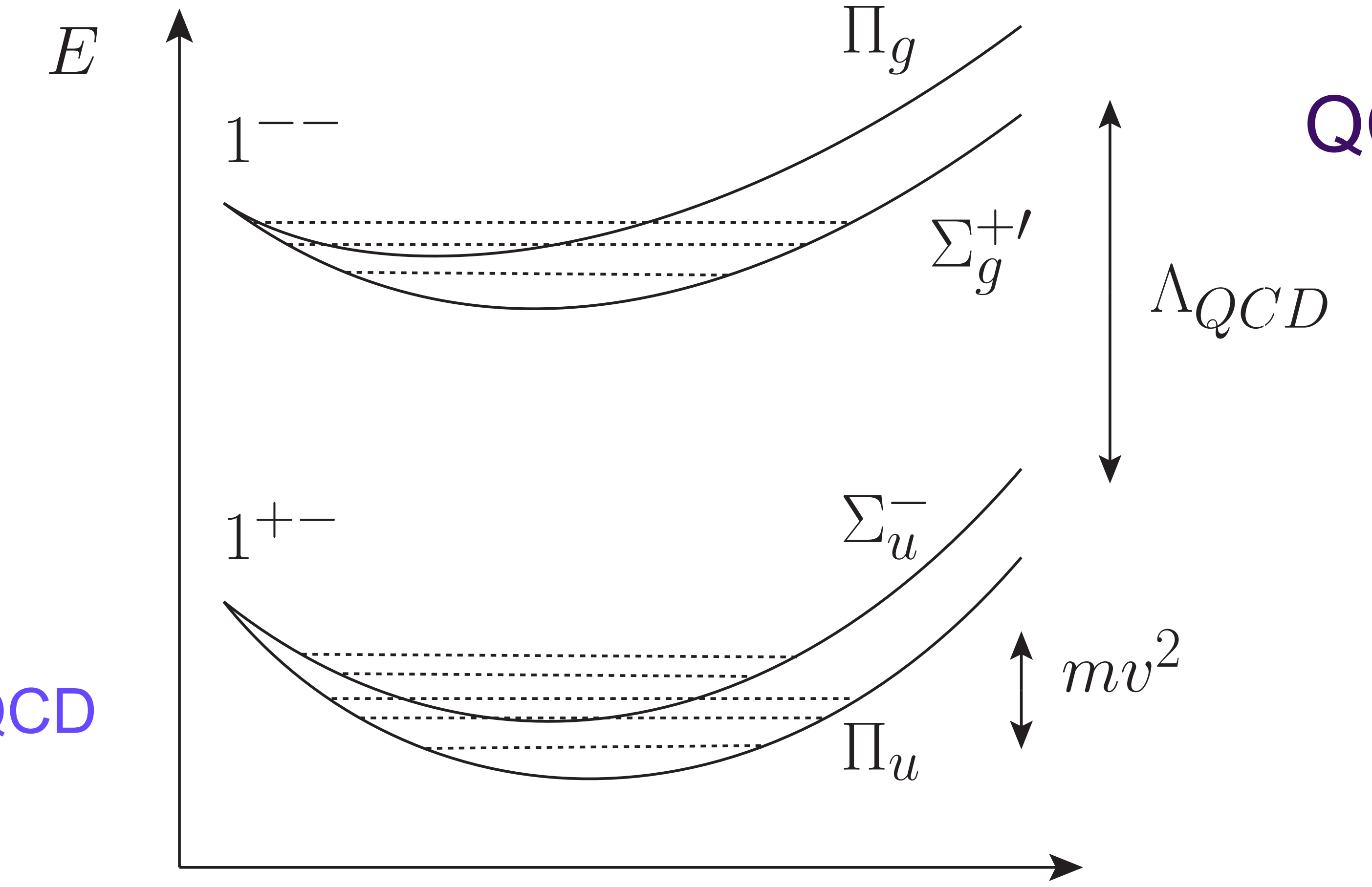


$$\Lambda_{\mathrm{QCD}} \gg mv^2$$

Braaten PRL 111 (2013) 162003
Braaten Langmack Smith PRD 90 (2014) 014044

Born Oppenheimer
Description

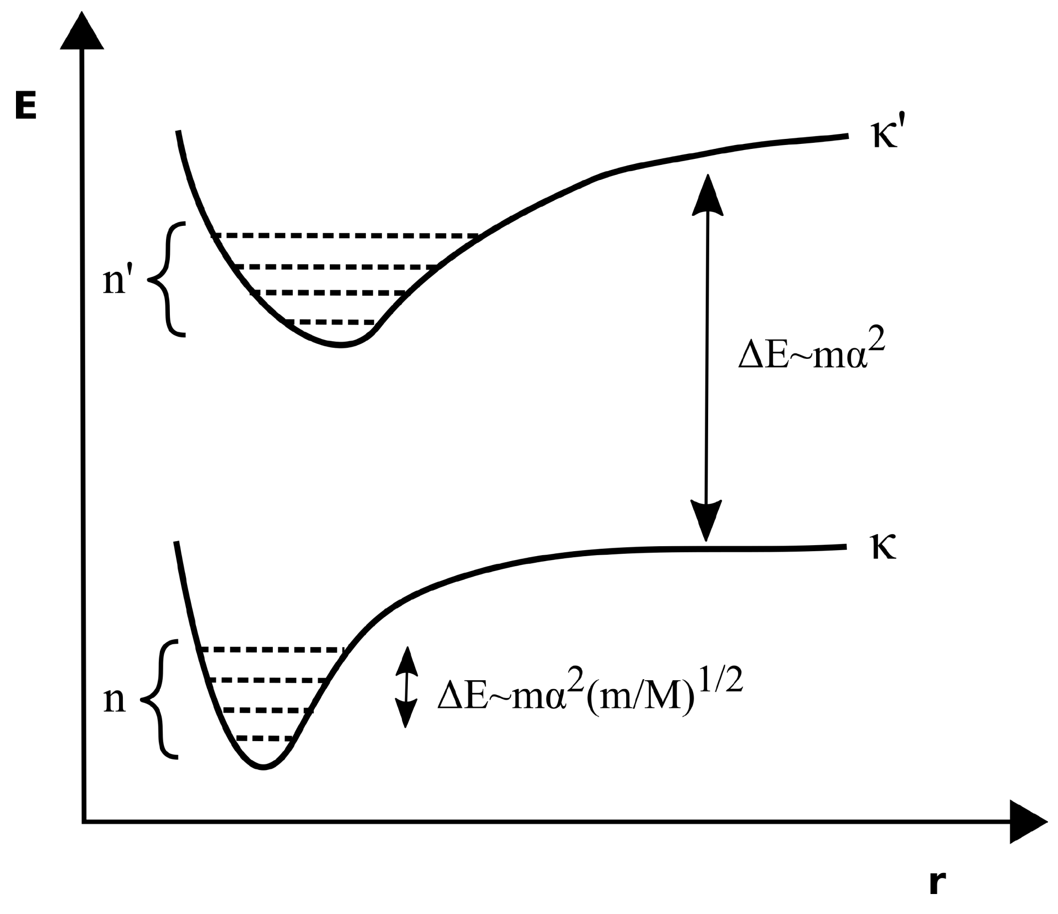
Higher excitations
develop a gap of order Lambda_QCD



QCD

Introducing a finite mass m:

- The spectrum of the mv^2 fluctuations around the lowest static energy is the quarkonium spectrum
- The spectrum of the mv^2 fluctuations around the higher excitations is the exotic spectrum (hybrids and tetraquarks)



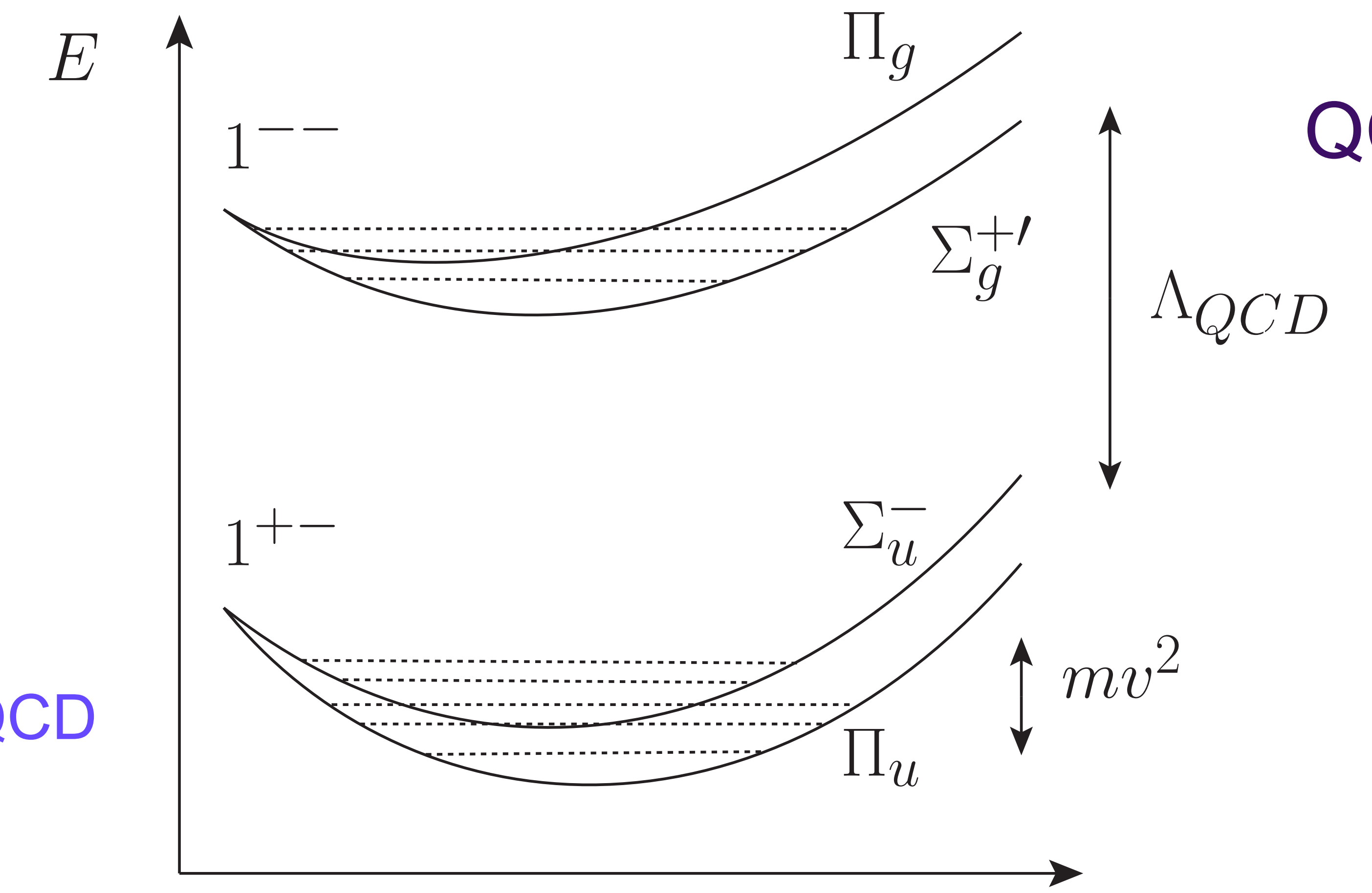
QED —

$$\Lambda_{\rm QCD} \gg mv^2$$

Braaten PRL 111 (2013) 162003
Braaten Langmack Smith PRD 90 (2014) 014044

Born Oppenheimer Description

Higher excitations
develop a gap of order Lambda_QCD



QCD

Introducing a finite mass m:

- The spectrum of the mv^2 fluctuations around the lowest static energy is the quarkonium spectrum
- The spectrum of the mv^2 fluctuations around the higher excitations is the exotic spectrum (hybrids and tetraquarks)

Nonperturbative matching to the pNREFT

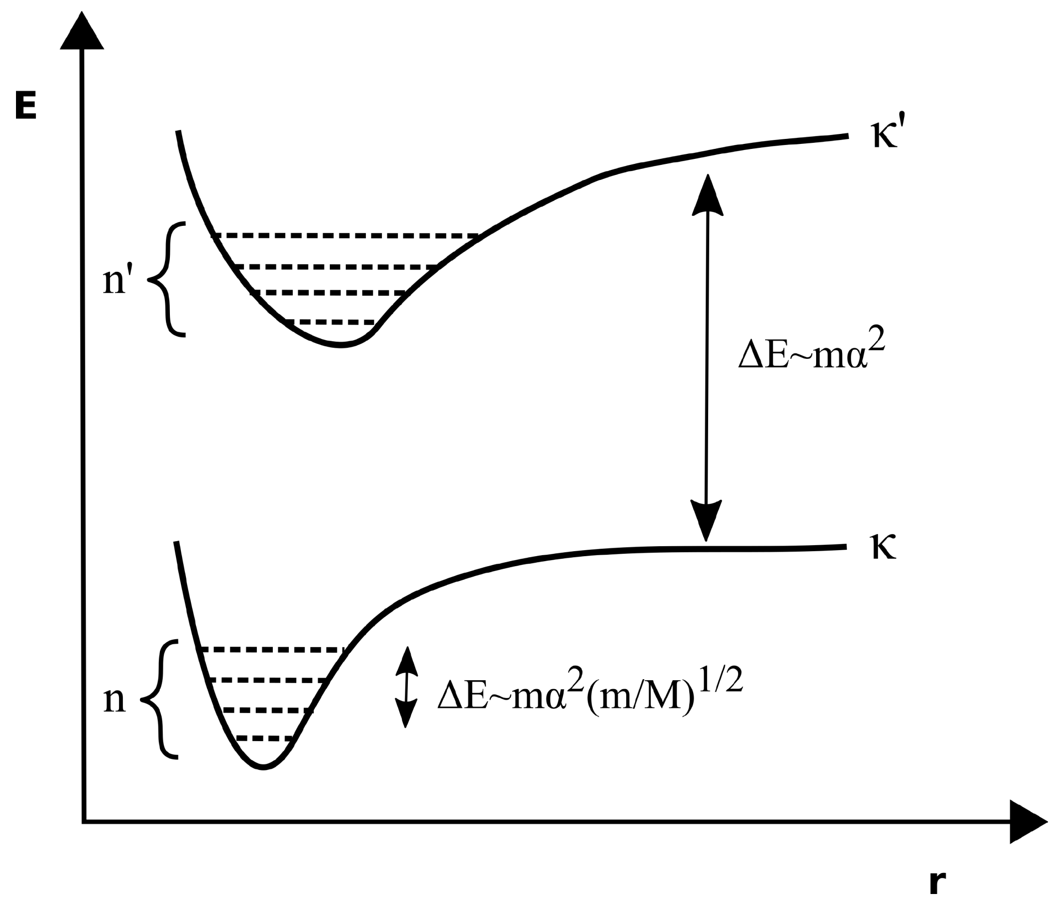
$$|\underline{0}; \mathbf{x}_1\mathbf{x}_2\rangle -> |(Q\bar{Q})_1\rangle \rightarrow \text{Quarkonium Singlet}$$

$$E_0(r) -> V_0(r)$$ (Strongly coupled)pNRQCD

$$|\underline{n} > 0; \mathbf{x}_1\mathbf{x}_2\rangle -> |(Q\bar{Q})g^{(n)}\rangle \rightarrow \text{Higher Gluonic Excitations}$$

$$|Q\bar{Q}q\bar{q}\rangle \quad \text{Tetraquarks}$$

$$E_n^{(0)}(r) -> V_n^{(0)}(r)$$ BOEFT



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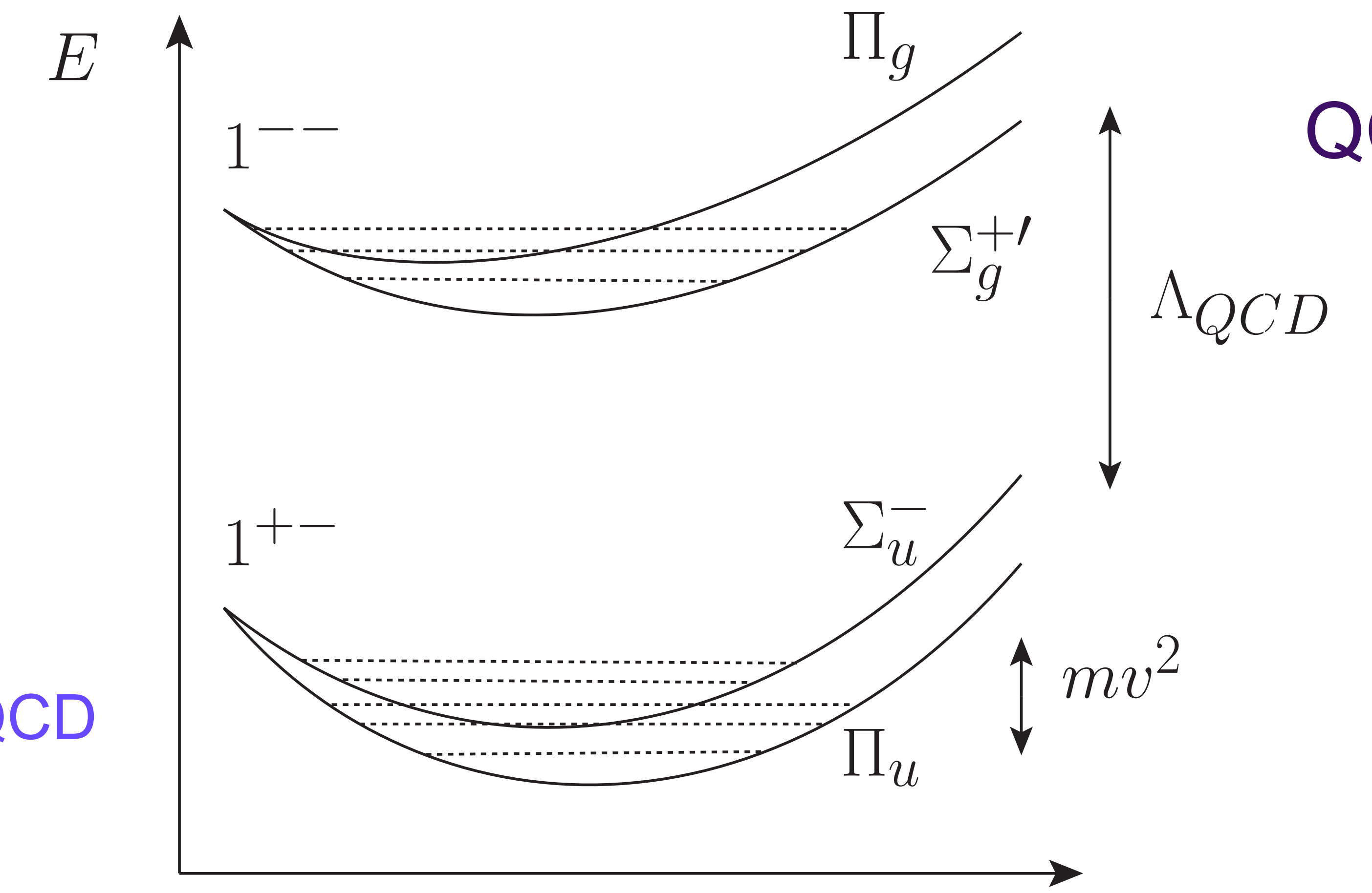


QCD

Introducing a finite mass m:

- The spectrum of the mv^2 fluctuations around the lowest static energy is the quarkonium spectrum
- The spectrum of the mv^2 fluctuations around the higher excitations is the exotic spectrum (hybrids and tetraquarks)

Nonperturbative matching to the pNREFT

systematically

$$\langle H|\mathcal{H}|H\rangle = \langle nljs|\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n}|nljs\rangle$$

expand quantomechanically NRQCD states and energies in 1/m around the zero order and identify the QCD potentials

$|\underline{0};\mathbf{x}_1\mathbf{x}_2\rangle -> \; |(Q\bar{Q})_1\rangle \rightarrow$ Quarkonium Singlet

$E_0(r) -> V_0(r)$ (Strongly coupled)pNRQCD

$|\underline{n}>0;\mathbf{x}_1\mathbf{x}_2\rangle -> \; |(Q\bar{Q})g^{(n)}\rangle \rightarrow$ Higher Gluonic Excitations

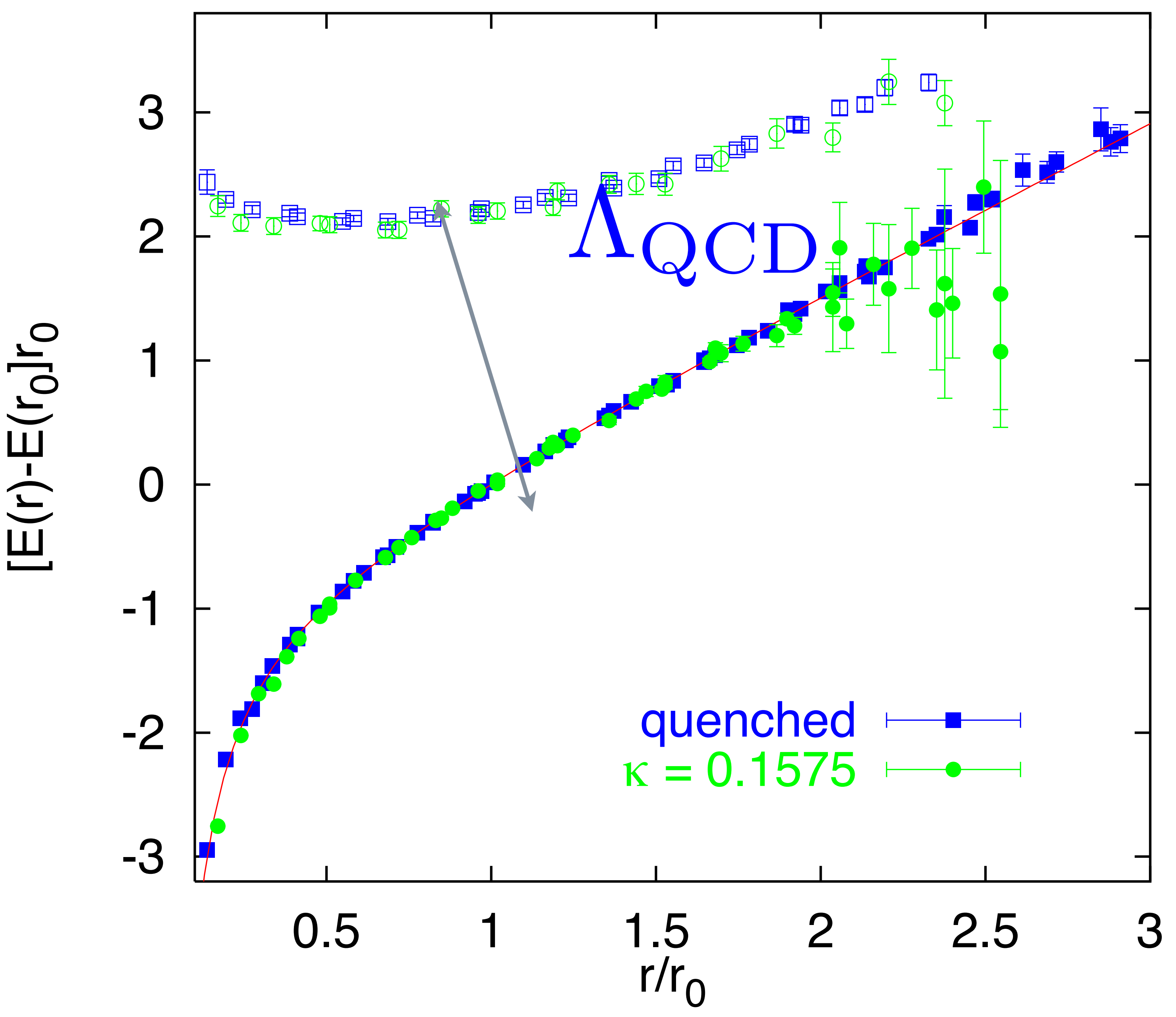
$|Q\bar{Q}q\bar{q}\rangle$ Tetraquarks

$E_n^{(0)}(r) -> V_n^{(0)}(r)$ BOEFT

Strongly coupled pNRQCD for quarkonium

$mv \sim \Lambda_{QCD}$ below the threshold

pNRQCD and the potentials come from integrating out all scales up to mv^2



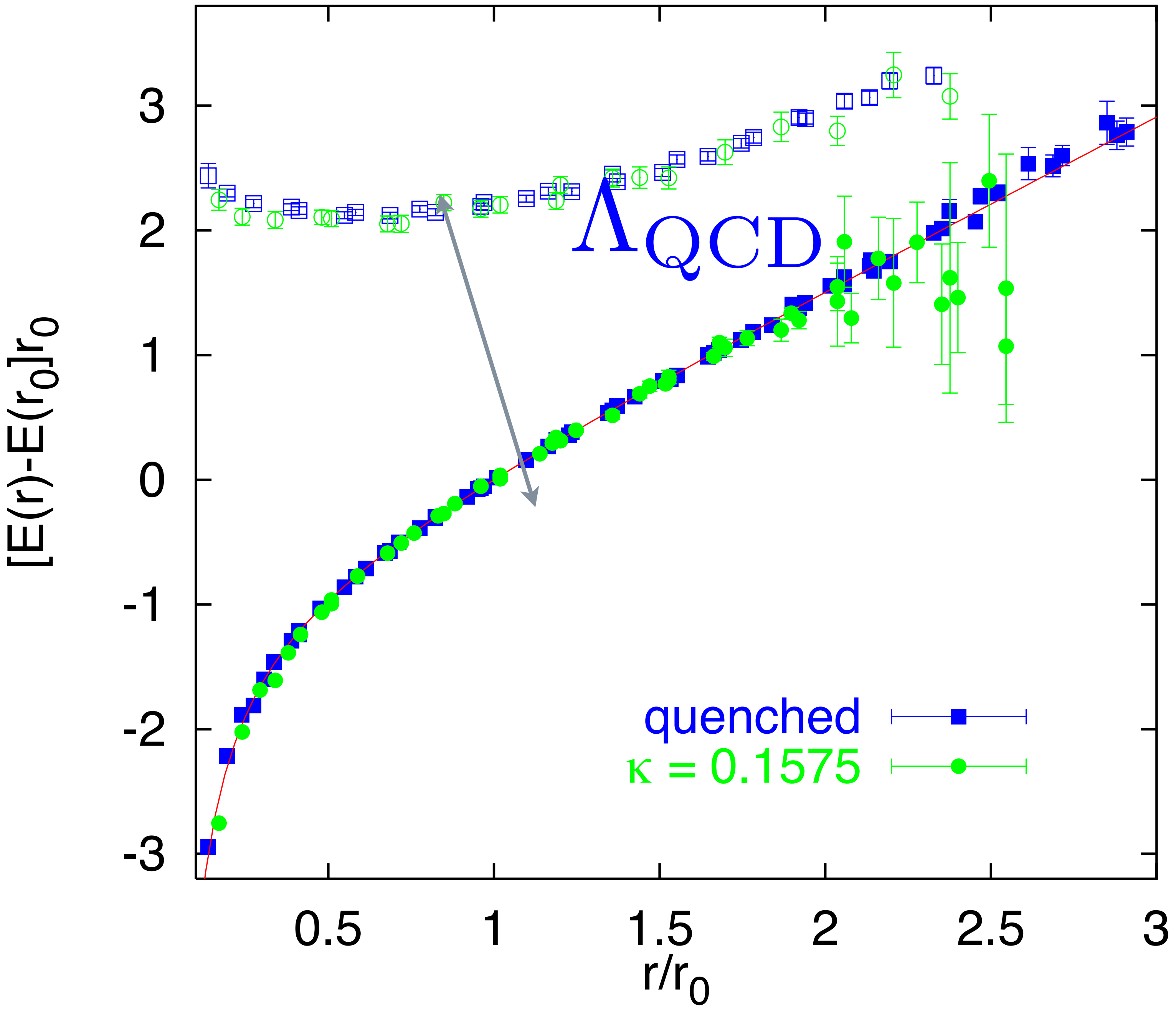
Bali et al. 98

- gluonic excitations develop a gap $\Lambda_{\rm QCD}$ and are integrated out

Brambilla Pineda Soto Vairo 00

Strongly coupled pNRQCD for quarkonium

$mv \sim \Lambda_{QCD}$ below the threshold



Bali et al. 98

pNRQCD and the potentials come from integrating out all scales up to mv^2

- gluonic excitations develop a gap $\Lambda_{\rm QCD}$ and are integrated out

$\Rightarrow$ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

Brambilla Pineda Soto Vairo 00

$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\} \quad + \Delta\mathcal{L}(\text{US light quarks})$$

- A pure potential description emerges from the EFT **however this is not the constituent quark model, alphas and masses are the QCD fundamental parameters**
- The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays
- We obtain the form of the nonperturbative potentials V in terms of generalized Wilson loops (stat that are low energy pure gluonic correlators: all the flavour dependence is pulled out

Applications regard: Spectrum, decays, production at LHC, studies of confinement

The singlet potential has the general structure

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

static spin dependent velocity dependent

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

$$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t \left\langle \boxed{} \right\rangle$$

E(t)

gauge invariant wilson loops can be calculated also in QCD vacuum model and large N

Pineda Vairo PRD 63 (2001) 054007

Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

E(t)

$$V^{(2)}_{\rm SD} = -\frac{r^k}{r^2} c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \left\langle \boxed{\mathtt{i}\ \ \mathtt{j}} \right\rangle \mathbf{L}_1 \cdot \mathbf{S}_2 + (1 \leftrightarrow 2)\ |V^{(2)}_{LS}$$

B

$$-\frac{r^k}{r^2}\left(c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \left\langle \boxed{\mathtt{i}\ \ \mathtt{j}} \right\rangle - \frac{2c_F - 1}{2} \nabla^k V^{(0)} \right) \mathbf{L}_1 \cdot \mathbf{S}_1 + (1 \leftrightarrow 2)\ |V^{(1)}_{LS}$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\left\langle \boxed{\mathtt{i}\ \ \mathtt{j}} \right\rangle - \frac{\delta_{ij}}{3} \left\langle \boxed{} \right\rangle \right) \left(\mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}}) \right)\ |V_T$$

$$+\left(\frac{2}{3} c_F^2 i \int_0^\infty dt \left\langle \boxed{} \right\rangle - 4\left(d_{sv} + \frac{4}{3} d_{vv} \right) \delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1 \cdot \mathbf{S}_2\ |V_S$$

The singlet potential has the general structure

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static spin dependent velocity dependent

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$$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t \,\Big\langle \overset{E(t)}{\boxed{\bullet\ \bullet}} \Big\rangle$$

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Pineda Vairo PRD 63 (2001) 054007

Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

$$V_{\rm SD}^{(2)} = -\frac{r^k}{r^2} c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \Big\langle \underset{B}{\overset{E(t)}{\boxed{i\quad j}}} \Big\rangle\, \mathbf{L}_1 \cdot \mathbf{S}_2 + (1 \leftrightarrow 2)\ |V_{LS}^{(2)}$$

$$-\frac{r^k}{r^2}\left(c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t\, \Big\langle \boxed{i\quad j} \Big\rangle - \frac{2c_F - 1}{2}\nabla^k V^{(0)}\right) \mathbf{L}_1 \cdot \mathbf{S}_1 + (1 \leftrightarrow 2)\ |V_{LS}^{(1)}$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\Big\langle \boxed{i\quad j} \Big\rangle - \frac{\delta_{ij}}{3} \Big\langle \boxed{} \Big\rangle \right) \left(\mathbf{S}_1 \cdot \mathbf{S_2} - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S_2} \cdot \hat{\mathbf{r}}) \right)\ |V_T$$

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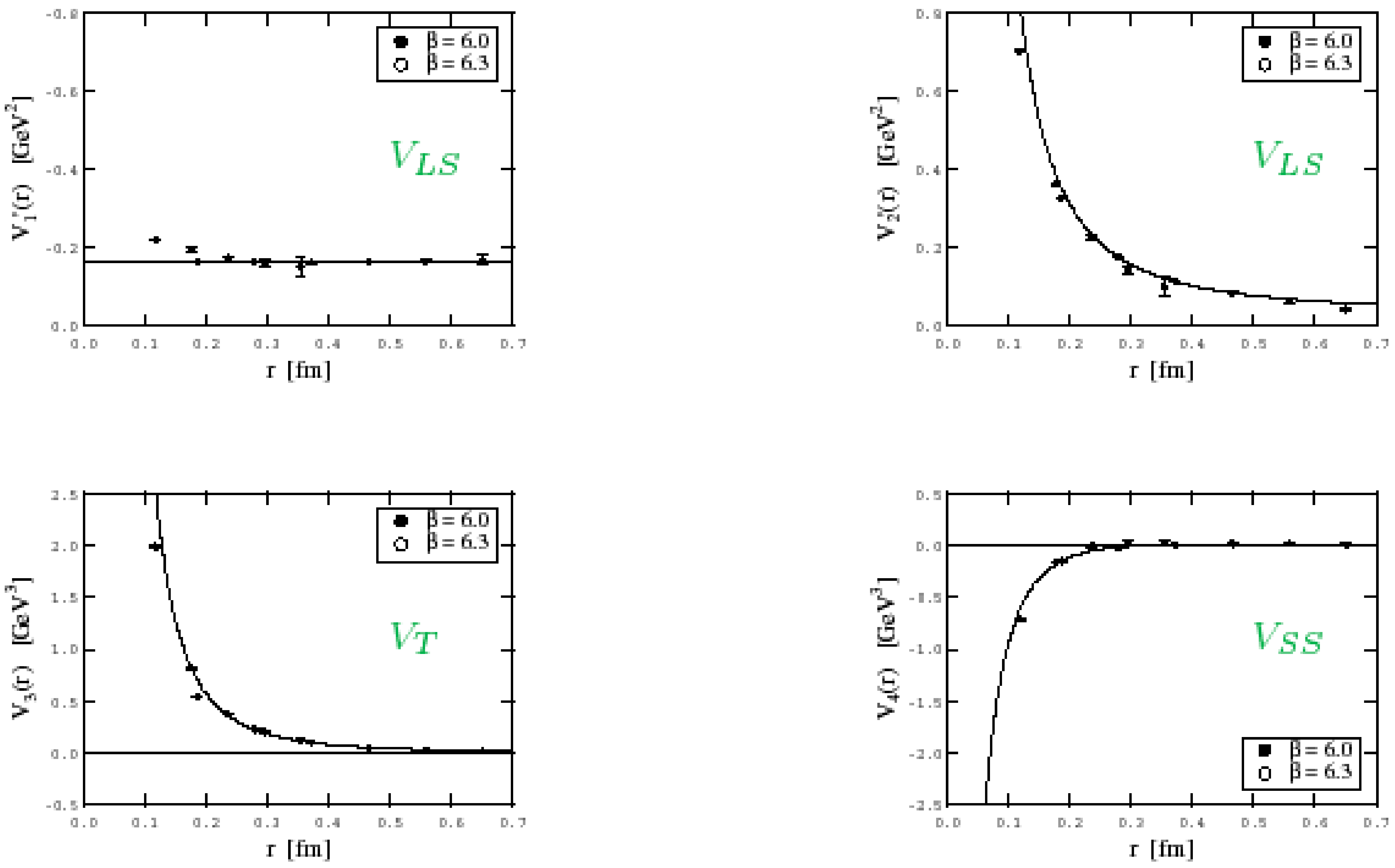
- the potentials contain the contribution of the scale m inherited from NRQCD matching coefficients—> they cancel any QM divergences, good UV behaviour
- the flavour dependent part is extracted in the NRQCD matching coefficients
- the nonperturbative part is universal:factorized and depends only on the glue —> only one lattice calculation to get the dynamics and the observables instead of an ab initio calculation of multiple Green functions

Lattice evaluation of the spin dependent potentials



Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

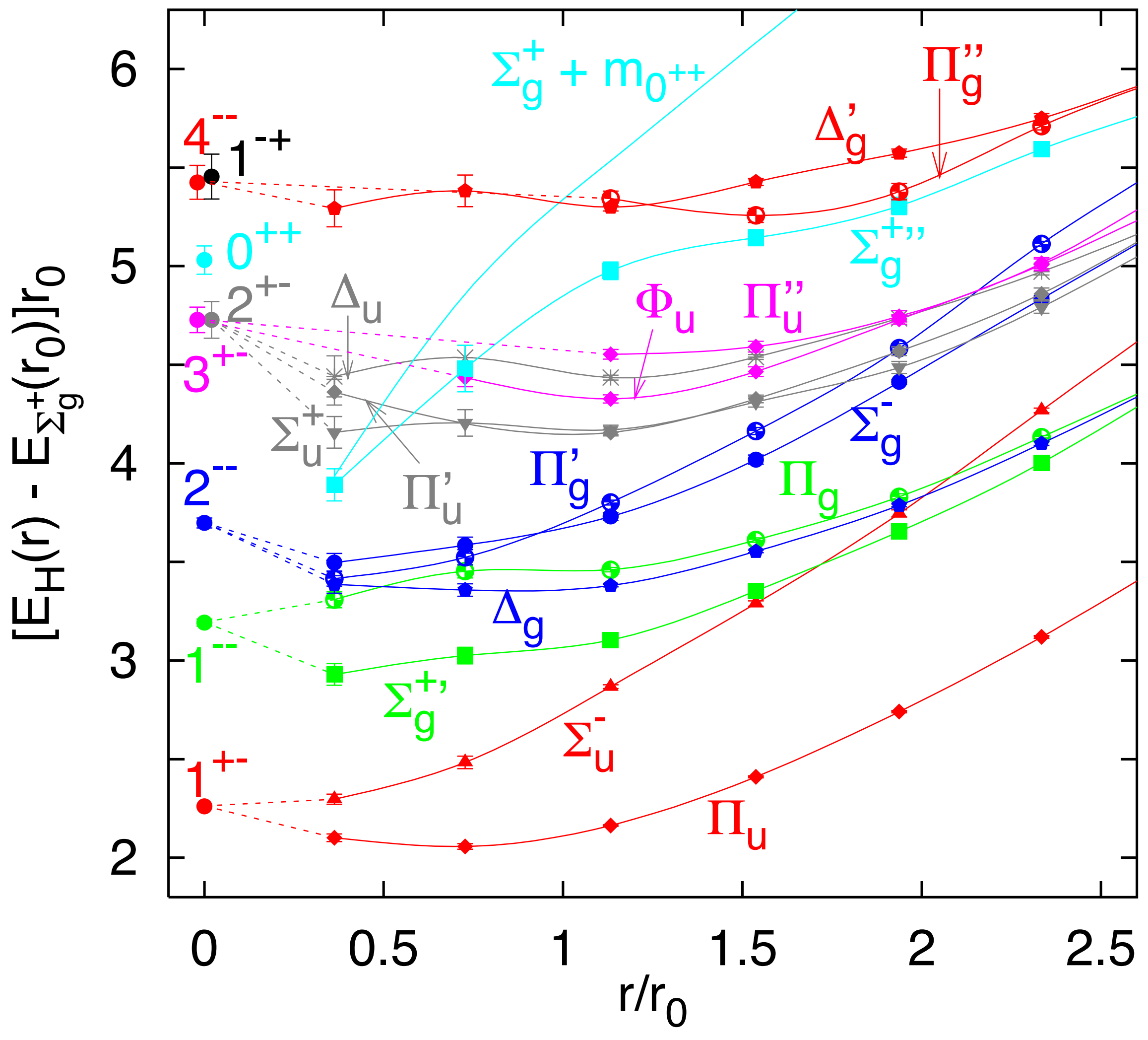
Such data can distinguish different models for the dynamics of low energy QCD e.g. effective string model

N. B., Martinez, Vairo 2014

BOEFT for HYBRIDS

Hybrids static energies at short distances

We can calculate the perturbative behaviour of the potential for short distance



In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.

the hybrid static energy can be written as a (multipole) expansion in r:

octet potential

non perturbative coefficient

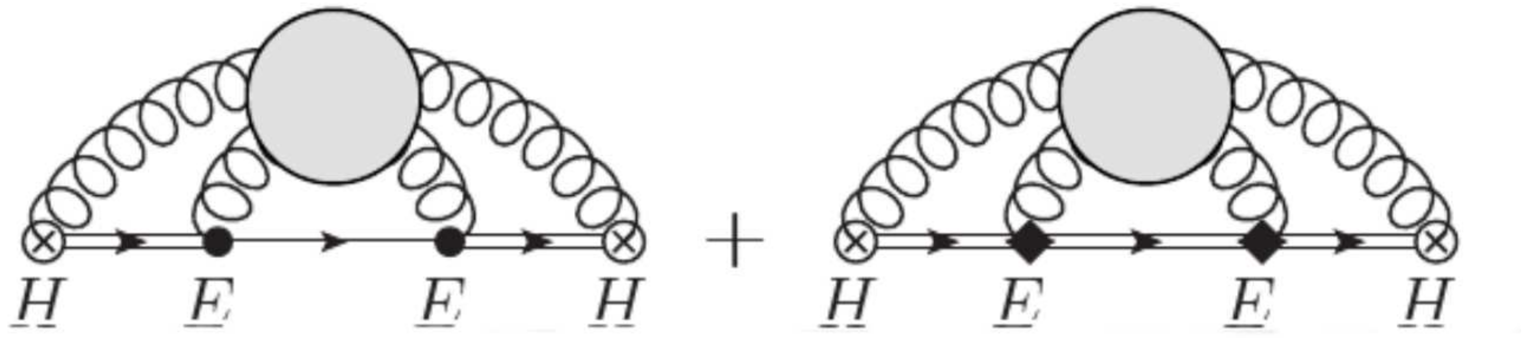
$$E_g = \frac{\alpha_s}{6r} + \Lambda_g + a_g r^2 + \ldots$$

Λ_g is the gluelump mass: $\Lambda_g = \lim_{T\to\infty} \frac{i}{T} \ln\langle H^a(T/2)\phi^{\text{adj}}_{ab}(T/2,-T/2)H^b(-T/2)\rangle$

calculated on the lattice

Foster Michael PRD 59 (1999) 094509
Bali Pineda PRD 69 (2004) 094001
Lewis Marsh PRD 89 (2014) 014502

a_g can be expressed as field correlators (single line = singlet, double line = octet), e.g.,



Gluonic excitation operators up to dim 3

Λ_η^σ	K^{PC}	H^a
Σ_u^-	1^{+-}	$\mathbf{r}\cdot\mathbf{B}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1^{+-}	$\mathbf{r}\times\mathbf{B}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\mathbf{r}\cdot\mathbf{E}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1^{--}	$\mathbf{r}\times\mathbf{E}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$
Σ_g^-	2^{--}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g'	2^{--}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g	2^{--}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+	2^{+-}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Π_u'	2^{+-}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u	2^{+-}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

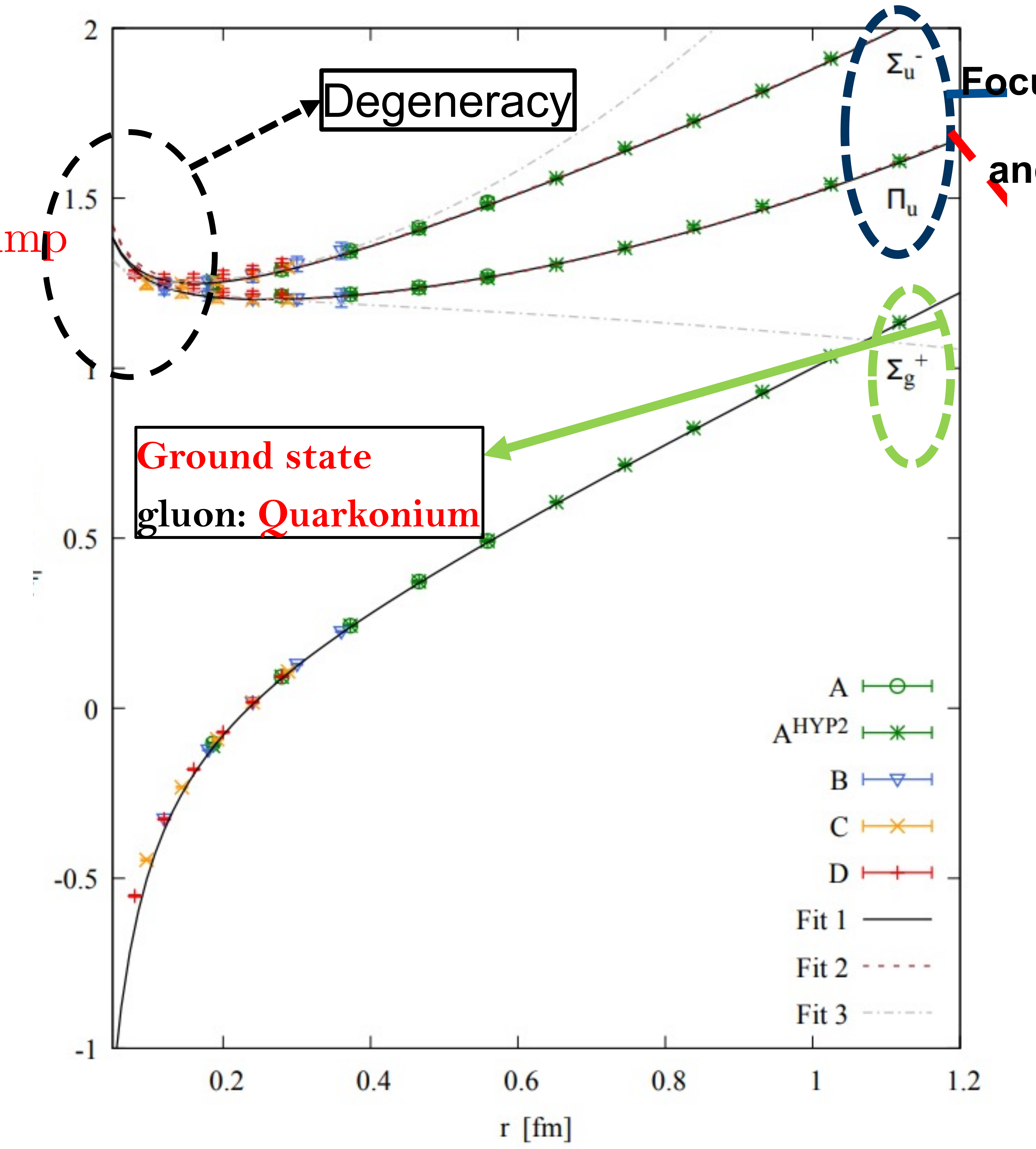
In the limit $r\to 0$ more symmetry: $D_{\infty h} \to O(3)\times C$

- Several Λ_η^σ representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r\to 0$ limit.

The gluelump multiplets Σ_u^-, Π_u; $\Sigma_g^{+\prime}$, Π_g; Σ_g^-, Π_g', Δ_g; Σ_u^+, Π_u', Δ_u are degenerate.

BOEFT for HYBRIDS

1^{+-} gluelump



Focus on the first hybrid static energy and construct BOEFT In this case

Schlosser and Wagner Phys. Rev. D. 105, (2022) The first hybrid static energy excitation

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
Oncala Soto PRD 96 (2017) 014004
Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016

$P^{i\dagger}_{\kappa\lambda} O^a(\mathbf{r},\mathbf{R},t) H^{ia}_{\kappa}(\mathbf{R},t) = Z_\kappa \Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$

$$\mathcal{L}_{\text{BOEFT for } 1+-} = \int d^3r \sum_{\lambda\lambda'} \mathrm{Tr}\left\{\Psi^\dagger_{1+-\lambda}\left(i\partial_0 - V_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_{\lambda}\frac{\boldsymbol{\nabla}^2_r}{m}\hat{r}^{i}_{\lambda'}\right)\Psi_{1+-\lambda'}\right\}$$

- $\lambda = \pm 1, 0; \quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}.$
- $V_{1+-\lambda\lambda'} = V^{(0)}_{1+-\lambda\lambda'} + \frac{V^{(1)}_{1+-\lambda\lambda'}}{m} + \frac{V^{(2)}_{1+-\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1+-\lambda\lambda'} = \delta_{\lambda\lambda'} V^{(0)}_{1+-\lambda}$, with $V^{(0)}_{1+-0} = E_{\Sigma_u^-}$, $V^{(0)}_{1+-\pm 1} = E_{\Pi_u}$

fitted from the lattice hybrids static energies

The first hybrid static energy excitation

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

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fitted from the lattice hybrids static energies

The LO e.o.m. for the fields $\Psi^\dagger_{1+-\lambda}$ are a set of coupled Schrödinger equations:

$$i\partial_0 \Psi_{1+-\lambda} = \left[\left(-\frac{\boldsymbol{\nabla}^2_r}{m} + V^{(0)}_{1+-\lambda}\right)\delta_{\lambda\lambda'} - \sum_{\lambda'} C^{\text{nad}}_{1+-\lambda\lambda'}\right]\Psi_{\kappa\lambda'}$$

The eigenvalues $\mathcal{E}_N$ give the masses M_N of the states as $M_N = 2m + \mathcal{E}_N$.

$$\hat{r}^{i\dagger}_\lambda \left(\frac{\boldsymbol{\nabla}^2_r}{m}\right)\hat{r}^i_{\lambda'} = \delta_{\lambda\lambda'}\frac{\boldsymbol{\nabla}^2_r}{m} + C^{\text{nad}}_{1+-\lambda\lambda'}$$

with $C^{\text{nad}}_{1+-\lambda\lambda'} = \hat{r}^{i\dagger}_\lambda \left[\dfrac{\boldsymbol{\nabla}^2_r}{m}, \hat{r}^i_{\lambda'}\right]$ called the nonadiabatic coupling.

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  Oncala Soto PRD 96 (2017) 014004
  Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016

$$\mathcal{L}_{\text{BOEFT for } 1^{+-}} = \int d^3r \sum_{\lambda\lambda'} \mathrm{Tr}\left\{\Psi^\dagger_{1^{+-}\lambda}\left(i\partial_0 - V_{1^{+-}\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}^2_r}{m}\hat{r}^i_{\lambda'}\right)\Psi_{1^{+-}\lambda'}\right\}$$

- $\lambda = \pm 1, 0;\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- $V_{1^{+-}\lambda\lambda'} = V^{(0)}_{1^{+-}\lambda\lambda'} + \dfrac{V^{(1)}_{1^{+-}\lambda\lambda'}}{m} + \dfrac{V^{(2)}_{1^{+-}\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1^{+-}\lambda\lambda'} = \delta_{\lambda\lambda'}V^{(0)}_{1^{+-}\lambda}$, with $V^{(0)}_{1^{+-}0} = E_{\Sigma_u^-}$, $V^{(0)}_{1^{+-}\pm 1} = E_{\Pi_u}$.

fitted from the lattice hybrids static energies

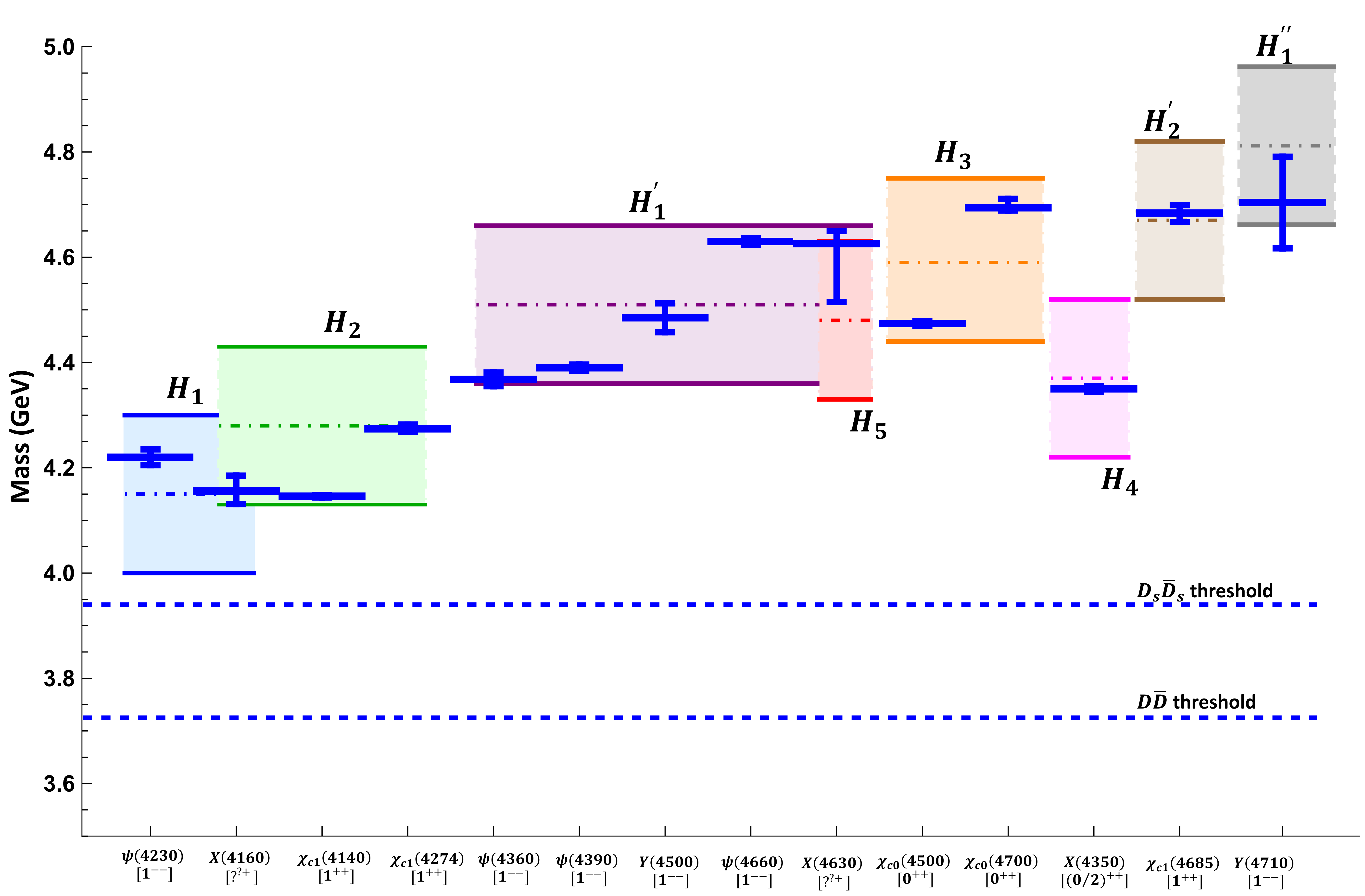
$$\left[-\frac{1}{mr^2}\partial_r r^2\partial_r + \frac{1}{mr^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E^{(0)}_\Sigma & 0 \\ 0 & E^{(0)}_\Pi\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_\Sigma \\ \psi^{(N)}_{-\Pi}\end{pmatrix} = \mathcal{E}_N\begin{pmatrix}\psi^{(N)}_\Sigma \\ \psi^{(N)}_{-\Pi}\end{pmatrix}$$

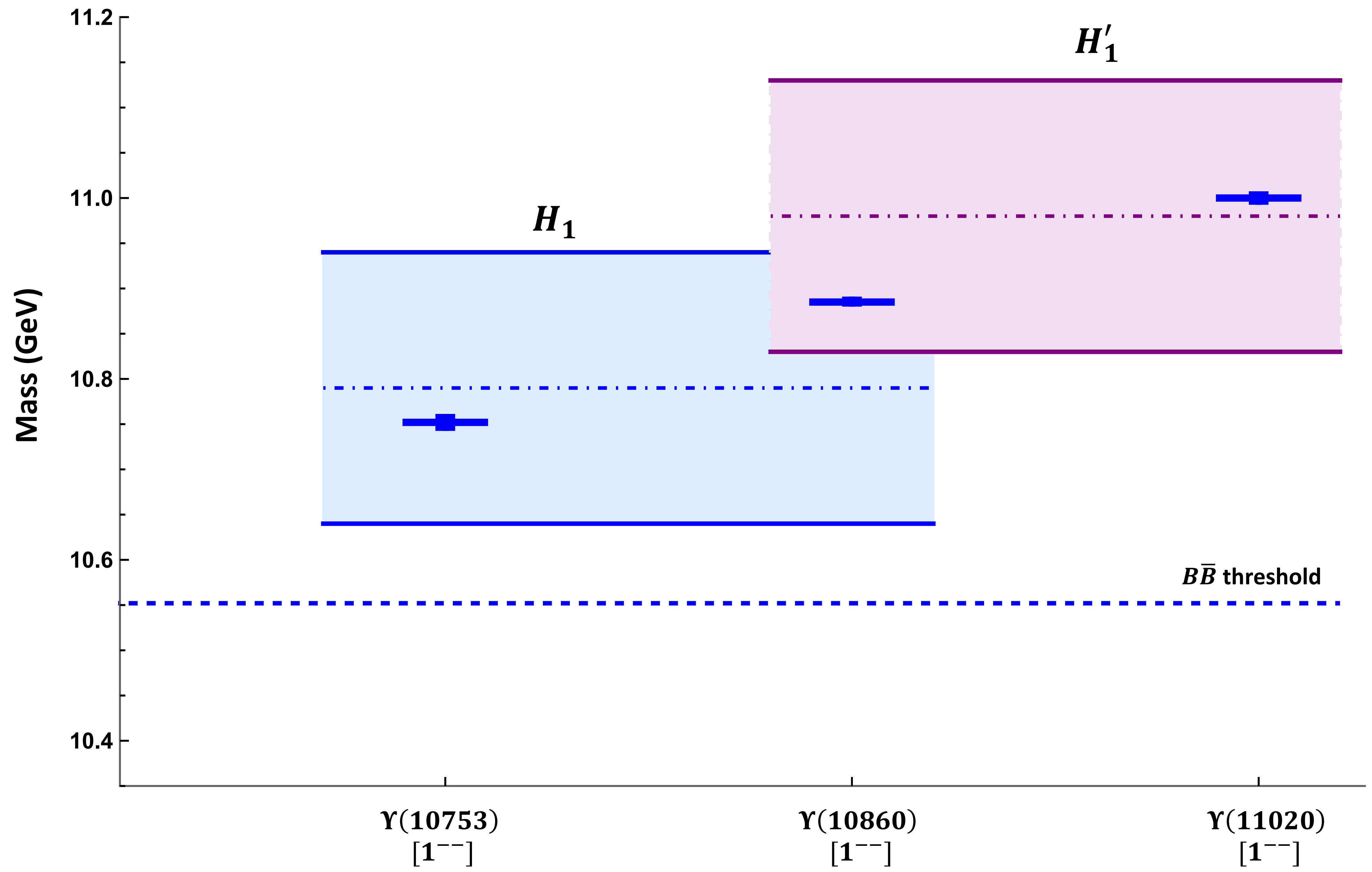
$$\left[-\frac{1}{mr^2}\,\partial_r\, r^2\,\partial_r + \frac{l(l+1)}{mr^2} + E^{(0)}_\Pi\right]\psi^{(N)}_{+\Pi} = \mathcal{E}_N\,\psi^{(N)}_{+\Pi}$$

Mixing remove the degeneration among opposite parity states: ->Lambda doubling existing also in molecular physics

- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = \left(\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g\right)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$

Hybrid multiplets as predicted by BOEFT (coloured rectangles) compared to the neutral isoscalar states observed in charmonium/bottomonium sector (crosses)





Note: Band in the mass value for each multiplet is due to the error (150 Mev) on the gluelump mass measured on the lattice

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

N. B. W. K. Lai, A- Mohapatra, A. Vairo 2212.09187

Spin dependent interactions

The **BOEFT** gives a prescription to calculate the **hybrids spin dependent potentials at order 1/m and 1/m^2**

1/m

$$V^{(1)}_{1+-\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

$$S_{12} = 12(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}}) - 4(\mathbf{S}_1\cdot\mathbf{S}_2)$$

1/m^2

$$V^{(2)}_{1+-\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V^{(2)}_{LS\,a}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{L}\,\hat{r}^{i}_{\lambda'}\right)\cdot\boldsymbol{S} + V^{(2)}_{LS\,b}(r)\hat{r}^{i\dagger}_{\lambda}\left(L^{i}S^{j}+S^{i}L^{j}\right)\hat{r}^{j}_{\lambda'} + V^{(2)}_{S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,a}(r)S_{12}\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,b}(r)\hat{r}^{i\dagger}_{\lambda}\hat{r}^{j}_{\lambda'}\left(S^{i}_{1}S^{j}_{2}+S^{i}_{2}S^{j}_{1}\right)$$

$\left(K^{ij}\right)^{k} = i\epsilon^{ikj}$ is the angular momentum of the spin one gluons

$\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

The **BOEFT** gives a prescription to calculate the **hybrids spin dependent potentials at order 1/m and 1/m^2**

1/m

$$V^{(1)}_{1+-\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

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1/m^2

$$V^{(2)}_{1+-\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V^{(2)}_{LS\,a}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{L}\,\hat{r}^{i}_{\lambda'}\right)\cdot\boldsymbol{S} + V^{(2)}_{LS\,b}(r)\hat{r}^{i\dagger}_{\lambda}\left(L^{i}S^{j} + S^{i}L^{j}\right)\hat{r}^{j}_{\lambda'} + V^{(2)}_{S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,a}(r)S_{12}\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,b}(r)\hat{r}^{i\dagger}_{\lambda}\hat{r}^{j}_{\lambda'}\left(S^{i}_{1}S^{j}_{2} + S^{i}_{2}S^{j}_{1}\right)$$

$\left(K^{ij}\right)^{k} = i\epsilon^{ikj}$ is the angular momentum of the spin one gluons

$\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

Features:

- New spin structures with respect to the quarkonium case: all terms at order 1/m and two terms at order 1/m^2

Differently from the quarkonium case, the hybrid potential gets a first contribution already at order $\Lambda^2_{\mathrm{QCD}}/m_h$. The corresponding operator does not contribute at LO to matrix elements of quarkonium states as its projection on quark-antiquark color singlet states vanishes. Hence, spin splittings are remarkably less suppressed in heavy quarkonium hybrids than in heavy quarkonia.

Hybrid spin dependent potentials at order 1/m and 1/m^2

1/m

$$V^{(1)}_{1+-\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

$$S_{12} = 12(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}}) - 4(\mathbf{S}_1\cdot\mathbf{S}_2)$$

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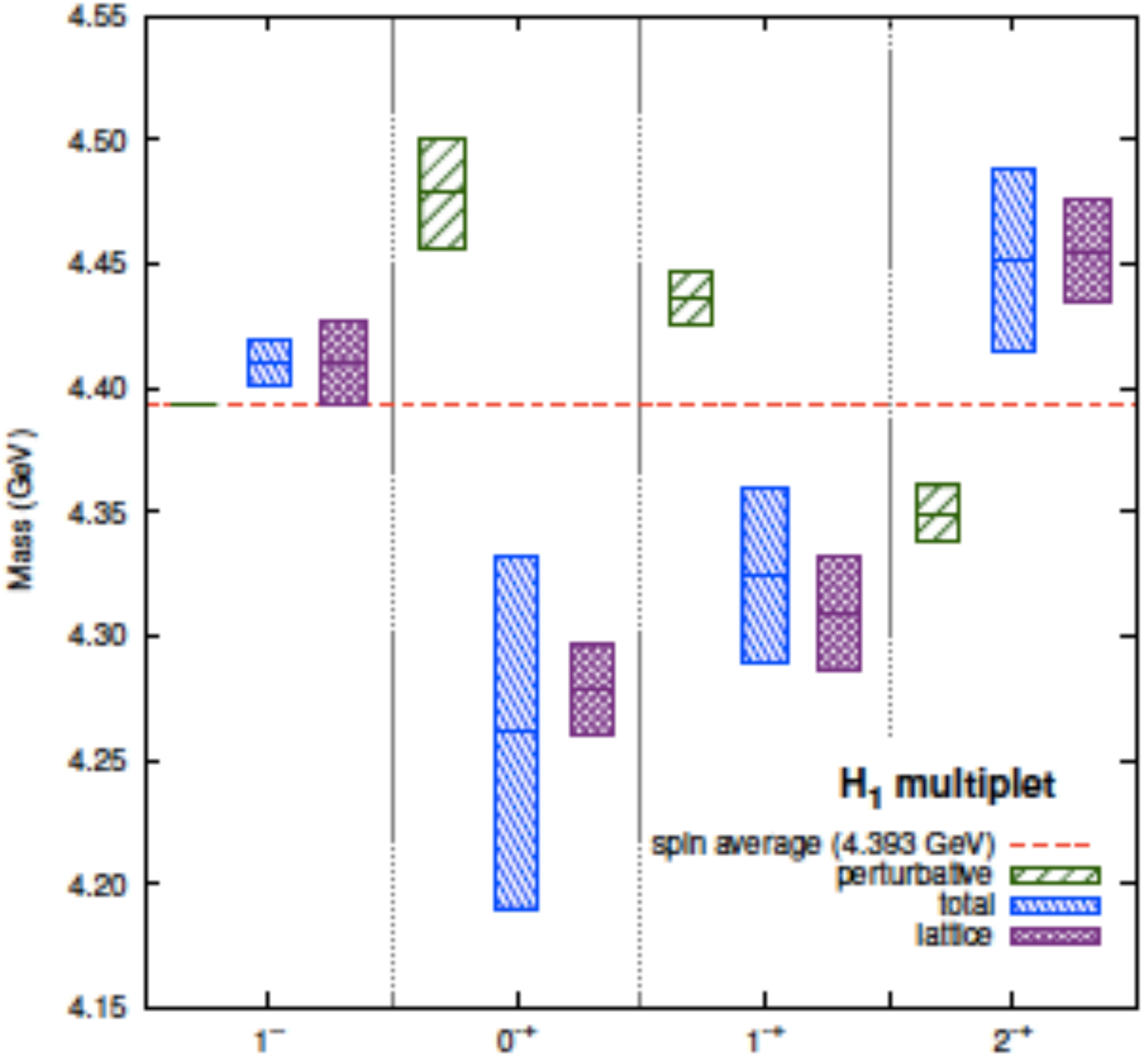
USE LATTICE CALCULATION OF THE CHARMONIUM SPIN MULTIPLETS TO EXTRACT the 6 UNKNOWNs and PREDICT THE BOTTOMONIUM SPIN MULTIPLETS, learn also about the DYNAMICS

Charmonium Hybrids Multiplets H_1



lattice data from (violet) from

G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, and D. Tims (Hadron Spectrum), JHEP **12**, 089 (2016), arXiv:1610.01073 [hep-lat].

with a pion of about 240 MeV

- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

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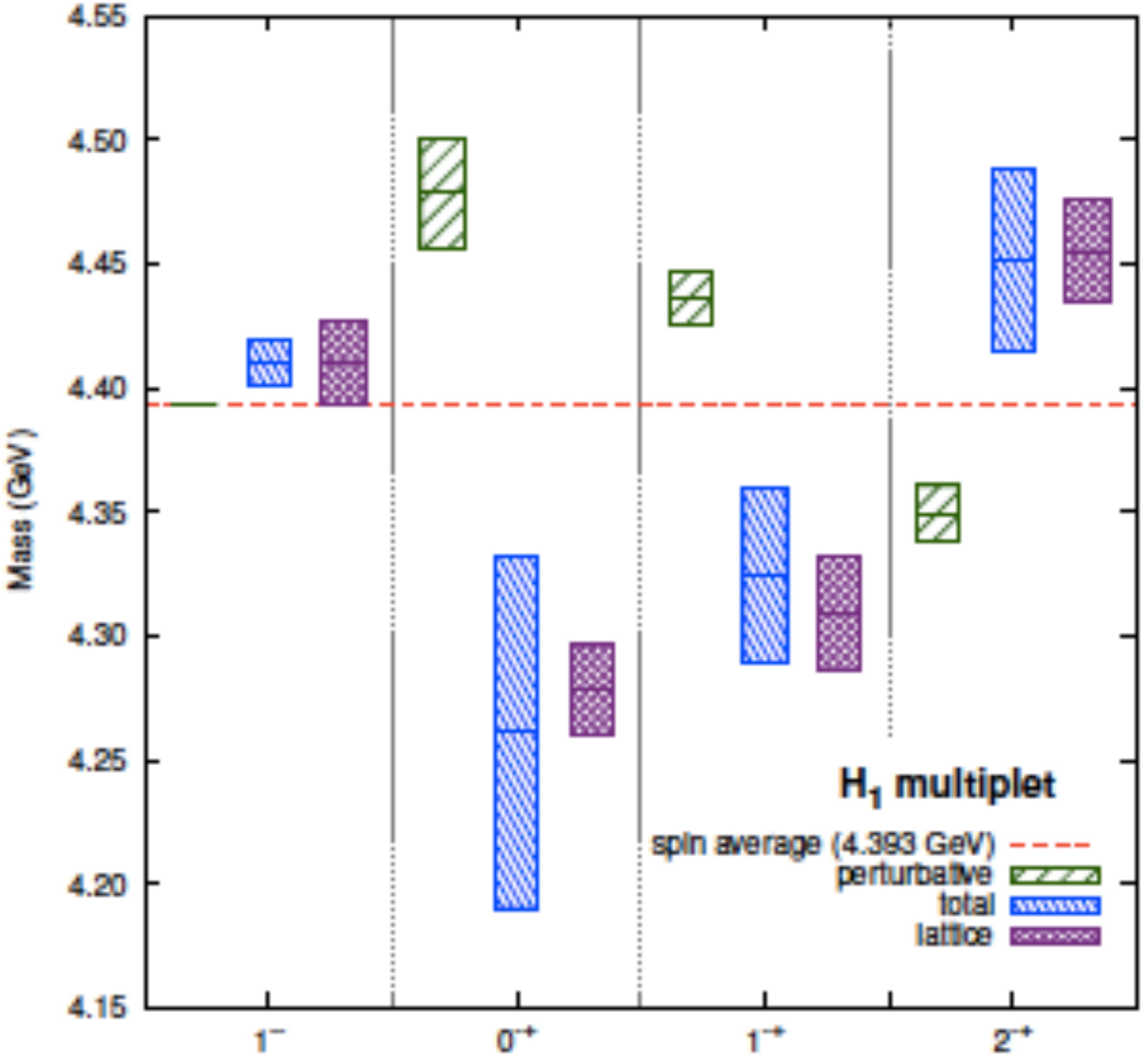

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estimated by the parametric size of higher order corrections, m alpha_s^5 for the perturbative part, powers of Lambda_qcd/m for the nonperturbative part, plus the statistical error on the fit

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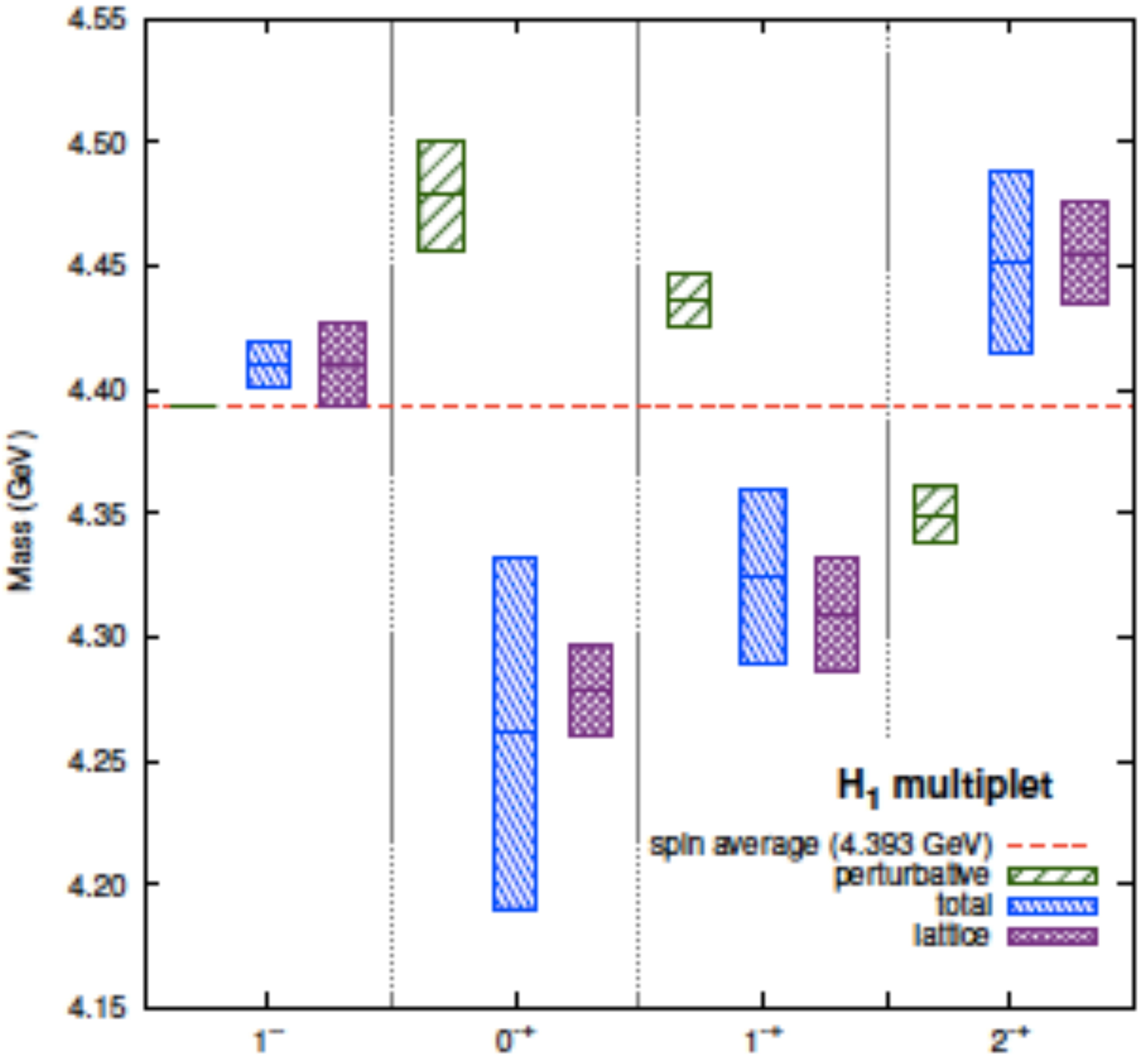

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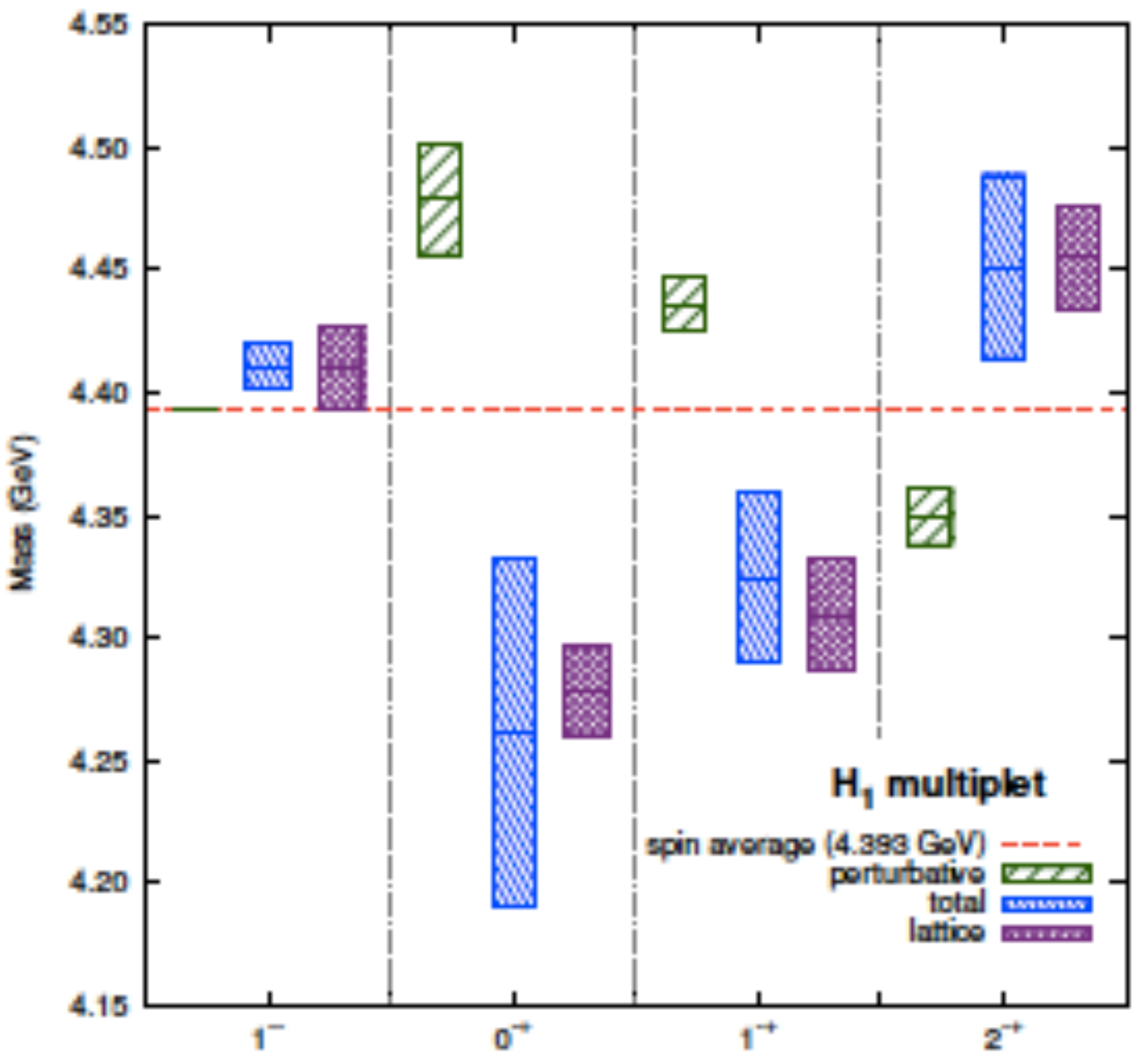
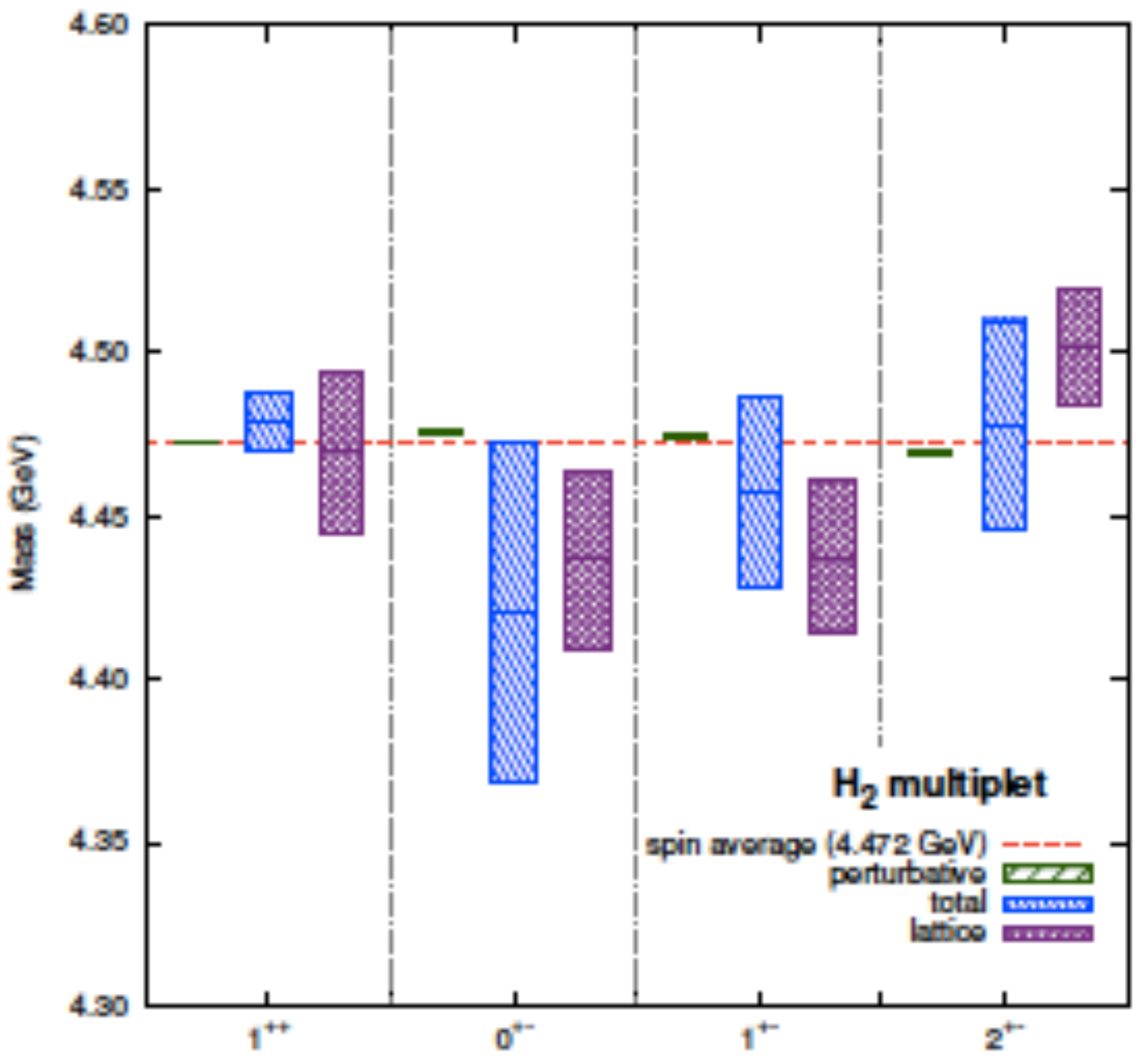
the perturbative part produces a pattern opposite to the lattice and to ordinary quarkonia —> discrepancy can be reconciled thanks to the nonperturbative parts, especially the one at order 1/m which goes like Lambda^2/m and is parametrically larger than the perturbative contribution at order m v^4

which is interesting as some models are taking the spin interaction from perturbation theory with a constituent gluon

○ Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

Charmonium Hybrids Multiplets H_1 and H_2





H_1 and H_2 corresponds to l=1 and are negative and positive parity resp. The mass splitting between H_1 and H_2 is a result of lambda-doubling

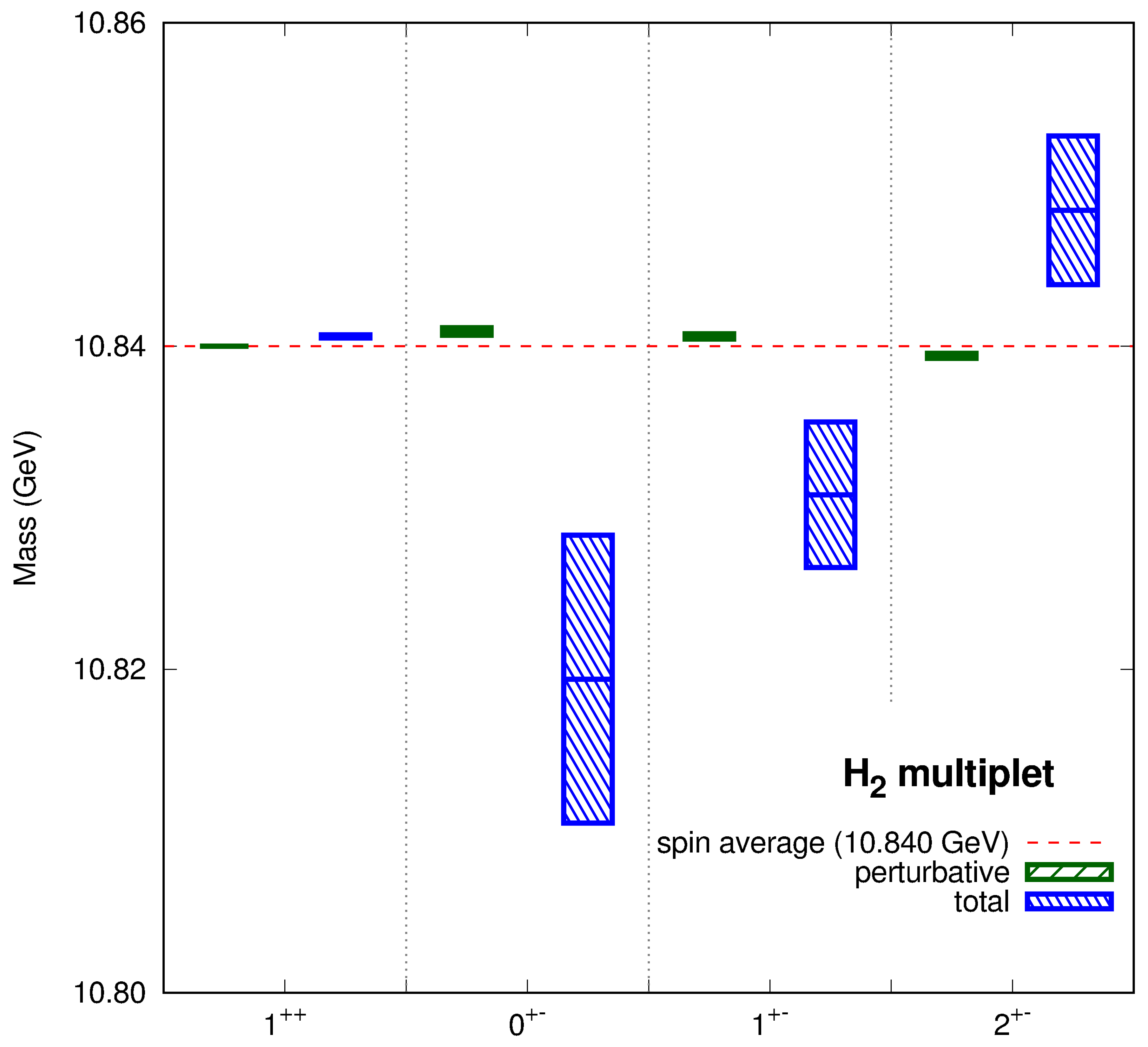
H_3 and H_4 are also calculated

- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

here you find predictions for all H multiplets

Bottomonium hybrid spin splittings

thanks to the BOEFT factorizatio we can fix the nonperturbative unknowns from a charmonium hybrid calculationthe nonperturbative low energy unknownsdo not depend on the flavor: we can predict the bottomonium hybrids splin splittings



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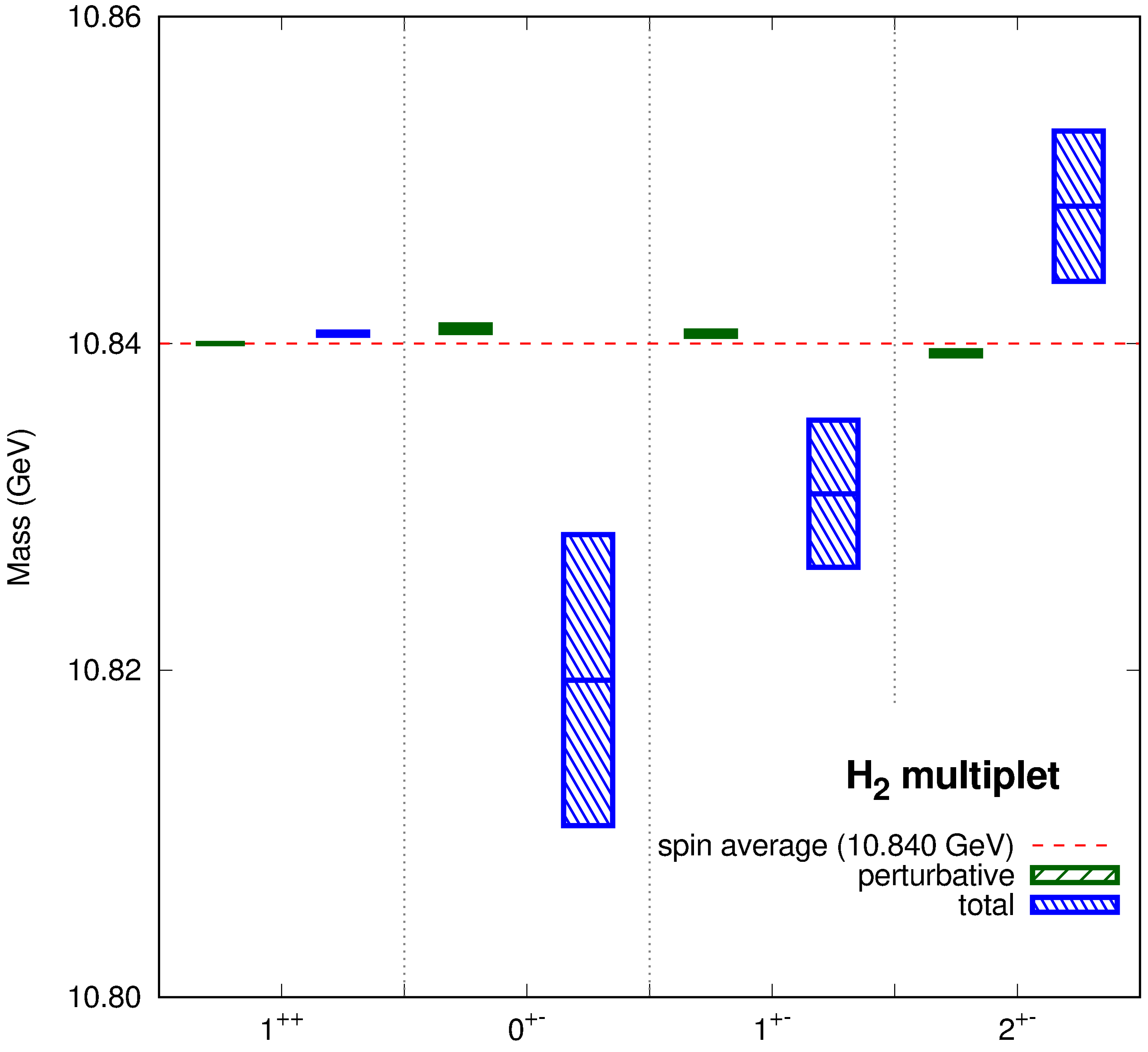

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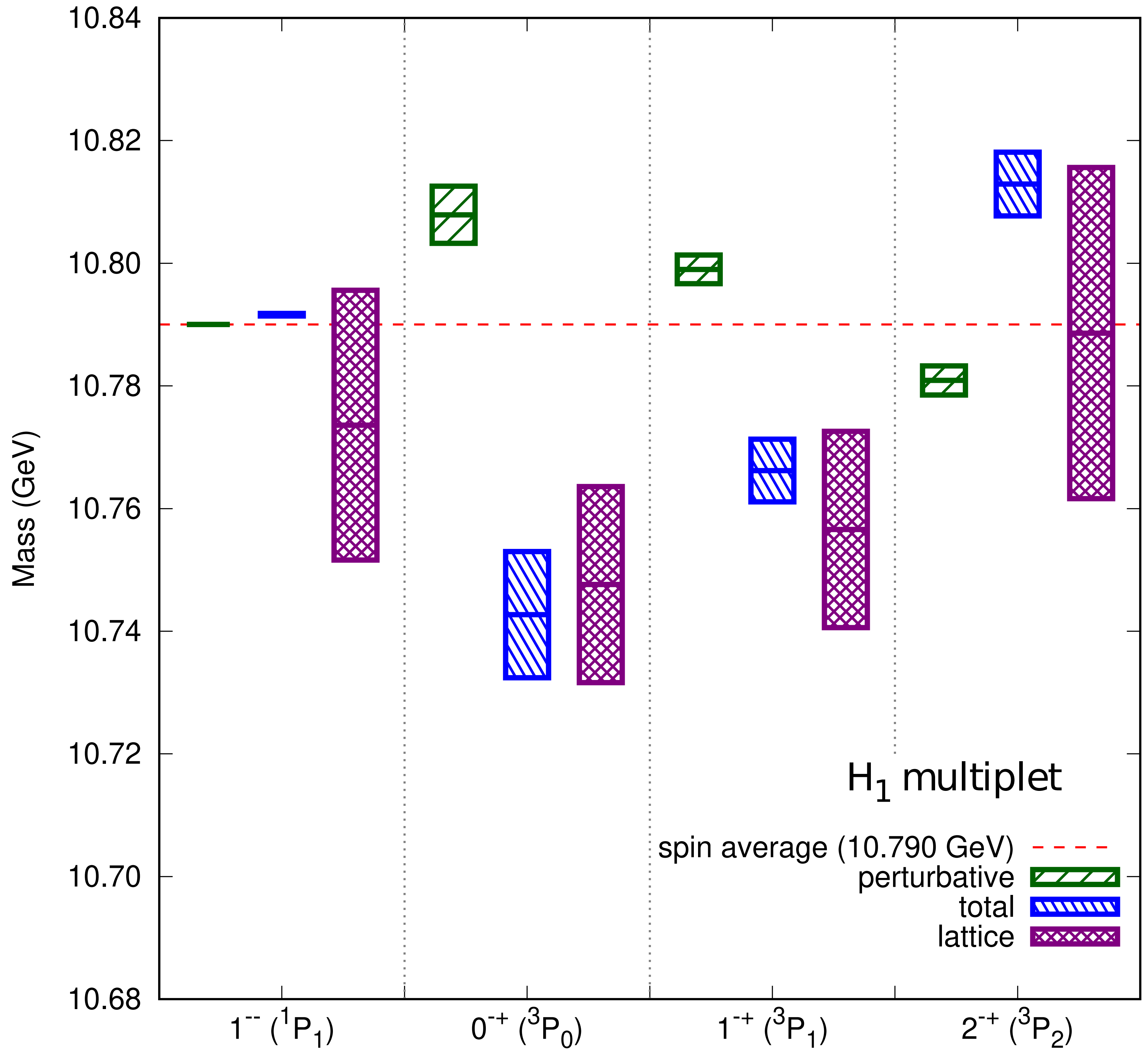
○ Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

Comparison of our prediction to the existing lattice data on H1

Bottomonium H_1 hybrid spin splittings



blue BOEFT predictions (more precise), violet actual lattice calculation

○ Ryan et al arXiv:2008.02656 [2+1 flavors, $m_\pi = 400$ MeV]
unpublished plot by J. Segovia and J. Tarrus

Spin effects

->difficult to insert in models

->this spin structure has huge impact in phenomenology : larger spin multiplets separation than in quarkonium

->less spin symmetry in decays due to quarkonium-hybrids mixing via a spin operator at 1/m

Oncala & Soto, Phys. Rev. D. 96, (2017)

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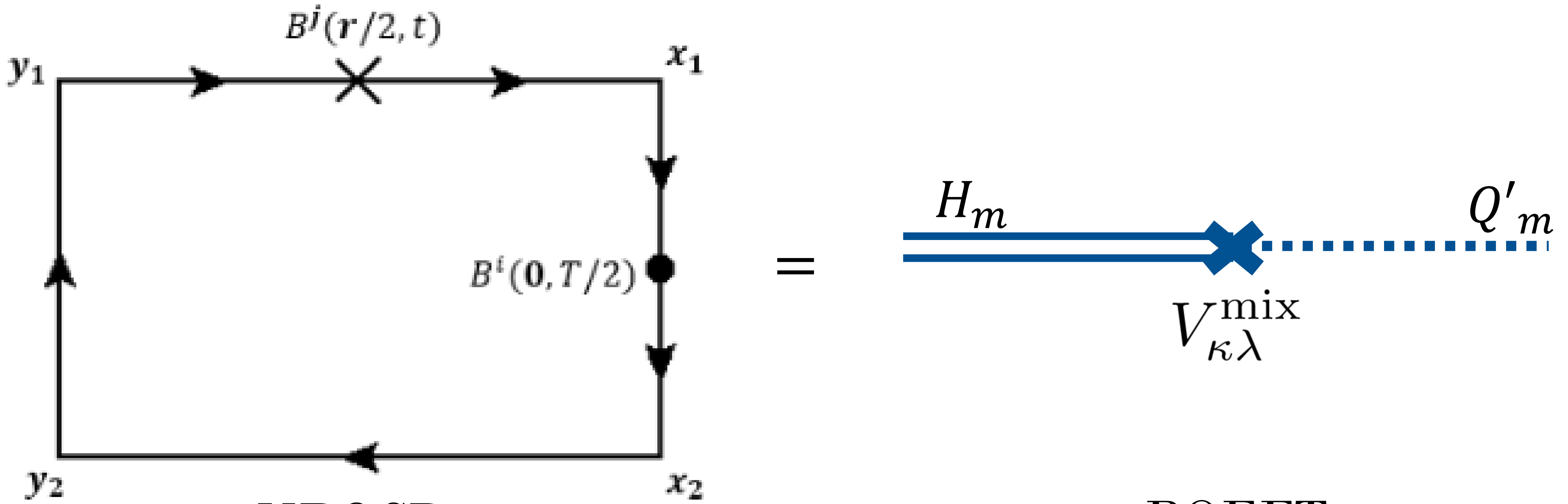
- Hybrid states in the same energy range and same quantum #'s as quarkonium can mix.

  Mixing impact spectrum and decay properties of hybrid. Implications on hybrid interpretation for exotics.

Ex. $H_1\left[1^{--}\right](4155) \leftrightarrow c\bar{c}\left[1^{--}\right](3S)$

Effect on decay: $H_m \leftrightarrow Q_m^{'} \rightarrow (\eta_c, J/\psi, \cdots) + (\gamma, \cdots)$

- Mixing potential $V_{\kappa\lambda}^{\text{mix}}$: determined from matching NRQCD and BOEFT at $O(1/m)$



NRQCD BOEFT

Expression after matching:

$$\boxed{V_{|\lambda|}^{\text{mix}} = -\frac{gc_F}{2m_Q}\,{}^{(0)}_{\lambda}\langle 1|B^j\left(\mathbf{r}/2, 0\right)|0\rangle^{(0)} P_\lambda^j,}$$

Above expression can be computed on lattice if we identify:

$$|0\rangle^{(0)} = |\Sigma_g^+\rangle$$

$$|1\rangle_{\lambda=0}^{(0)} = |\Sigma_u^-\rangle, |1\rangle_{|\lambda|=1}^{(0)} = |\Pi_u\rangle$$

BOEFT calculation of semi inclusive hybrids decays to quarkonium: H —> Quarkonium (S) plus X (anything light)

$$\Gamma_{H\to S} = -2\,\langle H|\mathrm{Im}\Delta V|H\rangle.$$

we calculated spin conserving and spin flipping decays they are same size

Decay to open threshold states not accounted

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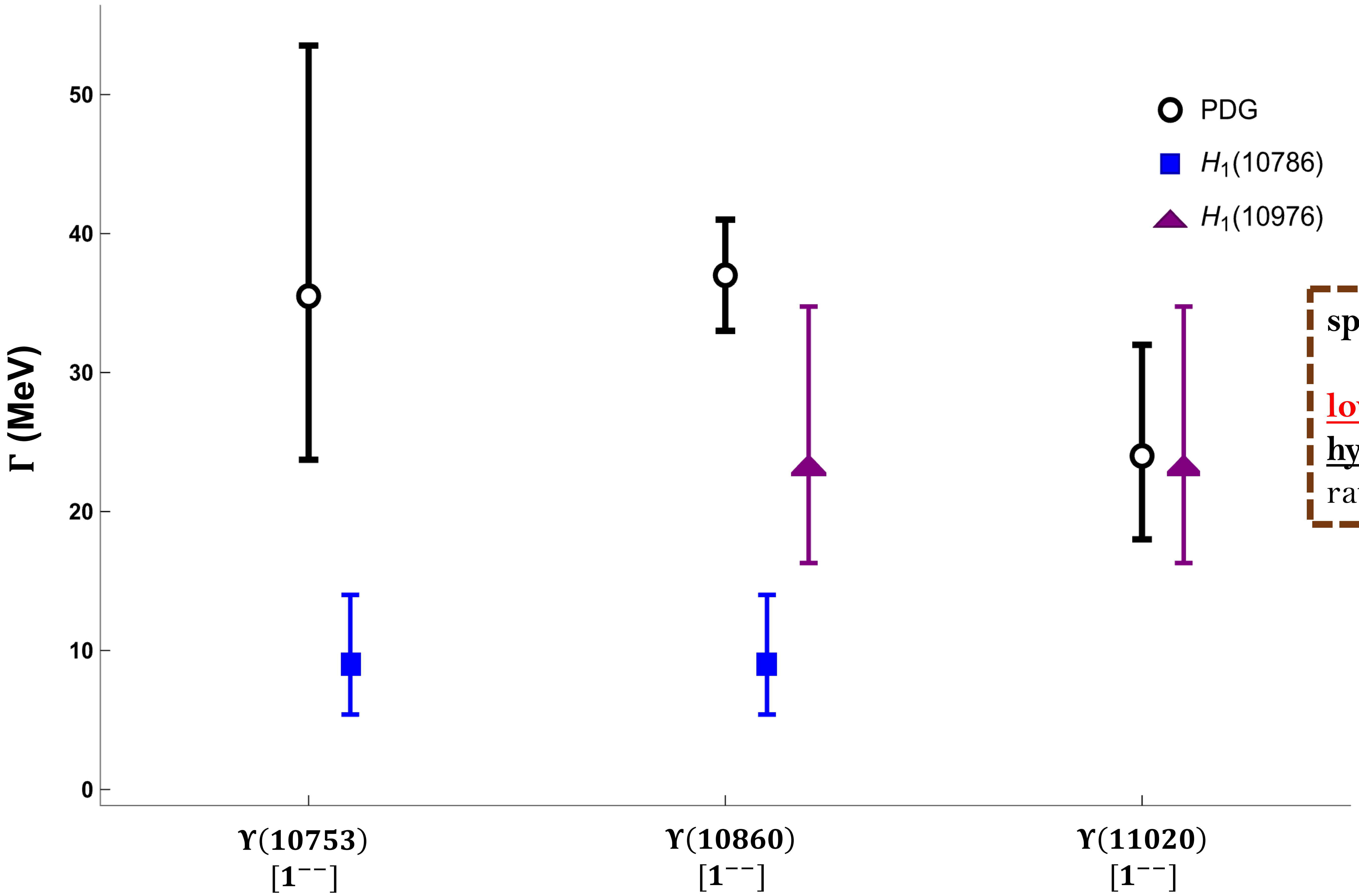
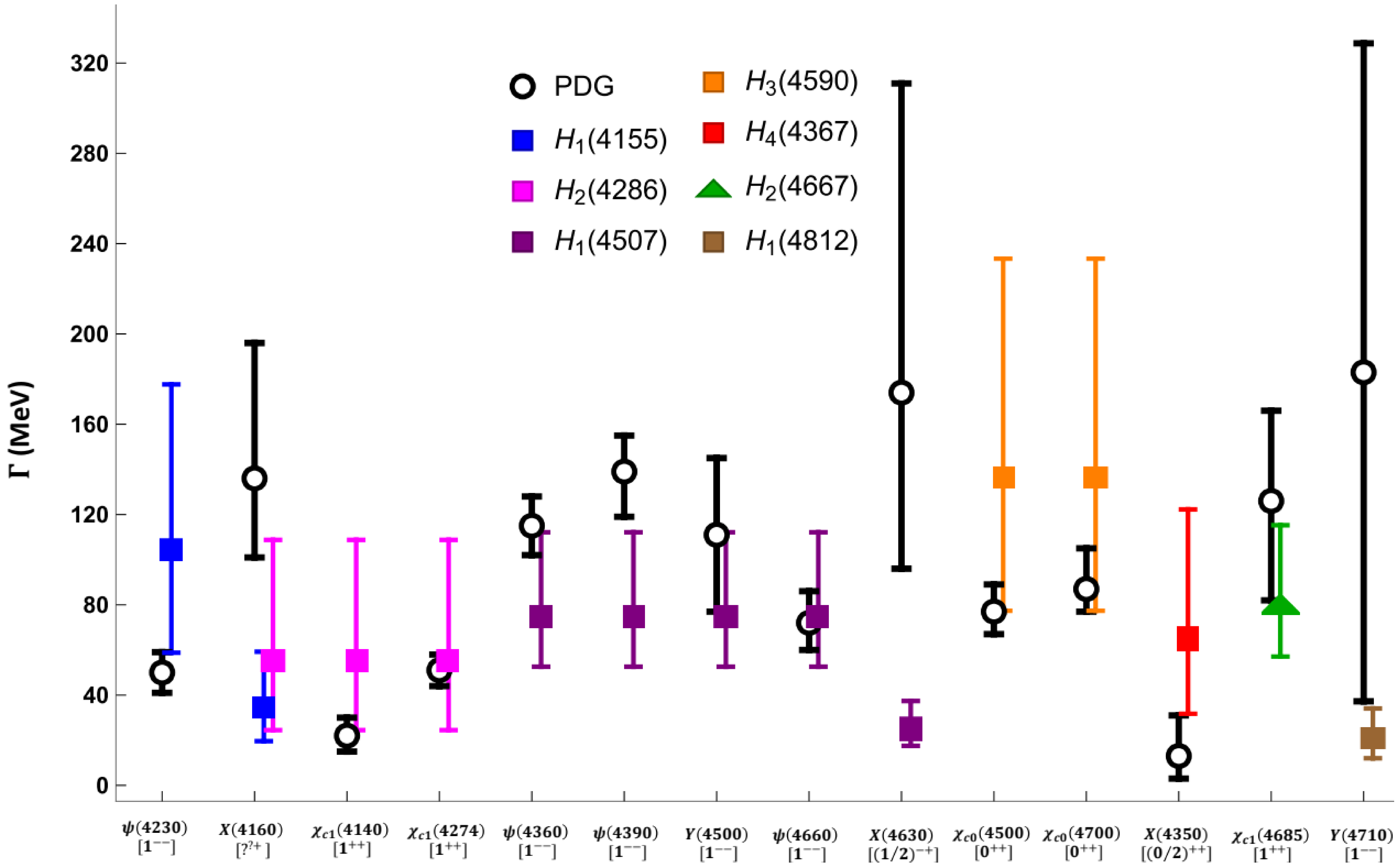
- Comparison: bottom exotic states with corresponding bottomonium hybrid state:



spin-conserving + spin-flipping decays
=
lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

BOEFT calculation of semi inclusive hybrids decays to quarkonium

Comparison: charm exotic states with corresponding charmonium hybrid state:



$$H_m \to Q_n + X$$

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Hybrid: Summary

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Hybrids ($Q\bar{Q}g$): Color singlet state of color octet $Q\bar{Q}$ + gluon. (Q = c, b)

✓ **Isoscalar neutral mesons (Isospin=0)**

✓ Candidates for hybrids based on **mass, quantum numbers,** and **decays** to **quarkonium**:

Charm sector:

- $X(4160)$: could be **charm hybrid** $H_1[2^{-+}](4155)$.
- $X(4630)$: could be **charm hybrid** $H_1[(1/2^{-+})](4507)$.
- $\psi(4390)$: could be **charm hybrid** $H_1[1^{--}](4507)$.
- $\psi(4710)$: could be **charm hybrid** $H_1[(1^{--})](4812)$.
- $X(4630)$: could be **charm hybrid** $H_1[(1/2^{-+})](4507)$.
- $\chi_{c1}(4685)$: could be **charm hybrid** $H_2[(1^{++})](4667)$.

Bottom sector:

- $\Upsilon(10753)$: could be **bottom hybrid** $H_1[(1^{--})](10786)$.

DISCLAIMER!!!

All the above interpretation can differ accounting for decays to meson-pair threshold states and hybrid-quarkonium mixing.

Hybrid Decays

Hybrid decays to meson-pair threshold states:

Conventional Wisdom: Hybrid decays to two S-wave mesons forbidden! $H_m \nrightarrow D^{(*)}\bar{D}^{(*)}$

Kou & Pene, Phys Lett B 631 (2005) **Page, Phys Lett B 407 (1997)** **Farina, Tecocoatzi, Giachino, Santopinto & Swanson, Phys Rev D 102 (2020)**

Born Oppenheimer quantum numbers for hybrids and ground state meson pair **does allow for decay to two s-wave mesons**. **Bruschini 2306.17120**

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Most quarkonium hybrids can decay into pair of s-wave mesons !

forbidden for decay into pair of s-wave mesons

Recent lattice computation for $c\bar{c}$ hybrid 1^{-+} **decay** to

$D_1\bar{D}$: 258(133) MeV

$D^*\bar{D}$: 88(18) MeV

$D^*\bar{D}^*$: 150(118) MeV

Shi et al 2306.12884

BOEFT for tetraquarks and pentaquarks

N.B. G. Krein, J. Tarrus, A. Vairo 1707.09647.
J. Tarrus 1901. 09761, J. Soto, J. Tarrus 2005.00552
N. B. A. Mohapatra, A. Vairo in preparation

BOEFT may be used to describe any system made by two heavy quarks bound adiabatically with some light quarks degrees of freedom (tetraquarks QQlight quarks, QQbar light quarks, pentaquarks)

In case of light quarks isospin quantum numbers should be added

Steps go as before:

—identify the symmetries, identify the interpolating operators O_n

$$\mathcal{O}_n(t,\,\boldsymbol{r},\,\boldsymbol{R}) = \chi(t,\,\boldsymbol{R}-\boldsymbol{r}/2)\phi(t,\,\boldsymbol{R}-\boldsymbol{r}/2,\boldsymbol{R})H_n(t,\,\boldsymbol{R})\phi(t,\,\boldsymbol{R},\boldsymbol{R}+\boldsymbol{r}/2)\psi^\dagger(t,\,\boldsymbol{R}+\boldsymbol{r}/2)$$

—define the static energies

$$E_n^{(0)}(r) = \lim_{T\to\infty}\frac{i}{T}\log\langle\mathcal{O}_n(T,\,\boldsymbol{r},\,\boldsymbol{R})|\mathcal{O}_n(0,\,\boldsymbol{r},\,\boldsymbol{R})\rangle$$

-obtain the coupled Schroedinger equations in BOEFT

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Notice:

-the perturbative part of the potentials can be calculated

-the structure of the spin corrections will be similar to the hybrids case (with a 1/m spin correction) calculation of decays will use the same technology

Tetraquarks QQbar case

N. B. A. Mohapatra, A. Vairo in preparation

$$\mathcal{O}^{Q\bar{Q}}_{\kappa,\lambda}(t,\,\boldsymbol{r},\,\boldsymbol{R}) = \chi^\dagger(t,\,\boldsymbol{x}_2)\phi(t,\,\boldsymbol{x}_2,\boldsymbol{R})H^{Q\bar{Q}}_{\kappa,\lambda}(t,\,\boldsymbol{R})\phi(t,\,\boldsymbol{R},\boldsymbol{x}_1)\psi(t,\,\boldsymbol{x}_1)$$

I=0

$$H_{0^{-+},0}(t,\boldsymbol{x}) = \left[\bar{q}(t,\boldsymbol{x})\gamma^5 T^a q(t,\boldsymbol{x})\right] T^a\,,$$

$$H_{1^{--},0}(t,\boldsymbol{x}) = \left[\bar{q}(t,\boldsymbol{x})\,(\hat{\boldsymbol{r}}\cdot\boldsymbol{\gamma})\,T^a q(t,\boldsymbol{x})\right] T^a\,,$$

$$H_{1^{--},\pm 1}(t,\boldsymbol{x}) = \left[\bar{q}(t,\boldsymbol{x})\,(\hat{\boldsymbol{r}}\times\boldsymbol{\gamma})\,T^a q(t,\boldsymbol{x})\right] T^a$$

$$E_{\kappa,\Lambda}\left(r\right) = V_o\left(r\right) + \underbrace{\Lambda_{H_\kappa}}_{\text{Adjoint meson}} + \mathcal{O}\left(r^2\right)$$

$Q\bar{Q}$ color state	Light spin $\boldsymbol{K}^{PC}$	Static energies	l	J^{PC} $\{S_Q=0, S_Q=1\}$	Multiplets
Octet	0^{-+}	$\{\Sigma_u^-\}$	0	$\{0^{++}, 1^{+-}\}$	T_1^0
			1	$\{1^{--}, (0,1,2)^{-+}\}$	T_2^0
			2	$\{2^{++}, (1,2,3)^{+-}\}$	T_3^0
	1^{--}	$\{\Sigma_g^{+\prime}, \Pi_g\}$	1	$\{1^{+-}, (0,1,2)^{++}\}$	T_1^1
		$\{\Sigma_g^{+\prime}\}$	0	$\{0^{-+}, 1^{--}\}$	T_2^1
		$\{\Pi_g\}$	1	$\{1^{-+}, (0,1,2)^{--}\}$	T_3^1
		$\{\Sigma_g^{+\prime}, \Pi_g\}$	2	$\{2^{-+}, (1,2,3)^{--}\}$	T_4^1

Coupled schroedinger eqs set up in BOEFT, need lattice input on the static tetra energies

Tetraquarks and Pentaquarks: lattice input needed

Tetraquark static energies

I=1 S. Prevlosek, H. Bahtiyar, J. Petrovich eprint: 1912.02656

I=0 Bicudo Cichy Peters Wagner PRD 93 (2016) 034501

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Mixing between quarkonium, hybrids; hybrids, tetraquarks

Preliminary studies N. B. , Schlosser, Wagner, Vairo

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Cross talk with the heavy light static energies

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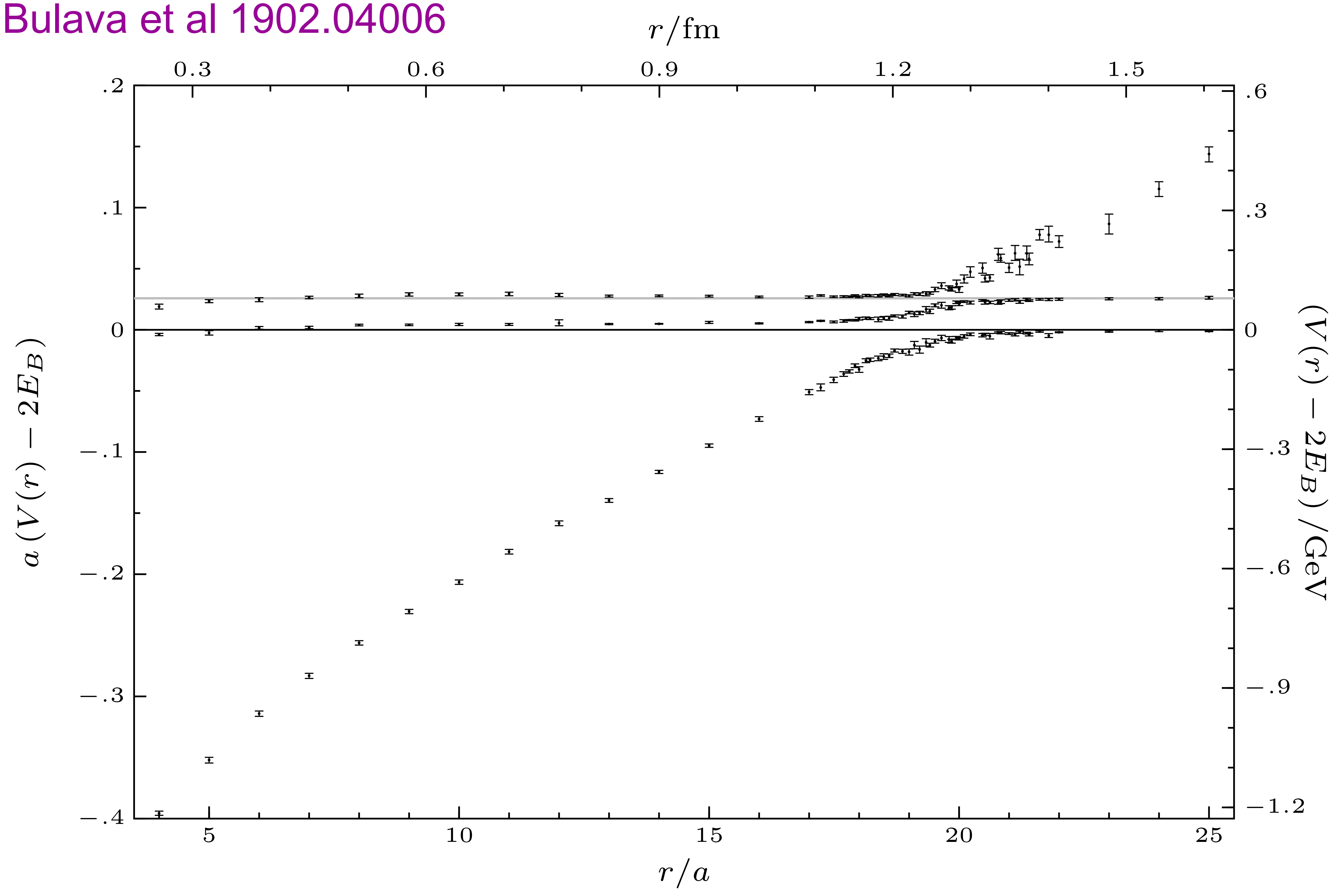
Adiabatic energy levels of the static energy of quarkonium and heavy-light, heavylight strange-> avoided level crossing

Bulava et al 1902.04006



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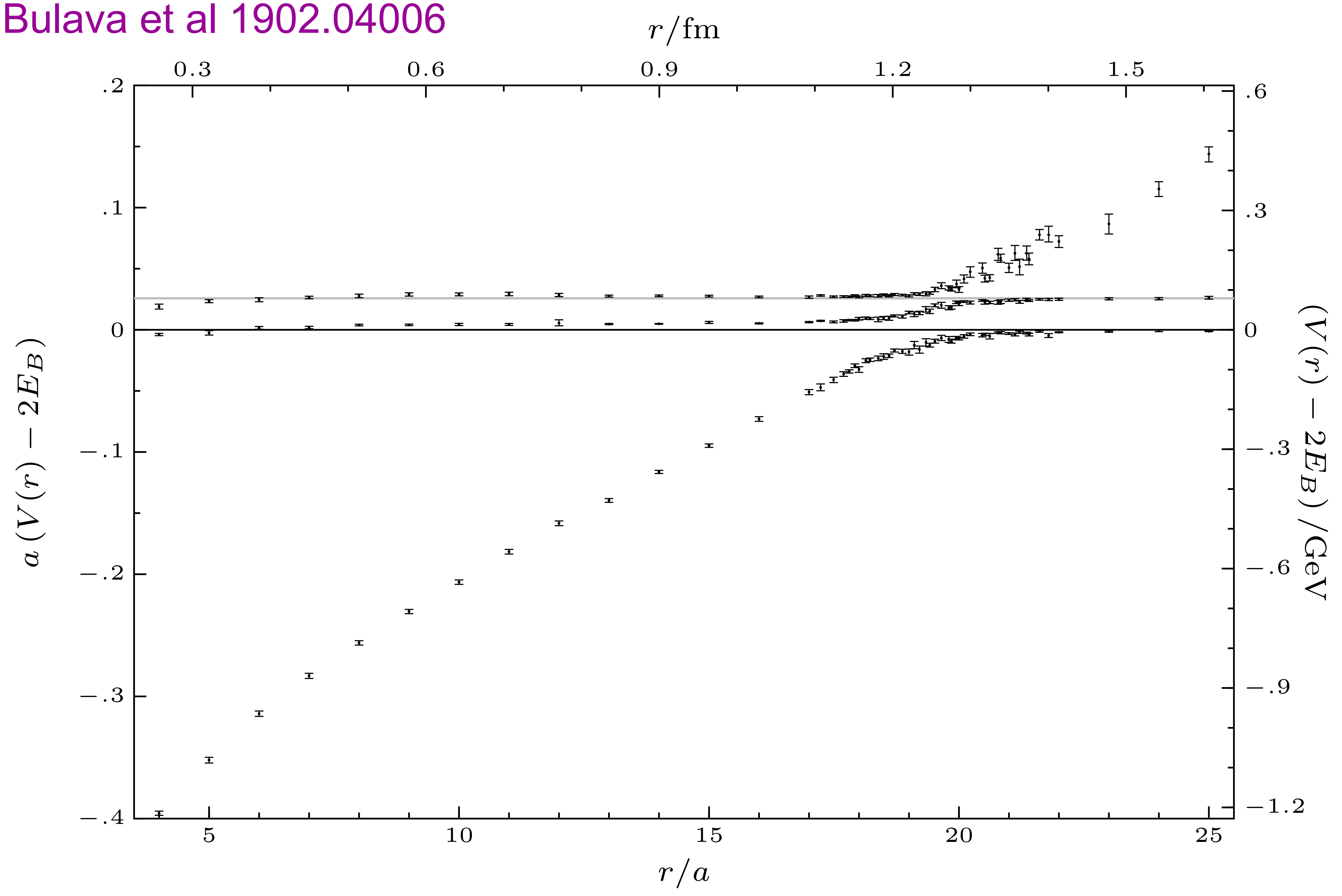

In the diabatic picture gives the coupling between quarkonium and heavy light states allowing to solve the coupled schoedinger eqs and determine the amount of quarkonium and molecular states

$$\begin{pmatrix} \hat{V}(r) & g_1 & g_2 \\ g_1 & \hat{E}_1 & 0 \\ g_2 & 0 & \hat{E}_2 \end{pmatrix}.$$

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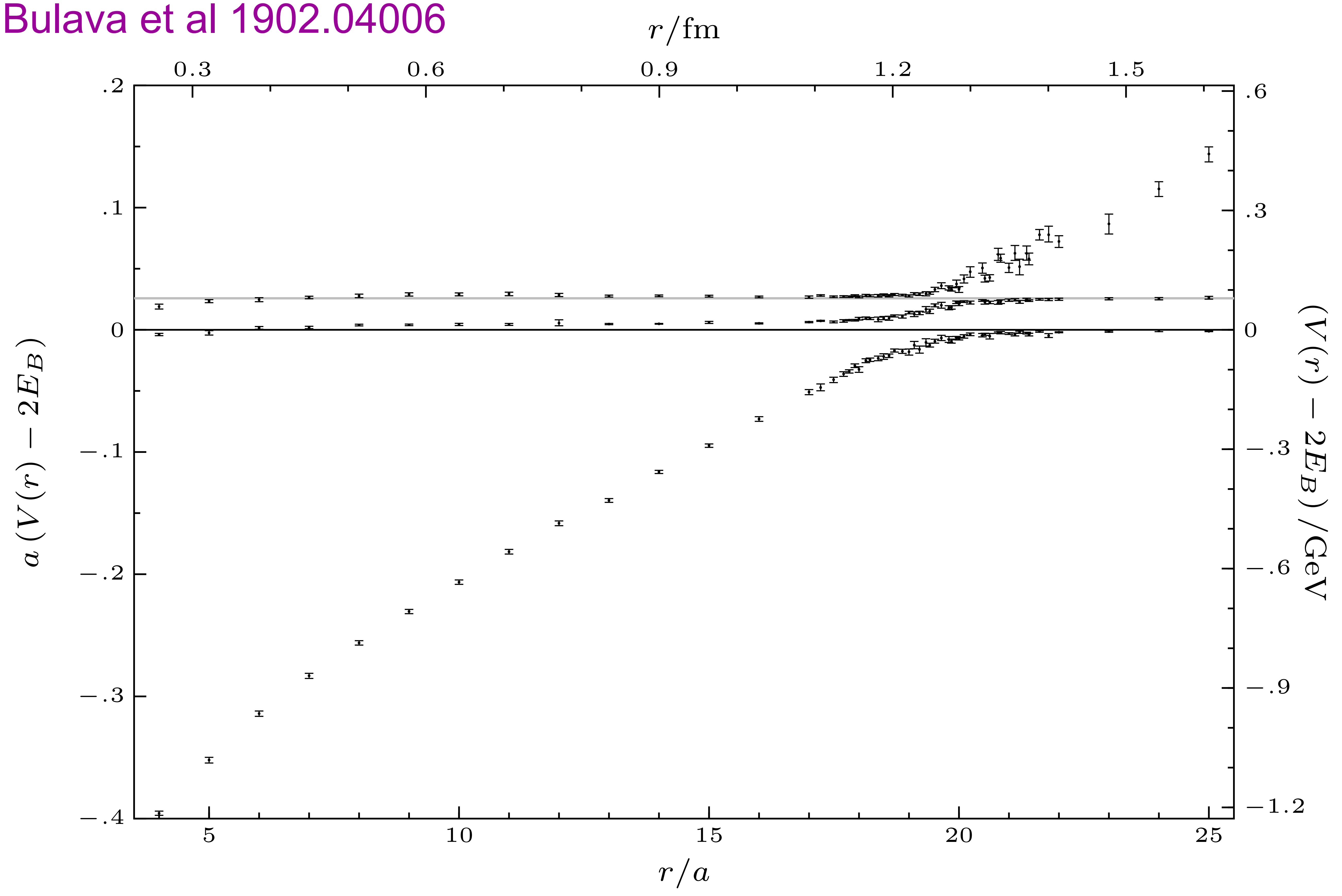

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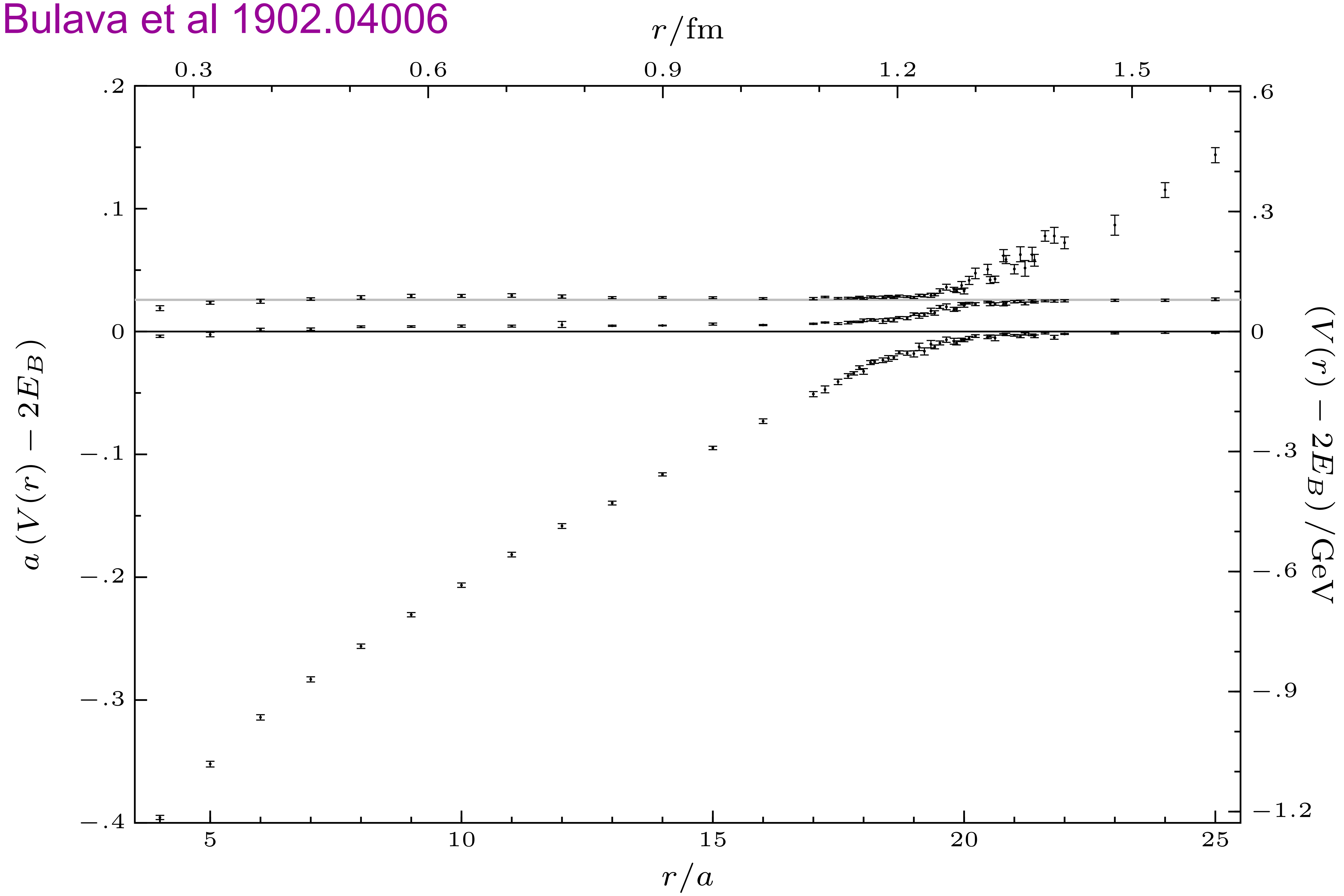

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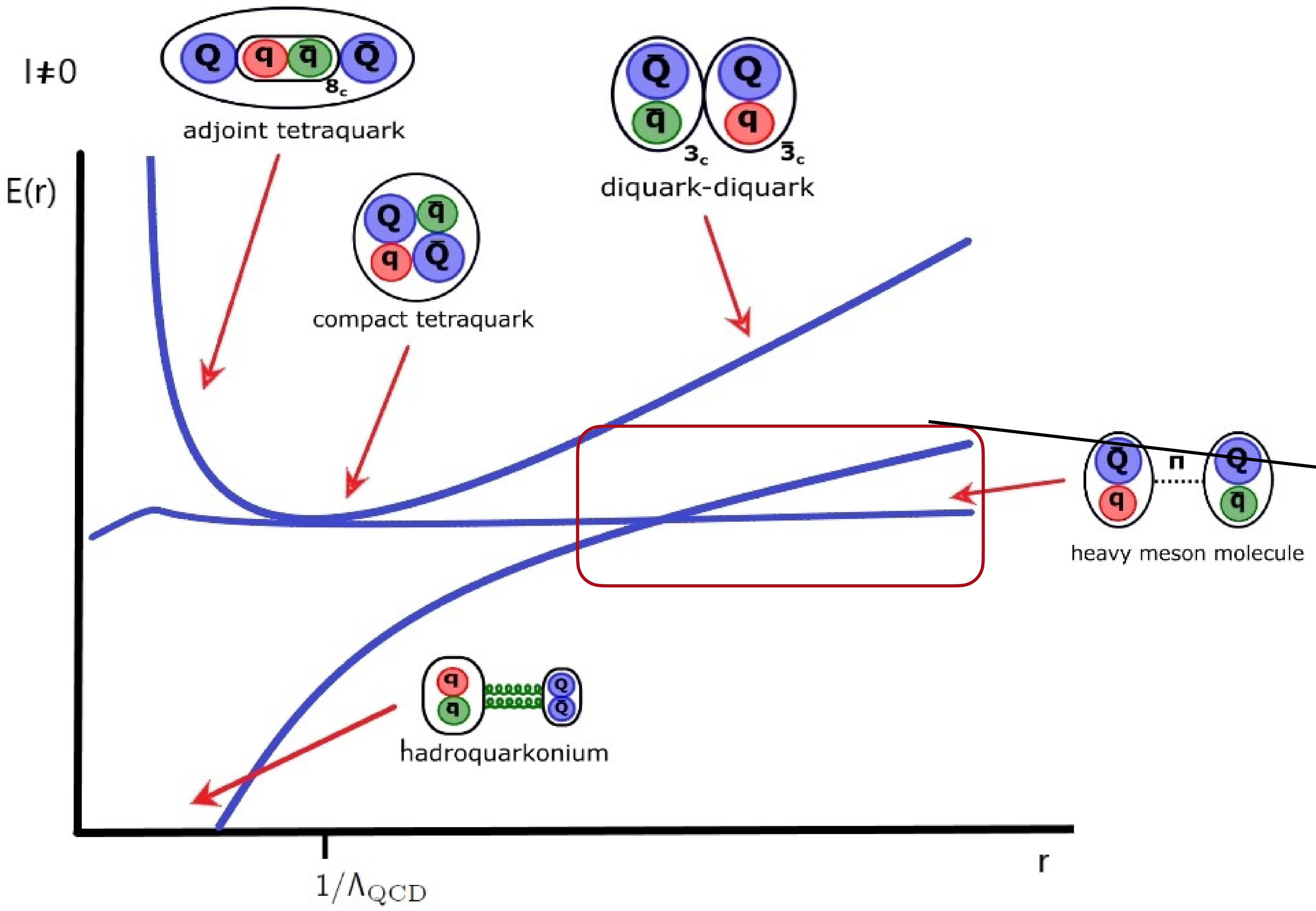
In this way special states with strong molecular components and characteristics like the X(3872) can be originated In BOEFT

Bruschini, Gonzalez
20172111.07653

The BOEFT contains all models: what dominates and where depends on the QCD dynamics

Static energies for $I \neq 0$ (schematic):



avoided crossing of the energy levels, mixing with open flavour meson-meson configurations

Bruschini, Gonzalez 2021

The static energies are defined in BOEFT that gives the appropriate set of operators to be used and could describe the short distance limit.
Being nonperturbative objects E(r) should be calculated on the lattice (or in QCD vacuum models)

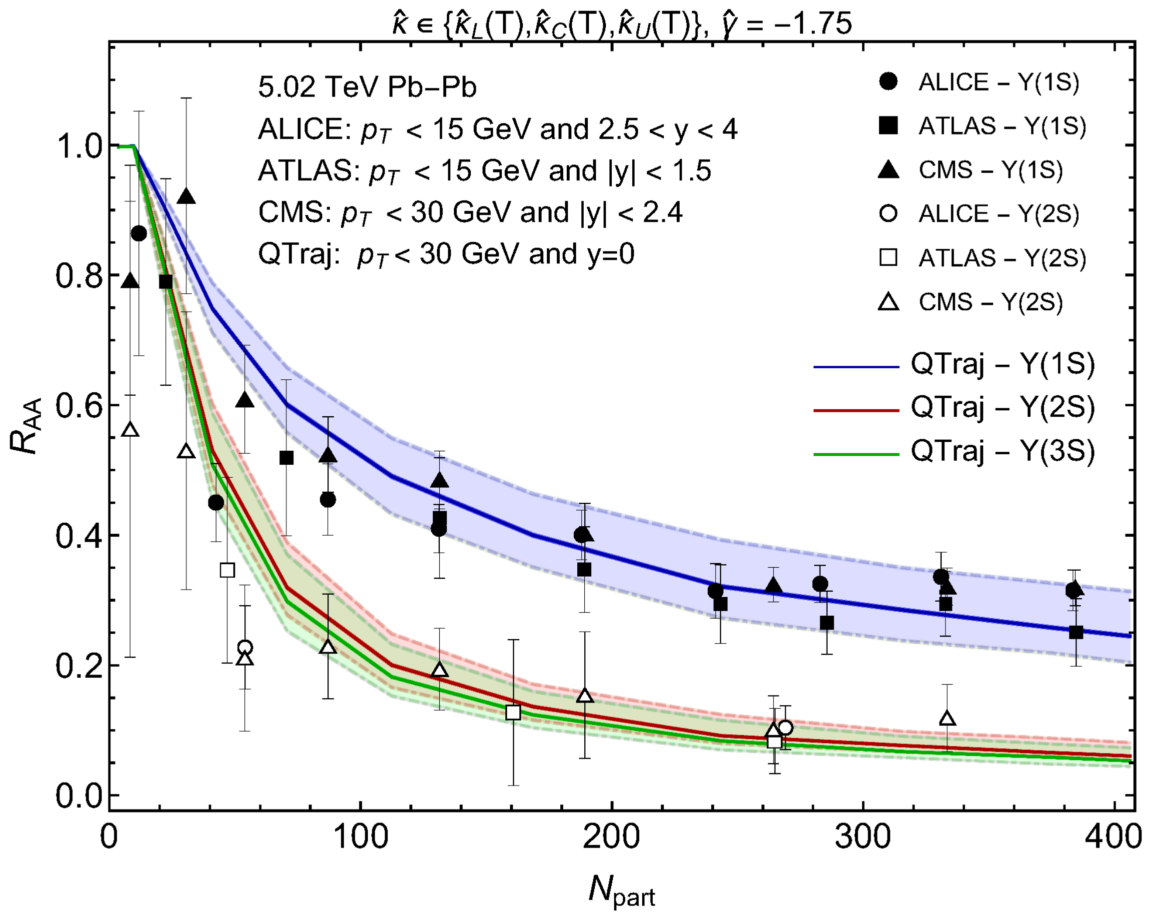
Figure from J. Tarrus

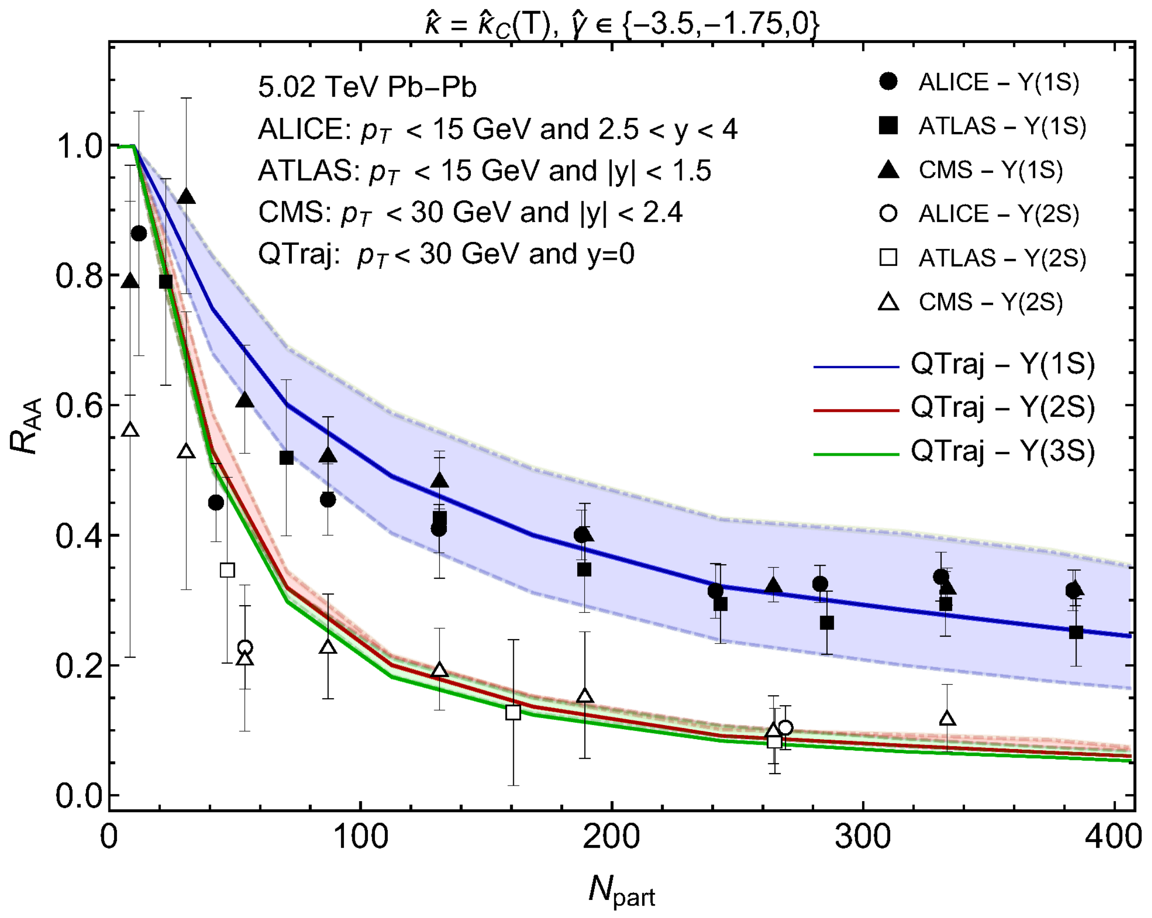
XYZ production and evolution in medium: may be studied with the EFT tools developed for quarkonium

Bottomonium Nuclear Modification factor

can be obtained using pNRQCD at finite temperature, density matrix,and open quantum systems

$$R_{AA}(nS) = \frac{\langle n, \mathbf{q} | \rho_s(t_F; t_F) | n, \mathbf{q} \rangle}{\langle n, \mathbf{q} | \rho_s(0; 0) | n, \mathbf{q} \rangle}$$





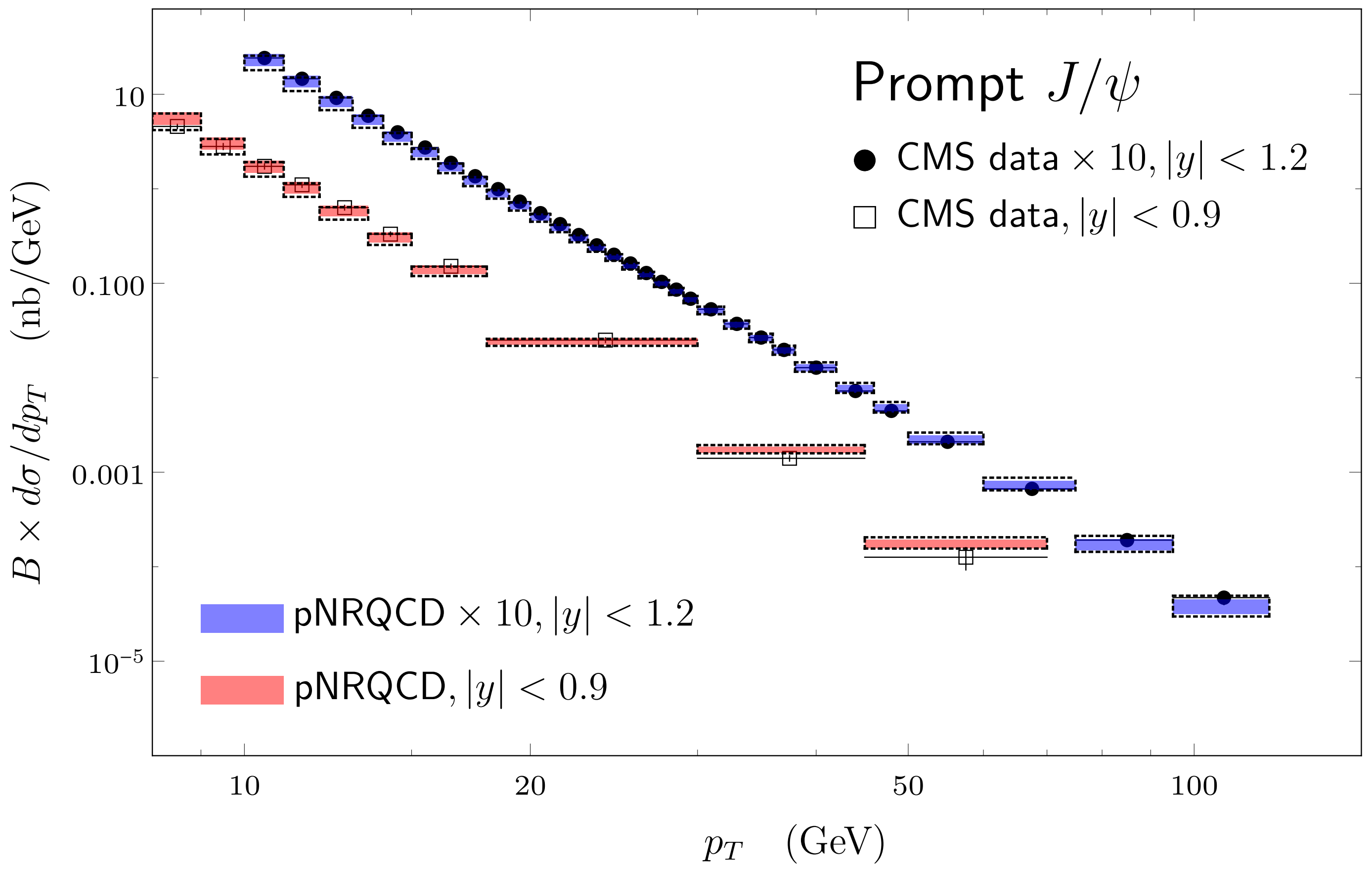
Bands are from the dependence in kappa and gamma parameters

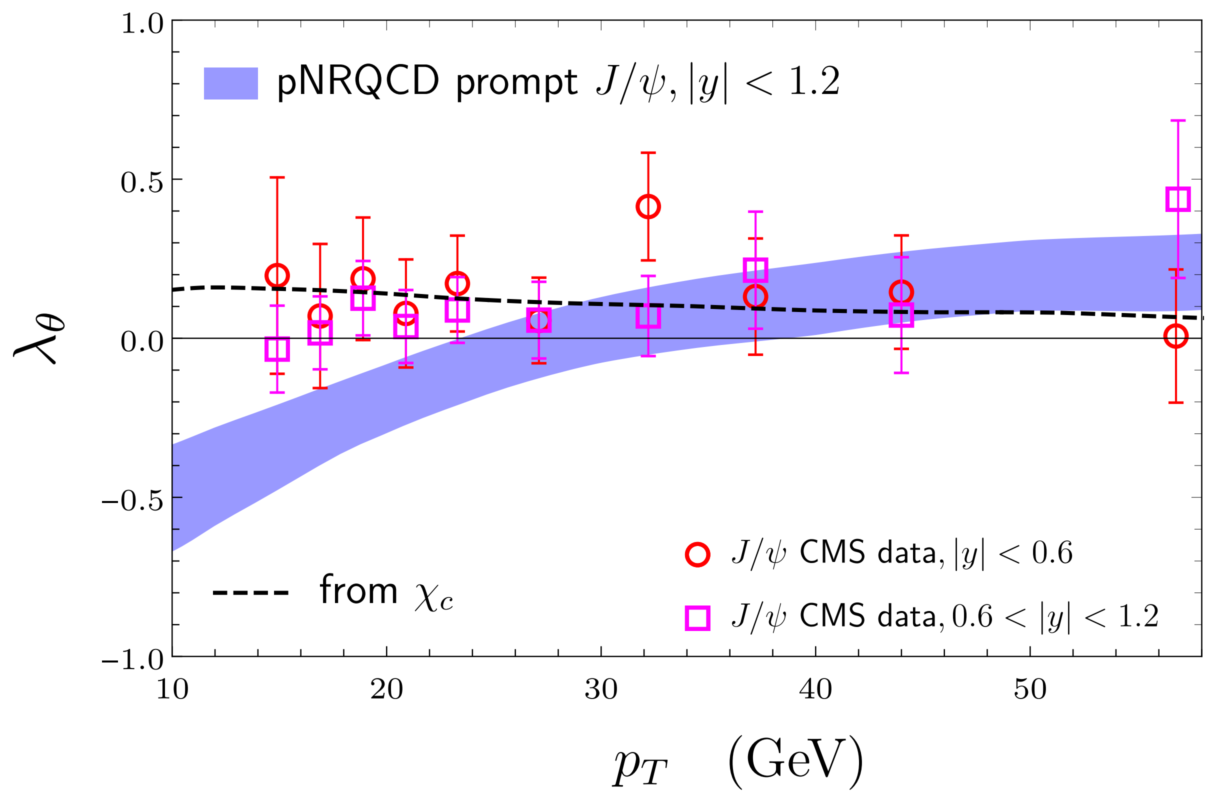
N.B., M. Escobedo, M. Strickland, A. Vairo P. VanderGriend et al

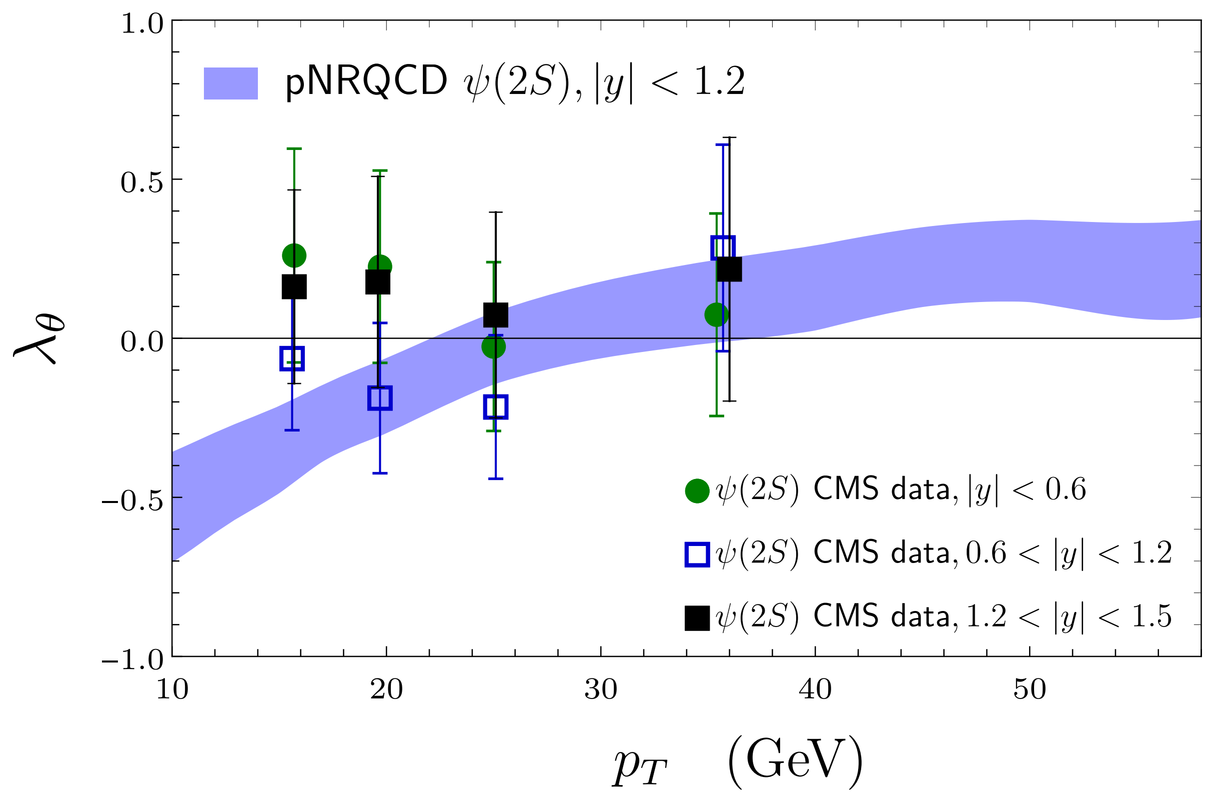
arXiv:2302.11826
arXiv:2107.06222
arXiv:1711.04515
arXiv:2205.10289

Quarkonium production can factorized and calculated in pNRQCD

N.B., Chung, Vairo, Wang 2210.17345







Outlook

NREFTs and lattice allows us to describe the physics of quarkonium away from the strong decay threshold in quantum field theory: higher order perturbative calculation can be performed and quarkonium can be used for precision physics/ factorisation allows to systematically study confinement

BOEFT allows to describe hybrids: new unexpected features are found (Lambda doubling, Spin structure, decays, mixing) that have important impact on the phenomenology

BOEFT allows to describe hybrids and calculate multiplets, mixing and decays: on going work

The same picture can be extended to tetraquarks and pentaquarks, once some lattice input on relevant correlators will be available.

NOTICE that the needed lattice calculations are simpler than the direct calculations of the X Y Z properties on the lattice, the knowledge of few correlators together with the BOEFT will allow to obtain many phenomenological information

NREFTs and lattice and open quantum system allows us to describe the nonequilibrium evolution quarkonium in the quark gluon plasma and production processes: same theory could be then used for XYZ production and evolution in medium in heavy ion collisions

This picture has the possibility to give a unified description to exotics and to leave the dynamics decide which configuration will dominate in a given range and can be tested and help to guide new discoveries at future tau-charm facilities

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BOEFT

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LATTICE

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Lattice calculation of the heavy quark transport coefficient

Fully heavy Tetraquarks

Tetraquarks	Order	$M_{Q\bar{Q}Q\bar{Q}}$ [GeV]	$B_{Q\bar{Q}Q\bar{Q}}$ [MeV]
$T_{cc\bar{c}\bar{c}}$	LO	6.1276(3)	16.6(4)
	NLO	6.078(2)	67.9(1)
	NNLO′	6.018(3)	144(2)
$T_{cc\bar{c}\bar{b}}/T_{bc\bar{c}\bar{c}}$	LO	9.294(3)	23.0(4)
	NLO	9.312(4)	72(2)
	NNLO′	9.259(5)	139(2)
$T_{bb\bar{c}\bar{c}}/T_{cc\bar{b}\bar{b}}$	LO	12.503(1)	23.7(4)
	NLO	12.457(4)	79(2)
	NNLO′	12.386(3)	157(3)
$T_{bc\bar{b}\bar{c}}$	LO	12.471(5)	19.5(8)
	NLO	12.417(5)	69(2)
	NNLO′	12.354(6)	139(2)
$T_{bb\bar{b}\bar{c}}/T_{bc\bar{b}\bar{b}}$	LO	15.652(6)	27.9(7)
	NLO	15.50(2)	87(2)
	NNLO′	15.37(7)	169(4)
$T_{bb\bar{b}\bar{b}}$	LO	18.8693(5)	31.2(6)
	NLO	18.8207(6)	83.6(1)
	NNLO′	18.7598(6)	151(1)

TABLE II. Predictions for tetraquark masses and binding energies for all combinations of tetraquarks involving only b and c quarks at each order of pNRQCD indicated. Pairs of tetraquarks in the same row have identical binding energies in our calculations due to charge conjugation.

Assi and Wagman 2311.01498

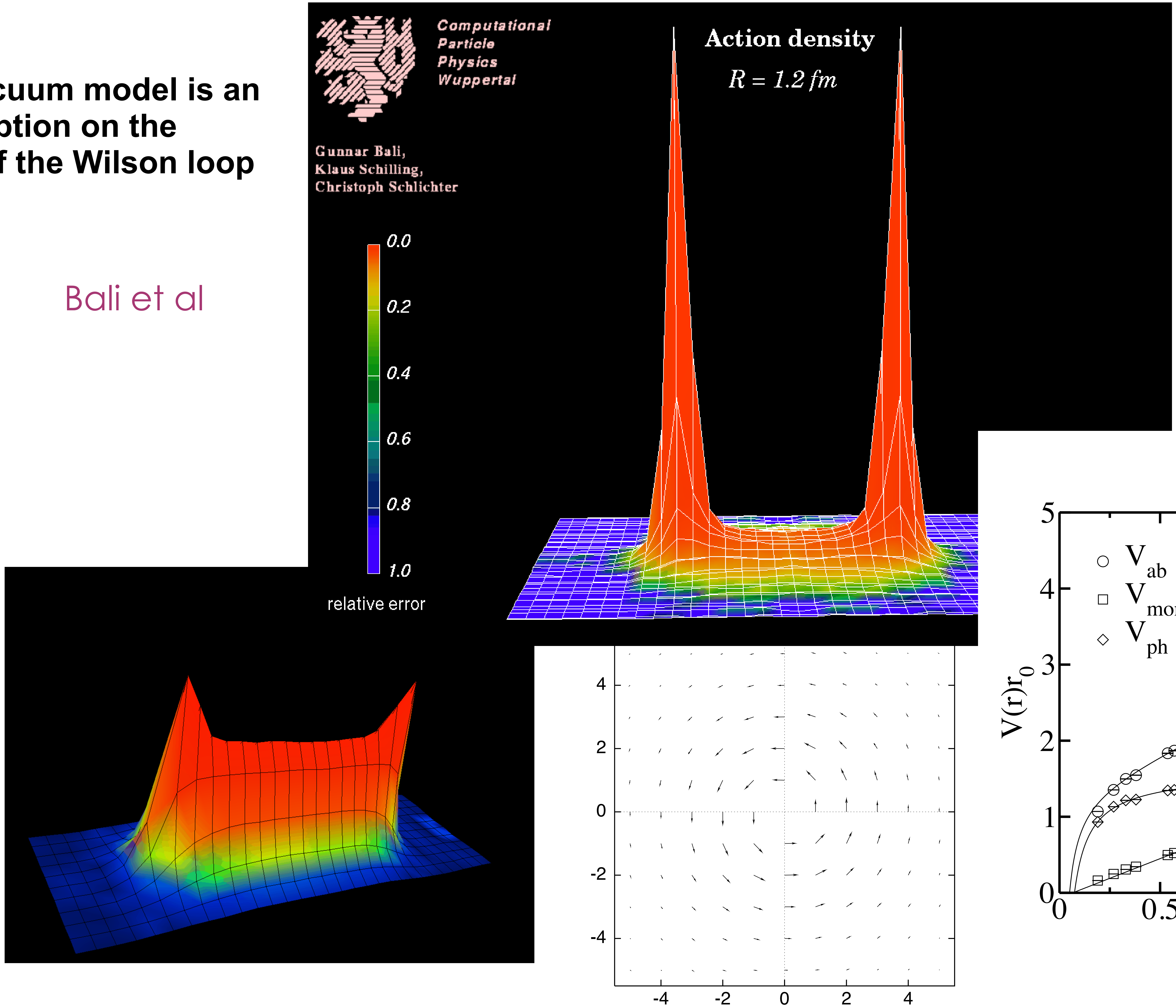
Variational and Green function Monte Carlo method based on Weakly coupled pNRQCD potential calculated at LO NLO and NNLO'
(prime means only two body forces are considered)

Decays may be calculated in the same framework

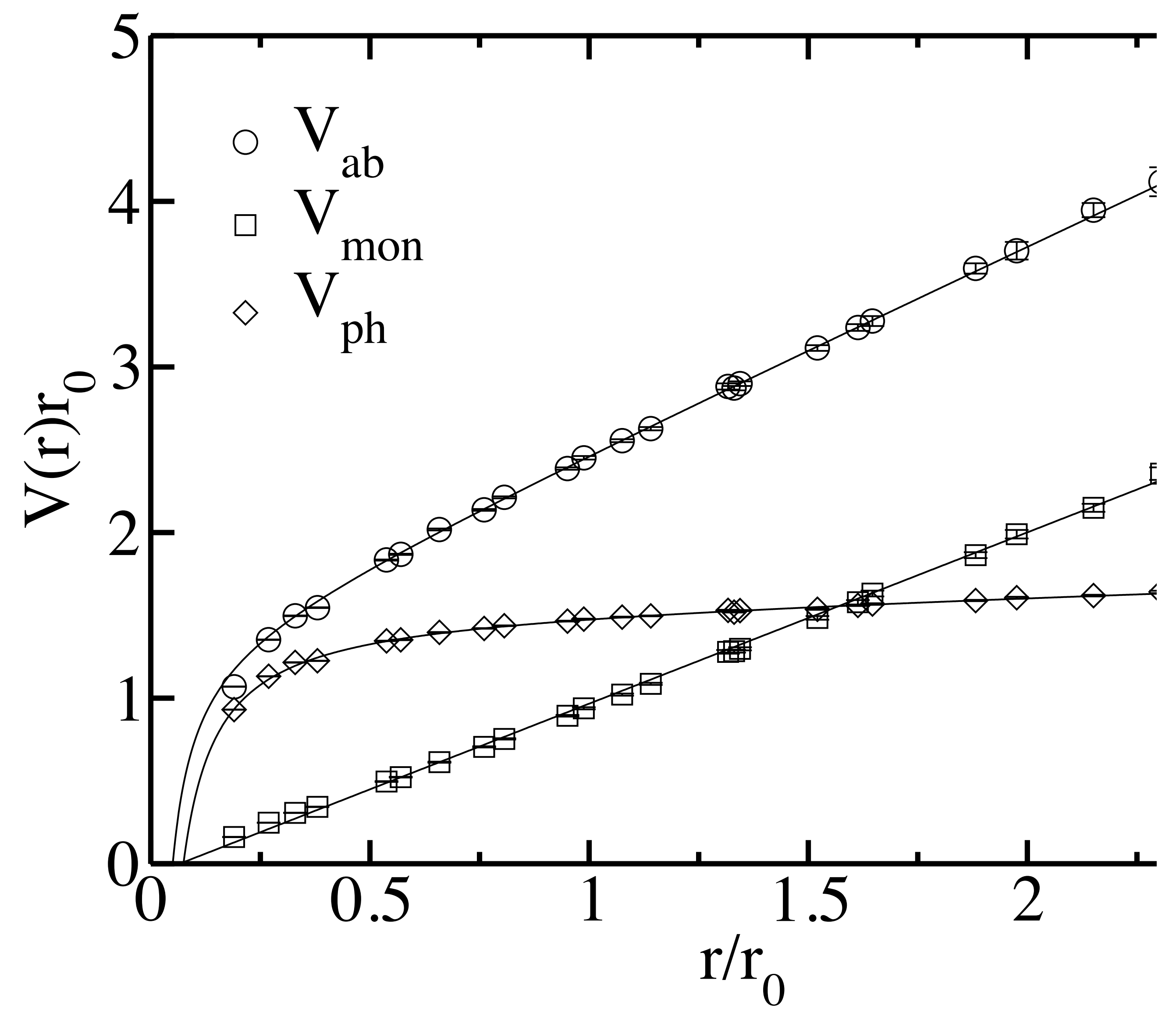
Low energy physics factorized in Wilson loops: can be used to probe the confinement mechanism

any QCD vacuum model is an assumption on the behaviour of the Wilson loop

Bali et al



Boryakov et al. 04



Spectrum: general consideration

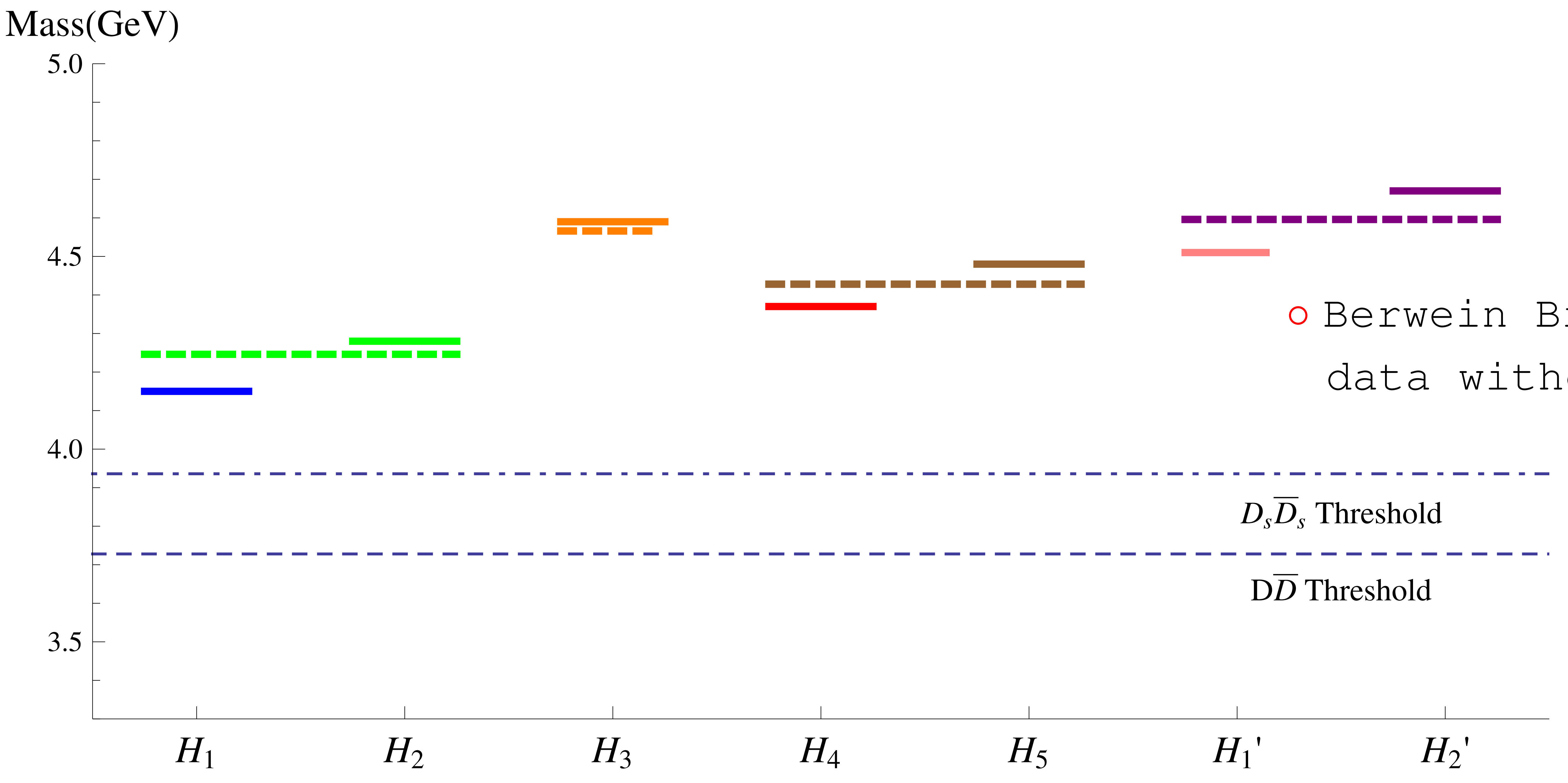
Multiplet	T	$J^{PC}(S=0)$	$J^{PC}(S=1)$	E_Γ
H_1	1	1^{--}	$(0,1,2)^{-+}$	$E_{\Sigma_u^-}$, E_{Π_u}
H_2	1	1^{++}	$(0,1,2)^{+-}$	E_{Π_u}
H_3	0	0^{++}	1^{+-}	$E_{\Sigma_u^-}$
H_4	2	2^{++}	$(1,2,3)^{+-}$	$E_{\Sigma_u^-}$, E_{Π_u}

Spin degenerated

Spectrum: with mixing and Λ-doubling

- The Schrödinger equation mixes states with the same parity.
  A consequence is Λ-doubling, i.e., the lifting of degeneracy between states with opposite parity. This happens also in molecular physics, however, there Λ-doubling is a subleading effect, while it is a LO effect in the quarkonium hybrid spectrum.
- The eigenstates are organized in the multiplets H_1, H_2, ... Neglecting off-diagonal terms, the multiplets H_1 and H_2 would be degenerate.
- We compute the spectrum using quark masses in the renormalon subtraction (RS) scheme: $m_{c\,\mathrm{RS}} = 1.477(40)$ GeV and $m_{b\,\mathrm{RS}} = 4.863(55)$ GeV.

  The gluelump masses, which enter in the normalization of the hybrid potentials, have been computed in the same scheme and assigned an uncertainty of ± 0.15 GeV, which is the largest source of uncertainty in the hybrid masses.



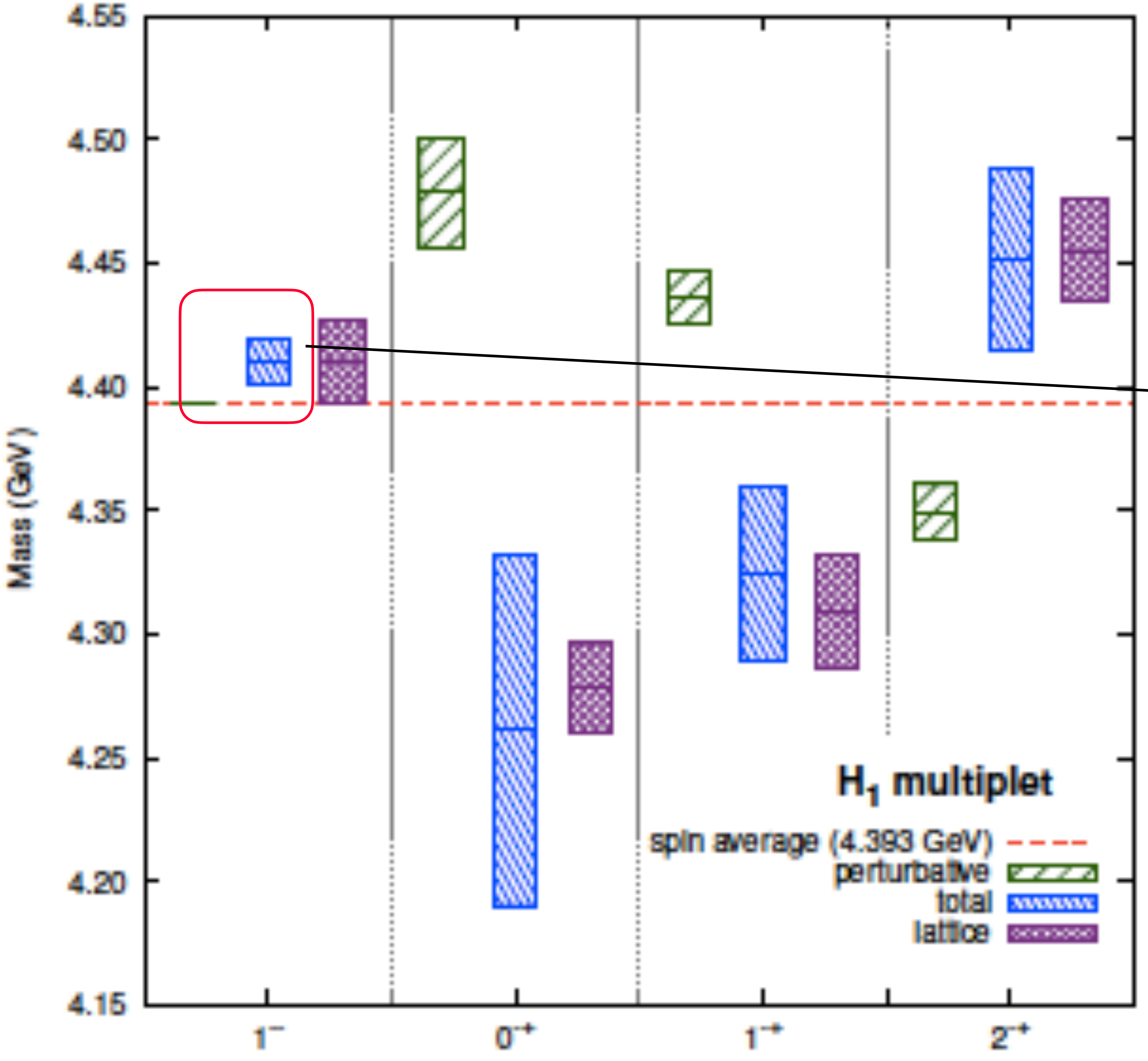
charmonium hybrids

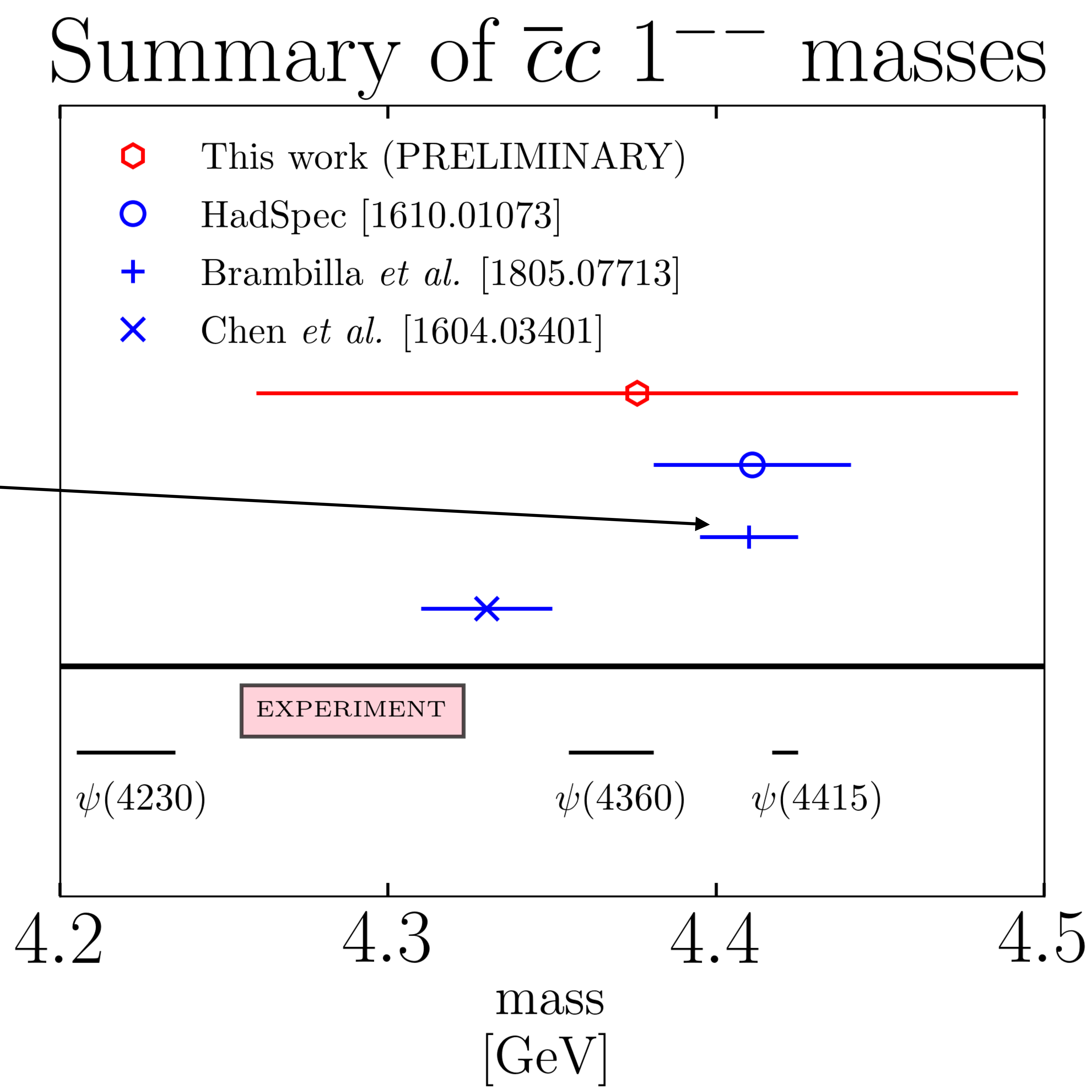
Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
data without mixing (dashed) from Braaten et al PRD 90 (2014)

Without Lambda-doubling masses of opposite parity states are degenerate

Charmonium Hybrids Multiplets H_1

HISQ lattice action with 2+1+1 sea quarks





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