ON ELEMENTARY AND COMPOSITE PARTICLES: THE CASE OF EXOTIC HADRONS.

AD POLOSA, SAPIENZA UNIVERSITY OF ROME

THE $X(3872)$ TUNING

- If interpreted as a molecule, the $X(3872)$ is thought to be a $D^0\bar{D}^{*0}$ bound state, with $J^{PC} = 1^{++}$ and $B \lesssim 100$ keV.
  Such a small value of B makes the X an outlier wrt to other X, Y, Z states.

- There must be a tuning of the strong interactions in the $D\bar{D}^*$ system ("molecule") making a large (and positive) so that $B = 1/(2ma^2) \sim 0$.

- Most of the states are found within 10-20 MeV from meson-meson thresholds – most with central values above threshold but within Γ.

THE RADIATIVE DECAYS OF $X(3872)$

$$\mathcal{R} \equiv \frac{\mathcal{B}(X \to \gamma\,\psi(2S))}{\mathcal{B}(X \to \gamma\,\psi(1S))} \simeq 6 \pm 4 \quad \text{(PDG)}$$

The phase space ratio $\Phi(2S)/\Phi(1S) \simeq 0.26$ would favor a small $\mathcal{R}$ – which is still possible with the numbers given above.

We assume that X has no significant charmonium component and we distinguish between a **compact** $c\bar{c}q\bar{q}$ and a **molecular** $D\bar{D}^*$ interpretation.

We find that $\mathcal{R}$ predicted in the compact case is (at least) 30 times larger, $\mathcal{R} \gtrsim 1$, than that predicted for a molecule, $\simeq 0.04$.

B. Grinstein, L. Maiani, A.D. Polosa, to appear soon.

THE RADIATIVE DECAYS OF $X(3872)$

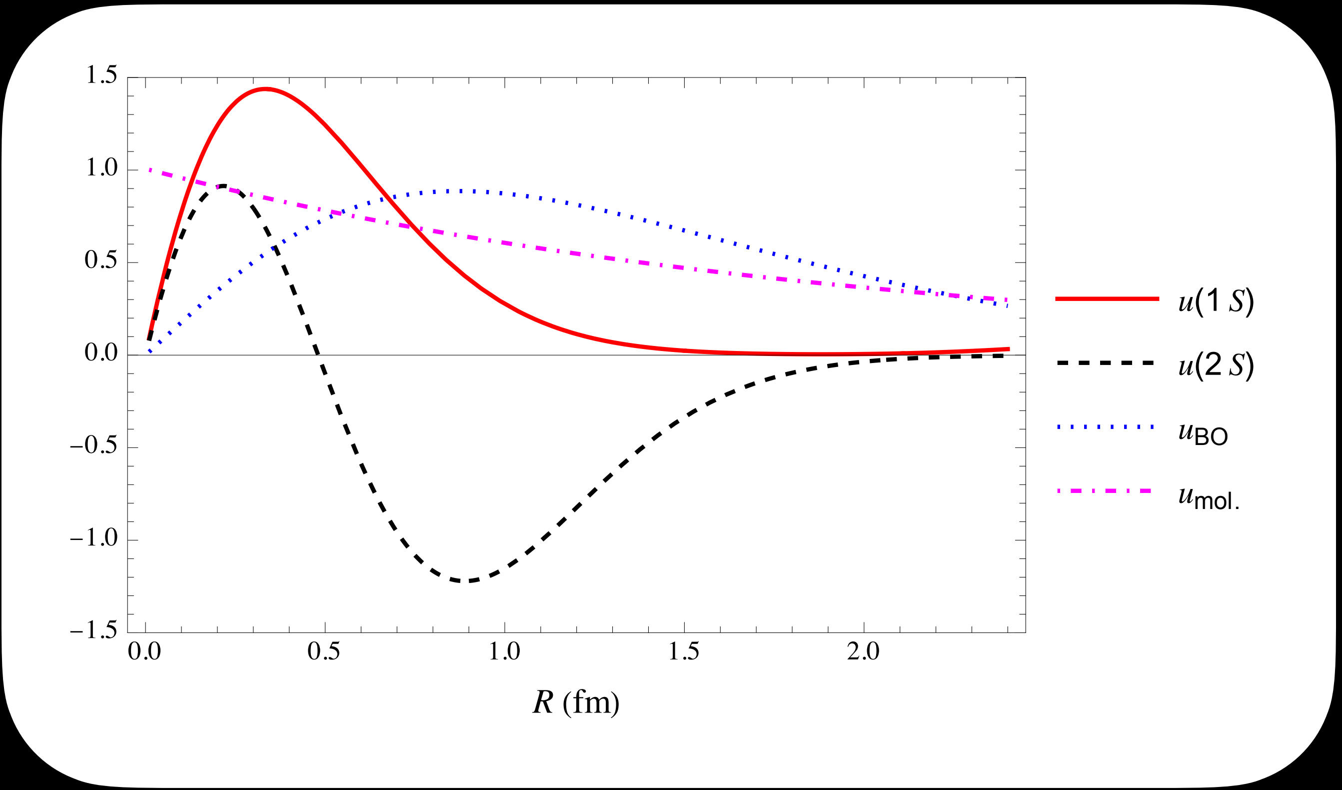
- The universal wavefunction used in the molecular picture amplifies small distances enhancing the J/ψ wrt $\psi(2S)$ – the latter has a larger spatial extent than the former.

- The diquark in the compact tetraquarks tends to be larger than a D or a $\bar{D}^*$ meson since the binding force is weaker.
  We also find $\mathscr{R}$ to be a rapidly increasing function of the size of the $D^{(*)}$ mesons.

- The large size $R \sim 10$ fm of the molecule has a minor role.

B. Grinstein, L. Maiani, A.D. Polosa, to appear soon.

- The universal wavefunction in the molecular picture amplifies small distances enhancing the J/ψ wrt $\psi(2S)$.



Reduced wavefunctions $u(r) = rR(r)$

B. Grinstein, L. Maiani, A.D.P., to appear soon.

THE UNIVERSAL WAVEFUNCTION $\psi_{\text{mol.}}(r)$

By **universal** w.f., we mean it does not depend on the details of the potential. For small $\boldsymbol{B}$ it is expected to be broader than the potential range, so a $\boldsymbol{\lambda\,\delta^3(r)}$ potential might be used to find it

$$\psi_{\text{mol.}}(r) = \left(\frac{2mB}{4\pi^2}\right)^{1/4} \frac{\exp(-r\sqrt{2mB})}{r}$$

E. Braaten and M. Kusunoki, PRD69, 074005 (2004)

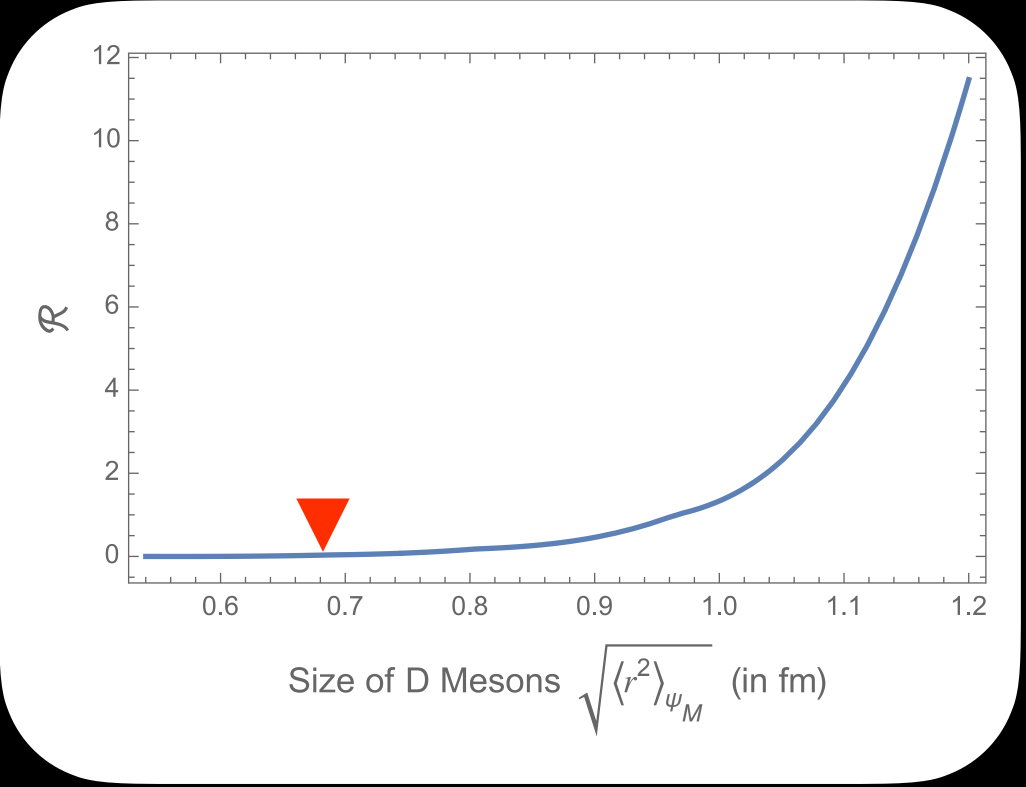
Corresponds to the $\boldsymbol{E = -B}$ bound state wf of the $\boldsymbol{\lambda\,\delta^3(r)}$ potential with the (renormalized) coupling

$$\lambda = \frac{2\pi}{m\sqrt{2mB}}$$

R. Jackiw, `Diverse topics in Theoretical and Mathematical Physics`, World Scientific

THE RADIATIVE DECAYS OF $X(3872)$

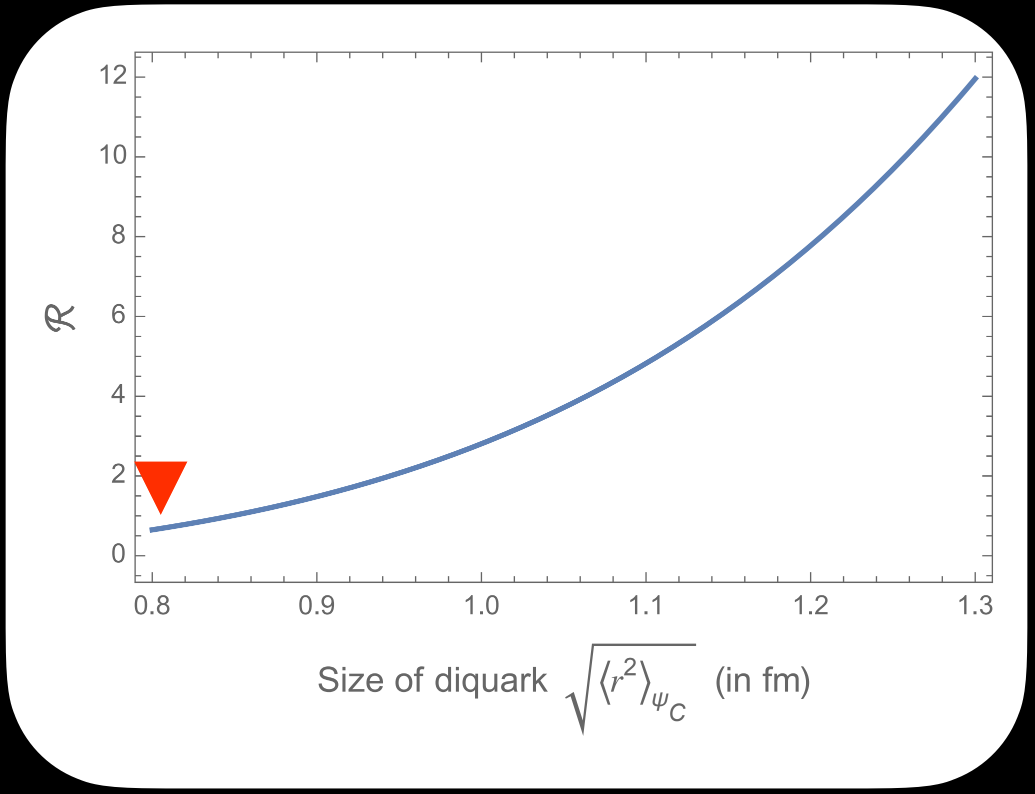
- We find $\mathscr{R}$ to be a rapidly increasing function of the size of $D^{(*)}$.



B. Grinstein, L. Maiani, A.D.P., to appear soon.
Isgur, Scora, Grinstein, Wise (ISGW-model)

THE RADIATIVE DECAYS OF $X(3872)$

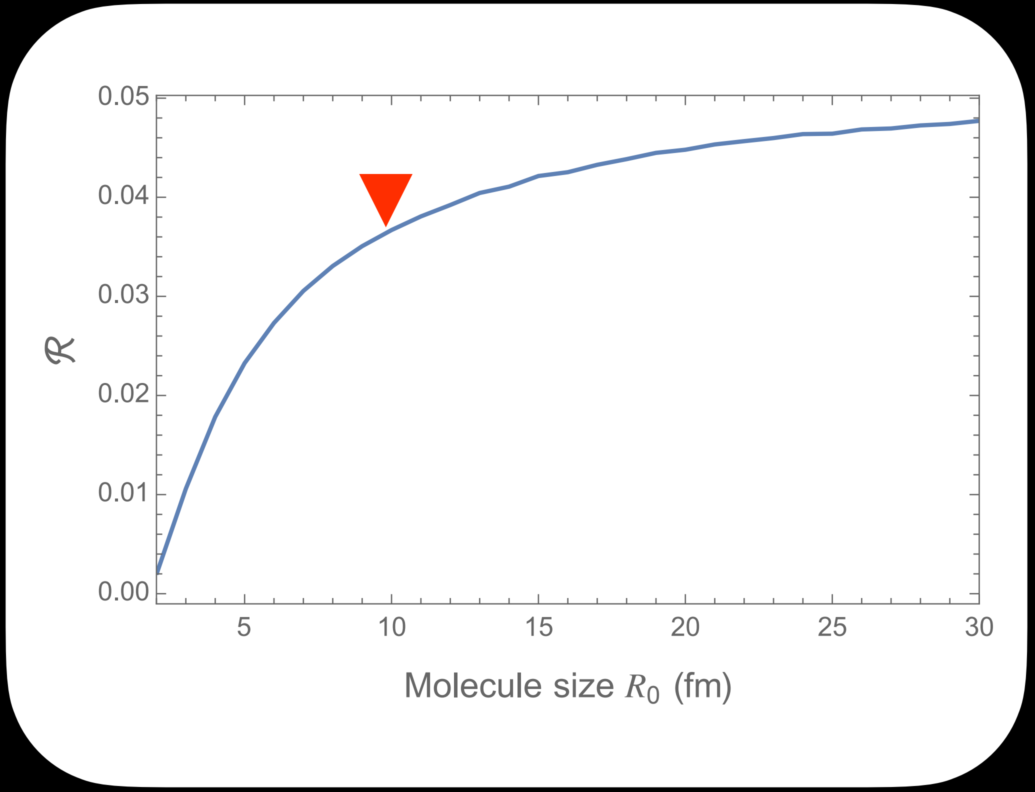
- The diquark in the compact tetraquarks tends to be larger than a D or a $\bar{D}^*$ meson since the binding force is weaker.



B. Grinstein, L. Maiani, A.D.P., to appear soon.

THE RADIATIVE DECAYS OF $X(3872)$

- The large size $R_0 \sim 10$ fm of the molecule has a minor role. We find in any case $\mathscr{R}_{\text{compact}} > \mathscr{R}_{\text{mol.}}$ with $\mathscr{R}_{\text{compact}} > 1$.



$$R_0 = 1/\sqrt{2mB}$$

B. Grinstein, L. Maiani, A.D.P., to appear soon.

RELATIVE MOMENTA IN MOLECULES

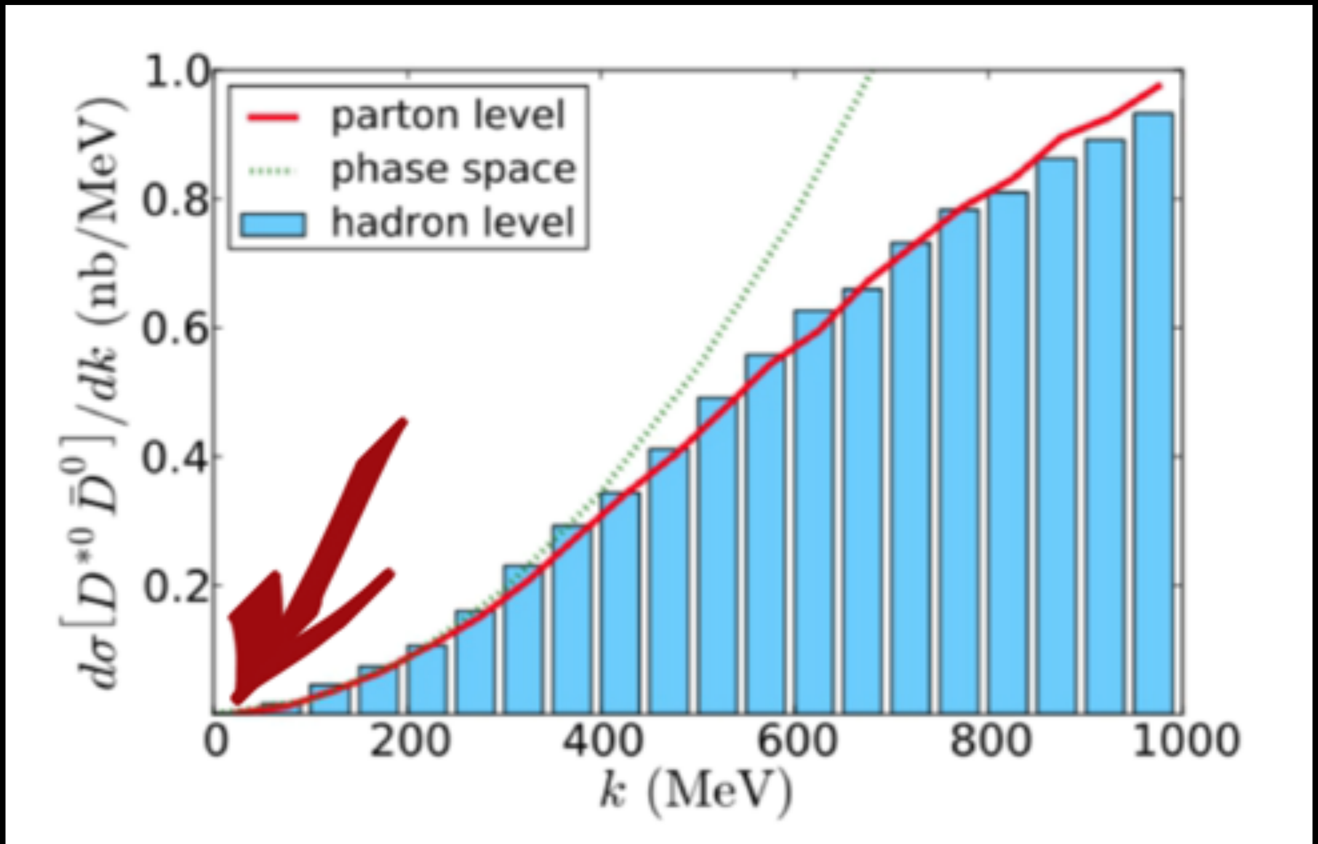
$$V(r) = \lambda \frac{\delta(r)}{4\pi r^2} \Rightarrow \langle V \rangle_{\psi_{\text{mol.}}} = 0$$

$$-B = \langle H \rangle = \langle T \rangle + \langle V \rangle = \langle T \rangle = \frac{\langle k^2 \rangle}{2m}$$

$$\langle k^2 \rangle = -\,2mB$$

as can also be computed directly by $\psi_{\text{mol.}}$ This corresponds to the pole in the shallow bound state scattering amplitude $k = i\sqrt{2mB}$ (pole in $E + B$). Here $|k| = \sqrt{2mB} \simeq 14$ MeV

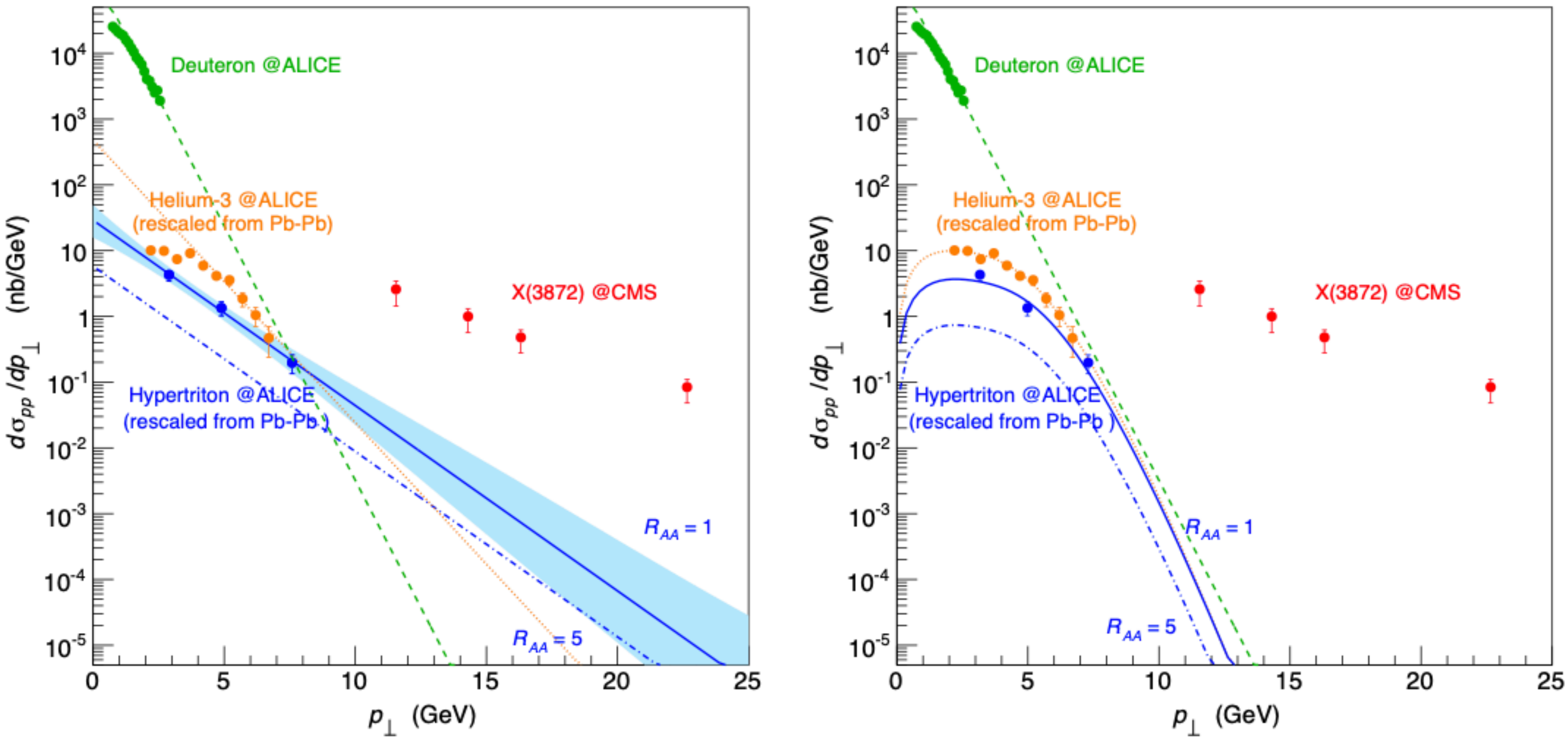
RELATIVE MOMENTA IN MOLECULES



Braaten and Artoisenet, PRD81103 (2010) 114018

Bignamini, Grinstein, Piccinini, ADP, Sabelli, PRL103 (2009) 162001

THE X BY A $c\bar{c}$ CORE



Esposito, Guerrieri, Maiani, Piccinini, Pilloni, ADP, Riquer, *Phys. Rev. D* 92 (2015) 3, 034028

THE X BY A $c\bar{c}$ CORE

The typical conclusion is that the hadronization into X proceeds via the production of a $c\bar{c}$ pair (e.g. recoiling a gluon)

- In the compact picture we have in mind here a $(c\bar{c})_8(q\bar{q})_8$ state is formed and its dynamics could be described in the Born-Oppenheimer approximation (fast light quarks and slow heavy quarks). This explains the use we did before of a ψ_{BO}.

- According to others the $c\bar{c}$ core combines with light quarks evolving in a $D\bar{D}^*$ loosely bound molecule.

$D\bar{D}^*$ SCATTERING

From scattering theory it is known that the scattering amplitude of the molecule constituents has a pole at a shallow level $\boldsymbol{E = -B}$ with $\boldsymbol{B > 0}$ (if $\boldsymbol{E = -B}$ is on the non-physical sheet one speaks of *virtual state*) with

$$f = -\frac{A_0^2}{2m}\frac{1}{E+B}$$

with the *reduced* normalized wf (the universal wf discussed above) of the corresponding stationary state

$$\chi = A_0 \exp(-r\sqrt{2mB})$$

$D\bar{D}^*$ SCATTERING

Indeed using (suggested from the $\delta^3(r)$ potential)

$$\frac{A_0^2}{2m} = \frac{\sqrt{2mB}}{m}$$

$$f(\alpha \to \beta) = -\frac{A_0^2}{2m}\frac{1}{E+B} = -\frac{\sqrt{2mB}}{m}\frac{1}{E+B}$$

This is obtained by A_0 in

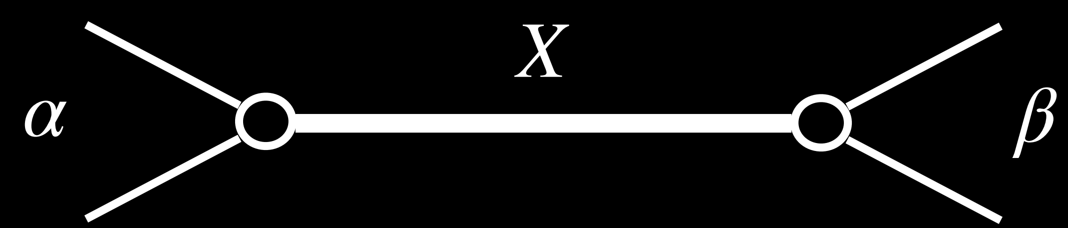
$$\chi = A_0 \exp(-r\sqrt{2mB})$$

which, including Y_0^0, gives the ψ found before in $\lambda\delta^3(r)$

$$\psi_{\text{mol.}}(r) = \left(\frac{2mB}{4\pi^2}\right)^{1/4} \frac{\exp(-r\sqrt{2mB})}{r}$$

THE POLAR FORMULA FOR THE $D\bar{D}^*$ SCATTERING

Introduce the coupling to X



Neglecting terms of order B^2 and E^2 $(E = k^2/2m)$ one finds in the case of the X

$$f(\alpha \to \beta) = -\frac{1}{8\pi m_X}\frac{g^2}{(p_D + p_{D^*})^2 + m_X^2 - i\epsilon} \simeq -\frac{1}{16\pi m_X^2}\frac{g^2}{E+B}$$

ADP Phys. Lett. B746, 248 (2015)

LANDAU ARGUMENT

The potential scattering of two slow particles $(kR \ll 1)$ described by an attractive potential U, with range R, featuring a shallow bound state at $-B$ has a **universal** scattering amplitude

$$f(ab \to ab) = -\frac{1}{\sqrt{2m}}\frac{\sqrt{B} - i\sqrt{E}}{E + B}$$

obtained by $\cot\delta_0 = -\sqrt{B/E}$. This is independent on the details of V and affected only by the value of B. A comparison with the pole formula

$$f(\alpha \to \beta) \simeq -\frac{1}{16\pi m_X^2}\frac{g^2}{E + B}$$

can be done at $k = i\sqrt{2mB}$ where the numerator in the first is $2\sqrt{B}$

recap of this in ADP Phys. Lett. B746, 248 (2015)

LANDAU ARGUMENT

This leads to

$$g^2 = \frac{16\pi m_X^2}{m}\sqrt{2mB}$$

$$f(\alpha \to \beta) = -\frac{1}{16\pi m_X^2}\frac{g^2}{E+B} = -\frac{\sqrt{2mB}}{m}\frac{1}{E+B}$$

Which is the same formula found before: the independency on the form of the potential.

L.D. Landau, JETP 39, 1865 (1960)

WEINBERG ARGUMENT

The previous formula for g^2 is valid only if the X is purely molecular, or $Z = 0$

$$g^2 = \frac{16\pi m_X^2}{m}\sqrt{2mB} = 8mm_X^2 \times (g_W)_{Z=0}$$

with

$$g_W^2 = \frac{2\pi\sqrt{2mB}}{m^2}(1 - Z)$$

and

$$|X\rangle = \sqrt{Z}\,|\mathfrak{X}\rangle + \int_{\boldsymbol{k}} C_{\boldsymbol{k}} \underbrace{|D\bar{D}^*(\boldsymbol{k})\rangle}_{|\alpha\rangle}$$

S. Weinberg Phys. Rev. 137, B672 (1965)

Neglecting terms of order B^2 and E^2 ($E = k^2/2m$) one finds in the case of the X

$$f(\alpha \to \beta) = -\frac{1}{8\pi m_X}\frac{g^2}{(p_D + p_{D^*})^2 + m_X^2 - i\epsilon} \simeq -\frac{1}{16\pi m_X^2}\frac{g^2}{E+B}$$

From this we have that

$$\frac{A_0^2}{2m} = \frac{g^2}{16\pi m_X^2} = \frac{m g_W^2}{2\pi}$$

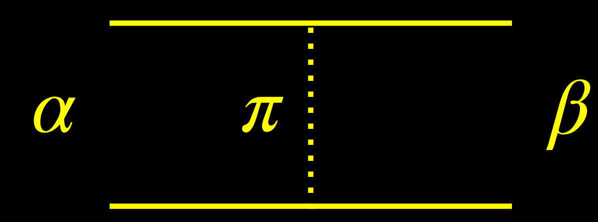
Finding the residue at the pole of the amplitude in eff. range exp.

$$\frac{1}{A_0^2} = \frac{1}{2\sqrt{2mB}} - \frac{1}{2} r_0$$

r_0 AND a FORMULAE

Solving the previous formula for $\boldsymbol{r_0}$

$$r_0 = -\frac{Z}{1-Z}R + O\left(\frac{1}{\Lambda}\right)$$



$$R = \frac{1}{\varkappa} = \frac{1}{\sqrt{2mB}}$$

The (positive!) scattering length is obtained using the expression of $\boldsymbol{r_0}$ given above into $\left(-\varkappa_0 + \frac{1}{2}r_0k^2 - ik\right)_{k=i\varkappa} = 0$

$$a = \frac{2(1-Z)}{2-Z}R + O\left(\frac{1}{\Lambda}\right)$$

(scattering length > 0)

S. Weinberg Phys. Rev. 137, B672 (1965)

THE Λ SCALE

In the case of the deuteron $\boldsymbol{d}$

$$\Lambda = m_\pi \Rightarrow \frac{1}{\Lambda} \simeq 1 \text{ fm}$$

because the pion can be integrated out given that

$$m_n - m_p \ll m_\pi$$

In the case of the $\boldsymbol{X}$, pion interactions between $\boldsymbol{D}$ and $\boldsymbol{\bar{D}^*}$ (u-channel)

$$\Lambda^2 = m_\pi^2 - \underbrace{(m_{D^*} - m_D)^2}_{q_0^2} \simeq (44 \text{ MeV})^2$$

giving

$$\frac{1}{\Lambda} \simeq 4.5 \text{ fm}$$

THE SIGN OF r_0 IN A ATTRACTIVE V

Scattering in the presence of shallow bound states generated by *purely attractive potentials* in NRQM are characterized by

$$r_0 \geq 0$$

even if there is a repulsive core, but in a *very* narrow region around the origin. Therefore the 1 fm estimated above is +1 fm

$$r_0 \simeq -\frac{Z}{1-Z}R + 1 \text{ fm} = r_0^{\text{exp.}} = +1.74 \text{ fm}$$

So we conclude that $Z \simeq 0$. The deuteron is a molecule! Only a "large" (wrt 1 fm) and negative r_0 would have been the token of the elementary deuteron.

Esposito, Maiani, Pilloni, ADP, Riquer, 2108.11413, *Phys. Rev. D*105 (2022) 3, L031503

DATA ON X: LHCB ANALYSIS

arXiv:2005.13419

For small kinetic energies

$$f(X \to J/\psi\pi\pi) = -\frac{(2N/g)}{(2/g)(E - m_X^0) - \sqrt{2\mu_+\delta} + E\sqrt{\mu_+/2\delta} + ik}$$

$$-\frac{1}{a} = \frac{2m_X^0}{g} + \sqrt{2\mu_+\delta} \simeq -6.92 \text{ MeV}$$ positive a

$$r_0 = -\frac{2}{\mu g} - \sqrt{\frac{2\mu_+}{2\mu^2\delta}} \simeq -5.34 \text{ fm}$$ negative r_0

using $E = k^2/2\mu$, μ being the reduced mass of the neutral $D\bar{D}^*$ pair, and taking g (LHCb) and m_X^0 (stable determination) from the experimental analysis. Since g can be larger, $r_0 \leq -2$ fm.

DETERMINATION OF Z

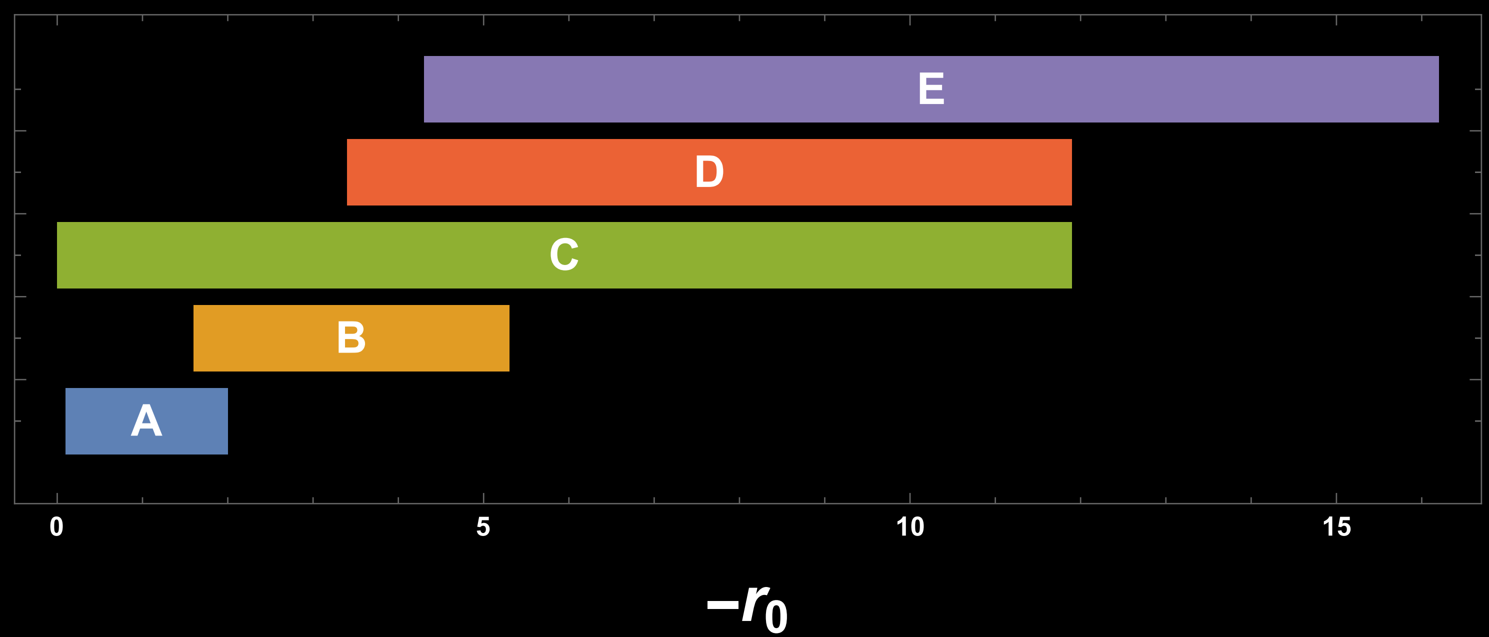
Neglect for the moment $O(1/\Lambda)$ corrections

$$r_0 = -\frac{Z}{1-Z}R = -5.34 \text{ fm}$$
$$a = \frac{2(1-Z)}{2-Z}R = 197/6.92 \text{ fm}$$

Gives $Z = 0.15 \neq 0!$ and $B = 20$ keV

Including ± 5 fm makes quite a difference depending on the sign.
In the case of -5 fm we might have $Z = 0$ even with $r_0^{\text{exp}} = -5.32$ fm!
In the case of $+5$ fm, a negative experimental r_0 is the proof of the compact state.
However we shall see that in the molecular case $O(1/\Lambda) \to -0.2$ MeV

$(-r_0)$ ACCORDING TO SOME ESTIMATES



A: Baru et al., 2110.07484

B: Esposito et al., 2108.11413

C: LHCb, 2109.01056

D: Maiani & Pilloni GGI-Lects

E: Mikhasenko, 2203.04622

H. Xu, N. Yu and Z. Zhang 2401.00411: $r_0 \approx -14$ fm combining LHCb and Belle data (for the X)

r_0 FROM LATTICE

M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 2202.10110

Applying the lattice Lüscher method, the authors study the $D\bar{D}^*$ scattering amplitude and make a determination of the scattering length and of the effective range for $\mathcal{T}_{cc}$

$$a = -1.04(29)\ \text{fm}$$
$$r_0 = +0.96^{+0.18}_{-0.20}\ \text{fm}$$

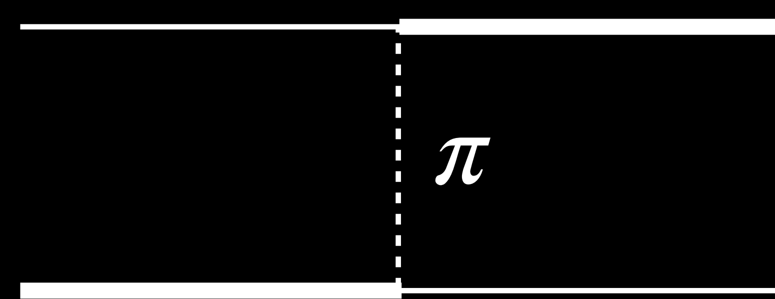
The mass of the pion is $m_\pi = 280$ MeV, to keep the D^* stable.
This result, for the moment, is compatible with a *virtual state* because of the negative a – like the singlet deuteron.
As for LHCb (2109.01056 p.12)

$$a = +7.16\ \text{fm}$$
$$-11.9 \le r_0 \le 0\ \text{fm}$$

r_0 IN THE MOLECULAR PICTURE

$$H_{DD*} = \frac{\boldsymbol{p}_{D*}^2}{2m_{D*}} + \frac{\boldsymbol{p}_D^2}{2m_D} - \lambda_0\, \delta^3(\boldsymbol{r})$$

A perturbation to the $\delta^3(\boldsymbol{r})$ potential derives from



Esposito, Glioti, Germani, ADP, Rattazzi, Tarquini, PLB847, 138285 (2023).

Potential = FT of the propagator in NR approximation

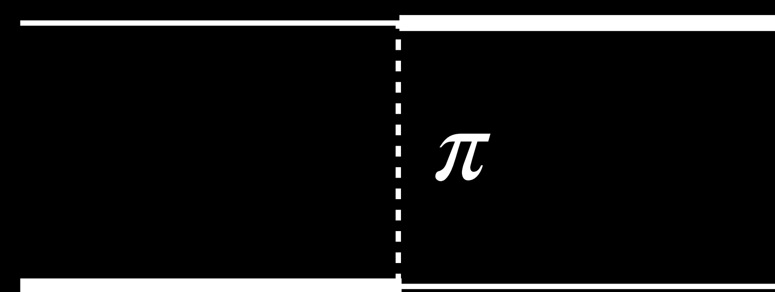
$$\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{q^2 + m_\pi^2 - i\epsilon}\, d^3q \xrightarrow[\text{NR}]{} \int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - \mu^2 - i\epsilon}\, d^3q \approx \int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - i\epsilon}\, d^3q$$

$$\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - i\epsilon}\, d^3q = -\frac{(2\pi)^3}{4\pi}\left(\frac{3\hat{r}_i\hat{r}_j}{r^3} - \frac{\delta_{ij}}{r^3} - \frac{4\pi}{3}\delta^3(\boldsymbol{r})\right)$$

r_0 IN THE MOLECULAR PICTURE

$$H_{DD*} = \frac{\boldsymbol{p}_{D*}^2}{2m_{D*}} + \frac{\boldsymbol{p}_D^2}{2m_D} - \lambda_0\, \delta^3(\boldsymbol{r})$$

A perturbation to the $\boldsymbol{\delta^3(r)}$ potential derives from

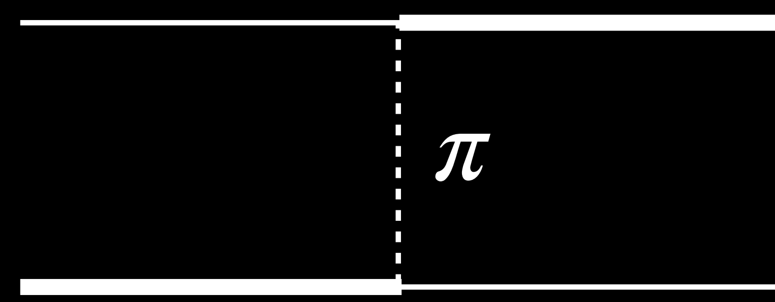


Esposito, Glioti, Germani, ADP, Rattazzi, Tarquini, PLB847, 138285 (2023).

In $\boldsymbol{S}$-wave we have to include the condition $\langle \hat{r}_i \hat{r}_j \rangle = \frac{1}{3}\delta_{ij}$ which, for $\boldsymbol{\mu = 0}$, leaves only an extra $\boldsymbol{\delta^3(r)}$ potential term.

But $\mu^2 = (m_{D*} - m_D)^2 - m_\pi^2 \simeq 44$ MeV, and this requires an extra, complex potential term.

THE COMPLEX POTENTIAL



Esposito, Glioti, Germani, ADP, Rattazzi, Tarquini, PLB847, 138285 (2023).

Keep μ finite! Are the corrections to r_0 of the size $O(1/m_\pi)$ or $O(1/\mu)$?

$$V_w = -\frac{g^2}{2f_\pi^2}\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2-\mu^2-i\epsilon}\,\frac{d^3q}{(2\pi)^3} = -\underbrace{\frac{g^2}{6f_\pi^2}}_{\beta}\left(\delta^3(r)+\mu^2\frac{e^{i\mu r}}{4\pi r}\right)\delta_{ij}$$

The contraction with polarizations $e_i^{(\lambda)}\bar{e}_j^{(\lambda')}$ gives $\delta_{\lambda\lambda'}$.

As for the $\delta^3(r)$ potential from π alone, it has not the right weight to make the bound state at $E=-B$. But combined with the strong one, an overall λ can be defined to make it.

So we divide $\boldsymbol{V}$ into

$$V = V_s + V_w = -\underbrace{\left(\lambda_0 + 4\pi\beta\right)}_{\lambda}\delta^3(\boldsymbol{r}) - \alpha\mu^2\frac{e^{i\mu r}}{r}$$

To compute any amplitude, all orders in $\boldsymbol{V_s}$ are needed, and possibly only the first order in $\boldsymbol{V_w}$.

Can we find $\boldsymbol{r_0}$ as a result of the correction to $\boldsymbol{f}$ due to the complex potential?

DISTORTED WAVE BORN APPROXIMATION

$$f = \frac{1}{k\cot\delta(k) - ik} = f_s + f_w = \frac{1}{-\frac{1}{a} - ik} + f_w$$

$$f_w = -\frac{2m}{4k^2}\int V_w(r)\,\chi_s^2(r)\,dr$$

Where $\chi_s(r)$ are scattering w.f. of the $\delta^3(\boldsymbol{r})$ potential, and m is the invariant DD^* mass. Thus r_0 is determined by the $\boldsymbol{k}^2$ coefficient in the *double expansion* around $k = 0$ and $\alpha = 0$ of the expression

$$f^{-1} = \left(\frac{1}{-\frac{1}{a} - ik} - \frac{2m}{4k^2}\int V_w(r)\,\chi_s^2(r)\,dr\right)^{-1}$$

CALCULATION OF r_0 (DWBA)

$$r_0 = 2m\alpha\left(\frac{2}{\mu^2 a^2} + \frac{8i}{3\mu a} - 1\right)$$

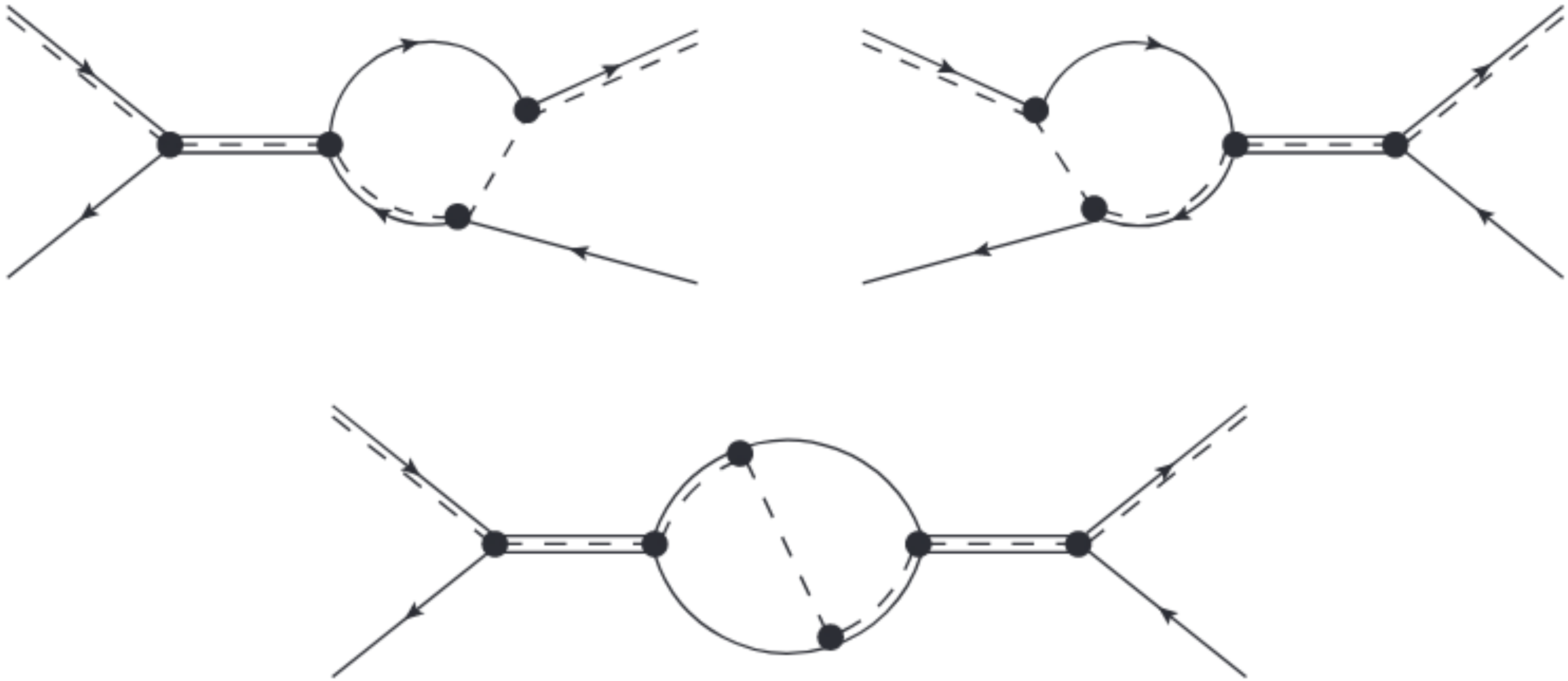
Esposito, Glioti, Germani, ADP, Rattazzi, Tarquini, PLB847, 138285 (2023).

$$-0.20 \text{ fm} \lesssim \text{Re}\, r_0 \lesssim -0.15 \text{ fm}$$

$$0 \text{ fm} \lesssim \text{Im}\, r_0 \lesssim 0.17 \text{ fm}$$

$$\alpha = \frac{g^2}{24\pi f_\pi^2} = \frac{5\times 10^{-4}}{\mu^2}$$

These results agree, analytically, with what found by Braaten et al. using EFT. It turns out that the real part of r_0 is just a tiny (negative!) fraction of a Fermi. This confirms the fact that the Weinberg criterion can be extended to the $X(3872)$ too.



Braaten, Galilean invariant XEFT, Phys. Rev. D 103, 036014 (2021), arXiv:2010.05801 [hep-ph]

SOME MORE TECHNICAL CONCLUSIONS

- It would be useful to have new comparative studies on the r_0 of the X(3872) and of the $\mathcal{T}_{QQ}$ particles, and to agree on the way to extract information from data (not easy).
- It would be of great relevance to learn more, on the experimental side, about deuteron production at high p_T .
- Some states are produced promptly in pp collisions, some are not. There is no clear reason why!
- Are there loosely bound molecules $B\bar{B}*$? Can we formulate more stringient bounds on $X^{\pm}$ particles?
- Derive Weinberg criterium in a modern language.
- More basically: are we on the right questions?