

Transport properties of quark-gluon-plasma from Lattice QCD

Hai-Tao Shu

Central China Normal University



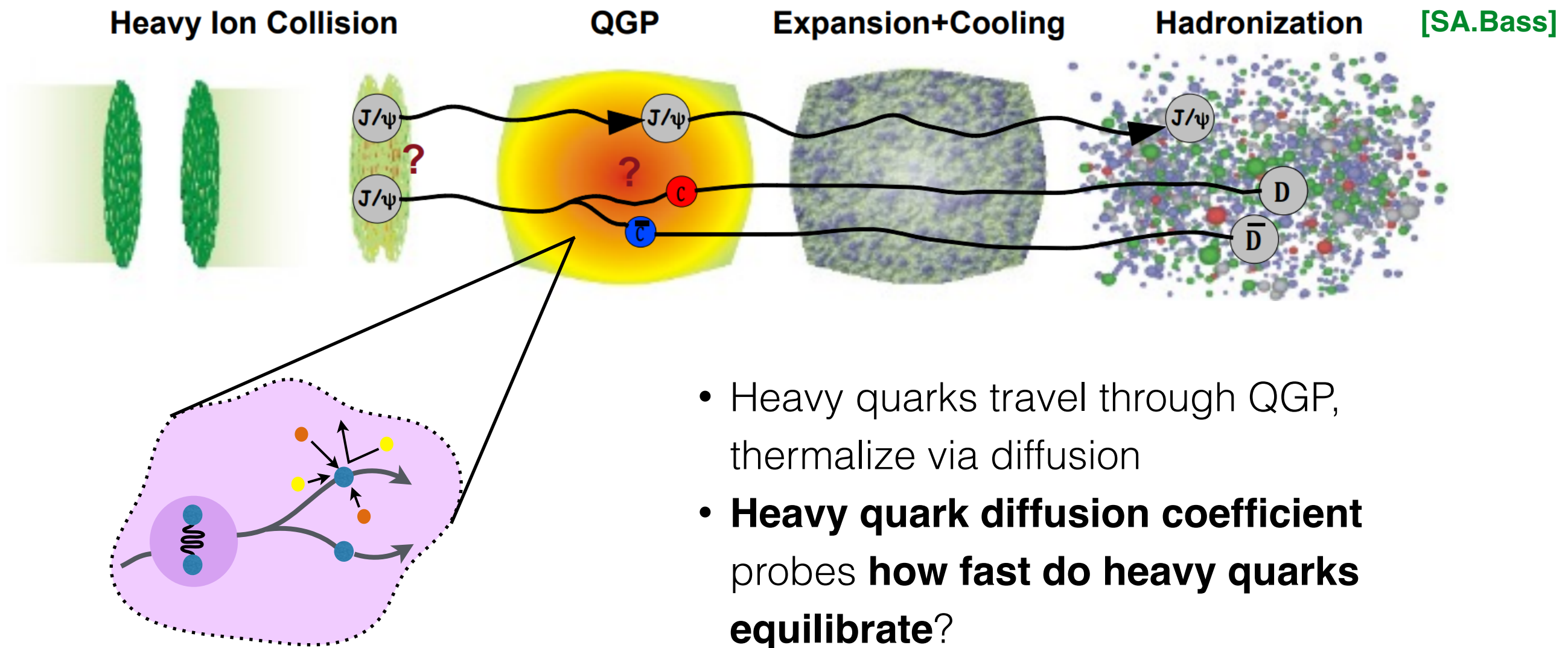
Nuclear Science
Computing Center at CCNU

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Jul. 10, 2026, Hefei, China

- ♦ Part 0: Background and motivation
- ♦ Part 1: Conserved current transport:
 - ➔ Heavy quark diffusion coefficient D_s
 - ➔ Electrical conductivity σ
- ♦ Part 2: Energy-momentum transport:
 - ➔ Shear viscosity η and bulk viscosity ζ
 - ➔ Jet transport coefficient $\hat{q}$
- ♦ Part 3: Summary and outlook

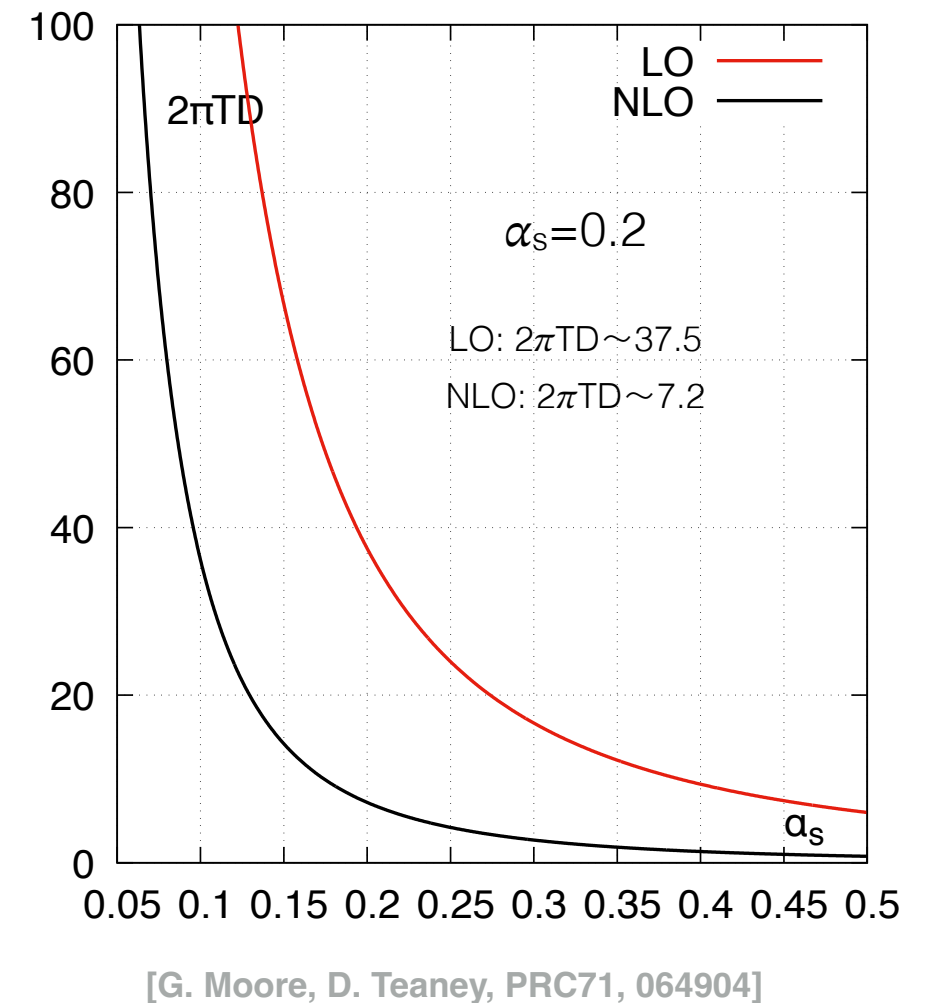
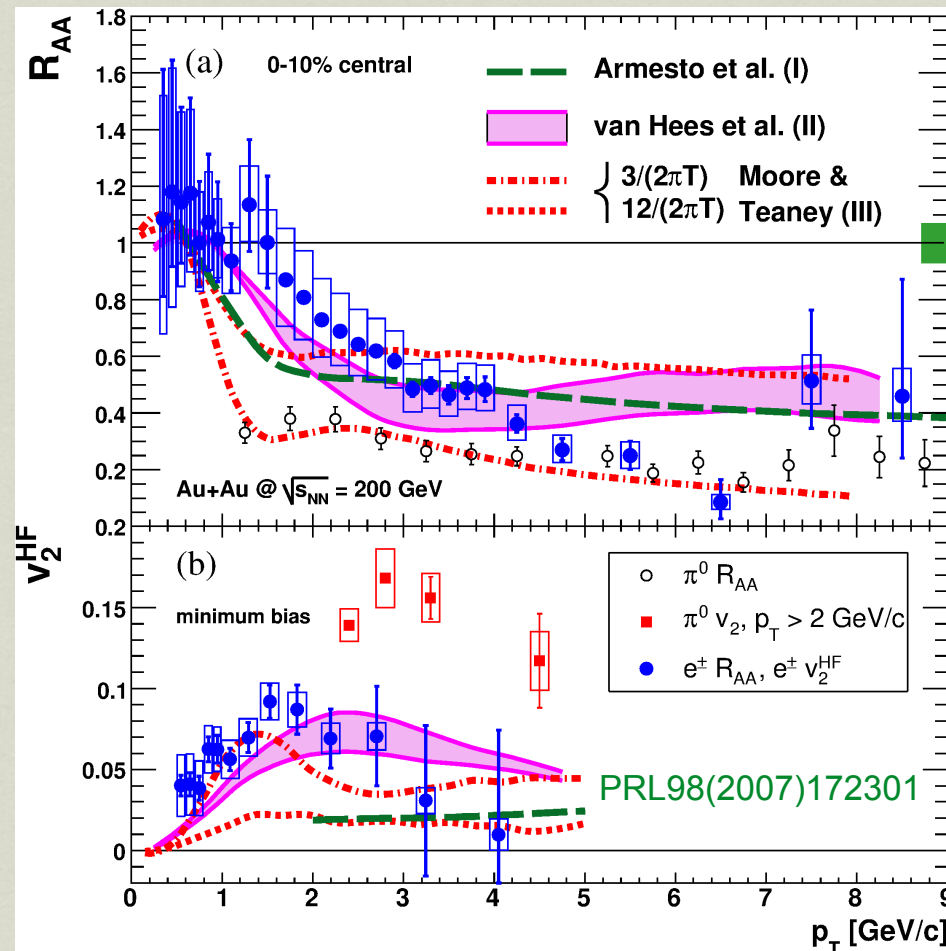
Transport of conserved current: HQ diffusion



How can we quantify the strength of the heavy quark diffusion?

Perturbative estimate of HQ diffusion

- Heavy quark diffusion coefficient



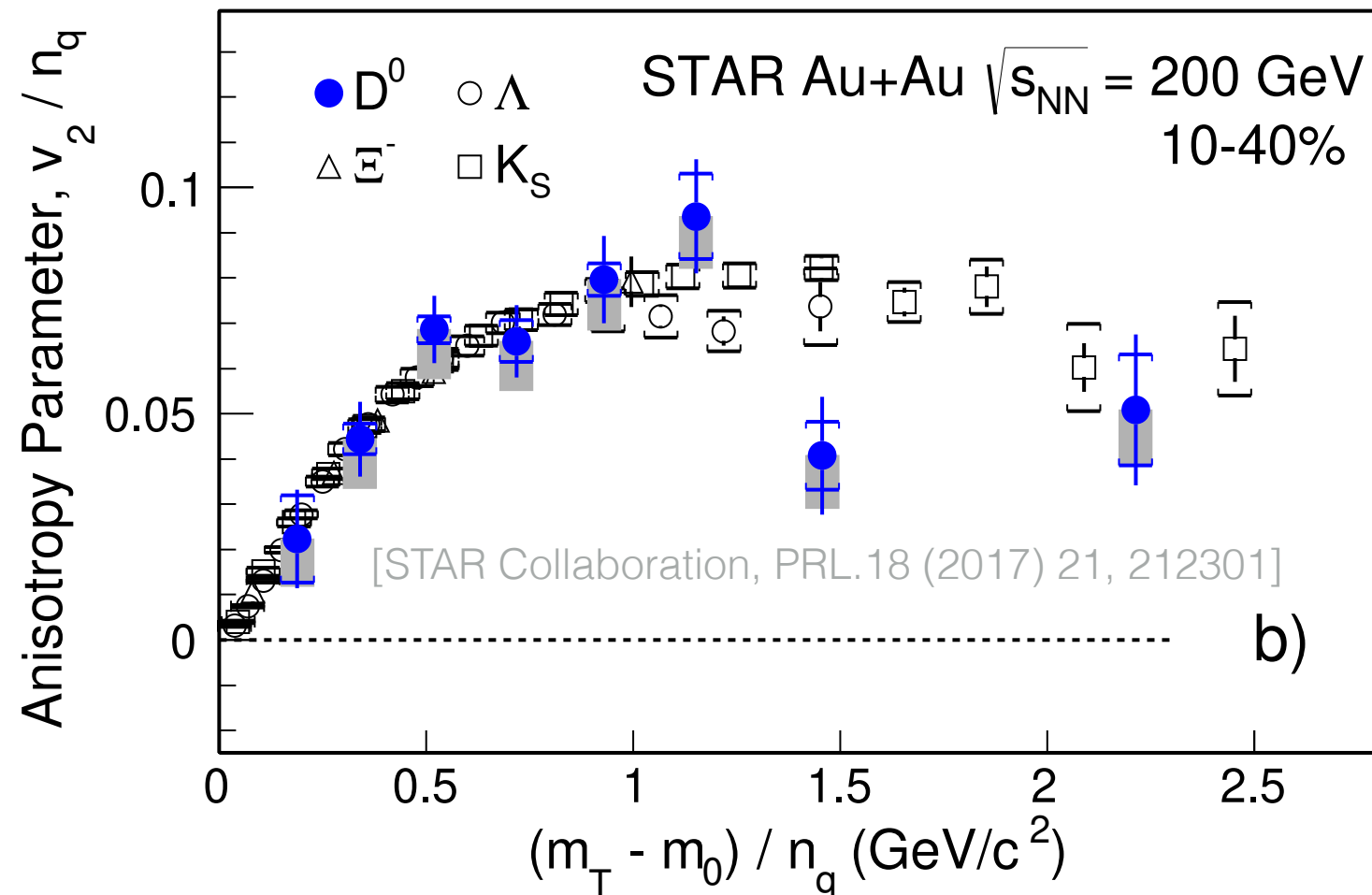
- HQ diffusion coefficient as crucial input for phenomenological studies
- Poor convergence from perturbative theory

Perturbative estimates:

[Moore, Teaney (2005); hep-ph/0412346
[Baier et al. (1994); arXiv:hep-ph/9411409]

$$\tau_{\text{kin,charm}} \sim 6 \text{ fm/c} \gg \tau_{\text{kin,light}} \sim 1 \text{ fm/c}$$

Experimental hints of HQ diffusion



Experimental estimates:

$$\tau_{\text{kin,charm}} \approx \tau_{\text{kin,light}}$$

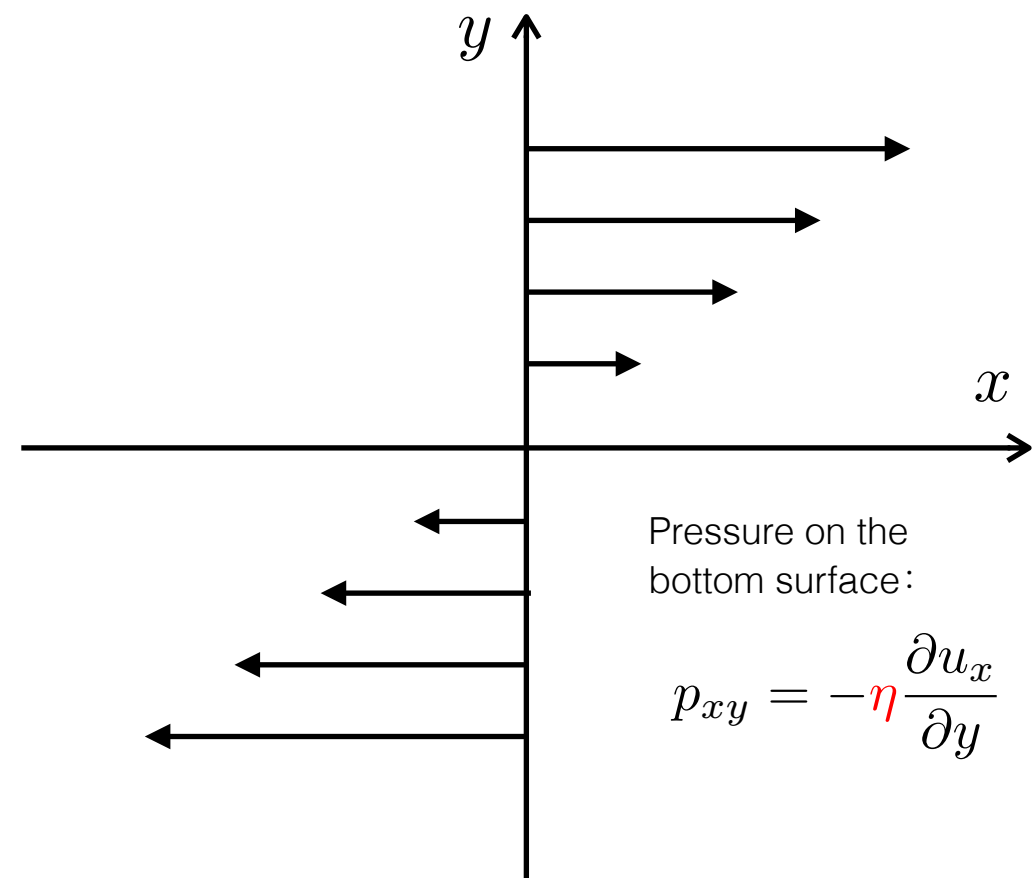
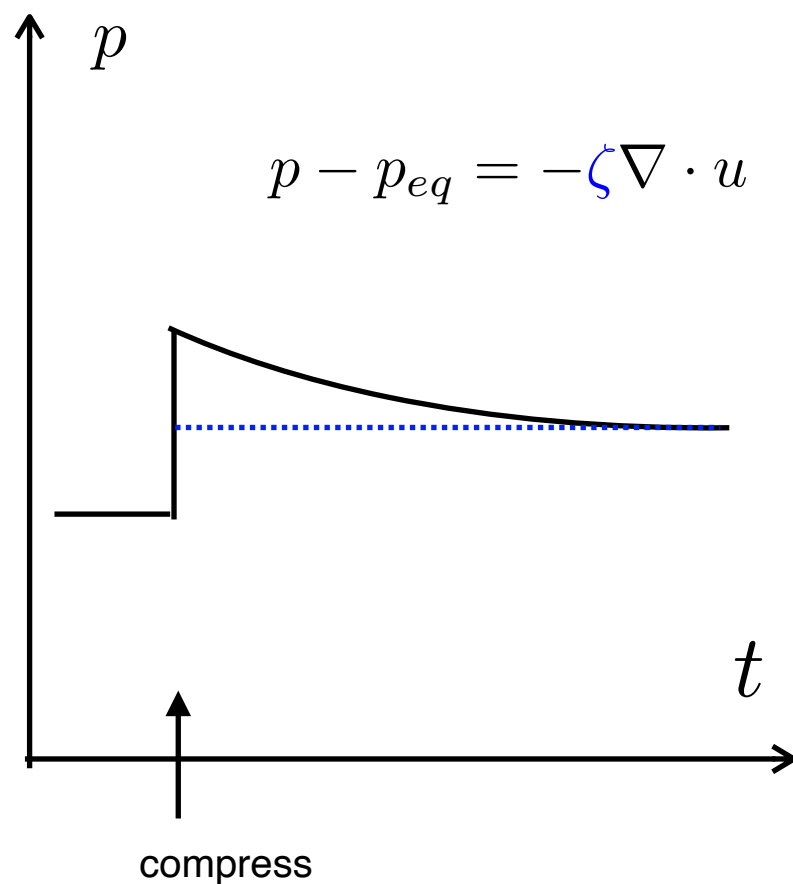
- Scaled D^0 v_2 is consistent with that of other light mesons
- Need non-perturbative ab-initio determination using lattice QCD

Transport of momentum/stress: viscosities

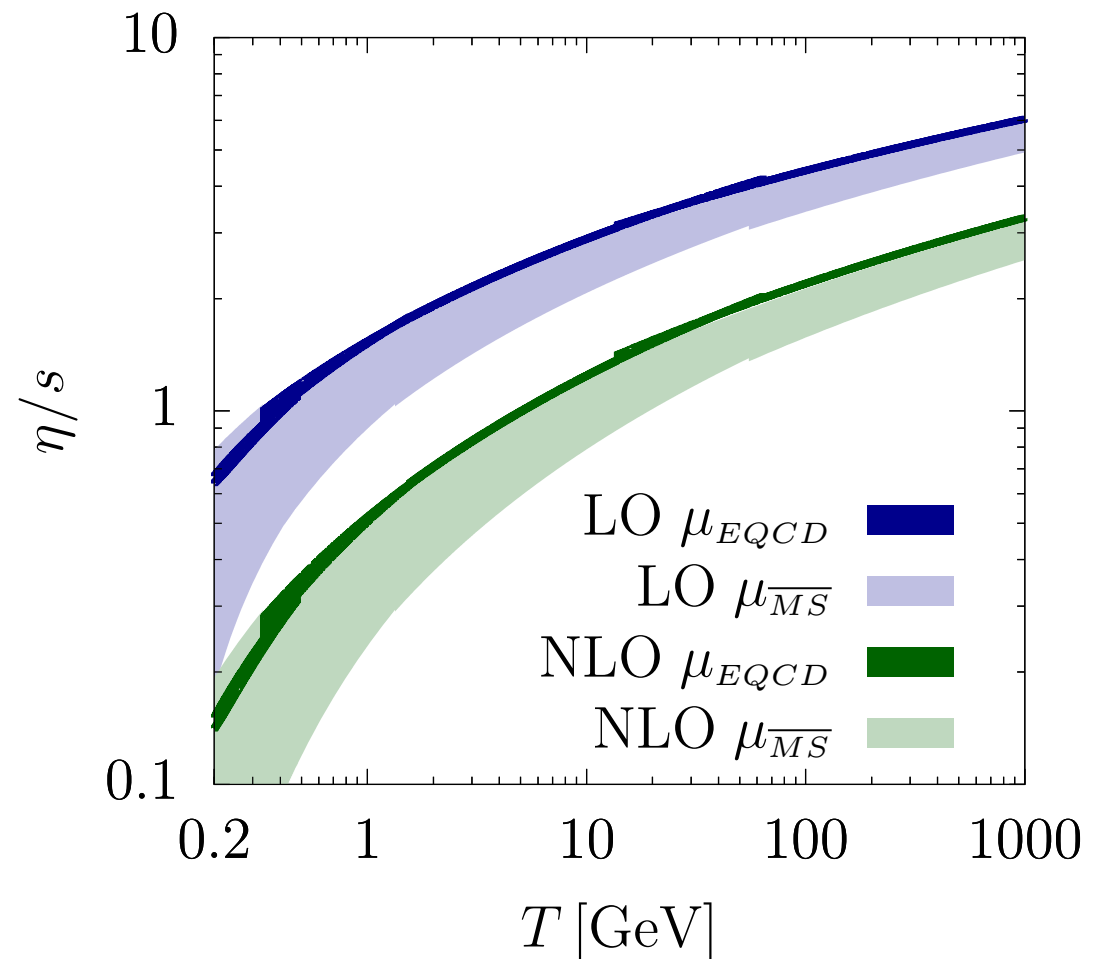
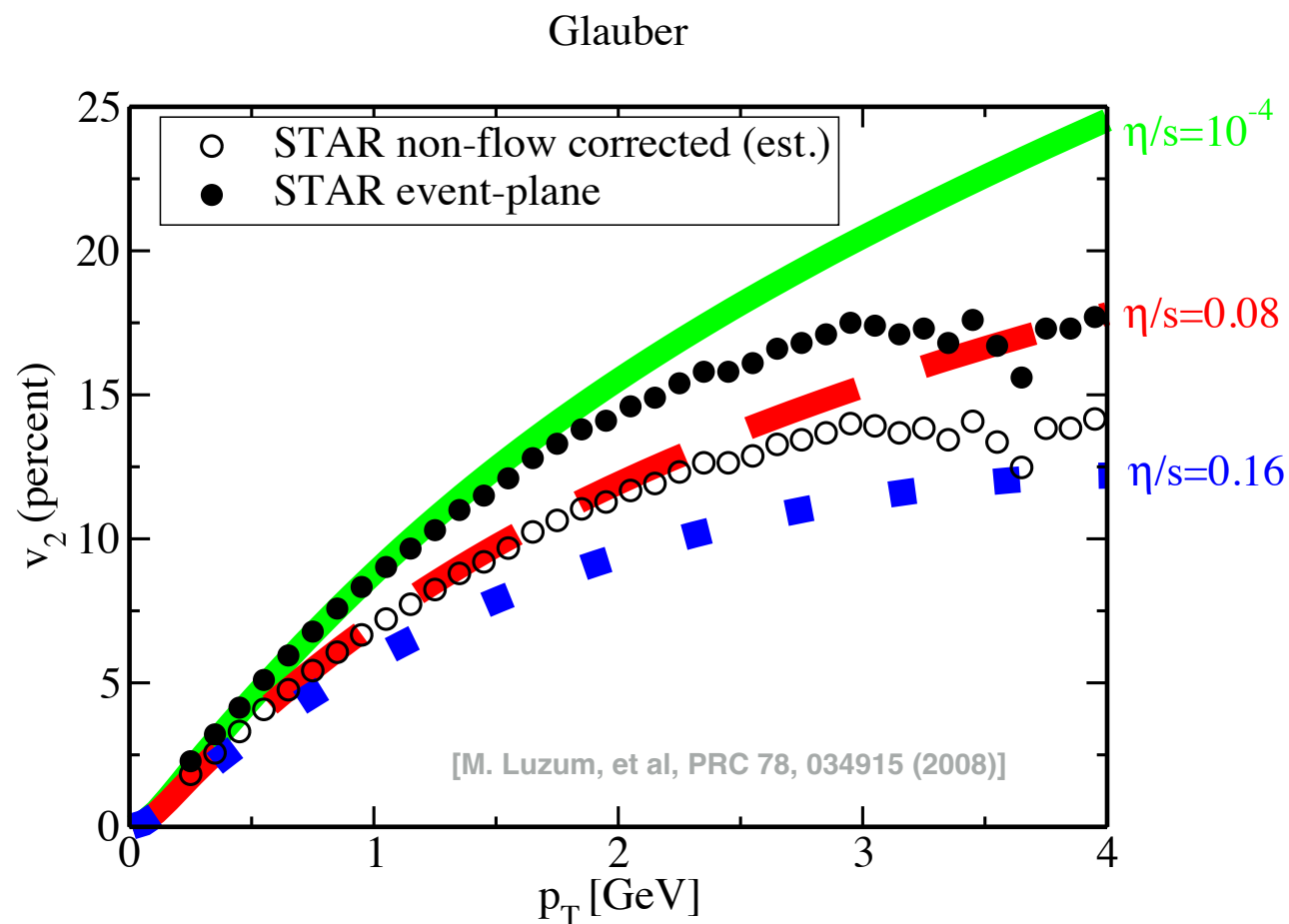
- Shear and bulk viscosity: the coefficients in the expansion of the EMT in terms of the velocity fields:

$$T^{\mu\nu} = -pg^{\mu\nu} + (\epsilon + p)u^\mu u^\nu + \Delta T^{\mu\nu}$$

$$\Delta T^{\mu\nu} = \eta(\Delta^\mu u^\nu + \Delta^\nu u^\mu) + \left(\frac{2}{3}\eta - \zeta\right)H^{\mu\nu}\partial_\rho u^\rho$$



Need for non-perturbative viscosities



[J. Ehiglieri, G. Moore, D. Teaney, JHEP 03 (2018) 179]

- High sensitivity of transport/phenomenological models to viscosity inputs
- Transport phenomena lie intrinsically in the non-perturbative regime
- Lattice QCD provides essential first-principles input

Introduction to lattice QCD

LQCD is designed for non-perturbative physics: hadron structure, QCD vacuum, hadron spectrum, **transport physics**, ...

$$S_F = \int d^4x \bar{\psi} [(\partial_\mu + iA_\mu)\gamma^\mu + m] \psi$$

$$S_G[C] = \frac{\beta}{3} \sum_{n \in \Lambda} \sum_{\mu \leq \nu} \text{Re Tr} [\mathbf{1} - U_{\mu\nu}(n)]$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \Pi_f \mathcal{D}[\psi] \mathcal{D}[\bar{\psi}] \mathcal{O} \exp(-S_{\text{LQCD}})$$

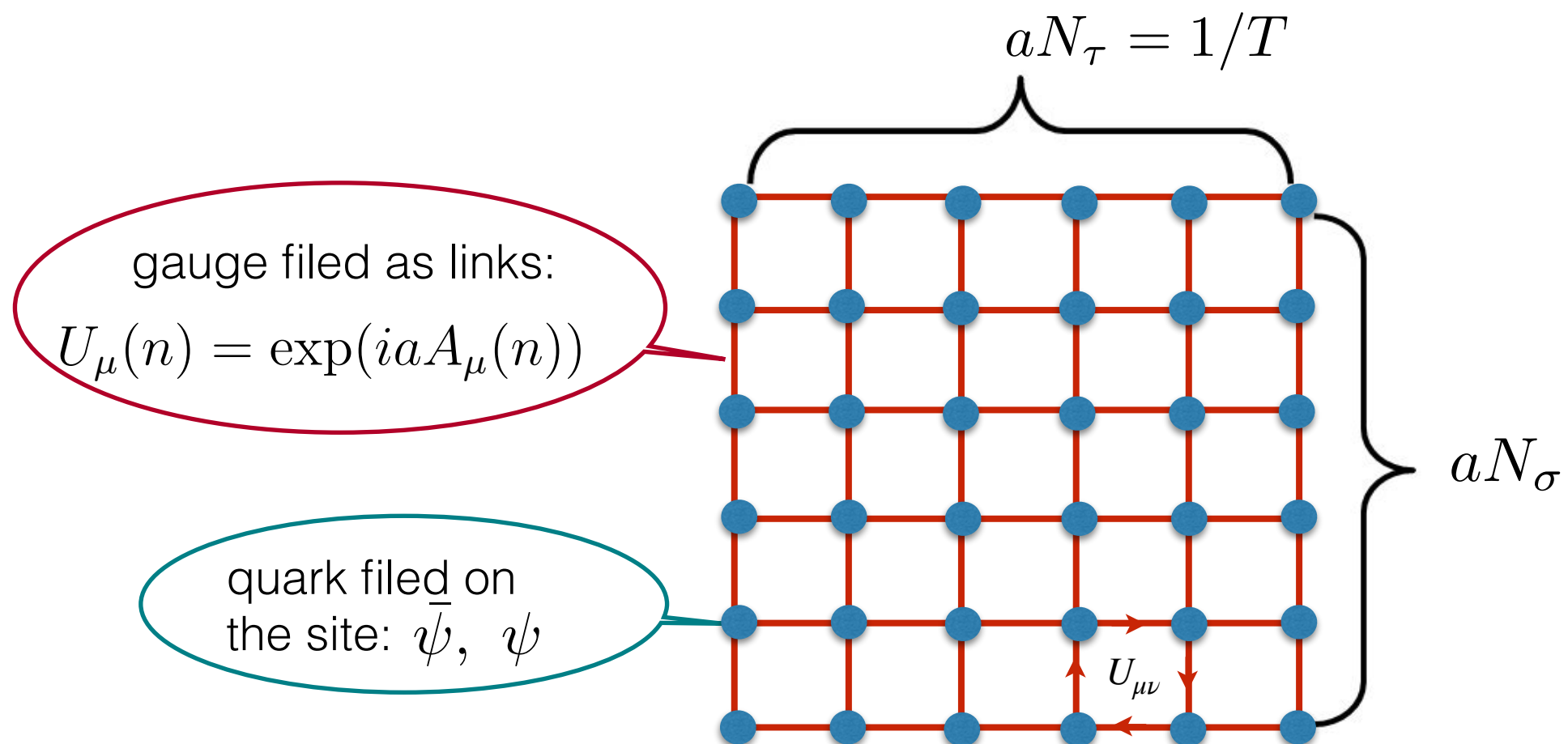
Diagram illustrating the components of the LQCD path integral:

- Partition function (points to Z)
- Path integral (points to $\int$)
- Gauge field (points to $\mathcal{D}[U]$)
- Quark field (points to $\mathcal{D}[\psi]$)
- Anti-quark field (points to $\mathcal{D}[\bar{\psi}]$)
- LQCD action (points to S_{LQCD})

LQCD as high-dimension numerical integration

- Basic quantities in LQCD:
 - * lattice volume
 - * lattice spacing
 - * gauge field SU(3) matrix
 - * quark field Dirac spinor (vector)
 - * quark mass

- 4 dimensional grid (lattice)
 - quarks live on lattice sites
 - gluons live on the links
- Typical sizes $24^3 \times 6$ to 256^4
 - parallelization over lattice sites



Ingredients of LQCD calculations

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \Pi_f \det[D_f] \mathcal{O} e^{-S_G[U]} \quad Z = \int \mathcal{D}[U] \Pi_f \det[D_f] e^{-S_G[U]}$$

Importance sampling

$$dP(U) = \frac{\mathcal{D}[U] \Pi_f \det[D_f] e^{-S_G[U]}}{Z}$$

$$\langle \mathcal{O} \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \mathcal{O}[U_n]$$

Gauge configurations



Procedures of LQCD calculations:

- Generation of gauge configurations
- Measure operators on the configurations
- Extract physical observables

$\Longleftrightarrow$

$\Longleftrightarrow$

$\Longleftrightarrow$

Experimental studies:

- Accelerator
- Detector
- Data analysis

Typical costs of a LQCD calculation

- Meson two-point correlation function

$$\langle O_1 O_2 \rangle = \frac{\int \mathcal{D}[U] \det[D_u] \det[D_d] \det[D_s] e^{-S_G[U]} \times O_1(U) O_2(U)}{\int \mathcal{D}[U] \det[D_u] \det[D_d] \det[D_s] e^{-S_G[U]}}$$

[LA, DC, OK, GDM, **HTS**, PRD 109 (2024) 11, 114505]

a (fm)	a^{-1} (GeV)	N_σ	N_τ	β	T/T_c	No. configuration
0.0215	9.187	80	20	7.0350	1.4734	10000
0.0178	11.11	96	24	7.1920	1.4848	10000
0.0140	14.14	120	30	7.3940	1.5118	10000
0.0117	16.88	144	36	7.5440	1.5042	10000

$$U_\mu(n)$$

$$3 \times 3 \times 2 \times 8 \text{ Byte} \times 4 \times 144^3 \times 36 = 58 \text{ GB}$$

$$D_f = m_f + (\partial_\mu + ig A_\mu^a t^a) \gamma^\mu$$

$$(3 \times 4) \times (3 \times 4) \times 2 \times 8 \text{ Byte} \times 144^3 \times 36 = 231 \text{ GB}$$



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Humongous costs needed to compute the fermion Dirac matrix !

Transport coefficient from LQCD: Kubo formula

Example: HQ diffusion coefficient

$$A \rightarrow J^\mu = \bar{\psi} \gamma^\mu \psi$$

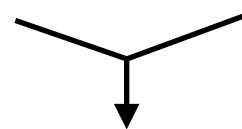
Legevin/diffusion dynamics

Non-equilibrium $\rightarrow$ Relaxation $\rightarrow$ Equilibrium

$$\langle A(\mathbf{x}) \rangle_{\text{eq}} = 0 \quad \partial_t \langle A(\mathbf{x}, t) \rangle = D \nabla^2 \langle A(\mathbf{x}, t) \rangle$$

Solution:

$$\langle A(\mathbf{k}, \omega) \rangle = \frac{i}{\omega + iD\mathbf{k}^2} \langle A(\mathbf{k}, t=0) \rangle$$



Kubo formula: $G_R(\mathbf{k}, \omega) = \frac{iD\mathbf{k}^2}{\omega + iD\mathbf{k}^2} \chi_q(\mathbf{k}) \quad \rho(\mathbf{k}, \omega) = \text{Im} G_R(\mathbf{k}, \omega)$

Linear response theory

Perturbation to Hamiltonian:

$$H(t) = H_0 - \int d\mathbf{x} A(\mathbf{x}) h(x) e^{\epsilon t} \Theta(-t)$$

Solution:

$$\frac{\partial}{\partial t} \left(\delta \langle A(\mathbf{k}, t=0) \rangle \right) = - \frac{G_R(\mathbf{k}, t)}{\chi_q(\mathbf{k})} \delta \langle A(\mathbf{k}, 0) \rangle$$

Transport coefficient can be determined from the transport peak of the spectral function (SPF)!

Spectral function from Euclidean correlation

- Spectral function and retarded correlation function

$$G^R(\omega) = \int_{-\infty}^{+\infty} \frac{d\omega'}{\pi} \frac{\rho(\omega')}{\omega' - \omega - i\eta}$$

- Lattice-computable Euclidean correlation function in imaginary time

$$G^E(\tau, \vec{p}) = \sum_{x,y,z} \exp(-i\vec{p} \cdot \vec{x}) \langle J_H(\tau, \vec{x}) J_H^\dagger(0, \vec{0}) \rangle$$

- Fourier transform of the Euclidean correlation function with analytic continuation $\tau = it$

$$G^E(i\omega_n) = \int_0^{1/T} d\tau e^{i\omega_n \tau} \int \frac{d\omega}{2\pi} e^{-\omega \tau} G^+(\omega)$$

- SPF as commutation of the mesonic currents

$$\begin{aligned} \rho(\omega) &= \int_{-\infty}^{+\infty} dt e^{i\omega t} \int d^3\mathbf{x} \left\langle \frac{1}{2} [J(\mathbf{x}, t), J(\mathbf{0}, 0)] \right\rangle \\ &= \frac{1}{2} [G^+(\omega) - G^-(\omega)] \end{aligned}$$

$$G^E(\tau) = \int \frac{d\omega}{\pi} K(\omega, \tau) \rho(\omega)$$

$$K(\omega, \tau, T) \equiv \frac{\cosh(\omega(\tau T - 1/2))}{\sinh(\omega/2T)}$$

(Anti-periodic boundary)

A way of extracting SPF from Euclidean correlators !

Various transport coefficients

- Kubo formula: Euclidean correlation functions $\longleftrightarrow$ spectral function

$$G_X(\tau) = \int d\vec{x} \langle \hat{\mathcal{O}}_X(\tau, \vec{x}) \hat{\mathcal{O}}_X(0, \vec{0}) \rangle = \int_0^\infty \frac{d\omega}{\pi} K(\omega, \tau, T) \rho_X(\omega, T)$$

- Transport coefficients encoded in the transport peak of the SPFs

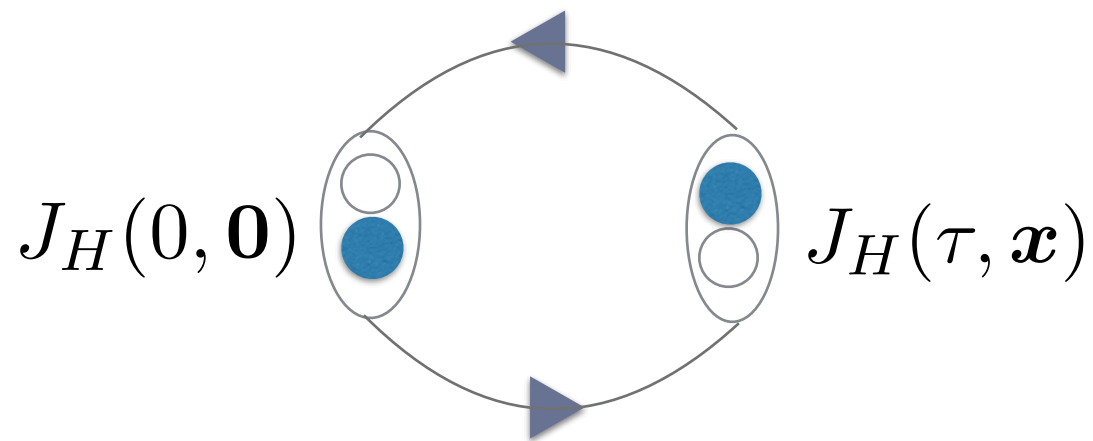
$$\text{Transport Coefficient} \propto \lim_{\omega \rightarrow 0} \frac{\rho_X(\omega)}{\omega}$$

$$D_s = \frac{1}{3\chi_q} \lim_{\omega \rightarrow 0} \frac{\rho_{ii}(\omega)}{\omega} \iff \hat{\mathcal{O}}_X = J^\mu$$

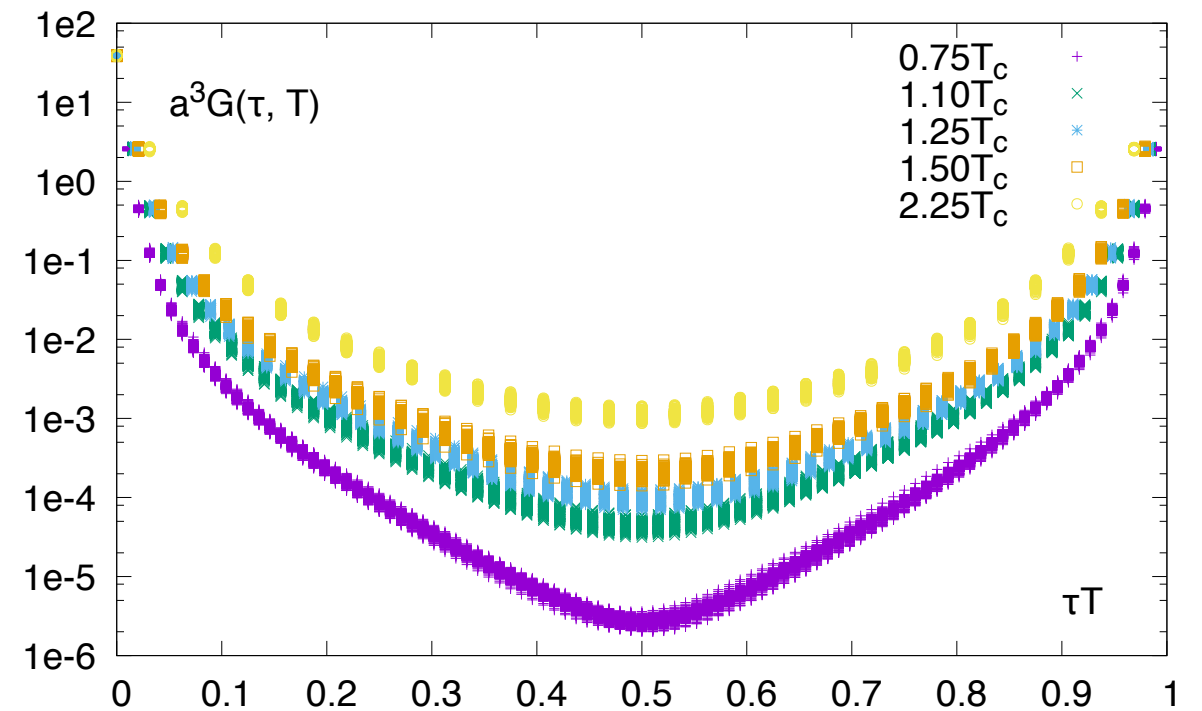
$$\eta = \lim_{\omega \rightarrow 0} \frac{\rho_{12,12}(\omega)}{\omega} \iff \hat{\mathcal{O}}_X = T_{12}$$

$$\zeta = \lim_{\omega \rightarrow 0} \frac{1}{9} \frac{\rho_{\theta,\theta}(\omega)}{\omega} \iff \hat{\mathcal{O}}_X = \theta$$

Computing charmonium 2pt on the lattice



$$J_H(x) = \bar{\psi}(x)\Gamma_H\psi(x)$$

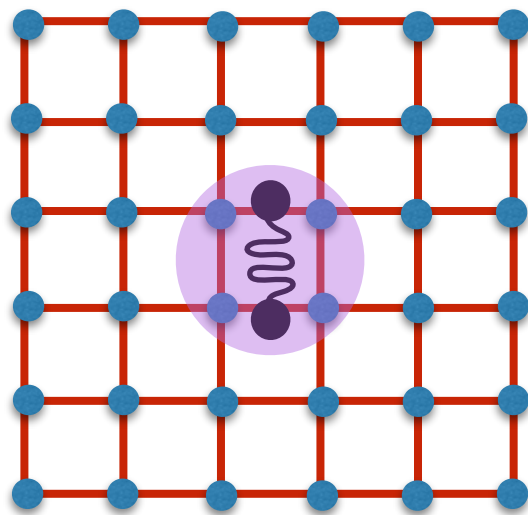


$$G_H(\tau, \vec{p}) = \sum_{x,y,z} \exp(-i\vec{p} \cdot \vec{x}) \langle J_H(0, \vec{0}) J_H^\dagger(\tau, \vec{x}) \rangle$$

- Euclidean 2pt correlation functions are typically available at only $\sim O(10)$ discrete time points.

Challenges of relativistic heavy flavor on the lattice

- Need high temperature to study thermal effects on heavy flavor
 $T = 1/(aN_\tau) \rightarrow$ small a with not-small N_τ
- Generic lattice quark action has discretization error in powers of am_Q
- Heavy flavor requires fine lattice for better resolution



Compton wavelength of heavy quarks: $\lambda_Q \sim \frac{1}{m_Q}$

Small λ_Q requires fine a to resolve the dynamics

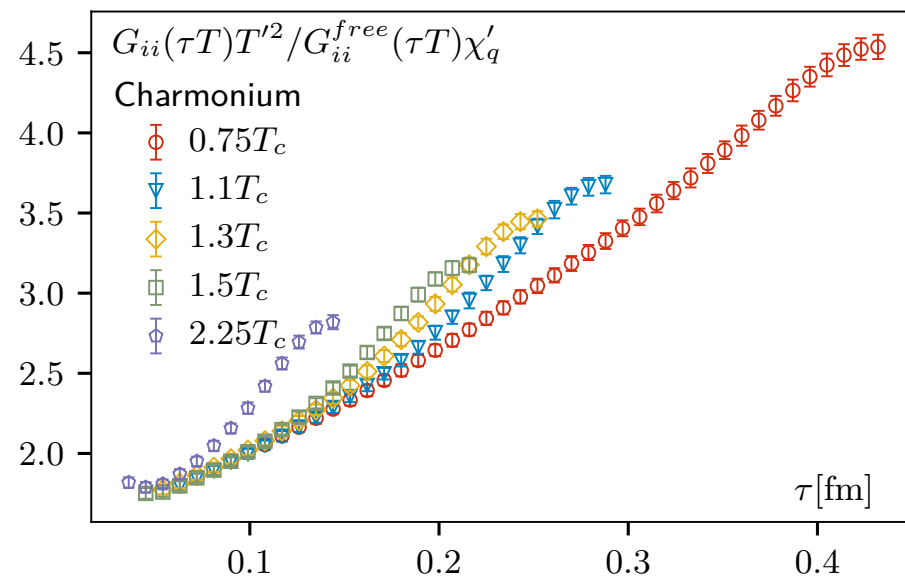
$N_\sigma^3 \times N_\tau$	β	a [fm]	T [MeV]	am_s
$48^3 \times 48$	7.455	0.0560	73.4	0.0222
$48^3 \times 24$	7.455	0.0560	146.8	0.0222
$48^3 \times 20$	7.455	0.0560	176.2	0.0222
$48^3 \times 18$	7.455	0.0560	195.8	0.0222
$48^3 \times 16$	7.455	0.0560	220.3	0.0222
$64^3 \times 64$	7.780	0.0420	73.4	0.0160
$64^3 \times 24$	7.780	0.0420	195.7	0.0160

$$aM_c = 0.042 \text{ fm} \times 1.273 \text{ GeV} \times 5.068 = 0.27 \ll 1$$

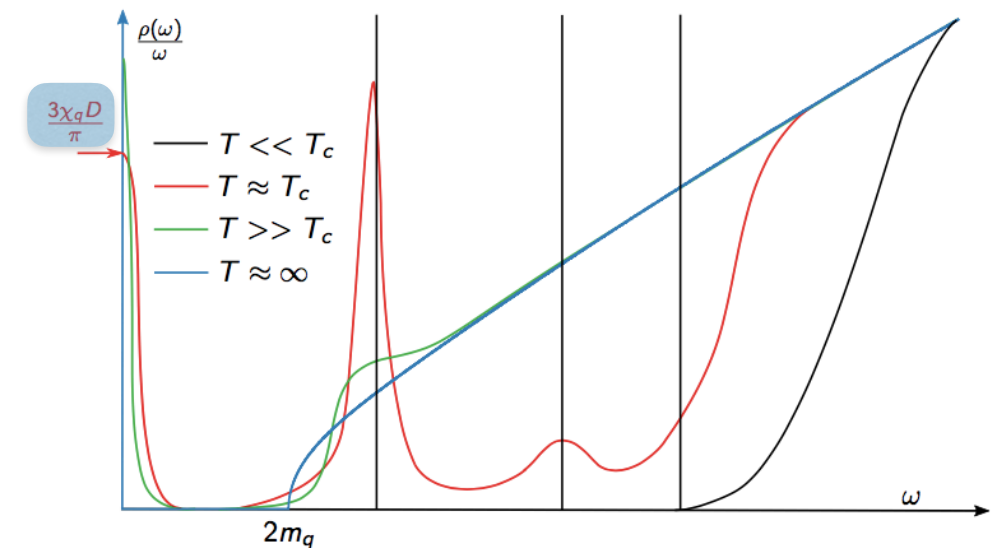
- Large & fine ensembles in generation by Wuhan LQCD group
- Discretization error of charm quarks under control
- LQCD study of relativistic charmonium on the way

Extracting transport coefficients from SPF

Temporal correlation function relates to spectral function:



meson correlators



meson SPF

ill-posed!

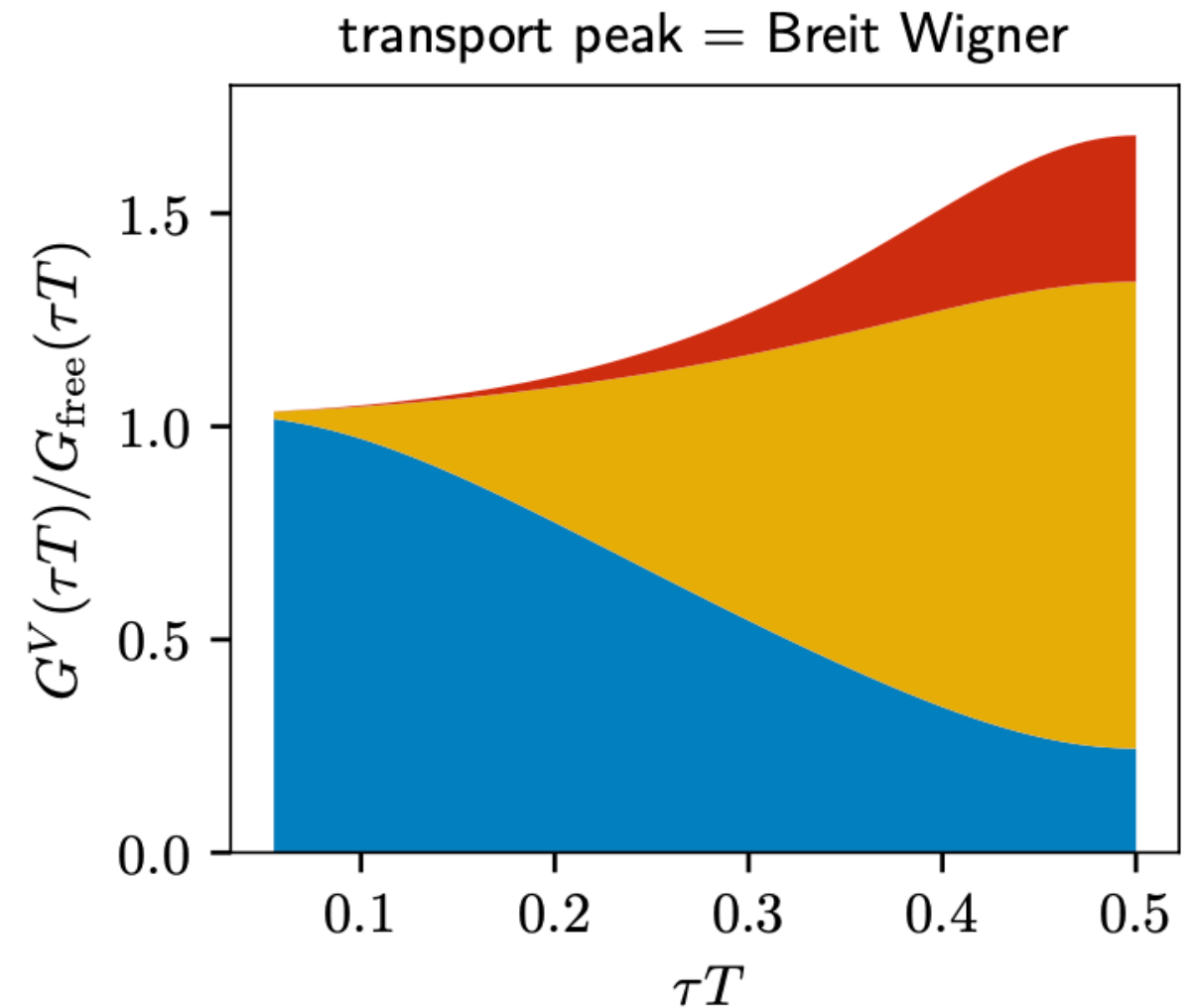
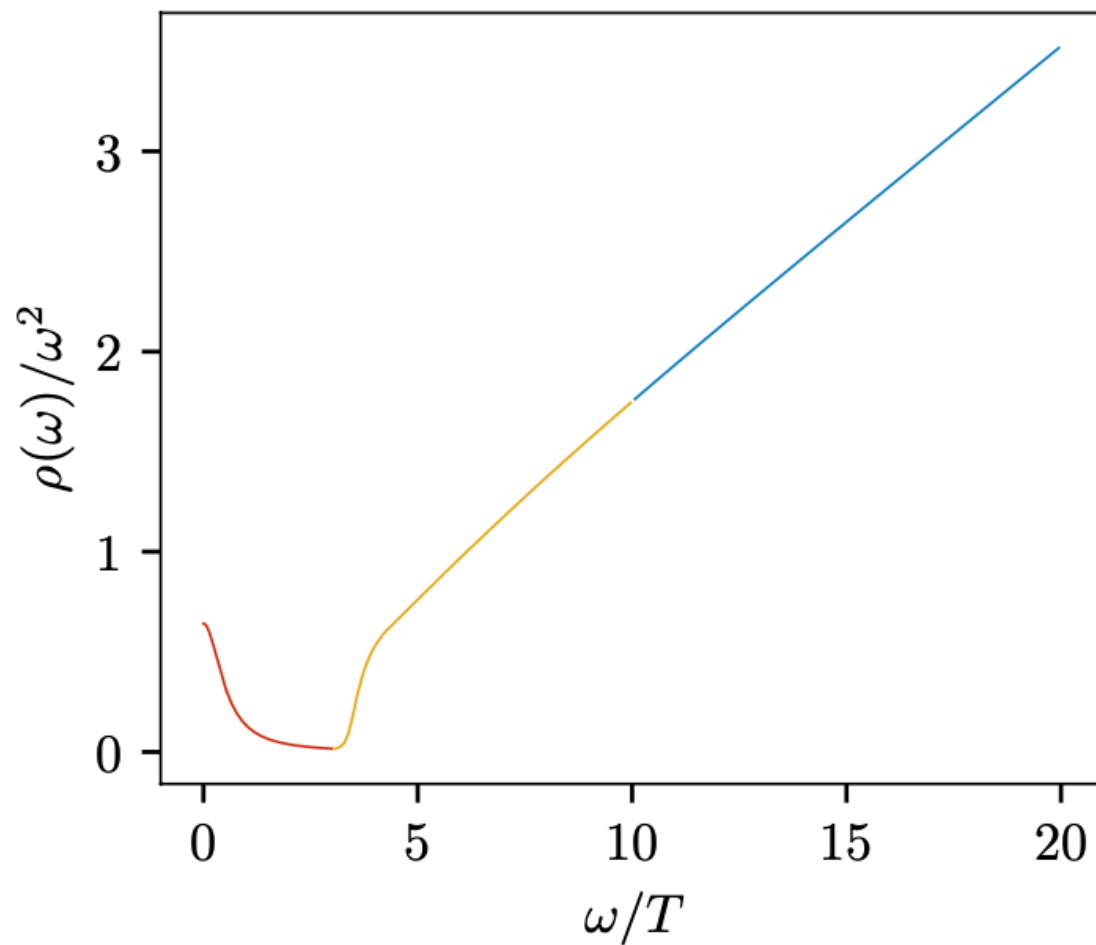
$$G_H(\tau, \vec{p}) = \int \frac{d\omega}{\pi} K(\omega, \tau, T) \rho_H(\omega, \vec{p}, T)$$

discretized
 $\sim O(10)$

continuous
 $\sim O(1000)$

- Need high precision lattice data to constrain the SPF
- Need to tackle the inverse problem

Challenges in reconstructing the transport peak



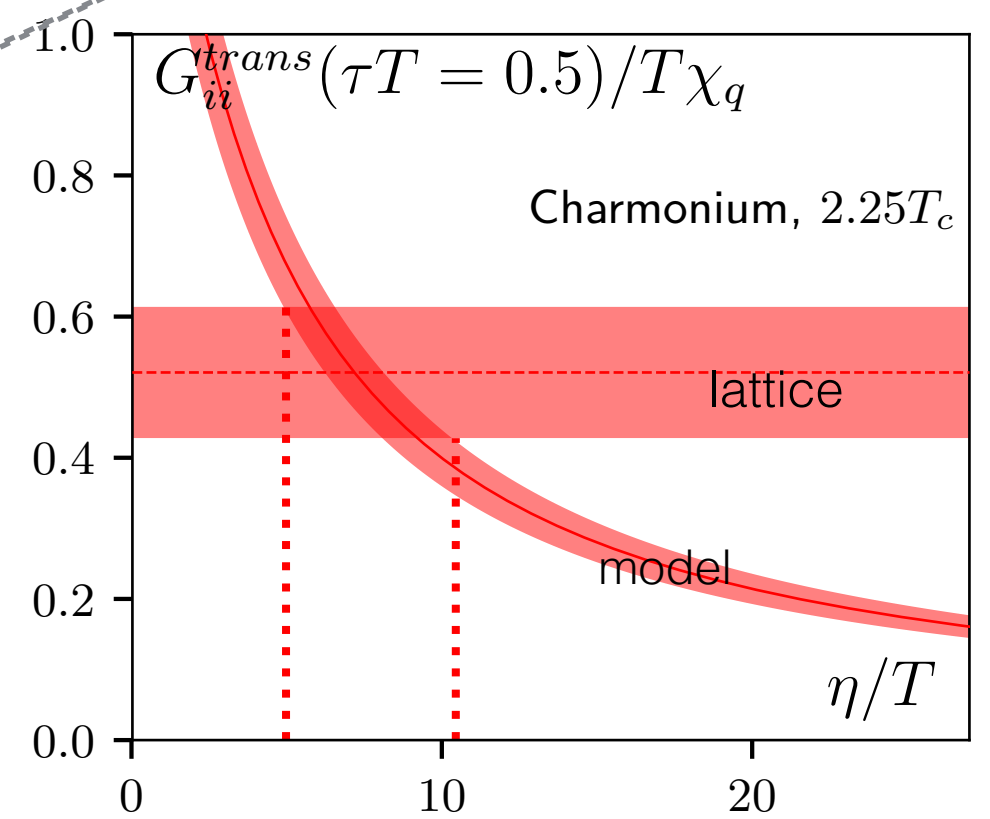
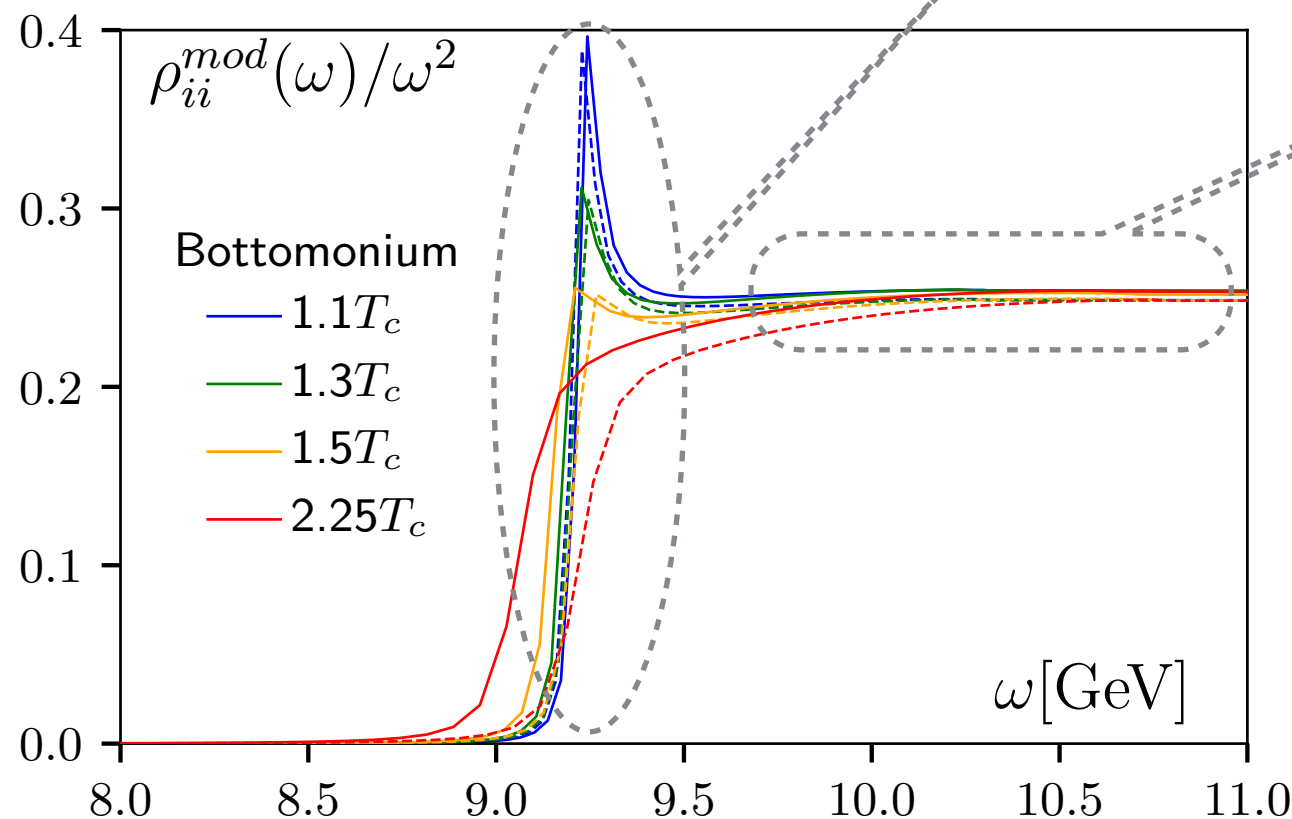
$$G_H(\tau, \vec{p}) = \int \frac{d\omega}{\pi} K(\omega, \tau, T) \rho_H(\omega, \vec{p}, T) \quad K(\omega, \tau, T) \equiv \frac{\cosh(\omega(\tau T - 1/2))}{\sinh(\omega/2T)}$$

Transport peak sensitive to large τT

Quarkonium SPF (in quenched approximation)

- pNRQCD calculations applicable around the threshold [M. Laine, JHEP05(2007)028]
- Ultraviolet asymptotics valid well above the threshold [Y. Burnier et al., EPJC72, 1902(2012)]
- Combine two parts by interpolation: [Y. Burnier et al., JHEP11(2017)206]

$$\rho_V^{pert}(\omega) = A^{match} \Phi(\omega) \rho_V^{pNRQCD}(\omega) \theta(\omega^{match} - \omega) + \rho_V^{vac}(\omega) \theta(\omega - \omega^{match})$$



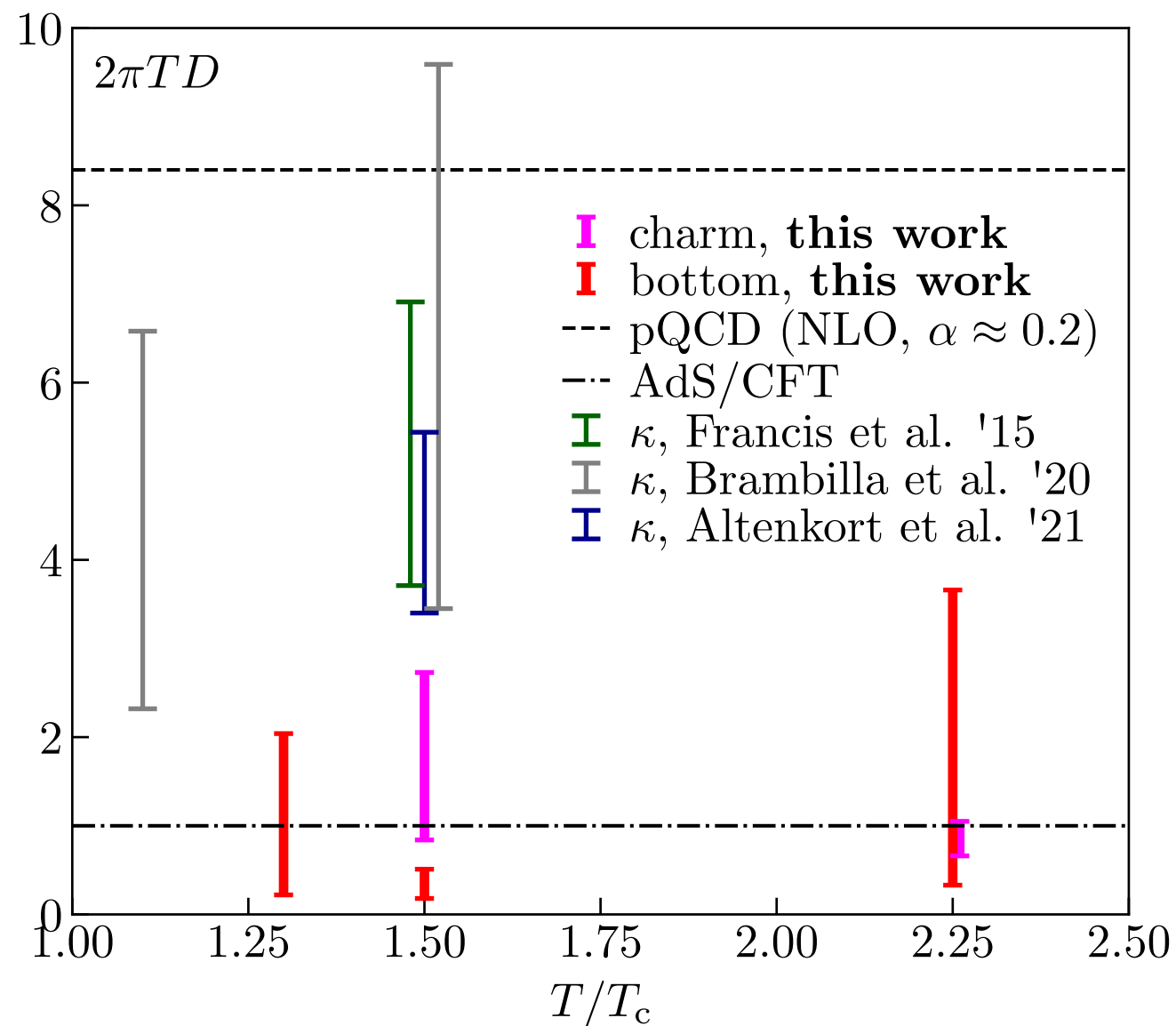
[HTD, OK, AL, HO, HS, 舒海涛, PRD104(2021) 11, 114508]

$$\rho_{ii}(\omega) = \rho_{ii}^{trans}(\omega) + \rho_{ii}^{mod}(\omega)$$

$$\rho_{ii}^{trans}(\omega) = 3\chi_q D \frac{\omega \eta^2}{\omega^2 + \eta^2}$$

$$G(\tau, T) = \int \frac{d\omega}{\pi} \frac{\cosh(\omega(\tau T - 1/2)/T)}{\sinh(\omega/(2T))} \rho(\omega, T)$$

Heavy quark diffusion from meson correlators



[HTD, OK, AL, HO, HS, 舒海涛, PRD104(2021) 11, 114508]

- Still quenched approximation (but in the continuum limit)
- Extension to full QCD on the way

Heavy quark diffusion via HQEFT

- Langevin equations of heavy quark motion

$$\partial_t p_i = -\eta_D p_i + \xi_i(t) \quad \langle \xi_i(t) \xi_j(t') \rangle = \kappa \delta_{ij} \delta(t - t')$$

- Mass dependent **momentum** diffusion coefficient

$$\kappa^{(M)} \equiv \frac{M^2 \omega^2}{3T \chi_q} \sum_i \frac{2T \rho_V^{ii}(\omega)}{\omega} \Big|_{\eta \ll |\omega| \lesssim \omega_{UV}}$$

- Large quark mass limit in HQ effective field theory

$$\kappa \equiv \frac{\beta}{3} \sum_{i=1}^3 \lim_{\omega \rightarrow 0} \left[\lim_{M \rightarrow \infty} \frac{M^2}{\chi_q} \int_{-\infty}^{\infty} dt e^{i\omega(t-t')} \int d^3 \vec{x} \left\langle \frac{1}{2} \{ \mathcal{F}^i(t, \vec{x}), \mathcal{F}^i(0, \vec{0}) \} \right\rangle \right]$$

J. Casalderrey-Solana and D. Teaney, PRD 74, 085012

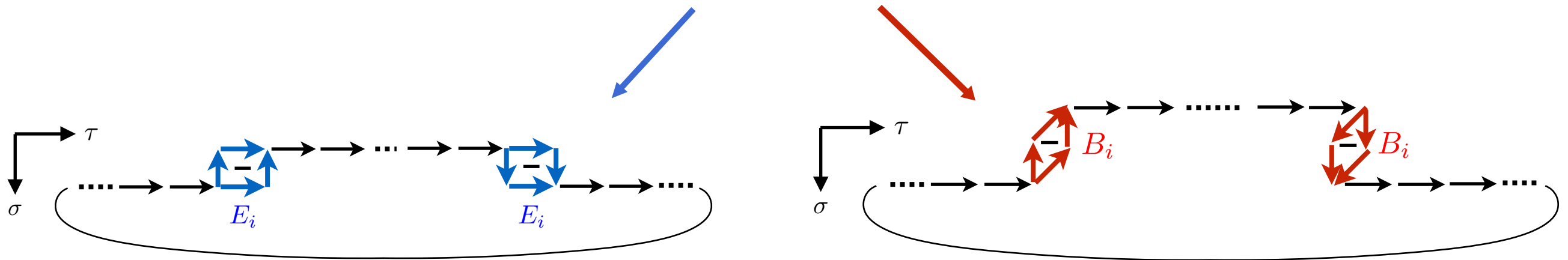
S. Caron-Huot et al., JHEP 0904 (2009) 053

A. Bouteffeux, M. Laine, JHEP 12 (2020) 150

$$\partial_t \mathbf{p} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = \mathbf{F}$$

Heavy quark mass limit and finite mass correction

$$\partial_t \mathbf{p} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = \mathbf{F}$$



$$G(\tau, T) = \int \frac{d\omega}{\pi} \frac{\cosh(\omega(\tau T - 1/2)/T)}{\sinh(\omega/(2T))} \rho(\omega, T)$$

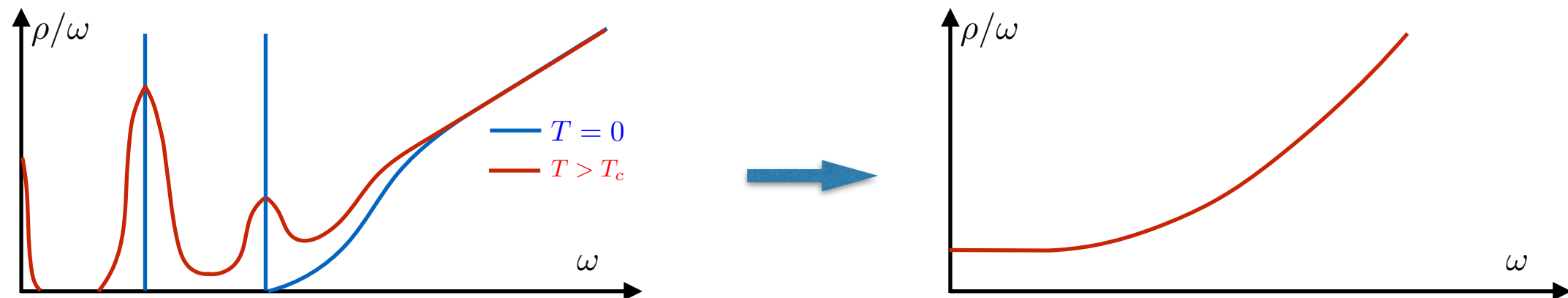
$$\frac{\kappa}{4\pi T^3} = \frac{1}{2\pi T^2} \lim_{\omega \rightarrow 0} \frac{\rho(\omega)}{\omega}$$

$$\kappa = \kappa_E + \frac{2}{3} \langle \mathbf{v}^2 \rangle \kappa_B \quad \longrightarrow \quad D_s = \frac{2T^2}{\kappa} \cdot \frac{\langle p^2 \rangle}{3MT}$$

$$2/3 \cdot \langle \mathbf{v}^2 \rangle_{\text{charm}} : 18\% \sim 30\%$$

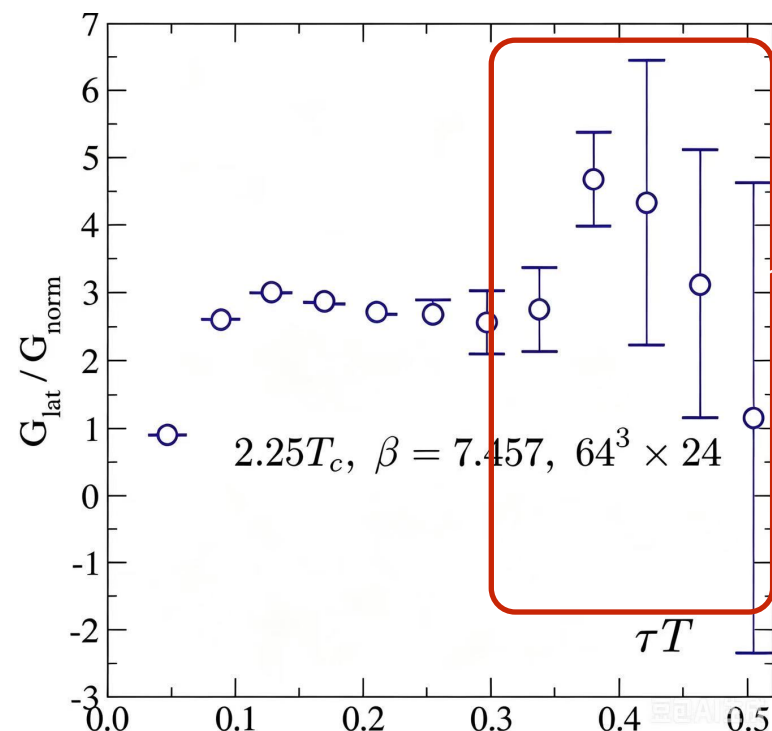
$$2/3 \cdot \langle \mathbf{v}^2 \rangle_{\text{bottom}} : 7\% \sim 13\%$$

Pros and cons via HQEFT



Advantages:

- Cheaper to measure on the lattice
- No peak structures in spectral functions
- Absence of transport peak



Desperate signal at large τT !

Challenges:

- Strong UV fluctuations in the correlators

Signal-to-noise ratio: a long-standing problem



Journal of High Energy Physics

A way to estimate the heavy quark thermalization rate from the lattice

To cite this article: Simon Caron-Huot *et al* JHEP04(2009)053

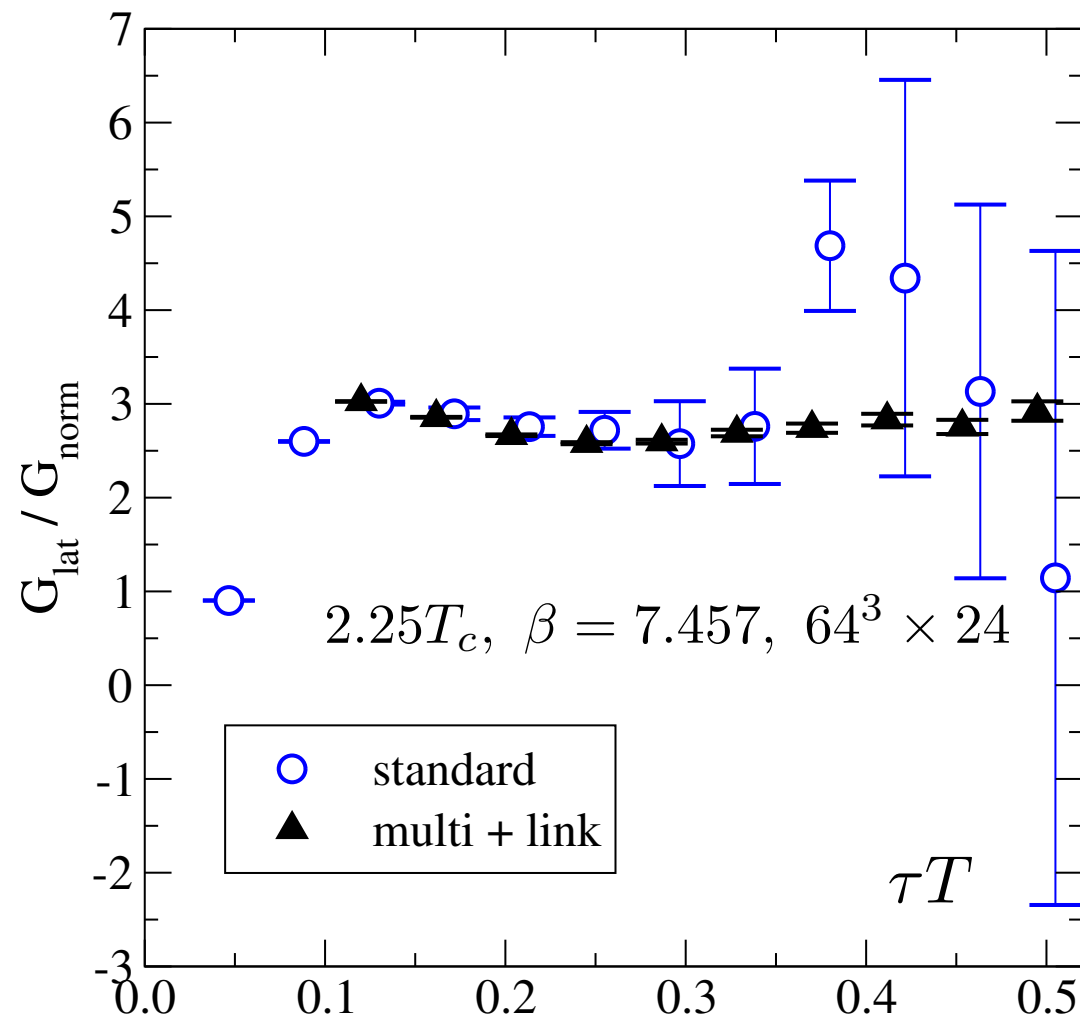
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Tomoki Hori, Takuya Mabuchi, Ikuya Kinefuchi et al.

A noise problem stuck for ~17 years!

Previous solution using multi-level algorithm



[A. Francis, et al, PRD92 (2015)116003]

ML: independent updates in each sub-lattice

[M. Luscher and P. Weisz, JHEP 09 (2001) 010]

A long history of lattice calculations, but all using **multi-level**:

[S. Caron-Huot, et al., JHEP 04 (2009) 053]

[D. Banerjee, et al., P.R.D 85 (2012) 014510]

[A. Francis, et al., PRD92 (2015)116003]

[D. Banerjee, et al., JHEP 08 (2022) 128]

[D. Banerjee, et al., Nucl.Phys.A.2023.122721]

[N. Brambilla, et al., PRD107 (2023) 054508]

- Multi-level algorithm reduces noise in correlators
- But only applicable in quenched approximation

Breaking down of Multi-level algorithm in QCD

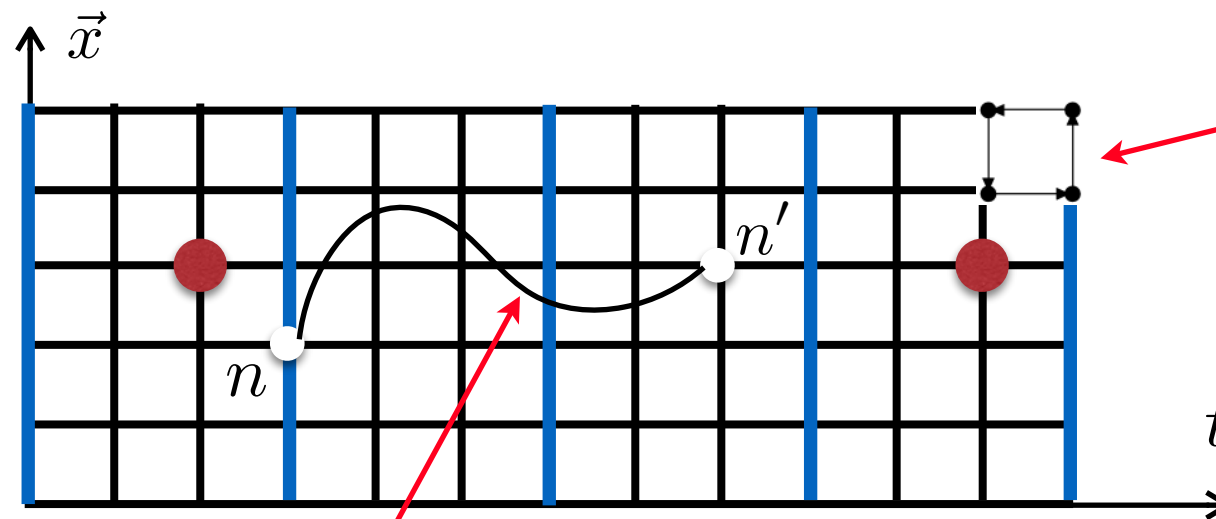
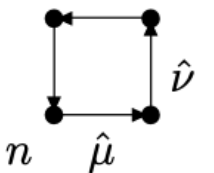
$$\mathcal{Z}(V, T) = \int [dU][d\psi][d\bar{\psi}] e^{-S_G[U] - S_F[U, \psi, \bar{\psi}]}$$

$$S_G[U] = \frac{1}{2g^2} \sum_{n, \mu, \nu} 2\text{Tr}[1 - P_{\mu\nu}(n)]$$

$$S_F[U, \psi, \bar{\psi}] = \bar{\psi} M_q[U] \psi$$

Action **local in quenched QCD** (sum of plaq.):

$$P_{\mu\nu}(n) = U_\mu(n) U_\nu(n + \hat{\mu}) U_\mu^\dagger(n + \hat{\nu}) U_\nu^\dagger(n) =$$



Action **non-local in full QCD**
(connection between any two sites):

$$M_q(n, n'; i, j)[U] = \hat{m}_q \delta_{n, n'} \delta_{ij} + \frac{1}{2} \sum_{\mu} \eta_{\mu}(n) \left((U_{\mu}(n))_{i, j} \delta_{n', n + \hat{\mu}} - (U_{\mu}^{\dagger}(n))_{i, j} \delta_{n, n' + \hat{\mu}} \right)$$

Smearing extendable to QCD — Gradient flow

Evolve fields according to diffusion equations:

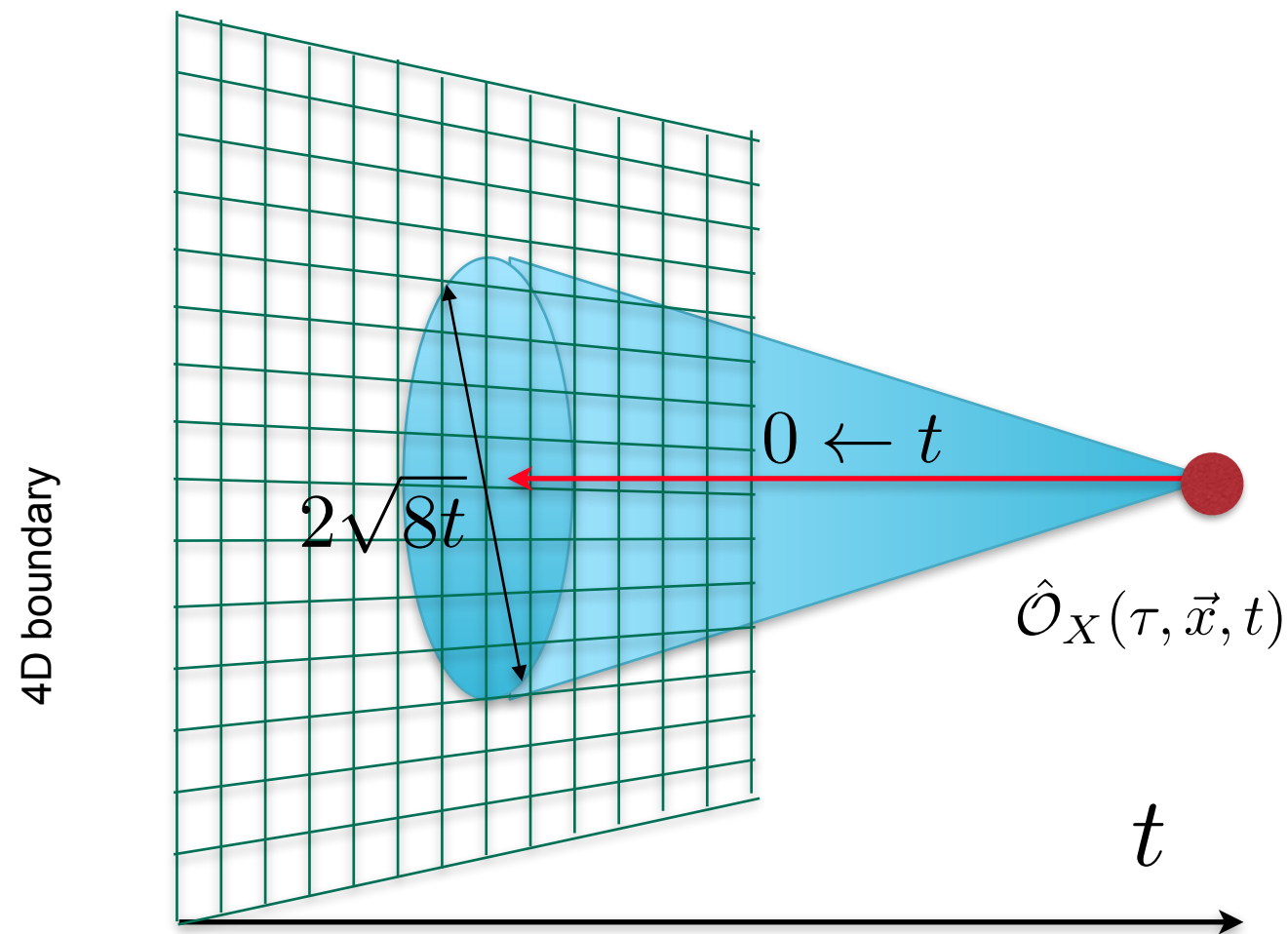
$$\frac{dB_\mu(x,t)}{dt} \sim -\frac{\delta S_G[B_\mu(x,t)]}{\delta B_\mu(x,t)} \sim D_\nu G_{\nu\mu}(x,t)$$

$$\partial_t \chi = \Delta \chi$$

$$\partial_t \bar{\chi} = \bar{\chi} \overleftarrow{\Delta}$$

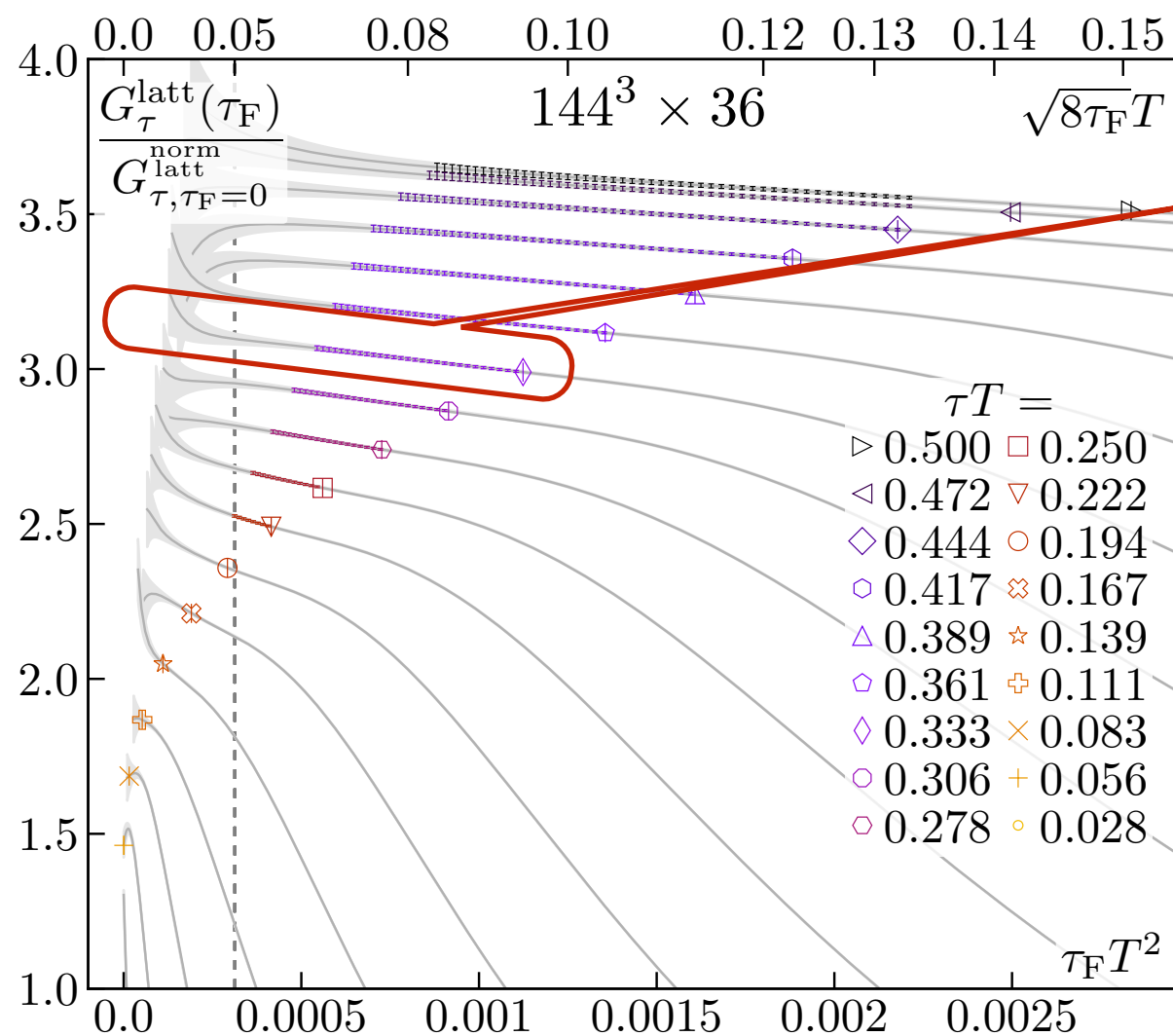
Luscher & Weisz, JHEP1102(2011)051

Narayanan & Neuberger, JHEP0603(2006)064

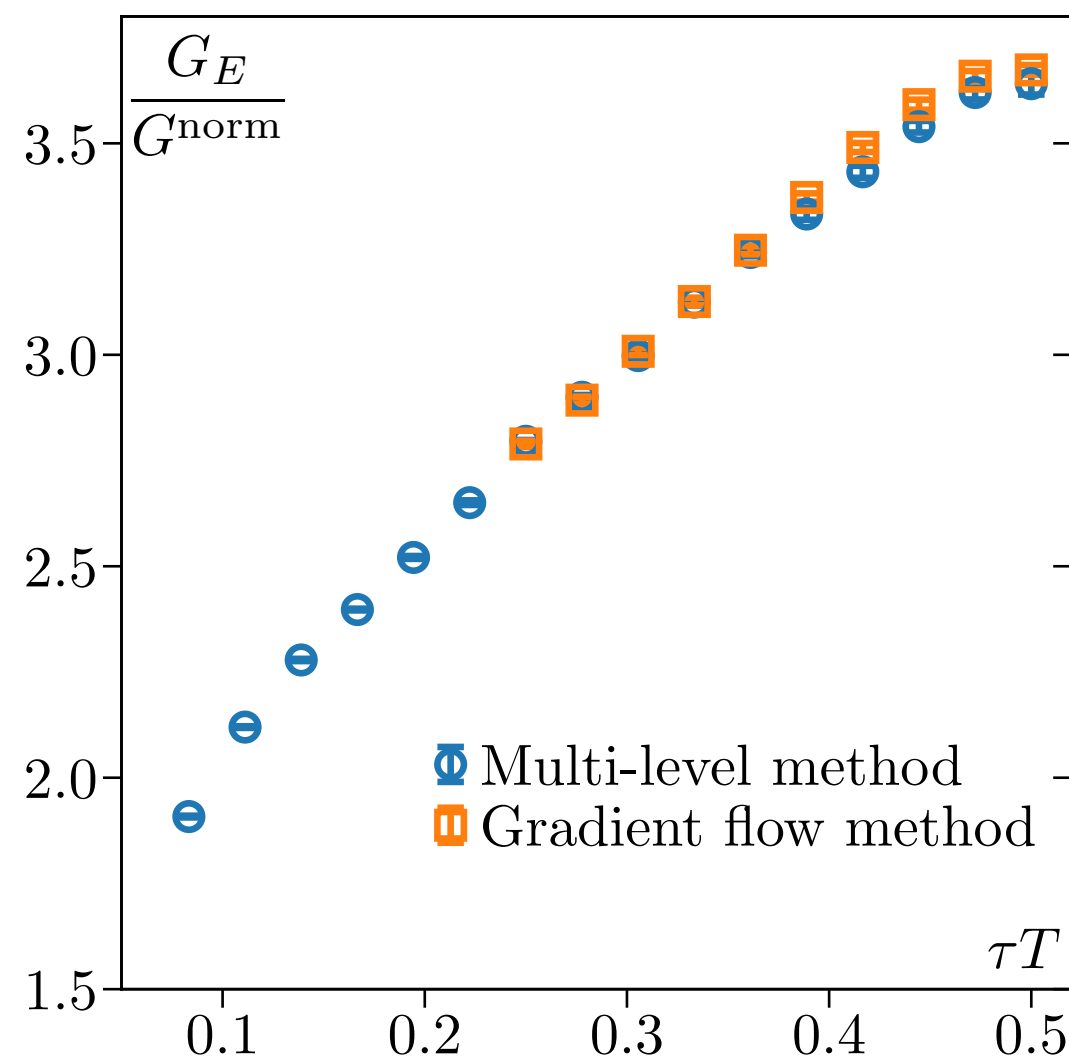


- Smear gauge field + quark field simultaneously

Flow smearing effects on the correlators



error bar
drops!

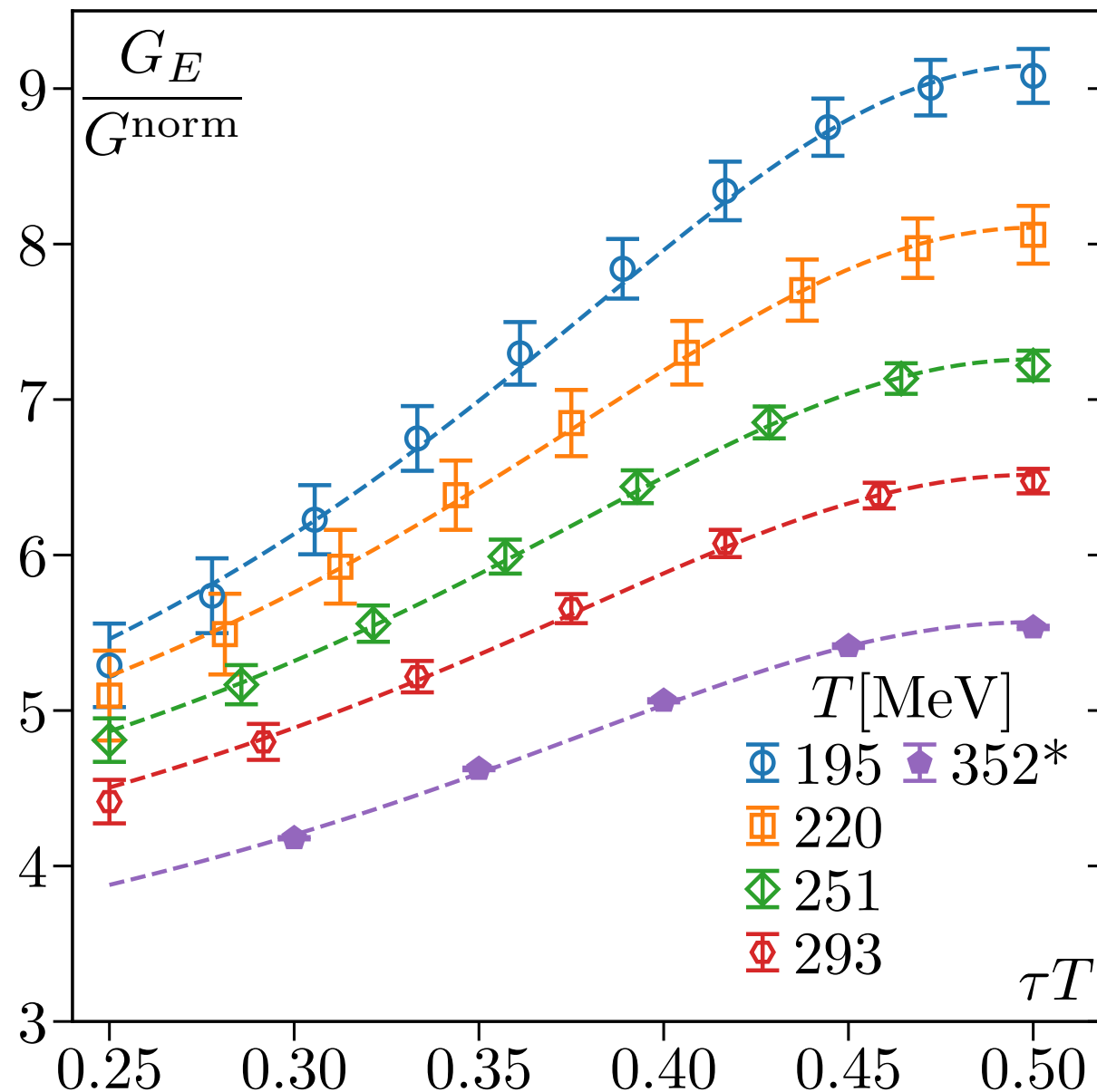


[LA, AME, OK, LM, GDM, 舒海涛, PRD 103 (2021) 1, 014511]

- Strong suppression of UV fluctuations under flow
- Well-defined renormalization
- Consistent quenched results from ML & GF

First full QCD calculation of EE correlators

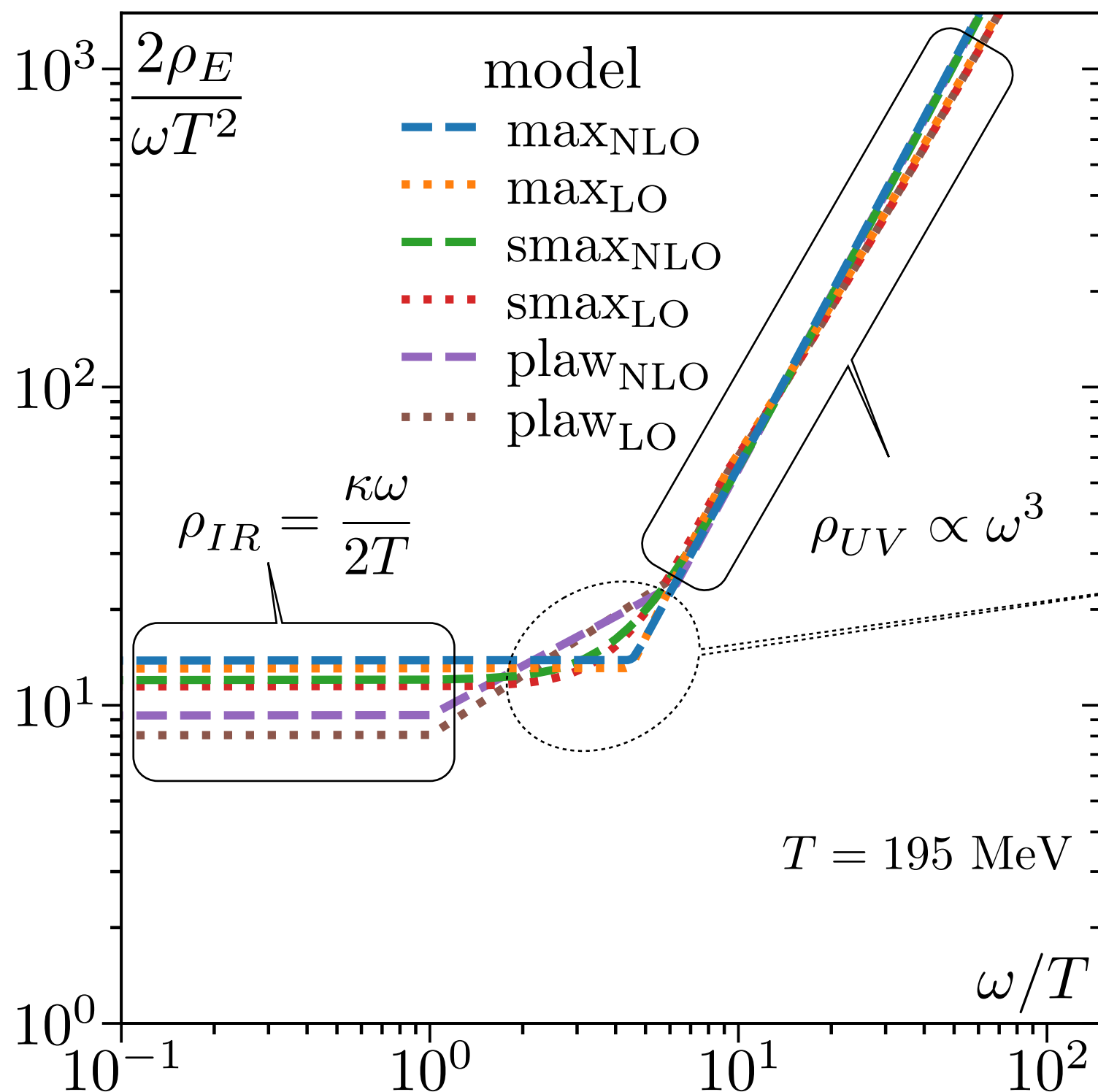
First full QCD calculation of κ , only possible via Gradient Flow!



- $195 \text{ MeV} \leq T \leq 352 \text{ MeV}$
- 2+1 flavor in the sea
- pion mass 320 MeV

[LA, OK, RL, SM, PP, 舒海涛, SS, PRL 130 (2023) 23, 231902]

Spectra analysis



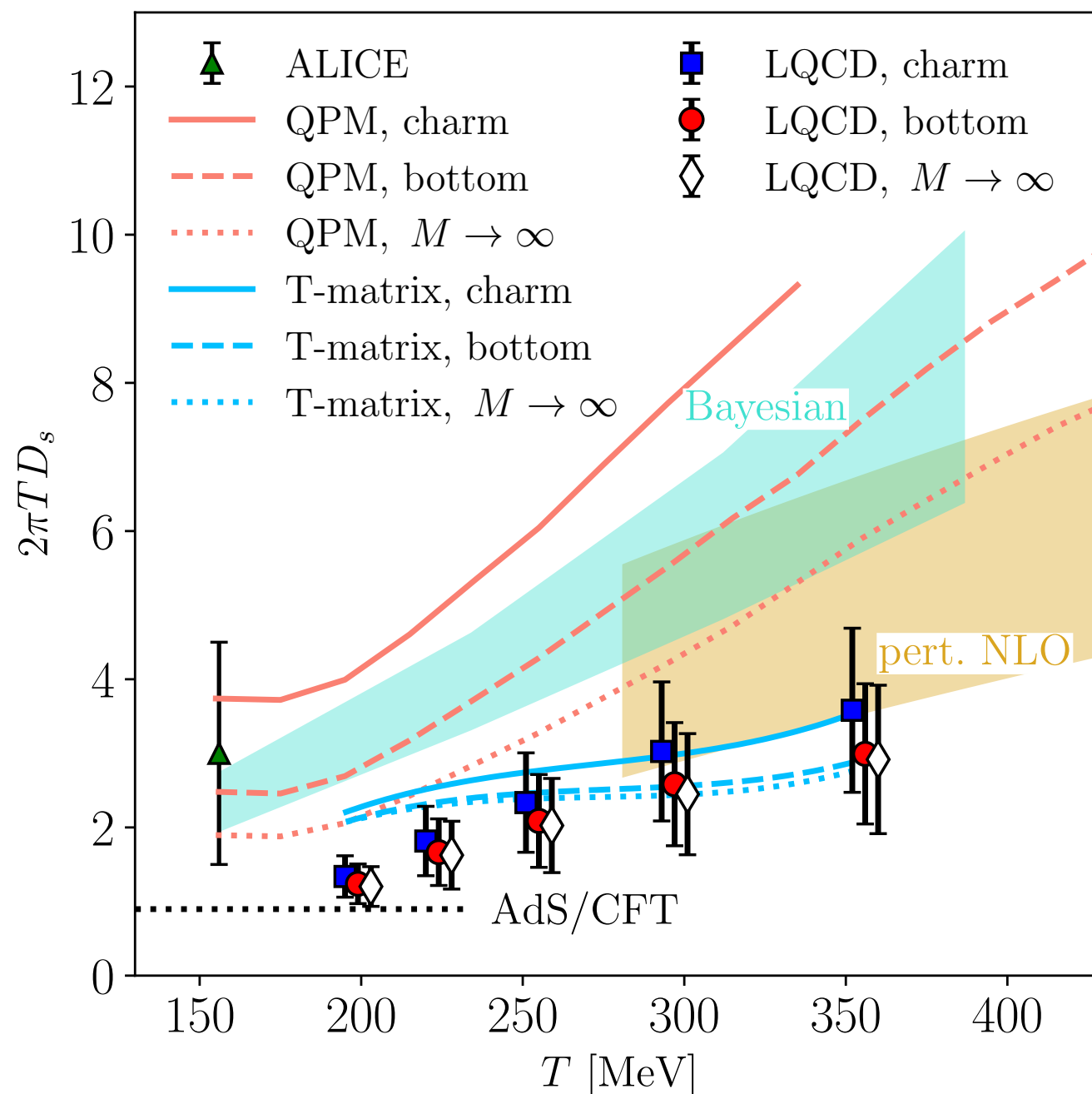
$$G(\tau, T) = \int \frac{d\omega}{\pi} K(\omega, \tau, T) \rho(\omega, T)$$

$$\left\{ \begin{array}{l} \rho_{\max} \equiv \max(\phi_{IR}, \phi_{UV}) \\ \rho_{\text{smax}} \equiv \sqrt{\phi_{IR}^2 + \phi_{UV}^2} \\ \rho_{\text{plaw}} \equiv \begin{cases} \phi_{IR} & \omega \leq \omega_{IR}, \\ a\omega^b & \text{for } \omega_{IR} < \omega < \omega_{UV}, \\ \phi_{UV} & \omega \geq \omega_{UV}, \end{cases} \end{array} \right.$$

[LA, OK, RL, SM, PP, 舒海涛, SS, PRL 130 (2023) 23, 231902]

Charm and Bottom quark diffusion

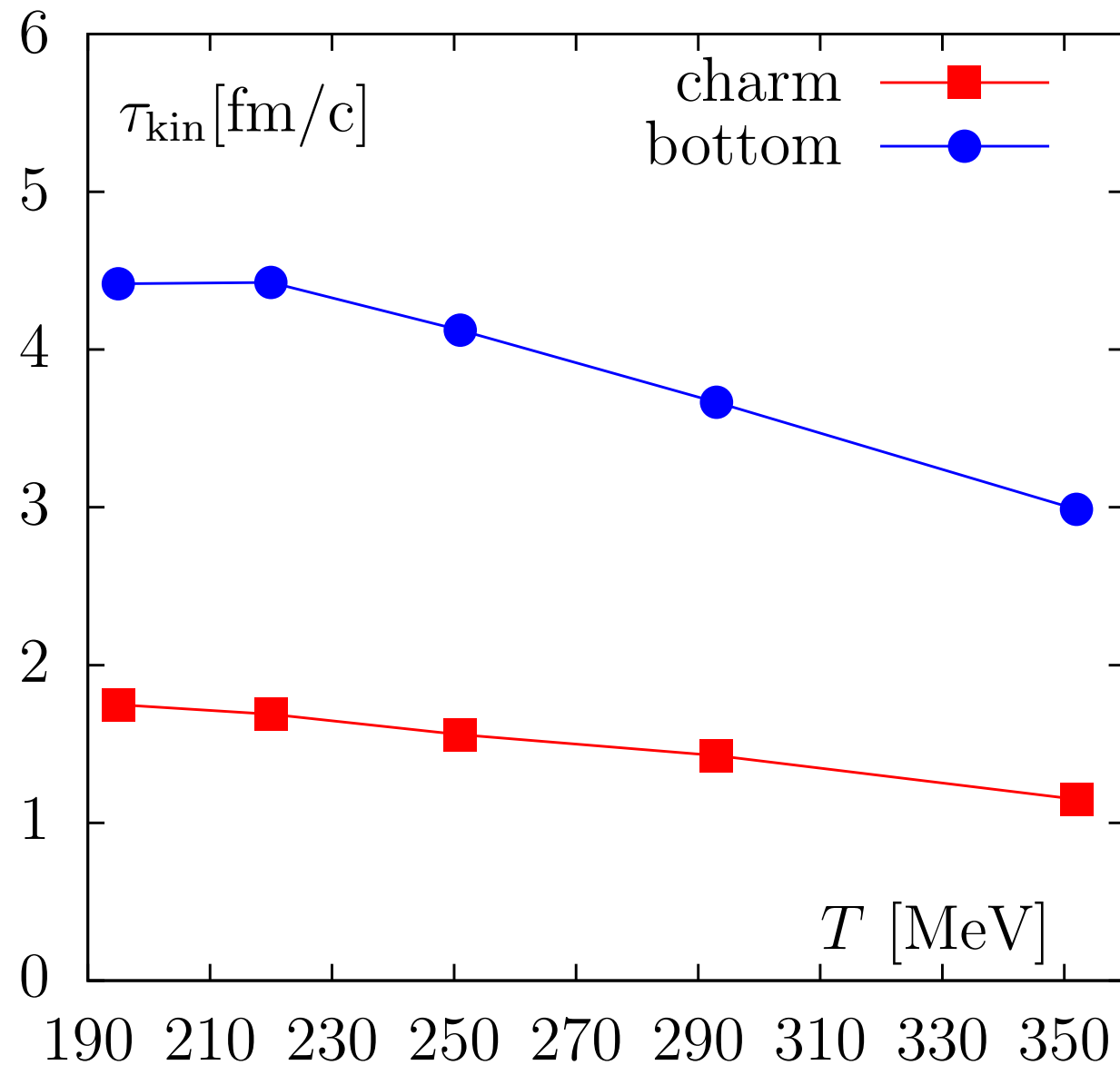
First full QCD determination of charm&bottom quark diffusion!



- HQ mass dependence of HQ diffusion: mild
- Universal change pattern with quark mass
- Weak quark mass dependence in LQCD & T-matrix
- Weaker than quasi-particle model (QPM) calculations

[LA, DC, OK, RL, GDM, SM, PP, 舒海涛, SS, PRL 132 (2024) 5, 051902]

Equilibration time of charm&bottom quark

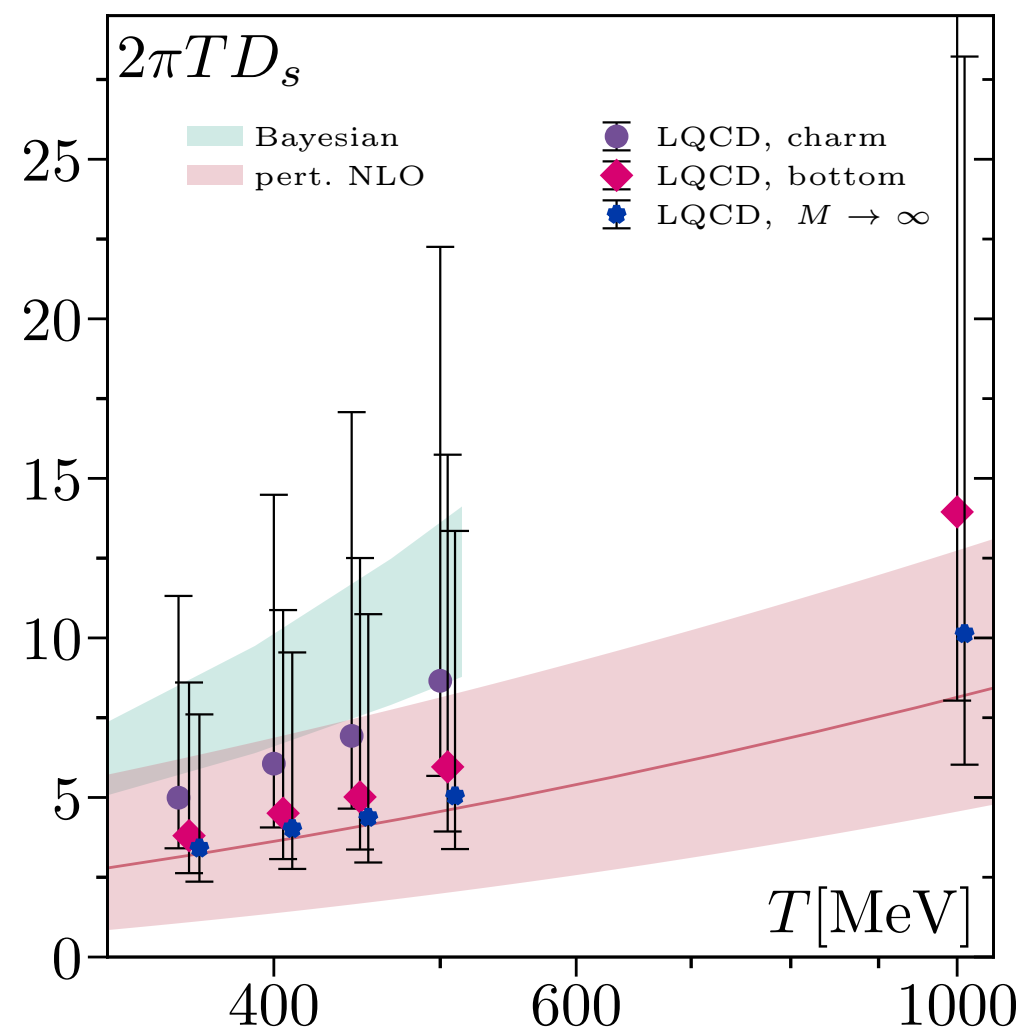
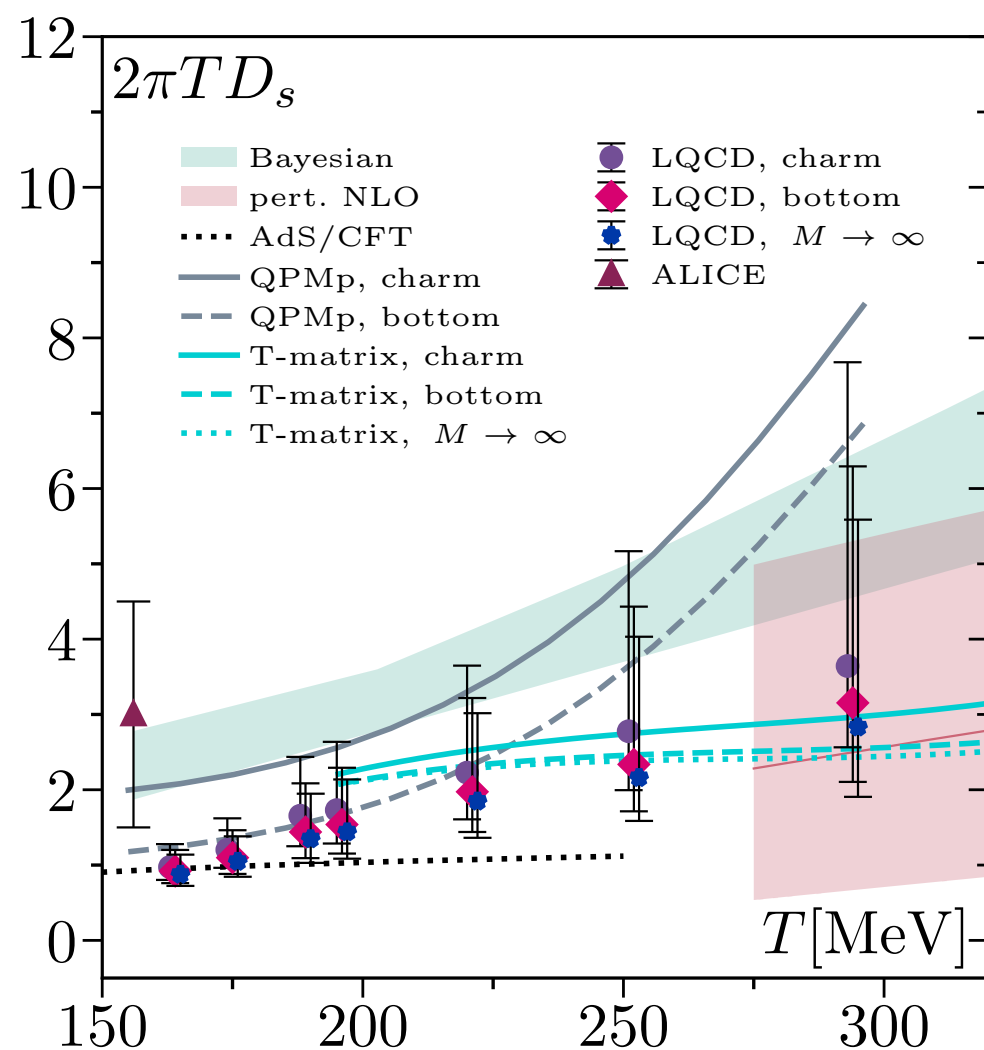


$$\tau_{\text{kin}} = \frac{1}{\eta_D} = \frac{1}{\kappa/T^3} \left(\frac{T_c}{T} \right)^2 \left(\frac{M}{1.5 \text{ GeV}} \right) \frac{3 \text{ GeV}}{T_c^2}$$

- Lattice provides short equilibration time for charm&bottom quark
- Rapid equilibrium $\longleftrightarrow$ QGP is near perfect fluid
- Lattice determination of $\tau_{\text{kin}}^{\text{charm}}$ favors the experimental estimate (~ 1 fm/c for all)

[LA, DC, OK, RL, GDM, SM, PP, **HTS**, SS, PRL 132 (2024) 5, 051902]

HQ diffusion at the physical point



[JDG, SM, PP, 舒海涛, JHW, et al., JHEP 09 (2025) 180]

$m_\pi \sim 320 \text{ MeV}$
 $195 \text{ MeV} \leq T \leq 352 \text{ MeV}$



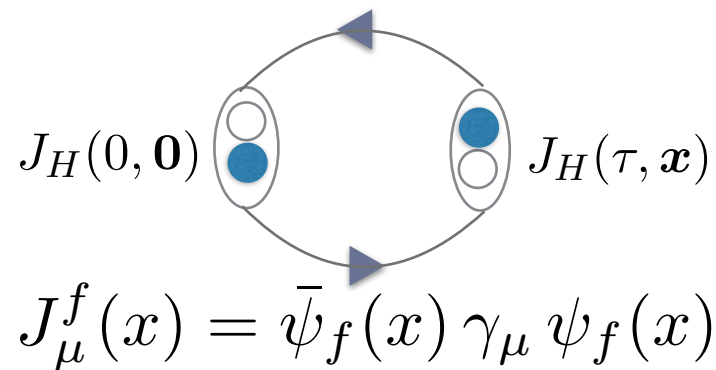
$m_\pi \sim 160 \text{ MeV}$
 $163 \text{ MeV} \leq T \leq 10000 \text{ MeV}$

- Similar magnitudes as previous studies
- Approaching AdS/CFT limit near T_c
- Consistent with NLO at high temperature

Heavy-flavor diffusion is insensitive to changes in the light degrees of freedom across the phase transition

Status of LQCD study of electrical conductivity

- Electrical conductivity also from meson correlator: $\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \rightarrow 0} \frac{\rho_{ii}(\omega, \vec{p} = \vec{0}, T)}{\omega T}$



$$J_\mu^f(x) = \bar{\psi}_f(x) \gamma_\mu \psi_f(x)$$

f = charm, bottom : heavy quark diffusion

f = light, strange : electrical conductivity

Challenge: numerical costs in simulating the **light** flavor

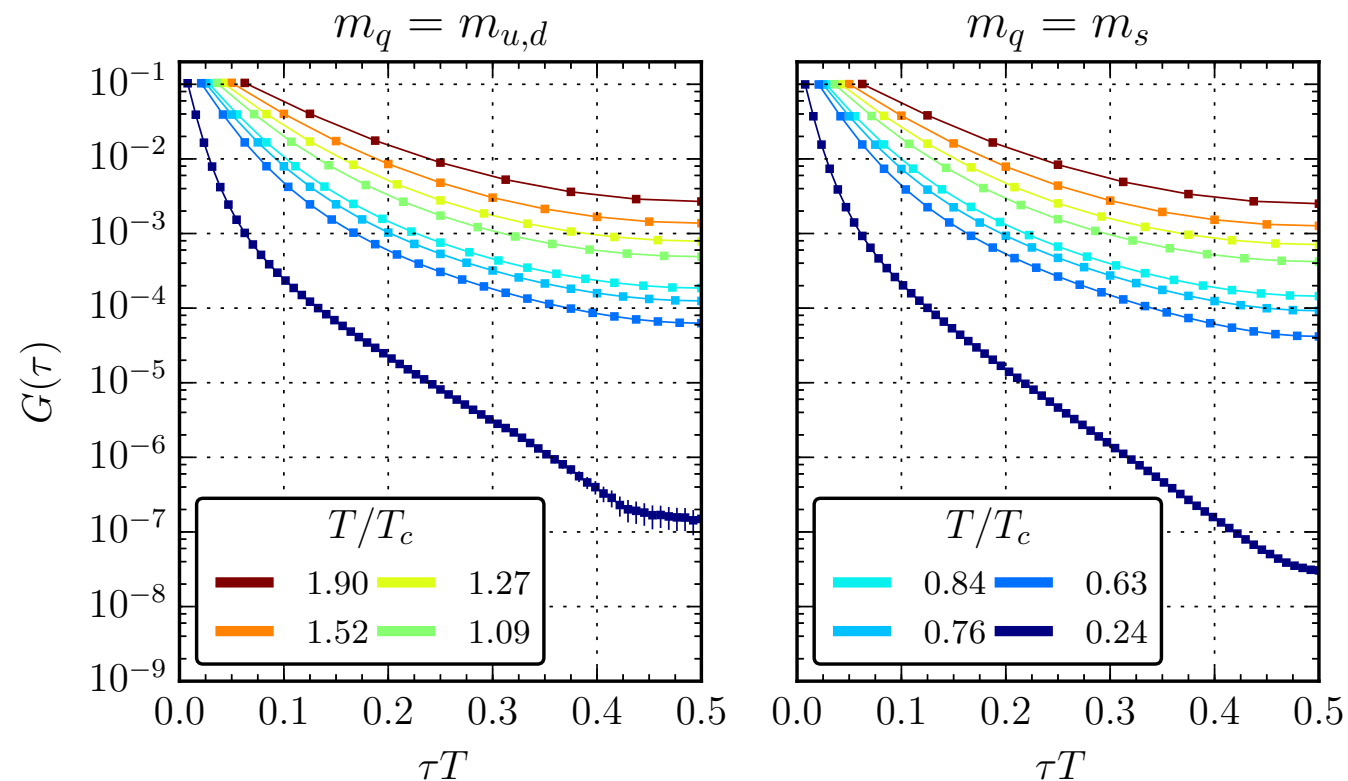
ref.	arXiv number	N_f (sea)	fermion type	m_π [MeV]	a_τ [fm]	a_s/a_τ	discretisation
[15]	hep-lat/0301006	0	quenched	—	$a_\tau \rightarrow 0$	1	continuum limit
[16]	hep-lat/0703008	0	quenched	—	0.0488, 0.0203	1	fixed cutoff
[17]	1012.4963	0	quenched	—	$a_\tau \rightarrow 0$	1	continuum limit
[18]	1112.4802	0	quenched	—	0.015	1	fixed scale
[19]	1212.4200	2	Wilson-clover	270	0.0486(4)(5)	1	fixed cutoff
[20]	1307.6763	2 + 1	Wilson-clover	384(4)	0.0350(2)	3.5	fixed scale
[21]	1412.6411	2 + 1	Wilson-clover	384(4)	0.0350(2)	3.5	fixed scale
[22]	1512.07249	2	Wilson-clover	270	0.0486(4)(5)	1	fixed scale
[23]	1604.06712	0	quenched	—	$a_\tau \rightarrow 0$	1	continuum limit
[24]	1910.08516	2 + 1	staggered	134.2(6)	0.0618, 0.0493	1	fixed cutoff

[G. Aarts and A. Nikolaev, EPJA 57 (2021) 4, 118]

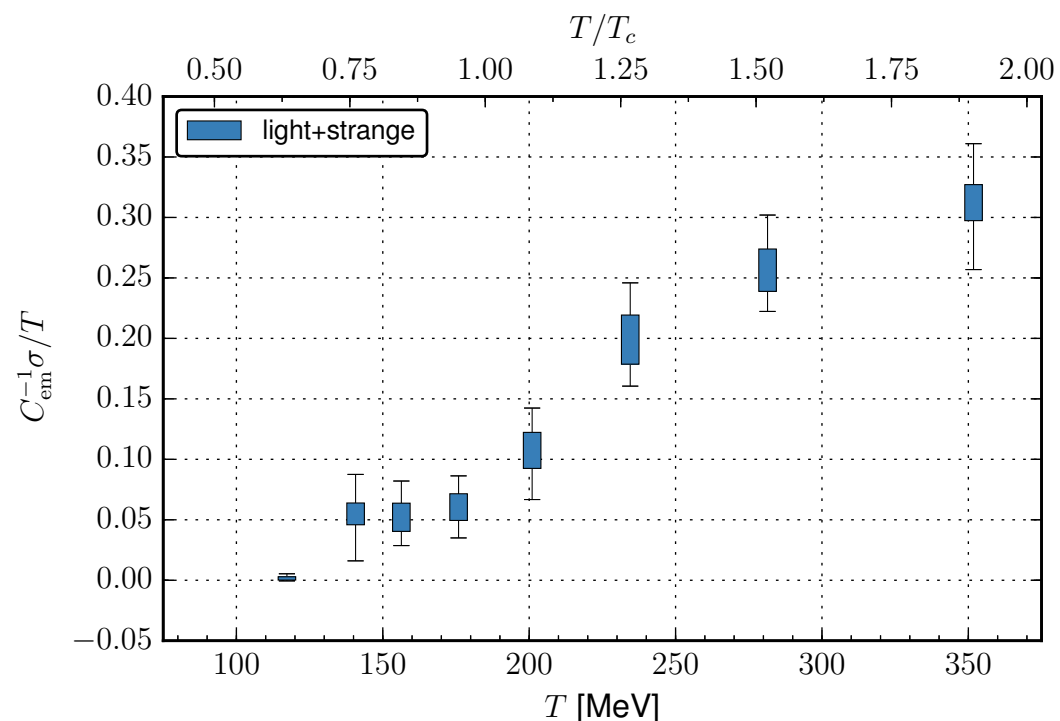
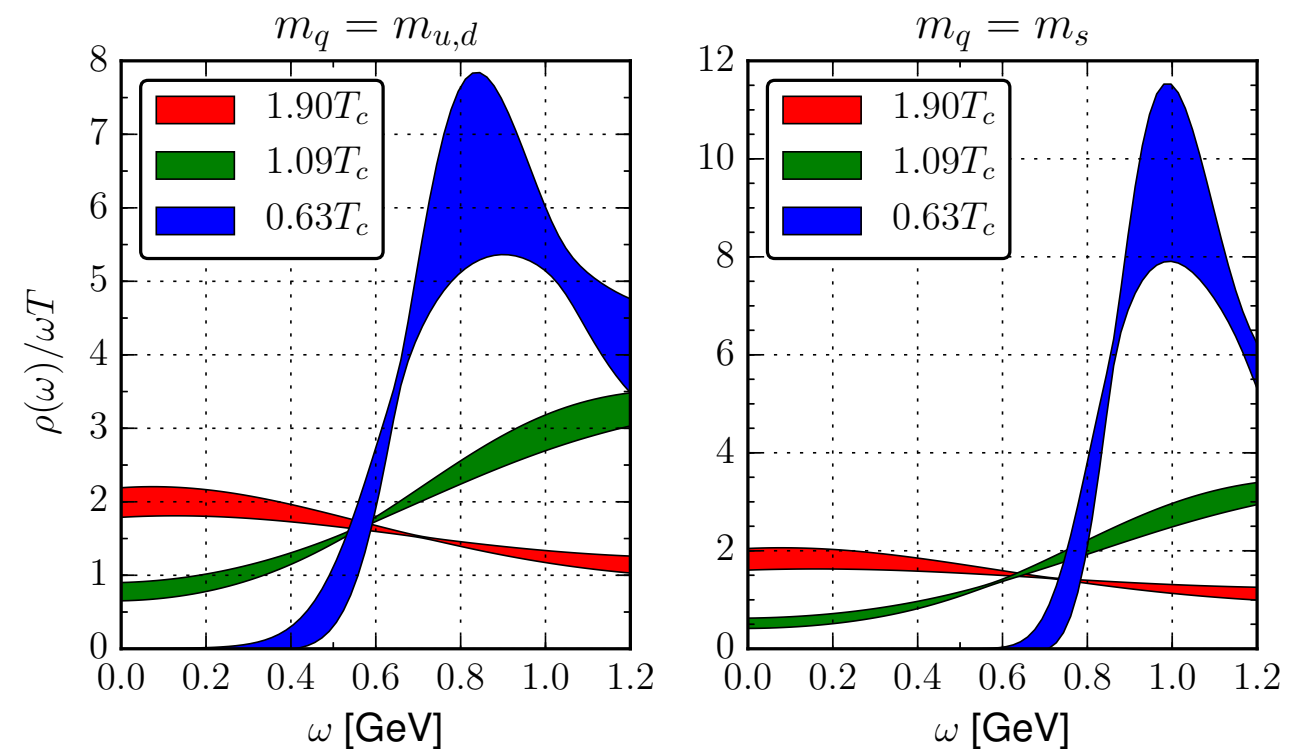
Status: renormalized full QCD results are available, but **no continuum limit**

Temperature dep. of electrical conductivity

- Correlators computed using conserved discretization in QCD



- SPF reconstructed using MEM



[G. Aart, et al., JHEP 02 (2015) 186]

- Similar values for light and strange flavor
- σ/T increases by a factor of 5–6 in the studied temperature range

Viscosities from lattice QCD

$$\eta(T) = \lim_{\omega \rightarrow 0} \frac{\rho_{\text{shear}}(\omega, T)}{\omega} \quad \zeta(T) = \frac{1}{9} \lim_{\omega \rightarrow 0} \frac{\rho_{\text{bulk}}(\omega, T)}{\omega}$$



$$G_{\text{shear}}(\tau) = \frac{1}{10} \int d^3x \left\langle \pi_{ij}(0, \vec{0}) \pi_{ij}(\tau, \vec{x}) \right\rangle = \int \frac{d\omega}{\pi} K(\omega, \tau) \rho_{\text{shear}}(\omega)$$

$$G_{\text{bulk}}(\tau) = \int d^3x \left\langle T_{\mu\mu}(0, \vec{0}) T_{\mu\mu}(\tau, \vec{x}) \right\rangle = \int \frac{d\omega}{\pi} K(\omega, \tau) \rho_{\text{bulk}}(\omega)$$



Gradient flow: renormalization scheme
& noise reduction for EMT

$$T_{\mu\nu}(t, x) = \check{c}_1(t) \tilde{\mathcal{O}}_{1,\mu\nu}(t, x) + \check{c}_2(t) \tilde{\mathcal{O}}_{2,\mu\nu}(t, x) + \check{c}_3(t) \tilde{\mathcal{O}}_{3,\mu\nu}(t, x) + \check{c}_4(t) \tilde{\mathcal{O}}_{4,\mu\nu}(t, x) + O(t)$$

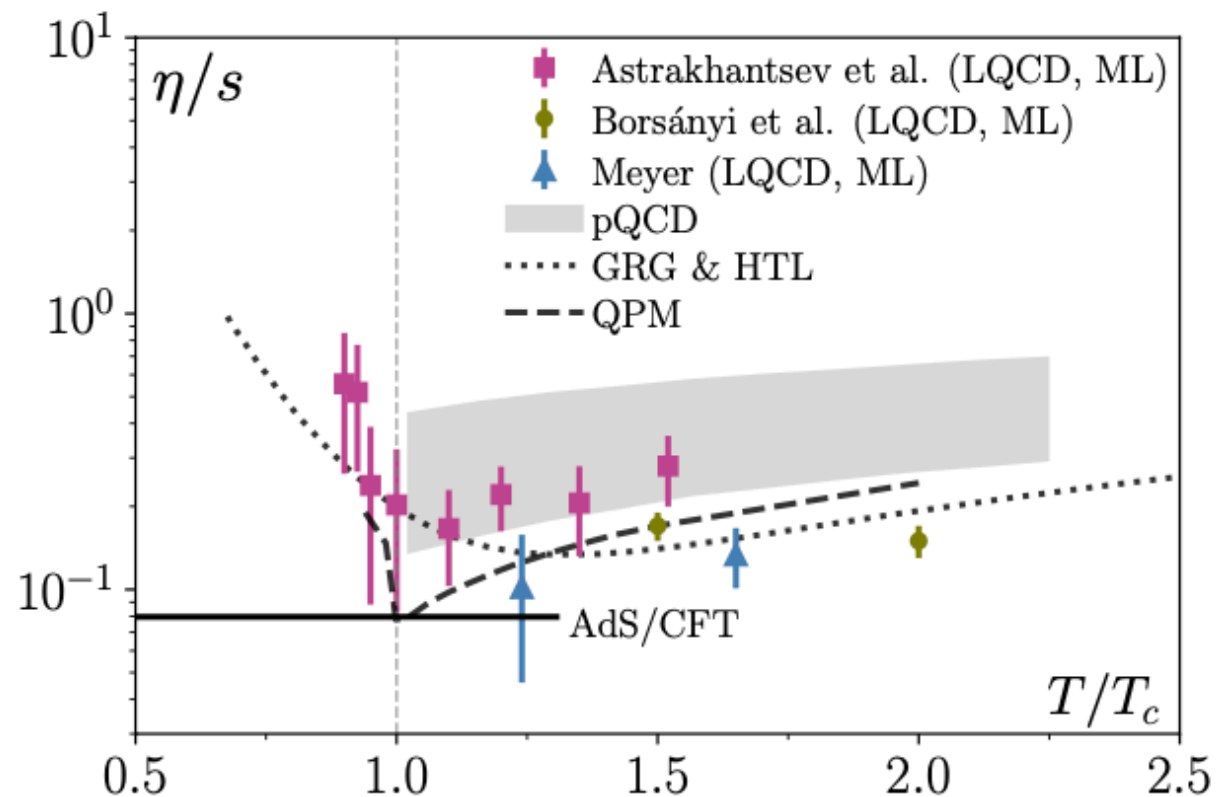
$$\tilde{\mathcal{O}}_{1,\mu\nu}(t, x) \equiv G_{\mu\rho}^a(t, x) G_{\nu\rho}^a(t, x)$$

$$\tilde{\mathcal{O}}_{2,\mu\nu}(t, x) \equiv \delta_{\mu\nu} G_{\rho\sigma}^a(t, x) G_{\rho\sigma}^a(t, x)$$

$$\tilde{\mathcal{O}}_{3,\mu\nu}(t, x) \equiv \sum_f \dot{\bar{\chi}}_f(t, x) \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) \dot{\chi}_f(t, x)$$

$$\tilde{\mathcal{O}}_{4,\mu\nu}(t, x) \equiv \delta_{\mu\nu} \sum_f \dot{\bar{\chi}}_f(t, x) \overleftrightarrow{D} \dot{\chi}_f(t, x)$$

Status of viscosities studies



Work	Method	Range
Meyer	ML	$1.24T_c, 1.65T_c$
Borsanyi et al.	ML	$1.5T_c, 2.0T_c$
Astrakhantsev et al.	ML	$0.9-1.5T_c$

- Previous LQCD calculations of viscosities are all in pure gauge

$$\tilde{\mathcal{O}}_{1,\mu\nu}(t, x) \equiv G_{\mu\rho}^a(t, x) G_{\nu\rho}^a(t, x)$$

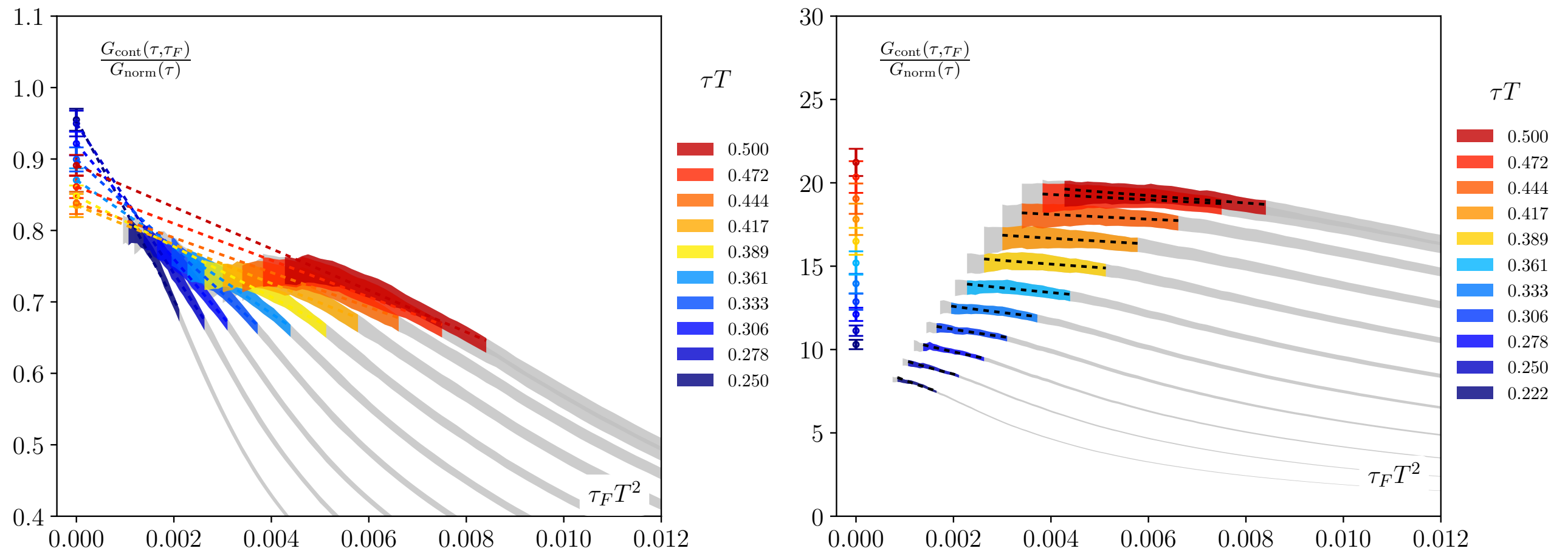
$$\tilde{\mathcal{O}}_{2,\mu\nu}(t, x) \equiv \delta_{\mu\nu} G_{\rho\sigma}^a(t, x) G_{\rho\sigma}^a(t, x)$$

~~$$\tilde{\mathcal{O}}_{3,\mu\nu}(t, x) = \sum_f \dot{\chi}_f(t, x) \left(\gamma_\mu \overleftrightarrow{D}_\nu - \gamma_\nu \overleftrightarrow{D}_\mu \right) \dot{\chi}_f(t, x)$$~~

~~$$\tilde{\mathcal{O}}_{4,\mu\nu}(t, x) = \delta_{\mu\nu} \sum_f \dot{\chi}_f(t, x) \overleftrightarrow{D} \dot{\chi}_f(t, x)$$~~

- Previous LQCD studies used the multi-level algorithm, not straightforwardly extendable to QCD

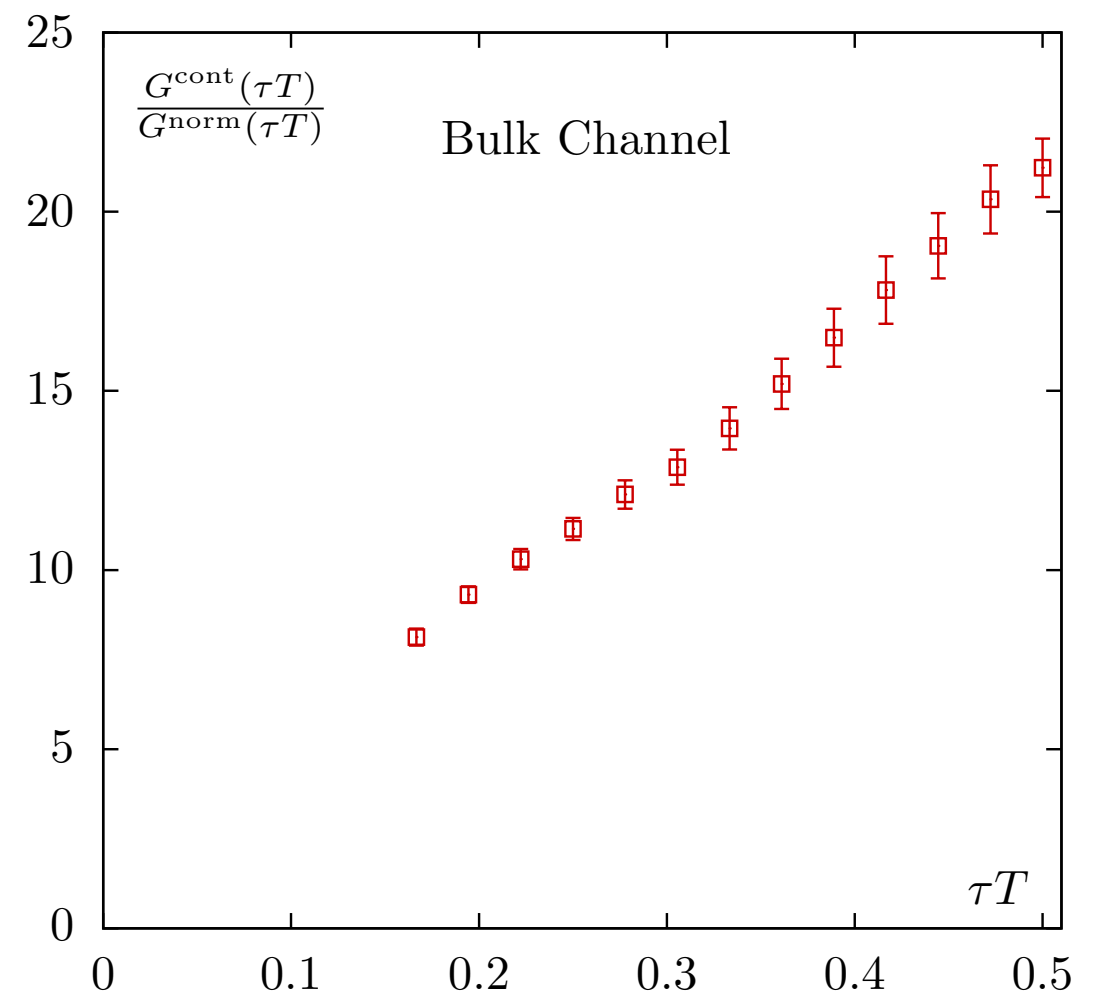
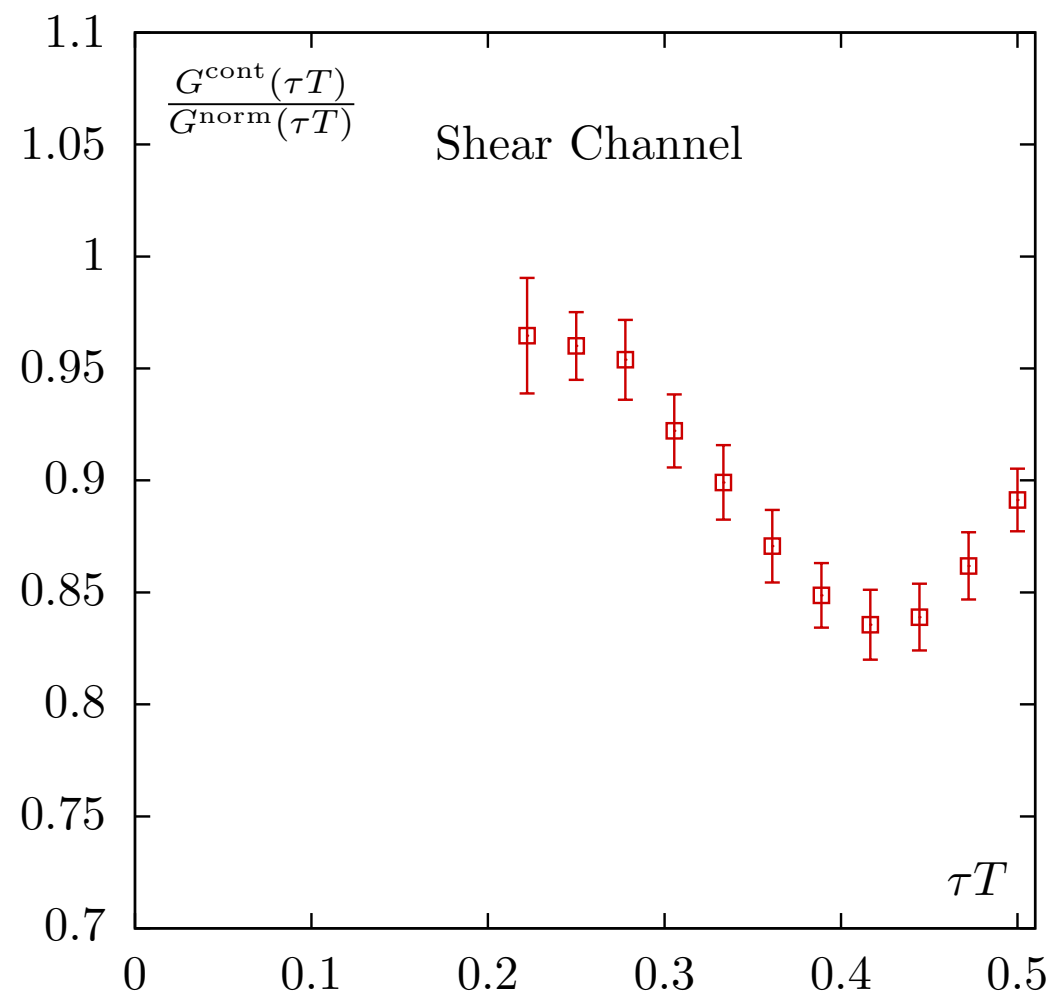
EMT correlators under flow at $1.5T_c$



[LA, AME, AF, OK, LM, GDM, 舒海涛, PRD 108 (2023) 1, 014503]

- Strong UV suppression for both channels
- Linear flow time dependence in certain flow time window

Physical EMT correlators at $1.5T_c$

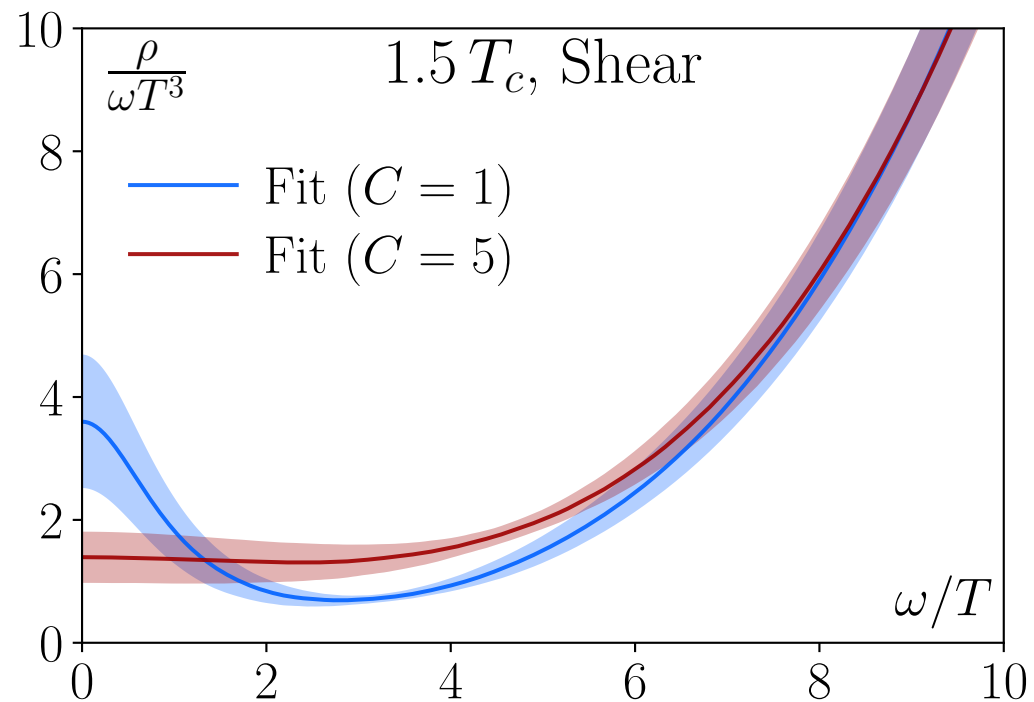


[LA, AME, AF, OK, LM, GDM, 舒海涛, PRD 108 (2023) 1, 014503]

- Achieved percent-level precision at large τT

Spectral function reconstruction

[LA, AME, AF, OK, LM, GDM, 舒海涛, PRD 108 (2023) 1, 014503]



- IR part: Lorentzian
- UV part: NLO spectral functions

[Y. Zhu & A. Vuorinen, JHEP 03, 002 (2013)]

[M. Laine et al., JHEP 09, 084 (2011)]

- ♦ A : transport strength
- C : peak width/lifetime
- B : UV normalization
- ♦ Scanning $C = 1$ and $C = 5$ brackets the dominant model uncertainty

$$G(\tau, T) = \int \frac{d\omega}{\pi} K(\omega, \tau, T) \rho(\omega, T)$$

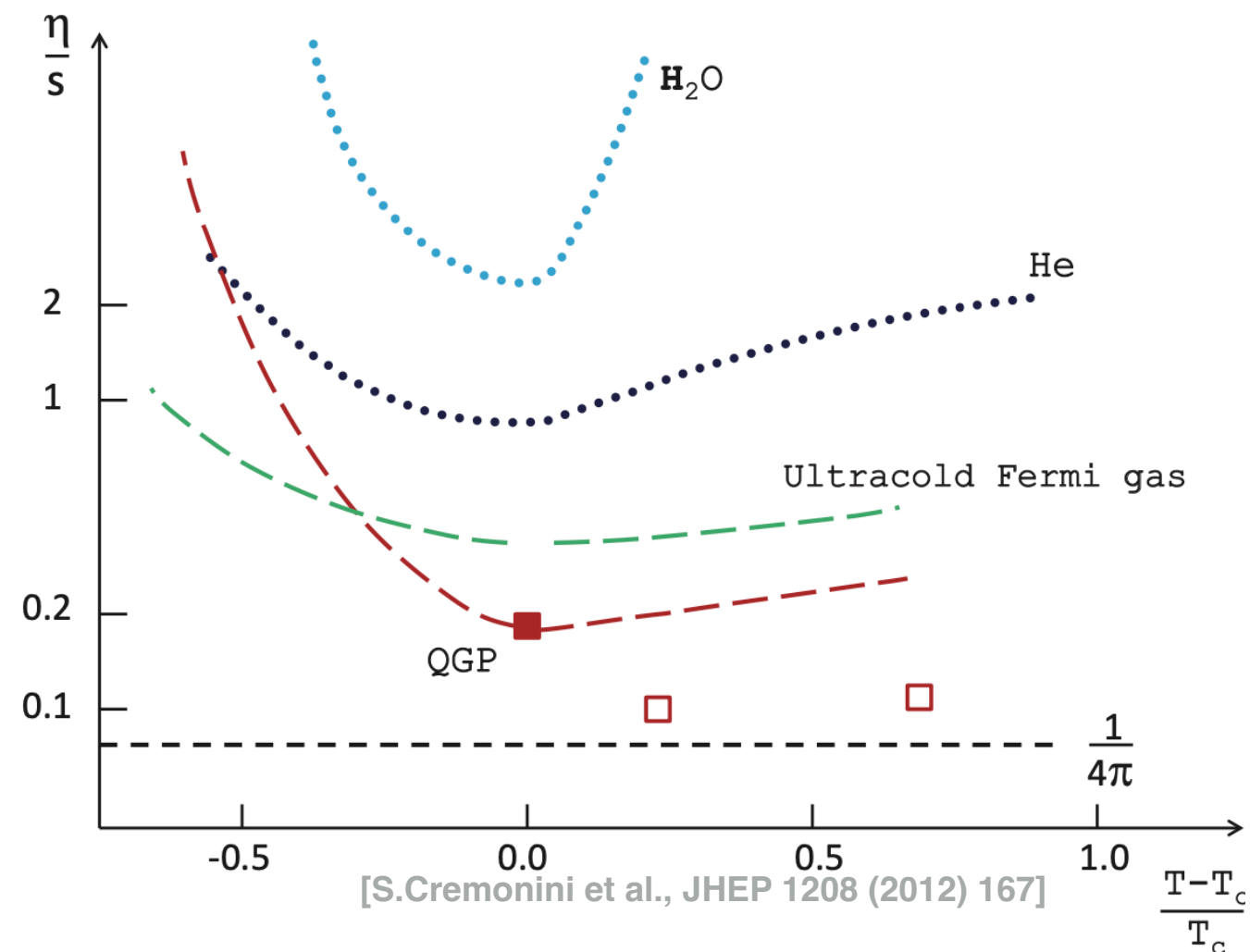
$$\frac{\rho_{\text{model}}(\omega)}{\omega T^3} = \frac{A}{T^3} \frac{C^2}{C^2 + (\omega/T)^2} + B \frac{\rho_{\text{pert}}(\omega)}{\omega T^3}$$

Viscosities:

$$\eta/T^3 = 0.765 - 2.45$$

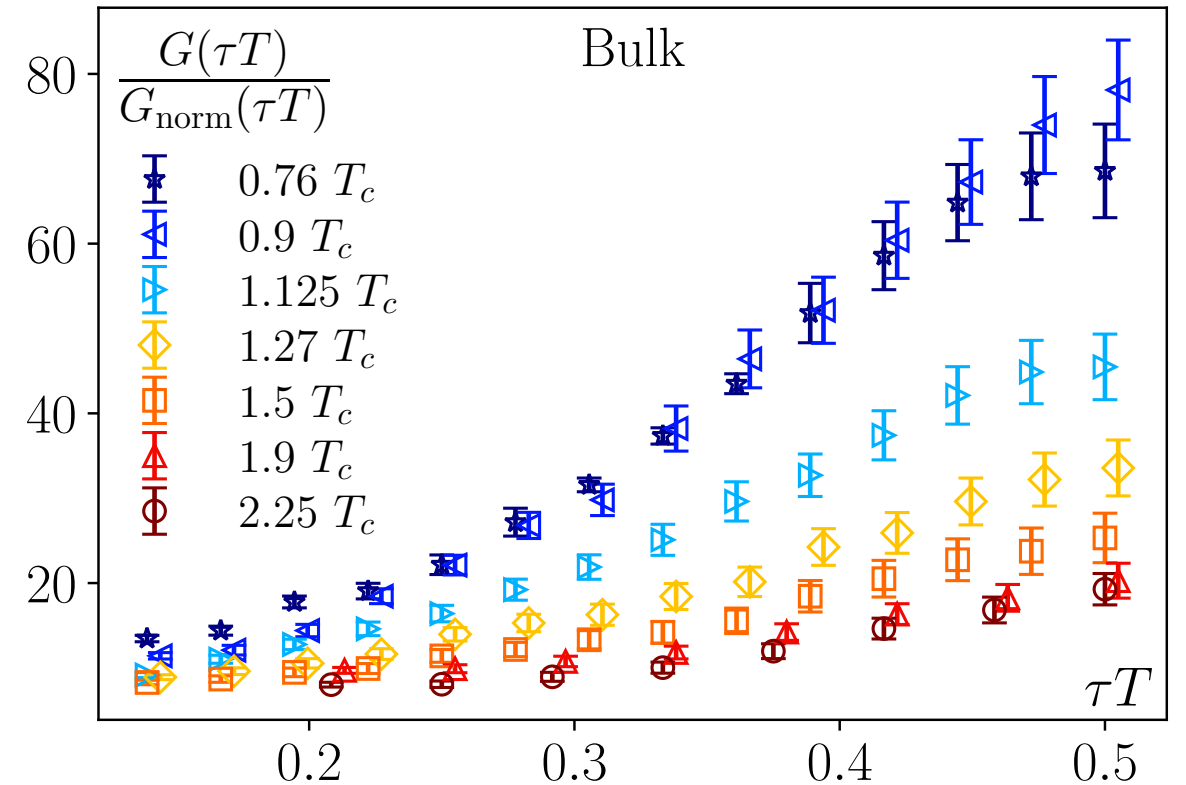
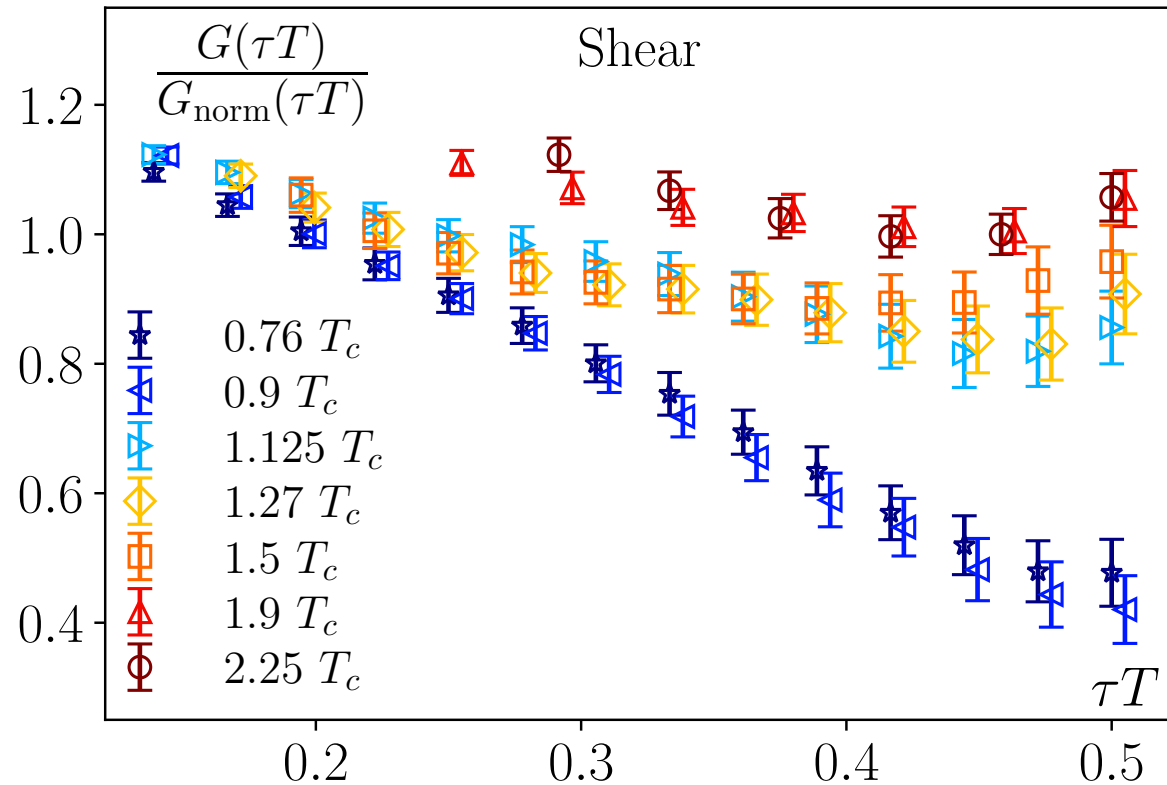
$$\zeta/T^3 = 0.087 - 0.301$$

Shear viscosity near the transition



- η/s is expected to be small in the strongly coupled region
- Across the phase transition, η/s probes the change of degree of freedom
- Temperature dependence of viscosities is crucial

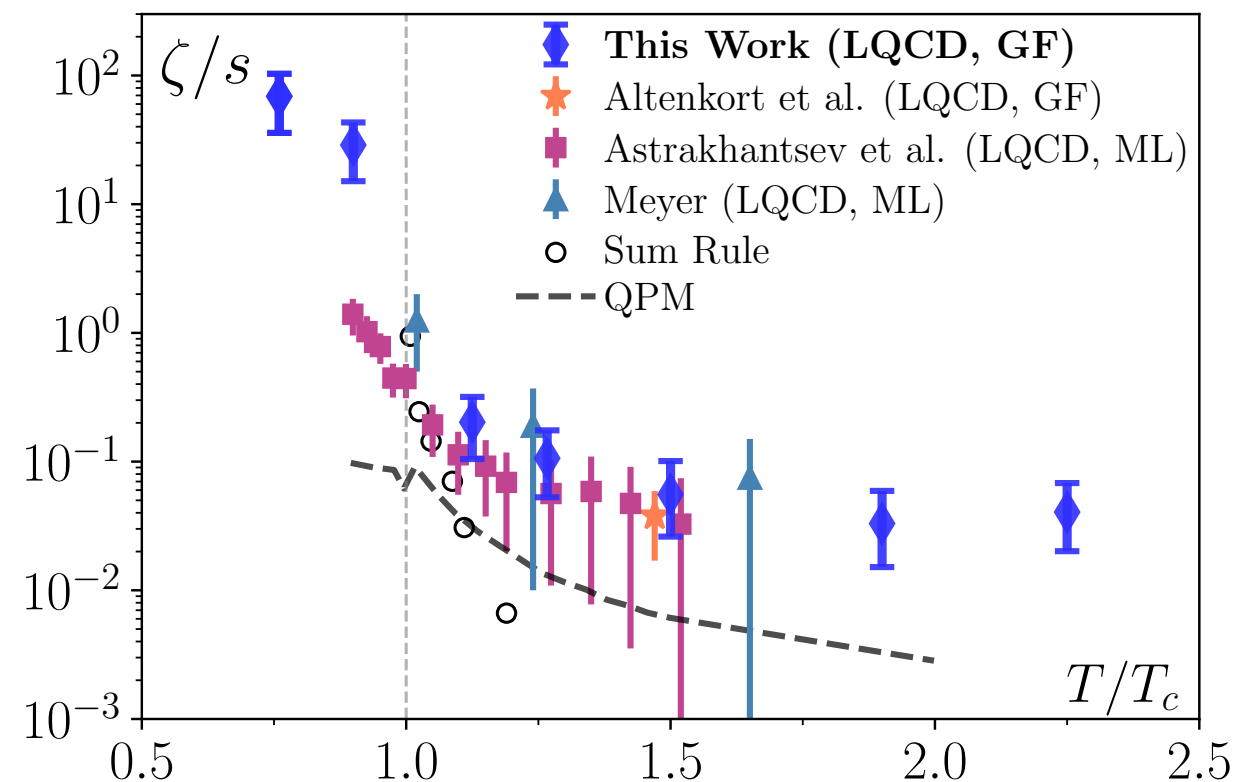
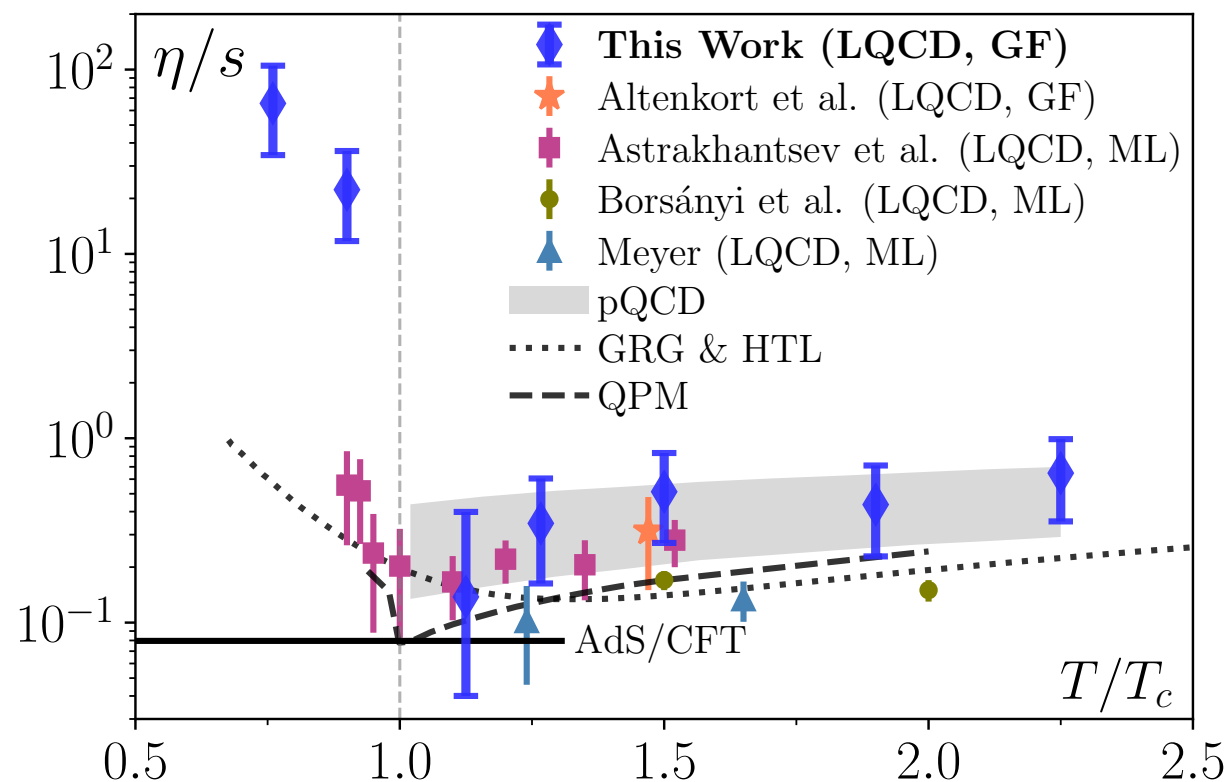
Temperature dependence of EMT correlators



[HTD, 舒海涛, CZ, PRD 113 (2026) 7, 074503]

- Both channels exhibit clear temperature dependencies
- Precision at percent level allows for controlled spectral reconstruction

Temperature dependence of viscosities



[HTD, 舒海涛, CZ, PRD 113 (2026) 7, 074503]

- η/s reaches a minimum in the transition region
- η/s consistent with NLO prediction at high T and close to the AdS/CFT bound near T_c
- ζ/s decreases with increasing T across the deconfined regime

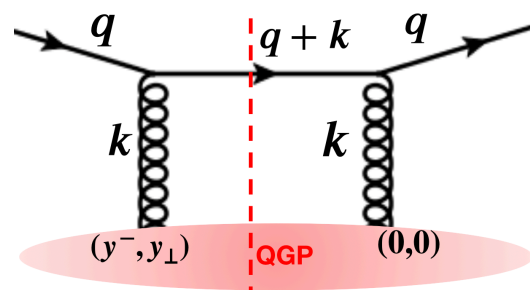
Extending to QCD is on the way!

$$T_{\mu\nu}(t, x) = \check{c}_1(t)\tilde{\mathcal{O}}_{1,\mu\nu}(t, x) + \check{c}_2(t)\tilde{\mathcal{O}}_{2,\mu\nu}(t, x) + \check{c}_3(t)\tilde{\mathcal{O}}_{3,\mu\nu}(t, x) + \check{c}_4(t)\tilde{\mathcal{O}}_{4,\mu\nu}(t, x) + O(t)$$

Jet quenching

- Jet exchanges transverse momentum squared with QGP when traversing it

$$\hat{q} = \frac{\sum_{i=1}^{N_{events}} \sum_{j=1}^{N^i(L)} [k_{\perp}^{i,j}]^2}{N_{events} \times L}$$



A forward hard quark scatters off undergoing a single gluon

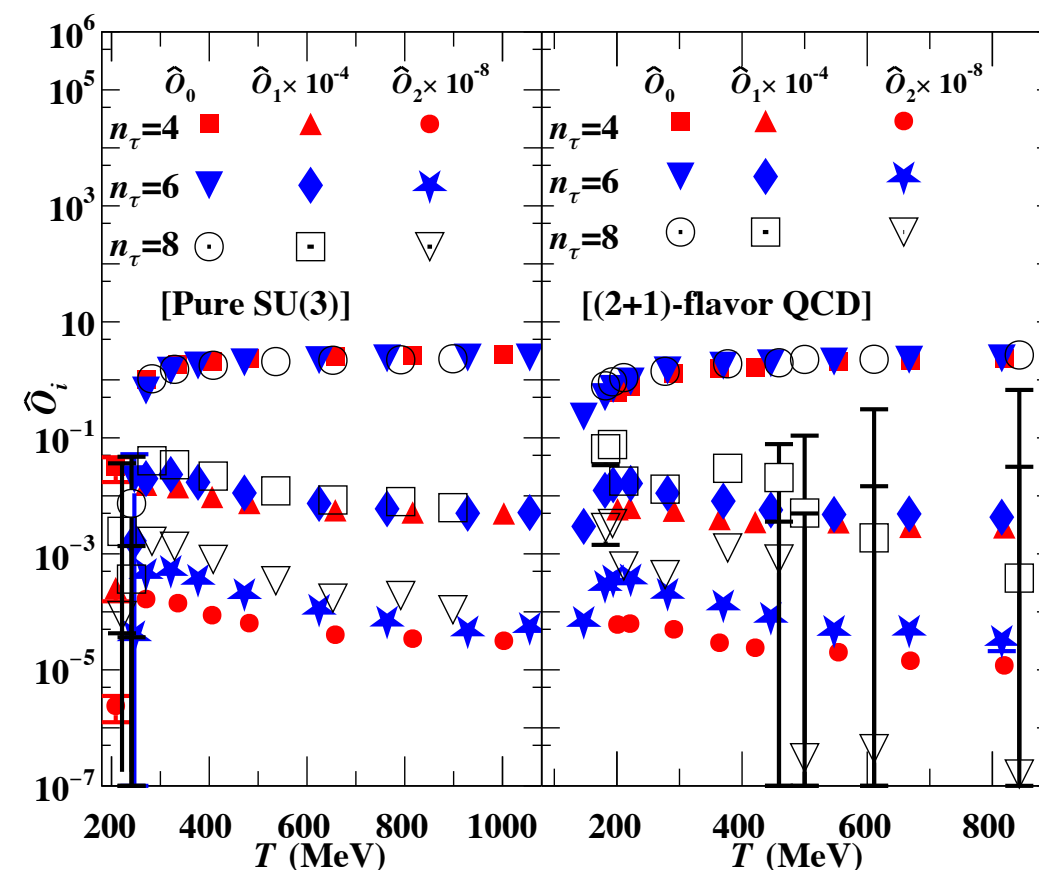
- High energy & virtuality parton with large q^- , single scattering per hard parton

$$\frac{\hat{q}}{T^3} = \frac{c_0 \sum_{n=0}^{\infty} \left(\frac{T}{q^-}\right)^{2n} \langle \frac{1}{T^4} \text{Tr} [F^{+j} \Delta^{2n} F_j^+] \rangle_{(T-V)}}{(T_1 + T_2)/T}$$

- Leading twist ($n=0$) dominates

→ in SU(3): $\hat{q} = \frac{2\pi\alpha_s}{N_c} s$

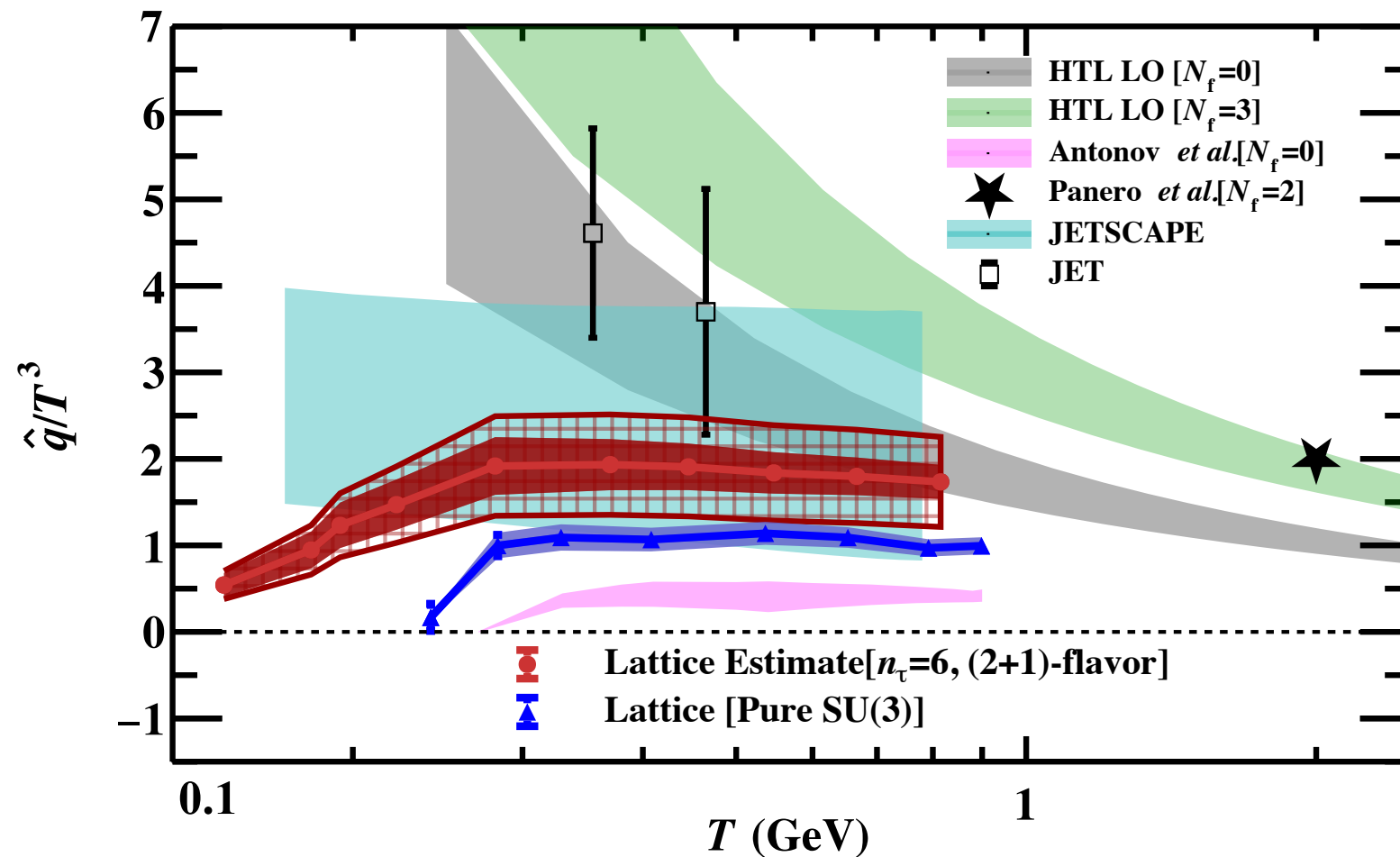
[A. Kumar, A. Majumder, J.H. Weber, PRD 106 (2022) 3, 034505]



SU(3): renormalized

QCD: w.o. matching

Jet transport coefficient



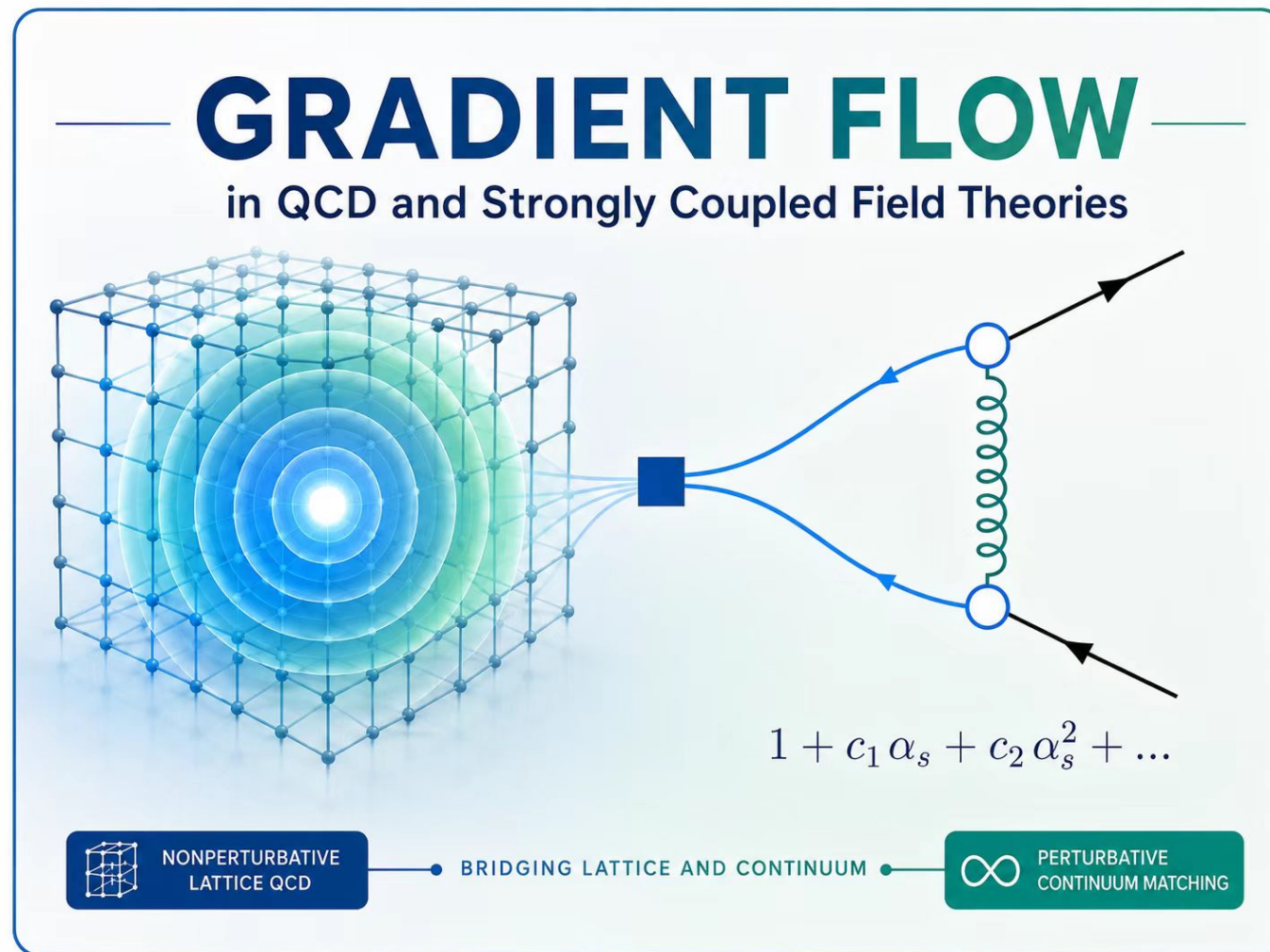
[A. Kumar, A. Majumder, J.H. Weber, PRD 106 (2022) 3, 034505]

- The lattice $\hat{q}$ are consistent with JETSCAPE phenomenological extractions, contrary to calculations using perturbation theory
- Possible improvement: gradient flow as smearing&renormalization+matching?

Summary

- ◆ Transport properties of the QGP are intrinsically non-perturbative
- ◆ Lattice QCD provides a reliable ab initio framework for determining transport coefficients
- ◆ Key challenges include precise control of Euclidean correlators and systematic uncertainties in spectral modeling
- ◆ For most transport coefficients, determinations in full QCD are now available
- * Gradient flow enables viscosity calculations in full QCD and improves jet-transport determinations

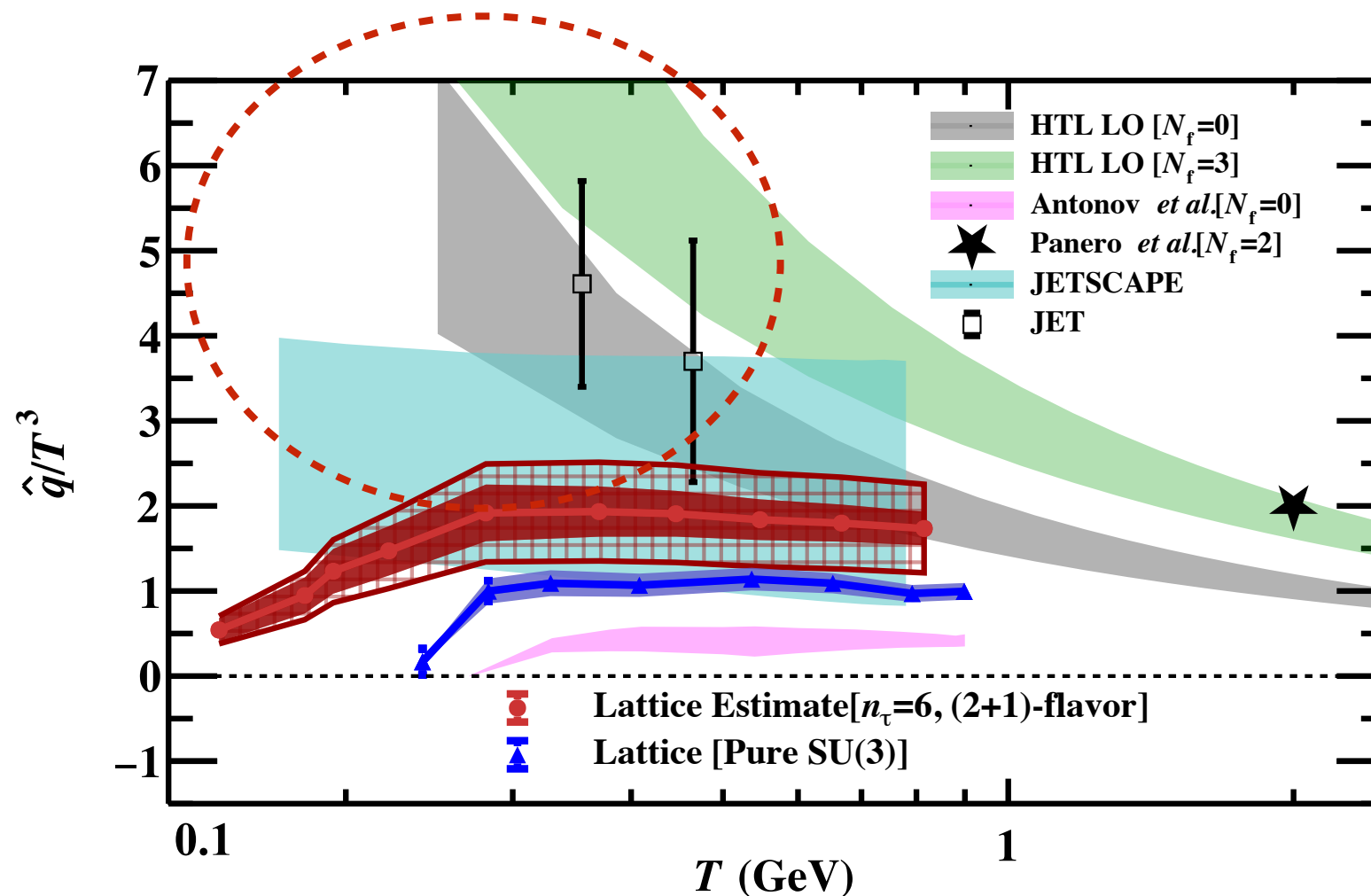
CCNU will host the
Gradient Flow in QCD and Strongly Coupled Field Theories
Mar. 2 - 5, 2027



<https://indico.ihep.ac.cn/event/30090/>

Backup: jet transport coefficient

Log in perturbative due to “one gluon exchange” diagram:

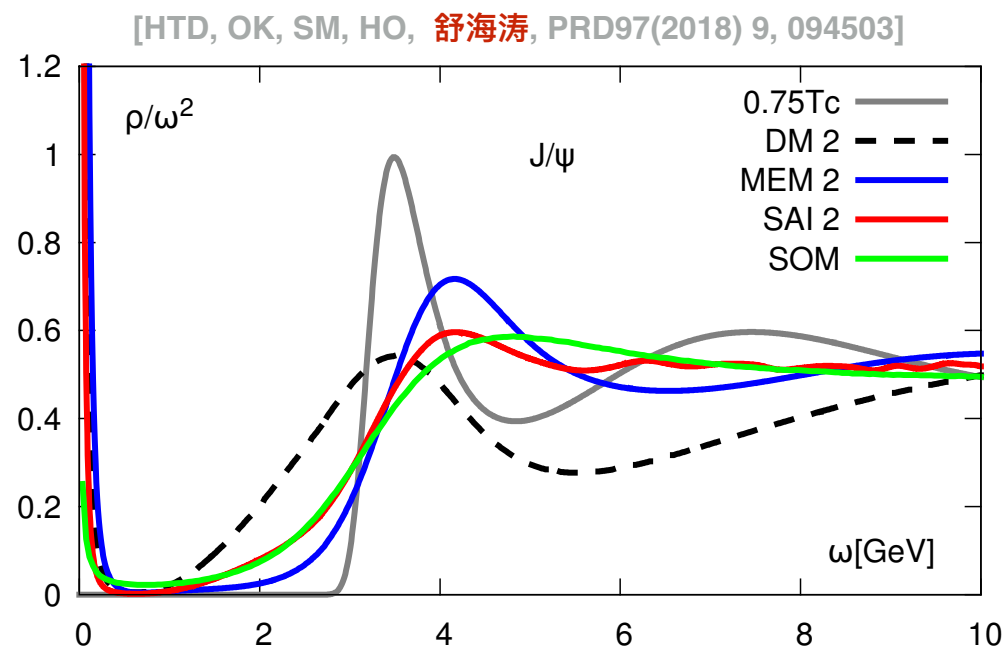
$$\hat{q} = \frac{N_C^2 - 1}{N_C} \alpha_s T^3 \left\{ \frac{8\pi^3}{45} \right\}$$


- JET uses models of energy loss from HTL effective theory
- JETSCAPE extraction applied an amalgam of theories for different epochs of the jet shower, with a data-driven (Bayesian) determination of $\hat{q}/T^3$, allowed to depend on T , the energy and scale of a given parton in the shower
- Both work with the OGE approximation $\rightarrow$ A log-like rise at low T is allowed in both frameworks

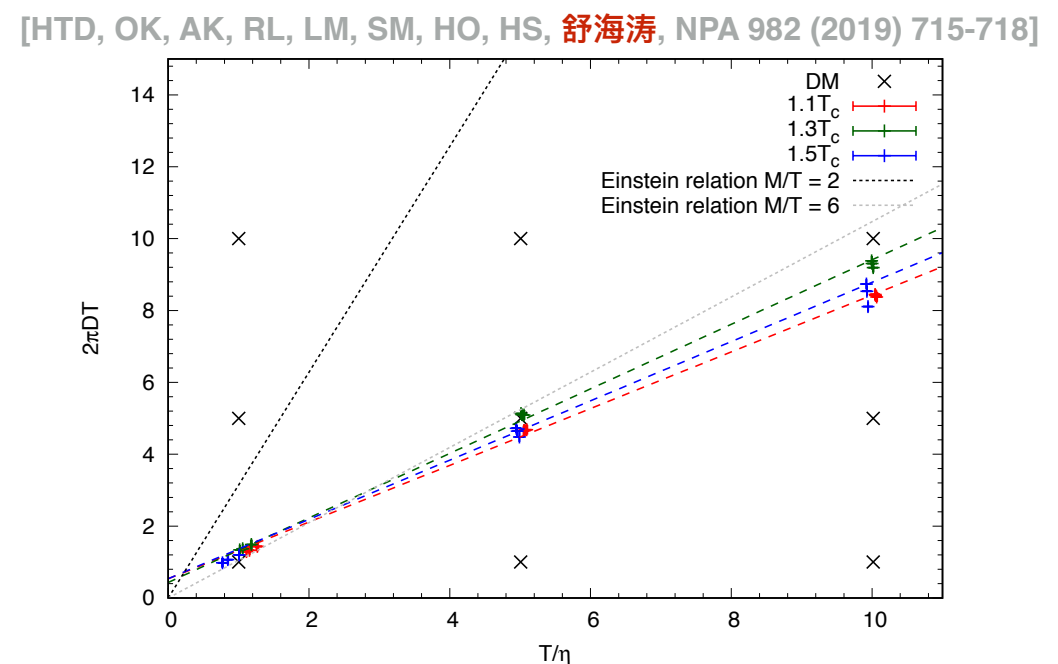
Backup: spectral Reconstruction Methods

Commonly used approaches

- * New Bayesian Method Y. Burnier and A. Rothkopf, PRL 111,18,182003
- * Backus-Gilbert Method B. B. Brandt, et al., PRD93, 054510(2016)
- * Maximum Entropy Method M. Asakawa, et al., PPNP. 46(2001) 445-508
- * Stochastic Approaches [HTD, OK, SM, HO, 舒海涛, PRD97(2018) 9, 094503]
- * ...



SPF reconstruction using SA&MEM at $1.5T_c$



Transport peak reconstruction using MEM

- Main limitations:
- Dependence on priors / assumptions
 - Limited spectral resolution

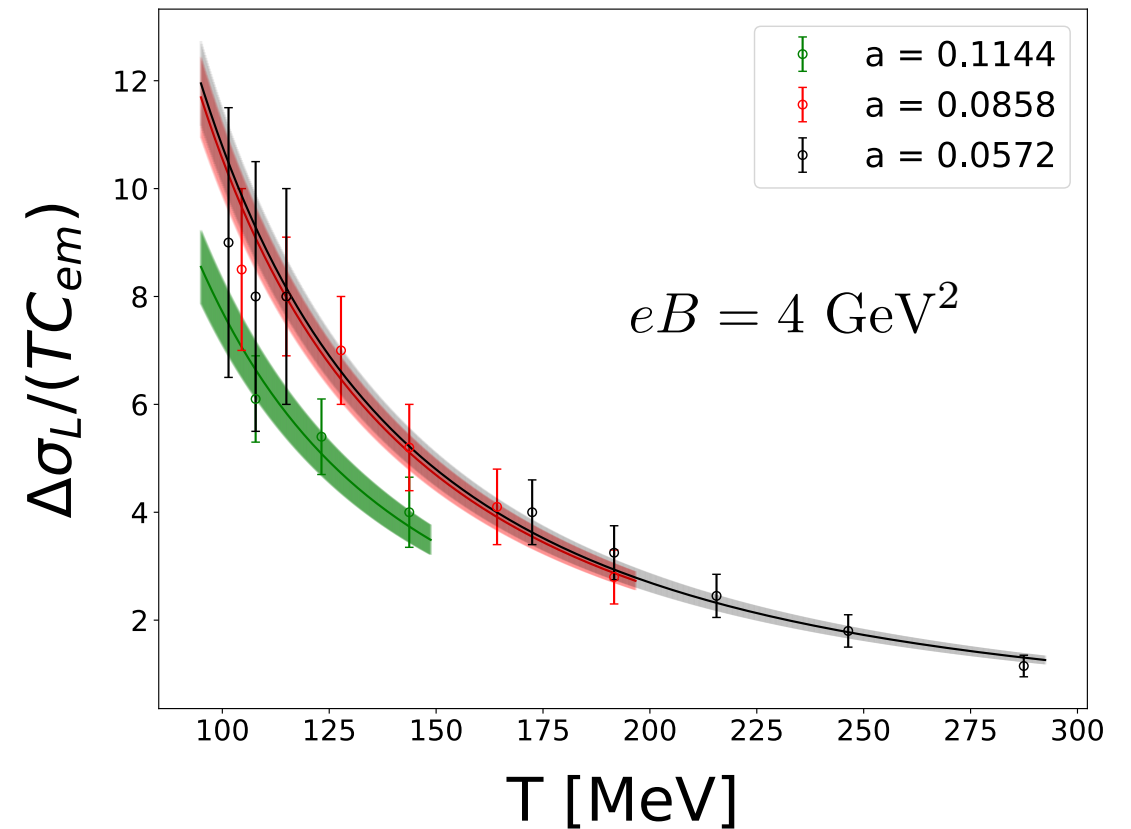
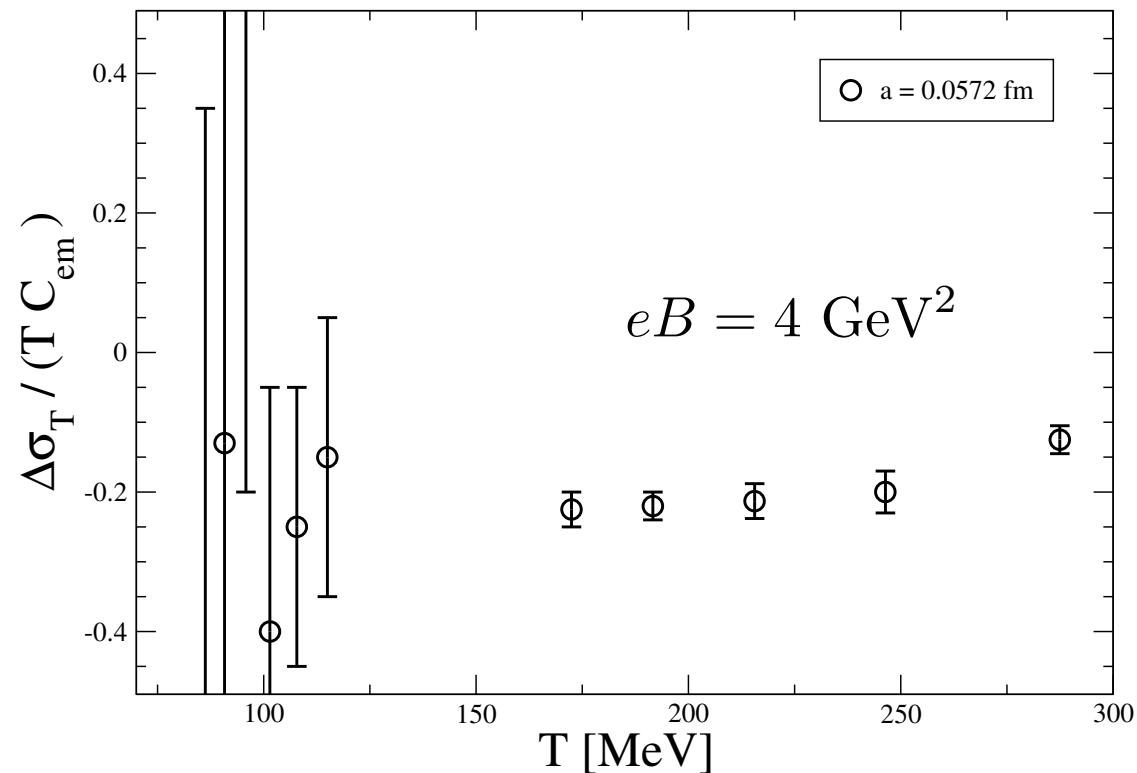
➔ **This talk: theory-inspired spectral fits**

Backup: electric conductivity in external B-field

$N_f=2+1$ calculations using HISQ

[G. Almirante, et al., PRD 111 (2025) 3, 034505]

σ/T extracted using Backus-Gilbert method



- Suppression of σ/T in the direction orthogonal to B

- Enhancement of σ/T in the direction parallel to B

- Motion in the direction orthogonal to B is restricted by Lorentz force $\rightarrow$ drop in σ/T
- The enhancement in the direction parallel to B can be interpreted through the chiral magnetic effect (CME)

$$\sigma_L^{CME} = \frac{N_c C_{em}^2}{2\pi^2 e^2} \frac{eB}{\sum_f |q_f|} \tau \quad (\text{extract the relaxation time})$$