

BESII

*Study of $D^0 \rightarrow K^+ K^- \omega$ and K_S^0
 K_L^0 asymmetry in $D^0 \rightarrow$
 $\omega K_S^0 / K_L^0$ decays on BESIII*

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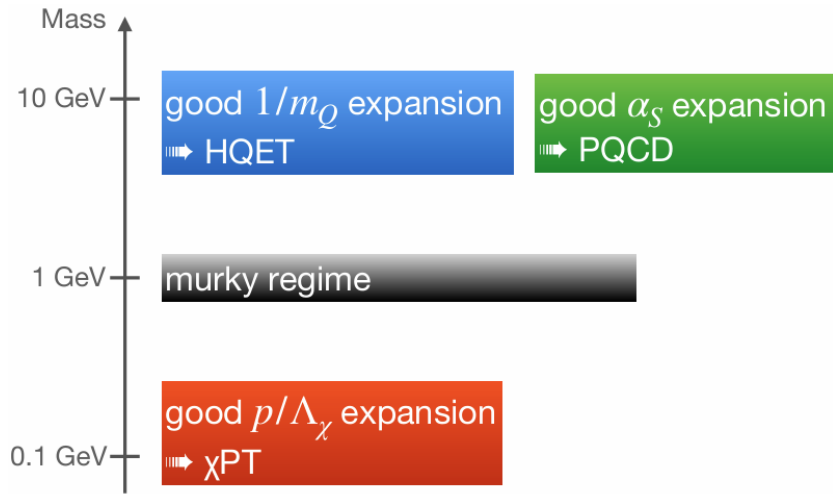
Thesis Mid-term Talk

2026.10.14



- Introduction
- Partial wave analysis of $D^0 \rightarrow K^+ K^- \omega$
- Study of $K_S^0 - K_L^0$ asymmetry in the decays $D^0 \rightarrow \omega K_S^0$ and $D^0 \rightarrow \omega K_L^0$
- Summary and Prospects

Peculiarities of Charm



□ $m_c \sim 1.3 \text{ GeV}/c^2$, neither heavy enough to allow for a sensible heavy quark mass expansion nor light enough for an application of chiral perturbation theory.

□ (semi-)leptonic D decays: lattice QCD

□ D decay dominantly into hadronic final states but no D meson hadronic decay theory based on QCD yet.... serve as a valuable laboratory for non perturbative QCD

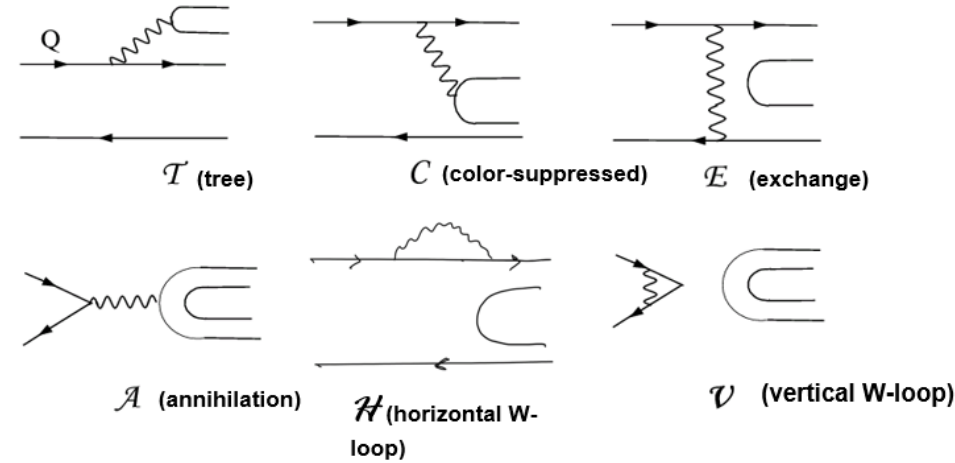
Mode	BR
PP	$\sim 10\%$
VP	$\sim 28\%$
VV	$\sim 10\%$
SP	$\sim 4.2\%$
AP	$\sim 10\%$
TP	$\sim 0.3\%$
2-body	$\sim 63\%$
hadronic	$\sim 84\%$
semileptonic	$\sim 16\%$

P: pseudoscalar meson
V: vector meson
A: axial vector meson
T: tensor meson

D Meson Decays

- Powerful theoretical models for charm hadronic decays: factorization-assisted topological-amplitude(FAT)approach, flavor or topological diagram approach (TDA)

- Topological information → Extract from experiments
- FSIs and non-factorizable contribution
- Predictions of observable quantities → Test by experiments



- Precise measurements to test theories and further understanding:

- Polarization puzzles in $D \rightarrow VV$
- $K_S^0 - K_L^0$ asymmetries in $D \rightarrow K^0 M$

... ..

- $D^0 \bar{D}^0$ samples produces at BESIII

- 20.3 fb^{-1} @ 3.773 GeV
- Near DD threshold
- $\sim 73.4\text{M}$ $D^0 \bar{D}^0$ pairs

BEPCII and BESIII



Double ring: e^+ and e^-
Cross angle: 22 mrad
 E_{cm} : 1.84 – 4.95 GeV
Peak luminosity: $1.1 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ @ $\psi(3770)$

Electromagnetic Calorimeter

CsI(Tl): $L = 28 \text{ cm}$

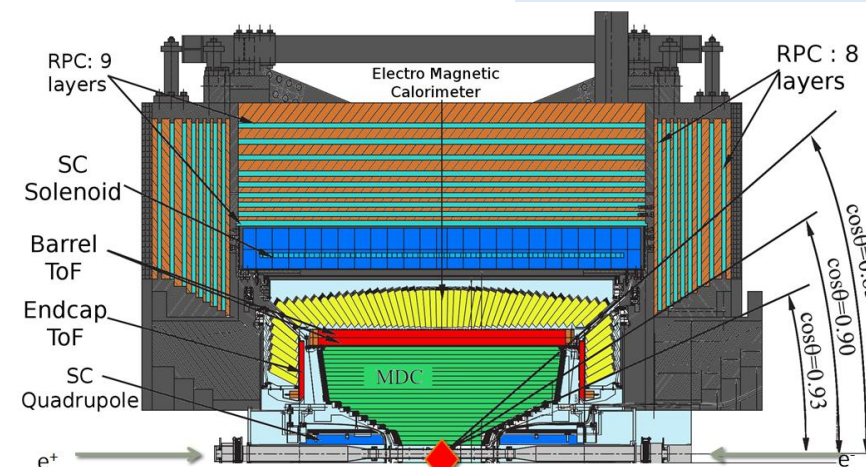
- Barrel $\sigma_E/E = 2.5\%$ @ 1 GeV
- Endcap $\sigma_E/E = 5.0\%$ @ 1 GeV

Muon Counter RPC

Barrel: 9 layers

Endcaps: 8 layers

- $\sigma_{\text{spatial}} = 1.48 \text{ cm}$



• BESIII Detector

Main Drift Chamber

Small cell, 43 layer

- $\sigma_{xy} = 130 \mu\text{m}$
- $dE/dx \sim 6\%$
- $\sigma_p/p = 0.5\%$ @ 1 GeV/c

Time Of Flight

Plastic scintillator

- $\sigma_T(\text{barrel}) = 68 \text{ ps}$
- $\sigma_T(\text{endcap}) = 110 \text{ ps}$
(update to 60 ps with MRPC)

Partial wave analysis of $D^0 \rightarrow K^+ K^- \omega$

Motivation

□ Polarization puzzles in $M \rightarrow VV$:

➤ $B \rightarrow VV[1][2]$: ($V = \phi, K^*, \rho$ and ω)

- Tree-dominated: Transverse polarization amplitudes suffer from the helicity-flipping suppression at the order of Λ_{QCD}/m_b . $f_L^{\text{theor.}} \sim 1 - O(\Lambda_{QCD}/m_b)$ confirmed by $B^0 \rightarrow \rho^+ \rho^-, B^+ \rightarrow \rho^+ \rho^0$; indicated the helicity conservation in $B \rightarrow VV$.
- Penguin-dominated or FSI: $f_L^{B \rightarrow \phi K^*} \sim 0.5$.

Comparison of the FAT approach
experimental results

Mode	$\mathcal{B}(10^{-6})$	$f_L(\%)$	$f_\perp(\%)$	$\phi_\parallel$ (rad)	$\phi_\perp$ (rad)
$B^- \rightarrow \rho^- \rho^0$	21.7 ± 5.1	95.5 ± 1.5	2.22 ± 0.64	-0.09 ± 0.05	-0.09 ± 0.05
Expt.	24.0 ± 1.9	95 ± 1.6			
$\bar{B}_0 \rightarrow \rho^+ \rho^-$	29.5 ± 6.5	92.6 ± 1.6	3.65 ± 0.91	-0.27 ± 0.08	-0.27 ± 0.08
Expt.	27.7 ± 1.9	$99.0^{+2.1}_{-1.9}$			
$\bar{B}^0 \rightarrow \rho^0 \rho^0$	0.94 ± 0.49	81.7 ± 10.8	9.21 ± 5.50	-0.04 ± 0.44	-0.03 ± 0.44
Expt.	0.96 ± 0.15	71^{+8}_{-9}			

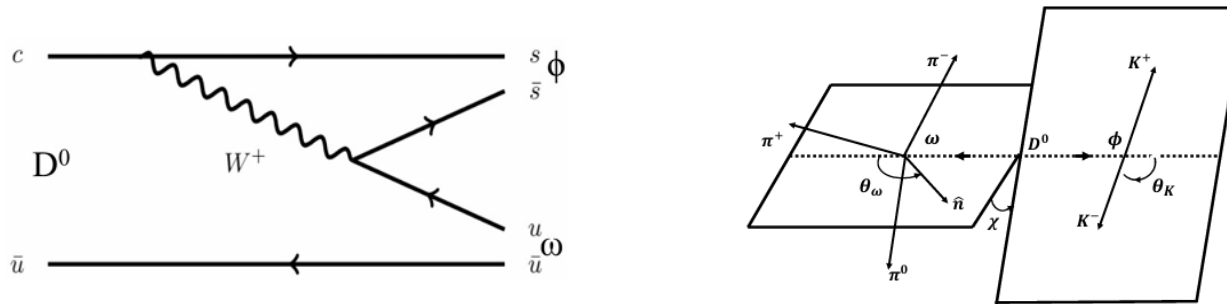
- consist between experiment measurements and theoretical predictions

[1] [PRL 91,171802\(2003\)](#), [2] [PRL 91,201801\(2003\)](#)

Motivation

➤ $D \rightarrow VV$:

- Perturbative QCD not valid, naive factorization model [3] and Lorentz invariant-based symmetry model [4] predict f_L not dominant in $D \rightarrow VV$ process: $f_L^{\text{theor.}} \sim 0.5^{[3]} 0.33^{[4]}$: $D^0 \rightarrow \bar{K}^{*0} \rho^0$.
- $f_L^{D^0 \rightarrow \rho^0 \rho^0} = 0.71 \pm 0.04 \pm 0.02$ (FOCUS Collaboration) $\rightarrow$ [D] wave dominant.
- In $D^0 \rightarrow \omega \phi$ [6], $f_L = 0.00 \pm 0.10 \pm 0.08$, $f_L^{U.L.} < 0.24$ (95% C.L.) are given by fitting to angular distributions (BESIII Collaboration) and shows larger difference from $D^0 \rightarrow \rho^0 \phi$.



- FSI through $K^{*+} K^{*-}$ or flavor-singlet contributions can lead to different, a more precise PWA result can help to confirm the theoretical models.

[3] [PRD 59,114013\(1999\)](#), [4] [JHEP 03,042 \(2014\)](#), [5] [PRD 75,052003\(2007\)](#), [6] [PRL 128,011803\(2022\)](#).

Motivation

- ❑ Still lack of knowledge about some crucial pieces of dynamics in D decay mechanisms. Polarization of $D \rightarrow VV$ provide valuable information to further understand D decays and non-perturbative QCD. Longitudinal polarization fraction(f_L) is important to test models.
- ❑ $20.3 \text{ fb}^{-1} e^+e^-$ annihilation data @ 3.773 GeV.
 - Larger $D^0\bar{D}^0$ samples $\rightarrow$ more precise results
 - Double-tag method $\rightarrow$ lower background level
 - Extract the S,P,D amplitudes of $D^0 \rightarrow \omega\phi$
- ❑ Other resonant on invariant mass structures

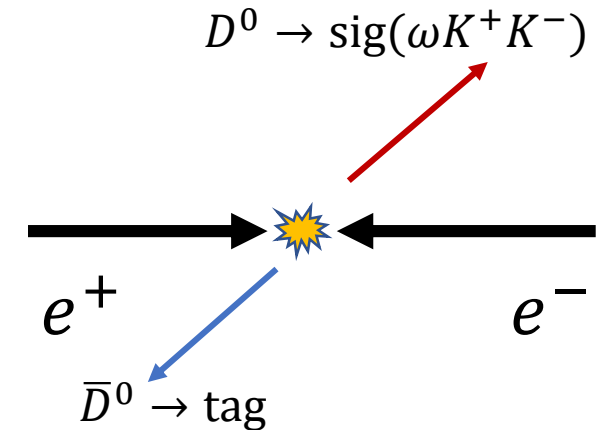
Dataset

❑ Quantum correlated $D^0\bar{D}^0$ produced at BESIII

- Data: 20.3 fb^{-1} @ $E_{cm} = 3.773 \text{ GeV}$
- Inclusive MC: 40× for background study
- Signal PHSP MC: 100× for amplitude analysis
- Signal DIY MC: 200× for amplitude analysis

❑ Double tag method

- Reconstruct both of $D^0\bar{D}^0$
- Tag mode: $\bar{D}^0 \rightarrow K^+\pi^-, K^+\pi^-\pi^0, K^+\pi^+\pi^-\pi^-$ (~25% of D^0 decay)





Event Selection - Preliminary Selection

➤ Good charged tracks:

- $|R_{xy}| \leq 1 \text{ cm}, |R_z| \leq 10 \text{ cm}, |\cos \theta| \leq 0.93$

➤ Good Photons :

- Barrel: $E > 25 \text{ MeV}, |\cos \theta| \leq 0.8$
- Endcap: $E > 50 \text{ MeV}, 0.86 \leq |\cos \theta| \leq 0.92$
- EMC time cut: $0 \leq T \leq 14 \text{ (50ns)}$
- Angle between shower and charged tracks larger than 10°

➤ PID(SimplePID package):

- π : $\text{Prob}(\pi) > \text{Prob}(K)$
- K : $\text{Prob}(K) > \text{Prob}(\pi)$

➤ π^0 Candidates:

- $0.115 < M_{\gamma\gamma} < 0.150 \text{ GeV}/c^2, \chi^2_{1c} < 50$

➤ Cosmic rays rejection for $\bar{D}^0 \rightarrow K^+ \pi^-$

➤ Tag D Reconstruction:

- $\Delta E = E_D - E_{Beam}, M_{BC} = \sqrt{(E_{Beam}^2 - p_D^2)};$
- Minimum $|\Delta E|$ is used to selected best candidate

➤ Ks veto for $\bar{D}^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ tag mode:

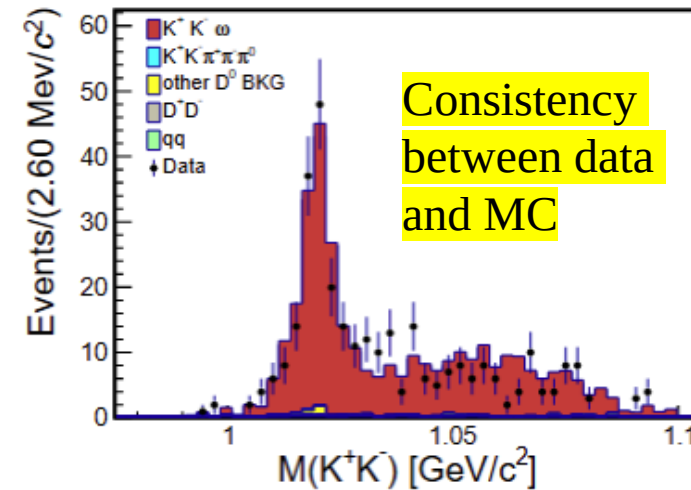
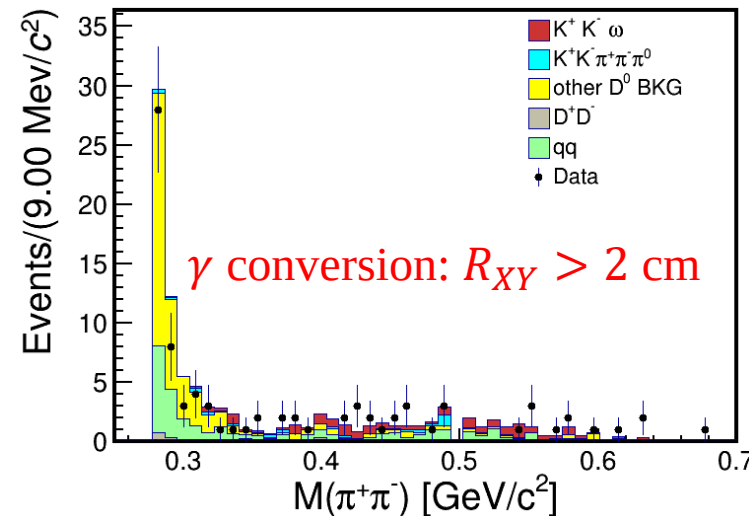
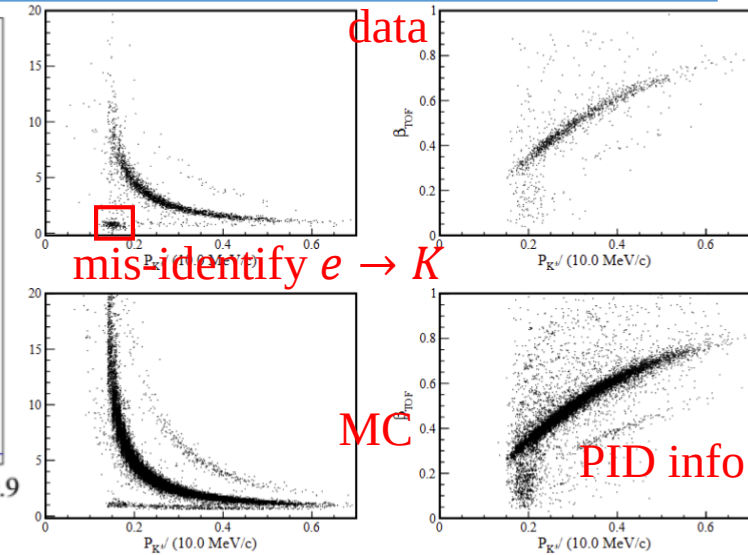
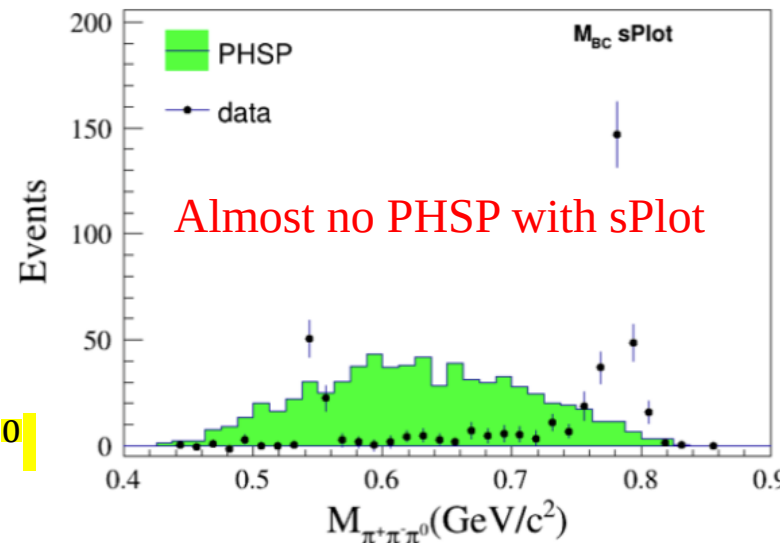
- $|M_{\pi^+ \pi^-} - 0.4976| > 0.03 \text{ GeV}/c^2.$

➤ The ST requirement follow the official document ([MEMO_D_ST_v4.0.pdf \(ihep.ac.cn\)](http://www.ihep.ac.cn/MEMO_D_ST_v4.0.pdf))

Event Selection - Further Selection

Case 1(Full Reconstruction)

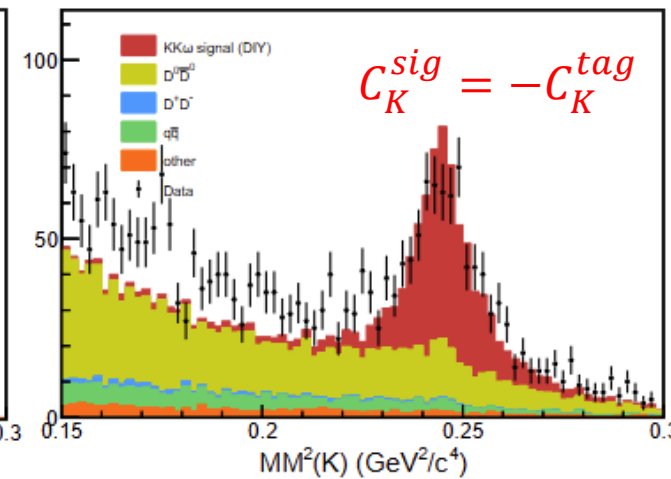
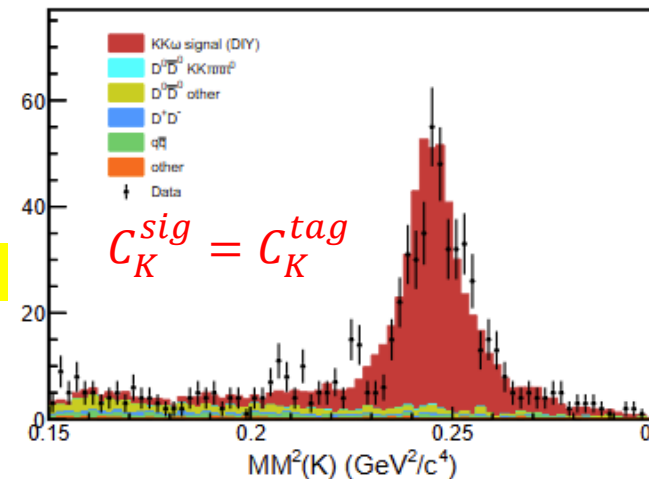
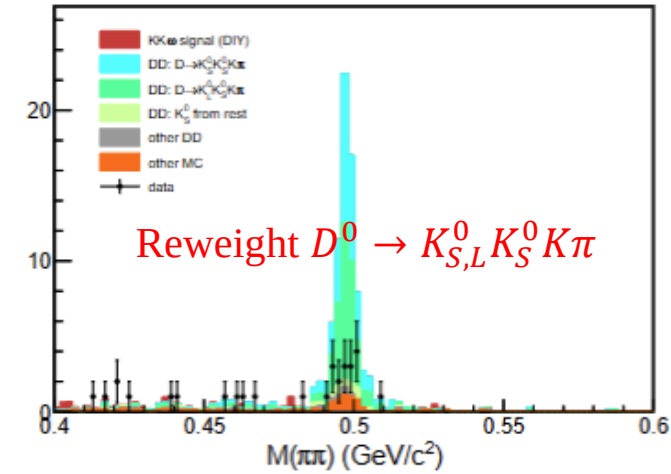
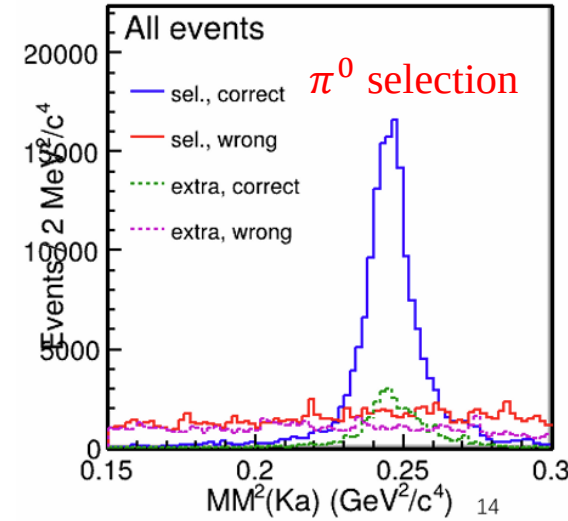
- Exactly four charged tracks: $\pi^+\pi^-K^+K^-$.
- π^0 candidate: Minimum $|\Delta E|$ to selected best candidate.
- ΔE requirement: $(-0.043, 0.035)$ GeV.
- Optimize of MC: remove $D^0 \Rightarrow K^+K^-\pi^+\pi^-\pi^0$ component that generated with PHSP mode.
- PID requirement: $prob(K) \geq 0.001$.
- γ conversion: $R_{XY} \leq 2$ cm for $\pi^+\pi^-$.
- $M_{\pi^+\pi^-\pi^0} > 0.74$ GeV/ c^2 .
- K_S^0 veto: $M_{\pi^+\pi^-} \notin (0.492, 0.504)$ GeV/ c^2 .
- $1C(M_{D^0}^{PDG})$ kinematic fit performed(only used in PWA).



Event Selection - Further Selection

Case 2(Miss one Kaon)

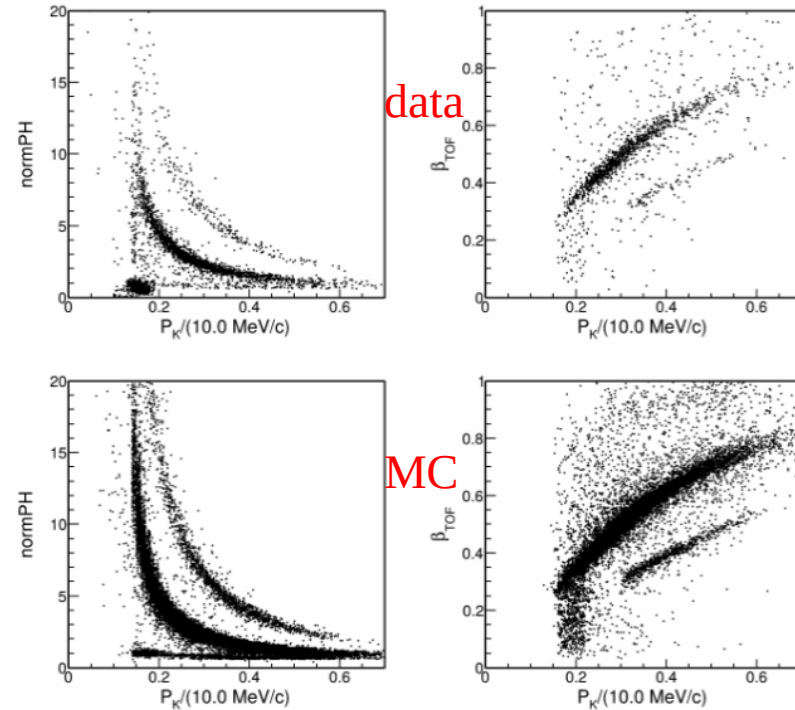
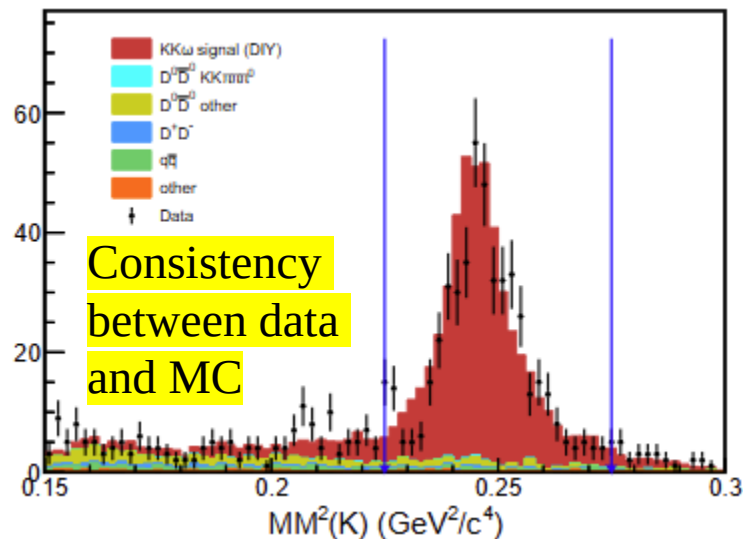
- Exactly three charged tracks: $\pi^+\pi^-K$.
- π^0 candidate: Minimum $|\Delta M_{miss}^2|$ to selected best candidate.
- Recoiled D^0 : $\vec{P}_{D^0}$ direction is opposite to ST $\vec{P}_{\bar{D}^0}$
- $|\vec{P}_{D^0}| = \sqrt{(E_{beam}^2 - M_{D^0}^2)}, E_{D^0} = E_{beam}$.
- $M_{D^0} - M_{found} > 0$.
- Suppress D^0 CF decays: $C_K^{sig} = C_K^{tag}$.
- M_{BC}^{tag} requirement: $(1.859, 1.873) \text{ GeV}/c^2$.
- Optimize of MC: Remove $D^0 \rightarrow K^+K^-\pi^+\pi^-\pi^0$ (PHSP). Reweight the $D^0 \rightarrow K_{S,L}^0 K_S^0 K \pi$ components.



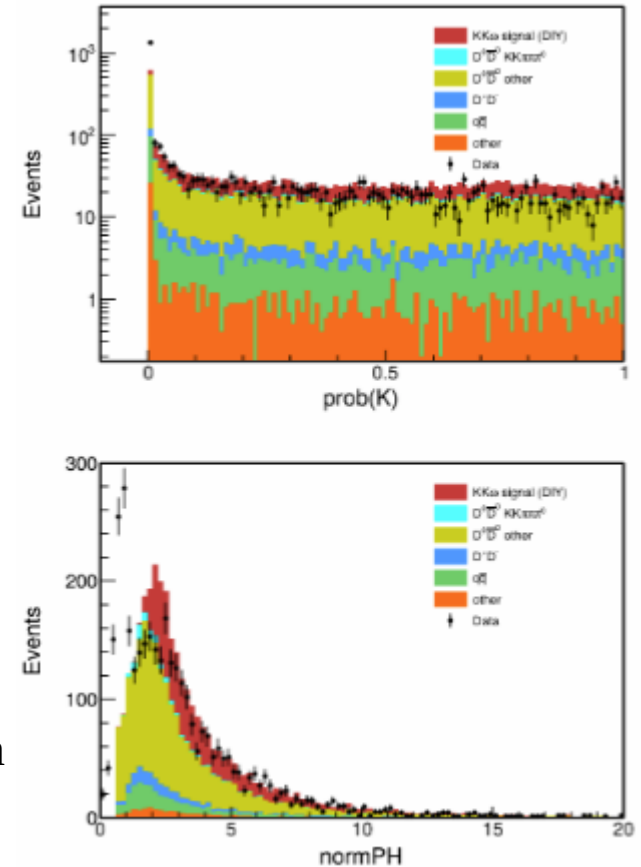
Event Selection - Further Selection

Case 2(Miss one Kaon)

- $M_{\pi^+\pi^-\pi^0} > 0.74 \text{ GeV}/c^2$.
- K_S^0 veto: $M_{\pi^+\pi^-} \notin (0.492, 0.504) \text{ GeV}/c^2$.
- PID requirement: $\text{prob}(K) \geq 0.003$.
- γ conversion: $R_{XY} < 2 \text{ cm}$ for $\pi^+\pi^-$.
- $4C(\vec{P}_{D^0}, E_{D^0})$ kinematic fit performed (PWA).



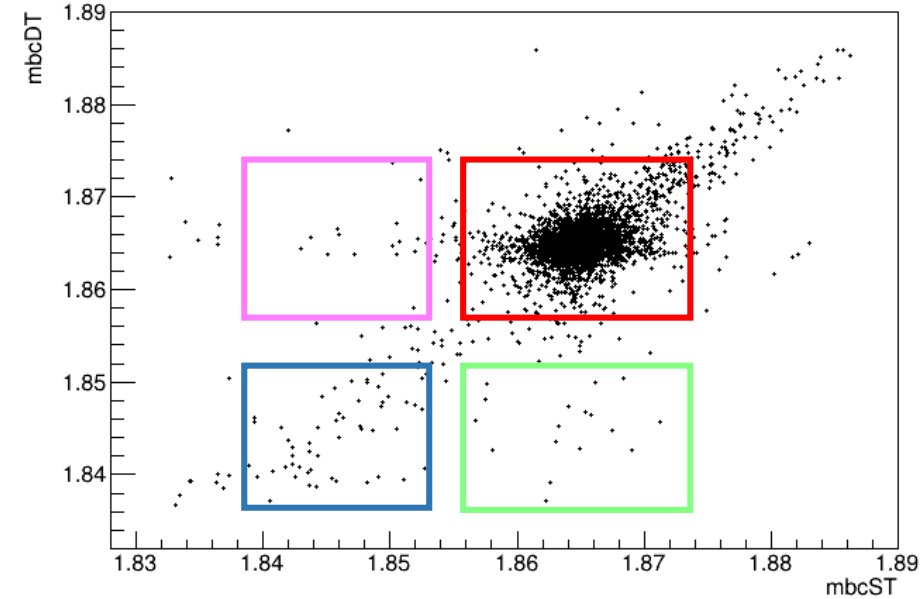
The PID inconsistent is even worse than that of case1.
We choose a more serious cut therefore.



Signal Purity - Full Reconstruction

- **Signal**: right tag and right signal D^0 , described by DIY MC shape.
- **BKGI**: background with right tag and wrong signal, described by $A_{x1}(x1; m_{x1}, z_{x1}, \rho_{x1}) \times S_{x2}$.
- **BKGI**: background with wrong tag and right signal, described by $A_{x2}(x2; m_{x2}, z_{x2}, \rho_{x2}) \times S_{x1}$
- **BKGI**: background in diagonal region. $T(x_1 - x_2; \mu, \sigma(x_1 + x_2), n) \times A_{x1}(x1; m_{x1}, z'_{x1}, \rho'_{x1}) \times A_{x2}(x2; m_{x2}, z'_{x2}, \rho'_{x2})$

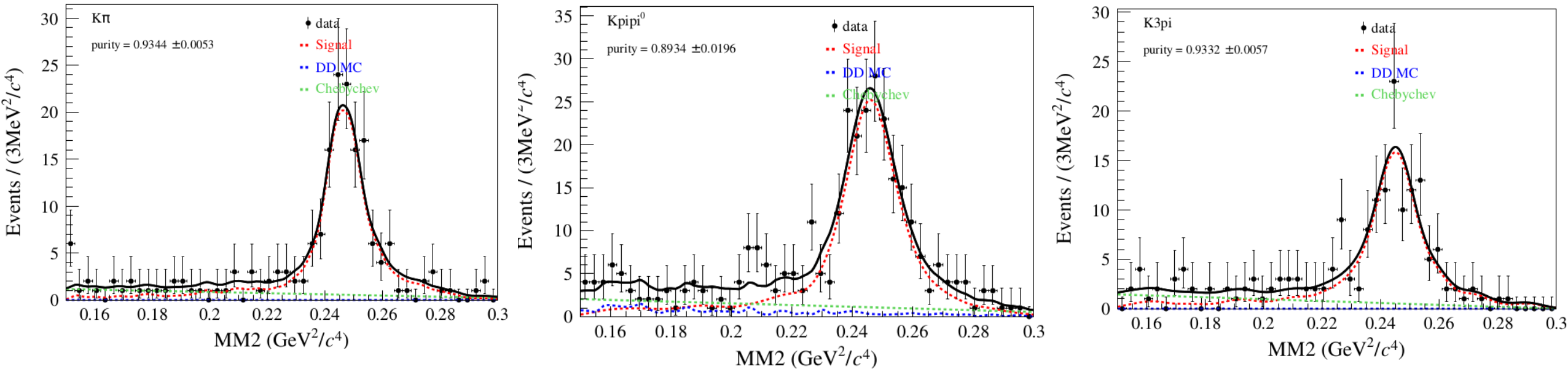
$$T(y; \mu, \sigma, n) = \frac{\Gamma(n/2+0.5)}{\sigma\sqrt{n\pi}\Gamma(n/2)} \left[1 + \frac{1}{2} \left(\frac{y-\mu}{\sigma} \right)^2 \right]^{-\frac{n+1}{2}}$$



Tag Mode	$K^+\pi^-$	$K^+\pi^-\pi^0$	$K^+\pi^-\pi^+\pi^-$
$N^{sig}(\text{signal region})$	84.9 ± 9.6	137.4 ± 14.2	67.3 ± 8.4
Purity(%)	99.9 ± 0.0	93.9 ± 1.3	91.9 ± 2.0

Signal Purity - Partially Reconstruction

- **Signal:** DIY MC shape $S \otimes$ Gauss function G .
- **Continuum Background:** described by 2nd -order Chebyshev polynomials.
- $D^0 \bar{D}^0$ Background: MC simulated shape.



Tag Mode	$K^+ \pi^-$	$K^+ \pi^- \pi^0$	$K^+ \pi^- \pi^+ \pi^-$
N^{sig} (signal region)	126.3 ± 14.0	193.9 ± 20.2	112.7 ± 14.18
Purity(%)	93.5 ± 1.8	89.4 ± 2.2	93.3 ± 2.0

Introduction of PWA

□ Likelihood Formula:

➤
$$\ln \mathcal{L} = \sum_{i=1}^3 \sum_k^{N_{Data,i}} \ln [\omega_{sig}^i f_s(p_j^k) + (1 - \omega_{sig}^i) f_B^i(p_j^k)]$$

- f_s : signal p.d.f
- ω_{sig} : fraction of signal (signal purity)
- p_j : four-momenta of daughter particles
- f_B : background p.d.f

□ Signal PDF:

➤
$$f_s(p) = \frac{\epsilon_s(p) |A_D(p)|^2 R_3(p)}{\int \epsilon_s(p) |A_D(p)|^2 R_3(p) dp}$$

MC integral

- $\epsilon_s(p)$: signal efficiency
- $R_3(p)$: phase space factor

□ Amplitude Construction:

➤ Constructed by covariant tensor method

$$A_{D^0}(p) = \sum_i \Lambda_i U_i(p)$$

- Λ_i : coupling factor

➤
$$U_i(p) = B_{L_D}(p) P_{R_1}(p) B_{L_{R_1}}(p) P_{R_2}(p) B_{L_{R_2}}(p) S_i(p)$$

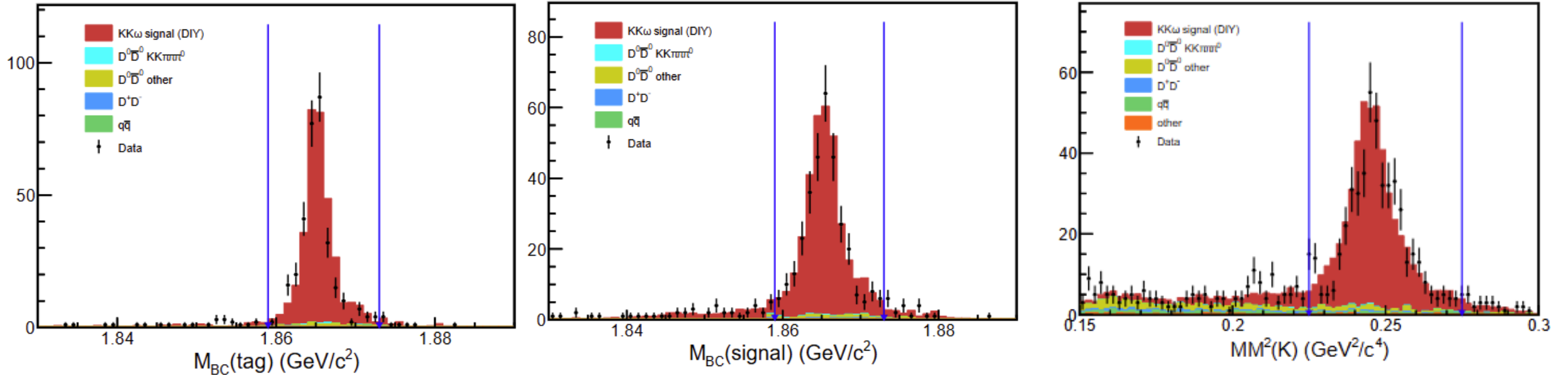
- B_L : Blatt-Weisskopf barrier factor
- P_R : Propagator of resonance R
- S_i : Spin factor of i -th decay amplitude

➤
$$A_{\bar{D}^0}(p) = \sum_i \Lambda_i \bar{U}_i(p) = \sum_i \Lambda_i U_i(\bar{p})$$

- $\bar{p}$: CP-conjugate of phase space point p

Background Estimation - f_B

- Consistency between Data and Inclusive MC in side-bands. And consistency of inclusive MC(without signal process) in signal region and side-bands is obtained.



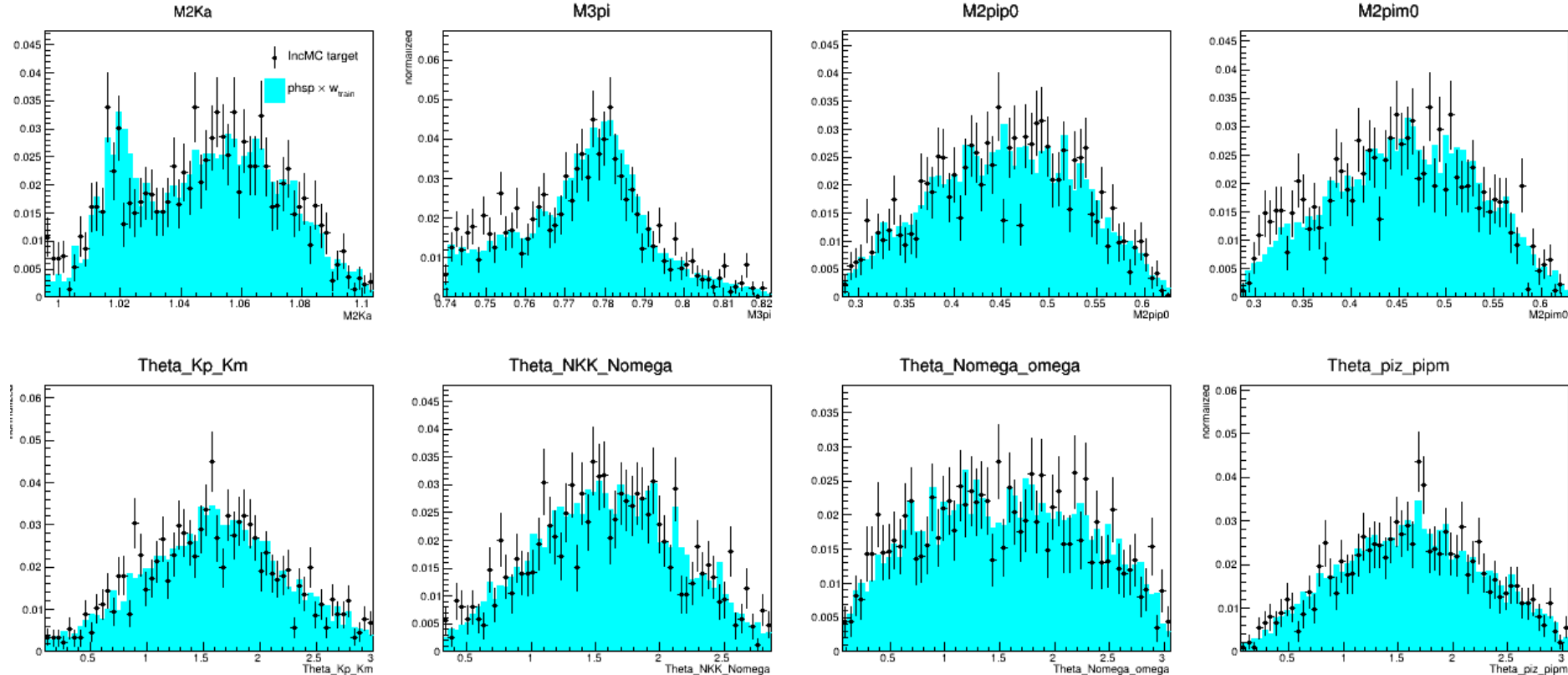
- By Mixed-sample Method:

$$\begin{aligned} & \triangleright M_{K^+K^-}, M_{\pi^+\pi^-\pi^0}, M_{\pi^+\pi^0}, M_{\pi^-\pi^0}, \\ & \theta_{N_{KK}, N_{\pi^+\pi^-\pi^0}}, \theta_{N_{\pi^+\pi^-\pi^0}, P_{\pi^+\pi^-\pi^0}}, \theta_{P_{\pi^0}, P_{\pi^+\pi^-}}, \theta_{P_{K^+}, P_{K^-}} \end{aligned}$$

χ of Data & IncMC	Full reconstruction	Miss one K
$\bar{D}^0 \rightarrow K^+\pi^-$	-	0.86
$\bar{D}^0 \rightarrow K^+\pi^-\pi^0$	1.11	-0.03
$\bar{D}^0 \rightarrow K^+\pi^-\pi^+\pi^-$	-1.18	0.63

Background Estimation - f_B

- p.d.f of background $f_B(p)$ is described with Inclusive MC, by performing Machine Learning on DIY MC(hep_ml package, XGboost model).
- Eight Variables used : $M_{K^+K^-}, M_{\pi^+\pi^-\pi^0}, M_{\pi^+\pi^0}, M_{\pi^-\pi^0}, \theta_{N_{KK}, N_{\pi^+\pi^-\pi^0}}, \theta_{N_{\pi^+\pi^-\pi^0}, P_{\pi^+\pi^-\pi^0}}, \theta_{P_{\pi^0}, P_{\pi^+\pi^-}}, \theta_{P_{K^+}, P_{K^-}}$



Amplitude Construction

□ Spin factor of $\omega \rightarrow \pi^+\pi^-\pi^0$:

$$A_\mu(\omega) = \varepsilon_{\mu\nu\rho\sigma} p_\omega^\sigma [t^\rho(\pi^+\pi^0, \pi^-) t^\nu(\pi^+\pi^0) - t^\rho(\pi^+\pi^-, \pi^0) t^\nu(\pi^+\pi^-) + t^\rho(\pi^0\pi^-, \pi^+) t^\nu(\pi^0\pi^-)]$$

$D \rightarrow S\omega, S \rightarrow P_1P_2$	$t^\mu(D)A_\mu(\omega)$
$D[S] \rightarrow V\omega, V \rightarrow P_1P_2$	$P^{\mu\nu}(D)t_\mu(V)A_\nu(\omega)$
$D[P] \rightarrow V\omega, V \rightarrow P_1P_2$	$\varepsilon_{\mu\nu\rho\sigma} p_D^\sigma t^\rho(D)t_\mu(V)A_\nu(\omega)$
$D[D] \rightarrow V\omega, V \rightarrow P_1P_2$	$t_{\mu\nu}(D)t^\mu(V)A^\nu(\omega)$
$D[P] \rightarrow T\omega, T \rightarrow P_1P_2$	$P^{\mu\nu\rho\sigma}(D)t_{\mu\nu}(T)t_\rho(D)A_\sigma(\omega)$
$D[D] \rightarrow T\omega, T \rightarrow P_1P_2$	$\varepsilon_{\mu\nu\rho\sigma} p_D^\sigma t^{\rho\alpha}(D)t_\alpha^\nu(T)A^\mu(\omega)$
$D[F] \rightarrow T\omega, T \rightarrow P_1P_2$	$t_{\mu\nu\rho}(D)t^{\mu\nu}(T)A^\rho(\omega)$
$D \rightarrow PP_1, P[P] \rightarrow \omega P_2$	$t^\mu(P)A_\mu(\omega)$
$D \rightarrow AP_1, A[S] \rightarrow \omega P_2$	$t^\mu(D)P_{\mu\nu}(A)A^\nu(\omega)$
$D \rightarrow AP_1, A[D] \rightarrow \omega P_2$	$t^\mu(D)t_{\mu\nu}(A)A^\nu(\omega)$
$D \rightarrow VP_1, V[P] \rightarrow \omega P_2$	$t^\mu(D)\varepsilon_{\mu\nu\rho\sigma} p_V^\sigma t^\nu(V)A^\rho(\omega)$
$D \rightarrow PT P_1, PT[P] \rightarrow \omega P_2$	$t^{\mu\nu}(D)P_{\mu\nu\rho\sigma}(PT)t^\rho(PT)A^\sigma(\omega)$
$D \rightarrow PT P_1, PT[F] \rightarrow \omega P_2$	$t^{\mu\nu}(D)t_{\mu\nu\rho}(PT)A^\rho(\omega)$
$D \rightarrow TP_1, T[D] \rightarrow \omega P_2$	$t^{\mu\alpha}(D)\varepsilon_{\mu\nu\rho\sigma} p_T^\sigma t_\alpha^\rho(T)A^\nu(\omega)$

➤ Spin projection operator

$$P^{(0)}(p_a) = 1,$$

$$P_{\mu\mu'}^{(1)}(p_a) = -g_{\mu\mu'} + \frac{p_{a\mu}p_{a\mu'}}{p_a^2},$$

$$P_{\mu\nu\mu'\nu'}^{(2)}(p_a) = \frac{1}{2}[P_{\mu\mu'}^{(1)}(p_a)P_{\nu\nu'}^{(1)}(p_a) + P_{\mu\nu}^{(1)}(p_a)P_{\nu\mu'}^{(1)}(p_a)] - \frac{1}{3}P_{\mu\nu}^{(1)}(p_a)P_{\mu'\nu'}^{(1)}(p_a).$$

➤ Blatt-Weisskopf barrier factor

$$B_0(q) = 1$$

$$B_1(q) = \sqrt{\frac{2}{q^2 + q_0^2}}$$

$$B_2(q) = \sqrt{\frac{13}{q^4 + 3q^2q_0^2 + 9q_0^4}}$$

q: breakup momentum

$$q_0 = 0.197321/R \text{ GeV}/c$$

$$R = 5 * 0.197321 \approx 1 \text{ fm for } D0$$

$$R = 3 * 0.197321 \approx 0.6 \text{ fm for other resonance}$$

➤ Propagator

Gounaris-Sakurai(GS) formula: $\rho(770)$

Relative Breit-Wigner(RBW): $f_2(1270)$

RBW with constant width $\Gamma(s)$: Other resonances

QC Correction

➤ Neglect $D^0 - \bar{D}^0$ mixing and CPV, the decay rates:

$$\frac{d\Gamma(f;g)}{d\Phi} \propto |A_{f\bar{g}}|^2 = |A_f\bar{A}_g - \bar{A}_fA_g|^2$$

➤ g Tag mode, f Signal mode, the decay rate of signal $D \rightarrow f$:

$$\frac{d\Gamma(D^0 \rightarrow f)}{d\Phi_f} \propto |A_f|^2 \int |\bar{A}_g|^2 d\Phi_g + |\bar{A}_f|^2 \int |A_g|^2 d\Phi_g - A_f\bar{A}_f^* \int \bar{A}_gA_g^* d\Phi_g - A_f^*\bar{A}_f \int \bar{A}_g^*A_g d\Phi_g$$

➤ Decay rate of $D \rightarrow f$ can be rewritten:

$$\begin{aligned} \frac{d\Gamma(D^0 \rightarrow f)}{d\Phi_f} &\propto |A_f|^2 + r_{\bar{g}}^2 |\bar{A}_f|^2 - r_{\bar{g}}R_{\bar{g}}e^{i\delta_{\bar{g}}}A_f\bar{A}_f^* - r_{\bar{g}}R_{\bar{g}}e^{-i\delta_{\bar{g}}}A_f^*\bar{A}_f \\ &= |A_f - r_{\bar{g}}R_{\bar{g}}e^{-i\delta_{\bar{g}}}\bar{A}_f|^2 + r_{\bar{g}}^2(1 - R_{\bar{g}}^2)|\bar{A}_f|^2. \end{aligned}$$

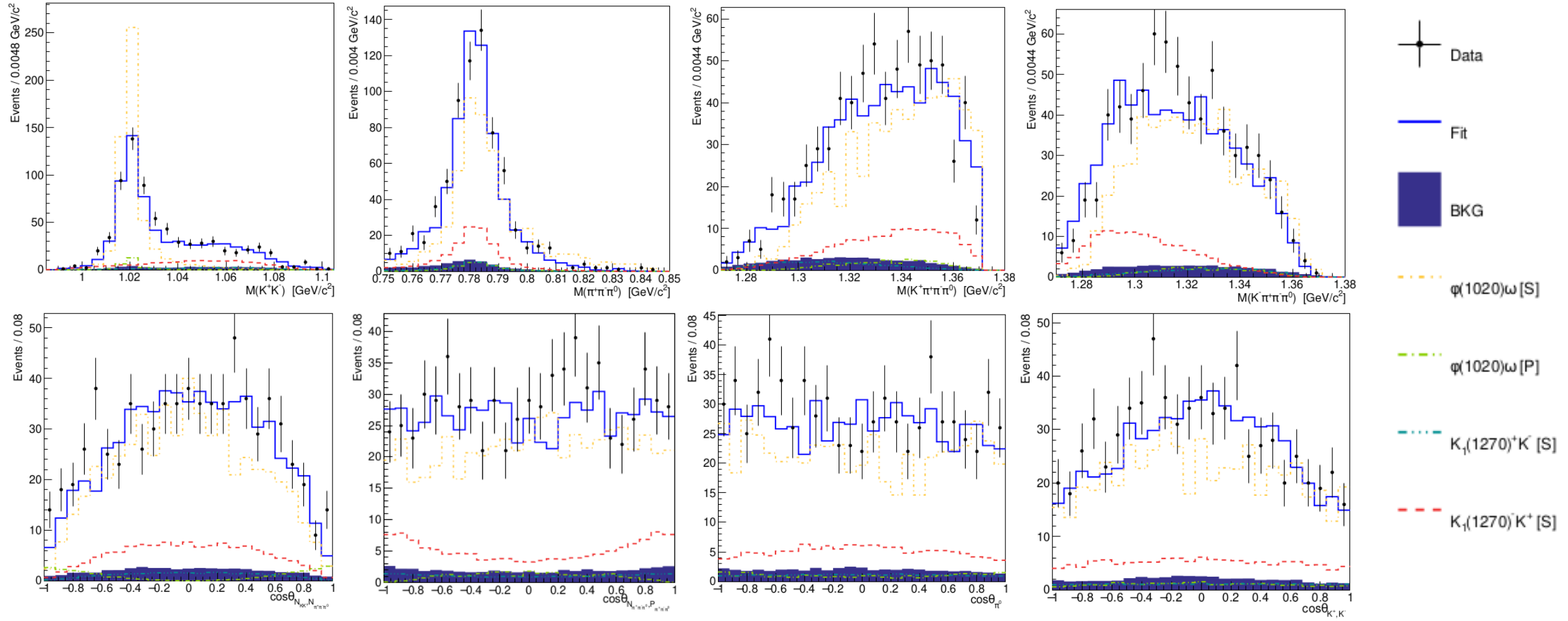
[1] LHCb Collaboration, R. Aaij *et al.*, “Simultaneous determination of the CKM angle γ and parameters related to mixing and CP violation in the charm sector,” LHCb-CONF-2025-003.

[2] BESIII Collaboration, M. Ablikim *et al.*, “Improved measurements of the coherence factors and strong-phase differences in $D \rightarrow K^-\pi^+\pi^+\pi^-$ and $D \rightarrow K^-\pi^+\pi^0$ with quantum-correlated $D\bar{D}$ decays,” J. High Energy Phys. **08**, 038 (2026).

$$r_g^2 = \frac{\int |\bar{A}_g|^2 d\Phi_g}{\int |A_g|^2 d\Phi_g}, \quad R_g e^{-i\delta_g} = \frac{\int A_g^* \bar{A}_g d\Phi_g}{\sqrt{\int |A_g|^2 d\Phi_g \int |\bar{A}_g|^2 d\Phi_g}}$$

Mode	r (%)	R	δ (°)
$K^-\pi^+$	5.857 ± 0.009 [1]	1	192.7 ± 2.3 [1]
$K^-\pi^+\pi^0$	4.41 ± 0.08 [2]	0.75 ± 0.03 [2]	209_{-8}^{+7} [2]
$K^-\pi^+\pi^-\pi^+$	5.51 ± 0.05 [2]	0.51 ± 0.04 [2]	182_{-13}^{+14} [2]

Fitting Results

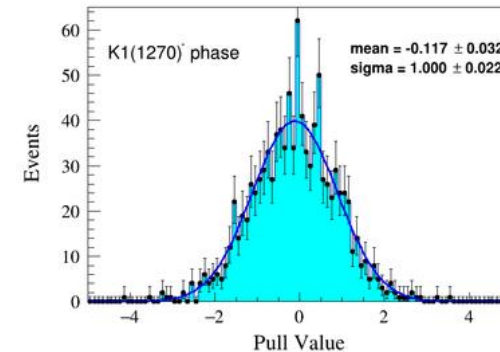
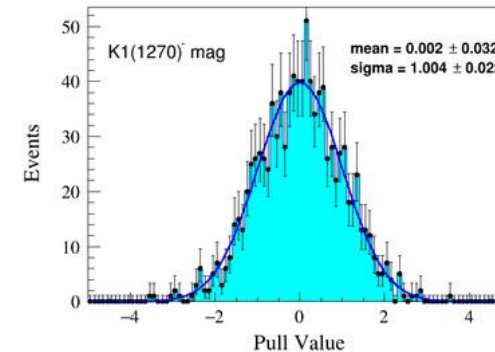
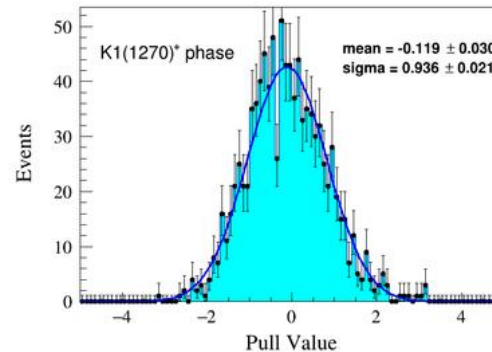
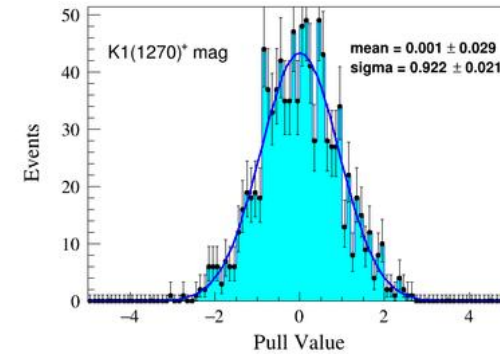
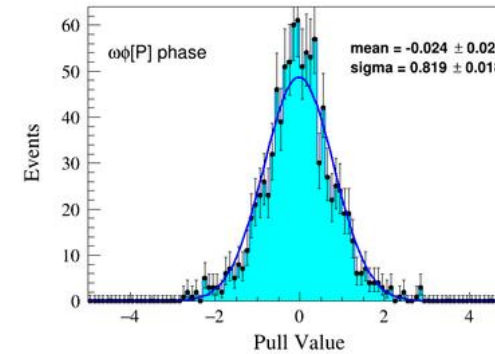
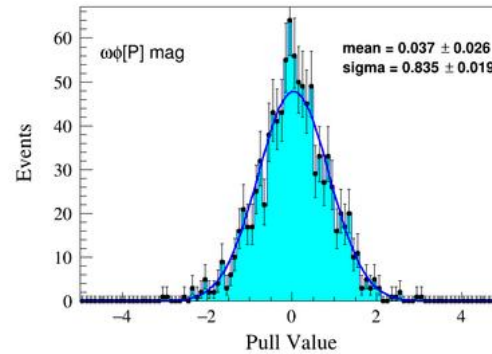


Fitting Results & IO Check

➤ Fitting results(magnitude and phase)

- I/O check is performed for 1000 times.
- The size of each sample is comparable with our data.

Resonance	FF(%)	mag	phase
$\omega\phi[S]$	80.4	1	0
$\omega\phi[P]$	3.90	0.071	0.193
$K_1(1270)^+K^-[S]$	4.36	1.511	5.160
$K_1(1270)^-K^+[S]$	15.3	2.838	1.824



Polarization of $D \rightarrow VV$

- For $D \rightarrow VV$ decay modes, these decays have a richer structure, as they have three partial-wave or helicity states.
- The polarized decay amplitudes can be expressed in several different but equivalent bases **helicity basis** (H_0, H_+, H_-), **transverse basis** ($A_0, A_\perp, A_\parallel$) and can be related to the partial wave amplitudes $[S], [P], [D]$.

$$A_0 = H_0 = -\frac{1}{\sqrt{3}} S + \sqrt{\frac{2}{3}} D ,$$

$$A_\parallel = \frac{1}{\sqrt{2}} (H_+ + H_-) = \sqrt{\frac{2}{3}} S + \frac{1}{\sqrt{3}} D$$

$$A_\perp = \frac{1}{\sqrt{2}} (H_+ - H_-) = P ,$$

$$S = \frac{1}{\sqrt{3}} (-A_0 + \sqrt{2} A_\parallel) = \frac{1}{\sqrt{3}} (-H_0 + H_+ + H_-)$$

$$P = A_\perp = \frac{1}{\sqrt{2}} (H_+ - H_-),$$

$$D = \frac{1}{\sqrt{3}} (\sqrt{2} A_0 + A_\parallel) = \frac{1}{\sqrt{6}} (2H_0 + H_+ + H_-),$$

Polarization of $D^0 \rightarrow \omega \phi$

$$\square \quad H_{\pm} = \sqrt{\frac{1}{3}} g_{00} \pm \sqrt{\frac{1}{2}} W r g_{11} + \sqrt{\frac{1}{6}} r^2 g_{22}, F_0 = \gamma_1 \gamma_2 \left(-\sqrt{\frac{1}{3}} g_{00} + \sqrt{\frac{2}{3}} r^2 g_{22} \right)$$

Longitudinal polarization fraction: $f_L = \frac{|H_0|^2}{|H_+ + H_0 + H_-|^2} \quad r = 2|q|$

$|q|$: momentum of V in D rest frame

γ : Lorentz factor of V in D rest frame

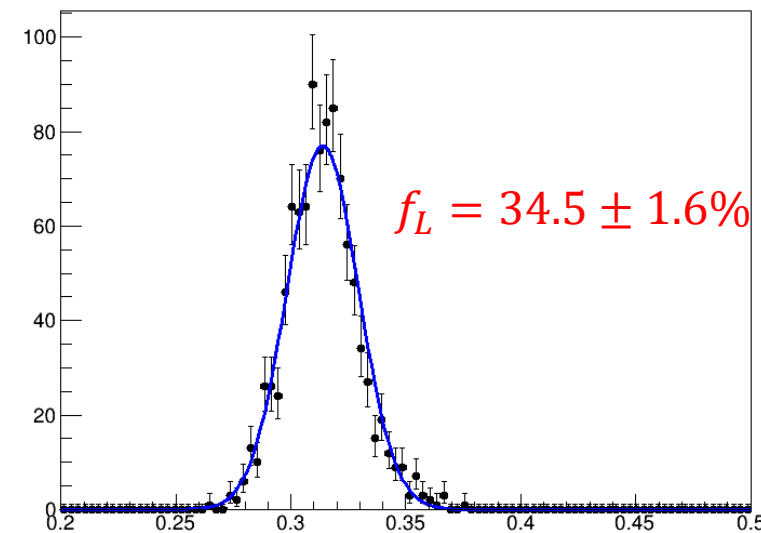
W : mass of D

The LS coupling parameters α_i in **covariant tensor** formula can be used by **covariant helicity** formula.

Covariant tensor: $\Sigma_{\sigma_1 \sigma_2} |\alpha_0 A(0,0) + \alpha_1 A(1,1) + \alpha_2 A(2,2)|^2$

Covariant helicity: $g_{ii} = f_i \alpha_i \rightarrow H_{\pm} = \sqrt{\frac{1}{3}} f_0 \alpha_0 \pm \sqrt{\frac{1}{2}} W r f_1 \alpha_1 + \sqrt{\frac{1}{6}} r^2 f_2 \alpha_2,$

$$H_0 = \gamma_1 \gamma_2 \left(-\sqrt{\frac{1}{3}} f_0 \alpha_0 + \sqrt{\frac{2}{3}} r^2 f_2 \alpha_2 \right)$$





Systematic uncertainty

- ❑ The systematic uncertainties can come from two parts.
- ❑ One is experimental systematic uncertainty which contains **background estimation**, **selection criteria** and **detection efficiency**.
- ❑ Another is model dependent uncertainty which contains the **resonance line shape**, the **radius of the centrifugal barrier**, uncertainties of **quantum coherence parameters**, **extra partial waves** and **fit bias**.



Resonance Line-shape

□ To study the effect of parameters in resonance line shape, we vary the fixed nominal masses and widths with 1σ of PDG value for the resonances with fixed parameters.

Large uncertainty
mainly from the
mass and width of
 $K_1(1270)^\pm$

wave	$K_1(1270)^+[S]K^-$	$K_1(1270)^-[S]K^-$	$\omega\phi[P]$
mag	0.1508(9.47%)	0.2663(8.99%)	0.0004(0.5%)
phase	0.0906(8.87%)	0.0864(4.50%)	0.0114(0.34%)

wave	$K_1(1270)^+[S]K^-$	$K_1(1270)^-[S]K^-$	$\omega\phi[P]$	$\omega\phi[S]$
Fit Fraction	0.0011(2.54%)	0.0016(1.08%)	0.0004(1.18%)	0.0018(0.22%)

Radius of the Centrifugal Barrier

- We assume that the probability distributions of radius are uniform, and obtain 1 GeV^{-1} as the approximative 1σ value of radius. Then, we vary the fixed radius with 1 GeV^{-1} for D^0 and other resonances respectively and repeat amplitude analysis. This uncertainty is calculated as the sum in quadrature of two parts.

Actually, as we performed 1C kinematic fit on D^0 , the B_L of D^0 is “1”, so the systematic uncertainty of barrier factor of D^0 should be ignored.

wave	$K_1(1270)^+[S]K^-$	$K_1(1270)^-[S]K^-$	$\omega\phi[P]$
mag	0.492(32.5%)	0.925(33.3%)	0.003(9.8%)
phase	0.021(2.0%)	0.016(0.9%)	0.006(0.3%)

wave	$K_1(1270)^+[S]K^-$	$K_1(1270)^-[S]K^-$	$\omega\phi[P]$	$\omega\phi[S]$
Fit Fraction	0.0000(0.17%)	0.0016(1.05%)	0.0005(1.20%)	0.0034(0.43%)

Quantum Coherence Correction

□ To study the effect of the quantum coherence correction, we change its input parameters by $\pm 1\sigma$ of the QC factors and repeat the amplitude analysis.

Mode	r (%)	R	δ ($^\circ$)
$K^- \pi^+$	5.857 ± 0.009 [1]	1	192.7 ± 2.3 [1]
$K^- \pi^+ \pi^0$	4.41 ± 0.08 [2]	0.75 ± 0.03 [2]	209^{+7}_{-8} [2]
$K^- \pi^+ \pi^- \pi^+$	5.51 ± 0.05 [2]	0.51 ± 0.04 [2]	182^{+14}_{-13} [2]

[1] LHCb-CONF-2025-003

[2] JHEP 08, 038 (2026)

wave	$K_1(1270)^+[S]K^-$	$K_1(1270)^-[S]K^-$	$\omega\phi[P]$
mag	0.0064(0.40%)	0.0036(0.12%)	0.0004(0.51%)
phase	0.0048(0.47%)	0.0025(0.13%)	0.0083(4.06%)

wave	$K_1(1270)^+[S]K^-$	$K_1(1270)^-[S]K^-$	$\omega\phi[P]$	$\omega\phi[S]$
Fit Fraction	0.0004(0.83%)	0.0003(0.20%)	0.0004(0.99%)	0.0004(0.05%)



Summary and next to do

□ Summary:

- ✓ Event Selection and Background estimation
- ✓ Simultaneous Fit with DIY MC
- ✓ Calculation of f_L based on PWA result
- ✓ IO check finished

□ Ongoing:

- Study of systematic uncertainty
- Memo preparation

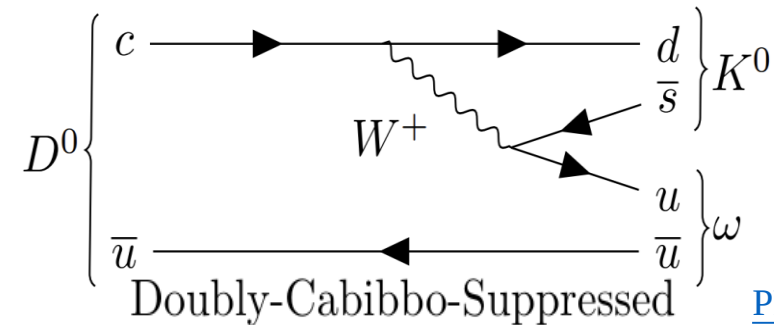
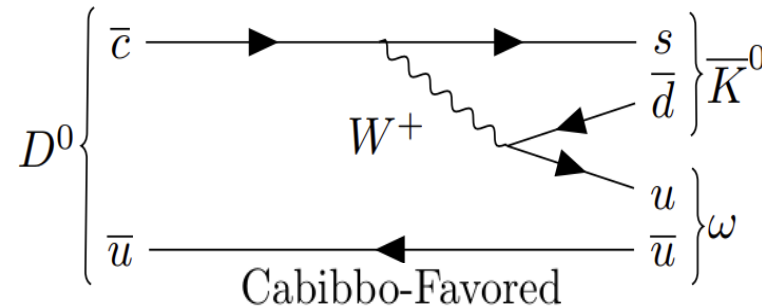
*Study of $K_S^0 - K_L^0$ asymmetry in the decays
 $D^0 \rightarrow \omega K_S^0$ and $D^0 \rightarrow \omega K_L^0$*

Motivation

□ The (quasi-)two-body D meson decays are important for understanding D decay mechanism, according to Glashow-Iliopoulos-Maiani (GIM): $V_{cd}^* V_{ud} + V_{cs}^* V_{us} + V_{cb}^* V_{ub} = 0 \not\Rightarrow$ CPV in D decays.

[PLB 349,363366\(1995\)](#)

- For the processes that involving a neutral K meson, Bigi and Yamamoto pointed out that interference between CF and DCS amplitudes in $D \rightarrow K^0 M$ can lead to experimentally observable $K_S^0 - K_L^0$ asymmetries .



[PRL 119.181802\(2017\)](#)

- CPV associated with those interference between CF and DCS amplitudes is studied to be $\sim 10^{-3}$.
- Measurement of $K_S^0 - K_L^0$ asymmetry $R(D^0) = \frac{\mathcal{B}(D^0 \rightarrow K_S^0 X) - \mathcal{B}(D^0 \rightarrow K_L^0 X)}{\mathcal{B}(D^0 \rightarrow K_S^0 X) + \mathcal{B}(D^0 \rightarrow K_L^0 X)} \simeq 2r_f \cos \delta_f$, can enable us to determine the amplitude of DCS processes and promote study of CPV.

Motivation

□ Compare of $R(D^0; X)$ from theories and experiments

- Different theoretical predictions (FAT, TDA...) of $R(D^0; X)$ show good consist with $D \rightarrow PP$ process but differ from $D \rightarrow VP$ process.
- $R(D^0; \omega)$ is calculated with $\mathcal{B}(D^0 \rightarrow K_L^0 \omega)$ measured by BESIII using 2.93 fb^{-1} data and $\mathcal{B}(D^0 \rightarrow K_S^0 \omega)$ from CLEO, the result shows large difference from FAT and TDA predictions.
- Measurement of $\mathcal{B}(D^0 \rightarrow K_S^0 \omega)$ and $\mathcal{B}(D^0 \rightarrow K_L^0 \omega)$ using a larger data sample can provide more precise results and provide important test of theoretical models.

	$D \rightarrow PP$		$D \rightarrow VP$	
$X = \pi^0, \eta, \eta', \phi, \omega$	π^0	η/η'	ϕ	ω
$R(D^0; X)_{\text{FAT}}^{[1]}(\%)$	11.3 ± 0.1			
$R(D^0; X)_{\text{F4}}^{[2]}(\%)$	10.7		10.1 ± 2.9	10.6 ± 3.4
$R(D^0; X)_{\text{F1}'}^{[2]}(\%)$	10.7		4.1 ± 2.9	8.9 ± 3.5
$R(D^0; X)_{\text{exp.}}(\%)$	$10.8 \pm 3.5^{[3]}$	$8 \pm 2.3^{[4]}$	$-0.1 \pm 4.7^{[4]}$	$-2.4 \pm 3.1^{[4]}$

Large difference between theory predictions and experiment measurement

[1] [PRD 95,073007\(2017\)](#), [2] [PRD 109,073008\(2024\)](#), [3] [PRL 100,091801\(2008\)](#), [4] [PRD 105,092010\(2022\)](#).

Summary of $D^0 \rightarrow \omega K_{S\backslash L}^0$

Results	$\mathcal{B}(D^0 \rightarrow K_S^0 \omega) \times 10^{-3}$	$\mathcal{B}(D^0 \rightarrow K_L^0 \omega) \times 10^{-3}$	$R(D^0; \omega) \times 10^{-2}$
This work	$11.73 \pm 0.18 \pm 0.25 \pm 0.47$	$11.01 \pm 0.14 \pm 0.22 \pm 0.44$	$3.2 \pm 1.0 \pm 0.9 \pm 2.80$
Previous results	$11.2 \pm 0.4 \pm 0.5$	$11.64 \pm 0.22 \pm 0.28$	-2.4 ± 3.1
STD(F4)	12.5 ± 0.6	10.5 ± 0.5	8.9 ± 3.5
GTD	11.6 ± 0.6	11.2 ± 0.5	1.6 ± 0.3

□ Summary

- ✓ Measurements of $\mathcal{B}(D^0 \rightarrow K_S^0 \omega)$, $\mathcal{B}(D^0 \rightarrow K_L^0 \omega)$ and $R(D^0; \omega)$.
- Submitted to PRD



Summary and Prospects

- ❑ Work I:PWA of $D^0 \rightarrow \omega K^+ K^-$
 - ✓ Obtained amplitude model & IO Check
 - Systemic uncertainty ongoing
 - Memo preparing
- ❑ Work II: $D^0 \rightarrow \omega K_S^0$ and $D^0 \rightarrow \omega K_L^0$
 - ✓ Submitted to PRD.(reply the 1st round of comments)
- ❑ Prospects:
 - Finish the analysis of $D^0 \rightarrow \omega K^+ K^-$, looking of other $D \rightarrow VV$ process.

Thank you !



Back UP

Standard Model (SM)

□ SM describes the **fundamental particles** that make up the visible world of matter and **how they interact with each other**.

➤ Still need to explain:

Dark matter and dark energy

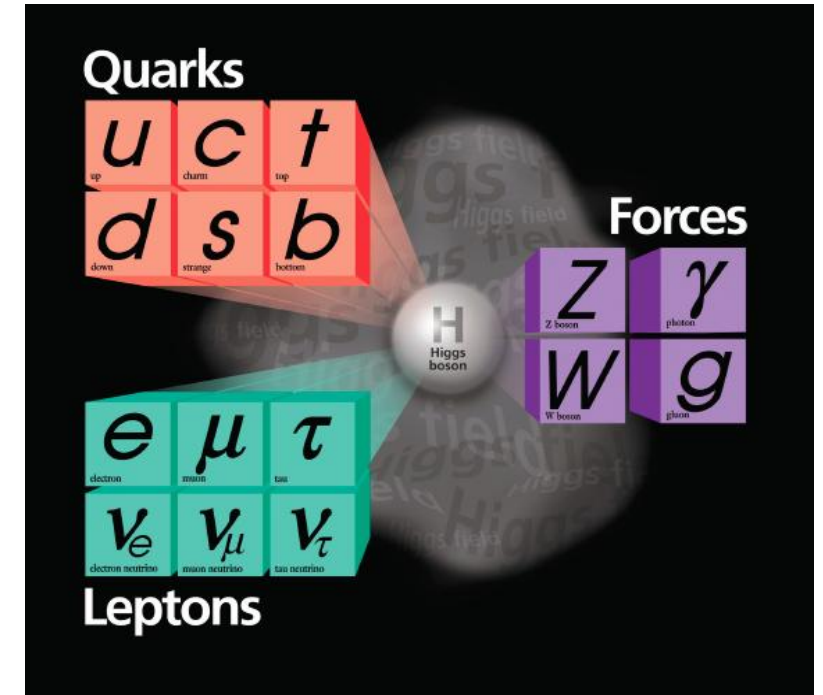
Mass of neutrinos

... ..



Experiments: test of SM, search for new physics beyond SM

SM phenomenology: Bridge between SM and experiments



➤ **Covariant helicity amplitude** for $D \rightarrow \phi\omega \rightarrow (K^+K^-)(\rho\pi) \rightarrow (K^+K^-)(\pi\pi_2\pi_3)$:

$$H_{\lambda_\phi, \lambda_\omega} = F_{\lambda_\phi, \lambda_\omega}^{JD} D_{\lambda_D, \lambda_\phi - \lambda_\omega}^{JD}(\Omega_D) \cdot T_\phi(m_\phi) F_{\lambda_{K^+}, \lambda_{K^-}}^{J_\phi}(\Omega_\phi^{\{D\}}) \cdot T_\omega(m_\omega) F_{\lambda_\rho, \lambda_\pi}^{J_\omega}(\Omega_\omega^{\{D\}}) D_{\lambda_\omega, \lambda_\rho - \lambda_\pi}^{J_\omega}(\Omega_\omega^{\{D\}}) \cdot T_\rho(m_\rho) F_{\lambda_{\pi_2}, \lambda_{\pi_3}}^{J_\rho} D_{\lambda_\rho, \lambda_{\pi_2} - \lambda_{\pi_3}}^{J_\rho}$$

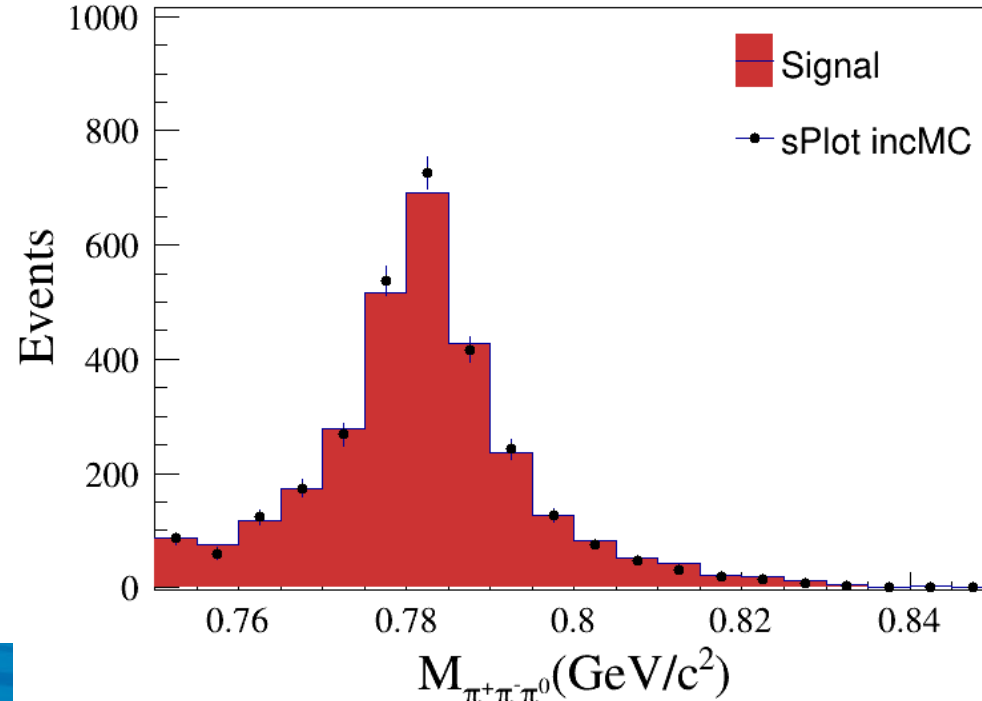
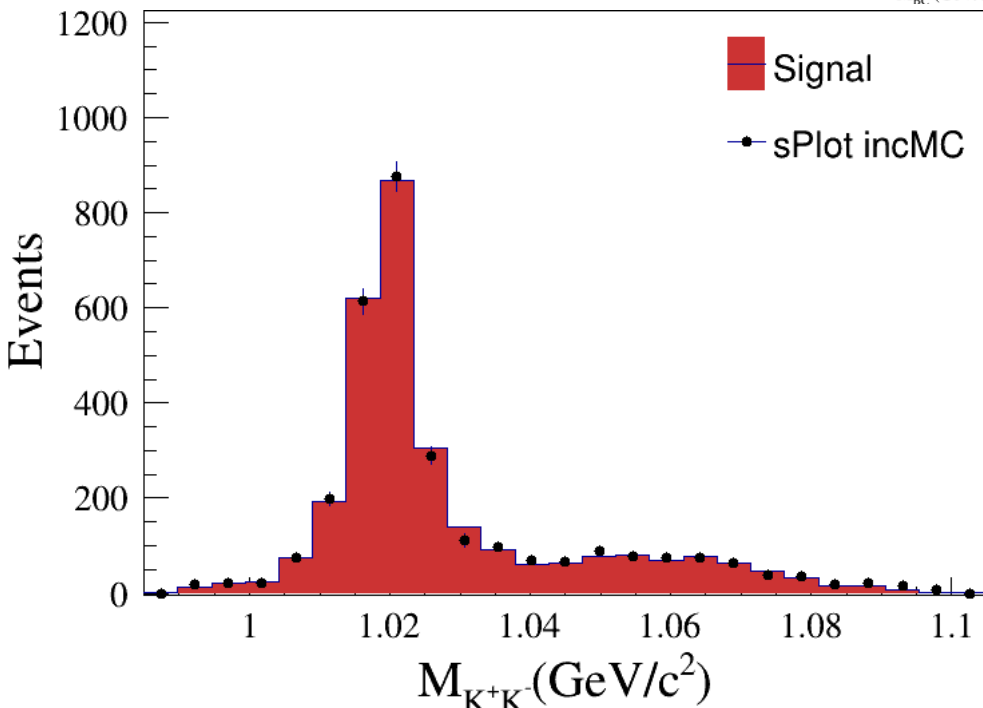
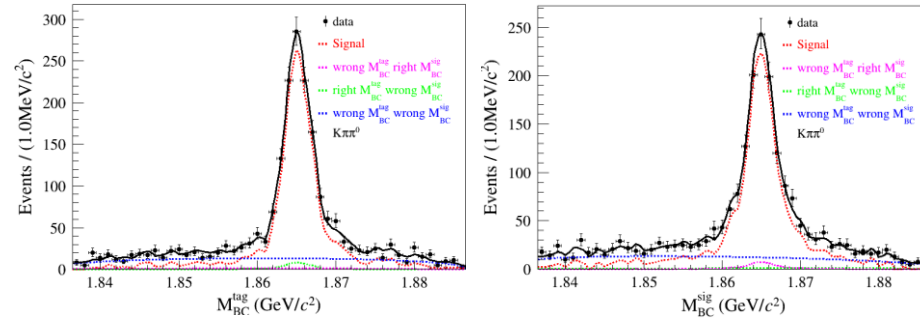
$$J_D = 0 \rightarrow \lambda_\phi = \lambda_\omega \rightarrow |A|^2 = \left| \sum_{\lambda_\phi \lambda_\omega} H_{\lambda_\phi \lambda_\omega} \right|^2 = |H_{11} + H_{-1-1} + H_{00}|^2$$

$$F_{\pm 1 \pm 1}^{JD} = \sqrt{\frac{1}{3}} g_{00} \pm \sqrt{\frac{1}{2}} W r g_{11} + \sqrt{\frac{1}{6}} r^2 g_{22}, \quad F_{00}^{JD} = \gamma_1 \gamma_2 \left(-\sqrt{\frac{1}{3}} g_{00} + \sqrt{\frac{2}{3}} r^2 g_{22} \right), \quad r + 2|q|$$

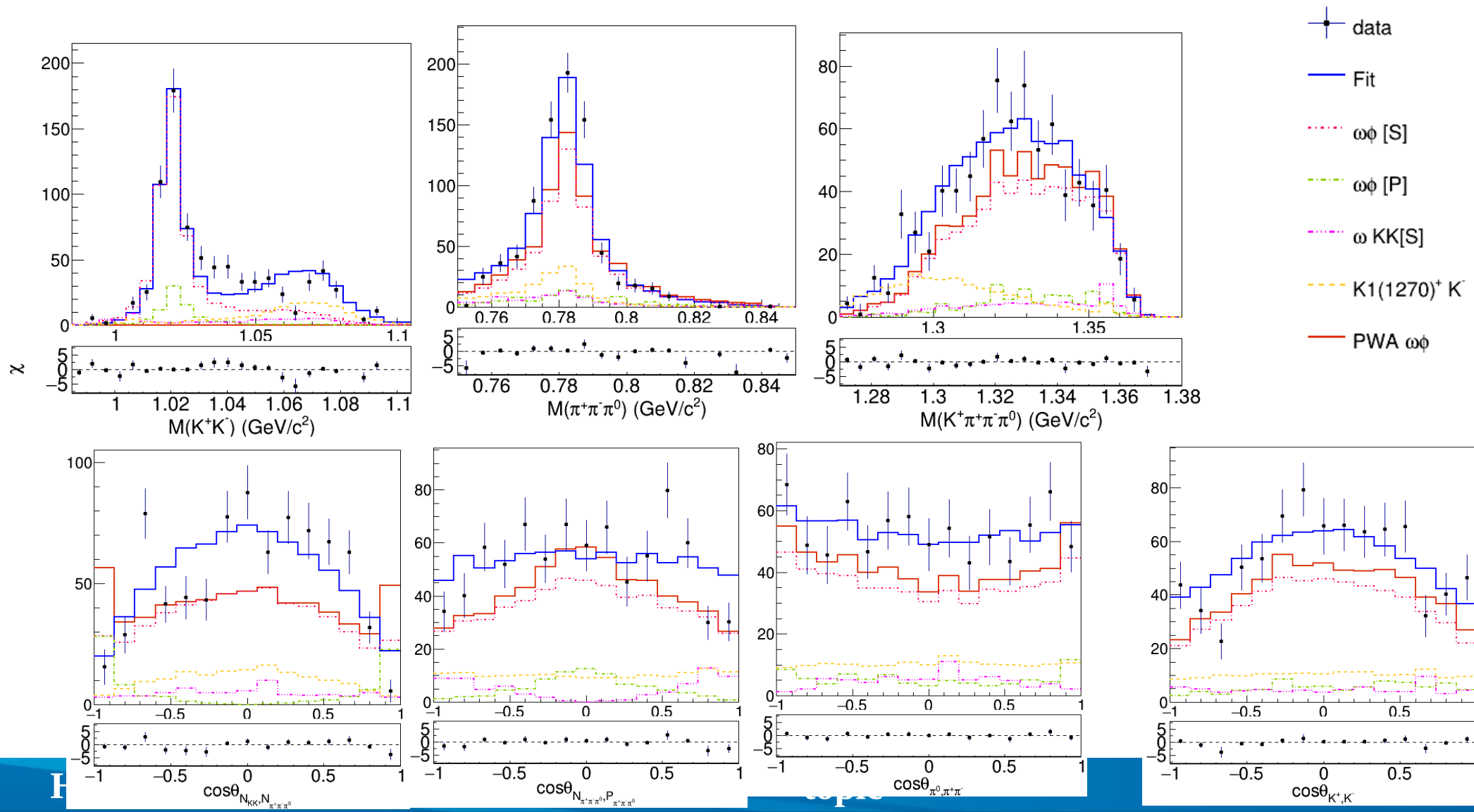
$|q|$: momentum of V in D rest frame γ : Lorentz factor of V in D rest frame W : mass of D

sPlosPlot on MC — Full Reconstruction

- To check the reliability of sPlot, 2-D M_{BC} fits are performed on inclusive MC samples. The comparisons between the signal components of inclusive MC and sPlot are shown below.

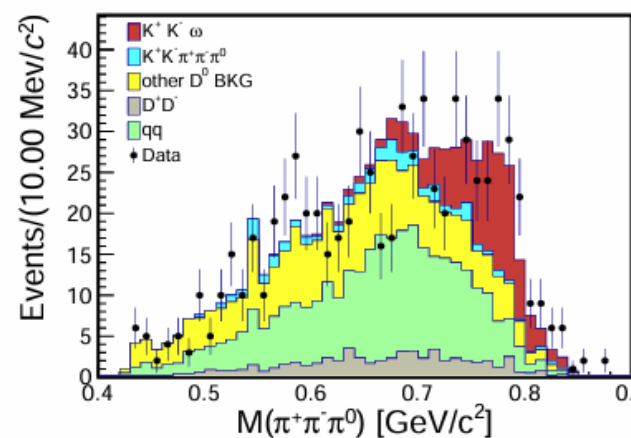
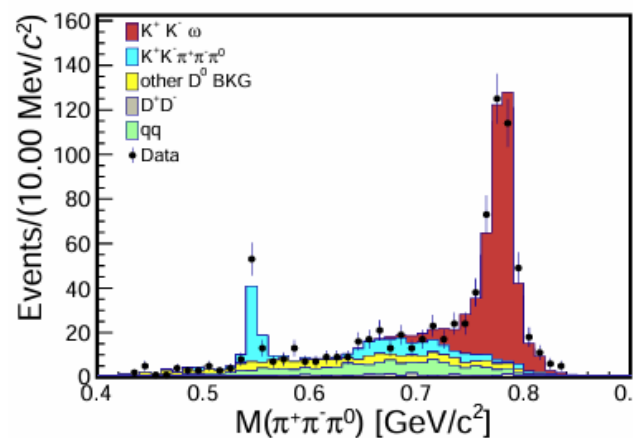
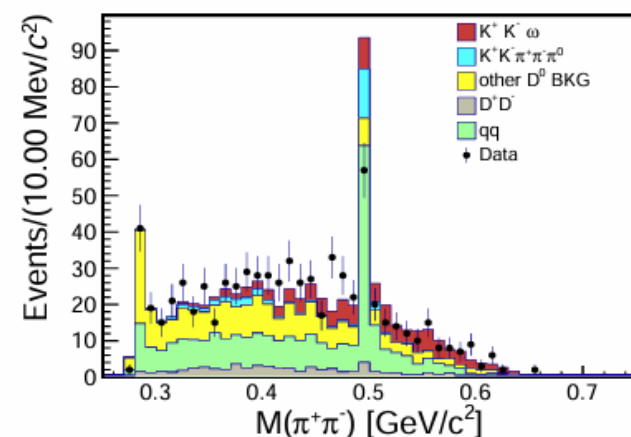
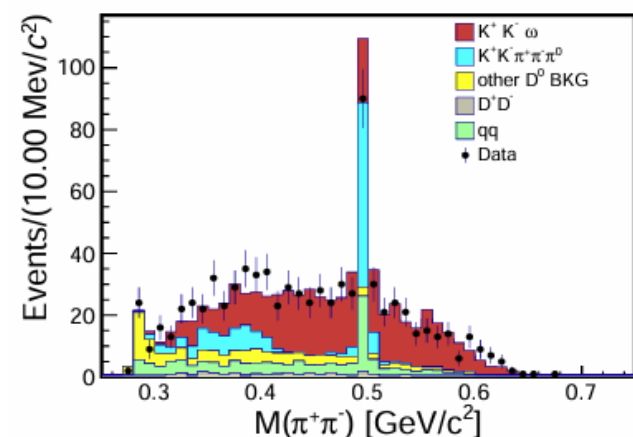
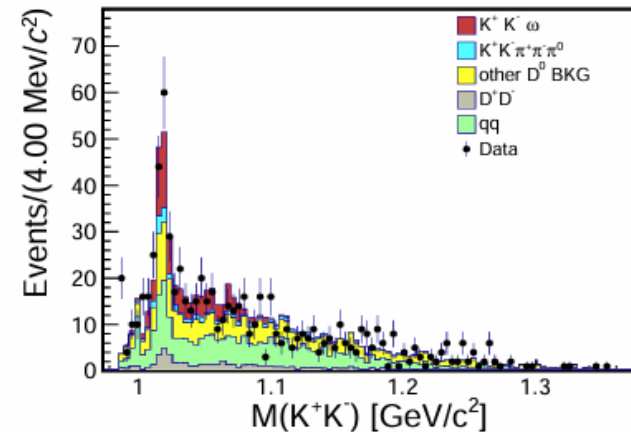
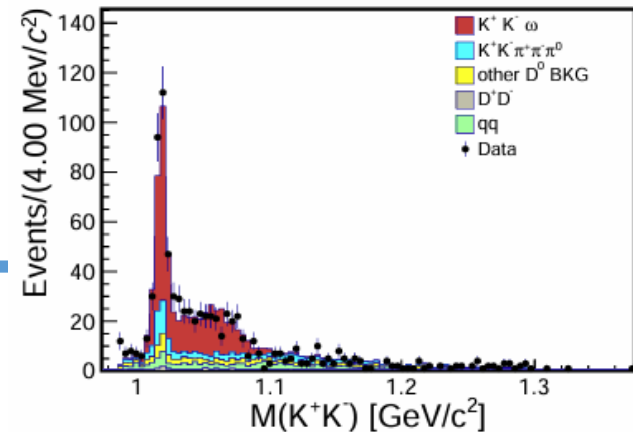
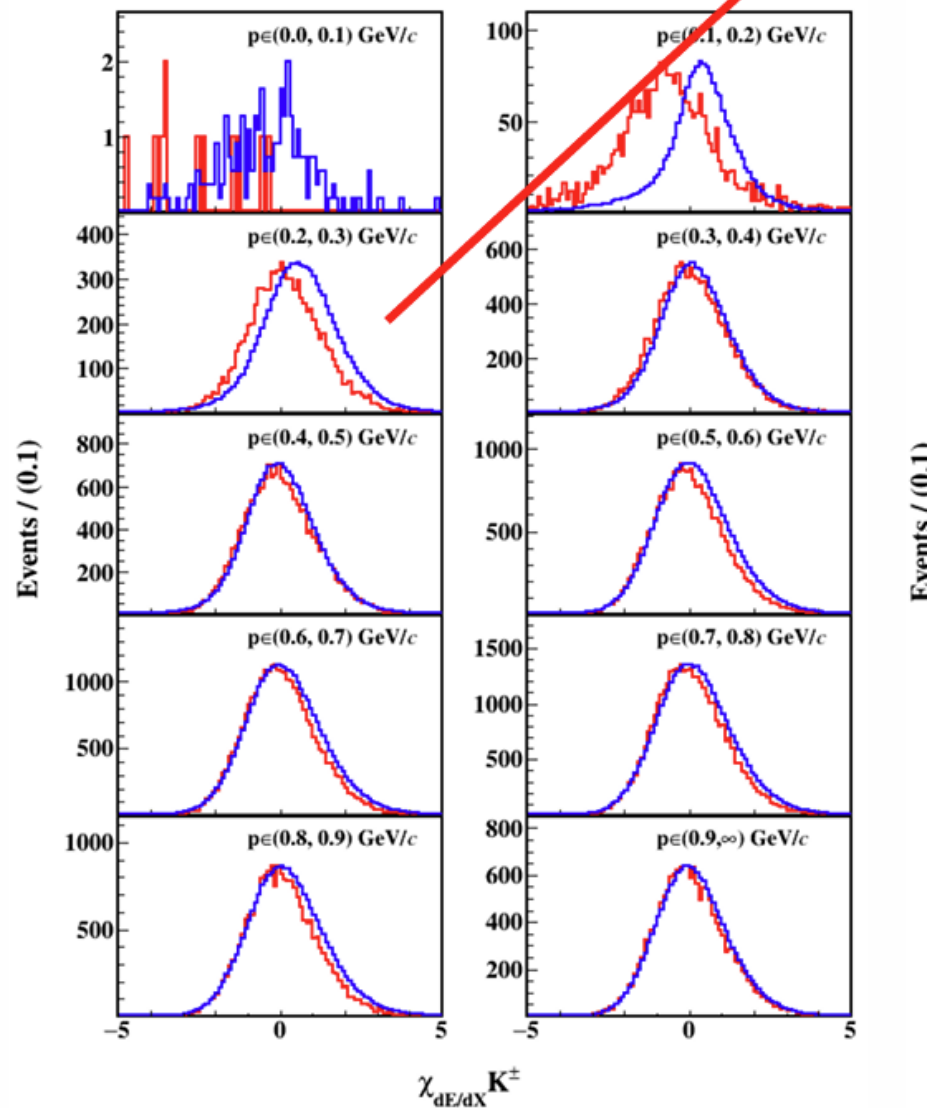


Fit on sPlot — Full reconstruction



$\chi_{dE/dx}$ of K sample

MC/data comparison

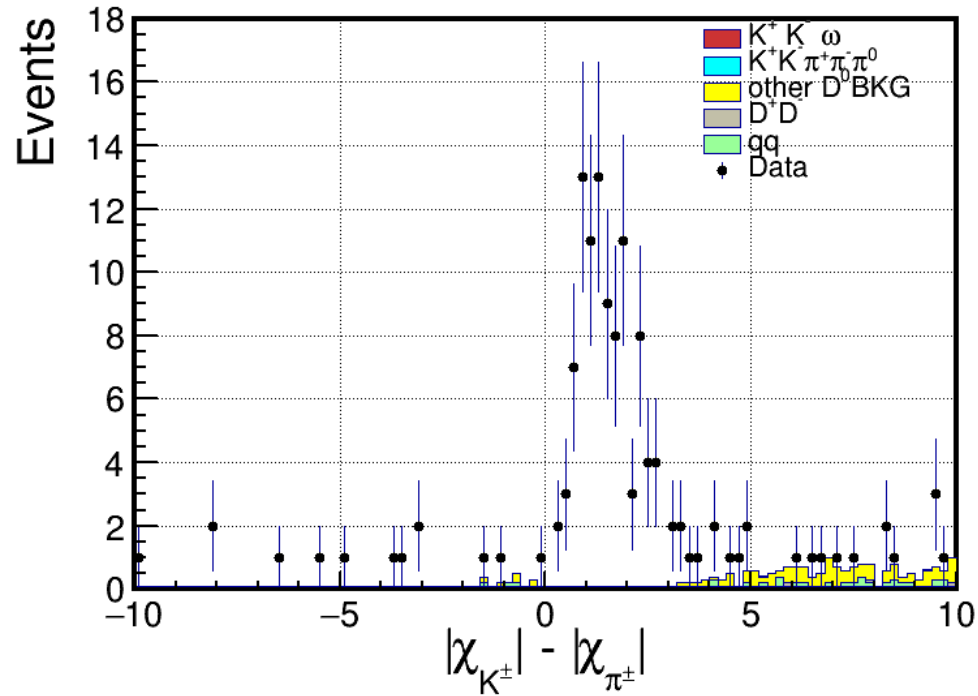


topic

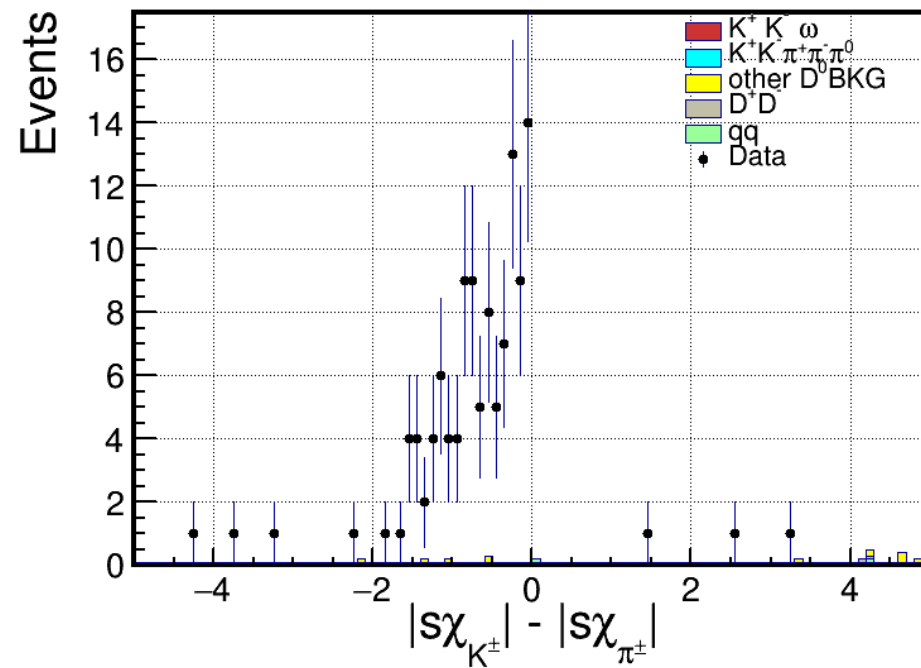
signal region(left)

non-signal region(right)

dE/dx info from simplePID

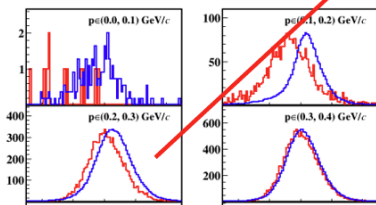


$\chi_{dE/dx}$ of K sample

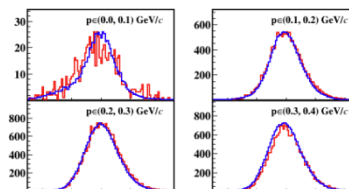


$\chi_{dE/dx}$ of π sample

MC/data comparison

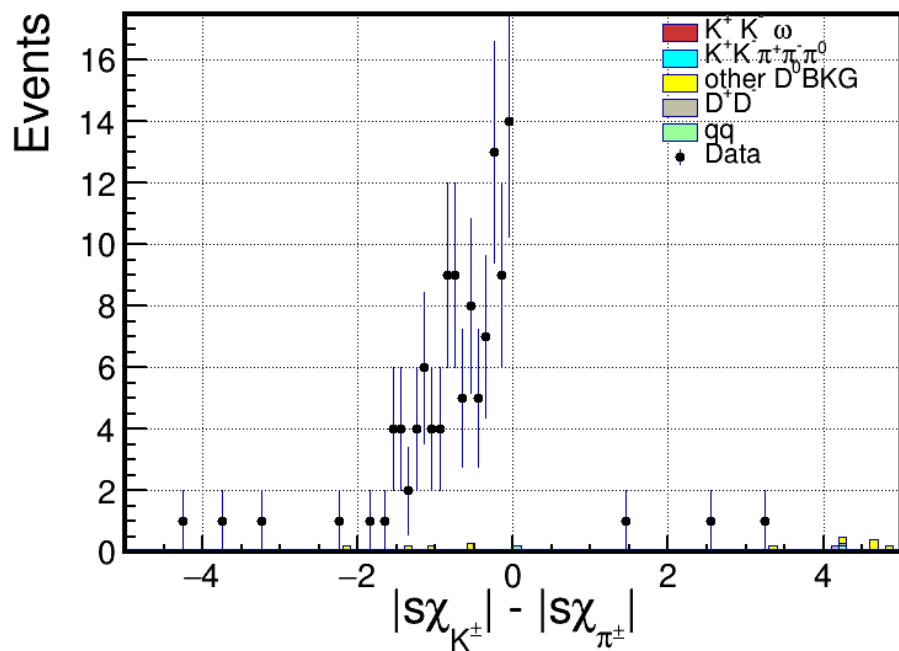


MC/data comparison



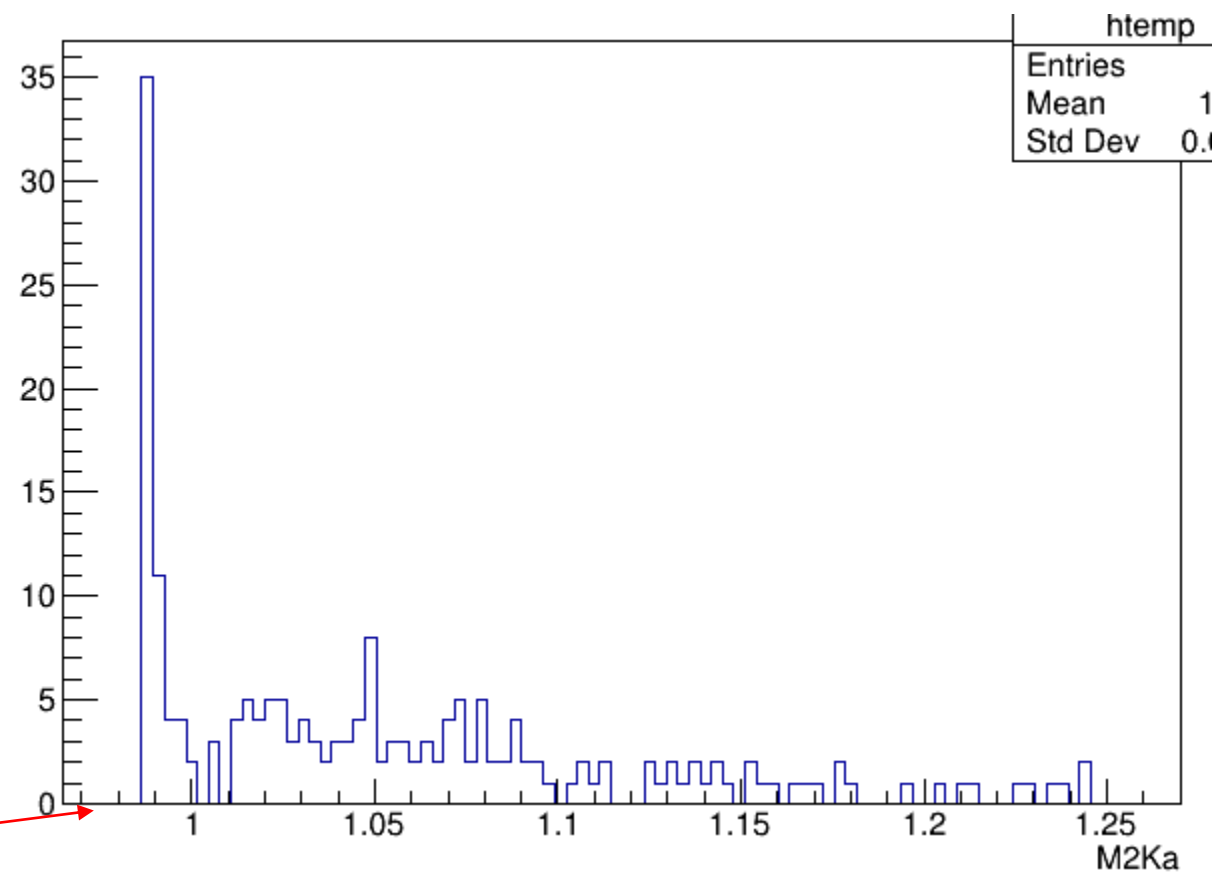
χ_K 的修正量要比 χ_π 大
原本 $|\chi_K| > |\chi_\pi|$ 的事例, 修正后 $|\chi_K| < |\chi_\pi|$ 了

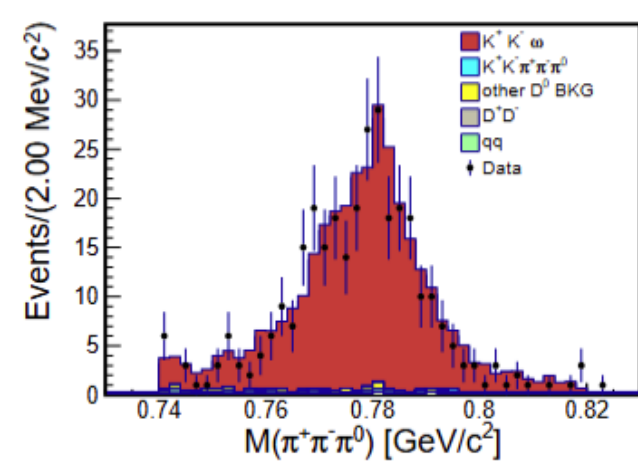
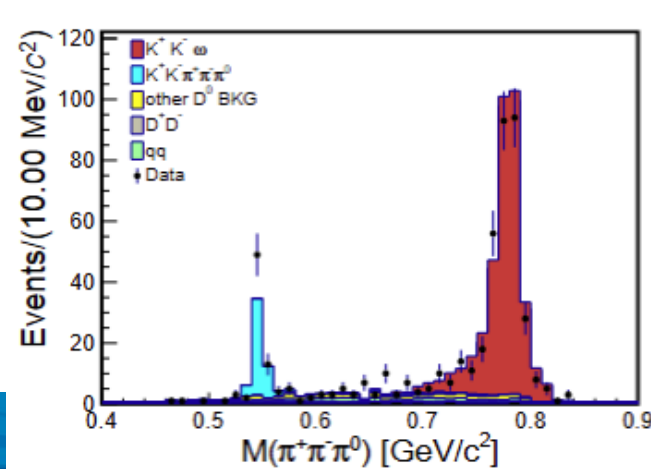
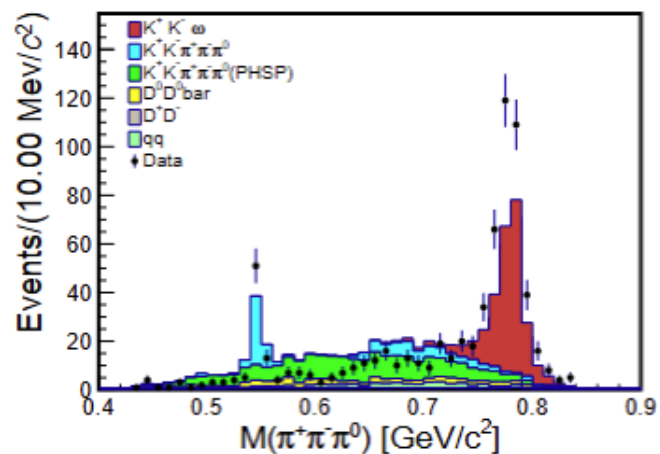
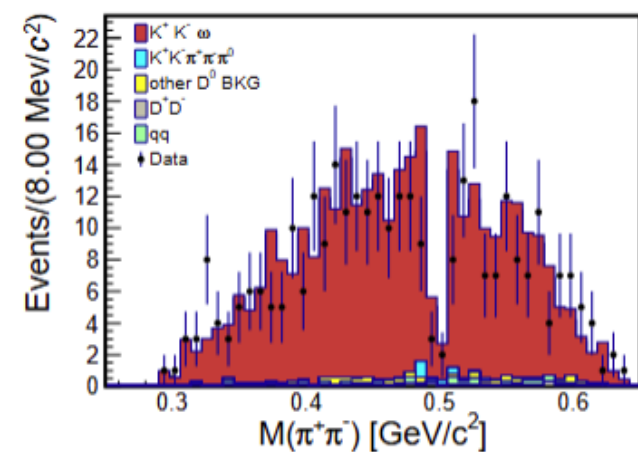
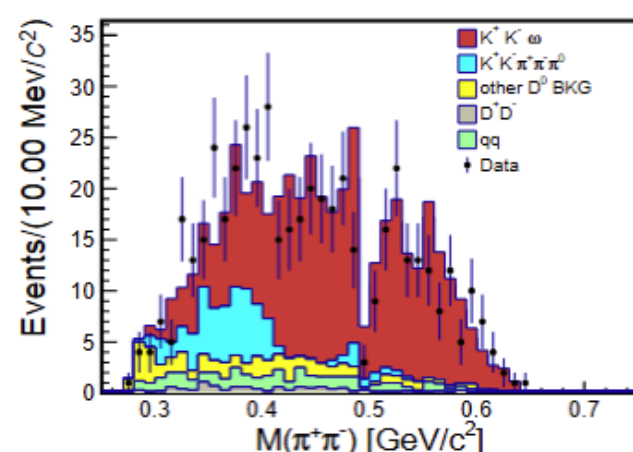
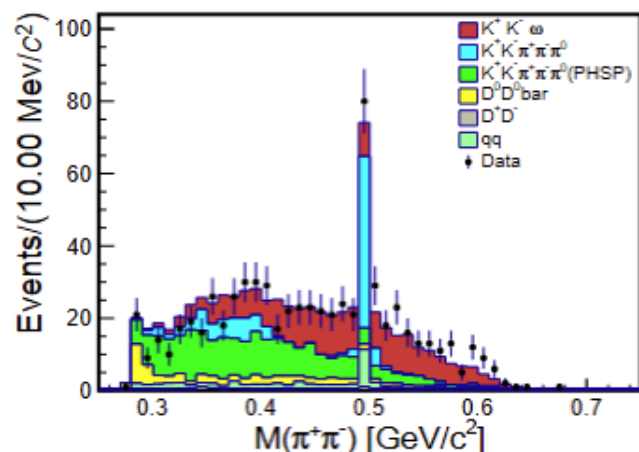
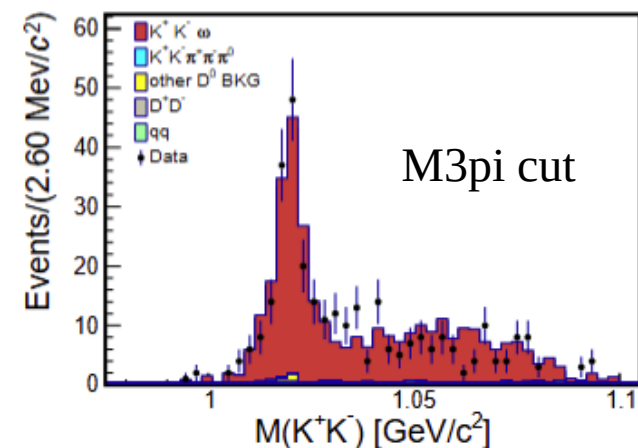
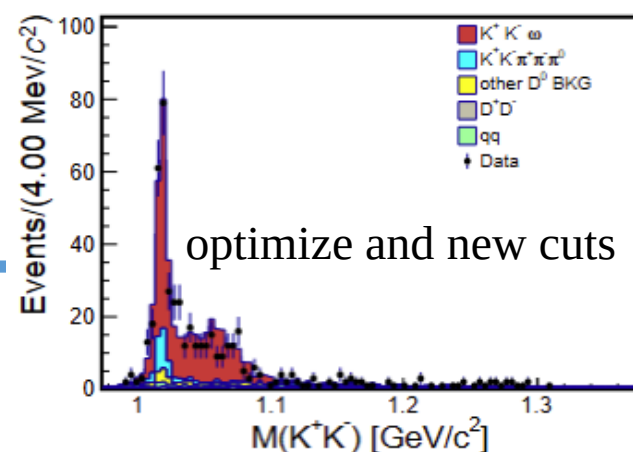
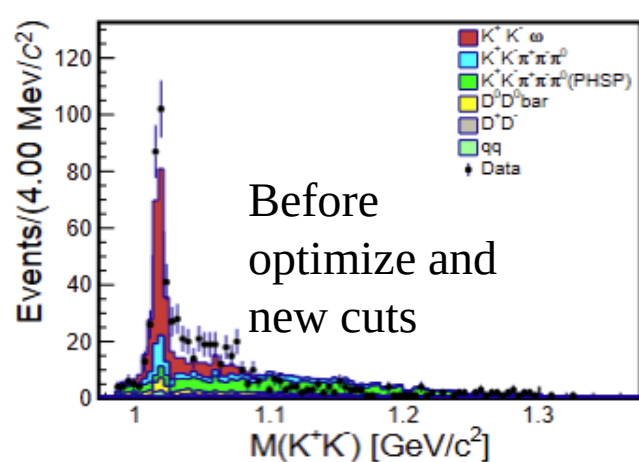
dE/dx info from simplePID



原本 $|\chi_K| > |\chi_\pi|$ 的事例，修正后 $|\chi_K| < |\chi_\pi|$ 了
对应的event包含了KK不变质量谱上的bump

$$\chi_K < -4, |s\chi_K| < |s\chi_{\pi}|$$





Amplitude	$ \tilde{c}_k $	$\arg(\tilde{c}_k)$ [rad]	Fit fraction [%]
$D^0 \rightarrow [\phi(1020)\rho(770)^0]_{L=0}$	1 (fixed)	0 (fixed)	$92.55 \pm 0.46 \pm 0.28$
$D^0 \rightarrow [\phi(1020)\omega(782)]_{L=0}$	$0.114 \pm 0.004 \pm 0.003$	$1.30 \pm 0.04 \pm 0.04$	$1.42 \pm 0.11 \pm 0.04$
Sum of fit fractions			$93.96 \pm 0.40 \pm 0.28$
$D^0 \rightarrow [\phi(1020)\rho(770)^0]_{L=1}$	1 (fixed)	0 (fixed)	$83.11 \pm 4.11 \pm 1.70$
$D^0 \rightarrow [\phi(1020)\omega(782)]_{L=1}$	$0.254 \pm 0.052 \pm 0.018$	$1.32 \pm 0.19 \pm 0.07$	$4.33 \pm 1.58 \pm 0.52$
Sum of fit fractions			$87.45 \pm 2.99 \pm 1.78$
$D^0 \rightarrow [\phi(1020)\rho(770)^0]_{L=2}$	1 (fixed)	0 (fixed)	$94.64 \pm 1.69 \pm 0.78$
$D^0 \rightarrow [\phi(1020)\omega(782)]_{L=2}$	$0.162 \pm 0.032 \pm 0.014$	$1.50 \pm 0.17 \pm 0.06$	$0.71 \pm 0.27 \pm 0.12$
Sum of fit fractions			$95.35 \pm 1.54 \pm 0.79$
$D^0 \rightarrow [K^+K^-]_{L=0}\rho(770)^0$	1 (fixed)	0 (fixed)	$85.41 \pm 5.89 \pm 3.49$
$D^0 \rightarrow [K^+K^-]_{L=0}\omega(782)$	$0.494 \pm 0.098 \pm 0.098$	$-0.95 \pm 0.19 \pm 0.15$	$9.24 \pm 3.26 \pm 3.64$
Sum of fit fractions			$94.65 \pm 5.03 \pm 5.04$
$K_1(1270)^+ \rightarrow [\rho(770)^0 K^+]_{L=0}$	1 (fixed)	0 (fixed)	$139.03 \pm 1.98 \pm 3.81$
$K_1(1270)^+ \rightarrow [\omega(782) K^+]_{L=0}$	$0.159 \pm 0.012 \pm 0.011$	$1.36 \pm 0.07 \pm 0.06$	$1.52 \pm 0.22 \pm 0.19$
Sum of fit fractions			$140.55 \pm 1.90 \pm 3.81$

*AMPs from LHCb
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confirm that $Br(D^0 \rightarrow \omega\phi) < Br(D^0 \rightarrow \rho^0\phi)$

Fitting Results

J^{PC}	$K^+ K^-$ resonance
0^{++}	$a_0(980), f_0(980), f_0(1370), KK$
1^{--}	$\phi(1020), \rho(1450), KK$
2^{++}	$f_2(1270)$

J^P	$K\omega$ resonance
0^-	$K(1460)$
1^+	$K_1(1270), K_1(1400), K_1(1650)$
1^-	$K_{st}(1410)$
2^-	$K_2(1580)$
2^+	$K_{st}(1430)$

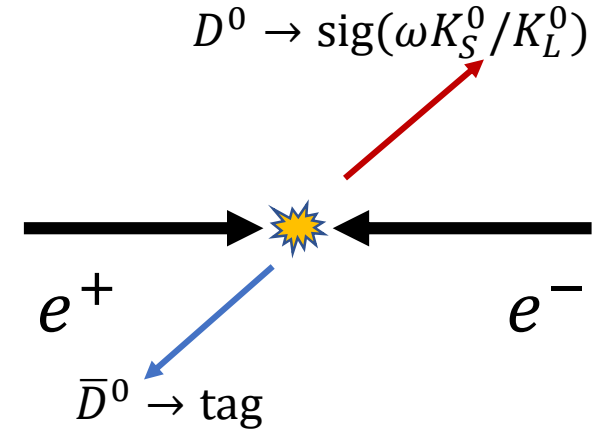
Amplitude	significance
$\omega\phi[S]$	$> 10\sigma$
$\omega\phi[P]$	3.5σ
$\omega\phi[D]$	2.0σ
$K_1(1270)^+ K^- [S]$	5.1σ
$K_1(1270)^- K^+ [S]$	8.1σ

Resonance	FF(%)	mag	phase
$\omega\phi[S]$	85.35	1	0
$\omega\phi[P]$	2.11	0.052	0.022
$K_1(1270)^+ K^- [S]$	3.70	0.999	-1.602
$K_1(1270)^- K^+ [S]$	10.69	1.978	1.286

Analysis Strategy

□ Data set: 7.93 fb^{-1} at 3.773 GeV:

- High precise measure of $\omega K_S^0 / K_L^0$.
- Cancel part systematic uncertainties.



□ Double Tag Method: $D^0 \bar{D}^0$ pair production.

- $\bar{D}^0$ reconstructed by tag mode i . (ST)
- D^0 reconstructed by sig mode $f(\omega K_S^0 / K_L^0)$. (DT)

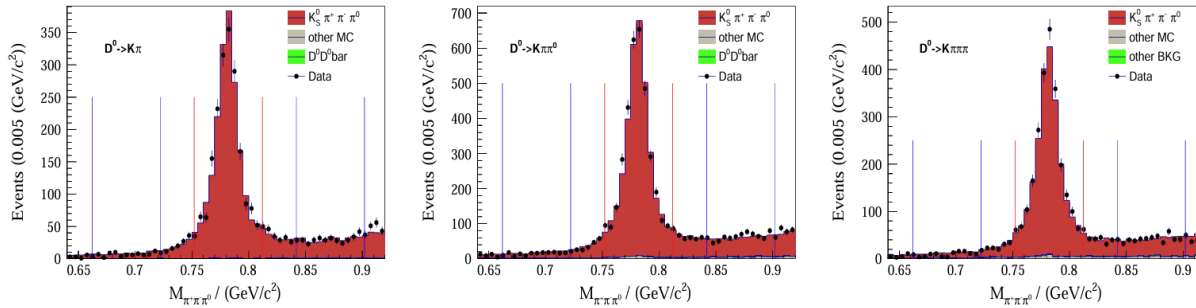
$$\text{➤ } \mathcal{B}_f^i = \frac{N_{\text{DT}}^i \cdot \epsilon_{\text{tag}}^i}{N_{\text{ST}}^i \cdot \epsilon_{\text{tag, sig}}^i \cdot [1 - \frac{2r_i R_i \cos \delta^i}{1+r_i^2} (2F_+^f - 1)]} ; \quad f_{\text{QC}}^f = \frac{1}{1 - \frac{2r_i R_i \cos \delta^i}{1+r_i^2} (2F_+^f - 1)} .$$

- Tag mode: $\bar{D}^0 \rightarrow K^+ \pi^-$, $\bar{D}^0 \rightarrow K^+ \pi^- \pi^0$, $\bar{D}^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$.

Background Analysis

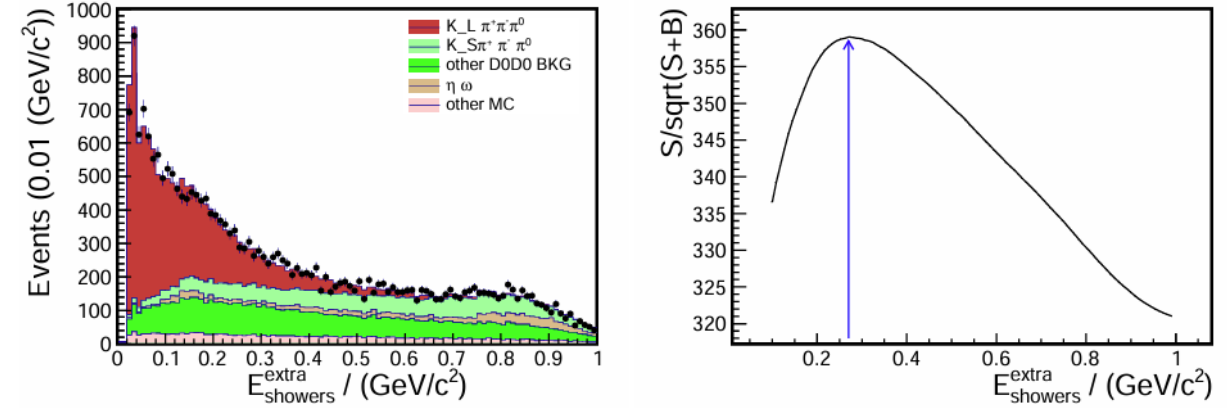
□ $D^0 \rightarrow \omega K_S^0$

➤ Low background level



- Peaking backgrounds non- ω resonance $K_S^0 \pi^+ \pi^- \pi^0$ processes
- Peaking backgrounds: wrong reconstruction of a soft γ of π^0

□ $D^0 \rightarrow \omega K_L^0$

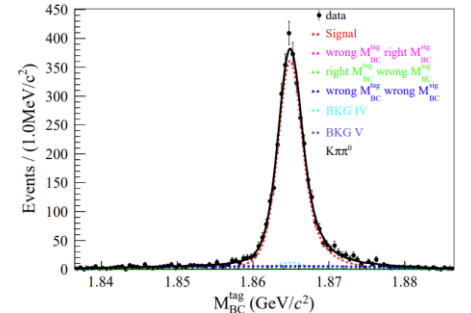
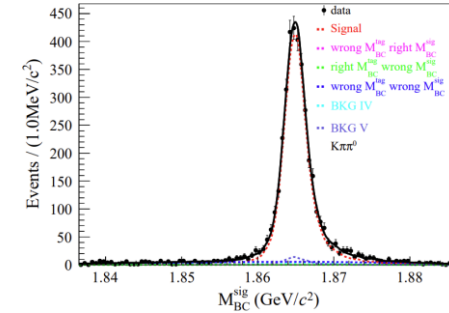


- Suppress background by requiring $E_{showers}^{extra}$
- Peaking backgrounds $\omega \eta$
- Peaking backgrounds ωK_S^0
- Peaking backgrounds non- ω resonance $K_{S/L}^0 \pi^+ \pi^- \pi^0$ processes
- Peaking backgrounds from $q\bar{q}$ process

DT Fitting Method & Results

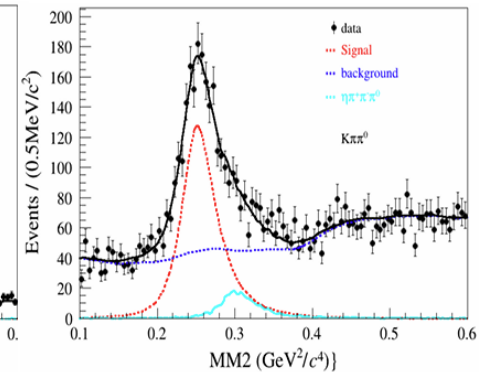
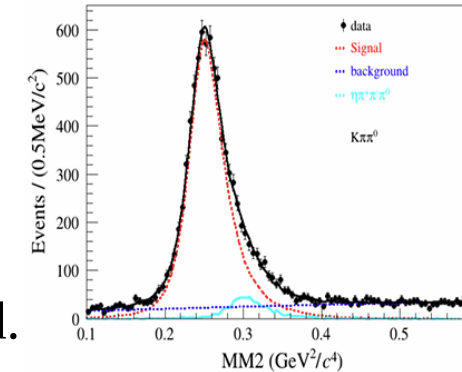
□ $D^0 \rightarrow \omega K_S^0$: Fully reconstructed; **2D-fit** to M_{BC}^{sig} v. s. M_{BC}^{tag} .

- Peaking BKGs from $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$: $M_{\pi^+ \pi^- \pi^0}$ side-band.
- Wrong reconstruction of a soft γ of π^0 : MC simulation.
- Signal shape modeled by MC.



□ $D^0 \rightarrow \omega K_L^0$: Missing mass method; fitting to M_{miss}^2 (MM2).

- Lateral peaks from η : MC simulation.
- Peaking BKGs from $D^0 \rightarrow K_{S/L}^0 \pi^+ \pi^- \pi^0$: $M_{\pi^+ \pi^- \pi^0}$ side-band.
- Peaking BKGs from $q\bar{q}$: estimated by M_{BC}^{tag} side-band and MC simulation.



Systematic Uncertainty

Source	$D^0 \rightarrow K_S^0 \omega(\%)$	$D^0 \rightarrow K_L^0 \omega(\%)$
$\pi^\pm$ tracking	1.0	0.8
$\pi^\pm$ PID	0.2	0.2
π^0 reconstruction	1.1	1.1
K_S^0 reconstruction	1.1	—
ST Yields	0.1	0.1
DT Fit	0.5	0.7
QC-correction	0.3	0.3
ω Mass window	0.5	0.7
$N_{\text{charged}}^{\text{extra}} = 0$	—	0.9
$E_\gamma^{\text{extra}} < 0.27 \text{ GeV}$	—	0.4
MC Statistics	0.3	0.3
Quoted BF's	0.7	0.7
Total	2.2	2.2

- Interference between the processes with and without ω intermediate state are not considered.
- A MC study indicates that this interference will affect the BF's of $D^0 \rightarrow \omega K_{S/L}$ with a factor of 4%, which will be regarded as the third uncertainties in the measurement.