



中国科学技术大学
University of Science and Technology of China

Cross Section Measurements of $e^+e^- \rightarrow K^+K^-$ via tagged ISR technique and study of Bose-Einstein correlations by $\pi^0\pi^0$ pairs at BESIII

Yijing Wang (王祎景)
Supervisor: Guangshun Huang
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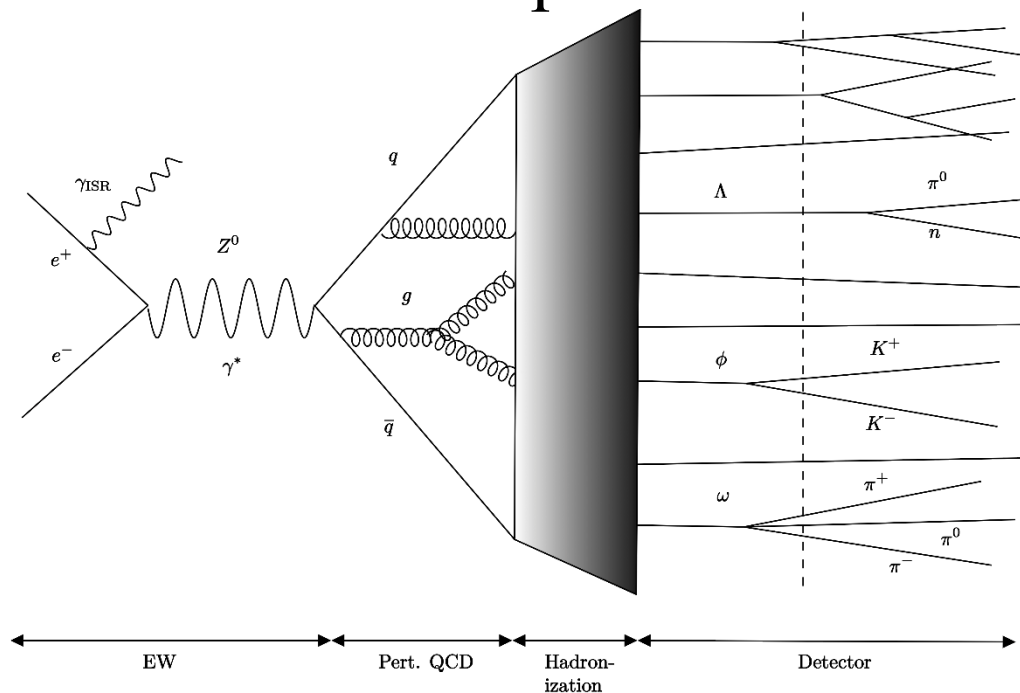
Outline

- **Introduction**
 - ❑ Overview
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- **Study of Bose-Einstein Correlations (BEC) by $\pi^0\pi^0$ pairs at BESIII**
 - ❑ Motivation
 - ❑ Analysis Strategy
 - ❑ Summary
- **Summary and prospect**

Introduction

Overview

➤ Process of hadron production:



- ❑ QED process (e^+e^- annihilation, ISR process, etc)
- ❑ Parton evolution in pQCD
- ❑ Non-pQCD hadronization, formation of **hadron source**
- ❑ Production of **final hadrons**

- The hadron source structure:
 - ❑ Model dependent and requires experimental constraints.
 - ❑ **Bose-Einstein correlations : constrain effective emission scales**
- Final hadron production for low energy process:
 - ❑ Cross section measurement (**muon anomaly a_μ** , hardronic spectroscopy...)
 - ❑ Electromagnetic form factor
 - ❑ Fragmentation Function.

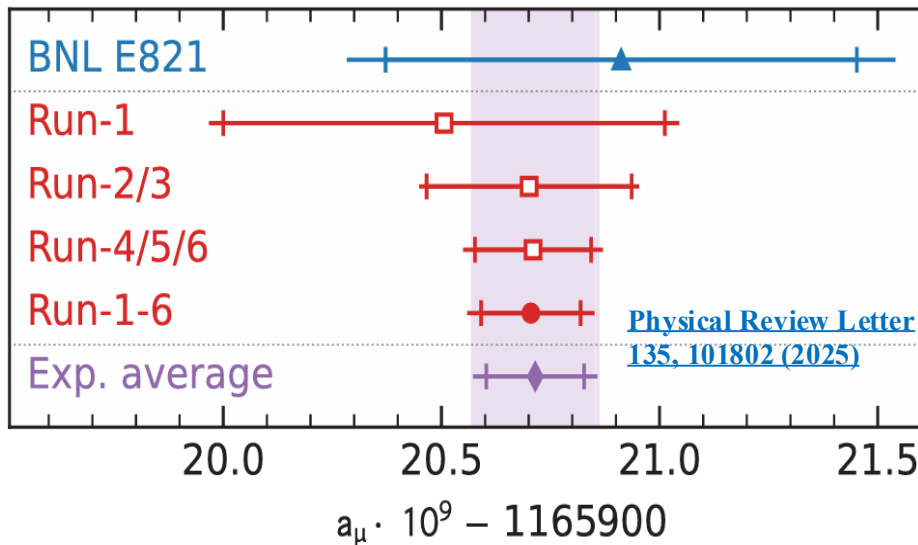
Muon anomaly

➤ Measurement of muon anomaly (a_μ)

- Precise experimental and theoretical value of a_μ :
stringent test of the SM.

- High precision for both experimental value (a_μ^{exp}) and theoretical calculations (a_μ^{SM}) $\cong 10^{-5}\%$

$$\bullet a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Had} (a_\mu^{HVP} + a_\mu^{LBL})$$



Item	Method	a_μ^{SM} portion	δa_μ^{SM} portion
a_μ^{QED}	perturbative	$\sim 99.99\%$	$\sim 0.001\%$
a_μ^{EW}	Perturbative	$\sim 1\text{ppm}$	$\sim 0.2\%$
a_μ^{HVP}	Non-perturbative	$\sim 59\text{ppm}$	$\sim 84\%$
a_μ^{LBL}	Non-perturbative	$\sim 1\text{ppm}$	$\sim 16\%$

- a_μ^{HVP} is the largest contributor to δa_μ^{SM}

- Two methods are used:

- Data driven, dispersive approach
- A first-principle, lattice-QCD

Muon anomaly

➤ Tension between a_μ^{exp} and a_μ^{SM}

- ❑ Prediction given by **LQCD** shows **consistency** with the experimental value.
- ❑ **5.8 σ tension** between **data driven prediction** (excluding CMD3 results) and a_μ^{exp}
- ❑ Results by CMD3 larger than others' and consistent with a_μ^{exp}

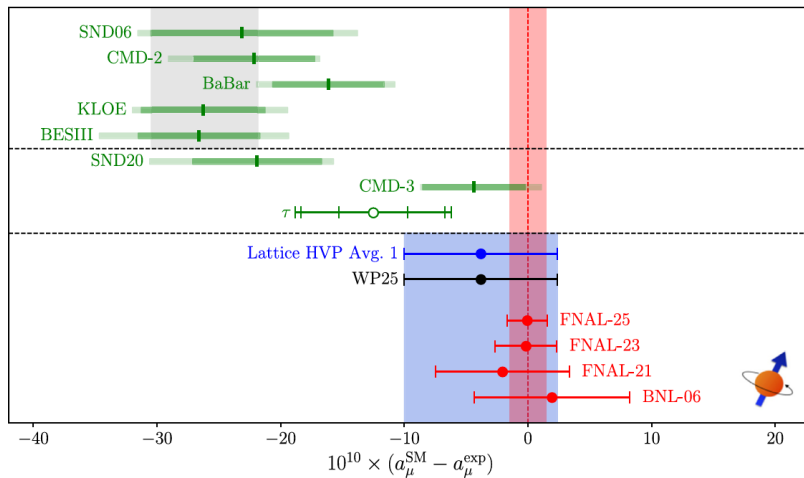
? How to calculate a_μ^{HVP} by data driven

$$a_\mu^{HVP,LO} = \frac{\alpha^2}{3\pi^2} \int_{m_\pi^2}^{+\infty} \frac{ds}{s} R(s) K(s)$$

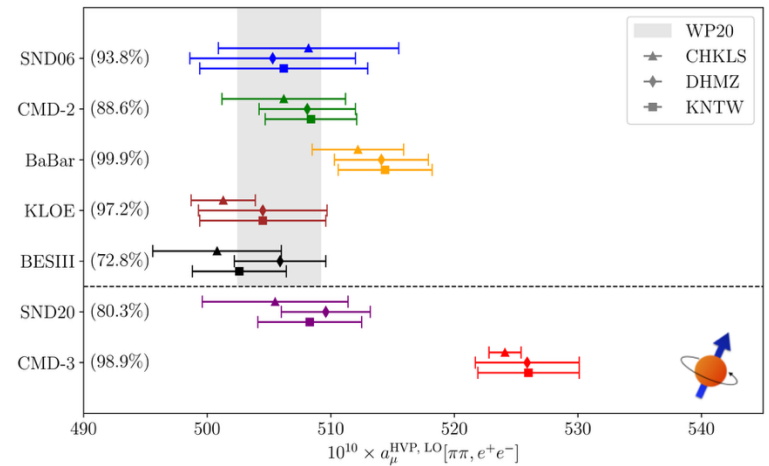
$R(s) = \frac{\sigma_{had}(s)}{4\pi\alpha^2/(3s)}$ is the hadronic R ratio

✓ Precise measured data required !

$\lesssim 2$ GeV: exclusive final states ($2\pi, 3\pi, 4\pi, 2K \dots$)



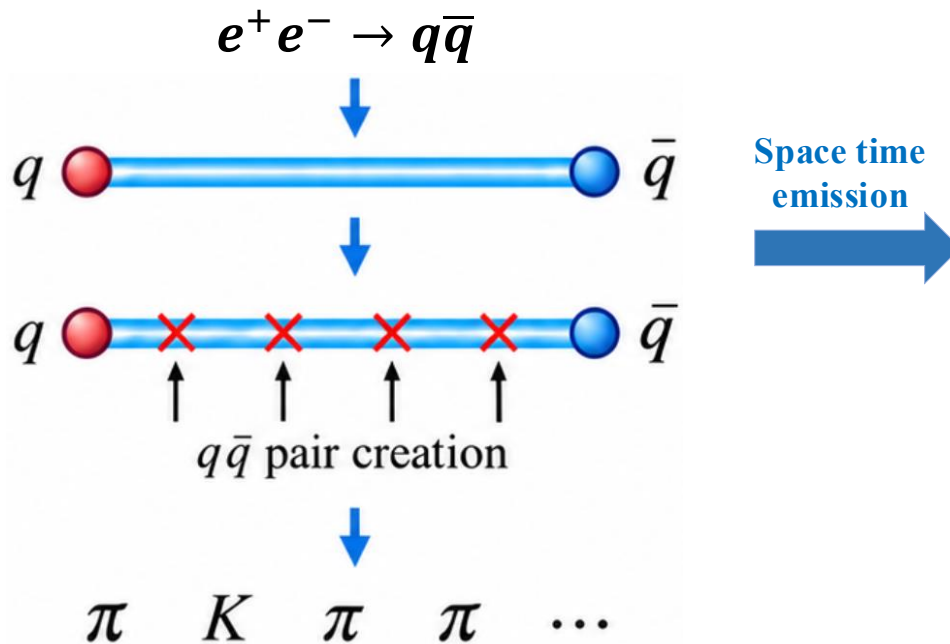
[Physics Reports 1143 \(2025\) 1–158](#)



Bose-Einstein correlations

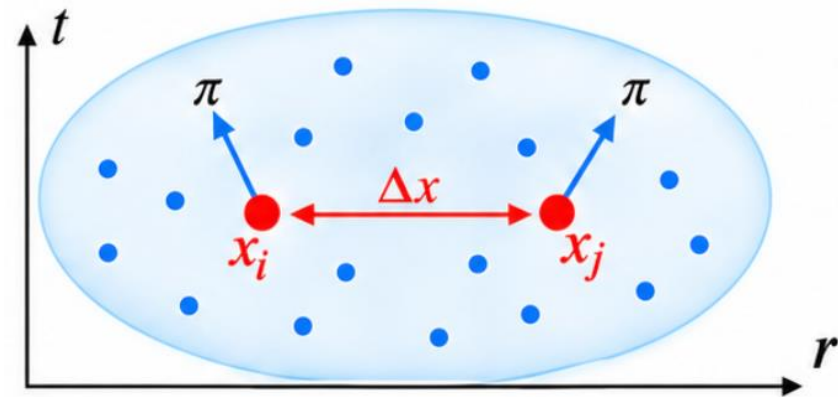
➤ String Fragmentation and the Hadronic Source

Hadron formation



Prompt hadrons + resonance decays

Emission function $S(x, p)$



[EPJC 78, 983 \(2018\)](#)

The space-time distribution of hadron emission points is the so-called “**hadronic source**”, which is described by :

$$S(x, p) = |f(x)|^2 \text{ for a fixed } p$$

Bose-Einstein correlations

➤ What and why Bose-Einstein correlations

❑ Origin: Wave function (WF) of a system of identical bosons symmetric under exchanging

❑ Higher probability of finding identical bosons in a
Bose-Einstein correlations (BEC)

❑ Space-time structure described by $f(x)$

$$A_{12} = \frac{1}{\sqrt{2}} (f(x_i) e^{ix_i p_1} f(x_j) e^{ix_j p_2} + f(x_i) e^{ix_i p_2} f(x_j) e^{ix_j p_1})$$

❑ Probability distributions of two particles:

$$P_{12} = \frac{\rho(x_i)\rho(x_j)}{2} (2 + e^{iq\Delta x} + e^{-iq\Delta x}) = \rho(x_i)\rho(x_j)(1 + \cos(q \cdot \Delta x))$$

$$= \int d^4x_i d^4x_j \rho(x_i)\rho(x_j) [1 + \cos(q \cdot \Delta x)]$$

$$= |R(0)|^2 + \text{Re}((\int d^4x_i \rho(x_i) e^{iq \cdot x_i})(\int d^4x_j \rho(x_j) e^{-iq \cdot x_j})) = |R(0)|^2 + |R(q)|^2$$

[Chin.Phys.C 48 \(2024\) 11, 113106](#)

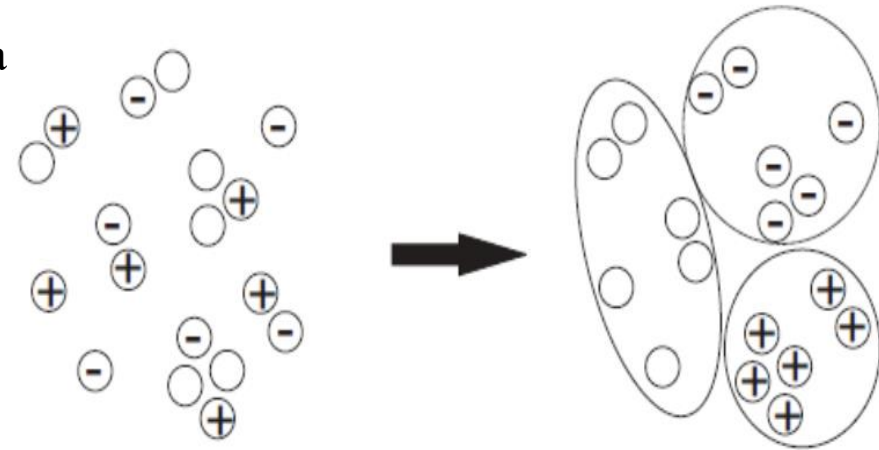
$R(0)$: uncorrelated part, got by unidentical bosons

❑ The correlation function is defined by:

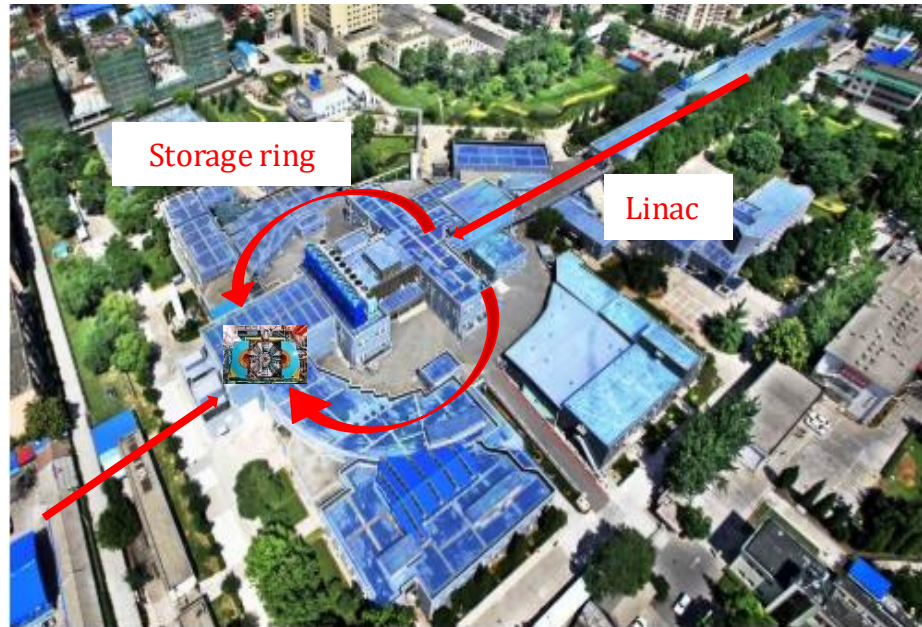
$$C(q) = \frac{P_{12}}{P_1 P_2} = \frac{|R(0)|^2 + |R(q)|^2}{|R(0)|^2} = 1 + \left| \frac{R(q)}{R(0)} \right|^2$$

BEC term, which straightly reflects the space-time structure of source!

Where $q = p_1 - p_2$, $x = x_k - x_i$, $R(q) = \int dx e^{iqx} \rho(x)$, $\rho(x) = |f(x)|^2$



Beijing Electron and Positron Collider II (BEPCII) Beijing Spectrometer III (BESIII)



Electromagnetic Calorimeter

CsI(Tl): $L = 28 \text{ cm}$

Barrel $\sigma_E = 2.5\%$

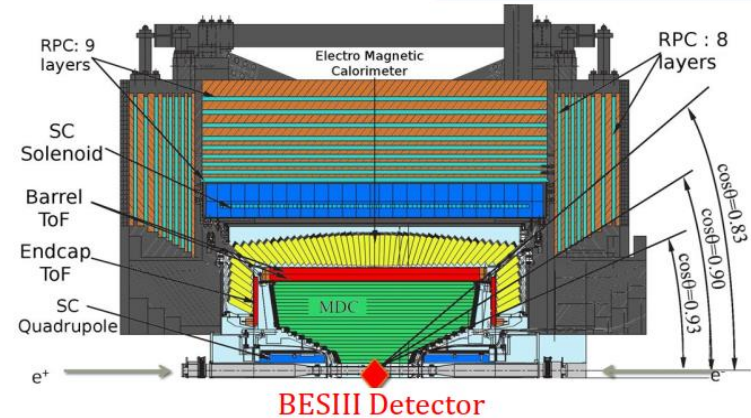
Endcap $\sigma_E = 5.0\%$

Muon Counter RPC

Barrel: 9 layers

Endcaps: 8 layers

$\sigma_{\text{spatial}} = 1.48 \text{ cm}$



Main Drift Chamber

Small cell, 43 layer

$\sigma_{xy} = 130 \mu\text{m}$

$dE/dx \sim 6\%$

$\sigma_p/p = 0.5\%$ at 1 GeV

Time Of Flight

Plastic scintillator

$\sigma_T(\text{barrel}) = 68 \text{ ps}$

$\sigma_T(\text{endcap}) = 110 \text{ ps}$

(update to 60 ps with MRPC)

$E_{\text{cm}} = 1.84 - 4.95 \text{ GeV}$

Peak luminosity @ $E_{\text{cm}} = 3.773 \text{ GeV}$:

$1.1 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$

Circumference: 237.53 m

Crossing angle: $2 \times 11 \text{ mrad}$

Topic1: Cross Section Measurements of $e^+e^- \rightarrow K^+K^-$ via tagged ISR technique

Motivation

➤ Why $e^+e^- \rightarrow K^+K^-$ channel?

❑ The **third** contribution for HVP

Channel	HVP contribution (10^{-10})	Channel	HVP contribution (10^{-10})
$\pi^+\pi^-$	5078.5 ± 33.8	$\pi^+\pi^-\pi^+\pi^-$	180.3 ± 5.5
$\pi^+\pi^-\pi^0$	462.1 ± 14.5	$\pi^+\pi^-\pi^0\pi^0$	136.8 ± 3.0
K^+K^-	230.8 ± 4.4	$K_S K_L$	128.2 ± 2.4

❑ The accuracy can be obtained in BESIII with 20 fb^{-1} datasets at $\psi(3770)$ peak

- ✓ The most accuracy results are from BaBar
- ✓ Around $\phi(1020)$: statistical uncertainty with **1.4%** →
- ✓ 1.2 – 2.0 GeV : statistical uncertainty with **2.2%** →
- ✓ 2.0 – 3.2 GeV : statistical uncertainty with **10%** →

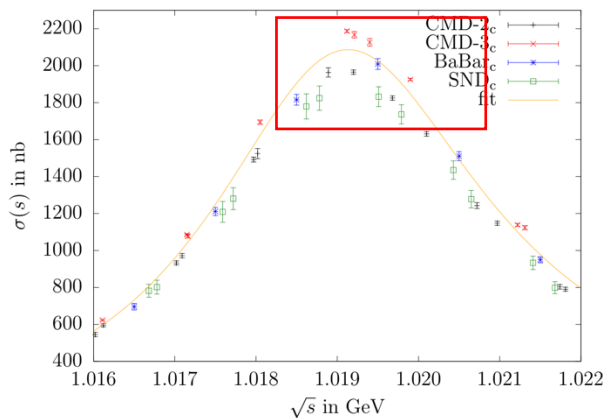
~1.0%
~1.5%
~4.5%

BESIII expected

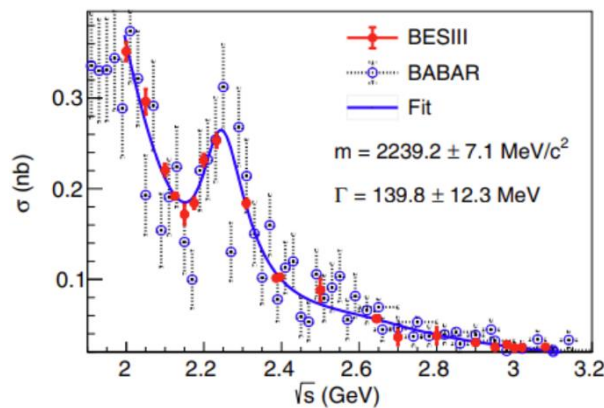
Motivation

➤ Puzzle $e^+e^- \rightarrow K^+K^-$ channel

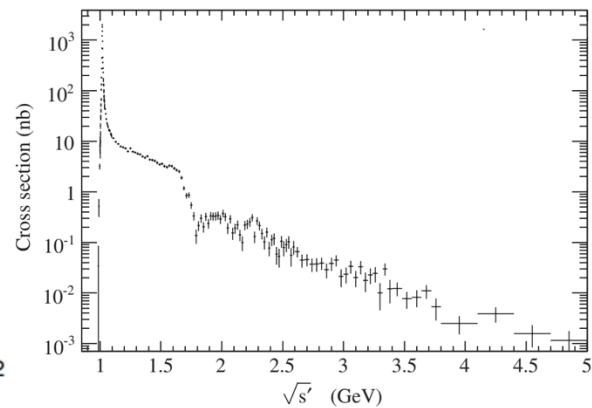
- Many results focusing on the $\phi(1020)$ peak region (SND, CMD-2, CMD-3, BABAR).
 - Tension** between the CMD-3's result and other groups', similar with the case for $\pi^+\pi^-$.
 - High precision** cross check could be offered by BESIII



[Physics Reports 1143 \(2025\) 1–158](#)



[Phys. Rev. D 99,032001\(2019\)](#)



[Phys. Rev. D 88, 032013 \(2013\)](#)

- Also fruitful results beyond the $\phi(1020)$
 - Substantial structure is evident in the center-of-mass energy range 1.1-2.4 GeV .
 - New resonance found in results by BESIII from 2.0-3.08 GeV

Event Selection

▣ Data Sets: $\sqrt{s}=3.773$ and 4.178 GeV, total luminosity: **23.452 fb⁻¹**

➤ Good charged tracks:

- $|\cos\theta| < 0.93$
- $V_r < 1$ cm, $|V_z| < 10$ cm
- $p/p_{\text{beam}} < 0.81$,
 p_{beam} is beam momentum
- Depth in MUC < 40 cm
- $N_{\text{Good}} = 2$

➤ Good photons:

- EMC time: $0 < t < 700$ ns
- $|\cos\theta_\gamma| < 0.8$, $E_\gamma > 25$ MeV for barrel
 $|\cos\theta_\gamma| \in (0.86, 0.92)$, $E_\gamma > 50$ MeV for end cap
- Angle from the nearest charged track $> 20^\circ$
- Good photon with the largest energy is the nominal ISR photon

➤ PID ($\frac{dE}{dx}$ + TOF):

- K : $\text{prob}(K) > \text{prob}(e)$
- $N_{K^+} = N_{K^-} = 1$

➤ Vertex fit and Kinematic fit

- Successful Vertex fit by two Kaons
- 4c kinematic fit and $\chi_{4c}^2 < 50$
- Looping good photon pairs to do 4c kinematic fit, choose the combination with minimum $\chi_{2\gamma, \min}^2$
- $\chi_{2\gamma, \min}^2 > \chi_{4c}^2$

➤ Special selection

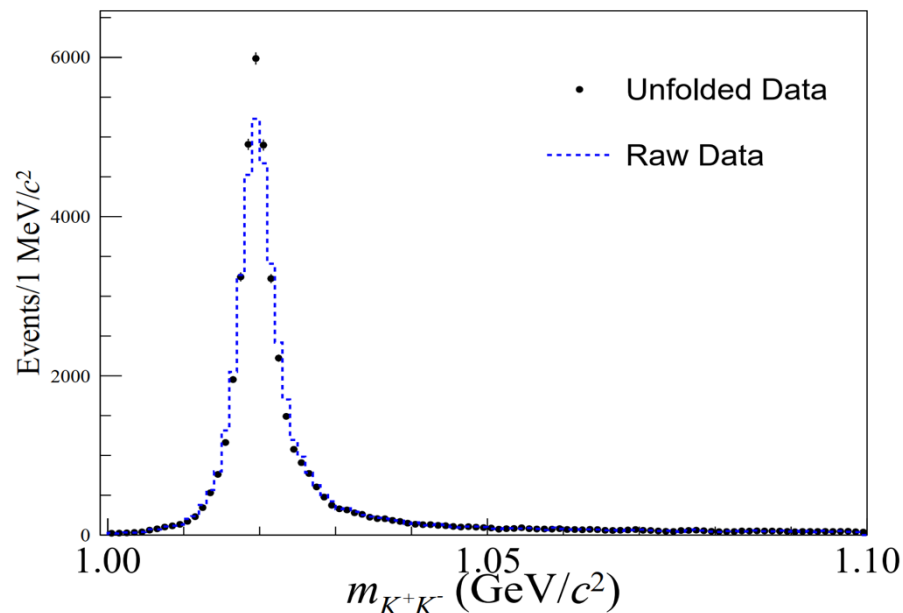
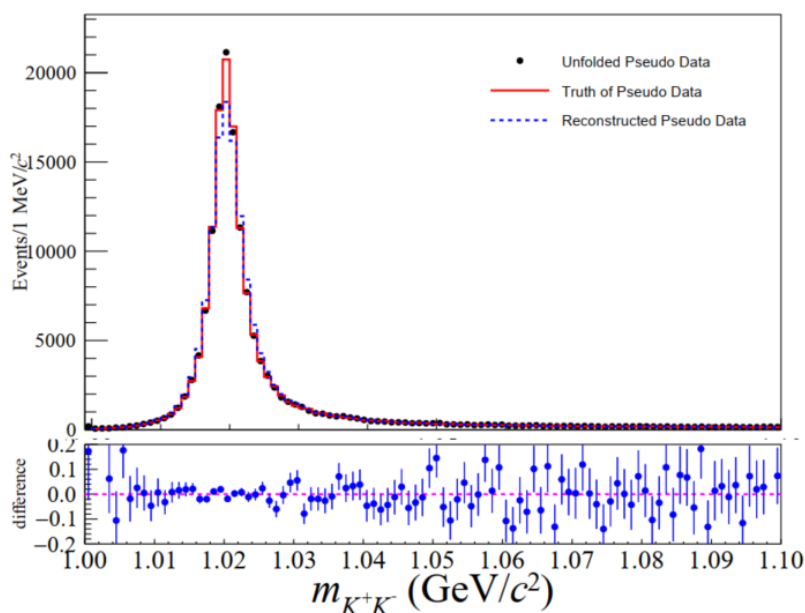
- For round03,04, $\sqrt{s}=3.773$ GeV data the $E/p < 0.8$ for the good charged tracks

Unfolding

➤ Why and how to do unfolding?

❑ High precision needed near ϕ resonance

❑ **Singular Value Decomposition (SVD)** is used to correct the smearing effects.



❑ SVD Effectively corrected the smearing effects, especially near the ϕ peak.

Systematic uncertainty

➤ Systematic uncertainty are listed below ($\sqrt{s} = 3.773$ GeV:

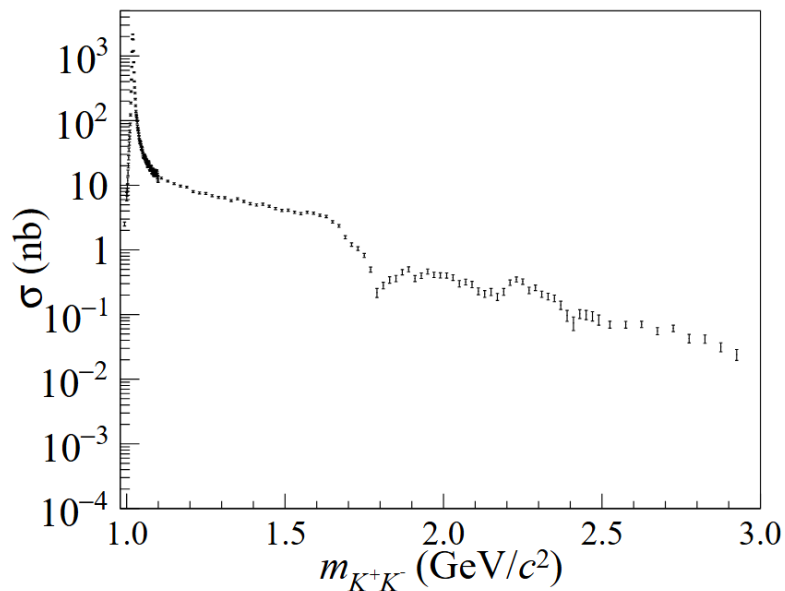
$M_{K^+K^-}$ (GeV) Uncertainty type	Threshold-1.1	1.1-2.5	2.5-3	Method
$K^\pm$ tracking efficiency	0.1%	1.2%	2.0%	Control sample: $J/\psi \rightarrow K_S K \pi$
$K^\pm$ PID efficiency	0.1%	0.8%	2.0%	Control sample: $J/\psi \rightarrow K_S K \pi$
Photon efficiency	0.2%	0.2%	0.2%	Control sample: $e^+e^- \rightarrow \mu^+\mu^-\gamma_{ISR}$
MUC efficiency	0.1%	0.4%	1.0%	Control sample: $J/\psi \rightarrow K^+K^-\pi^+\pi^-$
Unfolding	0.2%	0.0%	0.0%	Alternative line shape
Kinematic fit	0.5%	0.5%	0.5%	Correct the helicity parameters
ISR photon simulation	0.4%	0.4%	0.4%	Control sample: $e^+e^- \rightarrow K^+K^-$
VP correction	0.2%	0.2%	0.2%	Given by the theoretical calculation
Luminosity and Radiation function	0.7%	0.7%	0.7%	Given by the theoretical calculation and BESIII nominal result
overall	1.0%	1.7%	3.3%	

Cross section measurement

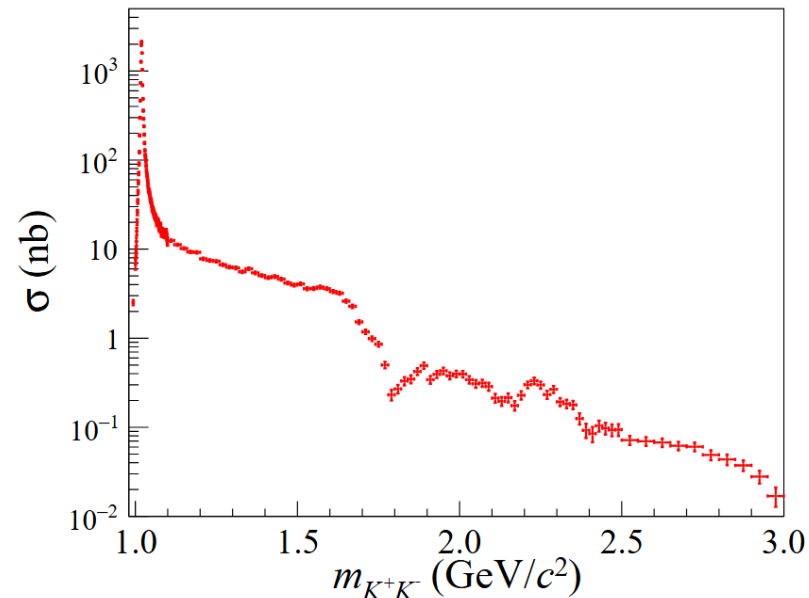
- In each interval, cross section is calculated by:

$$\sigma_{K^+K^-, \text{dressed}}(M_{K^+K^-}) = \frac{dN_{\text{sig}}/dM_{K^+K^-}}{\epsilon \cdot d\mathcal{L}_{\text{int}}/dM_{K^+K^-}}, \quad \frac{d\mathcal{L}_{\text{int}}}{dM_{K^+K^-}} = W(s, x) \mathcal{L}_{\text{int}}$$

$$\sigma_{K^+K^-, \text{Bare}} = \frac{\sigma_{K^+K^-, \text{dressed}}}{(\alpha(s)/\alpha(0))^2}$$



$\sigma_{K^+K^-, \text{dressed}}$



$\sigma_{K^+K^-, \text{Bare}}$

Cross section measurement

➤ Fit the lineshape

❑ The form factor is correlated to the $\sigma_{K^+K^-, \text{dressed}}$ by:

$$|F_K|^2(s') = \frac{3s'}{\pi\alpha^2(0)\beta_K^3} \frac{\sigma_{K^+K^-, \text{dressed}}(s')}{C_{FS}}, \quad C_{FS} \text{ is the final-state radiation correction factor}$$

❑ The lineshape is fitted by the following model:

$$F_K(s) = (a_\phi BW_\phi + a_{\phi'} BW_{\phi'} + a_{\phi''} BW_{\phi''}) / 3$$

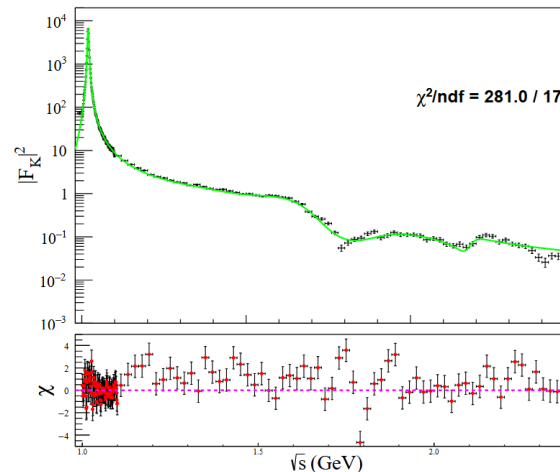
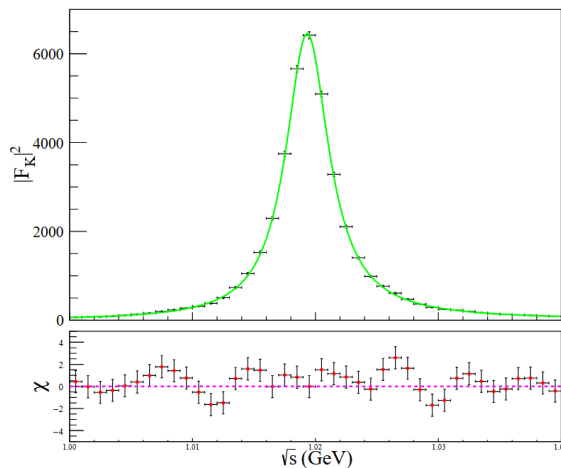
$$+ (a_\rho BW_\rho + a_{\rho'} BW_{\rho'} + a_{\rho''} BW_{\rho''} + a_{\rho'''} BW_{\rho'''}) / 2 \quad \text{With constraints}$$

$$+ (a_\omega BW_\omega + a_{\omega'} BW_{\omega'} + a_{\omega''} BW_{\omega''} + a_{\omega'''} BW_{\omega'''}) / 6$$

$$a_\phi + a_{\phi'} + a_{\phi''} = 1,$$

$$a_\rho + a_{\rho'} + a_{\rho''} + a_{\rho'''} = 1,$$

$$a_\omega + a_{\omega'} + a_{\omega''} + a_{\omega'''} = 1.$$

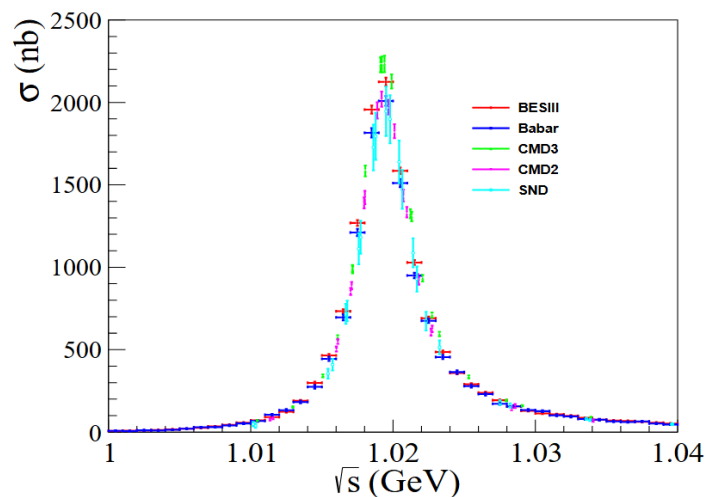


- Least χ^2 method is used to fit the line shape
- Reliable parameters of resonance near 2.2 is not obtained for the low statistics

Summary of K^+K^-

➤ Progress of K^+K^-

- ✓ Based on 23.452 fb^{-1} data sets, cross section for process $e^+e^- \rightarrow K^+K^-$ is from $2m_K$ to $3.00 \text{ GeV}/c^2$ with tagged ISR technique
- ✓ Being reviewed by the collaboration and preparing the draft
- ✓ Between results of BaBar and CMD3's and having improved precision near the $\phi(1020)$ (0.7% for statistics and 1.0% for systematics)
- ✓ Unable to yield reliable parameters of resonance near $2.2 \text{ GeV}/c^2$ due to low statistics.



➤ Next to do

- ❑ Publish the result

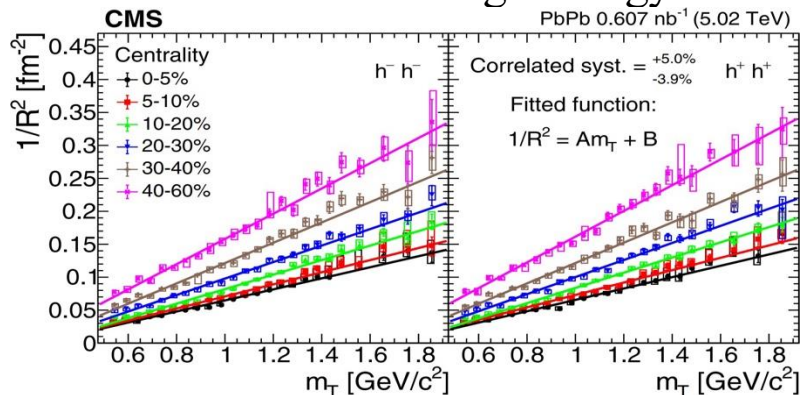
Topic2: Study of BEC by $\pi^0\pi^0$ pairs at BESIII

Motivation

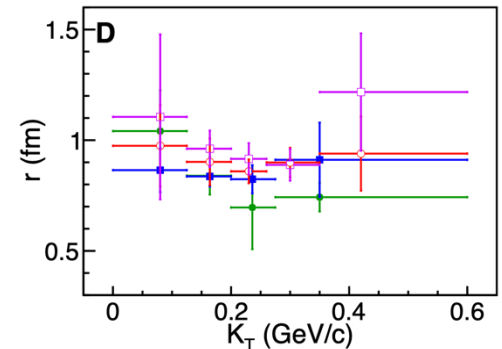
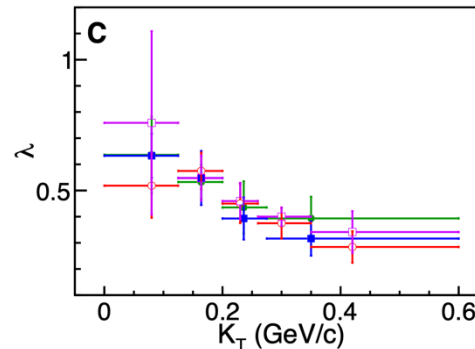
➤ Previous measurement of BEC

Representative measurements	
$\pi^\pm \pi^\pm$:	MARK II, AMY, OPAL; ZEUS, CMS, ATLAS
$\pi^0 \pi^0$:	L3 and OPAL in hadronic Z decays

- ❑ Studied by different experiments (MARKII, ZEUS, CMS.....)
- ❑ Fruitful results at high energy but **rare results at low energy and $\pi^0 \pi^0$ pairs**



Phys. Rev. C 109, 024914 (2024)



BESIII's preliminary results of $\pi^\pm \pi^\pm$ pairs

- ✓ BESIII fill the gap at few GeV based on the R-scan data from 2.0-3.08 GeV
- ✓ Preliminary results: by $\pi^\pm \pi^\pm$ pairs (Participate)
- ✓ **Different trend found indicating different source structures**

Motivation

➤ Why $\pi^0\pi^0$ pairs

- ❑ Different pictures in Lund fragmentation ($M_i = q_i \overline{q_{i+1}}$)
- ❑ Neighboring mesons inherit the flavor created at a string break.

Like-sign charged pions

$$\pi^+ = u\bar{d}$$

If $M_i = u\bar{d}$, the next meson starts with d .
It cannot be another $u\bar{d}$ at the adjacent rank.



Allowed example: $\pi^+ - \pi^- - \pi^+$

Neutral pions

$$\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$$

Adjacent flavor-neutral mesons can both project onto π^0 .



Adjacent $\pi^0\pi^0$ ranks are allowed

For Consequence in BEC:

- ✓ Broader neutral correlation in Q : $r_{00} < r_{\pm\pm}$
- ✓ Additional strength prediction: $\lambda_{00} > \lambda_{\pm\pm}$
- ✓ **BESIII: compare both states with matched kinematics and controlled backgrounds.**

Modulation and measurement of $R(Q^2)$

□ Two models often used:

- Gaussian source: $|R(Q^2)|^2 \propto \lambda e^{-r^2 Q^2}$, Exponential source: $|R(Q^2)|^2 \propto \lambda e^{-rQ}$
- λ , the correlation coefficient; r is the radius of the hadron source
- Long distance interaction parameter: $1 + \delta Q^2$
- Thus $1 + \left| \frac{R(Q^2)}{R(0)} \right|^2 = N(1 + \delta Q^2)(1 + \lambda e^{-r^2 Q^2})$ for gaussian source and
 $N(1 + \delta Q^2)(1 + \lambda e^{-rQ})$ for exponential source

□ Measurement method:

- Using the two identical particle correlation function:

$$C(Q^2) = \frac{\rho(Q^2)}{\rho_0(Q^2)}, \quad Q^2 = -(q_1 - q_2)^2,$$

$\rho(Q^2)$: distribution of Q^2 and $\rho_0(Q^2)$: distribution of Q^2 from reference sample

- Reference sample:
 - Like-sign pairs but opposite momentum of one particle (OHP)
 - Like-sign pairs but opposite transverse momentum of one particle (ROTA)
 - Particle pairs in different events (MIX)
 - Mix: mix events randomly
 - MixN: mix events with similar multiplicity of π^0
 - MixE: mix events with similar invariant mass of all π^0
- Double ratio: $R(Q^2) = \frac{C^{data}(Q^2)}{C^{MC}(Q^2)}$ to remove other possible correlations

Event Selection

- ❑ Data Sets: $\sqrt{s}=2.125, 2.396, 2.645, 2.900, 3.080$ GeV, total luminosity: $\sim 500 \text{ pb}^{-1}$,
- ❑ Two kinds of hadron MC samples used for analysis: Hybrid and Lualw

➤ Good charged tracks

- $|\cos\theta| < 0.93$
- $V_r < 0.5 \text{ cm}, |V_z| < 5 \text{ cm}$
- $p/p_{\text{beam}} < 0.94$, p_{beam} is beam momentum
- $\chi_{\text{prob}} = \left(\left| \frac{dE}{dx}_{\text{measured}} - \frac{dE}{dx}_{\text{proton}} \right| \right) / \sigma_{\text{proton}} > 10$
- $N_{\text{good}} > 0$

➤ Good photons:

- EMC time: $0 < t < 700 \text{ ns}$
- $|\cos\theta_\gamma| < 0.8, E_\gamma > 25 \text{ MeV}$ for barrel
- $|\cos\theta_\gamma| \in (0.86, 0.92), E_\gamma > 50 \text{ MeV}$ for end cap
- $N_\gamma \geq 4$

➤ Veto Bhabha and Di-gamma:

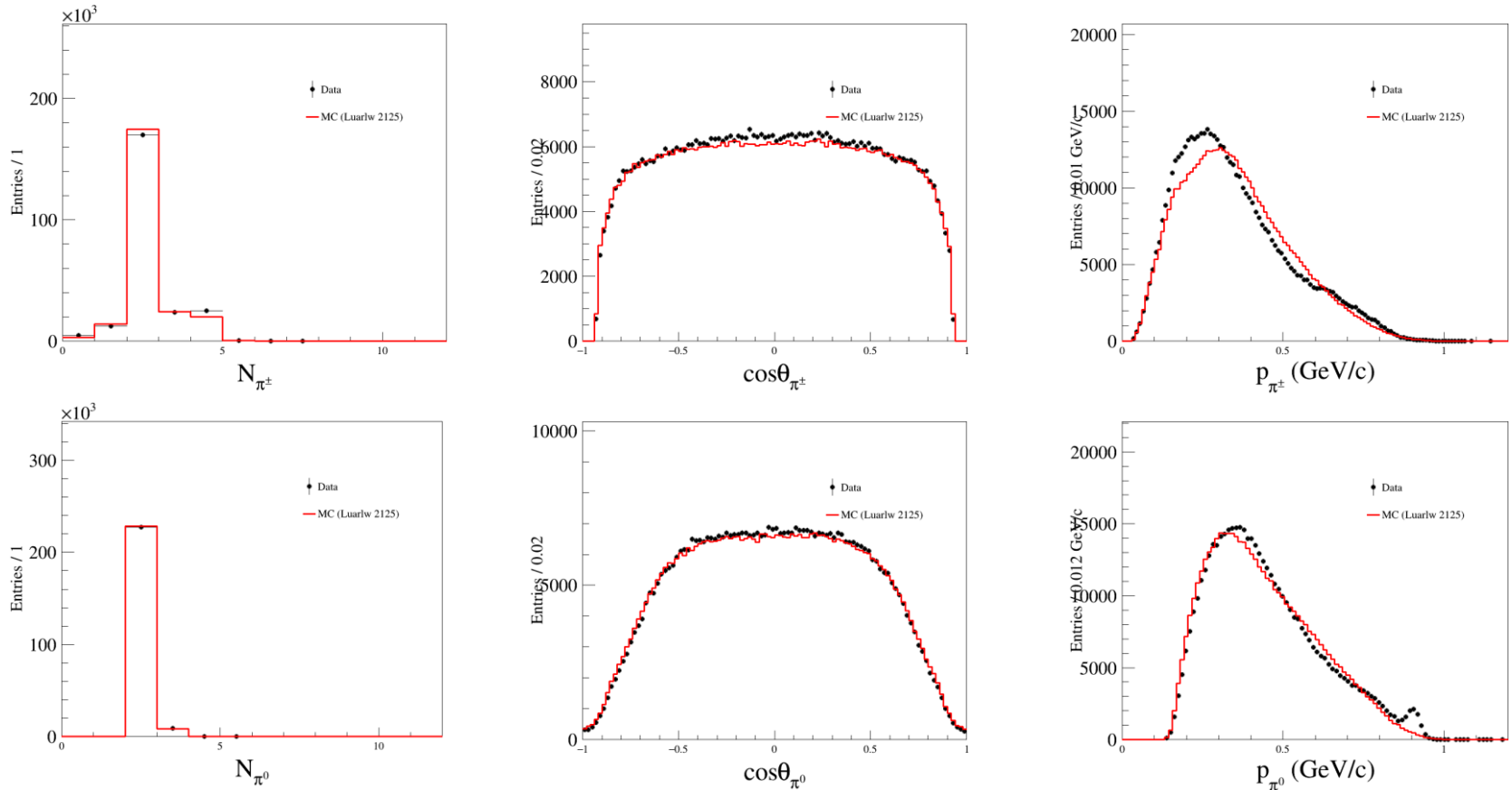
- Choose the two largest showers in EMC
- $E_{\text{largest}} < 0.65 E_{\text{beam}}$
- $|\delta\theta| = |\theta_1 + \theta_2 - 180^\circ| < 10^\circ$

➤ π^0 reconstruction

- All pairs of photons used to reconstruct π^0 list. For each π^0 : $m_{\pi^0} \in [120, 150] \text{ MeV}$
- 1C kinematic fit applied to each π^0 .
- Finding the largest π^0 subset without repeating photon and minimum $\sum \chi^2_{1c}$
- $N_{\pi^0} \geq 2$

Comparisons of data and MC

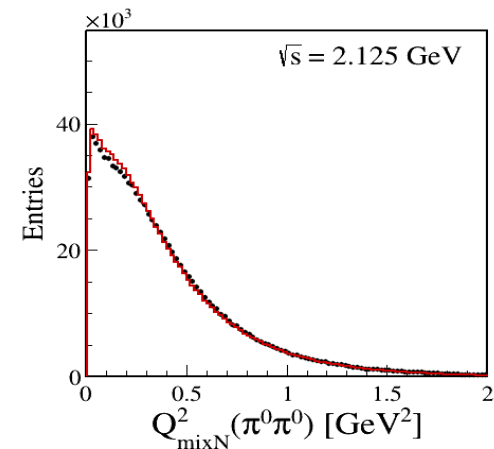
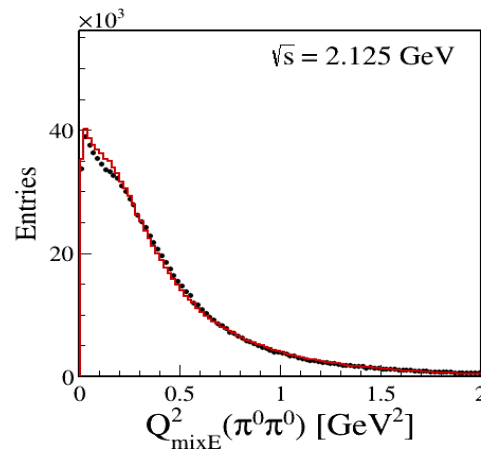
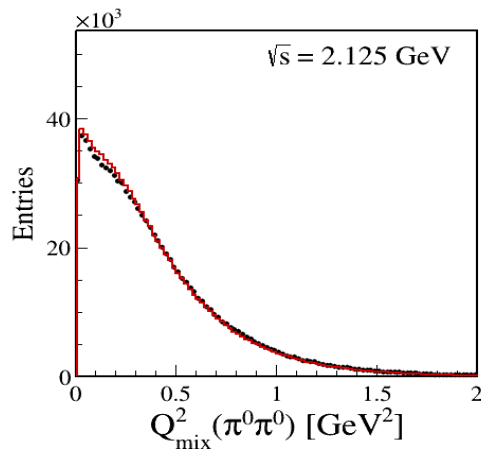
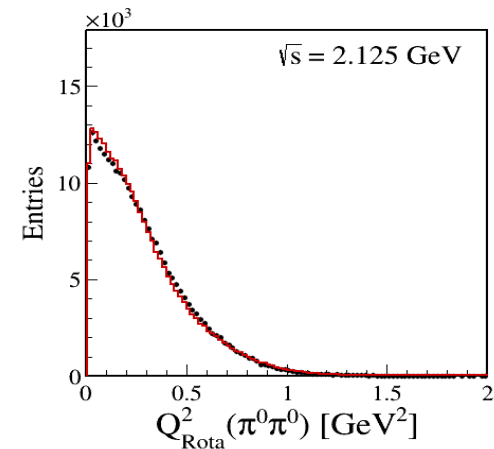
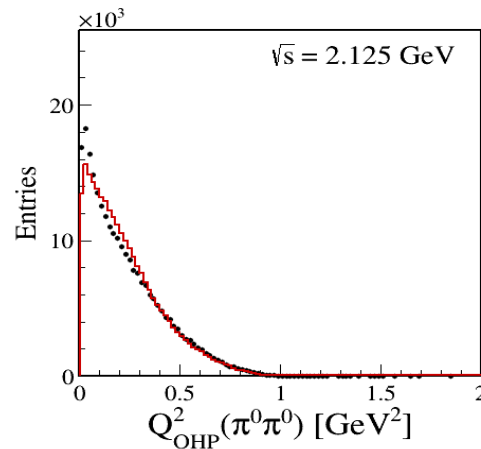
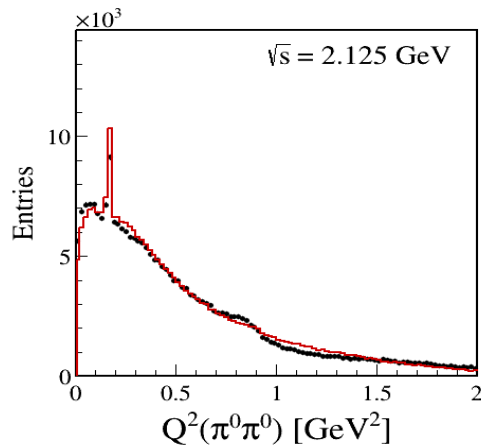
➤ Distributions of $\pi^\pm$ and π^0 at $\sqrt{s} = 2.125$ GeV as an example



- First row is distributions of $\pi^\pm$, and second row is π^0 's.
- Lualw MC describes data well

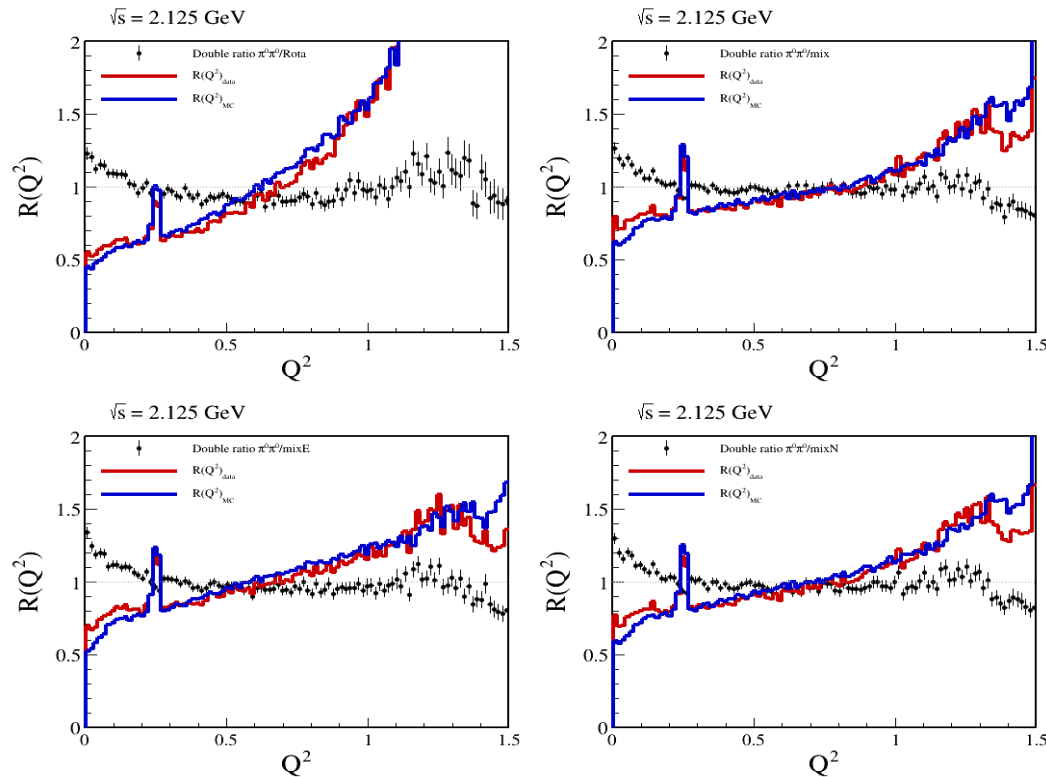
Comparisons of data and MC

► Distributions of Q^2



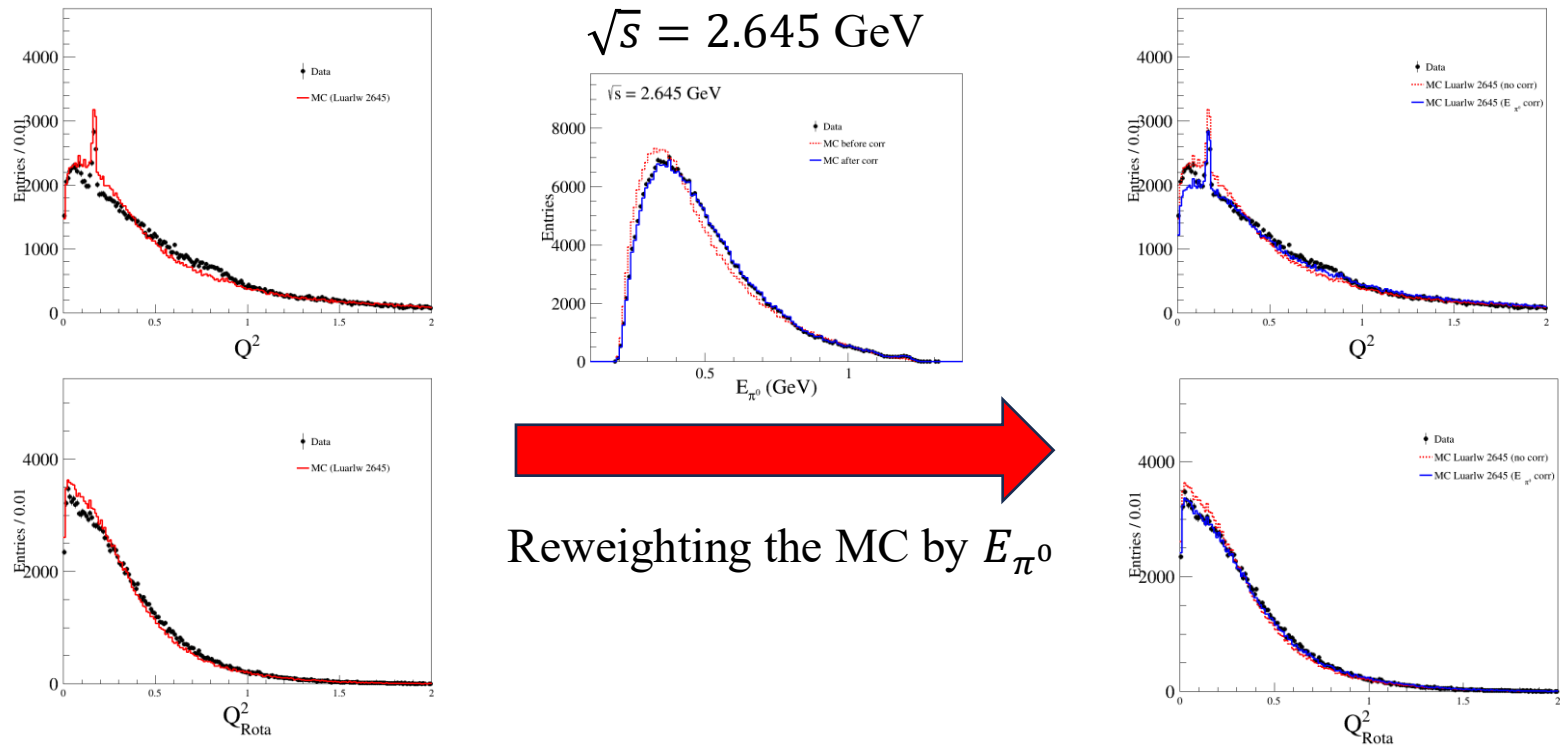
Comparisons of data and MC

➤ Double ratio



- ❑ Most reference samples are consistent with data (except for the OHP)
- ❑ BEC effect is observed
- ❑ Distinction between data and MC near $Q^2 \approx 0.8$

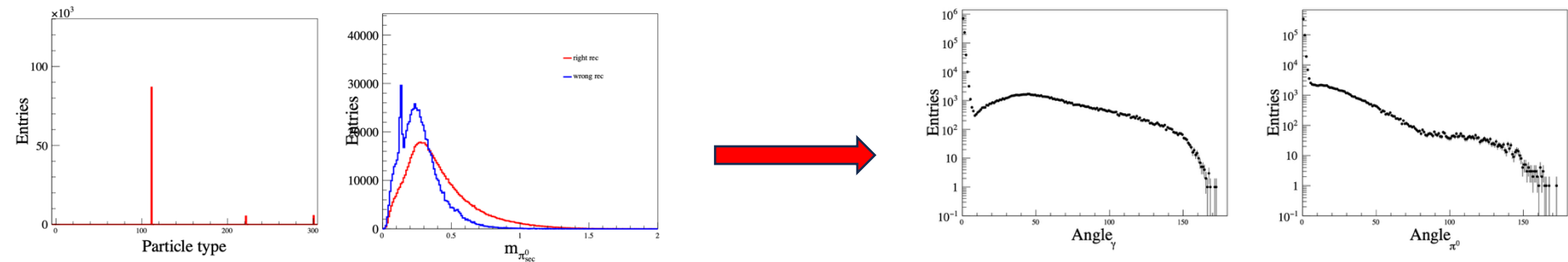
MC corrections: momentum corrections



- ❑ Tensions are found when the $\sqrt{s}$ comes up
- ❑ Event-by-event weighting: $\prod_{i=1}^{i=N} w_i$, w_i is the weight factor of $i^{\text{th}} \pi^0$
- ❑ Eliminating most differences between data and MC

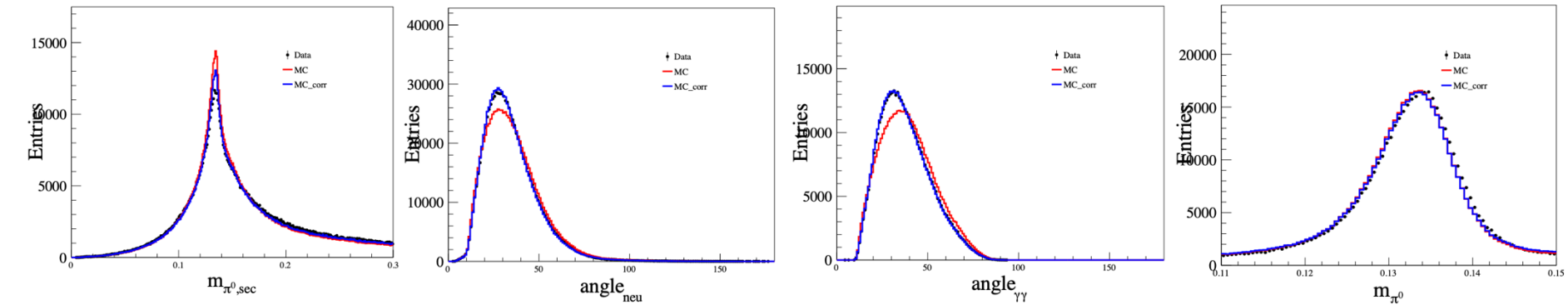
Study of fake π^0

➤ Match of π^0 and photons



- $R_{\pi^0_{\text{fake}}} = 7.0\%$, $R_{\gamma_{\text{fake}}} = 8.8\%$, here the “fake” means the truth match angle is larger than 20°
- Mainly caused by the wrong combination of photons from different π^0

➤ Check the effect of fake π^0

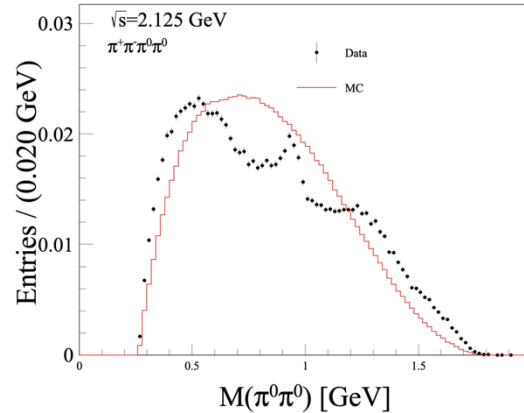


- The π^0 reconstruction of data and MC is similar after the corrections

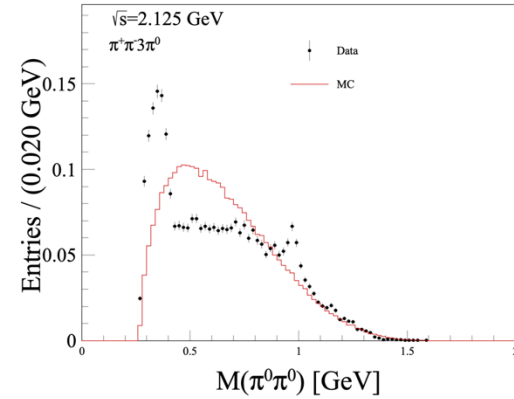
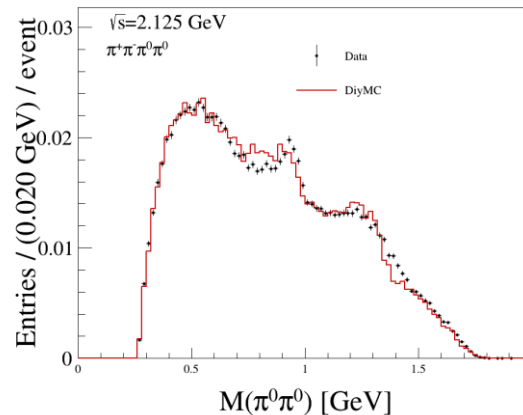
MC corrections: resonance effect

Channel	2.125 (%)
$\pi^+\pi^-2\pi^0$	42.0
$\pi^+\pi^-3\pi^0$	13.8
$\pi^+\pi^-4\pi^0$	6.3
$\pi^+\pi^-5\pi^0$	4.5
$2\pi^+2\pi^-2\pi^0$	14.3
$2\pi^+2\pi^-3\pi^0$	3.8
$2\pi^+2\pi^-4\pi^0$	1.2
$2\pi^+2\pi^-5\pi^0$	0.4
$K^+K^-2\pi^0$	0.6
$K^\pm\pi^\mp3\pi^0$	2.4
$\pi^+\pi^-2\pi^0\eta(\eta \rightarrow \gamma\gamma)$	1.7
$\pi^+\pi^-3\pi^0\eta(\eta \rightarrow \gamma\gamma)$	0.3
$3\pi^+3\pi^-2\pi^0$	0.2
$3\pi^+3\pi^-3\pi^0$	0.1
Sum over	91.6

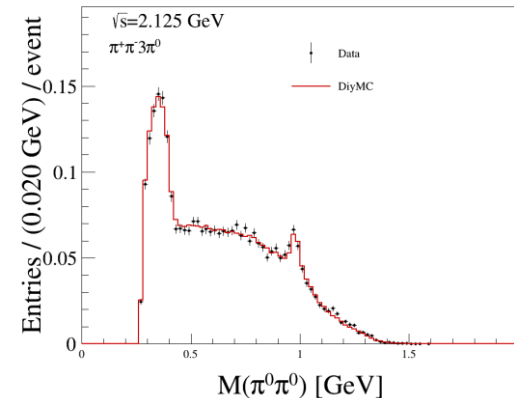
➤ PWA analysis of this two channels:



$\rho, \omega, f_0(500), f_0(980),$
 $f_2(1270), a_1(1260) \dots$



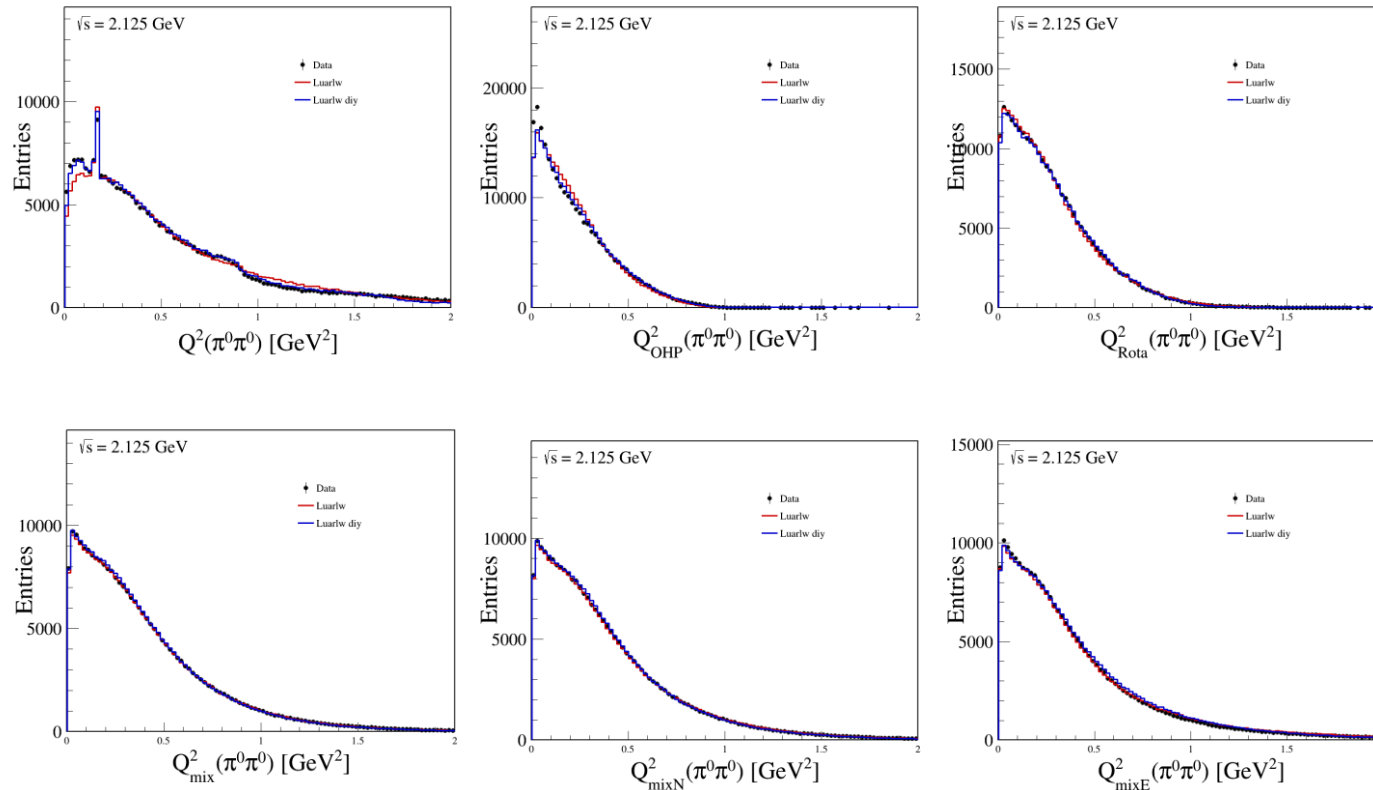
ρ, ω, η
 $f_0(980), \phi \dots$



Two major channels contributing to the resonances are checked:

$\pi^+\pi^-2\pi^0$ and $\pi^+\pi^-3\pi^0$

MC corrections: resonance effect



- ❑ Replacing the corresponding parts with DIYMC.
- ❑ The tension near 0.8 GeV become much smaller
- ❑ Other points are still ongoing

Summary of BEC

➤ Progress of BEC

- ✓ Feasibility study has been performed at 2.396 GeV and obvious BEC effect is observed.
- ✓ Effect of fake π^0 has been checked.
- ✓ Five Ref. samples have been checked
- ✓ MC tuning is performed by reweighting and DIY MC

➤ Next to do

- ❑ Finish the MC tuning and efficiency corrections
- ❑ Check the dependences on K_T , which is meaningful for the electron-positron collider.
- ❑ Systematic uncertainty and so on.

Summary and prospects

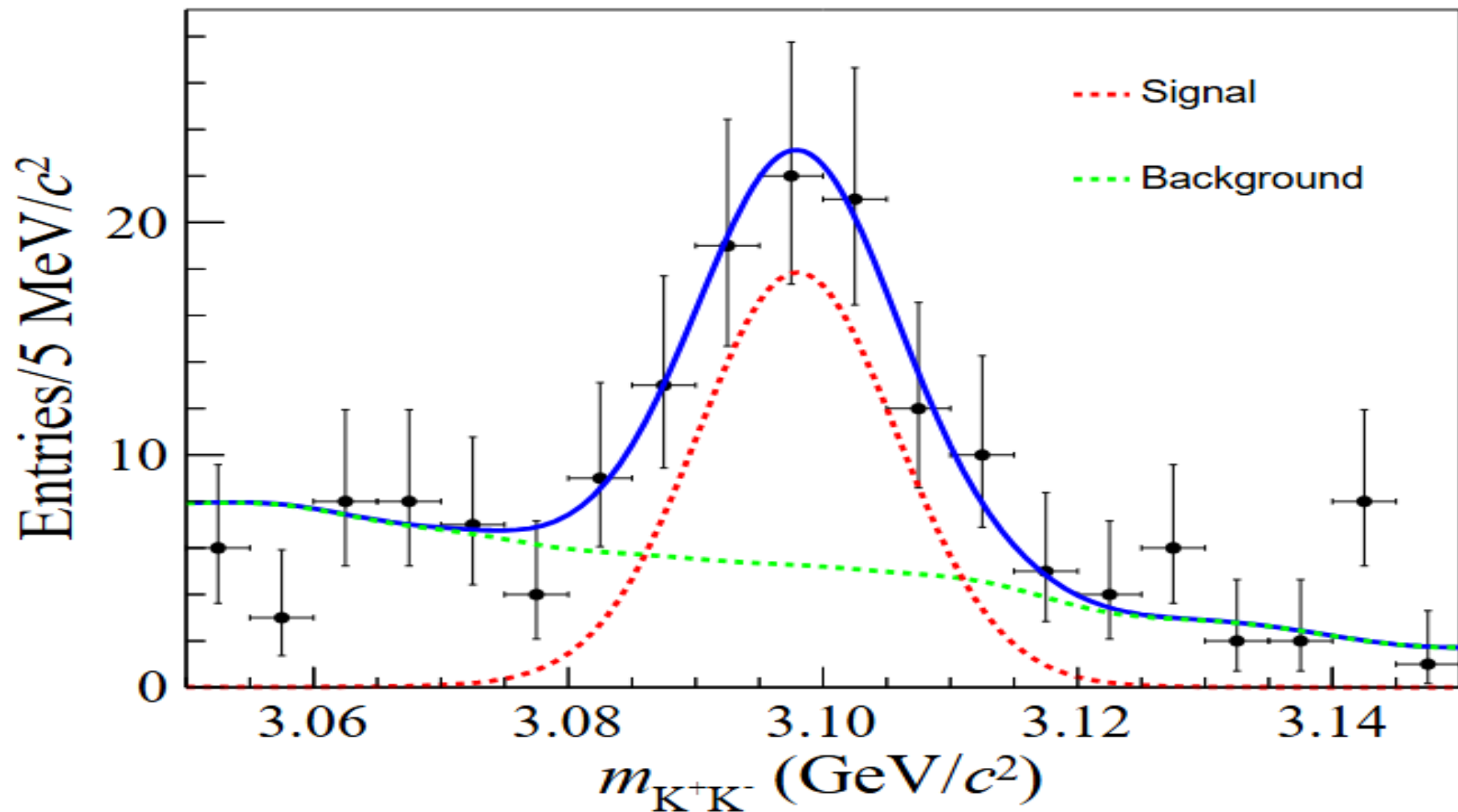
- ✓ Cross section measurement of $e^+e^- \rightarrow K^+K^-$ via initial state radiation technique has been completed and draft is being prepared
- Study of two-particle Bose-Einstein Correlations
 - Efficiencies corrections is ongoing
 - Dependence of K^T or other observable will be studied
 - Systematic uncertainties will be studied

Back up

Back up: Selection of $K^+K^-\pi^0$

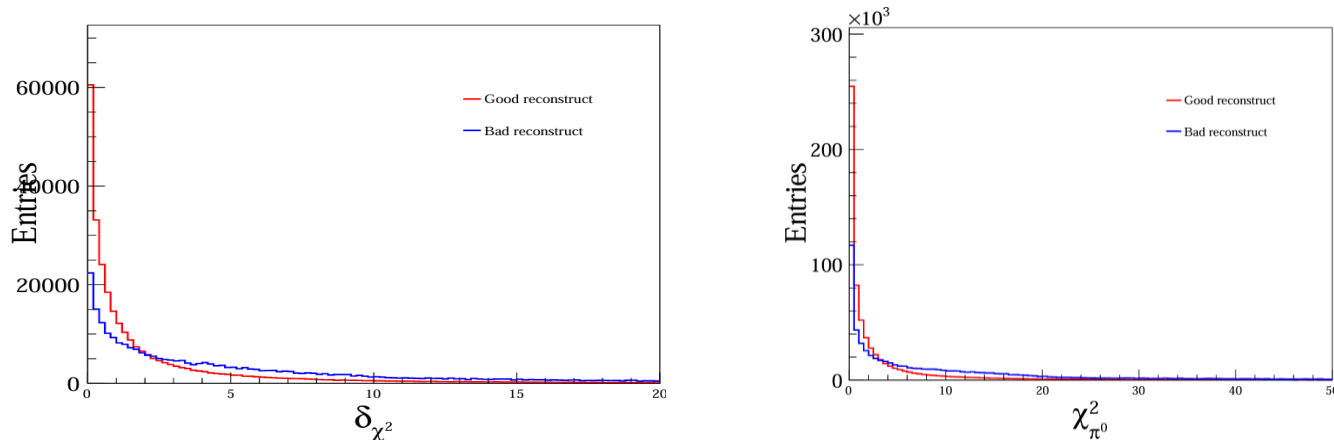
- Good charged tracks:
 - $|\cos\theta| < 0.93$
 - $V_r < 1 \text{ cm}, |V_z| < 10 \text{ cm}$
 - $N_{\text{Good}} = 2$
- Good photons:
 - EMC time: $0 < t < 700 \text{ ns}$
 - $|\cos\theta_\gamma| < 0.8, E_\gamma > 25 \text{ MeV}$ for barrel
 $|\cos\theta_\gamma| \in (0.86, 0.92), E_\gamma > 50 \text{ MeV}$ for end cap
 - Angle from the nearest charged track $> 10^\circ$
 - $N_\gamma \geq 2$
- PID ($\frac{dE}{dx} + \text{TOF}$):
 - $K: \text{prob}(K) > \text{prob}(\pi), \text{prob}(K) > \text{prob}(p)$
 - $N_{K^+} = N_{K^-} = 1$
- Vertex fit and Kinematic fit
 - Successful Vertex fit by two Kaons
 - 4c kinematic fit and $\chi_{4c}^2 < 60$
 - Looping single good photon to do 4c kinematic fit, choose the combination with minimum $\chi_{\gamma, \min}^2$
 - $\chi_{\gamma, \min}^2 > \chi_{4c}^2$
- Further selection
 - $M_{2\gamma} \in [120, 150] \text{ MeV}$

Back up: $J/\psi \rightarrow K^+ K^-$



Back up: $J/\psi \rightarrow K^+ K^-$

➤ An unbinned maximum likelihood fit used to extract signal yield:



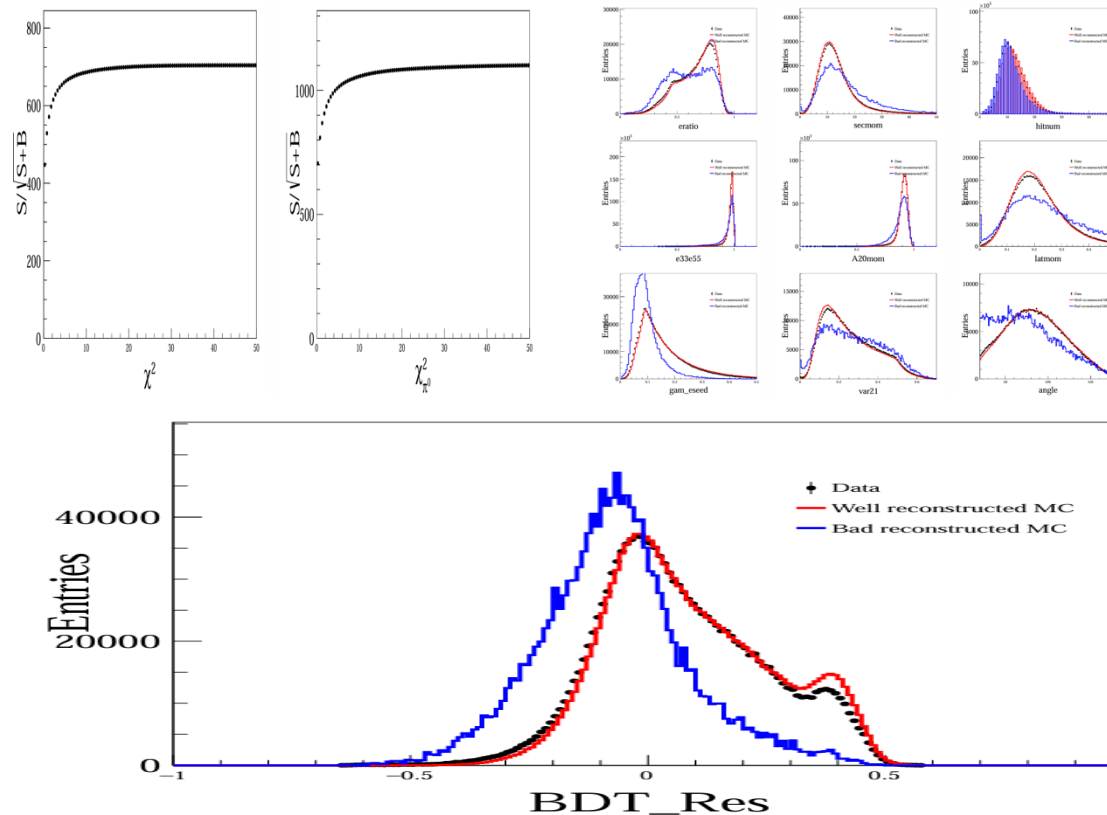
- Total fit = signal model $\otimes$ Gaussian + combined background model
- Summary of cross section of $e^+e^- \rightarrow \gamma^{ISR} J/\psi \rightarrow \gamma^{ISR} K^+ K^-$:

$\sqrt{s}(\text{GeV})$	$N_{J/\psi}$	ϵ	$W(s, x_0)$	$\Gamma(J/\psi \rightarrow e^+e^-) (\text{keV})$	$\mathcal{B}(J/\psi \rightarrow K^+K^-)$
3.773	71.1 ± 12.7	1.4%	0.171	5.529 ± 0.106	$(2.57 \pm 0.46) \times 10^{-4}$

- PDG: $(2.86 \pm 0.21) \times 10^{-4}$

Back up: efforts to suppress the fake π^0

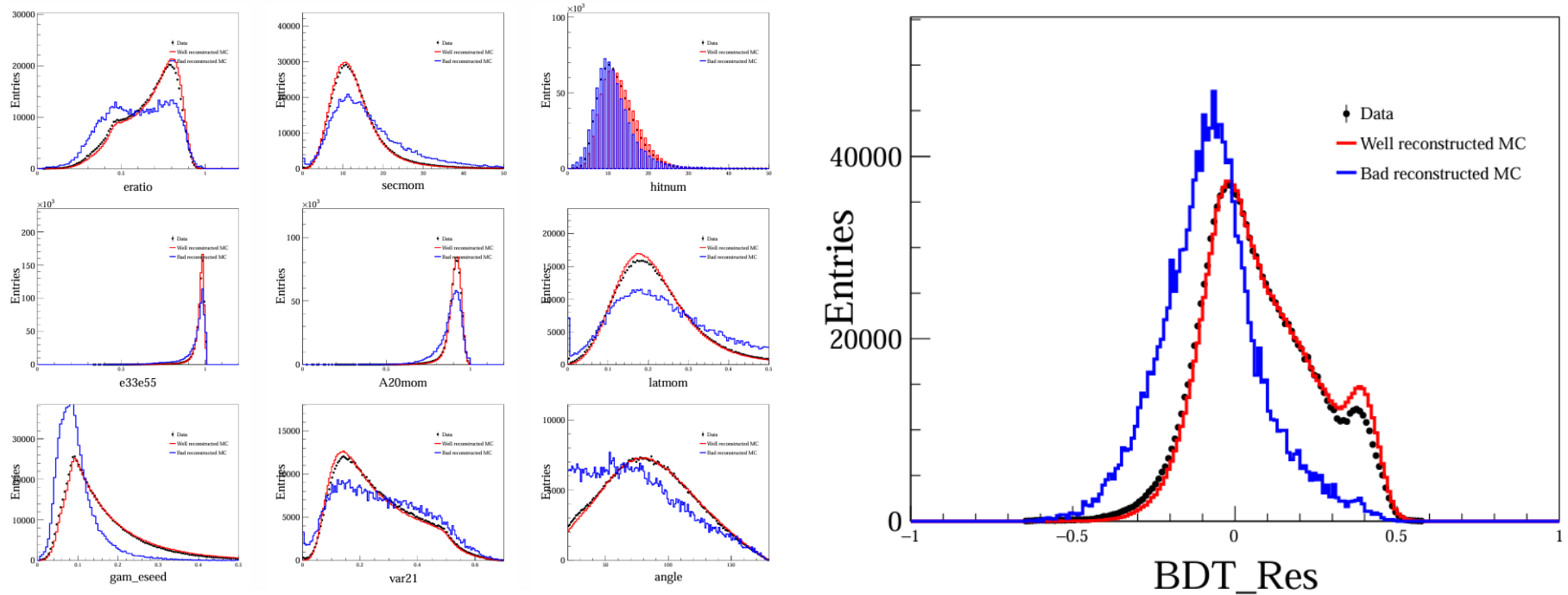
➤ Kinematic fit:



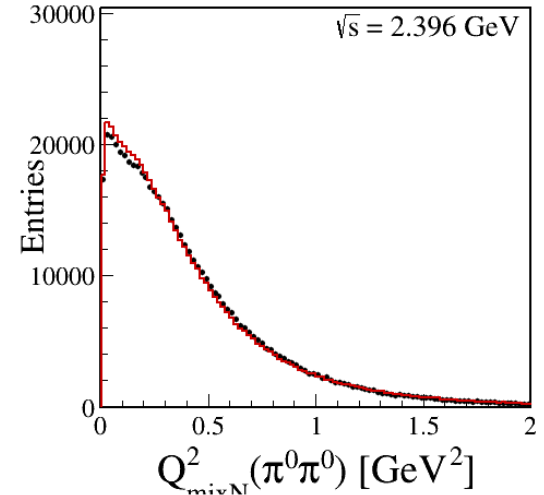
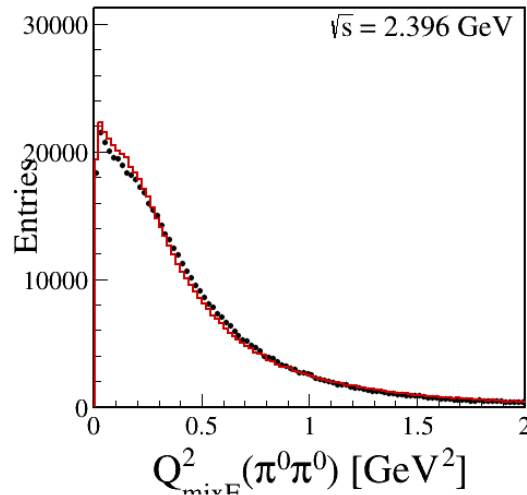
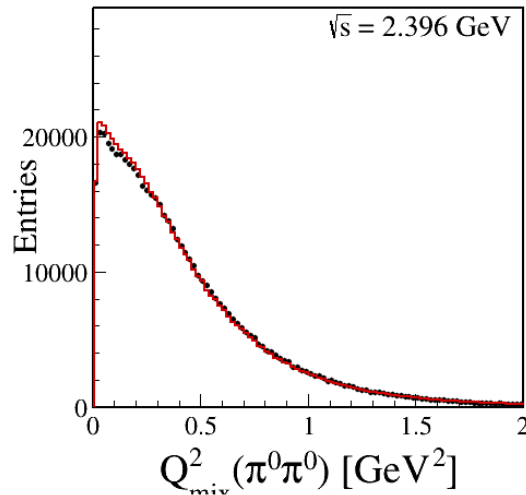
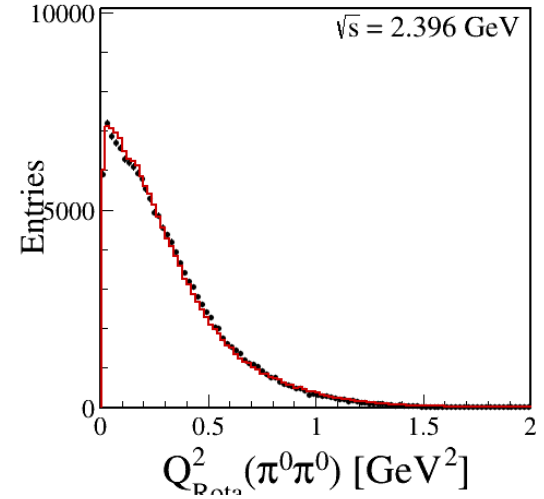
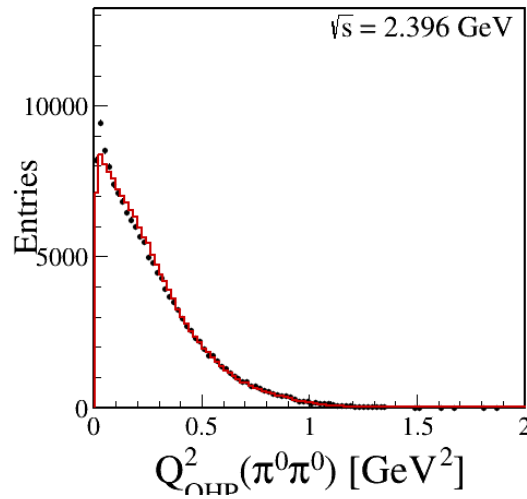
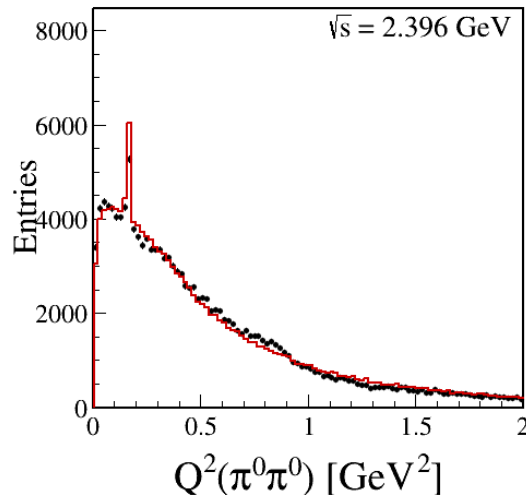
$$\delta_{\chi^2} = |\chi^2_{\text{full } \pi^0}/n_{\text{full}} - \chi^2_{\text{one less } \pi^0}/(n_{\text{full}}-1)|$$

Back up: efforts to suppress the fake π^0

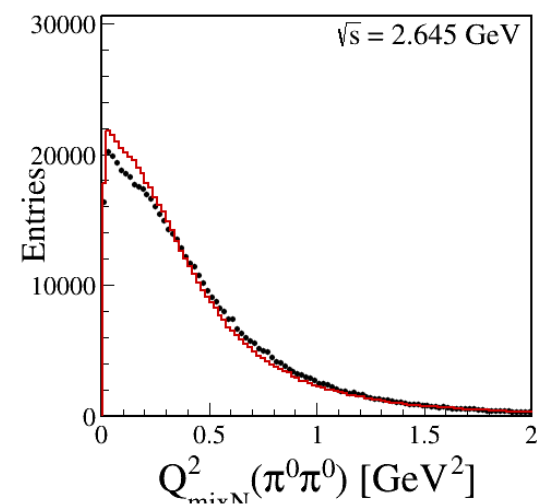
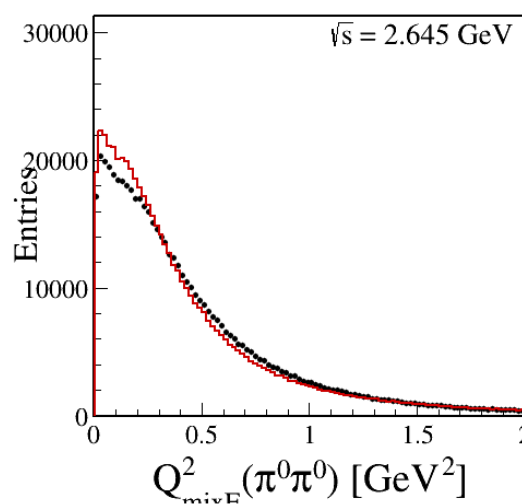
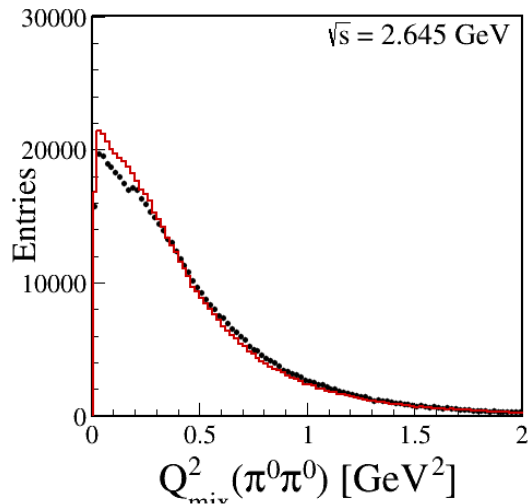
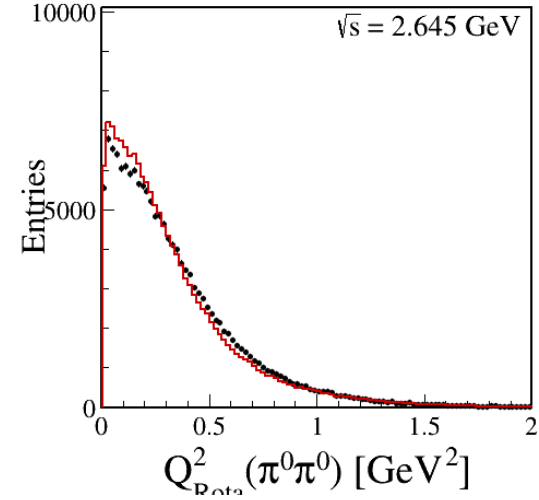
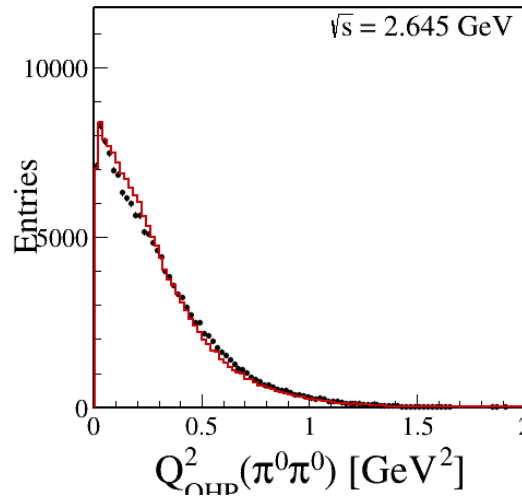
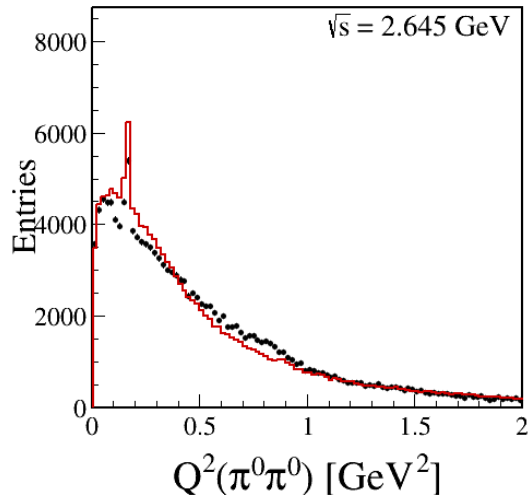
➤ BDT:



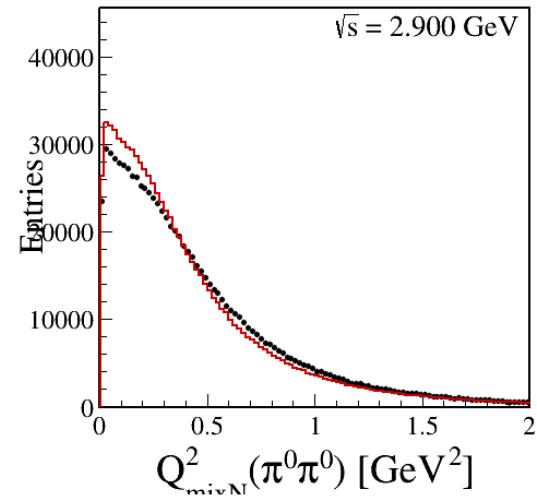
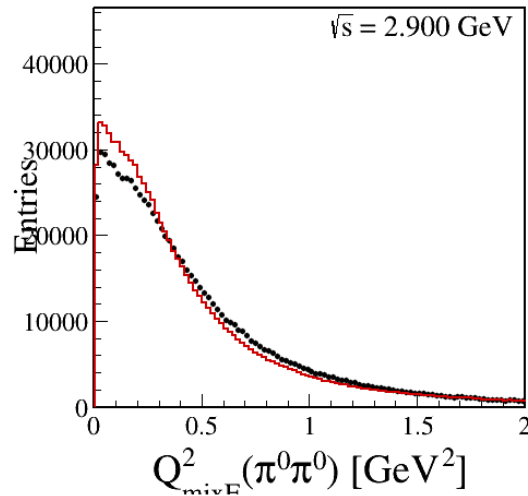
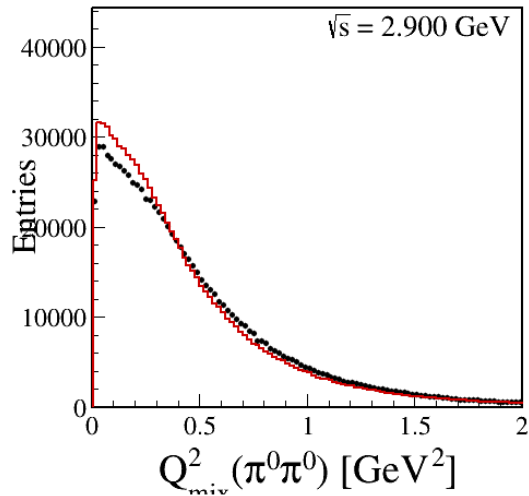
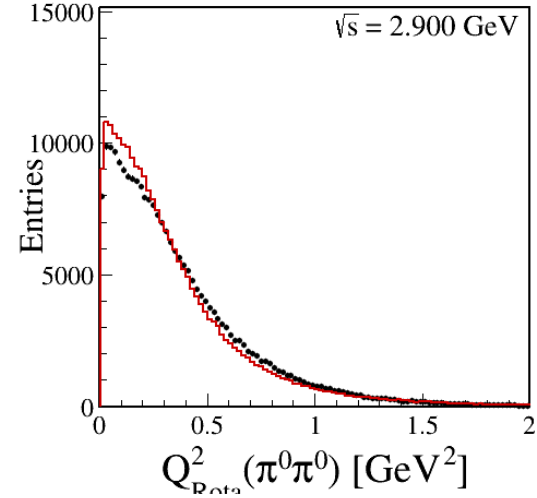
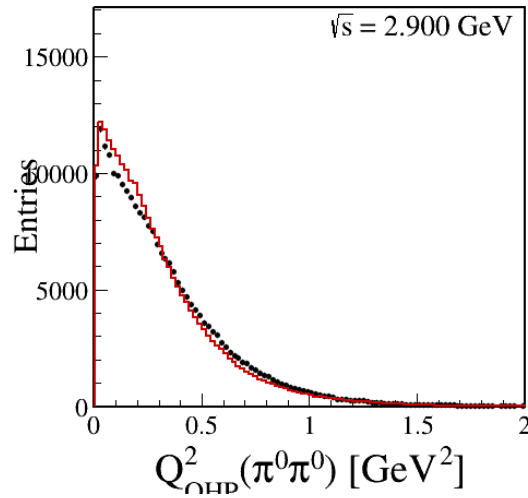
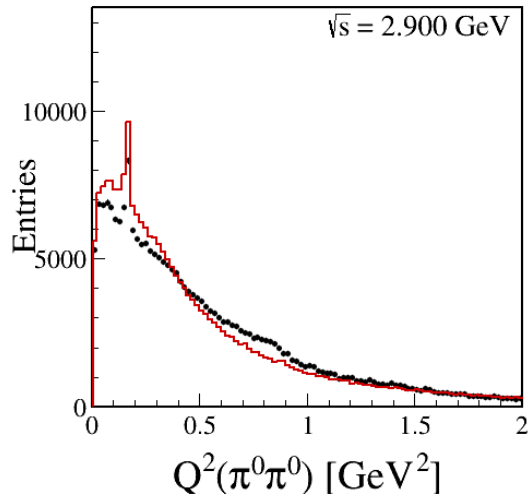
Back up: Q^2 and reference samples



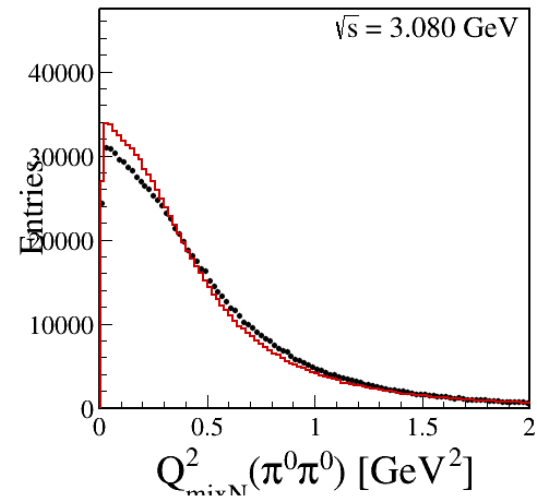
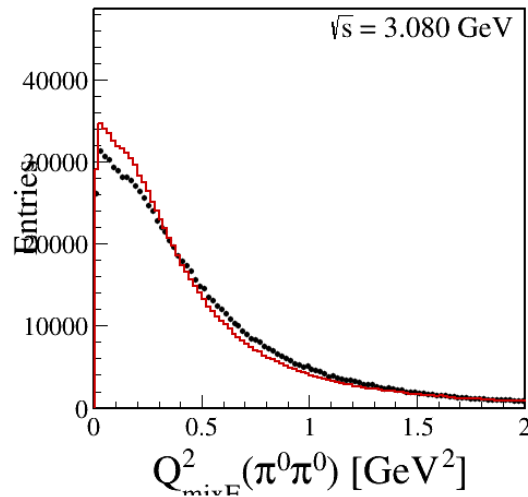
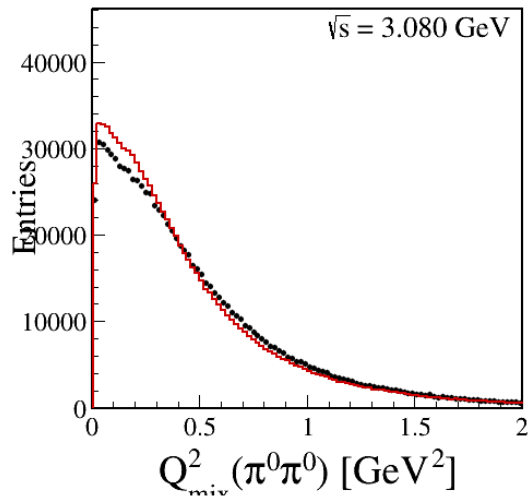
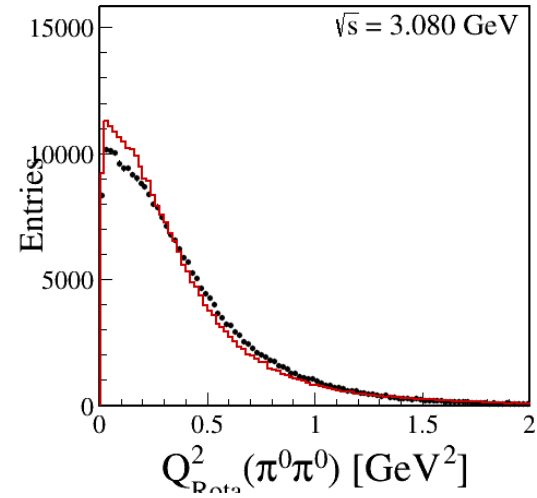
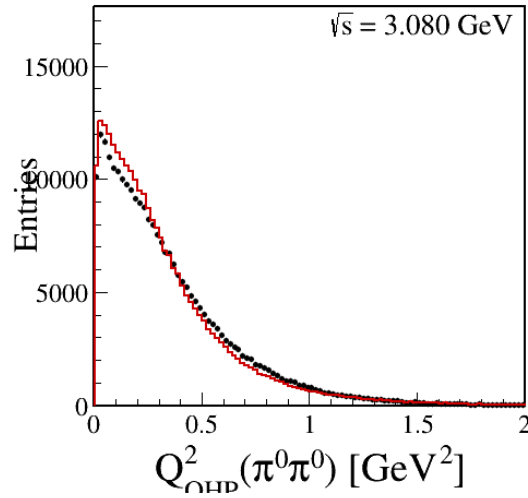
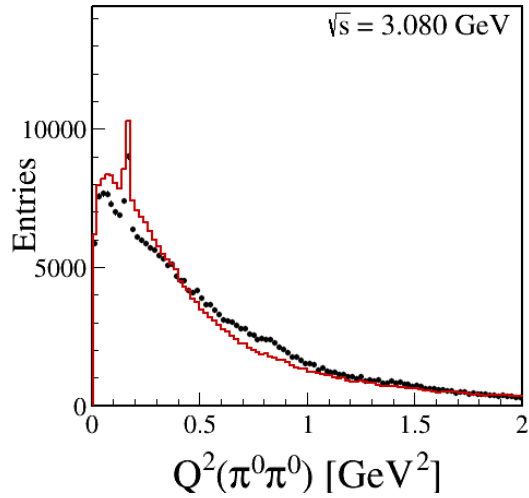
Back up: Q^2 and reference samples



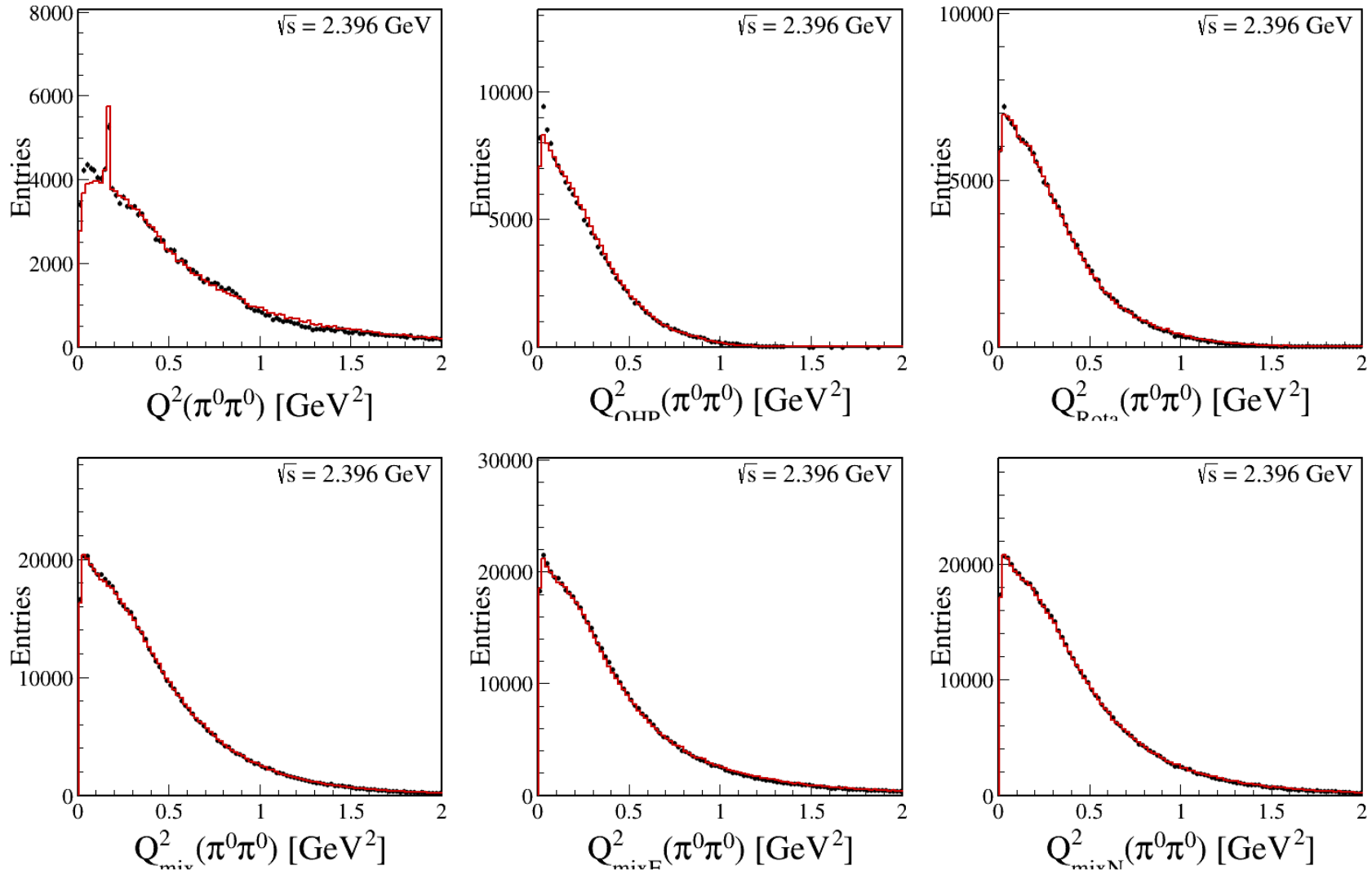
Back up: Q^2 and reference samples



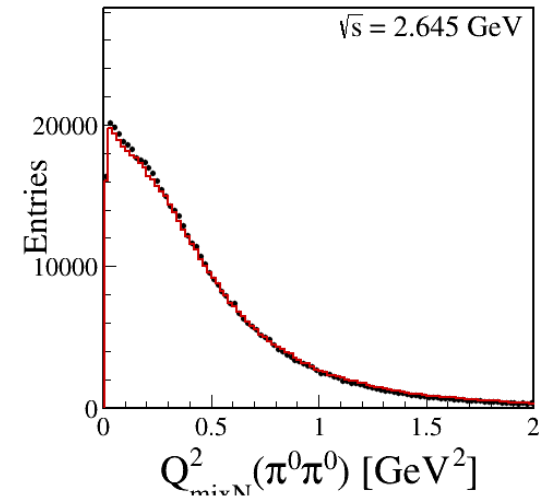
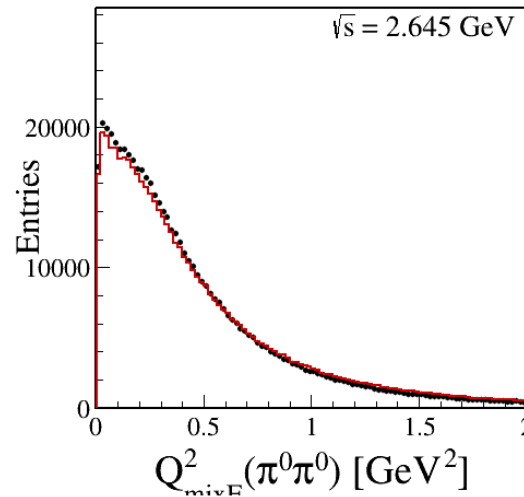
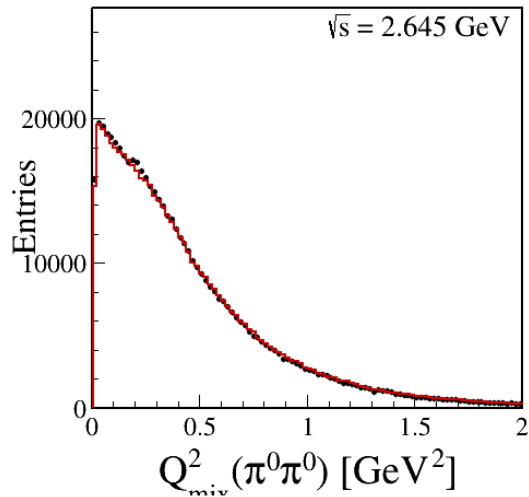
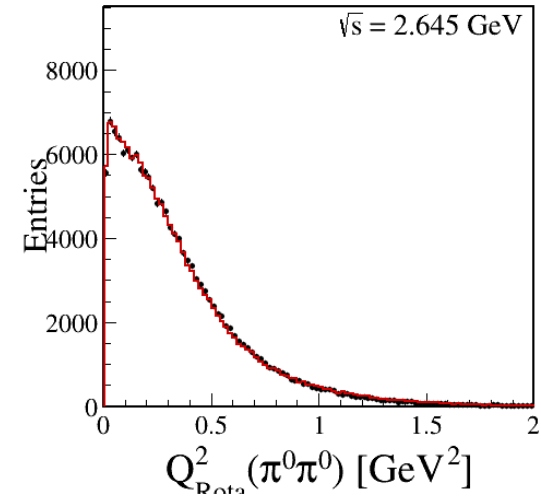
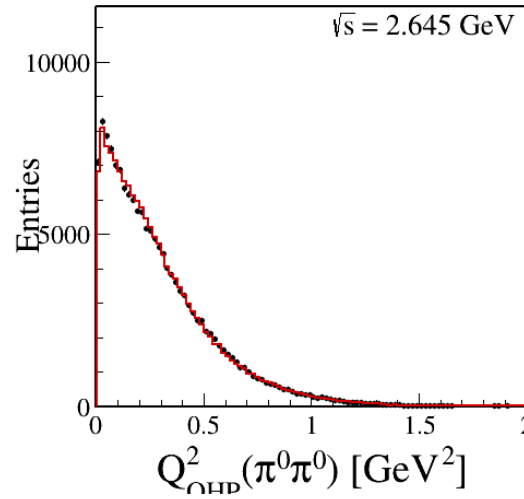
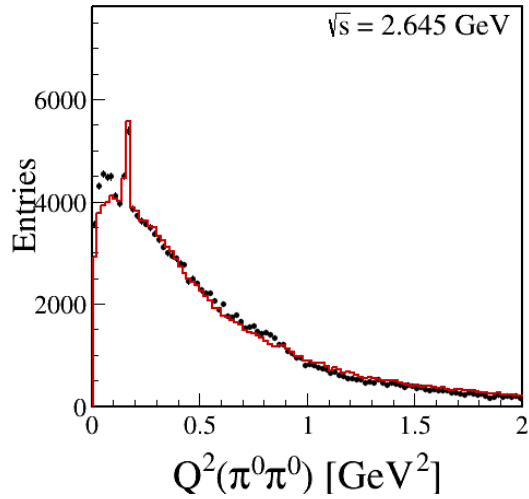
Back up: Q^2 and reference samples



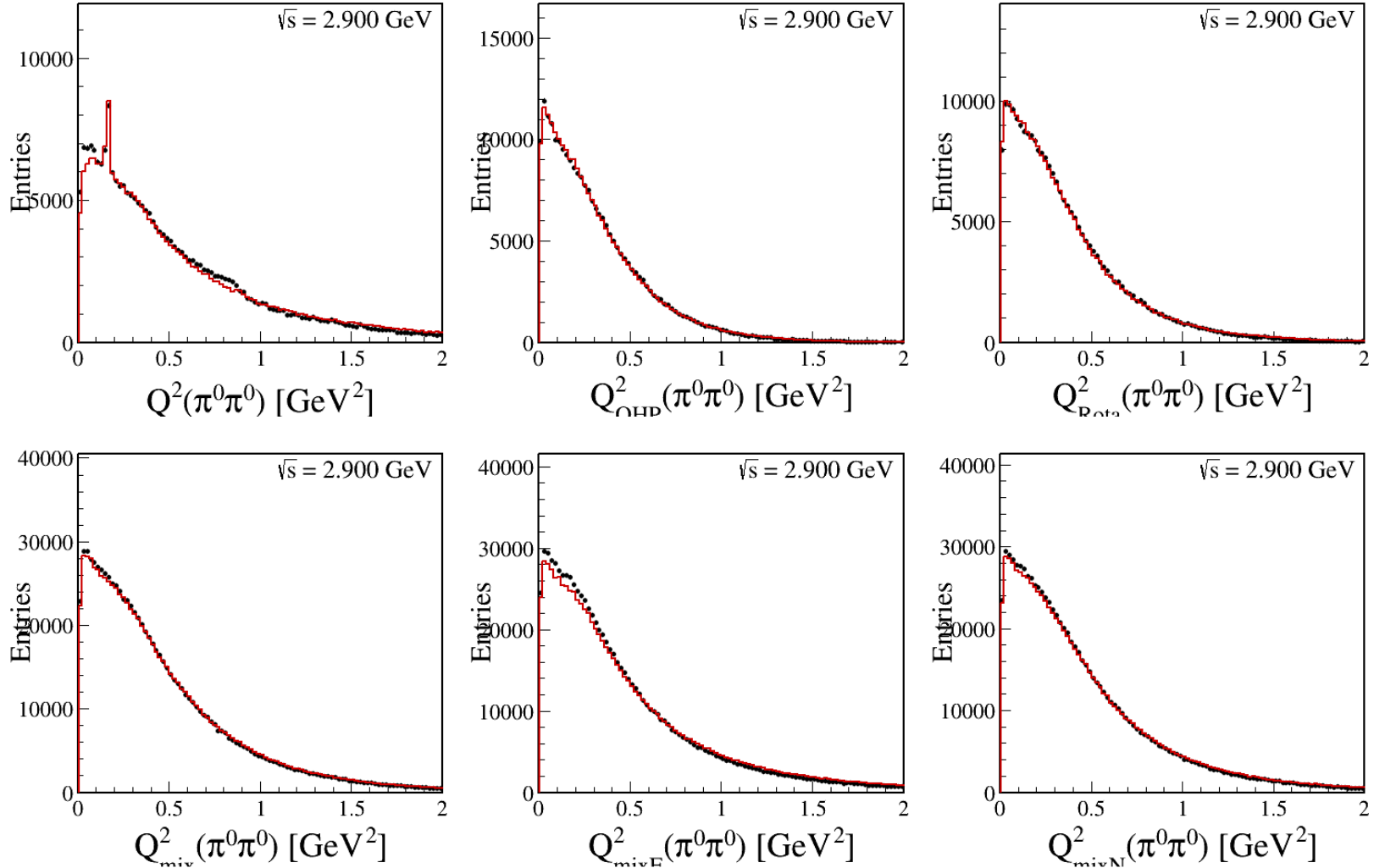
Back up: Q^2 and reference samples (corr)



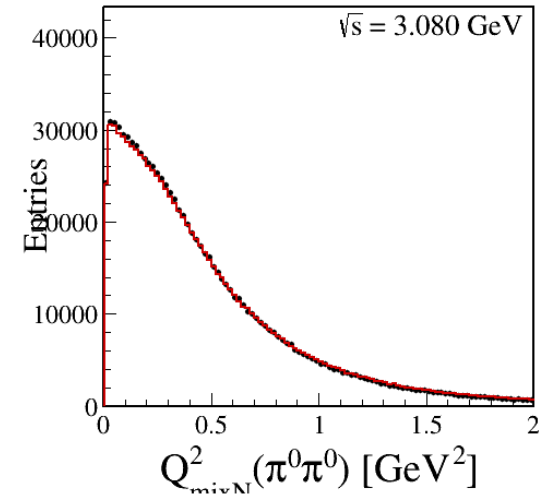
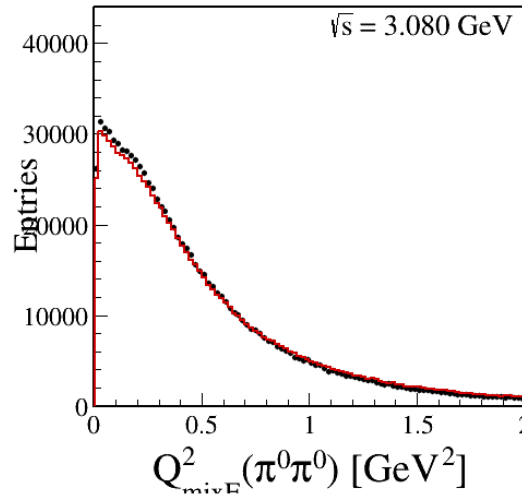
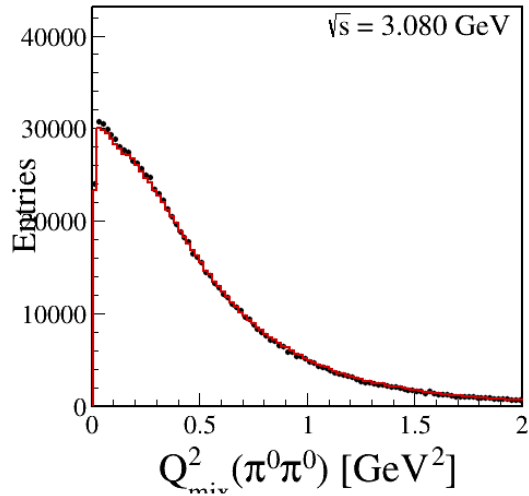
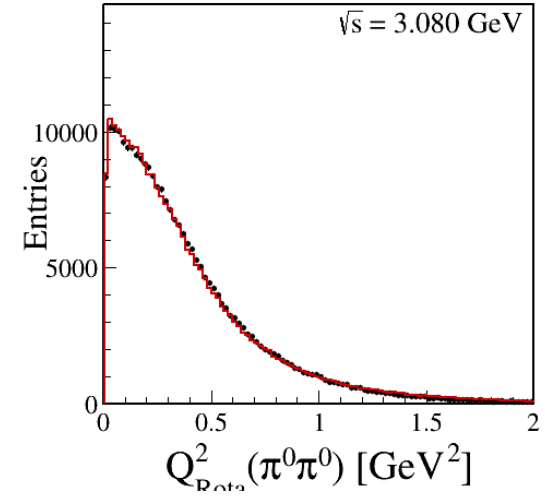
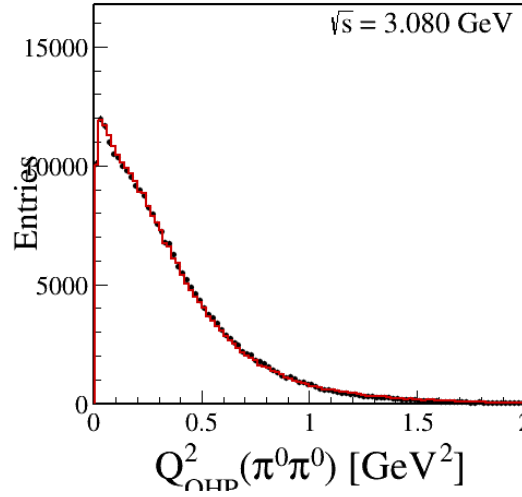
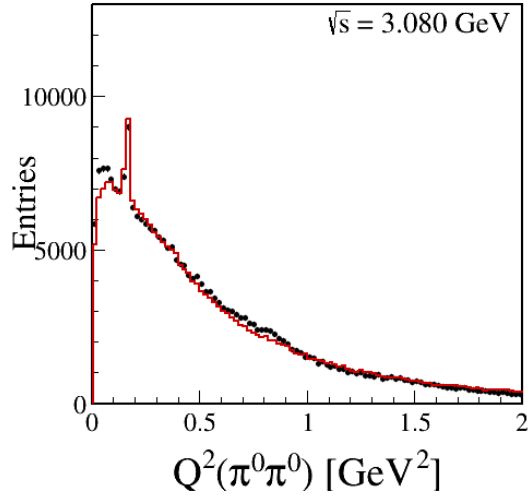
Back up: Q^2 and reference samples (corr)



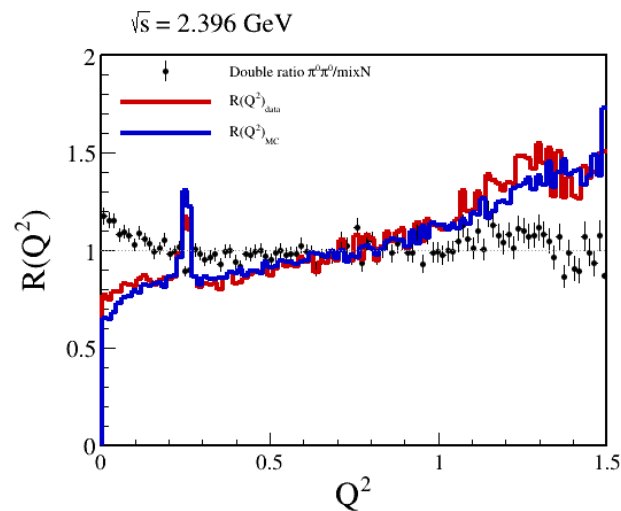
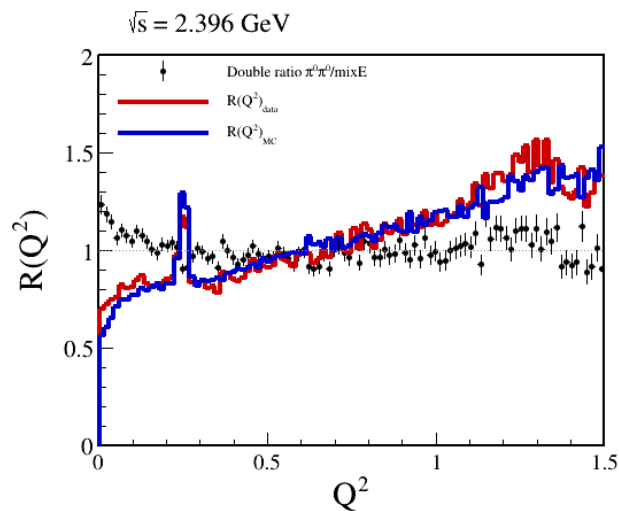
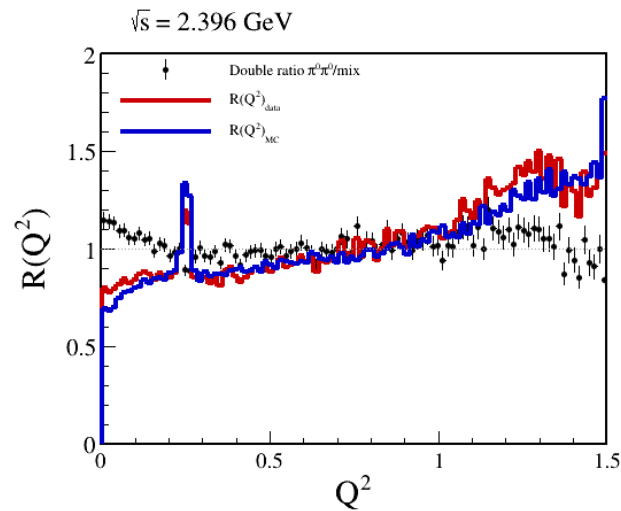
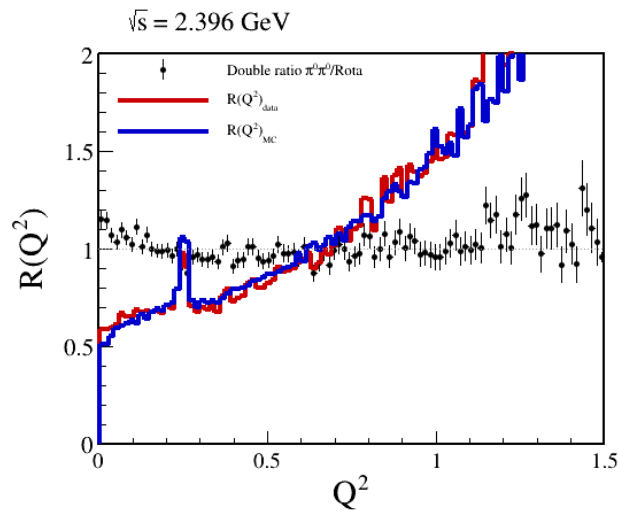
Back up: Q^2 and reference samples (corr)



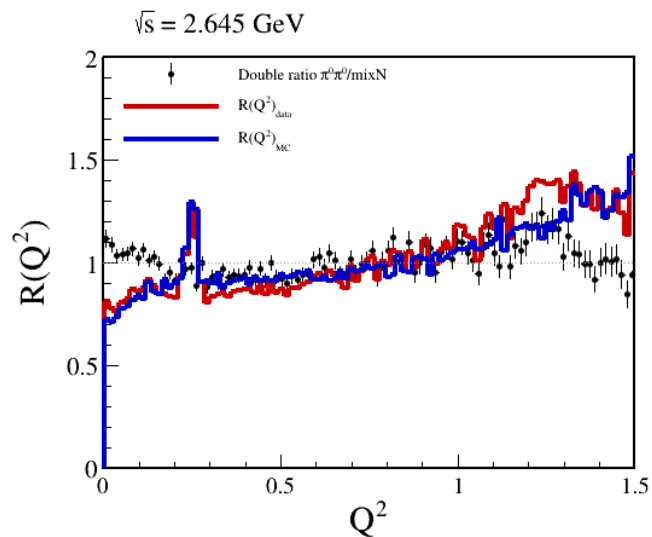
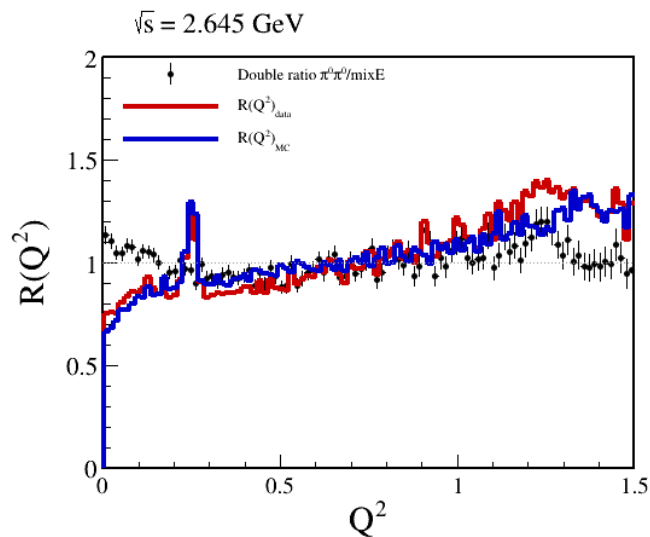
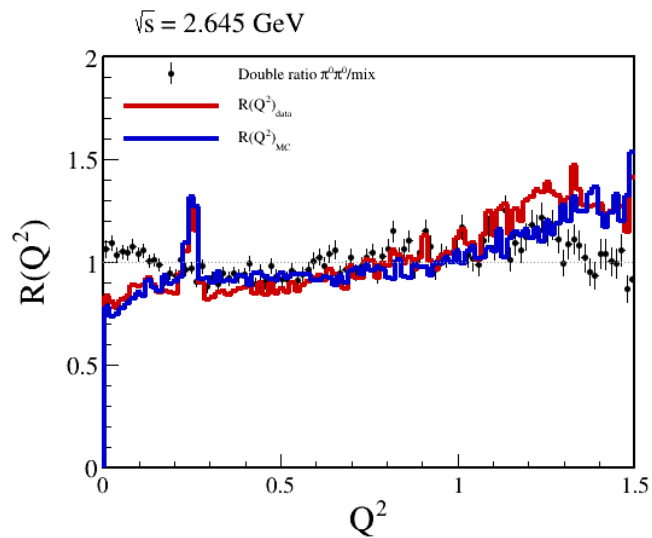
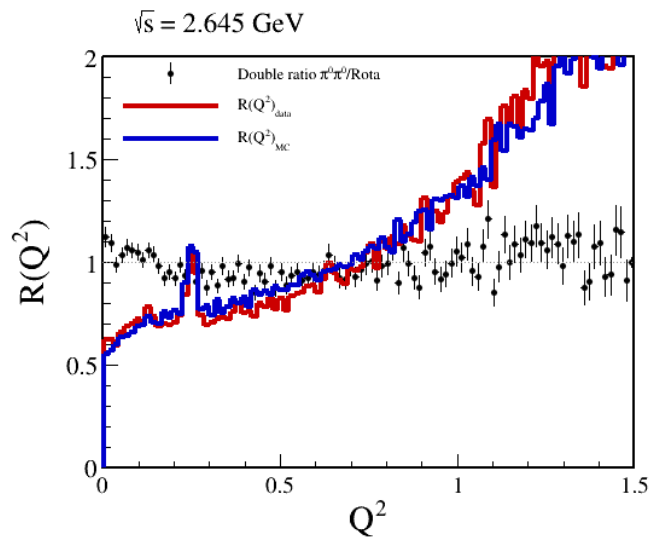
Back up: Q^2 and reference samples (corr)



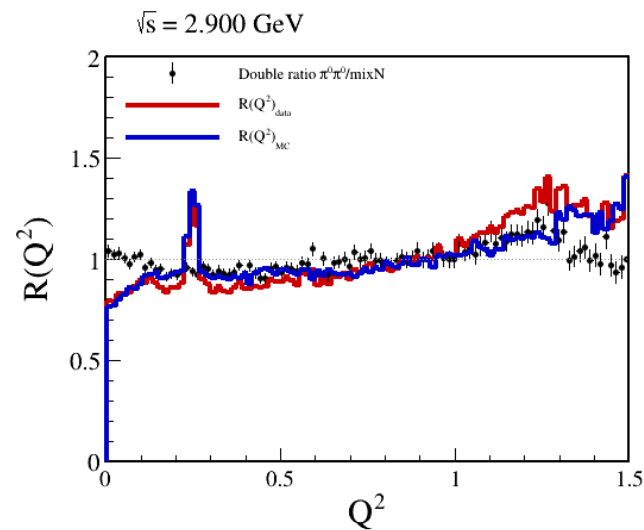
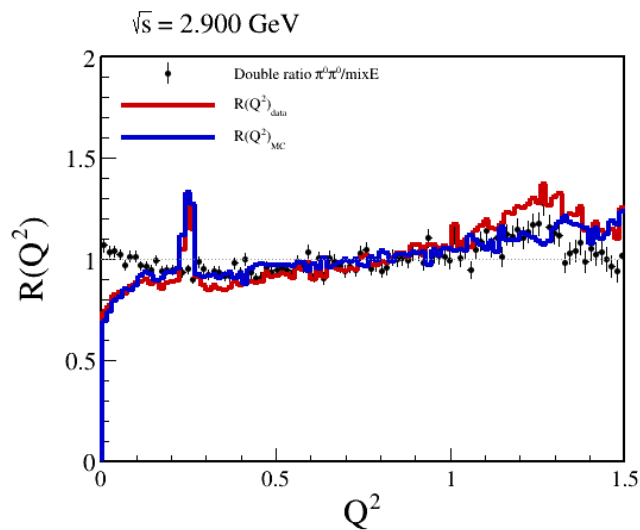
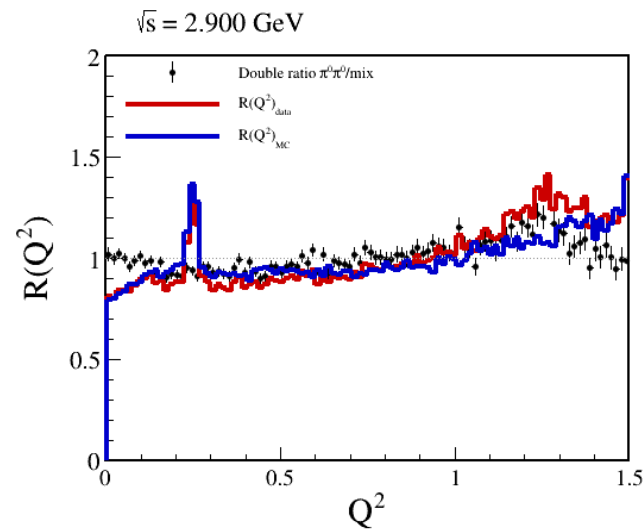
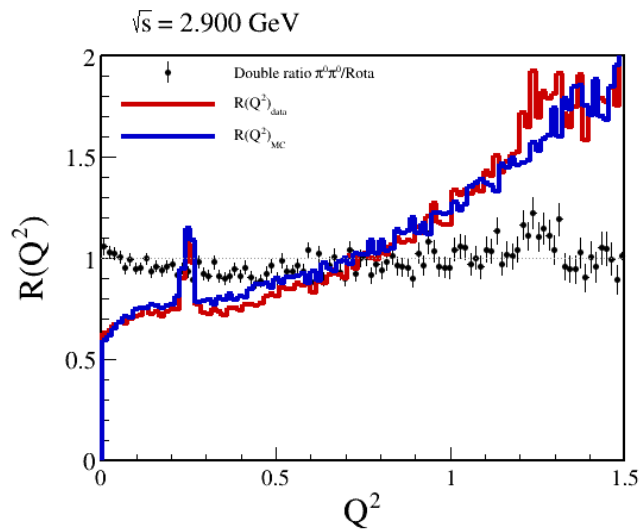
Back up: Double ratio



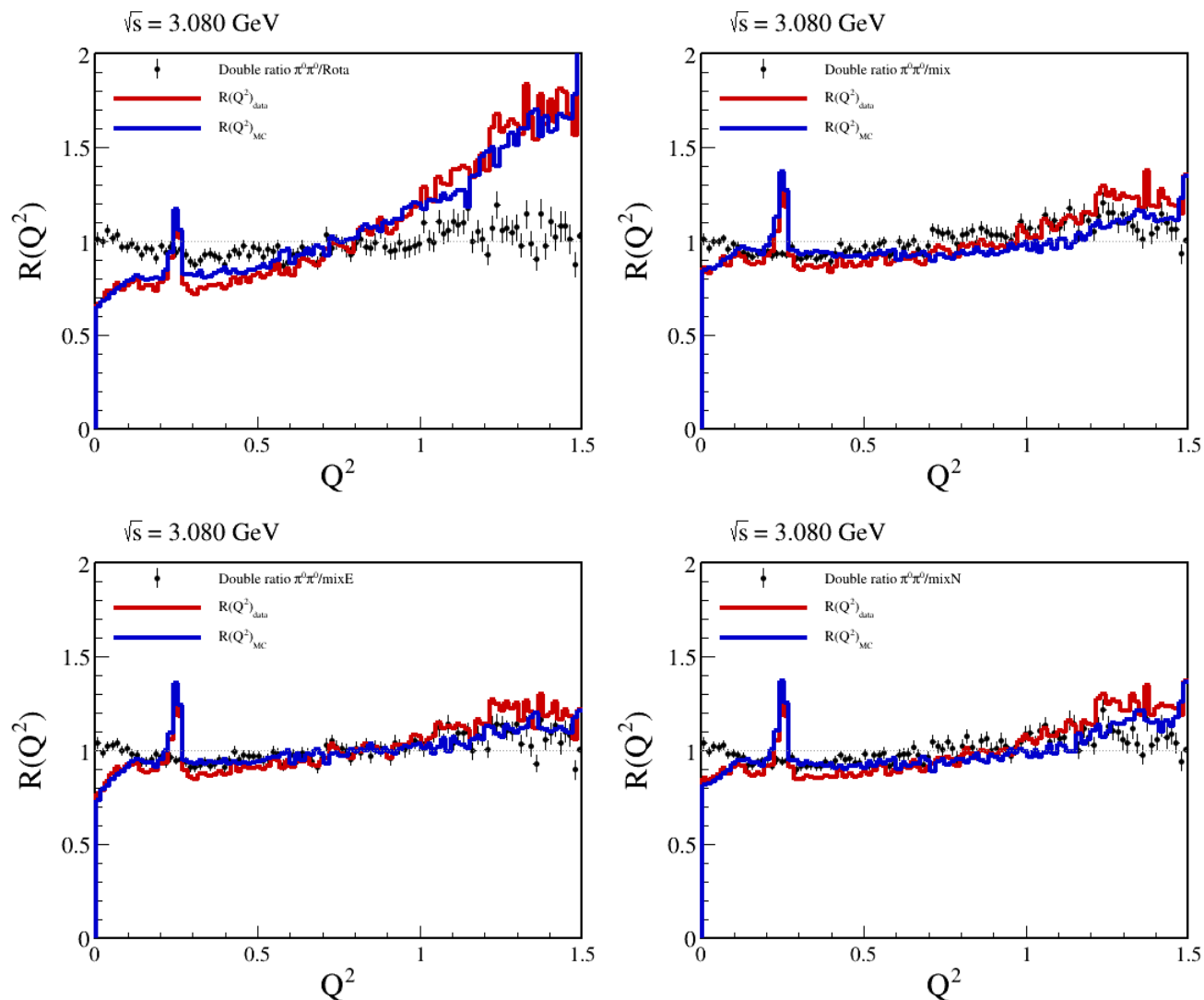
Back up: Double ratio



Back up: Double ratio



Back up: Double ratio



Back up: event selection for exclusive channels

Channel	2.125 (%)
$\pi^+\pi^-2\pi^0$	42.0
$\pi^+\pi^-3\pi^0$	13.8
$\pi^+\pi^-4\pi^0$	6.3
$\pi^+\pi^-5\pi^0$	4.5
$2\pi^+2\pi^-2\pi^0$	14.3
$2\pi^+2\pi^-3\pi^0$	3.8
$2\pi^+2\pi^-4\pi^0$	1.2
$2\pi^+2\pi^-5\pi^0$	0.4
$K^+K^-2\pi^0$	0.6
$K^\pm\pi^\mp3\pi^0$	2.4
$\pi^+\pi^-2\pi^0\eta(\eta \rightarrow \gamma\gamma)$	1.7
$\pi^+\pi^-3\pi^0\eta(\eta \rightarrow \gamma\gamma)$	0.3
$3\pi^+3\pi^-2\pi^0$	0.2
$3\pi^+3\pi^-3\pi^0$	0.1
Sum over	91.6

➤ Event selection:

- ❑ Good charged tracks:
 - $|\cos\theta| < 0.93$
 - $V_r < 1 \text{ cm}, |V_z| < 10 \text{ cm}$
- ❑ Good photons:
 - EMC time: $0 < t < 700 \text{ ns}$
 - $E_\gamma > 25 \text{ MeV}$ for barrel, $E_\gamma > 50 \text{ MeV}$ for end cap
 - Angle from the nearest charged track $> 10^\circ$
 - $N_\gamma \geq 4$ for $\pi^+\pi^-2\pi^0$, $N_\gamma \geq 6$ for $\pi^+\pi^-5\pi^0$
- ❑ PID ($\frac{dE}{dx} + \text{TOF}$):
 - π : $\text{prob}(\pi) > \text{prob}(e)$
 - $N_{\pi^+} = N_{\pi^-} = 1$
- ❑ Kinematic fit:
 - 4C kinematic fit, $\chi^2 < 200$
- ❑ π^0 mass window
 - $m_{\pi^0} \in [0.117, 0.153] \text{ GeV}/c^2$ for $\pi^+\pi^-2\pi^0$,
 - $m_{\pi^0} \in [0.115, 0.155] \text{ GeV}/c^2$ for $\pi^+\pi^-3\pi^0$,

Two major channels contributing to the resonances are checked:

$\pi^+\pi^-2\pi^0$ and $\pi^+\pi^-3\pi^0$