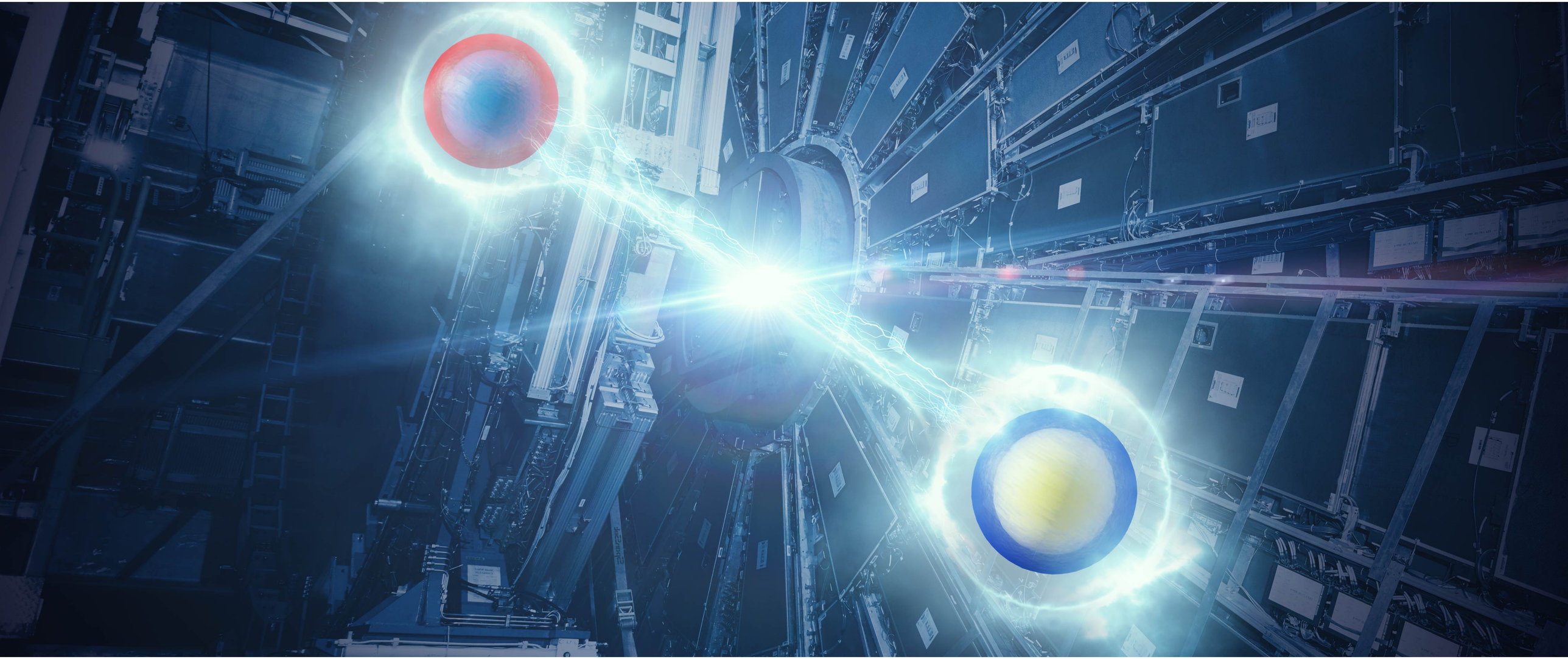
Quantum Entanglement and Quantum Nonlocality at High Energy Colliders

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For USTC “Jianwei” Seminar, Hefei, China

Not Beyond the Standard Model



The “Verschränkung”



Die gegenwärtige Situation in der Quantenmechanik.

Von E. SCHRÖDINGER, Oxford.

(Schluß[1].)

Inhaltsübersicht.

§ 11. Die Aufhebung der Verschränkung. Das Ergebnis abhängig vom Willen des Experimentators.
§ 12. Ein Beispiel.
§ 13. Fortsetzung des Beispiels: alle möglichen Messungen sind eindeutig verschränkt.
§ 14. Die Änderung der Verschränkung mit der Zeit. Bedenken gegen die Sonderstellung der Zeit.
§ 15. Naturprinzip oder Rechenkunstgriff?

§ 11. Die Aufhebung der „Verschränkung“. Das Ergebnis abhängig vom Willen des Experimentators.

Wir kehren wieder zum allgemeinen Fall der „Verschränkung“ zurück, ohne gerade den besonderen Fall eines Meßvorgangs im Auge zu haben, wie soeben. Die Erwartungskataloge zweier Körper A und B sollen sich durch vorübergehende Wechselwirkung verschränkt haben. Jetzt sollen die Körper wieder getrennt sein. Dann kann ich

kür des Experimentators anheimgestellt. Er braucht nicht besondere Variable auszuwählen, um die Konditionalsätze ausnützen zu können. Er darf sich ruhig einen Plan machen, der ihn zu maximaler Kenntnis von B führen würde, auch wenn er über B gar nichts wüßte. Es kann auch nichts schaden, wenn er diesen Plan zu Ende führt. Wenn er sich nach jeder Messung überlegt, ob er etwa schon am Ziel ist, so nur, um sich weitere überflüssige Arbeit zu ersparen.

Welcher A-Katalog sich solchermaßen indirekt ergibt, hängt selbstverständlich von den Maßzahlen ab, die an B auftreten (bevor die Verschränkung ganz gelöst ist; von den späteren, falls überflüssigerweise weitergemessen wird, nicht mehr). Gesetzt nun, ich hätte in einem bestimmten Fall auf solche Art einen A-Katalog erschlossen. Dann kann ich nachdenken und mir überlegen, ob

Quantum Entanglement

- Some basic facts of quantum entanglement.
- Is the state factorable when we separate the whole system into several parts (subsystems)?

$$|\Psi_{AB}\rangle \stackrel{?}{=} |\Psi_A\rangle \otimes |\Psi_B\rangle$$

- Example: the Bell states $|\Psi_{\pm}\rangle = 2^{-1/2}(|01\rangle \pm |10\rangle)$ are entangled (pure) states.
- The Neumann entropy: $S_{vN} = -\operatorname{tr}_A(\rho_A \log \rho_A)$, $\rho_A = \operatorname{tr}_B(|\Psi_{AB}\rangle\langle\Psi_{AB}|)$.

Quantum Entanglement

- Some basic facts of quantum entanglement.
- Is the state factorable when we separate the whole system into several parts (subsystems)?

$$\rho_{AB} \overset{?}{=} \sum_{i=1}^{N} p_i \rho_A^{(i)} \otimes \rho_B^{(i)}, \quad \left(\sum_{i}^{N} p_i = 1,\, p_i > 0 \right)$$

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- Not so easy.
- Bell states $\rho_\pm = |\Psi_\pm\rangle\langle\Psi_\pm| = 2^{-1}(|01\rangle\langle 01| \pm |10\rangle\langle 01| \pm |01\rangle\langle 10| + |10\rangle\langle 10|)$ are entangled (pure) states.

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- But their mixing $(\rho_+ + \rho_-)/2 = (|0\rangle\langle 0| \otimes |1\rangle\langle 1| + |1\rangle\langle 1| \otimes |0\rangle\langle 0|)/2$ is **not**!

Quantum Entanglement

- If the final state is a mixed state. How to verify the entanglement in the final state?

- Entanglement witness:

  *A **functional** which can distinguish a **specific** entangled state from separable ones.*

- Theorem: For every entangled state ρ, there exists a Hermitian operator A such that $\mathrm{Tr}(\rho A) < 0$, and $\mathrm{Tr}(\sigma A) \geqslant 0$ for all separable states σ.

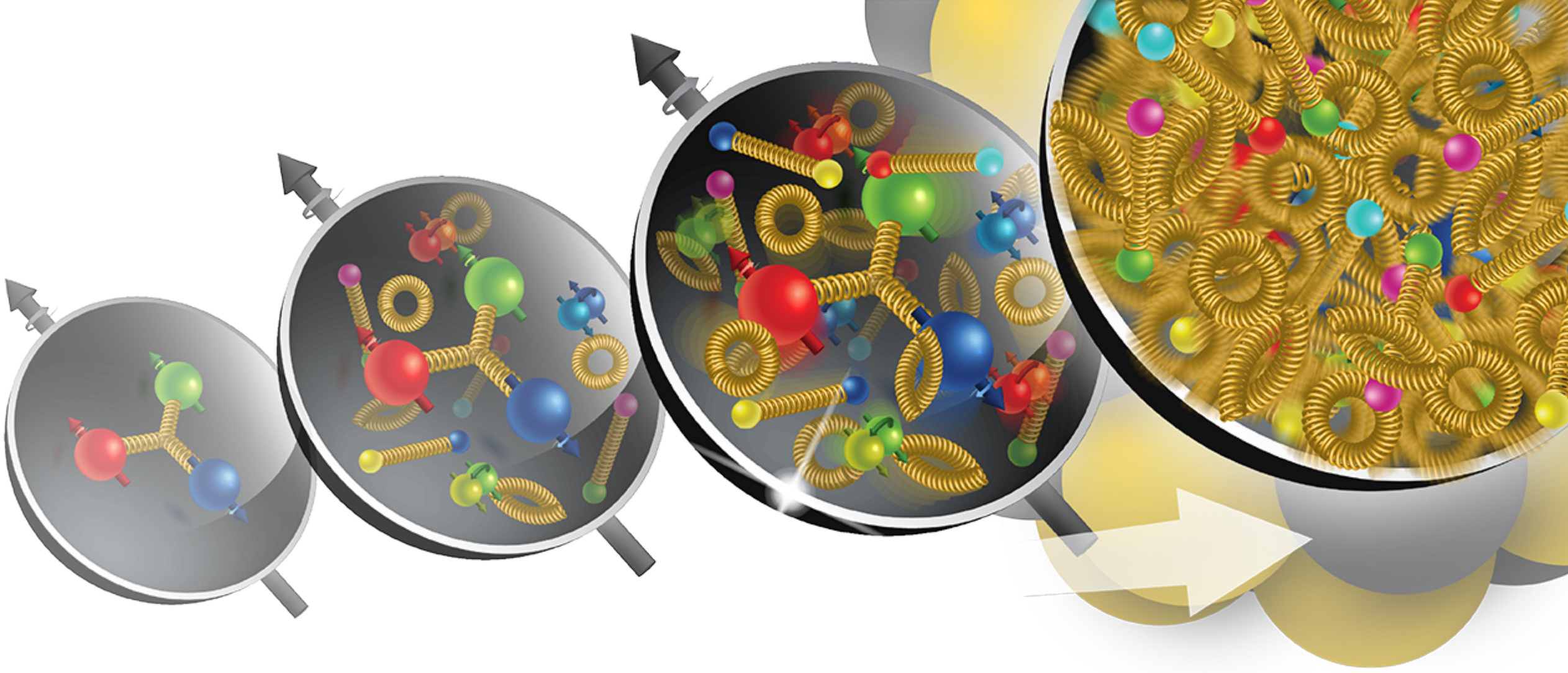
Quantum Entanglement

- If the final state is a mixed state. How to verify the entanglement in the final state?
- Entanglement witness:

  A functional that distinguishes entangled states from separable states

  The smart mathematicians can always tell us whether the answer we want exists, but never tell us how to find it.

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Quantum Entanglement

- The final state is a mixed state. How to verify the entanglement in the final state?
- For 2×2 and 2×3 system, it is solved.
- The concurrence: $\mathscr{C}[\hat{\rho}] = \max\{0,\, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\},\ \ 0 \leqslant \mathscr{C}[\hat{\rho}] \leqslant 1$

$$\lambda_i \in \sigma\left(\sqrt{\sqrt{\hat{\rho}}(\sigma_2 \otimes \sigma_2)\hat{\rho}^*(\sigma_2 \otimes \sigma_2)\sqrt{\hat{\rho}}}\right)$$

- A state is entangled **iff** $\mathscr{C}[\hat{\rho}] > 0$.

S. A. Hill, W. K. Wootters, Phys. Rev. Lett. 78 (1997) 5022.

Quantum Entanglement at Colliders

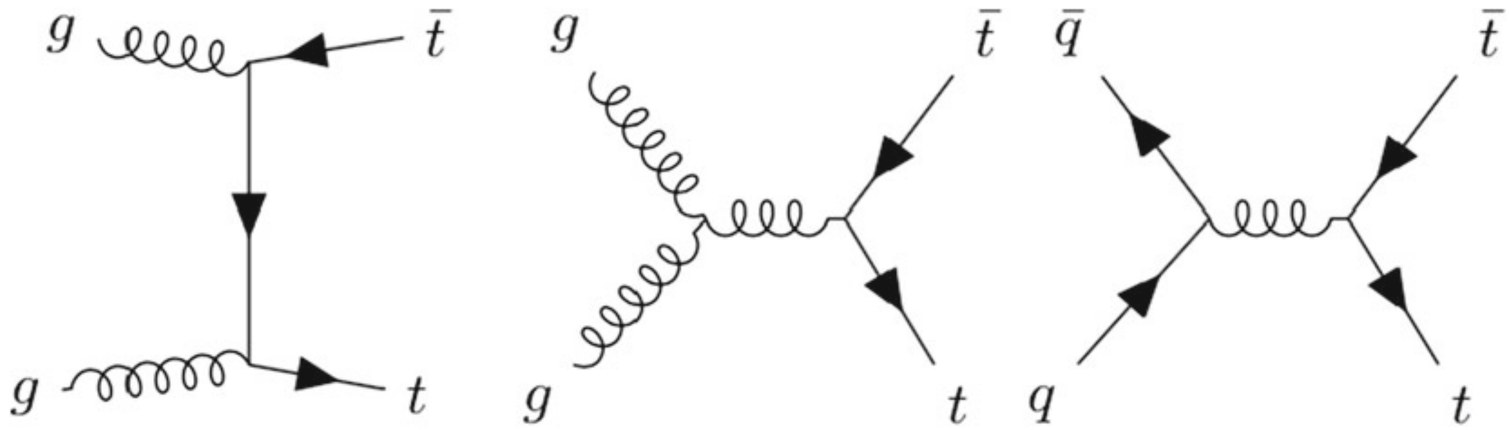
- Taking the top-antitop pair production as an example.
- One needs to reconstruct the “density matrix” in some basis in the spin space, e.g., $\{|t_\uparrow \bar{t}_\uparrow\rangle, |t_\uparrow \bar{t}_\downarrow\rangle, |t_\downarrow \bar{t}_\uparrow\rangle, |t_\downarrow \bar{t}_\downarrow\rangle\}$.
- Such a density matrix can be decomposed into

$$\rho = \frac{\mathbb{I}_2 \otimes \mathbb{I}_2 + B(t)_i \sigma^i \otimes \mathbb{I}_2 + B(\bar{t})_i \mathbb{I}_2 \otimes \sigma^i + C_{ij} \sigma^i \otimes \sigma^j}{4}$$

- The concurrence of the “state” is calculated with this “density matrix”.

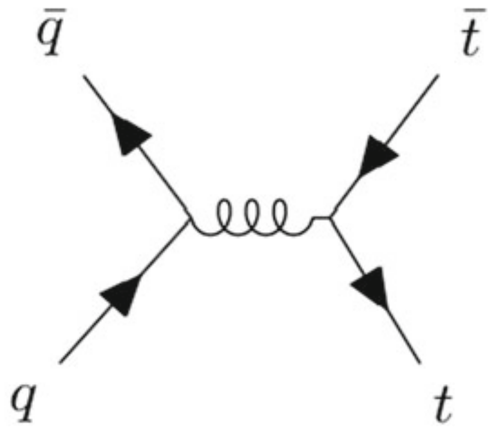
Quantum Entanglement at Colliders

- An example: top-pair production.
- At the Large Hadron Collider:



Quantum Entanglement at Colliders

- An example: top-pair production.
- At the Large Hadron Collider:



- s-channel:

  highly suppressed at high energy region

- quark-antiquark initial state:

  less in small-x region;

  boosted c.m. frame at high energy region.

Quantum Entanglement at Colliders

- An example: top-pair production.
- At the Large Hadron Collider:
  - Entanglement from quark-antiquark initial state

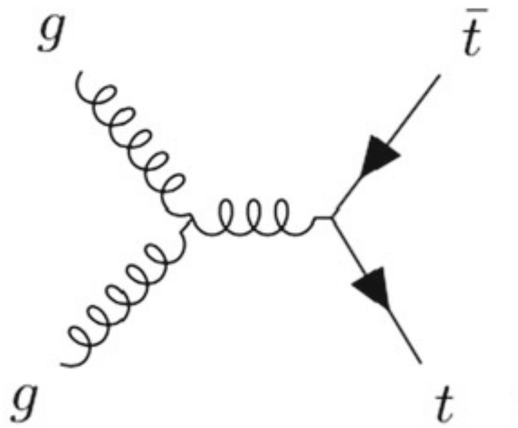


$$|t\bar{t}\rangle \sim (1 \pm \cos\theta)\,|\uparrow_{\hat{n}}\uparrow_{\hat{n}}\rangle + (1 \mp \cos\theta)\,|\downarrow_{\hat{n}}\downarrow_{\hat{n}}\rangle + \mathcal{O}(\gamma^{-1})$$

un(longitudinal)polarized initial state fermions, both left- and right-handed interactions

Quantum Entanglement at Colliders

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- s-channel:

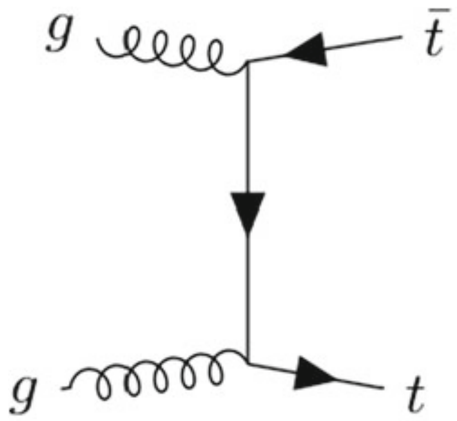
  highly suppressed at high energy region

- gluon-gluon initial state:

  more in small-x region.

Quantum Entanglement at Colliders

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- t-channel:

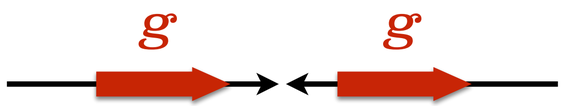
  dominant at high energy region

- gluon-gluon initial state:

  more in small-x region.

Quantum Entanglement at Colliders

- An example: top-pair production.
- At the Large Hadron Collider:
  - Entanglement from gluon-gluon initial state



$(+-)$

$$|t\bar{t}\rangle \sim (1 \pm \cos\theta)\,|\uparrow_{\hat{n}}\uparrow_{\hat{n}}\rangle + (1 \mp \cos\theta)\,|\downarrow_{\hat{n}}\downarrow_{\hat{n}}\rangle + \mathcal{O}(\gamma^{-1})$$

with another p-wave suppression factor.

Quantum Entanglement at Colliders

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- At the Large Hadron Collider:
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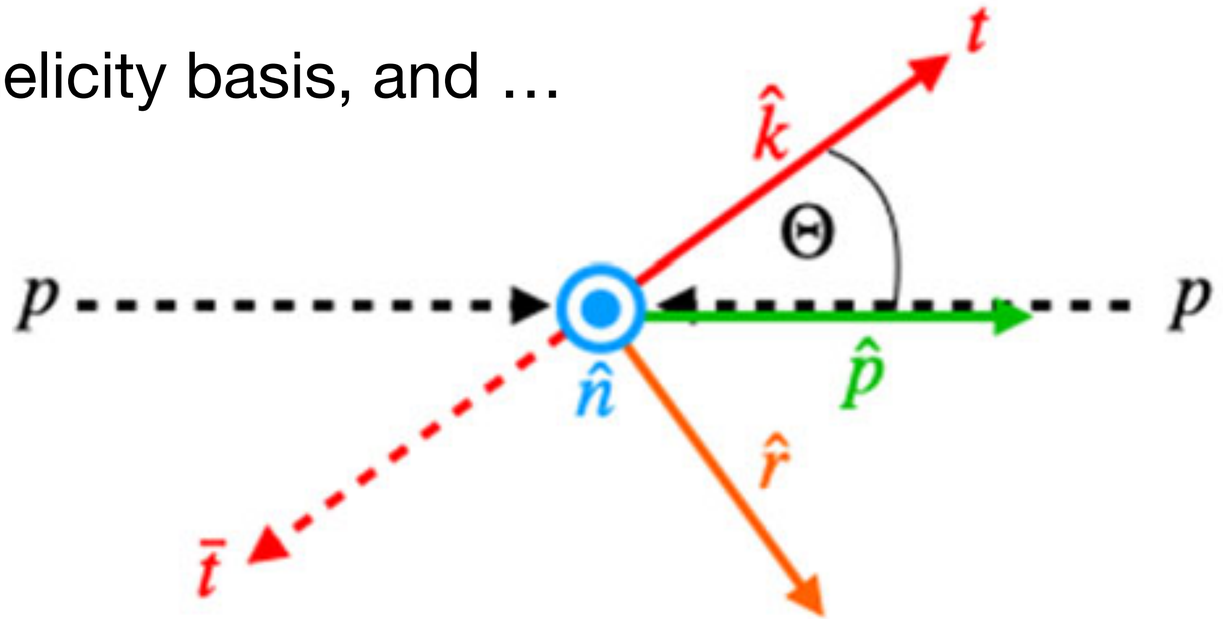
$$|t\bar{t}\rangle \sim (1 \pm \beta)\,|\uparrow_{\hat{n}}\downarrow_{\hat{n}}\rangle - (1 \mp \beta)\,|\downarrow_{\hat{n}}\uparrow_{\hat{n}}\rangle$$

Unfortunately, the direction $\hat{n}$ is different event by event.

Quantum Entanglement at Colliders

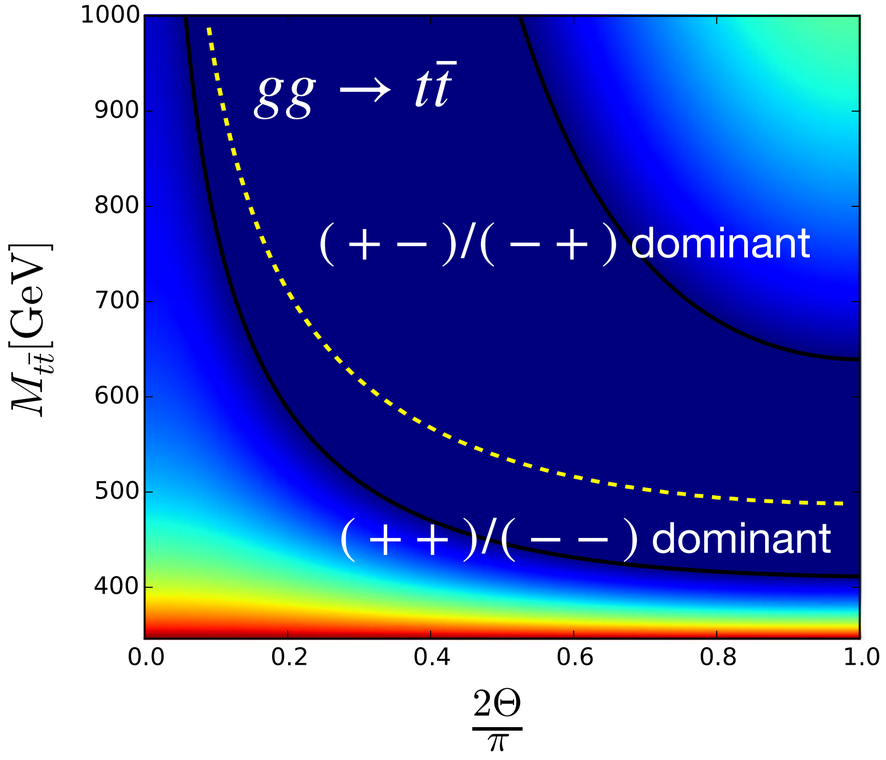
- An example: top-pair production.
- At the Large Hadron Collider: helicity basis, and …

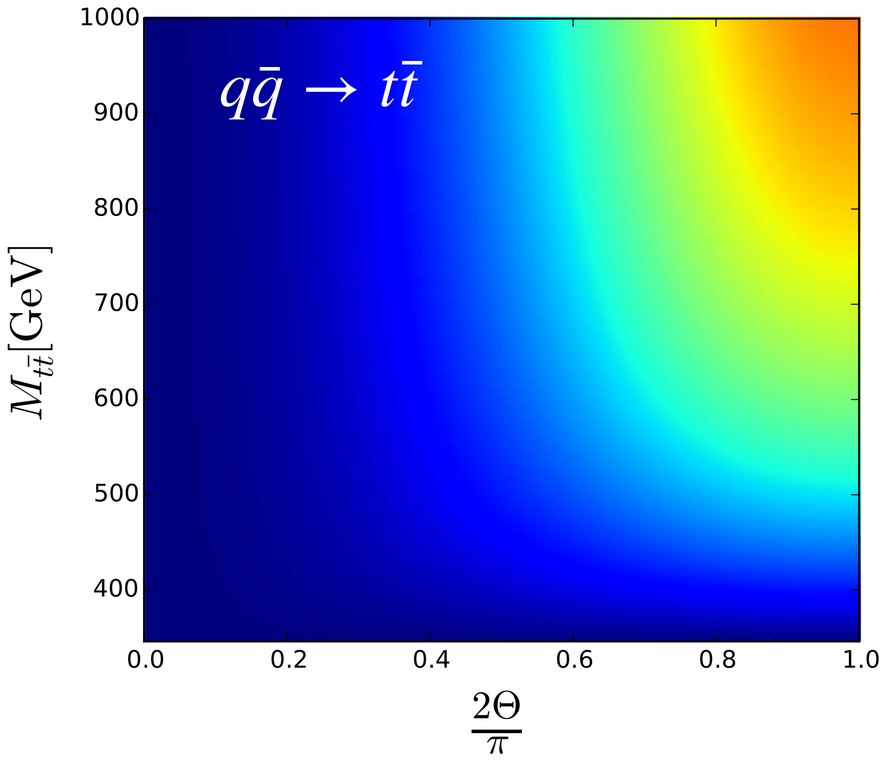
Fictitious State!



Quantum Entanglement at Colliders

- An example: top-pair production.
- At the Large Hadron Collider: helicity basis

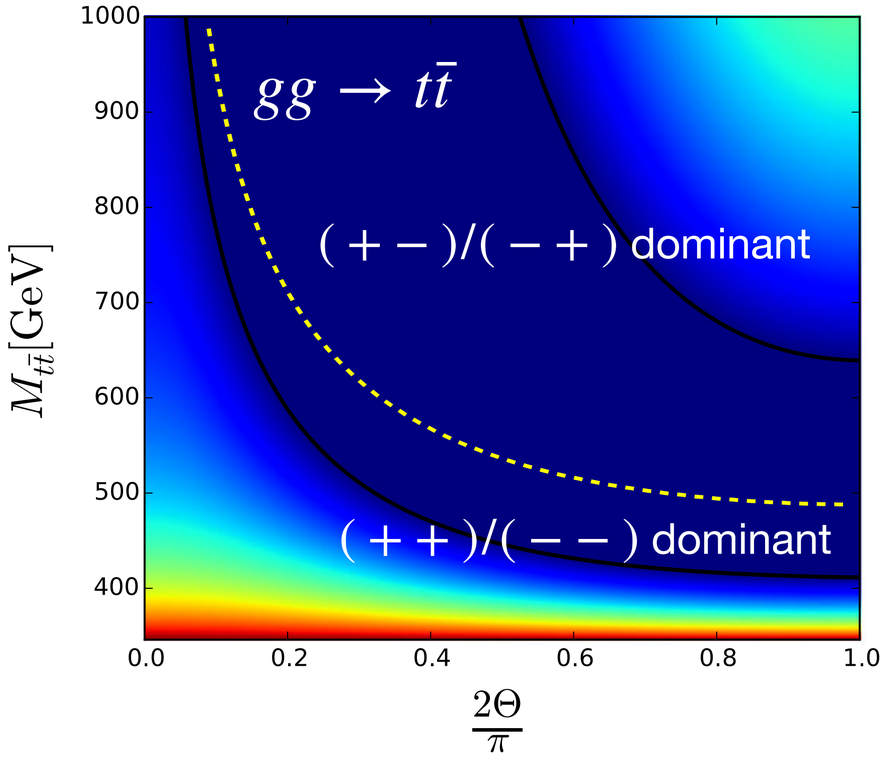


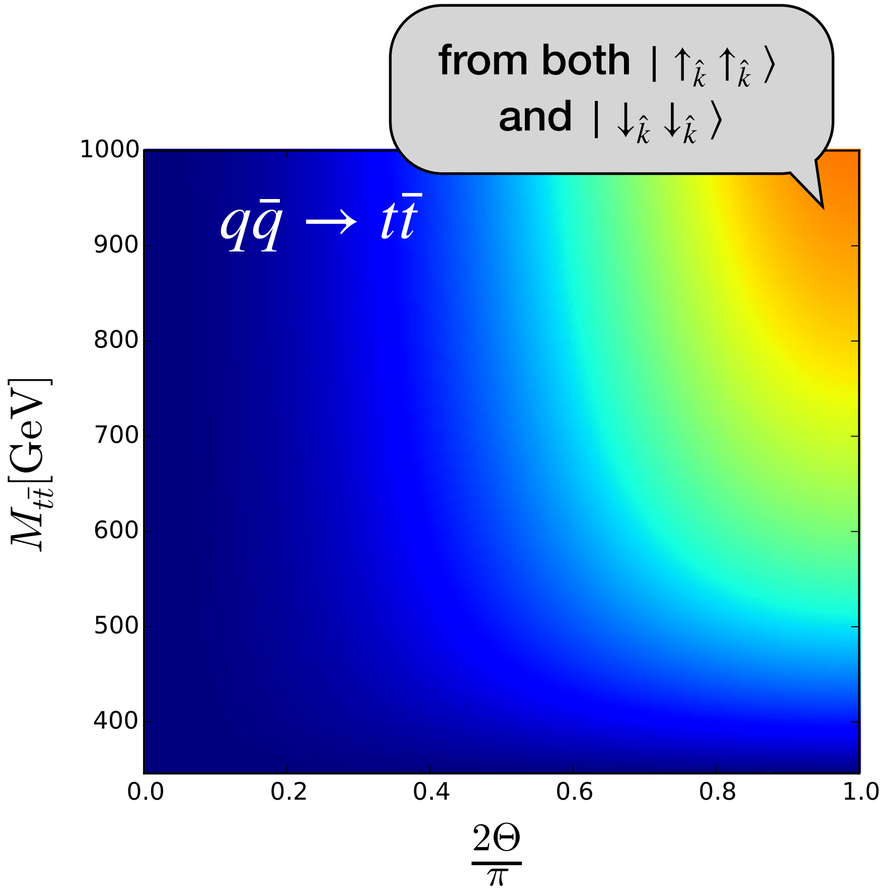


Y. Afik, J. R. M. de Nova, Eur. Phys. J. Plus, (2021) 136:907.

Quantum Entanglement at Colliders

- An example: top-pair production.
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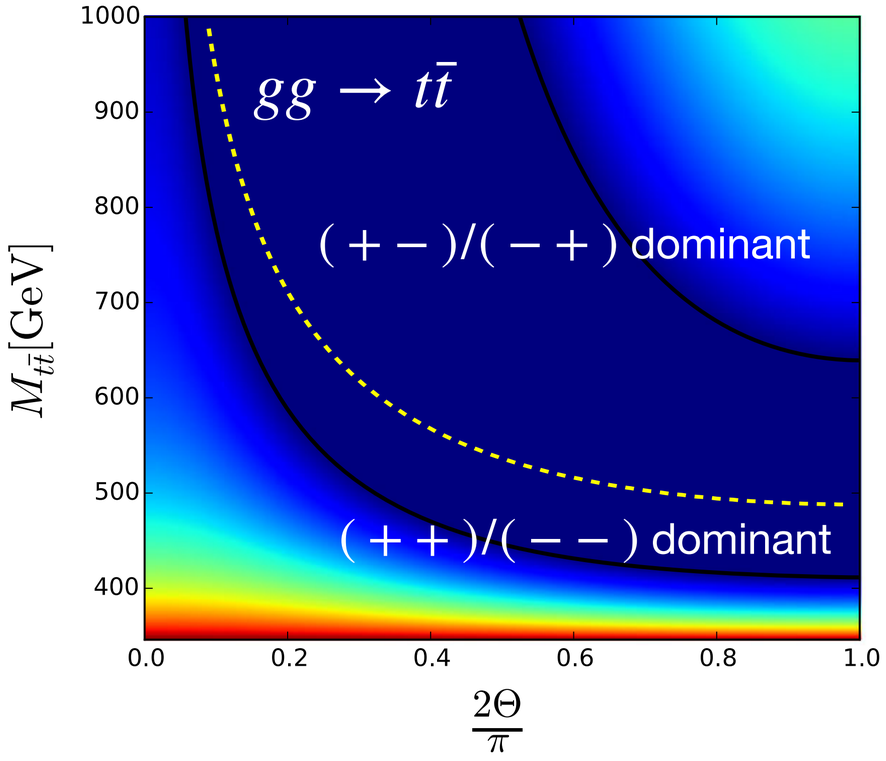


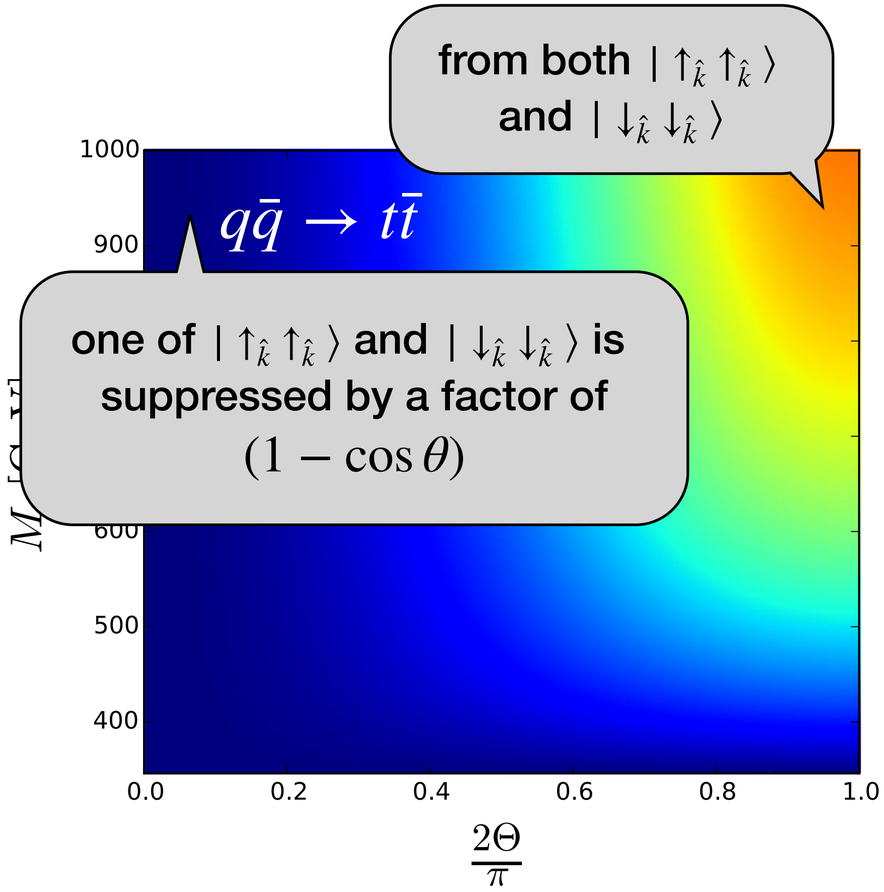


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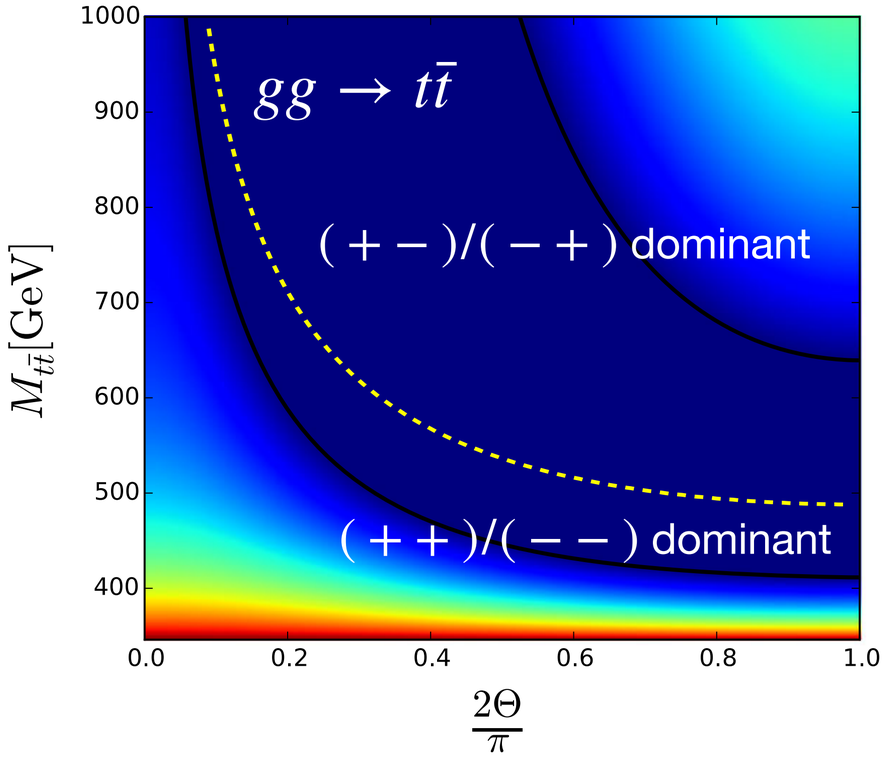


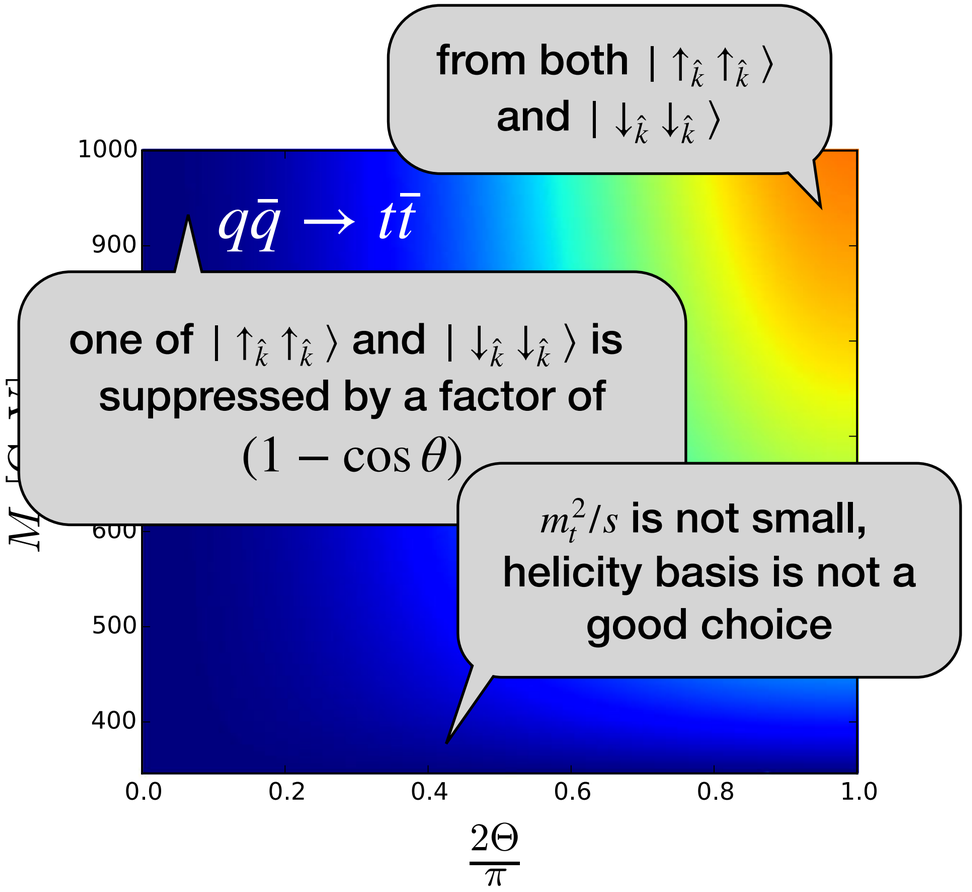


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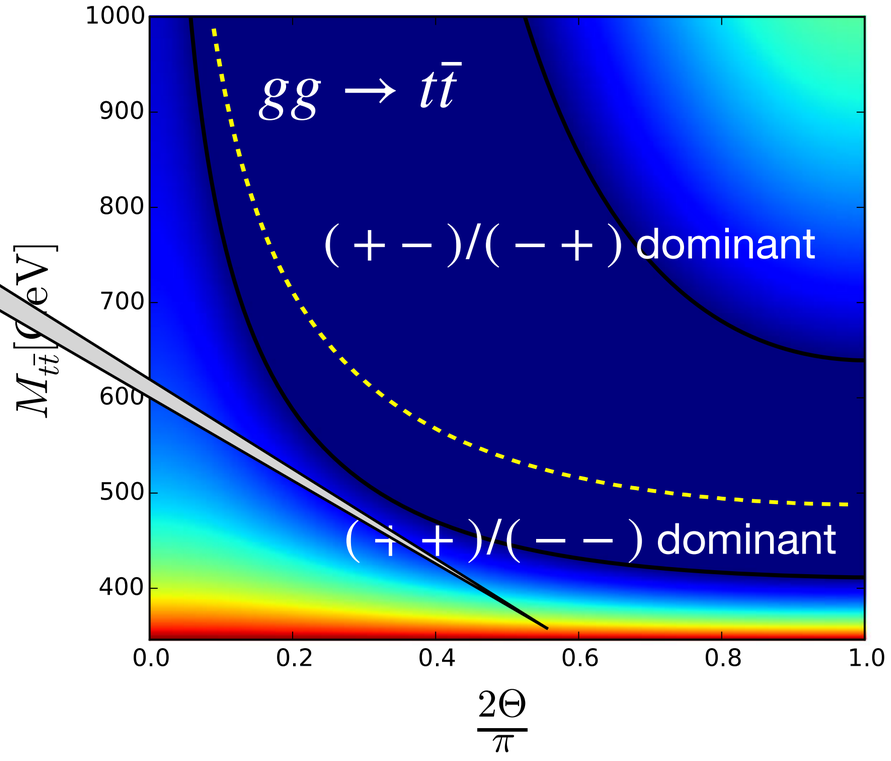


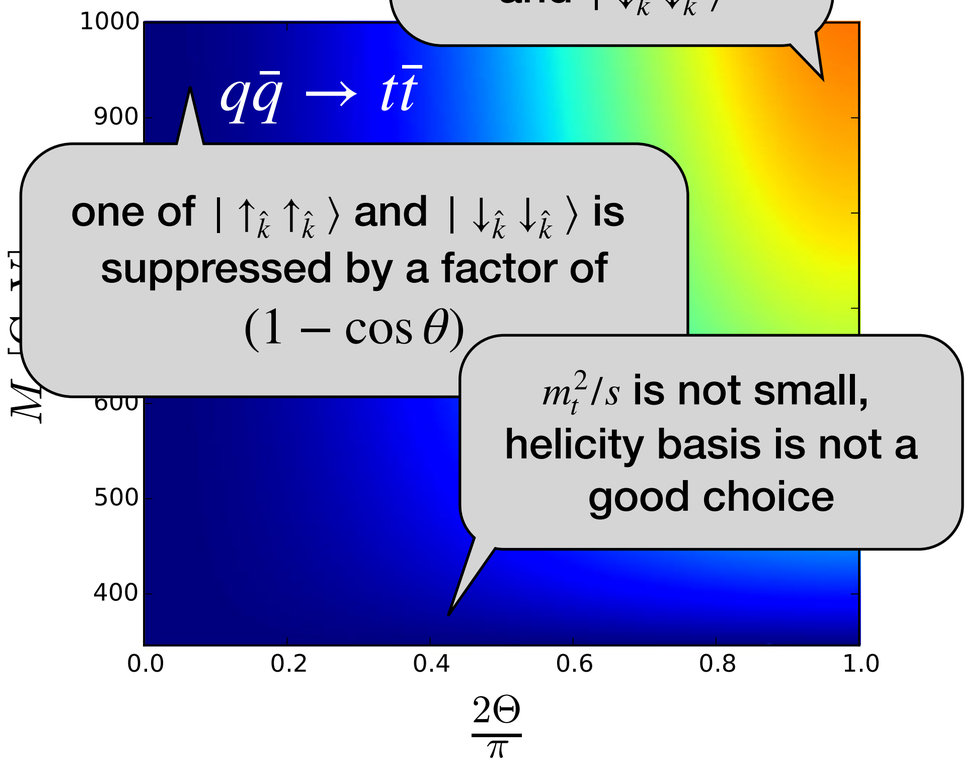
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Quantum Entanglement at Colliders

- An example: top-pair production.
- At the Large Hadron Collider: helicity basis

The most important contribution to the entanglement at the LHC

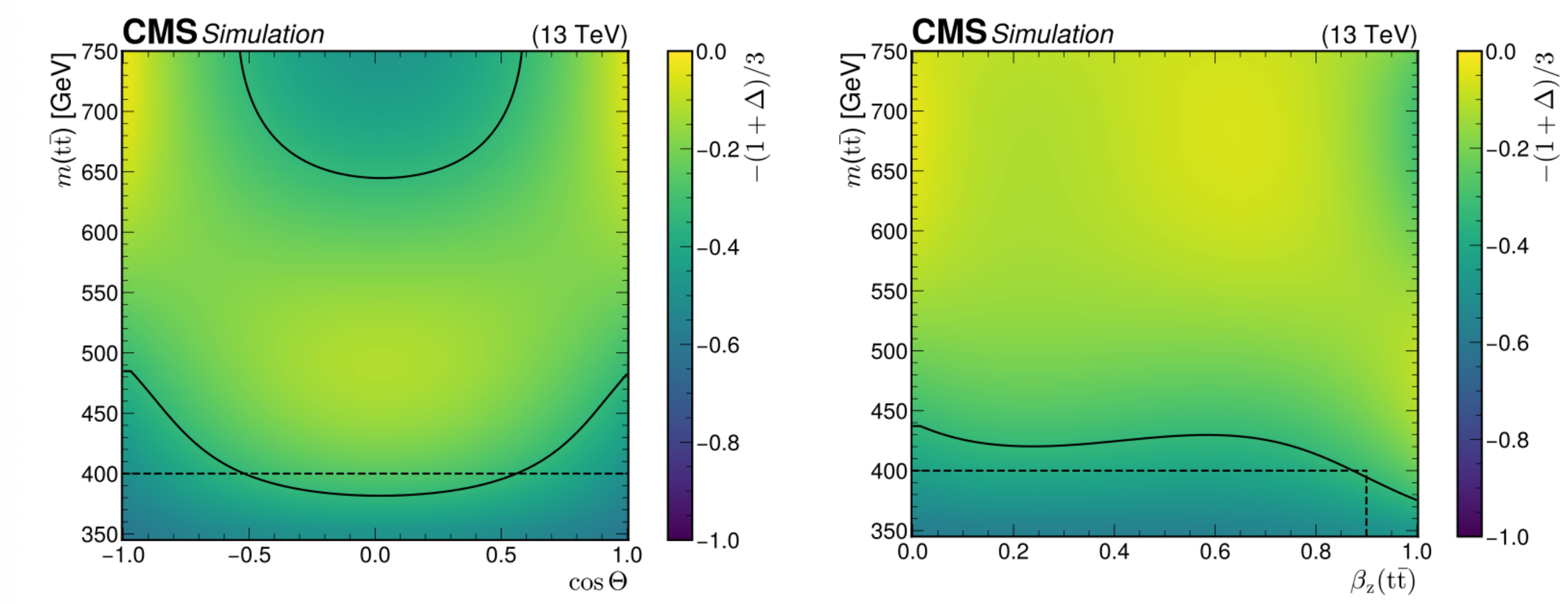




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Quantum Entanglement at Colliders

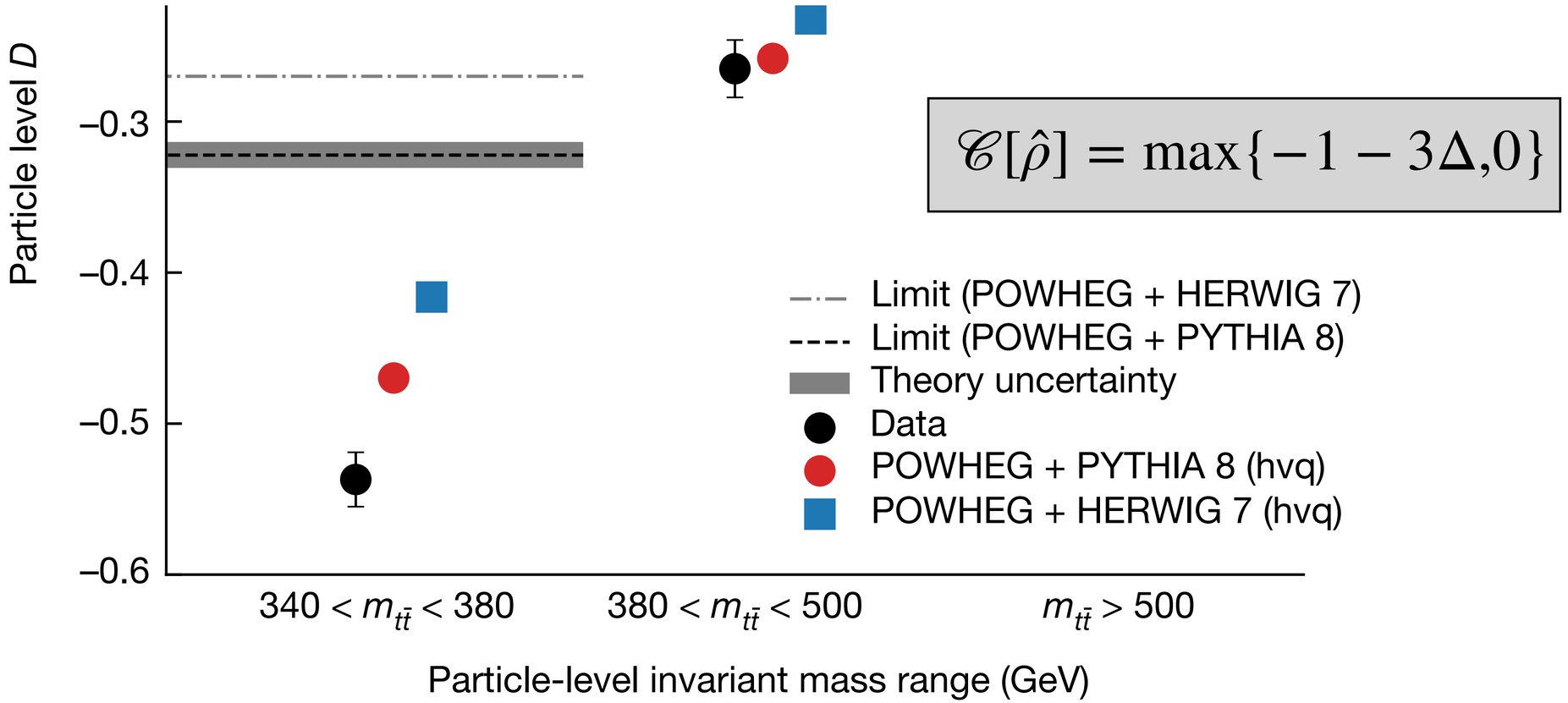
- An example: top-pair production.
- At the Large Hadron Collider: helicity basis



CMS collaboration, Rep. Prog. Phys. 87 (2024) 117801.

Quantum Entanglement at Colliders

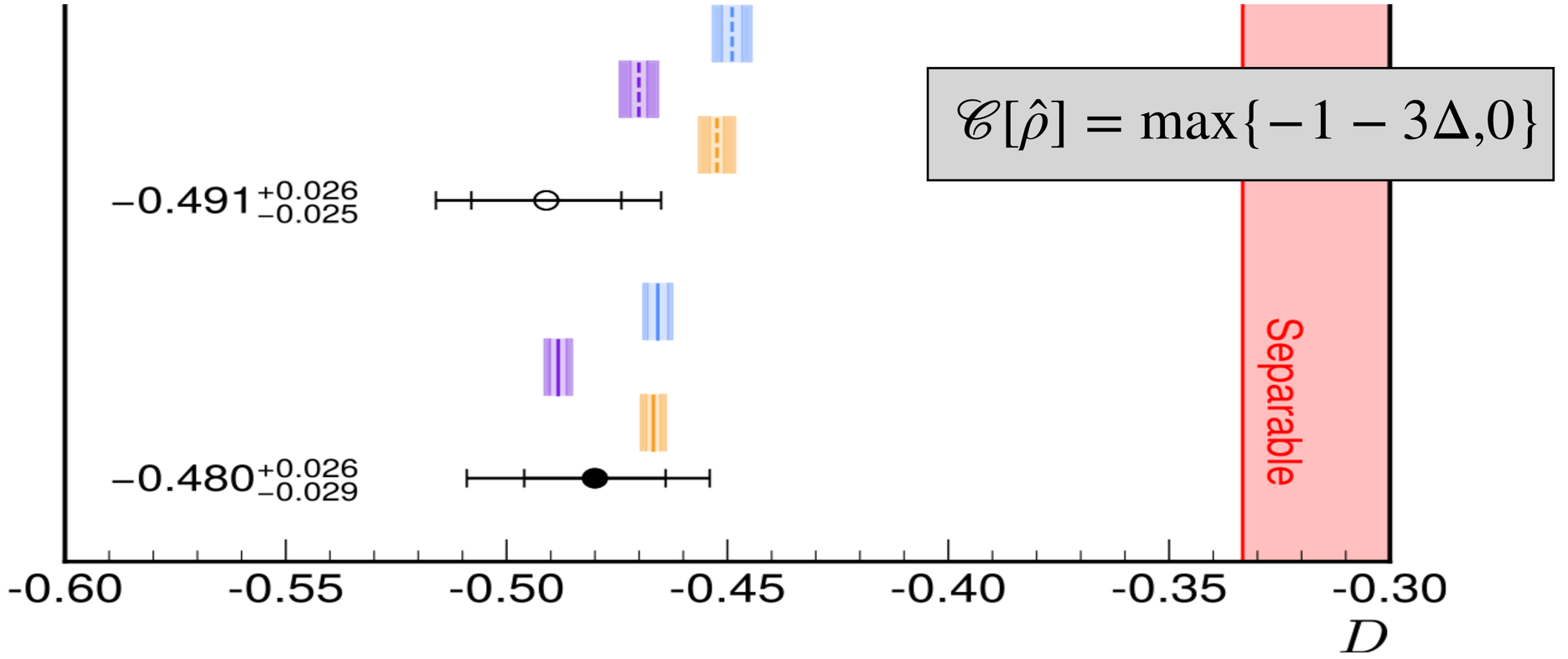
- The result



The ATLAS Collaboration, Nature 633 (2024) 542-547.

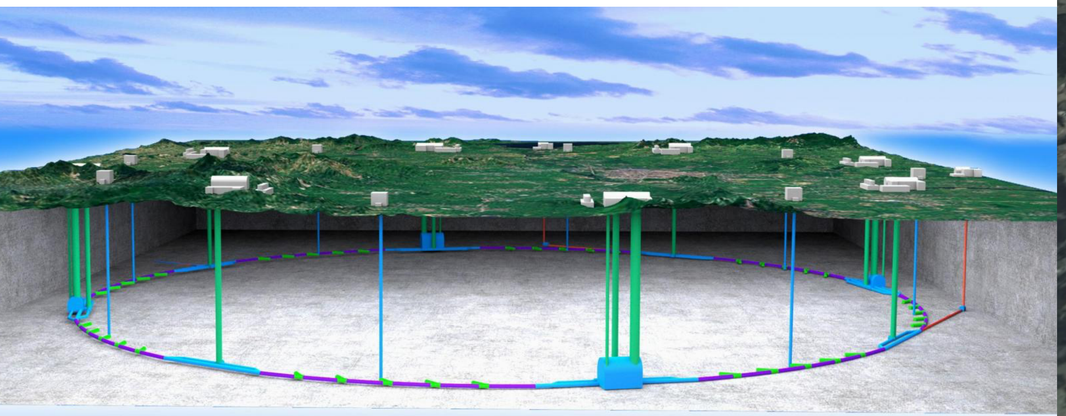
Quantum Entanglement at Colliders

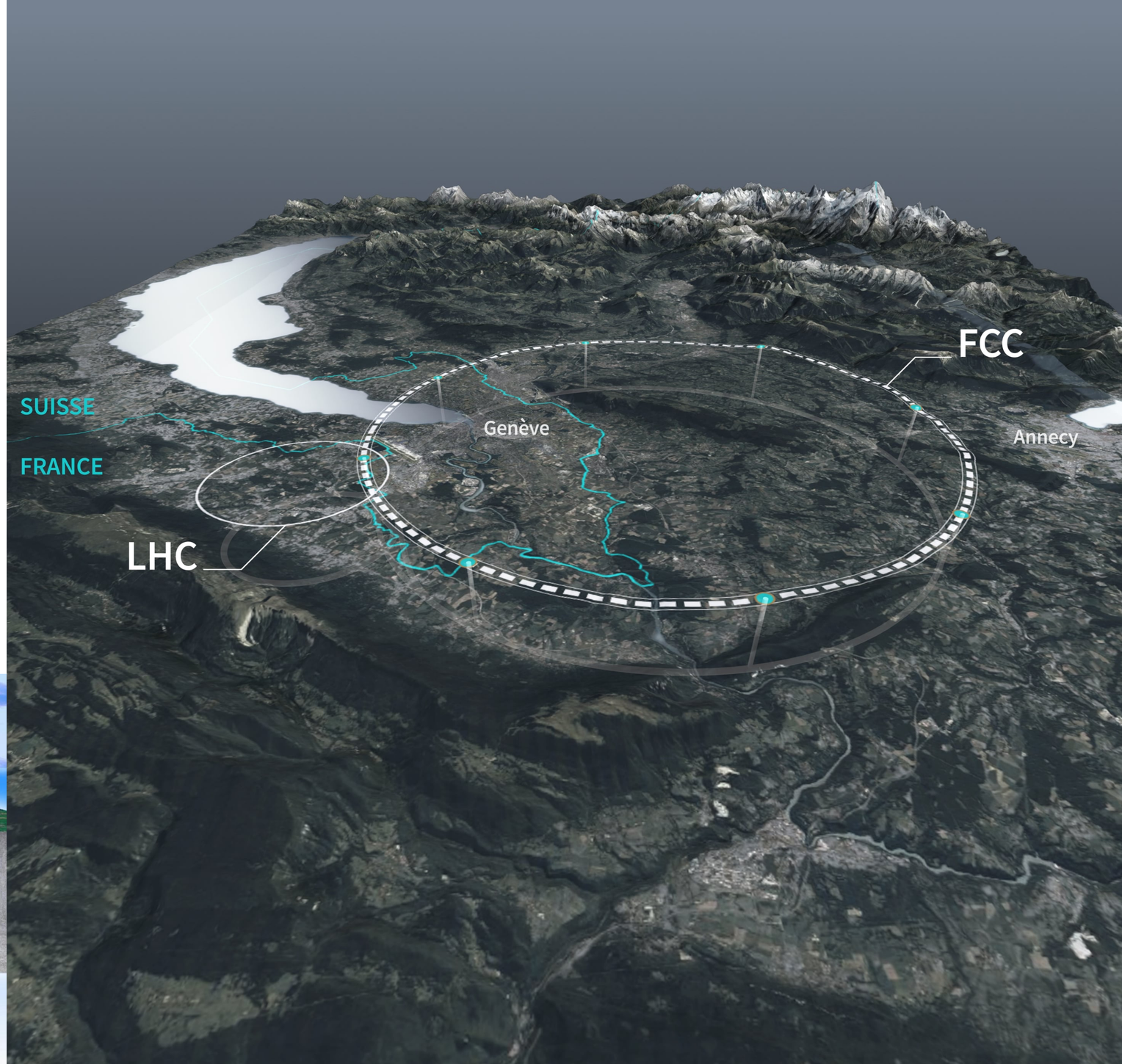
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The CMS Collaboration, Rep. Prog. Phys. 87 (2024) 117801.

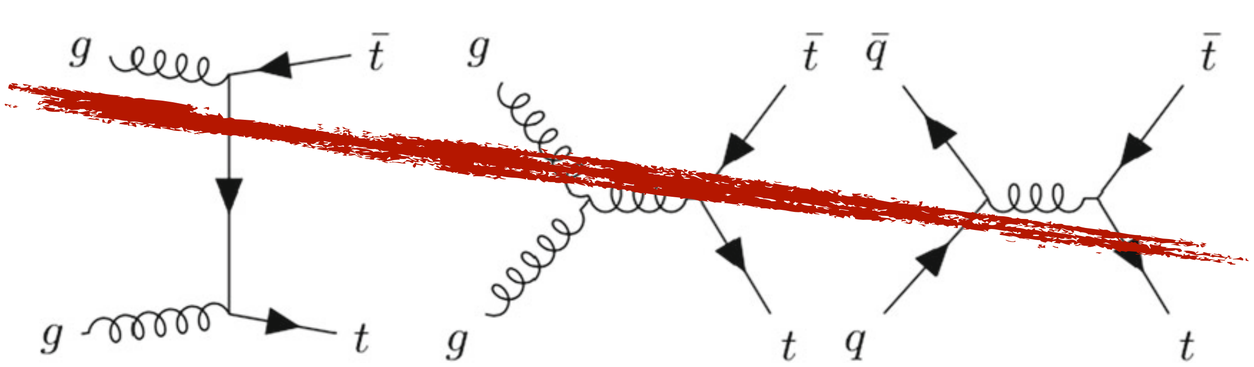
How about Higgs Factory?

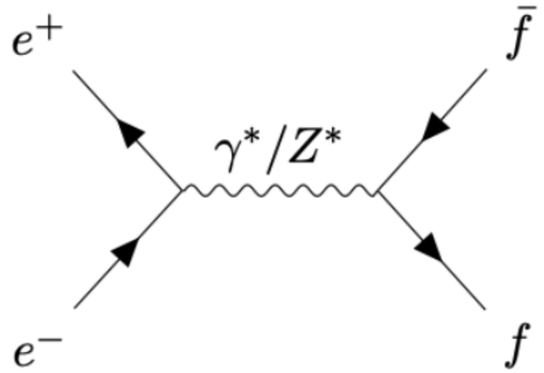




Quantum Entanglement at Colliders

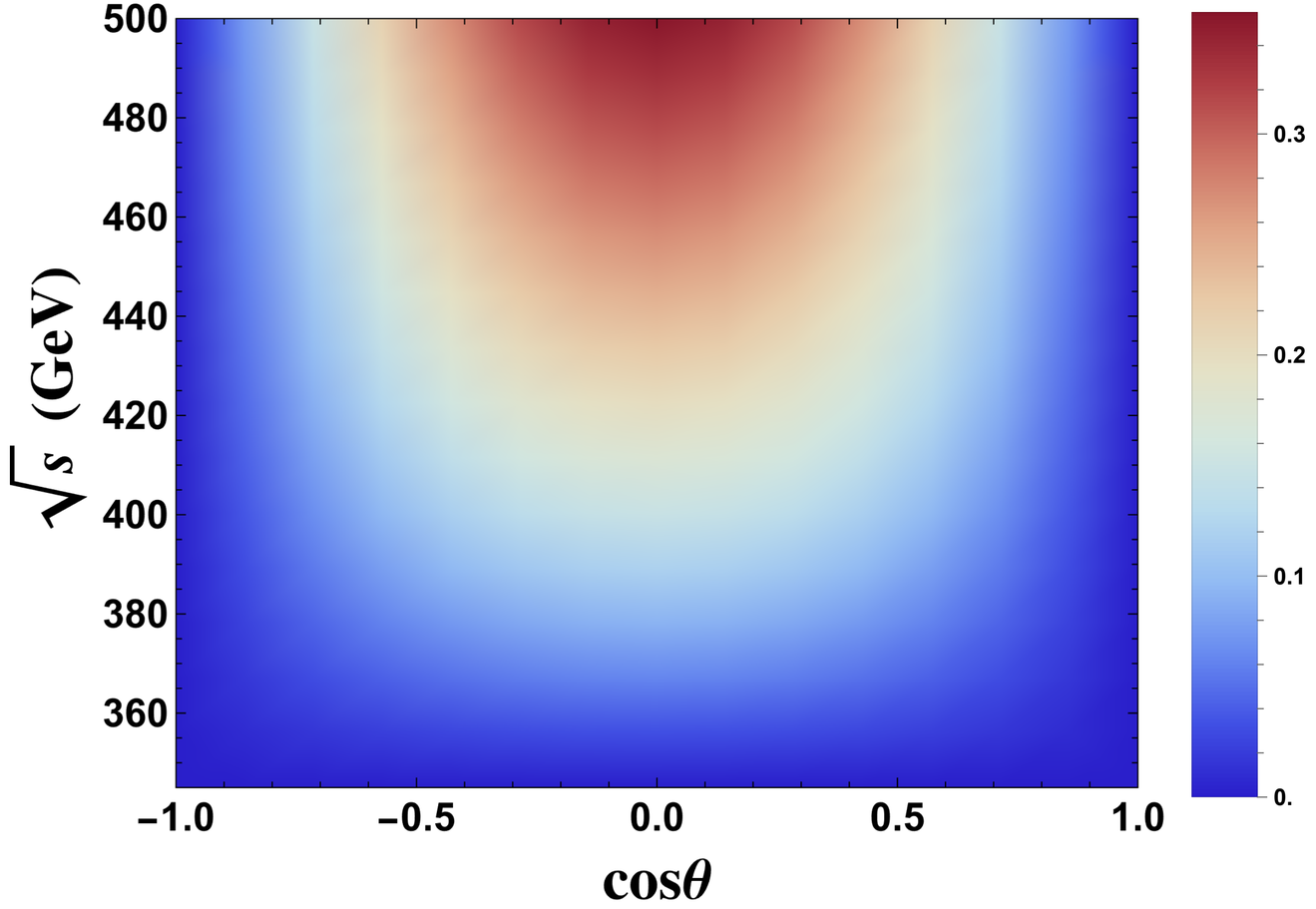
- The advantage at lepton colliders.
  - Simpler and clear contribution:





Quantum Entanglement at Colliders

- The advantage at lepton colliders.
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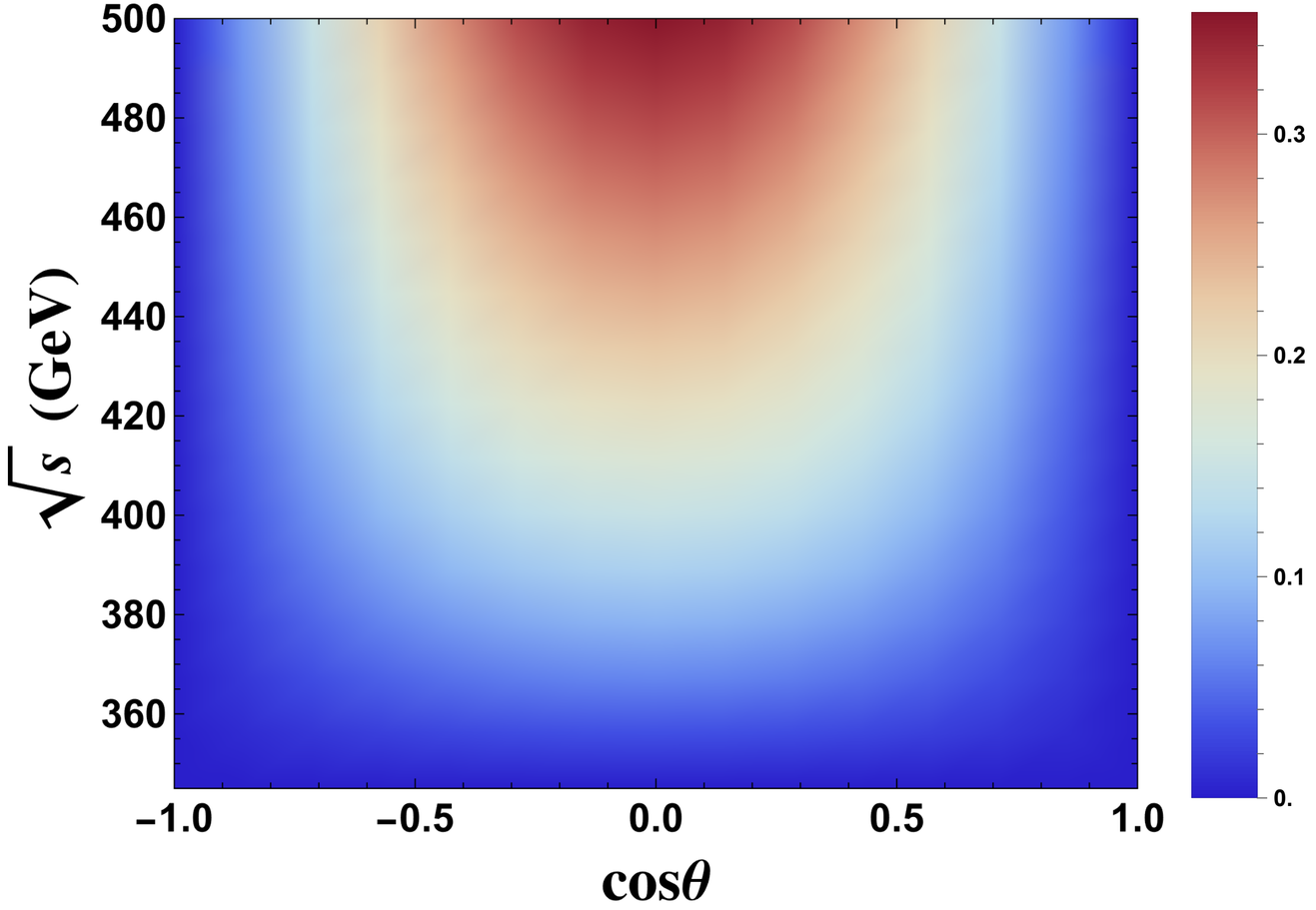
For simplicity and clearly, we do not show the contribution from the Z-boson in this figure, which will shift the distribution to left a little.

Quantum Entanglement at Colliders

- The advantage at lepton colliders.
  - Simpler and clear contribution:

The top-quarks are not energetic enough

The helicity basis is not good enough



For simplicity and clearly, we do not show the contribution from the Z-boson in this figure, which will shift the distribution to left a little.

Quantum Entanglement at Colliders

- It is helpful to use the transverse polarized initial state to increase the degree of the entanglement.
- In this case, the initial state is a Bell state selected by the interaction:

$$\frac{1}{2}(|\uparrow\rangle + |\downarrow\rangle) \otimes (|\uparrow\rangle + |\downarrow\rangle) = \frac{1}{2}(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

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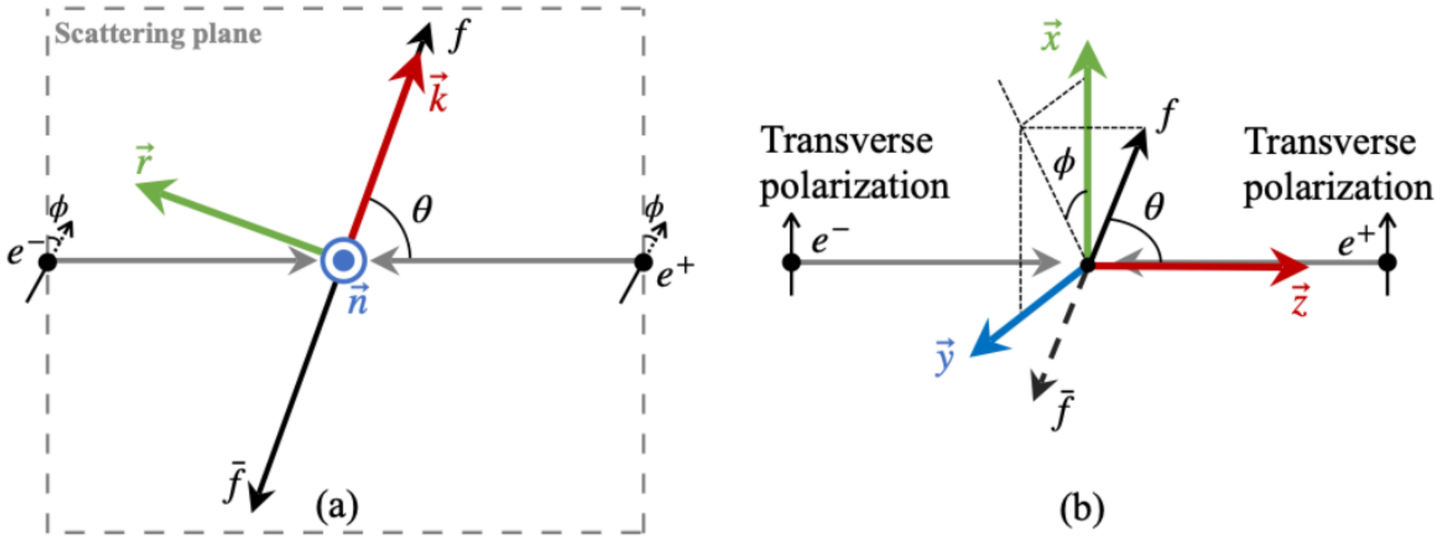
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Quantum Entanglement at Colliders

- It is helpful to use the transverse polarized initial state to increase the degree of the entanglement.
- The final state is always the Bell state along the direction $\hat{e}_2 = (\cos\theta\sin\phi, \cos\phi, \sin\theta\sin\phi\sqrt{1-\beta^2})$ in the helicity basis, which is just $\hat{y} - \left(1 - \sqrt{1-\beta^2}\right)\hat{k}$.

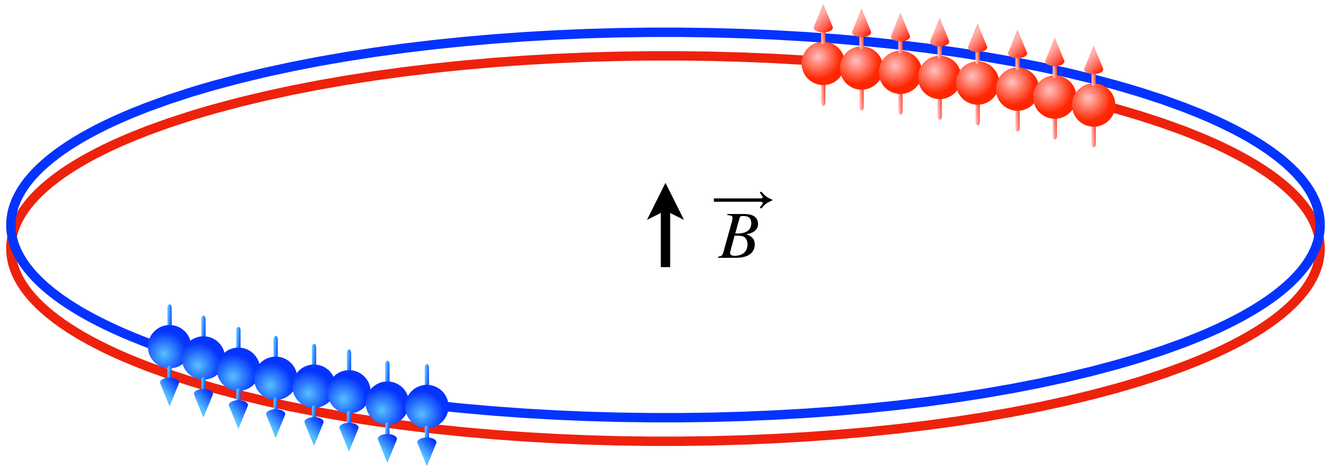


Quantum Entanglement at Higgs Factory

- At Higgs factory, transversely polarized initial state electron-positron beams could be get with the so-called Sokolov-Ternov effects.







Sokolov, A. A.; I. M. Ternov (1963). О поляризационных и спиновых эффектах в теории синхротронного излучения [Polarization and spin effects in the theory of synchrotron radiation]. Doklady Akademii Nauk SSSR (in Russian). 153 (5): 1052–1053

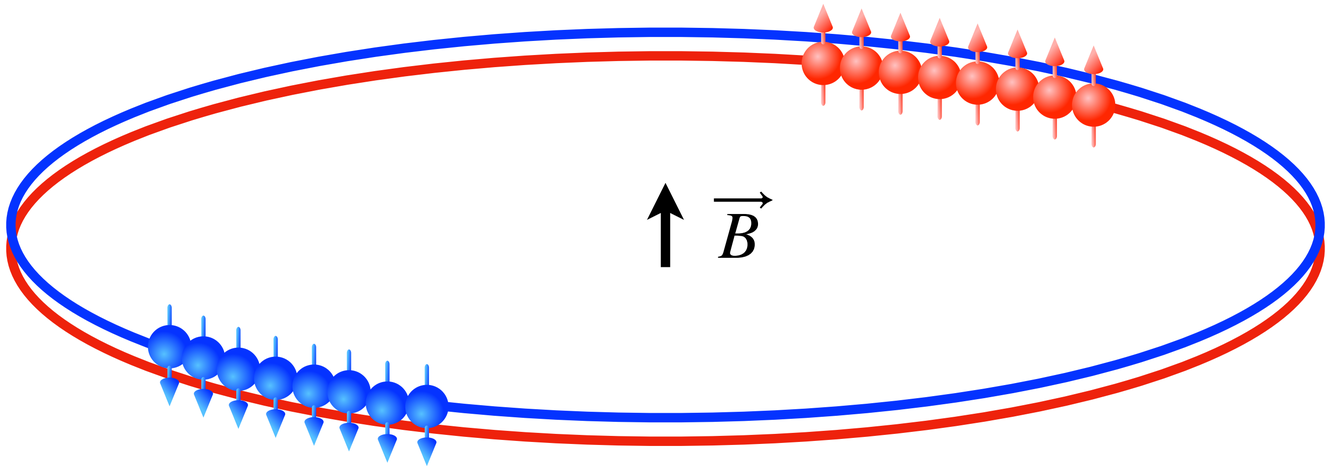
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$$B_x^{e^\pm} \to 92.4\,\% \text{ after several hours}$$







Sokolov, A. A.; I. M. Ternov (1963). О поляризационных и спиновых эффектах в теории синхротронного излучения [Polarization and spin effects in the theory of synchrotron radiation]. Doklady Akademii Nauk SSSR (in Russian). 153 (5): 1052–1053

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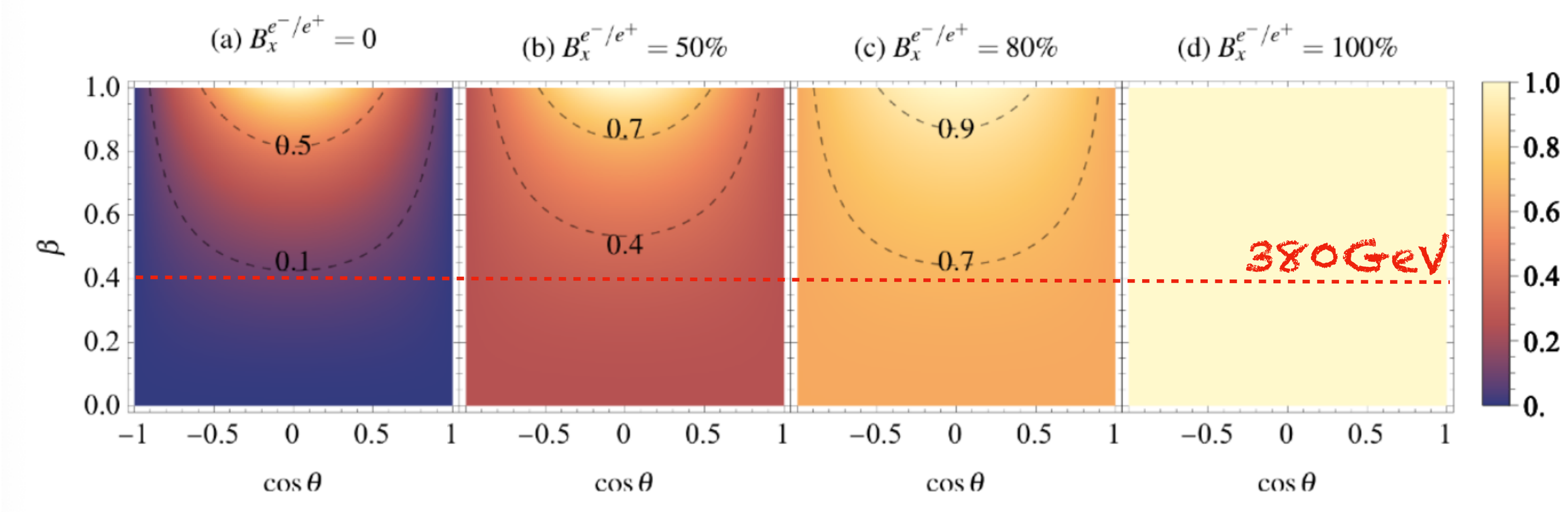
- The final state is maximal entangled!

$$\mathscr{C}[\hat{\rho}] = \sqrt{1 - \frac{4\left[1 - \left(B_{e^-}B_{e^+}\right)^2\right]\left(1 - \beta^2 \sin^2\theta\right)}{\left(2 - \beta^2 \sin^2\theta + B_{e^-}B_{e^+}\beta^2 \sin^2\theta \cos 2\phi\right)^2}}$$

*Base on the work in collaboration with Yi-Jing Fang, Amit Bhoonah, Kun Cheng, Tao Han, and Yandong Liu, to be published.

Quantum Entanglement at Colliders

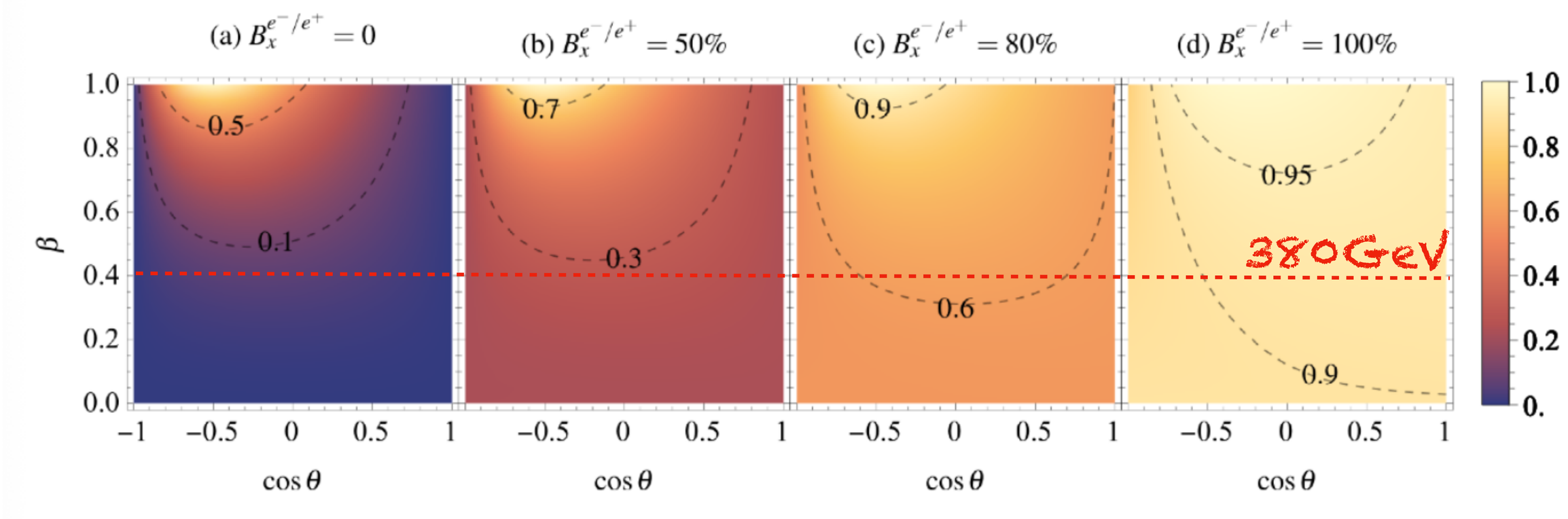
- It is helpful to use the transverse polarized initial state to increase the degree of the entanglement.



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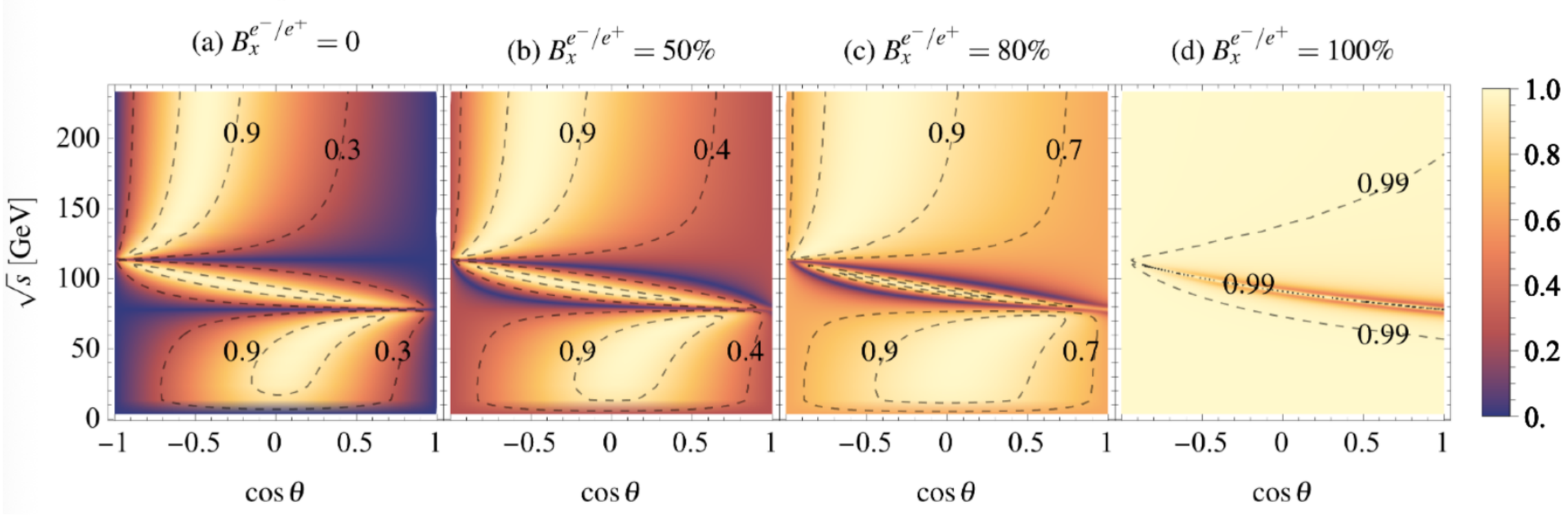
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Quantum Entanglement at Colliders

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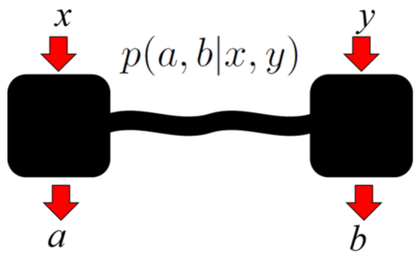
*Base on the work in collaboration with Yi-Jing Fang, Amit Bhoonah, Kun Cheng, Tao Han, and Yandong Liu, to be published.



From Quantum Entanglement to Quantum Nonlocality

The different correlations







“Alice”

“Bob”

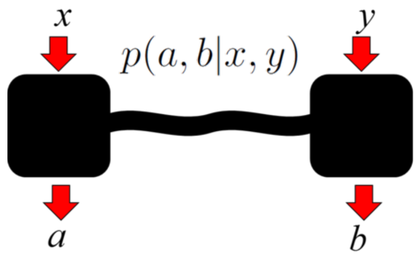
Local correlations

$$p(ab|xy) = \int_\Lambda d\lambda q(\lambda) p(a|x,\lambda) p(b|y,\lambda),$$

$$\sum_{a,b} ab\ p(ab|x_1y_1) + \sum_{a,b} ab\ p(ab|x_1y_2) + \sum_{a,b} ab\ p(ab|x_2y_1) - \sum_{a,b} ab\ p(ab|x_2y_2) \leqslant 2, \quad a,b \in \{+1,-1\}$$

The different correlations







“Alice”

“Bob”

Local correlations

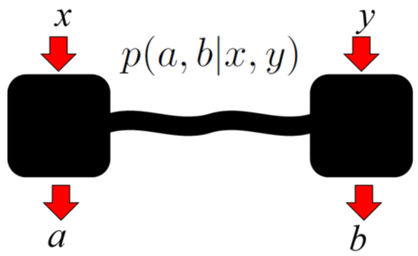
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$$\sum_{a,b} ab\ p(ab|x_1y_1) + \sum_{a,b} ab\ p(ab|x_1y_2) + \sum_{a,b} ab\ p(ab|x_2y_1) - \sum_{a,b} ab\ p(ab|x_2y_2) \leqslant 2, \quad a,b \in \{+1,-1\}$$

The hidden variables do not only determine the state of the system, but also determine the result of the measurement BEFORE it is measured!

The different correlations







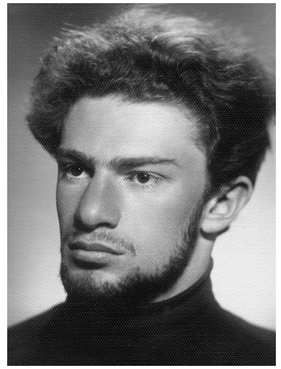
“Alice”

“Bob”

Quantum correlations-II

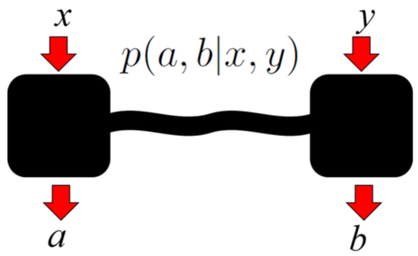
$$p(ab\,|\,xy) = \rho_{AB}(M_{a|x}M_{b|y}),\ [M_{a|x}, M_{b|y}] = 0$$

$$\|\hat{x}_1\hat{y}_1 + \hat{x}_1\hat{y}_2 + \hat{x}_2\hat{y}_1 - \hat{x}_2\hat{y}_2\| \leqslant \sqrt{4 + \|[\hat{x}_1, \hat{x}_2]\| \cdot \|[\hat{y}_1, \hat{y}_2]\|} = 2\sqrt{2}$$



The different correlations







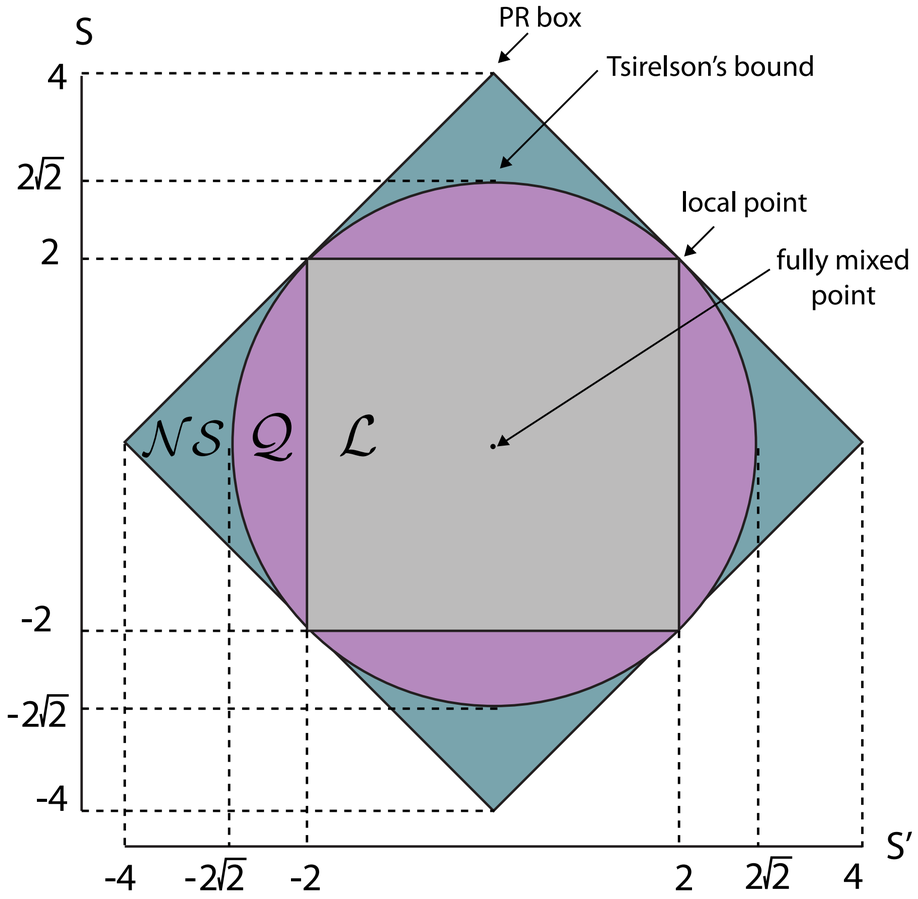
“Alice”

“Bob”

No-signaling correlations

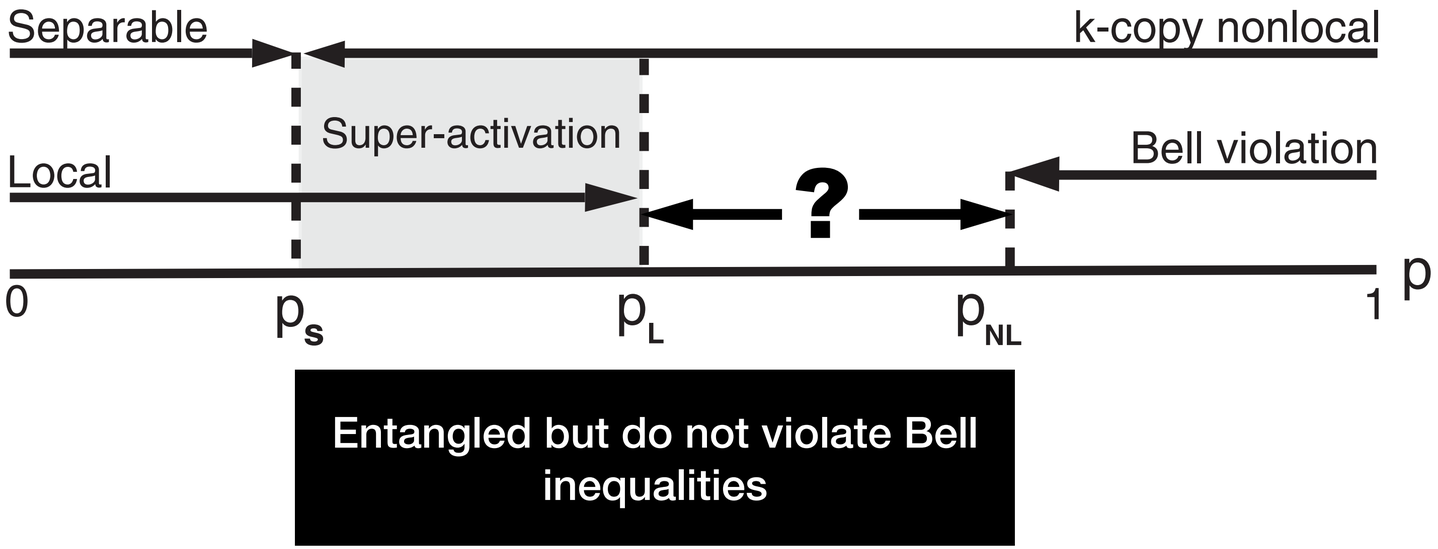
$$\sum_{b=1}^{\Delta} p(ab|xy) = \sum_{b=1}^{\Delta} p(ab|xy'), \quad \text{for all } a, x, y, y',$$

$$\sum_{a=1}^{\Delta} p(ab|xy) = \sum_{a=1}^{\Delta} p(ab|x'y), \quad \text{for all } b, y, x, x'.$$



The Relation between the Different Concepts

- A famous example (the Isotropic state): $\rho_{\text{iso}} = p|\Phi_+\rangle\langle\Phi_+| + (1-p)\frac{\mathbb{1}}{d^2}$,



For the $t\bar{t}$ case ($d = 2$):

$p_s = 1/3$, $p_L = 0.66$ and $p_{NL} = 0.7052$, or, $\mathscr{C}_s = 0$, $\mathscr{C}_L = 0.25$ and $\mathscr{C}_{NL} = 0.3111$, or, $D_s = -1/3$, $D_L = -0.42$ and $D_{NL} = -0.4370$.

WW production at Higgs factory

- Verifying the generalized Bell inequality:

$$\max_{\hat{A}_1,\hat{A}_2,\hat{B}_1,\hat{B}_2} \mathcal{I}_3(\hat{A}_1,\hat{A}_2;\hat{B}_1,\hat{B}_2) > 2$$

Collins-Gisin-Linden-Massar-Popescu (CGLMP) inequality

$$\begin{aligned}\mathcal{I}_3 \equiv + \big[&P(A_1 = B_1) + P(B_1 = A_2 + 1) \\ &+ P(A_2 = B_2) + P(B_2 = A_1)\big] \\ - \big[&P(A_1 = B_1 - 1) + P(B_1 = A_2) \\ &+ P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)\big]\end{aligned}$$

D. Collins, N. Gisin, N. Linden, S. Massar, S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).

WW production at Higgs factory

- The density matrix (some technical details…)

$$\hat{\rho}_{WW} \propto \mathcal{M}(e^+e^- \to W^+W^-)\hat{\rho}_{e^+e^-}\mathcal{M}(e^+e^- \to W^+W^-)^\dagger$$

$$\begin{aligned}
\hat{\rho}_{WW} \;=\; & \frac{1}{9}\hat{I}_9 + \frac{1}{3}d_+^i \hat{S}_i^+ \otimes \hat{I}_3 + \frac{1}{3}d_-^i \hat{I}_3 \otimes \hat{S}_i^- \\
& + \frac{1}{3}q_+^{ij} \hat{S}_{\{ij\}}^+ \otimes \hat{I}_3 + \frac{1}{3}q_-^{ij} \hat{I}_3 \otimes \hat{S}_{\{ij\}}^- \\
& + C_d^{ij} \hat{S}_i^+ \otimes \hat{S}_j^- + C_{d,q}^{i,jk} \hat{S}_i^+ \otimes \hat{S}_{\{jk\}}^- \\
& + C_{q,d}^{ij,k} \hat{S}_{\{ij\}}^+ \otimes \hat{S}_k^- + C_q^{ij,k\ell} \hat{S}_{\{ij\}}^+ \otimes \hat{S}_{\{k\ell\}}^-
\end{aligned}$$

Q. Bi, Q.-H. Cao, K. Cheng, HZ, Phys. Rev. D 109 (2024) 036022.

WW production at Higgs factory

- The density matrix (some technical details…) is “easy” to calculate
- $\beta \to 0$?

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WW production at Higgs factory

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    - *s*-channel is *p*-wave and suppressed by a factor of β.

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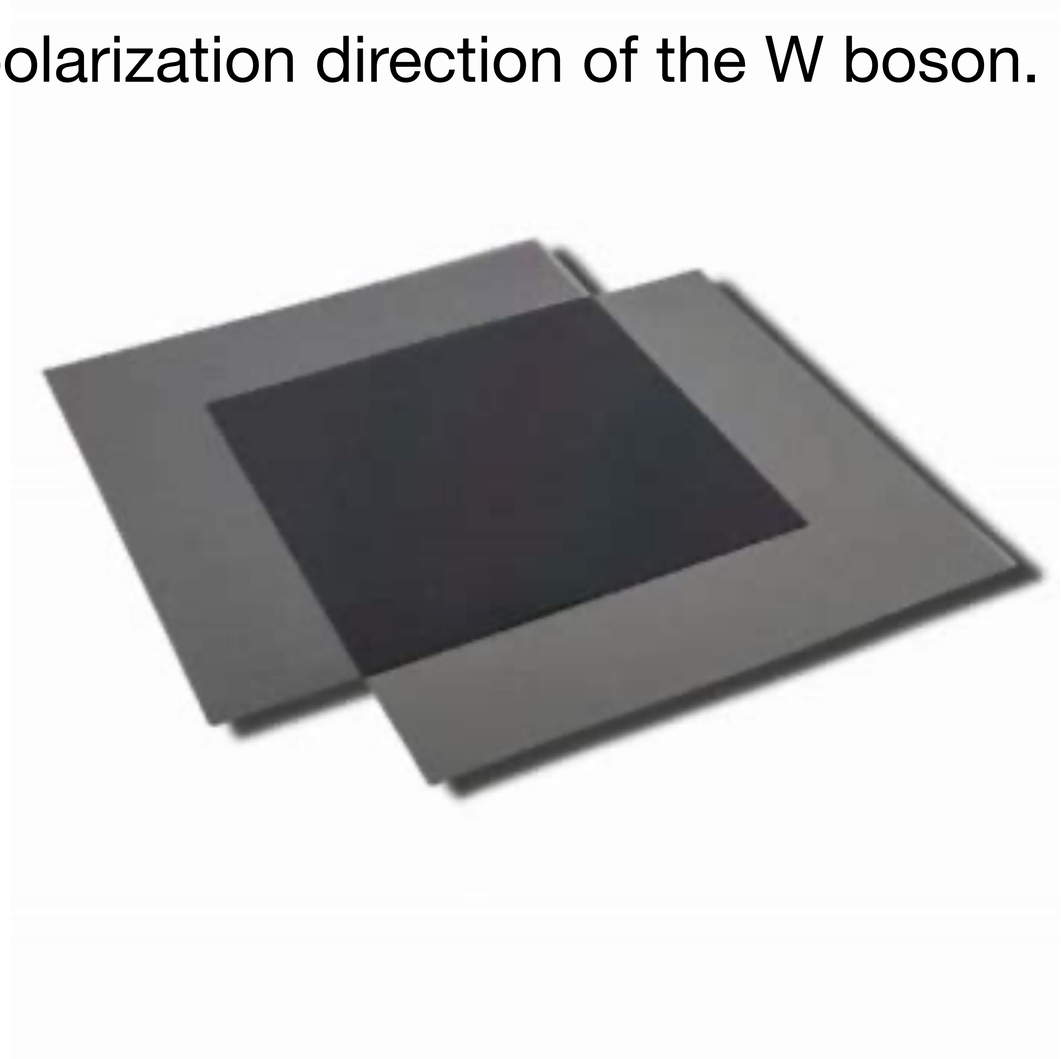
WW production at Higgs factory

- The density matrix (some technical details…) is “easy” to calculate
- $\beta \to 0$?
  - *s*-channel is *p*-wave and suppressed by a factor of β.
  - *t*-channel is purely left-handed current so that the initial state is selected by the (weak) interaction to be a pure state $|e_L^-\rangle \otimes |e_L^+\rangle$.

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WW production at Higgs factory

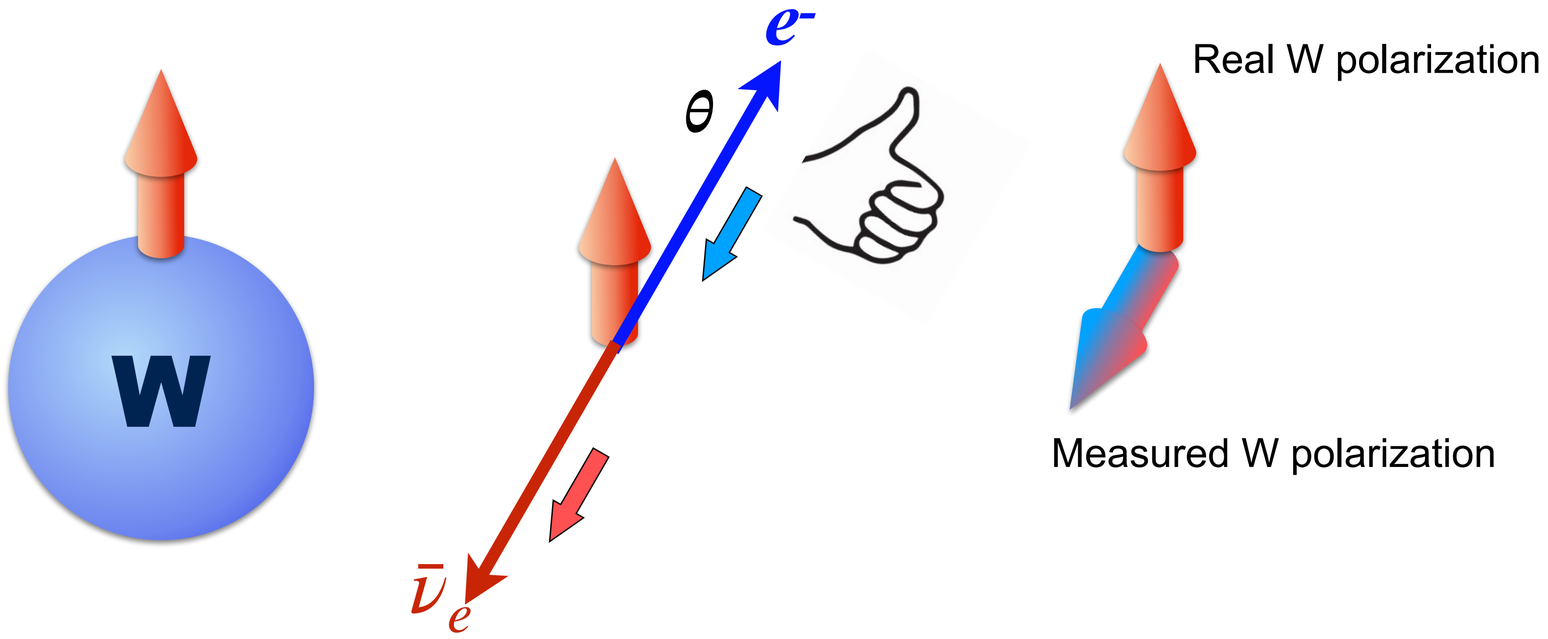
- “Measuring” the polarization direction of the W boson.



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WW production at Higgs factory

- “Measuring” the polarization direction of the W boson.



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WW production at Higgs factory

- Reconstruct the density matrix and “do” a Bell exp with the density matrix.

$$P(A_1 = B_1) = \sum_{\lambda=-1}^{1} \mathrm{Tr}[\hat{\rho}_{WW}\hat{\Pi}_{|A_1=\lambda,B_1=\lambda\rangle}],$$

$$d_i = \langle \mathfrak{n}_i \rangle, \qquad q_{ij} = \frac{5}{2}\langle \mathfrak{q}_{ij} \rangle, \quad \mathfrak{q}_{ij} \equiv \mathfrak{n}_i\mathfrak{n}_j - \frac{1}{3}\delta_{ij}$$

$$C^{d}_{ij} = \langle \mathfrak{n}^+_i \mathfrak{n}^-_j \rangle, \qquad C^{dq}_{i,jk} = \frac{5}{2}\langle \mathfrak{n}^+_i \mathfrak{q}^-_{jk} \rangle,$$

$$C^{q}_{ij,k\ell} = \frac{25}{4}\langle \mathfrak{q}^+_{ij} \mathfrak{q}^-_{k\ell} \rangle. \qquad C^{qd}_{ij,k} = \frac{5}{2}\langle \mathfrak{q}^+_{ij} \mathfrak{n}^-_k \rangle.$$

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WW production at Higgs factory

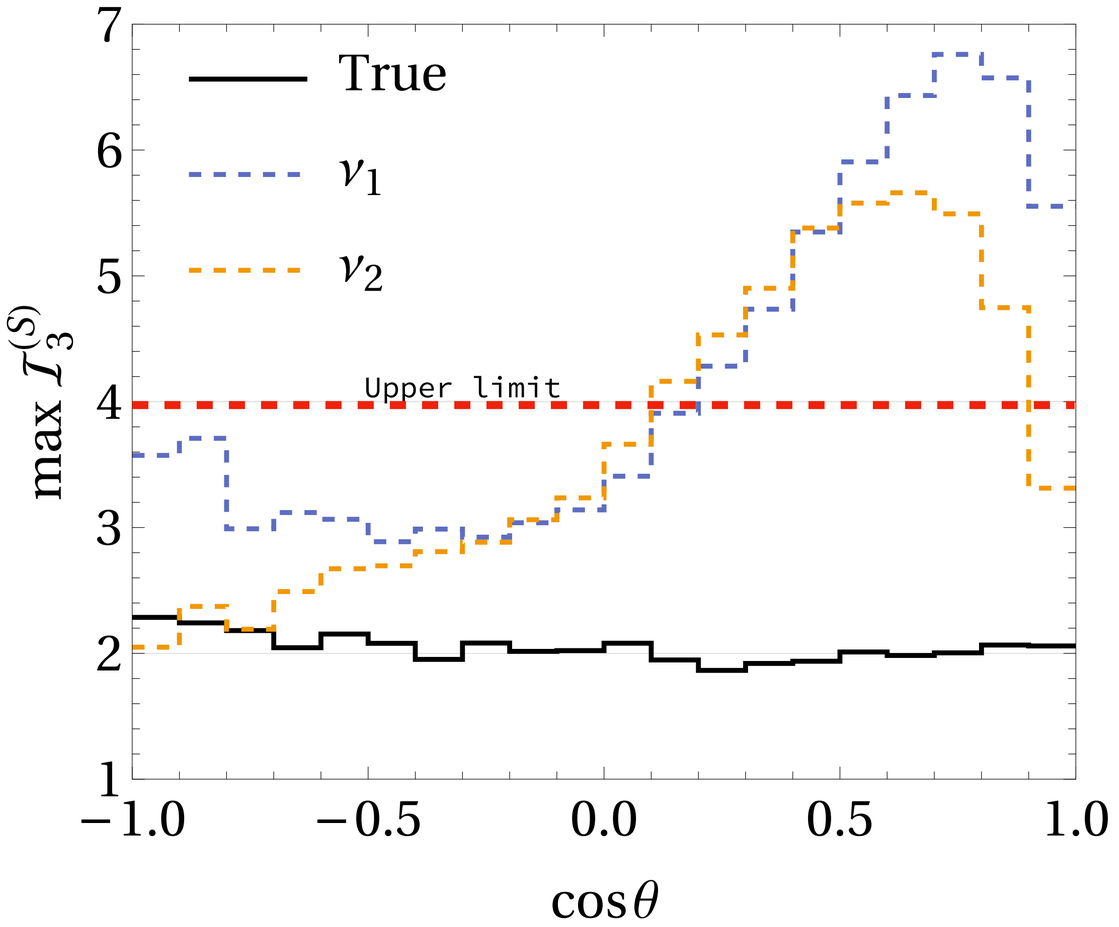
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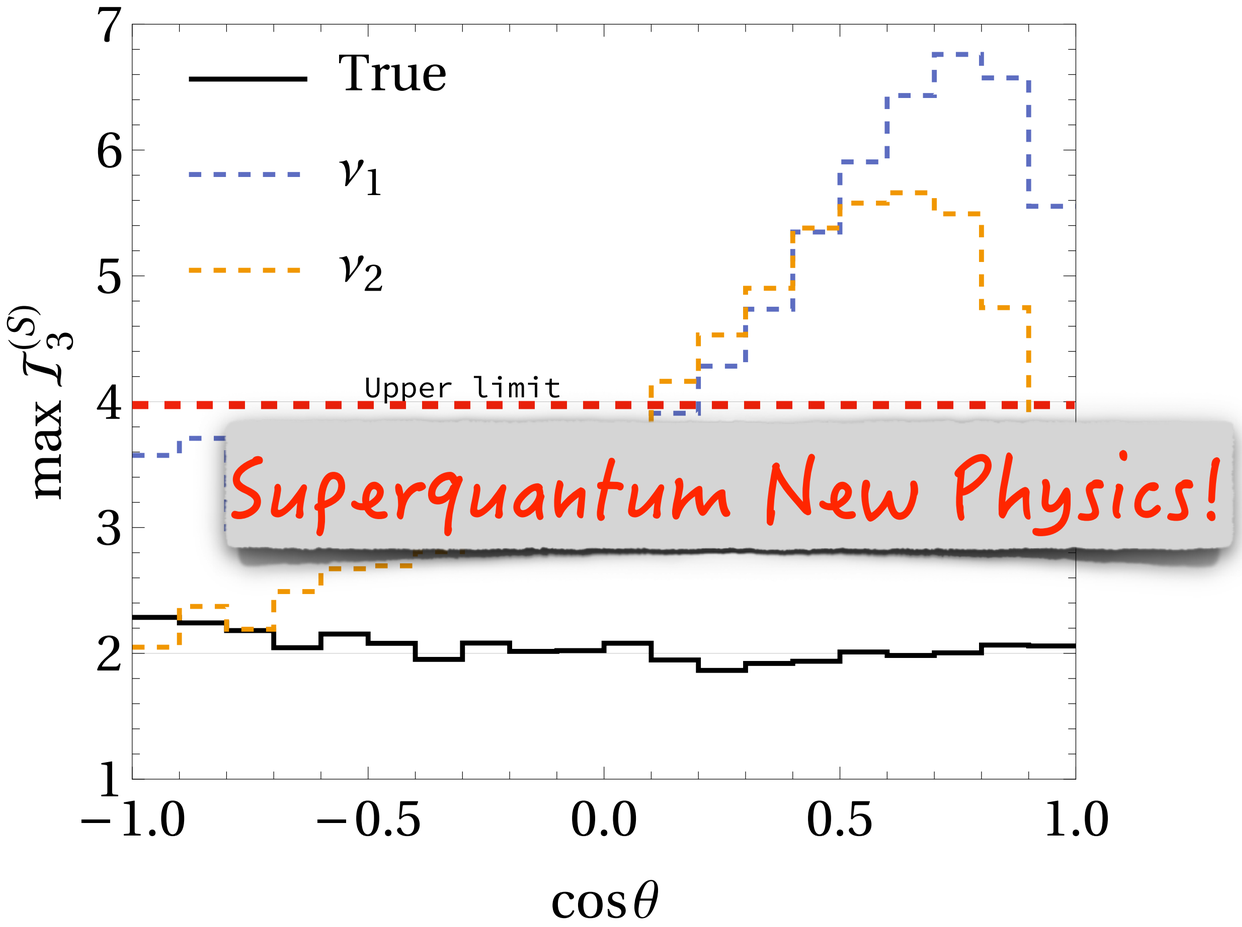
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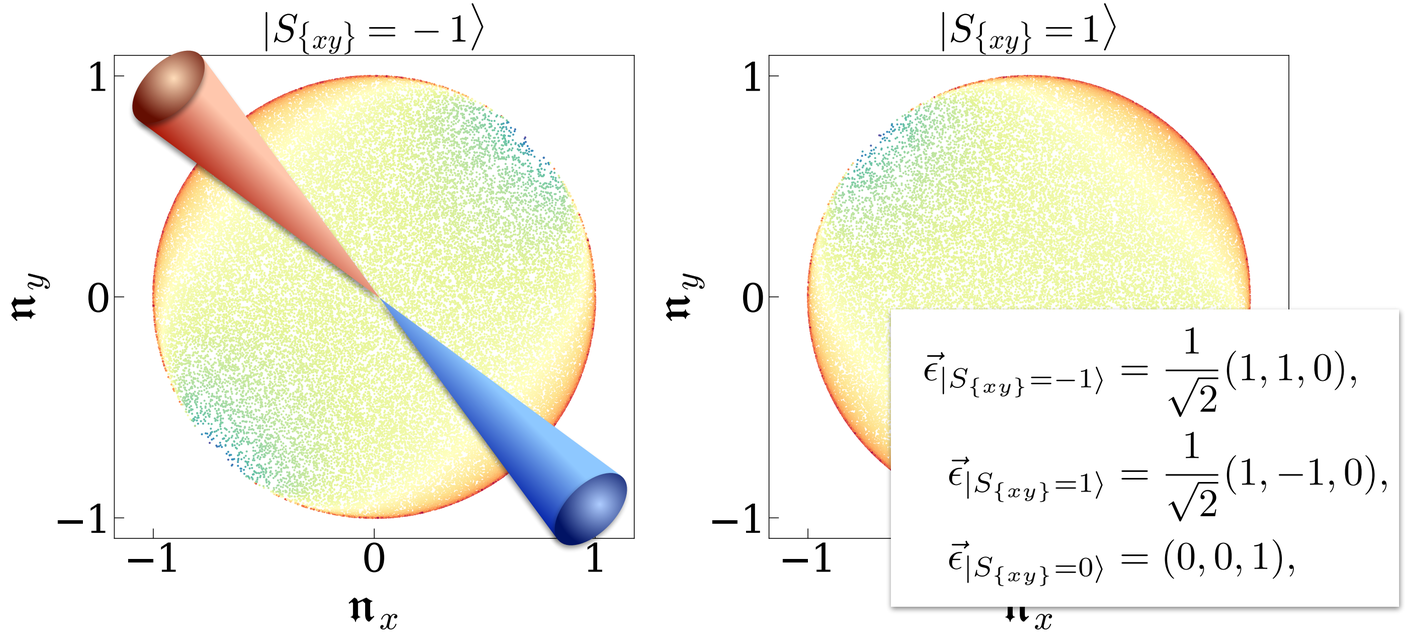
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WW production at Higgs factory

- Collider phenomenology: from dilepton channel to semi-leptonic channel.
- Circular polarization → linear polarization. $\hat{\Pi}_{\mathbf{n}} = \hat{I}_3 - \hat{S}^2_{\mathbf{n}}$



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WW production at Higgs factory

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- Circular polarization → linear polarization.

$$\mathcal{I}_3(\hat{S}_{\vec{a}_1}, \hat{S}_{\vec{a}_2}; \hat{S}_{\{x_3y_3\}}, \hat{S}_{\{x_4y_4\}}) \equiv + \left[P(S_{\vec{a}_1} = S_{\{x_3y_3\}}) + P(S_{\{x_3y_3\}} = S_{\vec{a}_2} + 1) + P(S_{\vec{a}_2} = S_{\{x_4y_4\}}) + P(S_{\{x_4y_4\}} = S_{\vec{a}_1})\right]$$

$$- \left[P(S_{\vec{a}_1} = S_{\{x_3y_3\}} - 1) + P(S_{\{x_3y_3\}} = S_{\vec{a}_2}) + P(S_{\vec{a}_2} = S_{\{x_4y_4\}} - 1) + P(S_{\{x_4y_4\}} = S_{\vec{a}_1} - 1)\right]$$

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WW production at Higgs factory

- Calculating the generalized Bell observable

$$
\begin{aligned}
P(S_{\vec{a}_1} = S_{\{x_3y_3\}}) &= \sum_{\lambda=-1}^{1} \mathrm{Tr}[\hat{\rho}_{WW}\hat{\Pi}_{|\vec{a}_1=\lambda, S_{\{x_3y_3\}}=\lambda\rangle}] \\
&= \mathrm{Tr}[\hat{\rho}_{WW}\cdot\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=-1)\otimes\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=-1)] + \mathrm{Tr}[\hat{\rho}_{WW}\cdot\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=1)\otimes\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=1)] \\
&\quad + \mathrm{Tr}[\hat{\rho}_{WW}\cdot\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=0)\otimes\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=0)] \\
&= 1 - 2q^{-}_{ij}\epsilon_{3i}\epsilon_{3j} - 2C^{dq}_{i,jk}a_{1i}(\epsilon_{1j}\epsilon_{1k}-\epsilon_{2j}\epsilon_{2k}) + 2C^{q}_{ij,kl}a_{1i}a_{1j}(-\epsilon_{1k}\epsilon_{1l}-\epsilon_{2k}\epsilon_{2l}+2\epsilon_{3k}\epsilon_{3l}).
\end{aligned}
$$

$$
\begin{aligned}
&\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=-1) = \frac{1}{2}(-\hat{S}_{\vec{a}_1}+\hat{S}^2_{\vec{a}_1}), && \hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=-1) = \hat{I}_3 - \hat{S}^2_{\vec{\epsilon}_1},\ \vec{\epsilon}_1 = \frac{\hat{x}_3+\hat{y}_3}{\sqrt{2}}, \\
&\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=1) = \frac{1}{2}(\hat{S}_{\vec{a}_1}+\hat{S}^2_{\vec{a}_1}), && \hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=1) = \hat{I}_3 - \hat{S}^2_{\vec{\epsilon}_2}, \quad \vec{\epsilon}_2 = \frac{\hat{x}_3-\hat{y}_3}{\sqrt{2}}, \\
&\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=0) = \hat{I}_3 - \hat{S}^2_{\vec{a}_1}. && \hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=0) = \hat{I}_3 - \hat{S}^2_{\vec{\epsilon}_3}, \quad \vec{\epsilon}_3 = \hat{x}_3\times\hat{y}_3.
\end{aligned}
$$

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WW production at Higgs factory

- If Alice “measure” the spin of the W^+ along the direction $\vec{a}_1$ and $\vec{a}_2$, Bob “measure” the linear polarization of the W^- along the direction x_3y_3 and x_4y_4, the result “should” be (calculated with the measured density matrix elements)

$$\begin{aligned}\mathcal{I}_3(\hat{S}_{\vec{a}_1},\hat{S}_{\vec{a}_2};\hat{S}_{\{x_3y_3\}},\hat{S}_{\{x_4y_4\}}) = {} & 2q^-_{ij}(\omega_{1i}\omega_{1j}+\omega_{2i}\omega_{2j}-2\omega_{3i}\omega_{3j})+2C^{dq}_{i,jk}a_{1i}(2\epsilon_{1j}\epsilon_{1k}-\epsilon_{2j}\epsilon_{2k}-\epsilon_{3j}\epsilon_{3k}+\omega_{1j}\omega_{1k}\\ & -2\omega_{2j}\omega_{2k}+\omega_{3j}\omega_{3k})+2C^{dq}_{i,jk}a_{2i}(-2\epsilon_{1j}\epsilon_{1k}+\epsilon_{2j}\epsilon_{2k}+\epsilon_{3j}\epsilon_{3k}+2\omega_{1j}\omega_{1k}\\ & -\omega_{2j}\omega_{2k}-\omega_{3j}\omega_{3k})+6C^{q}_{ij,kl}a_{1i}a_{1j}(-\epsilon_{2k}\epsilon_{2l}+\epsilon_{3k}\epsilon_{3l}-\omega_{1k}\omega_{1l}+\omega_{3k}\omega_{3l})\\ & +6C^{q}_{ij,kl}a_{2i}a_{2j}(\epsilon_{2k}\epsilon_{2l}-\epsilon_{3k}\epsilon_{3l}-\omega_{2k}\omega_{2l}+\omega_{3k}\omega_{3l})\end{aligned}$$

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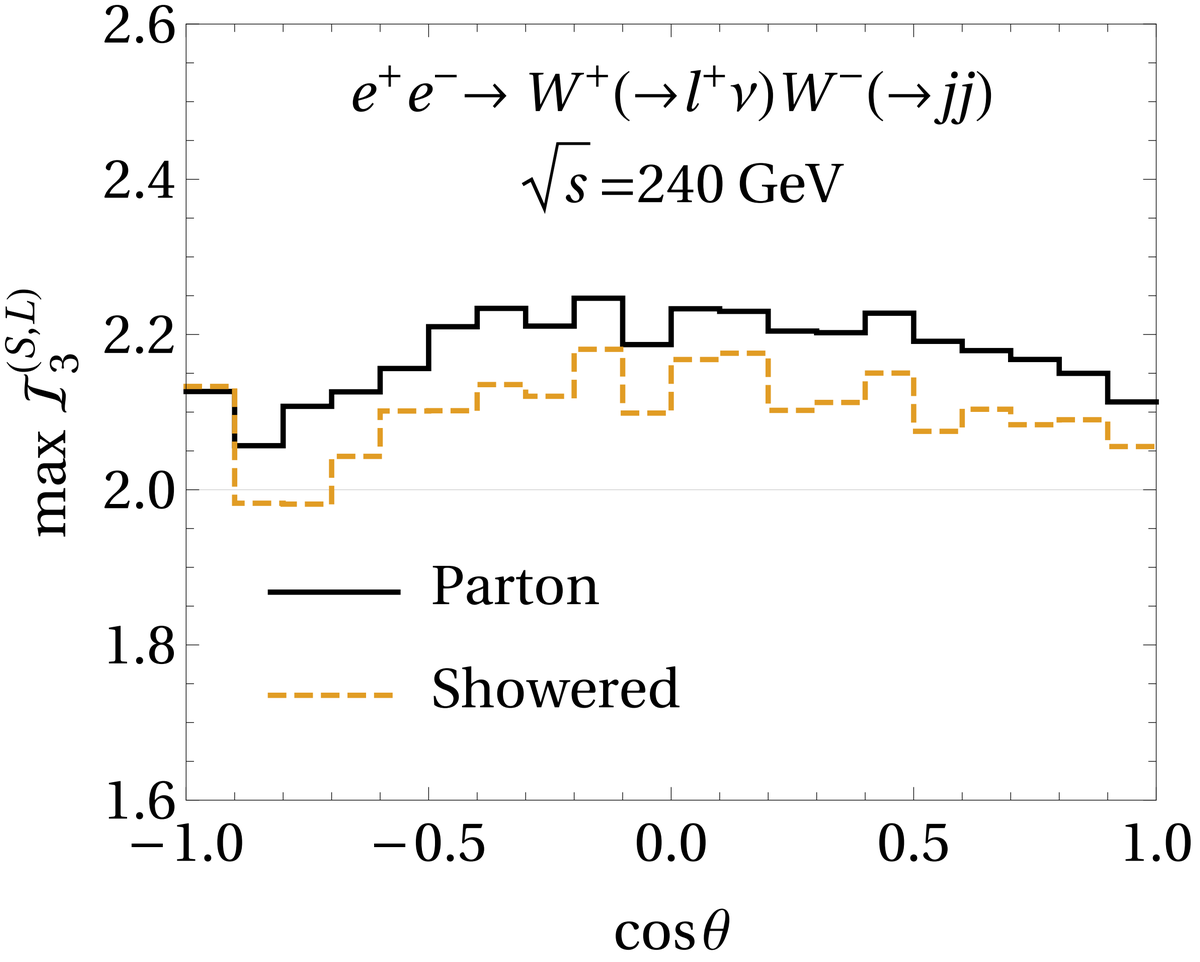
WW production at Higgs factory

- Some details ($e^+e^- \to W^+W^- \to \ell^\pm \nu jj$)
  - 240GeV electron-positron collider
  - (LO) MADGRAPH5_AMC@NLO+PYTHIA8+FASTJET
  - 2 Exclusive jets with Durham algorithm ($E_j > 5\mathrm{GeV}, |\eta_j| < 3.5$)
  - One isolated charged lepton ($e^\pm, \mu^\pm$) ($E_\ell > 15\mathrm{GeV}, \ |\cos\theta_\ell| < 0.98$)
  - Missing energy ($\cos\theta_{\ell\nu} < 0.2$)
  - Reconstructed *W* mass ($|m_{jj} - m_W| < 20\mathrm{GeV}$)

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WW production at Higgs factory

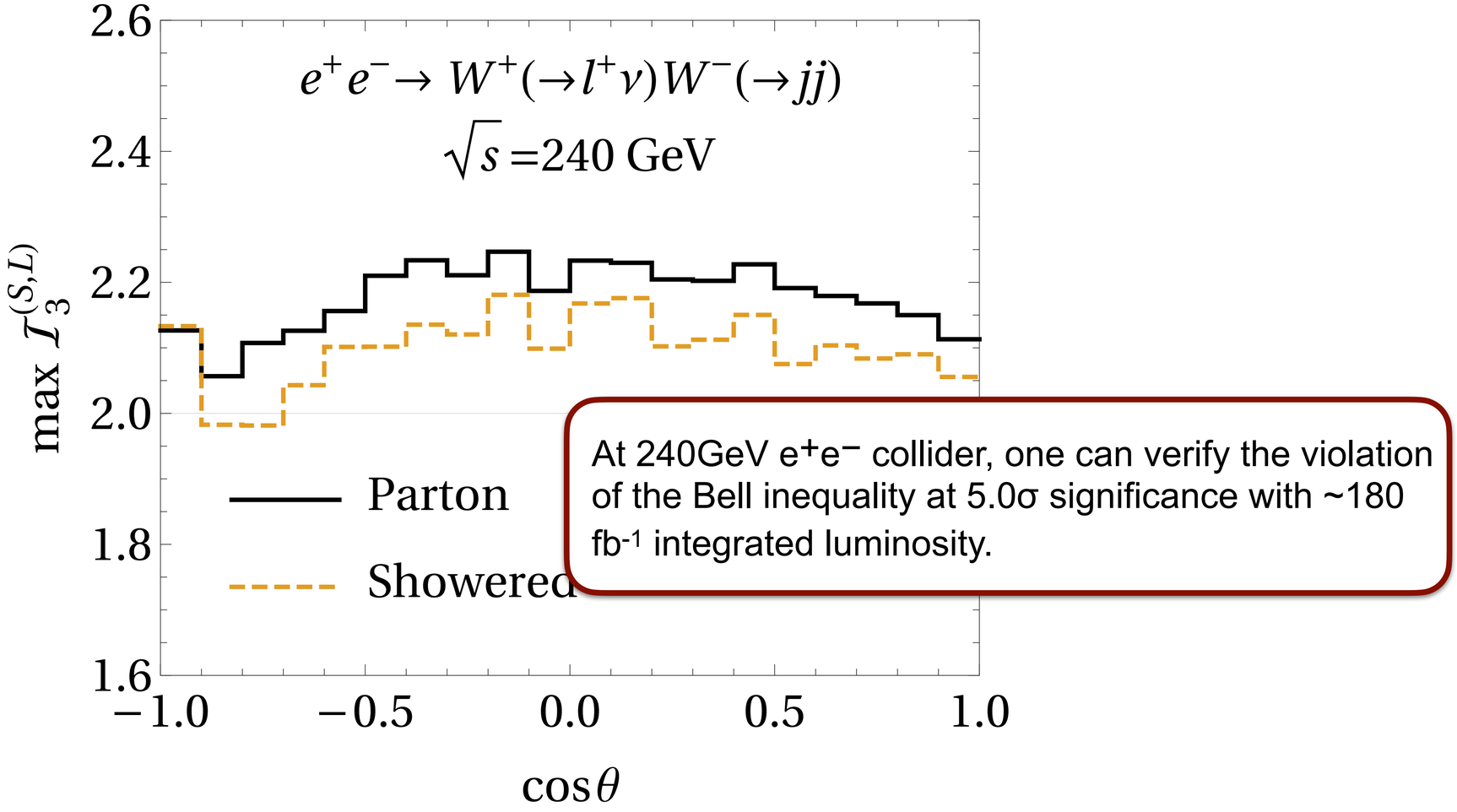
- The result



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WW production at Higgs factory

- The result



At 240GeV e^+e^- collider, one can verify the violation of the Bell inequality at 5.0σ significance with ~180 fb^{-1} integrated luminosity.

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Summary and Discussion

- Recently, a lot of works appear on the topic of quantum entanglement and quantum nonlocality in particle physics. (e.g., ttbar, Higgs decays into ZZ, Lambda-Lambdabar, neutrino oscillation, fragmentation, …)

- More?


Thank you!