Hadronic molecules and kinematic singularities

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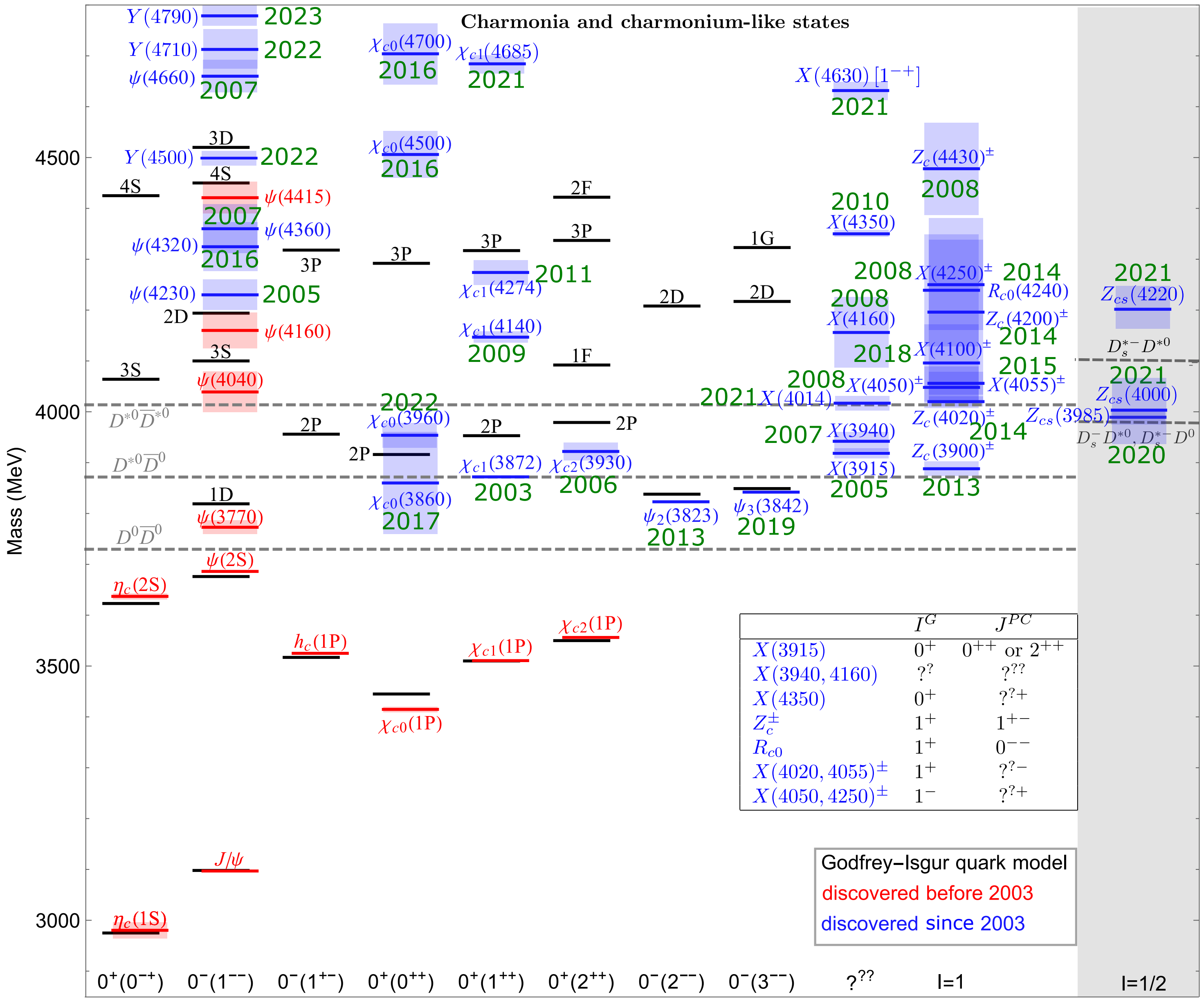
Charmonia and charmonium-like structures

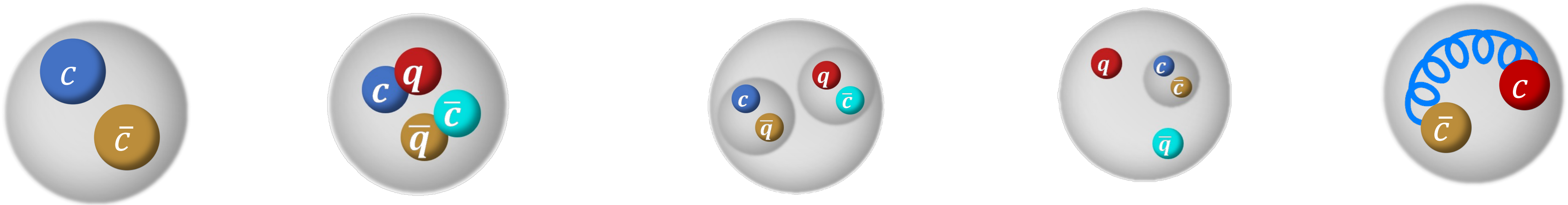
- Abundance of new states from peak hunting
  - ☐ b-hadron (B, Λ_b) decays
  - ☐ Hadron/heavy-ion collisions
  - ☐ $\gamma\gamma$ processes
  - ☐ e^+e^- collisions: vectors and states produced from vector decays
    - ➢ BESIII
    - ➢ Future tau charm facilities

- What are they?
  - ☐ Nonperturbative QCD ⇒ difficult!



Charmonia and charmonium-like structures

- Different pictures of the internal structure   see also talks by M. Karliner, A. Polosa

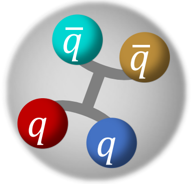


charmonium   compact tetraquark   hadronic molecule   hadro-charmonium   hybrid charmonium

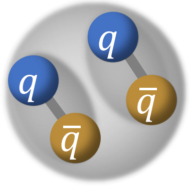
  - Nonperturbative, mixing is unavoidable ⇒ dominant component?
  - Other effects: threshold cusps and triangle singularities (see later)

- Theoretical methods
  - Lattice QCD
  - EFT based: heavy quark symmetry, flavor symmetry
  - Phenomenological approaches: quark model, QCD sum rules, …
- A unified description/classification is still missing

Different confinement pictures

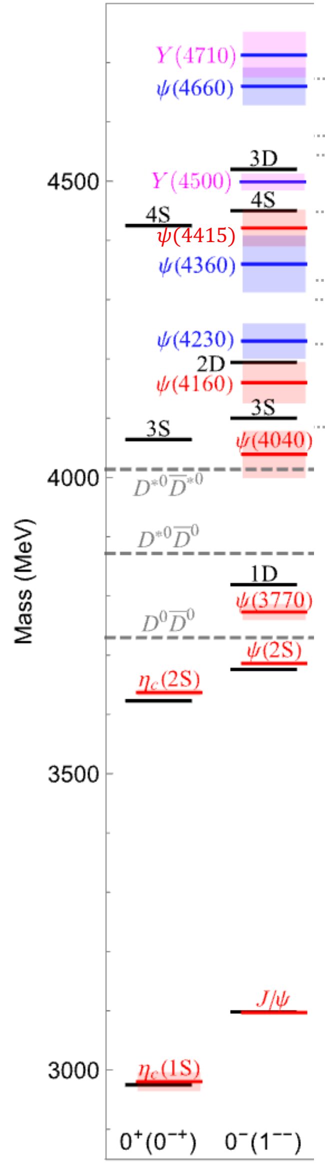


compact tetraquark



hadronic molecule

Many thresholds $\gtrsim 4$ GeV



$\psi' f_0(980)$

$D_{s1}\bar{D}_s^*, \Lambda_c^+\bar{\Lambda}_c^-$

$\chi_{c0,1,2}\phi$

$D_{s0}^*\bar{D}_s^*, D_{s1}\bar{D}_s$

$D_2(2460)\bar{D}^*$

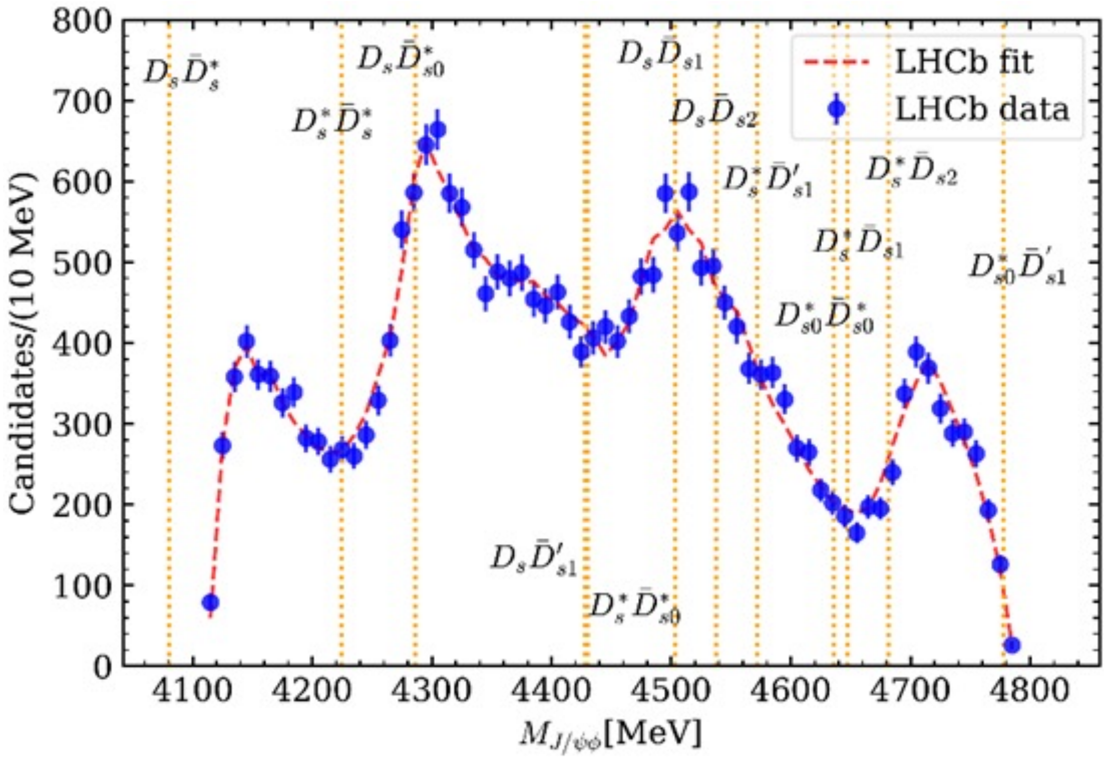
$D_1(2420)\bar{D}^*$

$D_1(2420)\bar{D}$

$\chi_{c0,1,2}\omega$

$J/\psi f_0(980)$

2-body S-wave thresholds



Data: LHCb, PRL 127 (2021) 082001

Plot: X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65 [arXiv:2101.01021]

Thresholds play an essential role

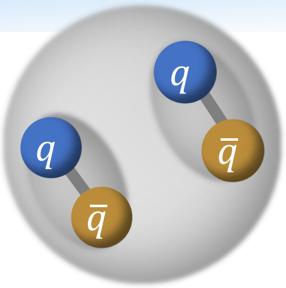
- Complicates extraction of resonance properties!

  Measurements on various final states are important

- Hadronic molecules?

Hadronic molecules

- **Composite systems of hadrons**
  - analogues of the deuteron ($\approx pn$ bound state)
  - bound by the residual strong force, more extended than $1/\Lambda_{\text{QCD}}$



- **Compositeness $1-Z$**

See Polosa's talk for more discussion

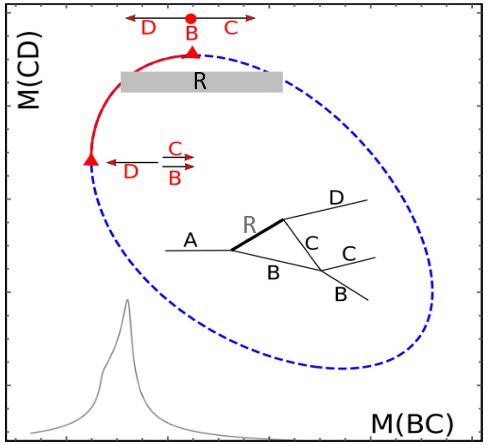
S. Weinberg (1965); V. Baru et al. (2004); T. Hyodo et al. (2012); F. Aceti, E. Oset (2012); G.-Y. Chen, W.-S. Huo, Q. Zhao (2015); Z.-H. Guo, J. Oller (2016); I. Matuschek et al. (2021); Esposito et al. (2022); J. Song et al. (2022); M. Albaladejo, J. Nieves (2022); … for reviews, see T. Hyodo, IJMPA 28 (2013) 1330045; FKG, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.-S. Zou, RMP 90 (2018) 015004

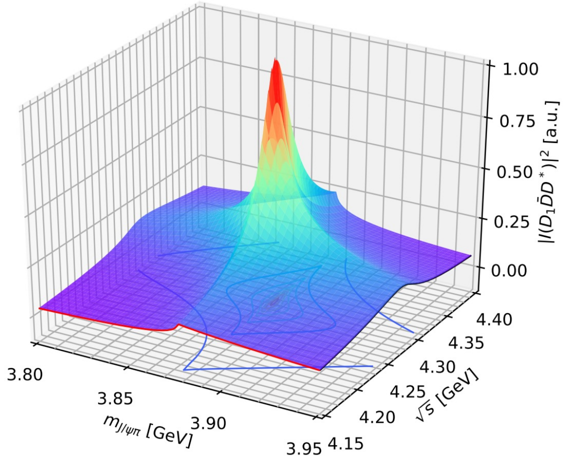
  - probability of finding the physical state in two-hadron component (S-wave loosely bound)
  - can be expressed in terms of low-energy observables ⇐ line shapes, scattering parameters
    - coupling constant $g_{\text{NR}}^2 \approx (1-Z)\frac{2\pi}{\mu^2}\sqrt{2\mu E_B}$ — E_B: binding energy; μ: reduced mass
    - ERE parameters (scattering length, effective range) $a \approx -\frac{2(1-Z)}{(2-Z)\sqrt{2\mu E_B}}, r_e \approx -\frac{Z}{(1-Z)\sqrt{2\mu E_B}}$ (for $r_e \leq 0$)
    - phase shift $1-Z = 1-\exp\left(\frac{1}{\pi}\int_0^\infty dE\,\frac{\delta(E)}{E-E_B}\right)$ — Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502
      - ✓ derived with separable T-matrix & pole-dominance approximation of $\delta(E)$
      - ✓ valid independent of the sign of r_e ⇒ $Z = 0$ for $r_e > 0$ with ERE up to $\mathcal{O}(p^2)$

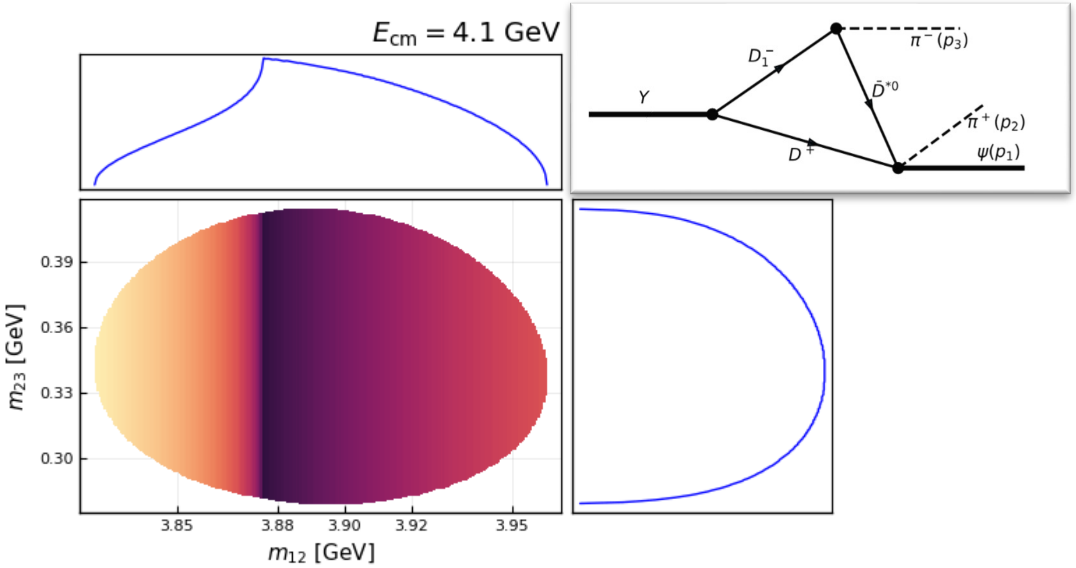
Complications due to kinematic singularities

- Singularities of S-matrix / scattering amplitudes
  - Dynamics ⇒ poles: bound states, resonances
  - Kinematics ⇒ additional singularities: threshold cusps, triangle singularities, …
- Triangle singularity (TS) L.D. Landau (1959); J.D. Bjorken (1959); J. Mathews (1959); N. Nakanishi (1959); Coleman, Norton (1965); Schmid (1967); …
  - Sensitive to energy
  - Crucial to have measurements at various energies, in different processes

For a review, see FKG, X.-H. Liu, S. Sakai, PPNP 112, 103757 (2020)

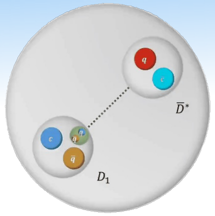






TS effects for $Z_c(3900)$: Q. Wang et al. (2013); M. Albaladejo et al. (2015); JPAC (2016); Q.-R. Gong et al. (2018); I. Danilkin et al. (2020); …
(more see later)

Binding mechanism of hadronic molecules



- Phenomenological picture: boson exchanges M. Voloshin, L. Okun, JETP Lett. 23 (1976) 333

☐ One-pion exchange

N.A. Tönqvist, ZPC 61 (1994) 525; …

➢ systems like $D\bar{D}, \Sigma_c\bar{D}$ unbound

Composite	J^{PC}	Deuson
$D\bar{D}^*$	0^{-+}	$\eta_c(\approx 3870)$
$D\bar{D}^*$	1^{++}	$\chi_{c1}(\approx 3870)$
$D^*\bar{D}^*$	0^{++}	$\chi_{c0}(\approx 4015)$
$D^*\bar{D}^*$	0^{-+}	$\eta_c(\approx 4015)$
$D^*\bar{D}^*$	1^{+-}	$h_{c0}(\approx 4015)$
$D^*\bar{D}^*$	2^{++}	$\chi_{c2}(\approx 4015)$
$B\bar{B}^*$	0^{-+}	$\eta_b(\approx 10545)$
$B\bar{B}^*$	1^{++}	$\chi_{b1}(\approx 10562)$
$B^*\bar{B}^*$	0^{++}	$\chi_{b0}(\approx 10582)$
$B^*\bar{B}^*$	0^{++}	$\eta_b(\approx 10590)$
$B^*\bar{B}^*$	1^{+-}	$h_b(\approx 10608)$
$B^*\bar{B}^*$	2^{++}	$\chi_{b2}(\approx 10602)$

☐ Vector-meson exchange S. Krewald, R. Lemmer, F. Sassen, PRD 69 (2004) 016003; …

➢ 0^{++} $D\bar{D}$ bound state predicted

Y.-J. Zhang, H.-C. Chiang, P.-N. Shen, B.-S. Zou, PRD 74 (2006) 014013;
D. Gamermann et al., PRD 76 (2007) 074016; …

✧ Lattice QCD S. Prelovsek et al., JHEP06 (2021) 035

But not seen in D. Wilson et al. [HadSpec], arXiv:2309.14070

➢ Hidden-charm pentaquarks above 4 GeV (including $\Sigma_c\bar{D}$) predicted J.-J. Wu, R. Molina, E. Oset, B.-S. Zou, PRL 105 (2010) 232001; …

☞ Survey of the molecular spectrum in a simple model

✧ light-vector-meson exchanges
✧ single channel
✧ neglecting mixing

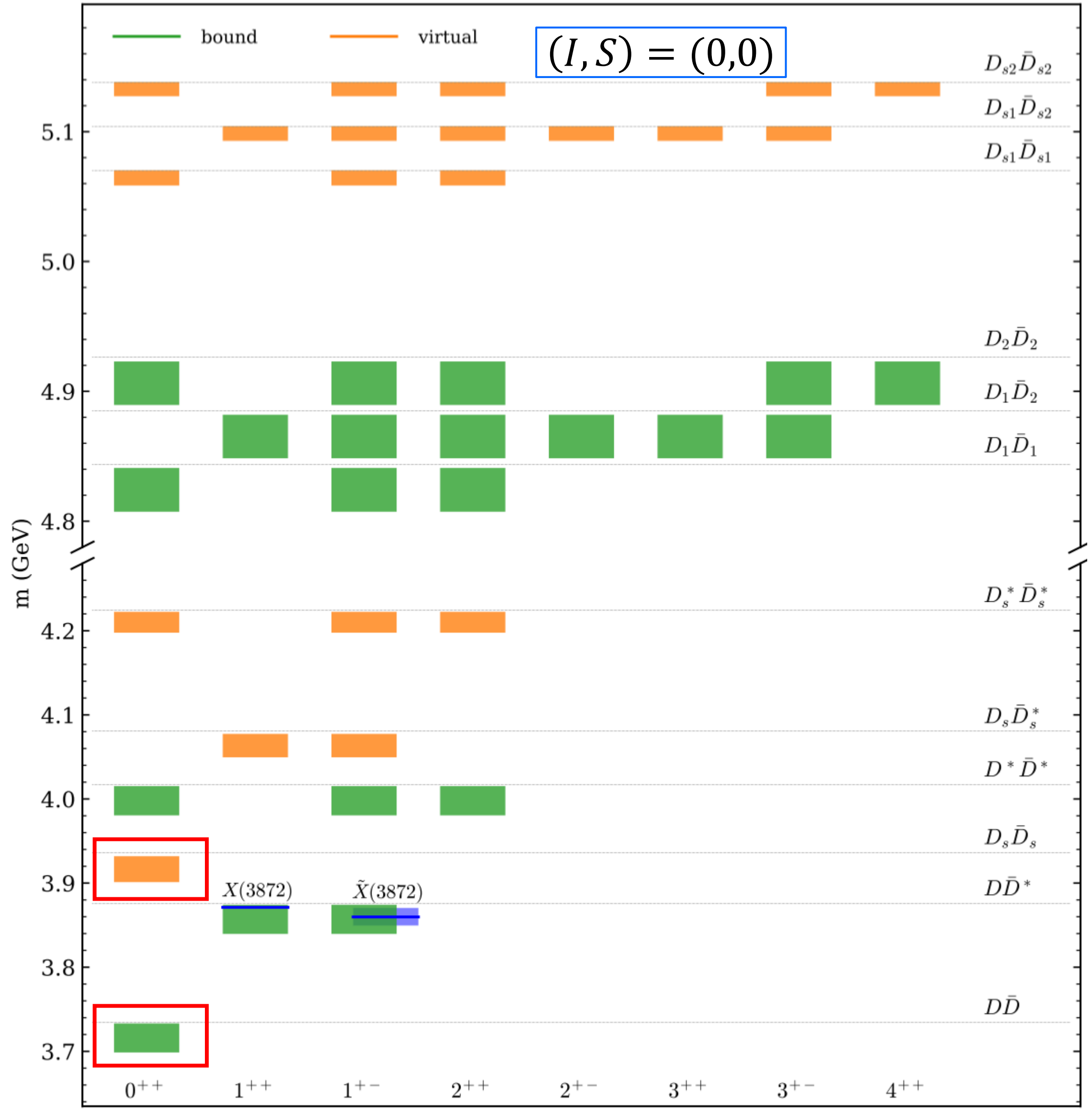
X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65;
CTP 73 (2021) 015201

Extension of the survey including more meson exchanges:
F.-Z. Peng, M. Sanchez-Sanchez, M.-J. Yan, M. Pavon Valderrama, PRD 105 (2022) 034028;
M.-J. Yan, F.-Z. Peng, M. Pavon Valderrama, arXiv:2304.14855; …

For a list of the literature on one-boson exchange models, see, e.g., Y.-R. Liu et al., PPNP 107 (2019) 237

Survey of hadronic molecules: hidden-charm mesons w/ $P = +$

X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65

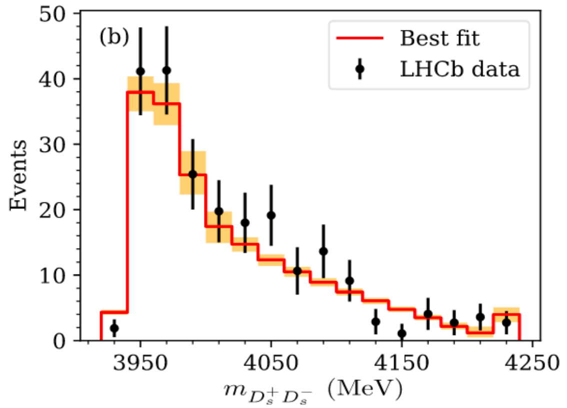


- ✓ > 200 hidden-charm hadronic molecules
- ✓ $X(3872)$ as a $\bar{D}D^*$ bound state
- ✓ $\tilde{X}(3872)$ COMPASS, PLB 783 (2018) 334
- ✓ $\bar{D}D$ bound state from lattice S. Prelovsek et al., JHEP06 (2021) 035

  & other models C.-Y. Wong, PRC 69 (2004) 055202; Y.-J. Zhang et al., PRD 74 (2006) 014013; D. Gamermann et al., PRD 76 (2007) 074016; J. Nieves et al., PRD 86 (2012) 056004; ...

  But not seen exp., not seen in HadSpec, arXiv:2309.14070

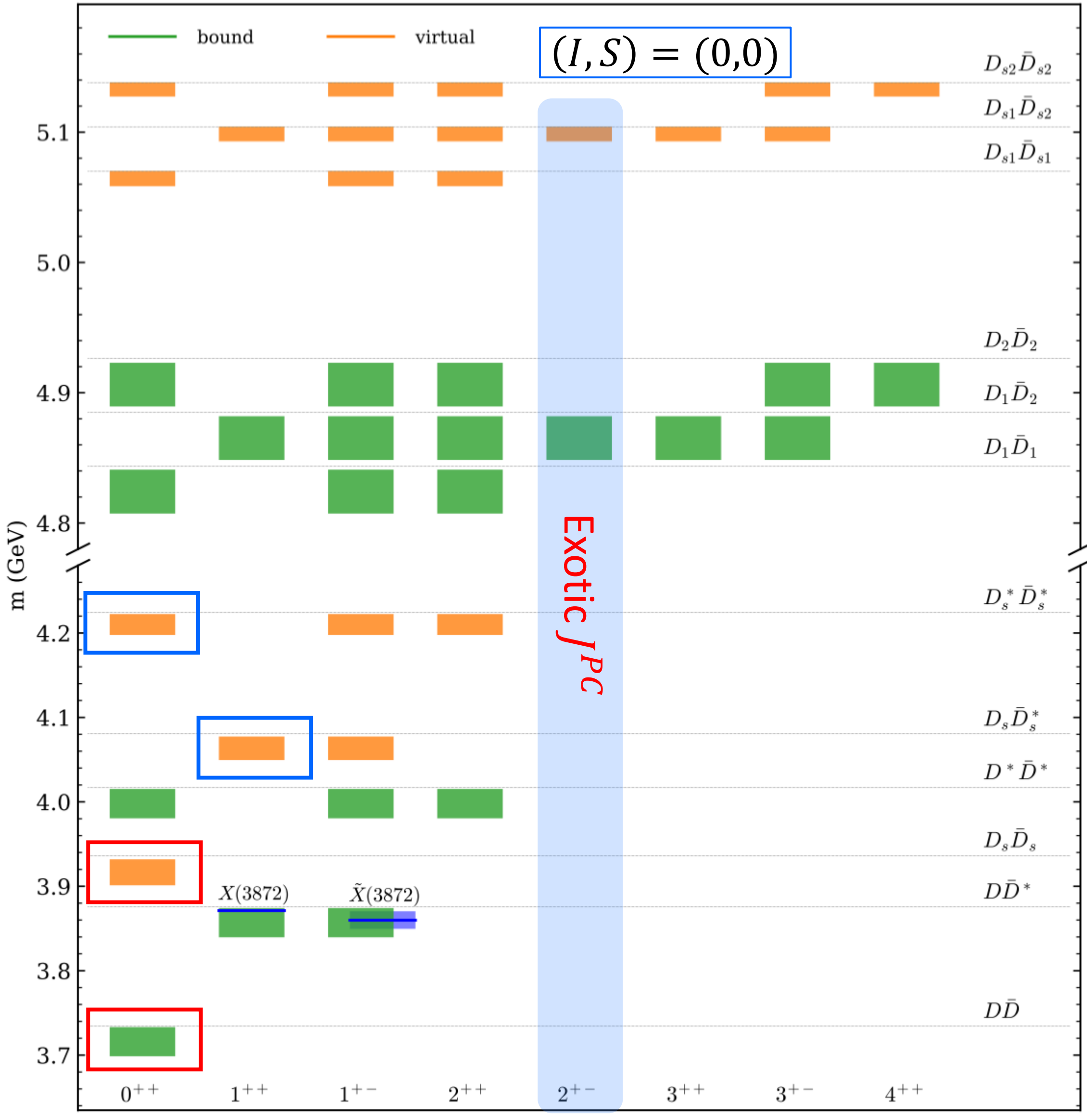
- ✓ $X(3960)$ in $B^+ \to D_s^+ D_s^- K^+$



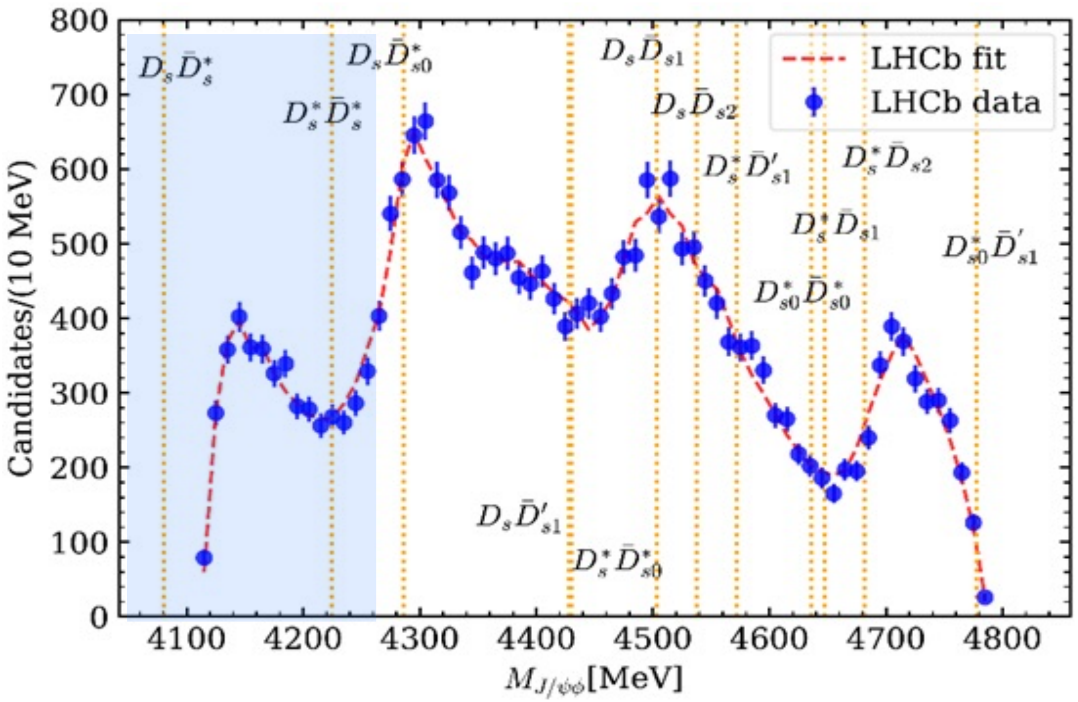
Data from LHCb, PRL 131 (2023) 071901;
Fit in
T. Ji, X.-K. Dong, M. Albaladejo, M.-L. Du, FKG, J. Nieves, B.-S. Zou, Sci. Bull. 68 (2023) 2056

Survey of hadronic molecules: hidden-charm mesons w/ $P = +$

X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65



✓ $D_s\bar{D}_s^*$, $D_s^*\bar{D}_s^*$ virtual states?



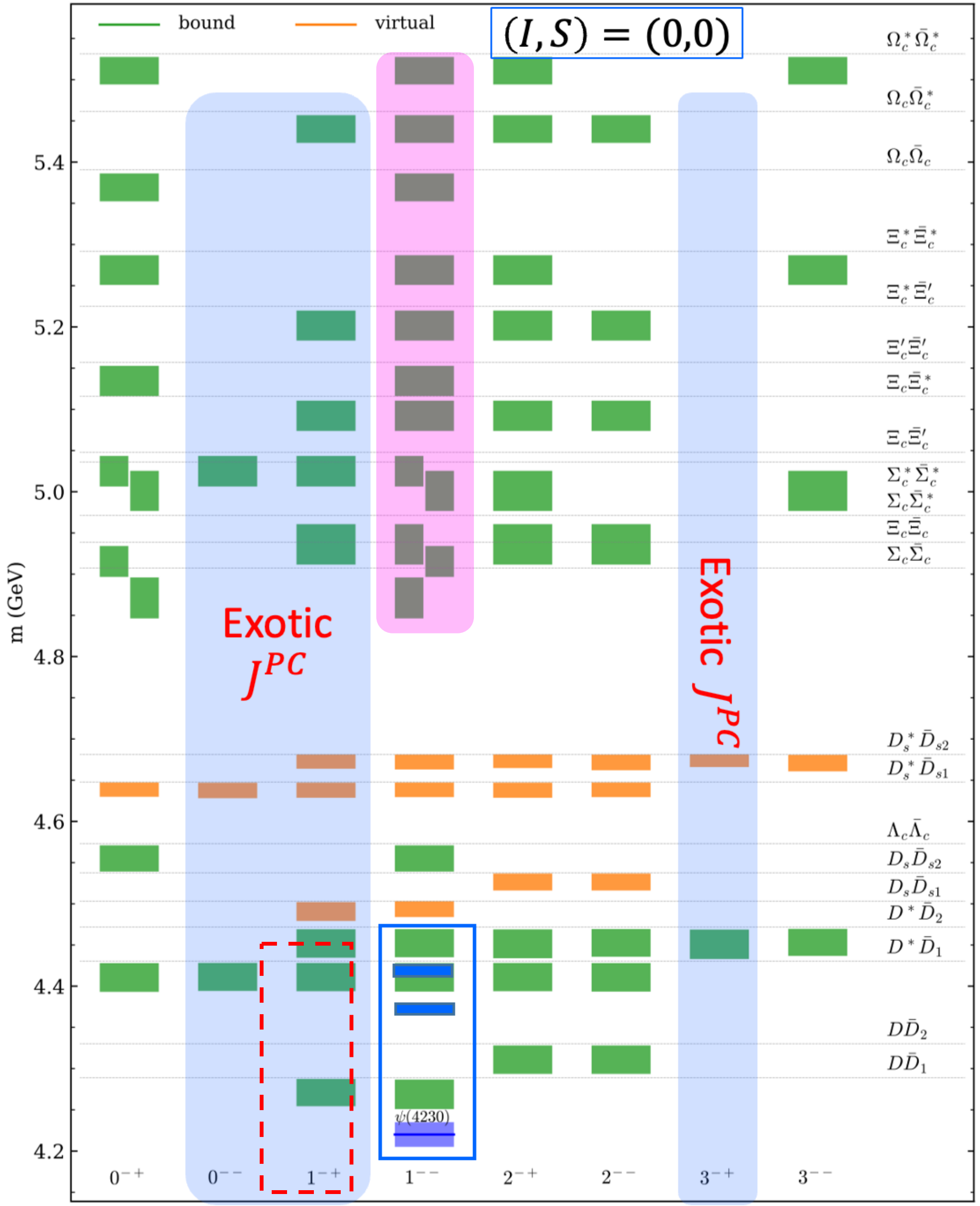
Data: LHCb, PRL 127 (2021) 082001

Virtual poles found from the fit in X. Luo, S.X. Nakamura, PRD 107 (2023) L011504

✓ Exotic $J^{PC} = 2^{+-}$ $[h_{c2}]$: $e^+e^- \to h_{c2}\pi\pi$

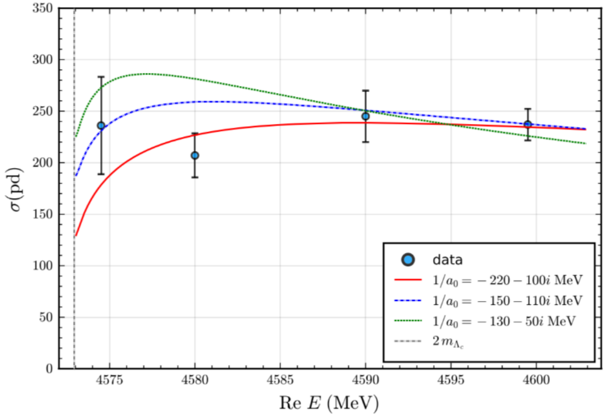
$\hookrightarrow J/\psi\pi\pi, \bar{D}D^*\pi, \ldots$

Hidden-charm mesons w/ $P = -$



- ✓ $Y(4260)/\psi(4230)$ as a $\bar{D}D_1$ bound state
- ✓ $\psi(4360), \psi(4415)$: $D^*\bar{D}_1, D^*\bar{D}_2$?
- ✓ Evidence for 1^{--} $\Lambda_c\bar{\Lambda}_c$ molecular state in BESIII data
  - Sommerfeld factor
  - near-threshold pole

  Data from BESIII, PRL 120 (2018) 132001; see also Q.-F. Cao et al., PRD 100 (2019) 054040



- ✓ Positive-C parity states: 0^{-+} $[\eta_c], 1^{-+}[\eta_{c1}], 2^{-+}[\eta_{c2}], 3^{-+}[\eta_{c3}]$

  Also predicted in Z.-P. Wang, F.-L. Wang, G.-J. Wang, X. Liu, arXiv:2312.03512

- ✓ Numerous states with exotic quantum numbers 0^{--} $[\psi_0], 1^{-+}[\eta_{c1}], 3^{-+}[\eta_{c3}]$

  e.g., $e^+e^- \to \gamma\eta_{c1,3}, \omega\eta_{c1,3}$; $\eta_{c1,3} \to D\bar{D}^*\pi, J/\psi\omega, \ldots$

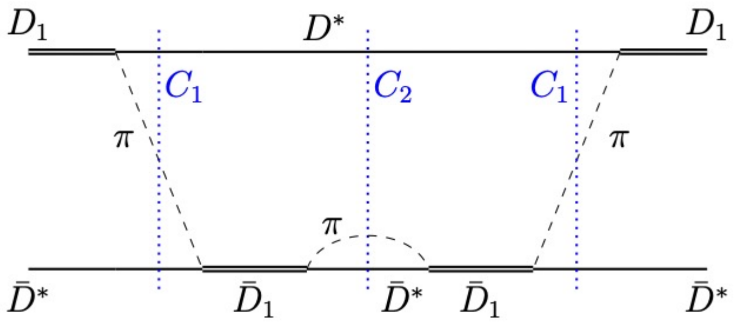
- ✓ Many 1^{--} states in [4.8, 5.6] GeV: future tau-charm facilities

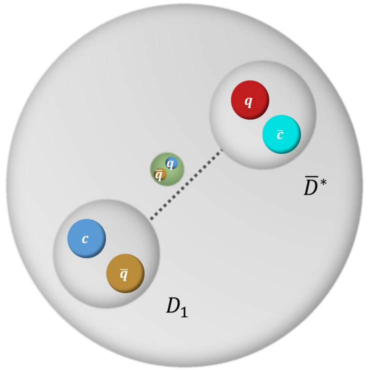
Closer look at the 0^{--} state

T. Ji, X.-K. Dong, FKG, B.-S. Zou, PRL 129 (2022) 102002

- 0^{--} spin partner $\psi_0(4360)$ $[D^*\bar{D}_1]$ of $\psi(4230), \psi(4360), \psi(4415)$ as $D\bar{D}_1, D^*\bar{D}_1, D^*\bar{D}_2$ hadronic molecules
- Robust against the inclusion of coupled channels and three-body effects

Molecule	Components	J^{PC}	Threshold	E_B
$\psi(4230)$	$\frac{1}{\sqrt{2}}(D\bar{D}_1 - \bar{D}D_1)$	1^{--}	4287	67 ± 15
$\psi(4360)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_1 - \bar{D}^*D_1)$	1^{--}	4429	62 ± 14
$\psi(4415)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_2 - \bar{D}^*D_2)$	1^{--}	4472	49 ± 4
ψ_0	$\frac{1}{\sqrt{2}}(D^*\bar{D}_1 + \bar{D}^*D_1)$	0^{--}	4429	**63 ± 18**

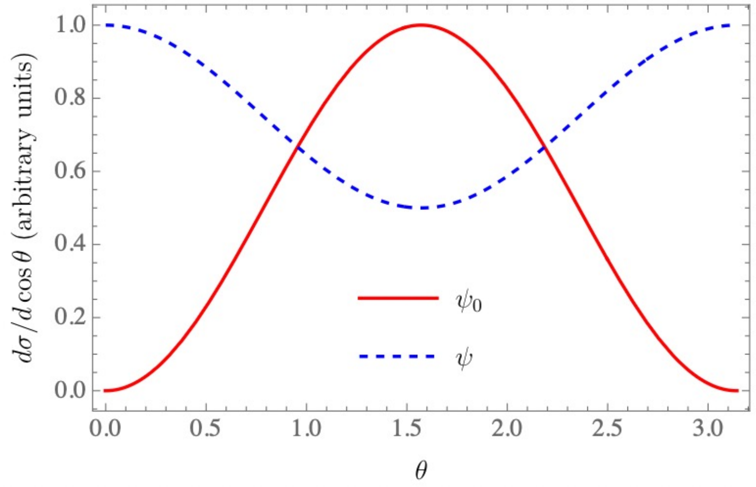




- May be searched for using $e^+e^- \to \psi_0\eta, \psi_0 \to J/\psi\eta, D\bar{D}^*, D^*\bar{D}^*\pi, \ldots$

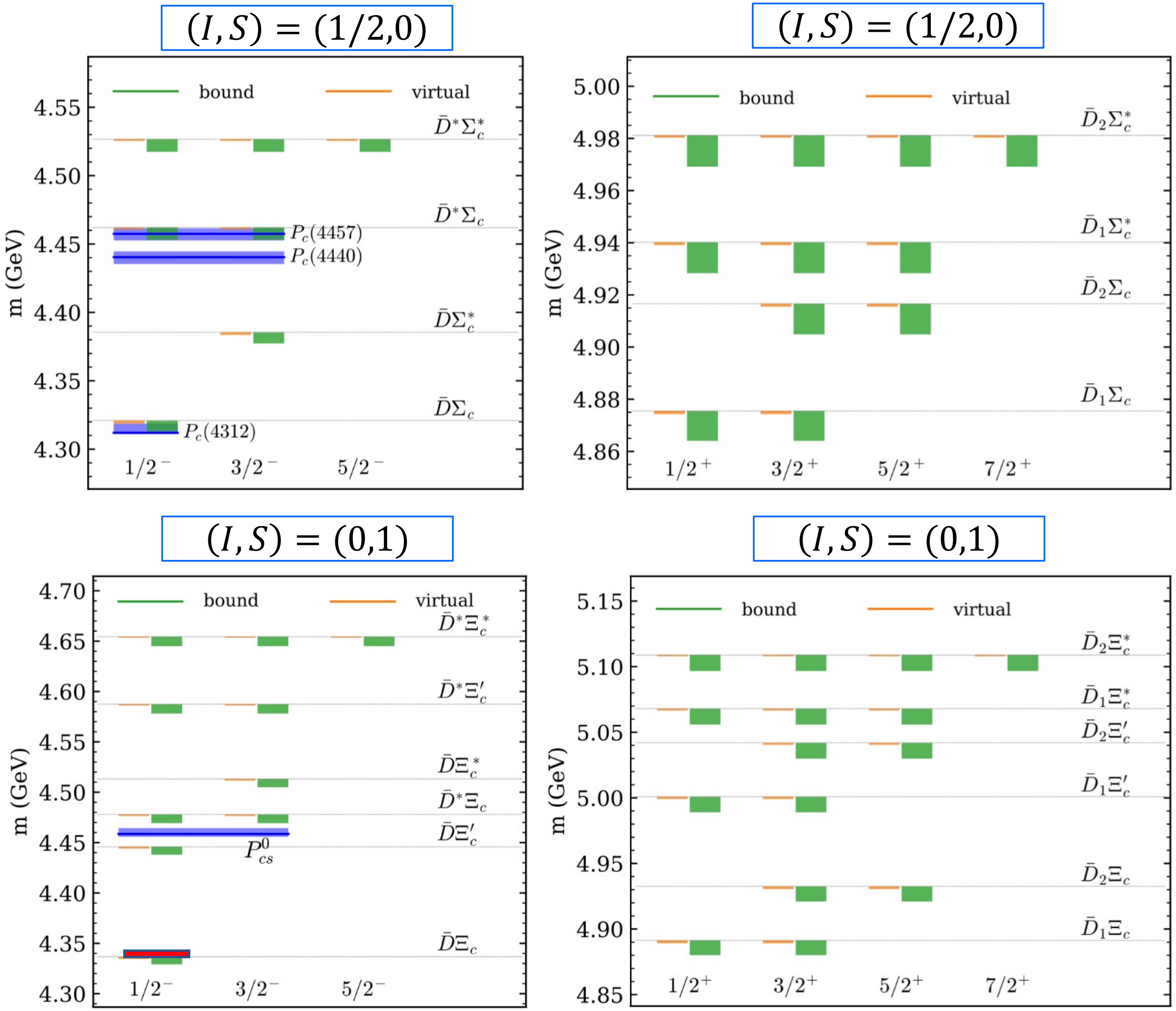
$M = (4366 \pm 18)$ MeV,

$\Gamma < 10$ MeV

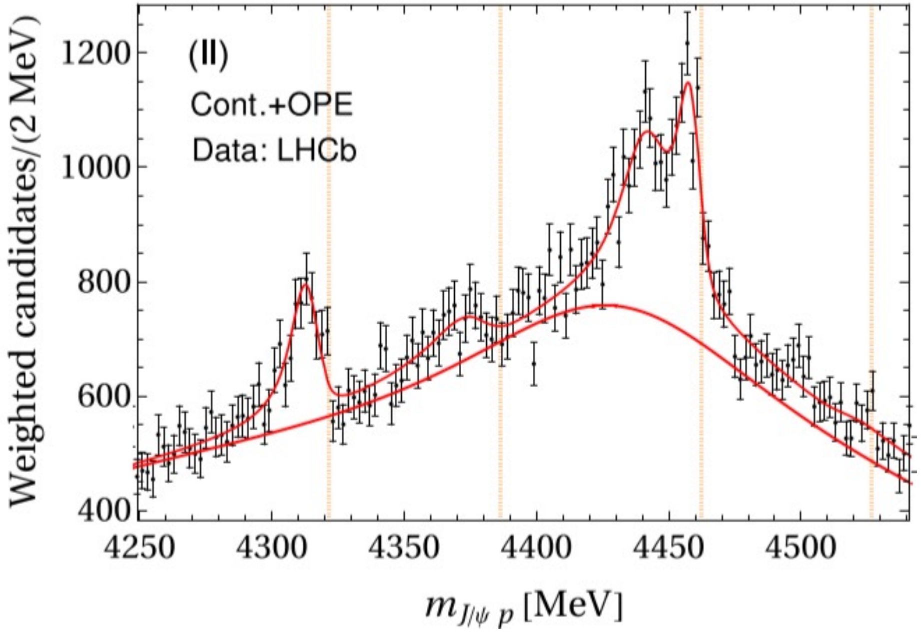


Hidden-charm pentaquarks

X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65



- ✓ P_c states as $\bar{D}^{(*)}\Sigma_c^{(*)}$ molecules
- ✓ The LHCb data can be well described in a pionful EFT



M.-L. Du et al., PRL 124 (2020) 072001; JHEP 08 (2021) 157

- ✓ $P_{cs}(4459)$: 2 $\bar{D}^*\Xi_c$ molecular states
- ✓ $P_{cs}(4338)$: $\bar{D}\Xi_c$ molecular state

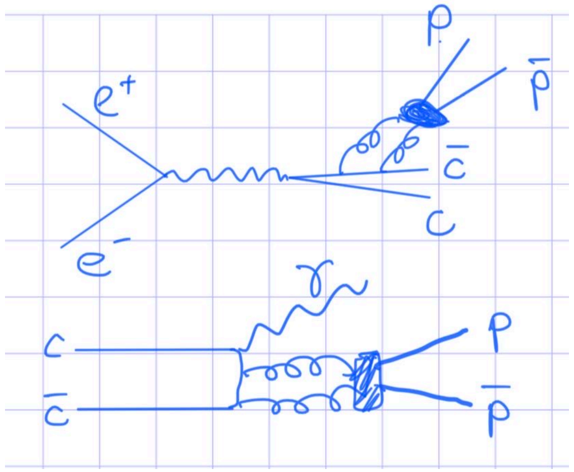
Cross section estimate for P_c

- Rough estimate of events number for $e^+e^- \to J/\psi p\bar{p}$ K.-T. Chao, FKG, Y.-J. Zhang, in preparation

$$\mathcal{A}(e^+e^- \to J/\psi\, p\bar{p}) \sim \mathcal{A}(e^+e^- \to J/\psi\, gg) \otimes \mathcal{A}(gg \to p\bar{p})$$

$$\mathcal{A}(J/\psi \to \gamma p\bar{p}) \sim \mathcal{A}(J/\psi \to \gamma gg) \otimes \mathcal{A}(gg \to p\bar{p})$$

$$\Rightarrow \frac{\sigma(e^+e^- \to J/\psi\, p\bar{p})}{\Gamma(J/\psi \to \gamma p\bar{p})} \approx \frac{\sigma(e^+e^- \to J/\psi\, gg)}{\Gamma(J/\psi \to \gamma gg)}$$



leads to

$$\sigma(e^+e^- \to J/\psi\, p\bar{p}) \approx \sigma(e^+e^- \to J/\psi\, gg) \times 4 \times 10^{-3}$$

$\approx \mathcal{O}(4\text{ fb})$ @ 6 GeV using NRQCD for $\sigma(e^+e^- \to J/\psi gg)$ from Y.-Q. Ma, Y.-J. Zhang, K.-T. Chao, PRL 102 (2009) 162002

With integrated luminosity of 1 ab^{-1}, $\mathcal{O}(4\times10^3)$ $J/\psi p\bar{p}$ events annually

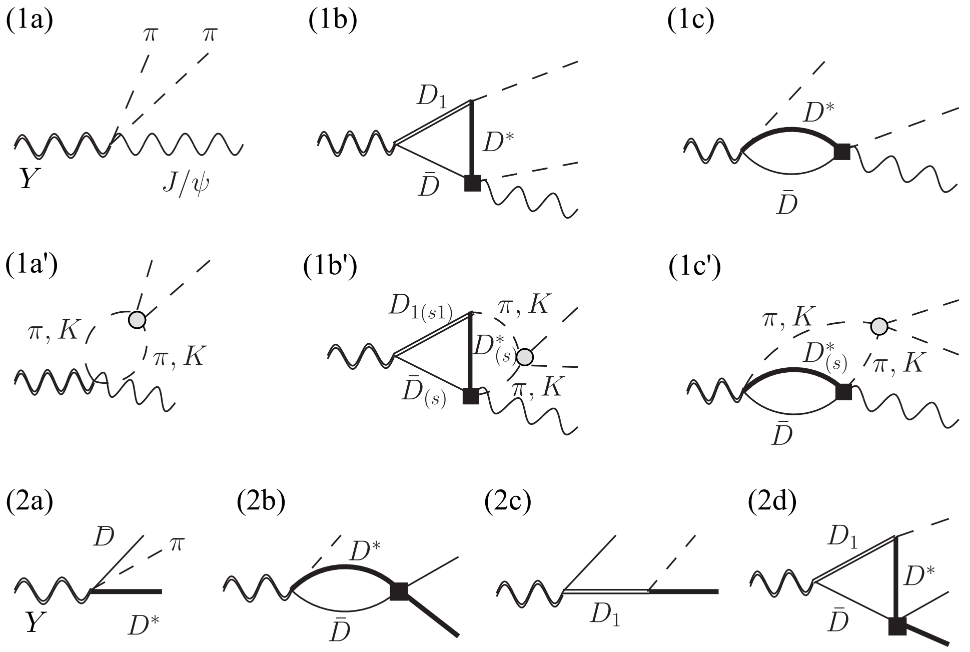
Assuming $\mathcal{B}(P_c \to J/\psi\, p) \sim$ a few per cent,

$\lesssim \mathcal{O}(10^5)$ $P_c\bar{p}$ events annually

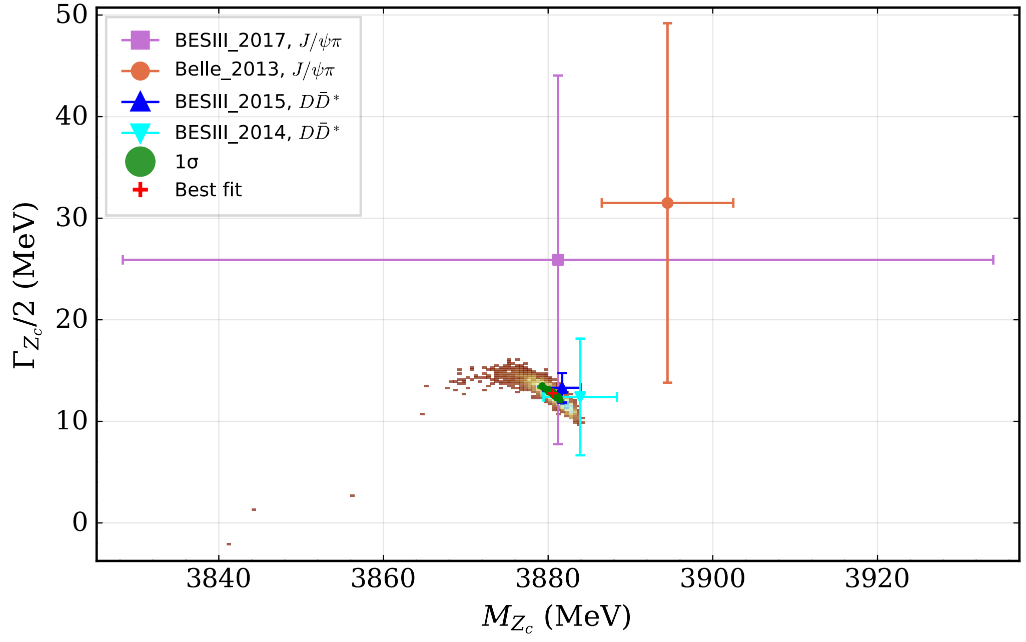
$$\sigma(e^+e^- \to P_c\bar{p}) \lesssim \frac{\sigma(e^+e^- \to J/\psi\, p\bar{p})}{\mathcal{B}(P_c \to J/\psi\, p)} = \mathcal{O}(0.1\text{pb})$$

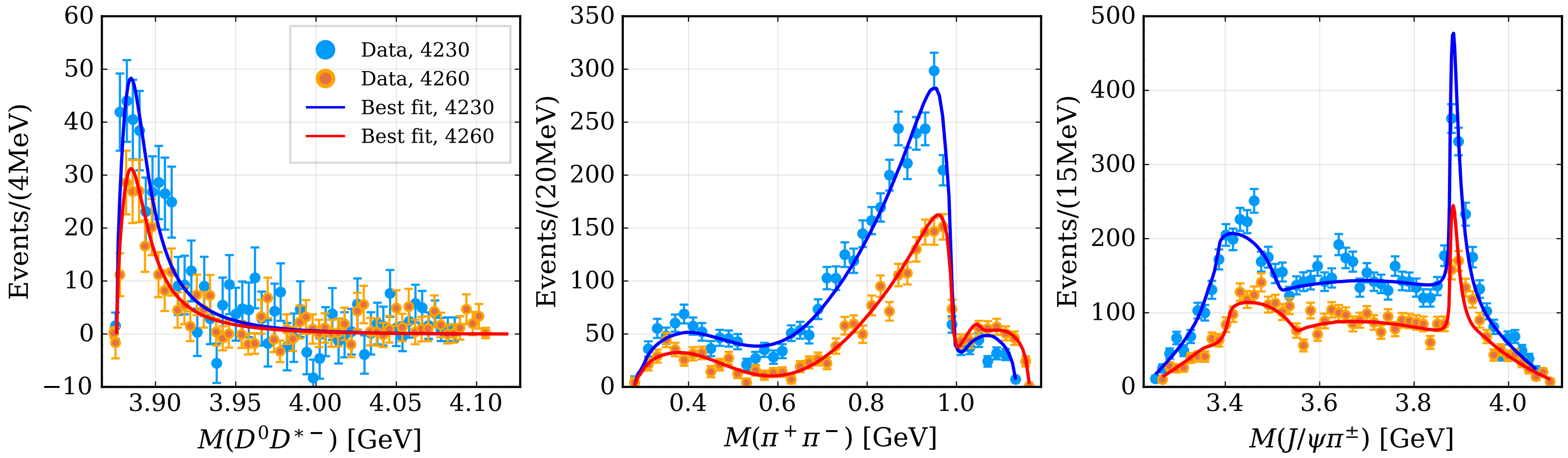
- Open-charm final states should have much larger cross sections + P_c states are expected to decay dominantly into open-charm states

Analysis w/ TS: example of the $Z_c(3900)$



Y.-H. Chen, M.-L. Du, FKG, arXiv:2310.15965





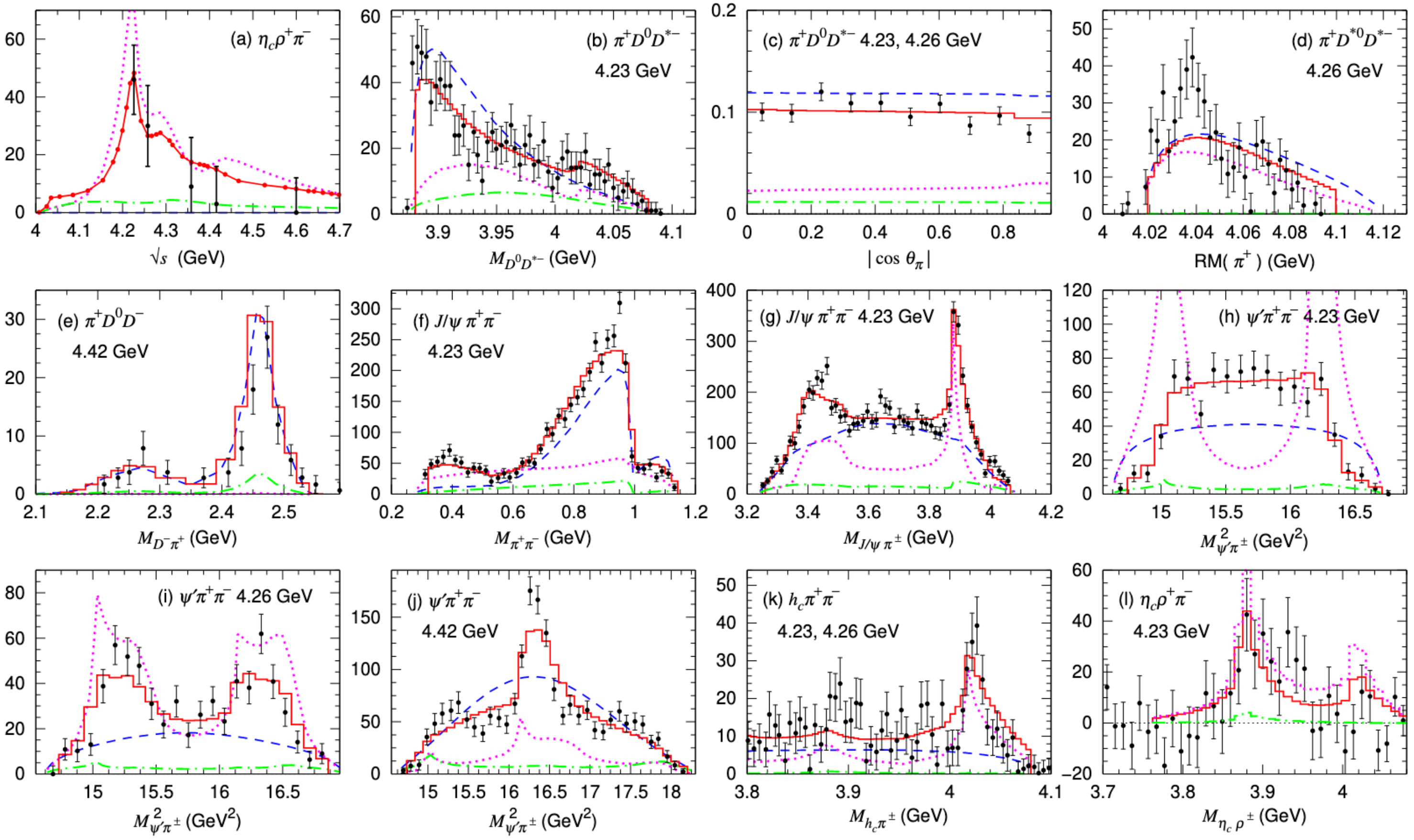
Conclusions:

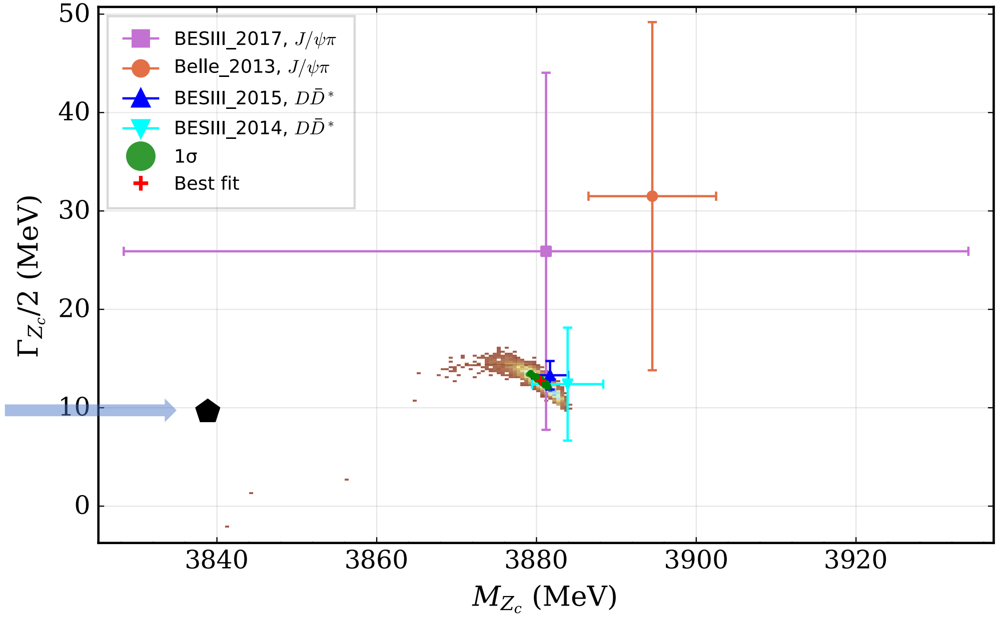
- needs a $Z_c(3900)$
- $D\bar{D}^*$ molecular and non-molecular components are of similar importance for $Z_c(3900)$

BESIII data: PRD 92 (2015) 092006; PRL 119 (2017) 072001

Analysis w/ TS: example of the $Z_c(3900)$

A global analysis with a phenomenological method S. Nakamura, X.-H. Li, H.-P. Peng, Z.-T. Sun, X.-R. Zhou, arXiv:2312.17658





+ many more e^+e^- cross section data

- ➢ more than 170 parameters
- ➢ error analysis?

Perspective at future tau-charm facilities

STCF CDR, Front. Phys. 19 (2024) 14701;
FKG, H.-P. Peng, J.-J. Xie, X.-R. Zhou, arXiv:2203.07141 (Snowmass)

- A rich program for hidden-charm resonances
  - High luminosity: to study the known vector states and Z_c, Z_{cs} in much more detail; their connections to X states
  - $E > 5\ \mathrm{GeV}$:
    - above thresholds of excited charm-meson pairs; to establish a hidden-charm family ⇒ a unified description
    - $e^+e^- \to \omega X(J^{\pm +}), \rho W_c$ (spin partners of Z_c)
    - $e^+e^- \to \phi X(J^{++})$, to study the heavier $PC = ++$ states observed in $\phi J/\psi$
    - $e^+e^- \to \eta X(J^{+-})$, to study the J^{++} and 1^{+-} states (spin multiplet structure is crucial)
    - production of hidden-charm states with exotic $J^{PC} = 0^{--}, 2^{+-}, 1^{-+}, 3^{-+}, \ldots$
    - $J/\psi p\bar{p}, \Lambda_c \bar{D}^{(*)}\bar{p}, \Sigma_c^{(*)}\bar{D}^{(*)}\bar{p}, \ldots$ accessible, hidden-charm pentaquarks, rich spectrum above $\Lambda_c \bar{D}$ threshold
  - Energy scan ⟹ handle of kinematic singularities in multi-hadron final states

Thank you for your attention!

Reviews in the last few years

- >>10 review articles:
  - H.-X. Chen et al., *The hidden-charm pentaquark and tetraquark states*, Phys. Rept. 639 (2016) 1
  - A. Hosaka et al., *Exotic hadrons with heavy flavors: X, Y, Z, and related states*, PTEP 2016 (2016) 062C01
  - J.-M. Richard, *Exotic hadrons: review and perspectives*, Few Body Syst. 57 (2016) 1185
  - R. F. Lebed, R. E. Mitchell, E. Swanson, *Heavy-quark QCD exotica*, PPNP 93 (2017)143
  - A. Esposito, A. Pilloni, A. D. Polosa, *Multiquark resonances*, Phys. Rept. 668 (2017) 1
  - FKG, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.-S. Zou, *Hadronic molecules*, RMP 90 (2018) 015004
  - A. Ali, J. S. Lange, S. Stone, Exotics: *Heavy pentaquarks and tetraquarks*, PPNP 97 (2017) 123
  - S. L. Olsen, T. Skwarnicki, *Nonstandard heavy mesons and baryons: Experimental evidence*, RMP 90 (2018) 015003
  - Y.-R. Liu et al., *Pentaquark and tetraquark states*, PPNP107 (2019) 237
  - N. Brambilla et al., *The XYZ states: experimental and theoretical status and perspectives*, Phys. Rept. 873 (2020) 154
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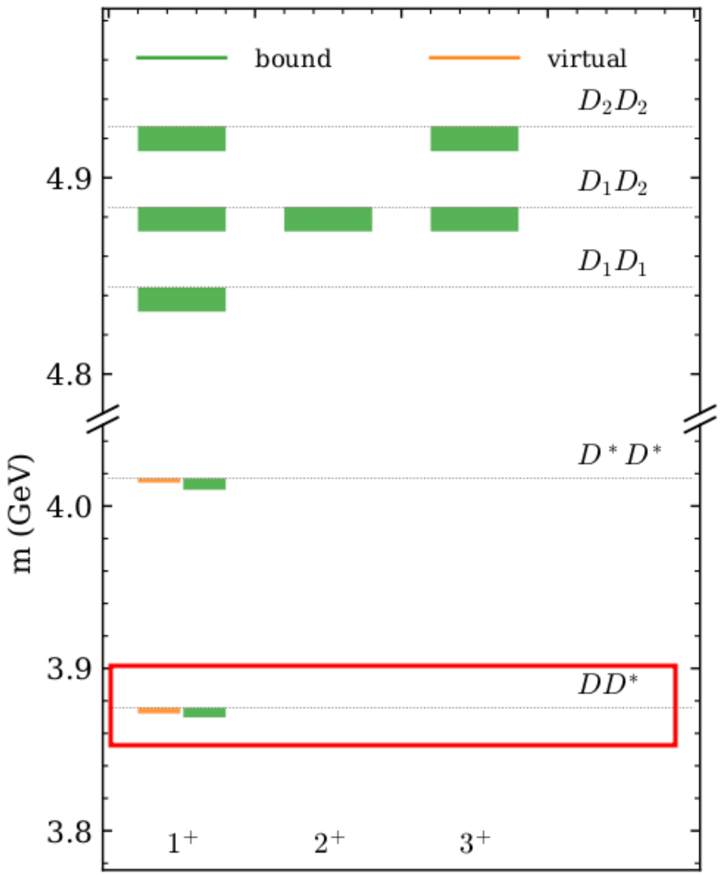
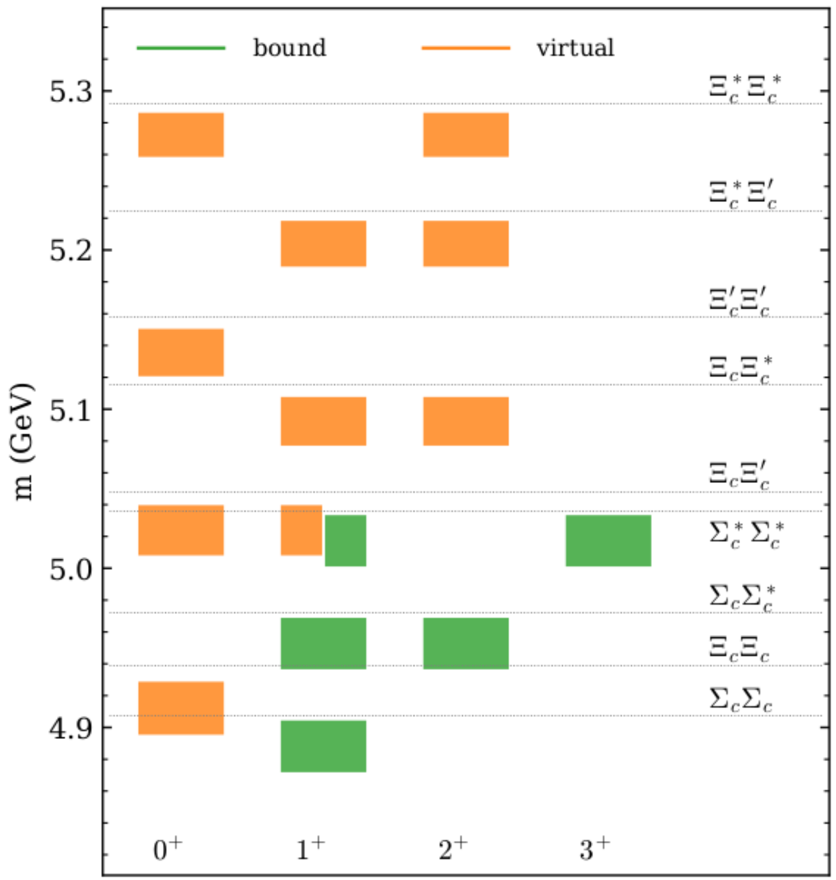
Double-charm tetraquarks and dibaryons

X.-K. Dong, FKG, B.-S. Zou, CTP 73 (2021) 125201

$(I, S, B) = (0,0,0)$

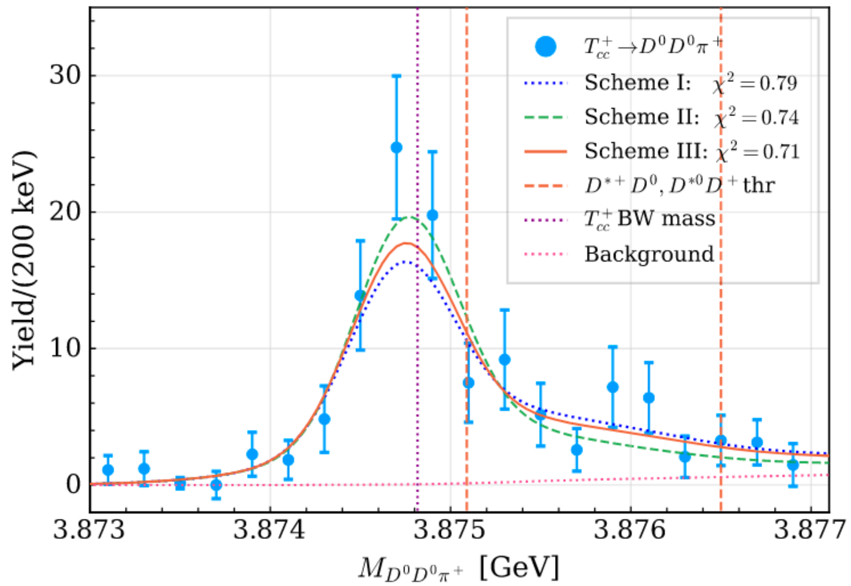


$(I, B) = (0,2)$



✓ $T_{cc}(3875)$ as D^*D molecule

✓ The LHCb data can be well described in a pionful EFT w/ 3-body effects



M.-L. Du et al., PRD 105 (2022) 014024

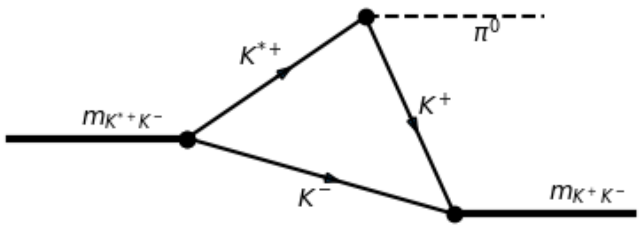
✓ isoscalar DD^* molecular state

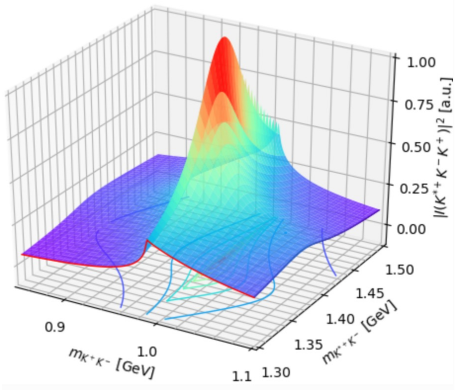
✓ It has a spin partner 1^+ D^*D^* state

✓ Many (> 100) other similar double-charm molecular states

More about TS

- Example: $K^*\bar{K}K$ triangle

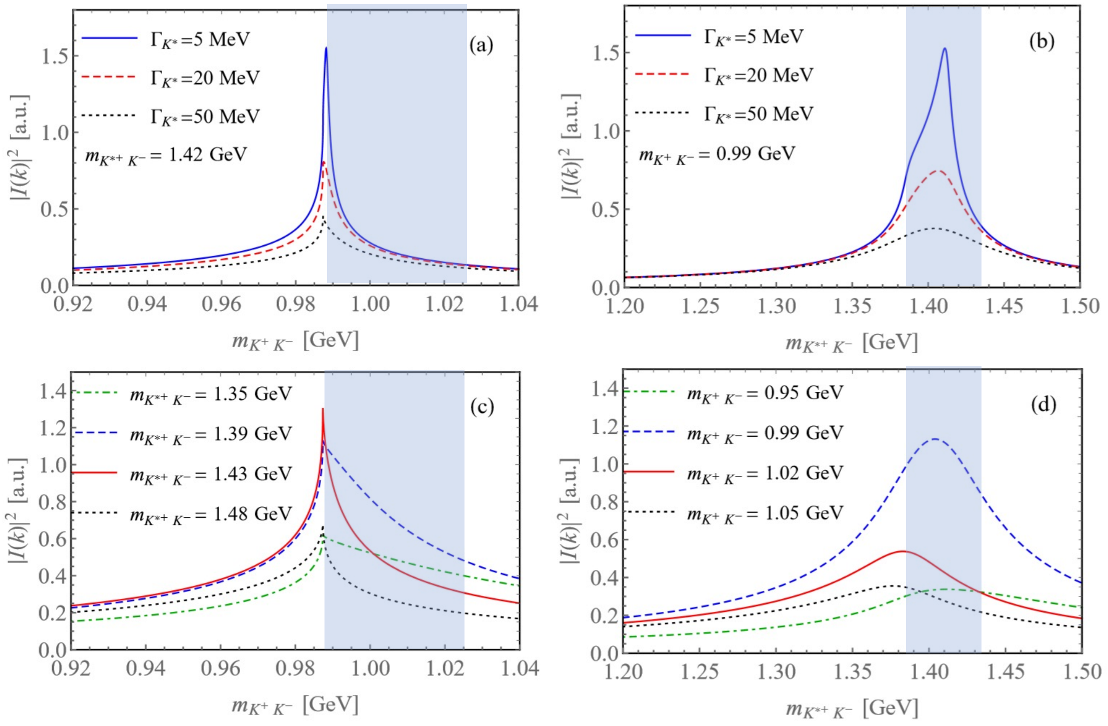




While a resonance would persist independent of energy.

- ➢ Counterclockwise argand plot, resembling that of a resonance

$K\bar{K}$ threshold



TS in the physical region