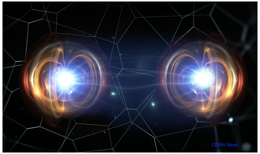
超子极化和量子纠缠
Lambda Polarization and Quantum Entanglement

Bo-Wen Xiao

CUHK-Shenzhen



The EPR Paradox and the Birth of Entanglement

Historical Development:

- **Einstein-Bohr Debates (1920s-30s)**
  - Einstein: **“God does not play dice”** vs Bohr: **“Stop telling God what to do”**
  - Determinism vs probability
- **Schrödinger (1932):** coined **“Entanglement”**
- **EPR Paper (1935)** [Phys. Rev. 47, 777]
  - Local realism; **“Spooky action at a distance”**

What is entanglement?

- Separable: $|\psi\rangle = |\phi\rangle_A \otimes |\chi\rangle_B$ (independent)
- Entangled: **cannot** be written as a product
- Canonical example (spin singlet): $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Bohr & Einstein

Schrödinger

EPR trio (1935)

EPR “paradox” revealed the profound nature of entanglement

Bell States and Bell's Theorem

- **Three views of quantum indeterminacy**
  - **Realism:** hidden variables; outcomes determined but unknown
  - **Copenhagen:** indeterminacy is fundamental
  - **Agnosticism:** only predictive power matters
- **Bell Nonlocality** [Bell, 1964]
  - Observable difference: Realism vs Copenhagen; eliminates the agnostic view
  - Decisive evidence supporting QM
- **CHSH Inequality** [Clauser et al., 1969]
  - Generalized Bell inequality;
  - Foundation of quantum information

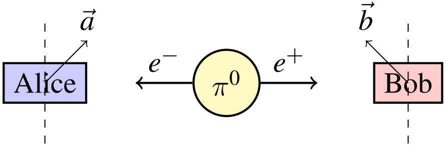
Bell States (Maximally Entangled):

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$
$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

John Stewart Bell

EPRB Experiment: Testing Bell Nonlocality

Einstein-Podolsky-Rosen-Bohm Experiment



Spin-singlet: $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Correlation: $E(\vec{a}, \vec{b}) = \langle A(\vec{a}) \cdot B(\vec{b}) \rangle$

Bell CHSH Inequality:

$$\mathbb{B}_{HT} = |E(a,b) - E(a,b') + E(a',b) + E(a',b')| \leq 2$$

$$\mathbb{B}_{QM} = |\cos\theta_{ab} - \cos\theta_{ab'} + \cos\theta_{a'b} + \cos\theta_{a'b'}| \leq 2\sqrt{2}$$

Local Hidden Variable Theory

- Pre-existing density $P(\lambda)$ for λ
- $A(\vec{a}, \lambda) = \pm 1$ predetermined
- $E(\vec{a}, \vec{b}) = \int P(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda) d\lambda$
- Local realism: $\mathbb{B}_{HT} \leq 2$

Quantum Mechanics

- No predetermined values
- $E(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b} = -\cos\theta_{ab}$
- Nonlocality: $2 < \mathbb{B}_{QM} \leq 2\sqrt{2}$
  Elementary proof with:
  $\alpha \cos\theta + \beta \sin\theta \leq \sqrt{\alpha^2 + \beta^2}$

QM violates Bell inequality $\Rightarrow$ Nature is nonlocal!

Separable vs Entangled States

How to distinguish these two states:
Positive Partial Transpose (PPT)

1. Separable (Unentangled) States:

$$\boxed{\rho = \sum_{i,j} P(a_i, b_j)\rho_a^i \otimes \rho_b^j}$$

- $P(a_i, b_j) \geq 0$ are probabilities and $\sum_{i,j} P(a_i, b_j) = 1$
- ρ_a^i, ρ_b^j are local density matrices

2. Entangled States:

Cannot be written in separable form
$\rho \neq \sum_{i,j} P(a_i, b_j)\rho_a^i \otimes \rho_b^j$

Peres Horodecki criterion:

- $\rho^{T_b} = \sum_{i,j} P(a_i, b_j)\rho_a^i \otimes \left(\rho_b^j\right)^T$
- If ρ is separable, all eigenvalues of ρ^{T_b} must be non-negative;
- Otherwise, ρ is definitely entangled.
- For 2×2 and 2×3, PPT is necessary and sufficient for separability.
- Higher *dim*, there are PPT entangled states. (PPT not sufficient)

Quantify entanglement [Wootters, 98]

- Use time reversal operation $\mathcal{T} = -i\sigma^y\mathcal{K}$ (Anti-Unitary) to flip spin.
- Define concurrence.

Time Reversal Operation and Kramers Degeneracy

Time Reversal for Spin-1/2

$$\boxed{\mathcal{T} = -i\sigma_y \mathcal{K}}$$

$$\mathcal{T}|\uparrow\rangle = |\downarrow\rangle \quad \mathcal{T}|\downarrow\rangle = -|\uparrow\rangle$$

- $\mathcal{K}$ is complex conjugation

$$\tilde{\chi} = \mathcal{T}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}-\beta^* \\ \alpha^*\end{pmatrix}$$

- Orthogonality: $\langle\tilde{\chi}|\chi\rangle = 0$

For arbitrary spin:

$$\mathcal{T} = e^{-i\pi S_y/\hbar}\mathcal{K}$$
$$\mathcal{T}^2 = -1 \text{ (for fermions)}$$

Kramers Degeneracy

For half-integer spin systems with time-reversal symmetry:
Every energy level is at least doubly degenerate

Concurrence: Measuring the Degree of Entanglement (Pure States)

Time Reversal Operation flips spins:

- $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
- $\mathcal{C}(|\psi\rangle) \equiv |\langle\tilde{\psi}|\psi\rangle|$, $|\tilde{\psi}\rangle = -\sigma_y \otimes \sigma_y|\psi^*\rangle$
- $|\tilde{\psi}\rangle = \delta^*|00\rangle - \gamma^*|01\rangle - \beta^*|10\rangle + \alpha^*|11\rangle$, the spin-flipped complex conjugate.
- $\mathcal{C}(|\psi\rangle) = 2|\alpha\delta - \beta\gamma|$ measures overlap with time-reversed state.
- $\mathcal{C} = 0$: Separable (no entanglement)
- $0 < \mathcal{C} < 1$: Partially entangled
- $\mathcal{C} = 1$: Maximally entangled

$\mathcal{C}$ = invariance under time reversal

Separable State

$|\psi_1\rangle = |00\rangle$ and $\alpha = 1, \beta = \gamma = \delta = 0$

$\mathcal{C} = 2|1 \cdot 0 - 0 \cdot 0| = 0$

Bell State (Maximally Entangled)

$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$\mathcal{C} = 2|\frac{1}{2} - 0| = 1$

Partially Entangled

$|\psi_2\rangle = \frac{1}{\sqrt{3}}|00\rangle + \sqrt{\frac{2}{3}}|11\rangle$

$\mathcal{C} = 2|\frac{1}{\sqrt{3}} \cdot \sqrt{\frac{2}{3}}| = \frac{2\sqrt{2}}{3} \approx 0.94$

Spin Density Matrix for Spin-1/2 Particles

Density Matrix Formalism

For a spin-1/2 particle, the density matrix is:

$$\boxed{\rho = \frac{\mathbb{1}_2 + \vec{n}\cdot\vec{\sigma}}{2}}$$

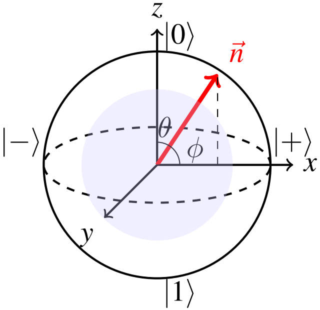
Bloch Vector: $n_i = \langle \sigma_i \rangle = \mathrm{Tr}(\rho\sigma_i)$

- $|\vec{n}| = 1$: Pure state
- $|\vec{n}| < 1$: Mixed state
- $\vec{n} = 0$: Maximally mixed state

For a heavy quark:

- **Production mechanism:** QCD processes determine initial Bloch vectors
- **Experimental access:** Weak decay measures spin projections $\langle \vec{n}\cdot\vec{\sigma}\rangle$

Bloch Sphere Representation



Geometry encodes quantum information

- $\vec{n} = (0,0,1)$: $\rho = |0\rangle\langle 0|$
- $\vec{n} = (1,0,0)$: $\rho = |+\rangle\langle +|$
- $\vec{n} = (0,0,0)$: $\rho = \frac{1}{2}\mathbb{1}_2$ (classical)

Density Matrix and Concurrence for Two-Qubit Systems

Extending to mixed states

Density Matrix Representation:

- Pure state: $\rho = |\psi\rangle\langle\psi|$
- Mixed state: $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
- General form in computational basis:

$$\rho = \begin{pmatrix} \rho_{00,00} & \rho_{00,01} & \rho_{00,10} & \rho_{00,11} \\ \rho_{01,00} & \rho_{01,01} & \rho_{01,10} & \rho_{01,11} \\ \rho_{10,00} & \rho_{10,01} & \rho_{10,10} & \rho_{10,11} \\ \rho_{11,00} & \rho_{11,01} & \rho_{11,10} & \rho_{11,11} \end{pmatrix}$$

Properties:

- Hermitian: $\rho^\dagger = \rho$
- Trace $\mathrm{Tr}(\rho) = 1$; Non-negative.

Concurrence in general:

[Hill, Wootters, 97; Wootters, 98]

- Define: $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$
- Compute: $\mathcal{R} = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$
- Eigenvalues of $\mathcal{R}$: $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ (descending order)

$$\mathcal{C}(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

Example - Werner State:

$$\rho_W = p|\Psi^-\rangle\langle\Psi^-| + \frac{1-p}{4}\mathbb{1}_4$$

- $p = 1$: Pure Bell state
- $\mathcal{C}(\rho_W) = \max\{0, \frac{3p-1}{2}\}$
- Entangled when $p > 1/3$

Spin Density Matrix: Physical Interpretation

The most general two-qubit density matrix:

$$\rho = \frac{1}{4}\left(\mathbb{1}_4 + B_i^+\sigma^i \otimes \mathbb{1}_2 + B_j^- \mathbb{1}_2 \otimes \sigma^j + C_{ij}\sigma^i \otimes \sigma^j\right)$$

Physical Quantities:

- $B_i^+ = \mathrm{Tr}\,\rho(\sigma_i \otimes \mathbb{1}_2)$
- $B_j^- = \mathrm{Tr}\,\rho(\mathbb{1}_2 \otimes \sigma_j)$
- $C_{ij} = \mathrm{Tr}\,\rho(\sigma_i \otimes \sigma_j)$
  Spin correlation /NB: Not $[\mathcal{C}]$

Special Case:

For Bell states: $B_i^+ = B_j^- = 0$
(No individual spin polarization)

Bell States & Correlation Matrices:

State	Correlation Matrix
$\lvert\Psi^-\rangle = \frac{1}{\sqrt{2}}(\lvert\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\rangle)$	$C_{ij} = \mathrm{diag}(-1,-1,-1)$
$\lvert\Psi^+\rangle = \frac{1}{\sqrt{2}}(\lvert\uparrow\downarrow\rangle + \lvert\downarrow\uparrow\rangle)$	$C_{ij} = \mathrm{diag}(1,1,-1)$
$\lvert\Phi^+\rangle = \frac{1}{\sqrt{2}}(\lvert\uparrow\uparrow\rangle + \lvert\downarrow\downarrow\rangle)$	$C_{ij} = \mathrm{diag}(1,-1,1)$
$\lvert\Phi^-\rangle = \frac{1}{\sqrt{2}}(\lvert\uparrow\uparrow\rangle - \lvert\downarrow\downarrow\rangle)$	$C_{ij} = \mathrm{diag}(-1,1,1)$

For singlet state:

$C_{ij} = -\delta_{ij}$ means spins are always anti-parallel.

Correlation matrix C_{ij} fully characterizes entanglement structure for Bell states

Entanglement and Bell Nonlocality Conditions

Starting from the spin density matrix: $\rho_{\alpha\alpha',\beta\beta'} = \frac{1}{4}\left(\mathbb{1}_{\alpha\alpha',\beta\beta'} + C_{ii}\sigma^i_{\alpha\beta} \otimes \sigma^i_{\alpha'\beta'}\right)$

- Anti-correlated spins: $C_{xx}, C_{yy}, C_{zz} < 0$
- Def: $D = (C_{xx} + C_{yy} + C_{zz})/3 = \mathrm{tr}C/3$
- $D = -1$: Perfect anti-correlation

Four eigen values of $\mathcal{R} = \rho$ (since $\tilde{\rho} = \rho$)

$$
\begin{aligned}
\lambda_1 &= \frac{1}{4}(1 - C_{xx} - C_{yy} - C_{zz}), \\
\lambda_2 &= \frac{1}{4}(1 + C_{xx} + C_{yy} - C_{zz}), \\
\lambda_3 &= \frac{1}{4}(1 + C_{xx} - C_{yy} + C_{zz}), \\
\lambda_4 &= \frac{1}{4}(1 - C_{xx} + C_{yy} + C_{zz}).
\end{aligned}
$$

Entanglement Condition

- Concurrence $\mathcal{C}[\rho] = \frac{1}{2}(-3D - 1) > 0$:

$$\boxed{D < -\frac{1}{3}}$$

Bell Nonlocality Condition

- For CHSH violation $\mathbb{B} > 2$: [Horodecki, et al, 95]

$$\boxed{D < -\frac{1}{\sqrt{2}} \approx -0.707}$$

Hierarchy: Bell Nonlocality $\subset$ Entanglement $\subset$ All Quantum States

Top Quark Weak Decay and Spin Transfer

Top Quark Decay: Choose its rest frame

$$t \to W^+ b \to \ell^+ \nu_\ell b, \ \bar{t} \to W^- \bar{b} \to \ell^- \bar{\nu}_\ell \bar{b}$$

Decay Spin Density Matrix:

$$\Gamma_\pm = \frac{\mathbb{1}_2 + \kappa_\pm \vec{\sigma}_t \cdot \hat{l}_\pm}{2}$$

Parity Violating Angular Distribution:

$$\frac{d\Gamma}{d\cos\theta} \propto 1 + \kappa_\pm \cos\theta$$

- Weak decay **Spin-momentum correlation** reveals spin directions.
- $\kappa_\pm = \pm 1$ **($t\bar{t}$) spin analyzing power**
- $\sigma_{l_+ l_-} \propto \mathrm{tr}[\Gamma_+ \otimes \Gamma_- \rho]$ NB $\mathrm{tr}[\sigma^i \sigma^j] = 2\delta^{ij}$

Correlation between di-leptons

$$\frac{d^2\sigma}{\sigma d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2}\left[1 - \hat{l}_+ \cdot C \cdot \hat{l}_-\right]$$

Entanglement Signature

$$\boxed{\langle \cos\varphi \rangle = -\frac{1}{3} D = -\frac{1}{9}\mathrm{Tr}(C)}$$

Experimental Reach:

- Extract $D = \mathrm{Tr}(C)/3$ parameter directly
- **Quantum Tomography**: all elements of ρ can be measured. [Bernreuther, Heisler, Si, 15; ATLAS, 1612.07004; CMS, 1907.03729]

First Observation of Quark Entanglement at the LHC

[ATLAS (Nature 2024):] First observation of entanglement in quarks at the highest-energy.

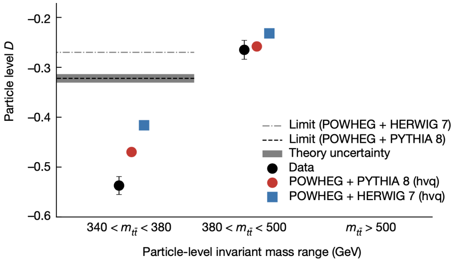
Entanglement Measure:

$$D = \mathrm{tr}[C]/3 = -3\langle \cos\phi \rangle$$

where ϕ is the angle between charged leptons in their parent top/antitop rest frames

Key Features:

- Spin transferred to decay products
- Measured near $t\bar{t}$ threshold
- From atomic physics to high-energy collisions: **A new frontier!**
- **CMS, STAR, BES-III** more to come.



$\sqrt{s} = 13$ TeV, 140 fb^{-1} data (2015-2018)

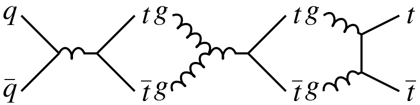
Measured: $D < -1/3$ (Entanglement criterion)
$D = -0.547 \pm 0.002$ (stat.) ± 0.021 (syst.)

- Observed: $> 5\sigma$ from no entanglement
- **Yet, Bell Nonlocality**: $D < -1/\sqrt{2}$

Theory vs Experiment: Top Quark Entanglement

Quantum State Tomography $\rho_{\alpha\alpha',\beta\beta'} = R_{\alpha\alpha',\beta\beta'}/\mathrm{tr}R$ [Afik, de Nova, 2022]

Top quark pair production



$$R_{\alpha\alpha',\beta\beta'} = \frac{1}{N}\sum \mathcal{M}^*_{t_\alpha \bar{t}_{\alpha'}} \mathcal{M}_{t_\beta \bar{t}_{\beta'}}$$

- **Measured $D \approx -0.54$** near threshold
- **Gluon fusion dominance** at LHC
- **Angular momentum conservation** determines spin correlations
- **Statistical mixture** of $q\bar{q}$ and gg

Near Threshold ($\beta \to 0$):

- $q\bar{q}$: Separable state ($\mathcal{C} = 0$), since $t\bar{t}$ spin (± 1) is equally mixed along beam.
- gg: Maximally entangled singlet Ψ^-

High Energy ($\beta \to 1$) with $\theta = \pi/2$:

- Both channels: Maximally entangled triplet Ψ^+ along $\hat{n}$ with nonzero OAM.

Mixed State at LHC

$$\rho = w_{q\bar{q}}\rho_{q\bar{q}} + w_{gg}\rho_{gg}$$

"Observation of Entanglement but not Bell Nonlocality due to Quark channel mixture"

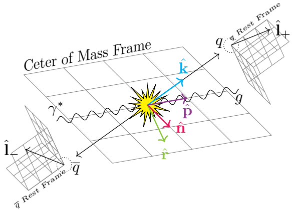
Quark Pair Production in Photon-Gluon Fusion: Longitudinal case

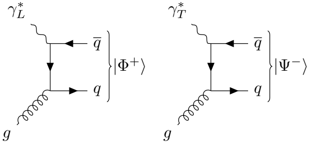
[Qi, Guo, Xiao] ▸ arXiv:2506.12889v1 [hep-ph]

Photon-Gluon Fusion Process

$$\gamma^*_{\lambda=\pm,0} + g \to q + \bar{q}$$

$$\rho_L = \frac{1}{4}\left(\mathbb{1}_4 + C_{ij}\sigma^i \otimes \sigma^j\right)$$





For $q\bar{q}$ with $\beta \to 0$ and $\theta = \frac{\pi}{2}$

Longitudinal photons contribution:

$$C_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\chi_1 & -\chi_2 \\ 0 & -\chi_2 & \chi_1 \end{pmatrix}$$

with $$\chi_1 = \frac{1 - 2z^2 + z^2\beta^2}{1 - z^2\beta^2}, \quad \chi_2 = \sqrt{1 - \chi_1^2}.$$

- ρ_L is given by a pure state $= |\Psi\rangle\langle\Psi|$, with

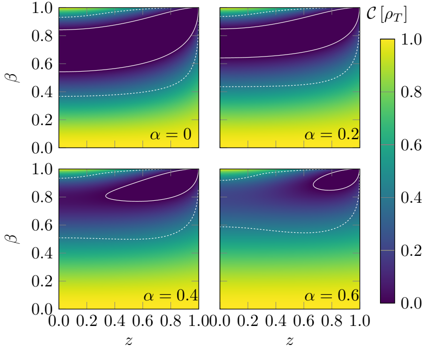
$$|\Psi\rangle = \frac{1}{2}\left(\sqrt{1+\chi_1}, \mathrm{i}\sqrt{1-\chi_1}, \mathrm{i}\sqrt{1-\chi_1}, \sqrt{1+\chi_1}\right).$$

- **Near Threshold** ($\beta \to 0$) with $\theta = \frac{\pi}{2}$: $|\Phi^+\rangle$.
- **High Energy** ($\beta \to 1$): $|\Phi^+\rangle$.
- $q\bar{q}$ has spin 1 with nonzero OAM and $\mathcal{C}[\rho_L] \equiv 1$!

Always Maximally Entangled! Very Special!

Quark Pair Production in Photon-Gluon Fusion: Transverse case

[Qi, Guo, Xiao] arXiv:2506.12889v1 [hep-ph] **Transverse photons**: similar to $gg \to q\bar{q}$ channel.



- Density plots of the concurrence for transverse photon as functions of β and $z = \cos\theta$ at given $\alpha \equiv Q^2/\hat{s}$.
- Solid lines (entanglement ($\mathcal{C}[\rho_T] = 0$)) and dashed lines (Bell nonlocality).
- **Near Threshold ($\beta \to 0$):** Maximally entangled singlet Ψ^-
- **High Energy ($\beta \to 1$) with $\theta = \pi/2$:** Maximally entangled triplet Φ^-.

- Low background and Maximal signal. Better to have LT separation! (Also UPC)
- Possible measurements: $b\bar{b}$ or $c\bar{c}$ or hyperon $\Lambda\bar{\Lambda}$.
- Diffractive DIS [Fucilla and Hatta, 2509.05267] L: Maximal Entanglement; T: entanglement is necessary and sufficient for Bell-nonlocality.
- QI at EIC [Cheng, Han, Trifinopoulos, 2510.23773] (Polarized ep collisions)

Λ Hyperon: Nature's Built-in Spin Analyzer

The Hyperon Decays

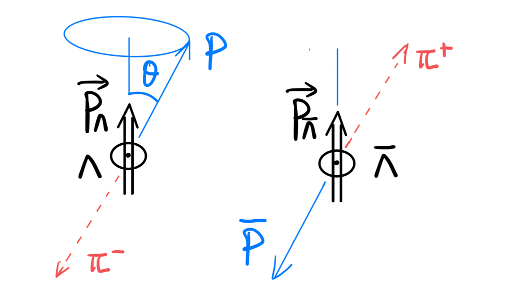
$$\Lambda \to p + \pi^-$$
$$\bar{\Lambda} \to \bar{p} + \pi^+$$

Anisotropic Angular Distribution:

$$\frac{dN_\Lambda}{N_\Lambda d\cos\theta} = \frac{1}{2}\left(1 + \alpha_\Lambda \vec{P}_\Lambda \cdot \hat{p}_p\right)$$
$$\frac{dN_{\bar{\Lambda}}}{N_{\bar{\Lambda}} d\cos\theta} = \frac{1}{2}\left(1 + \alpha_{\bar{\Lambda}} \vec{P}_{\bar{\Lambda}} \cdot \hat{p}_{\bar{p}}\right)$$

- Asymmetry parameter $\alpha_\Lambda \simeq -\alpha_{\bar{\Lambda}} = 0.75$
- Proton predominantly is going off in the direction of the spin of the Lambda.

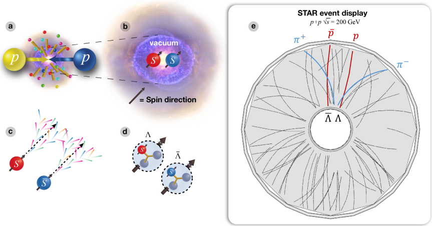
Self-Analyzing Property



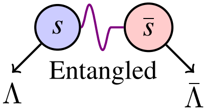
- Weak decay violates parity.
- Proton direction reveals Λ spin direction

First evidence of spin correlation in $\Lambda\bar{\Lambda}$ hyperon pairs

STAR Collaboration [Nature 2026; arXiv:2506.05499] $p + p$ collisions at $\sqrt{s} = 200$ GeV



Entanglement as a Tool



- QCD Confinement
- Chiral Symmetry
- Spin Dynamics
- Decoherence
- Bell Nonlocality
- Nuclear Medium !?

- Relative polarization (same as D): $P_{\Lambda\bar{\Lambda}} = (18 \pm 4)\%$
- Parallel: $1/3$; Antiparallel: -1; no spin correlation 0.
- Short-range pairs show maximal entanglement
- Long-range pairs: correlation vanishes (decoherence)
- Evidence for quantum entanglement in QCD vacuum

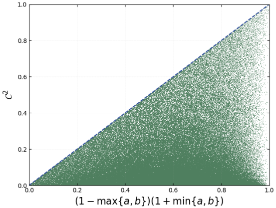
"Entanglement: A new paradigm for exploring QCD phenomena"

Upper Bound on Concurrence with Polarization

[Liu, Qi, He, Xiao] arXiv:2604.17756 [hep-ph] **Tighter Upper Bound:**

$$\boxed{\mathcal{C}_{\max} = \sqrt{(1 - \max\{\|\vec{B}^{+}\|, \|\vec{B}^{-}\|\})(1 + \min\{\|\vec{B}^{+}\|, \|\vec{B}^{-}\|\})}}$$

Numerical Verification:



2,000,000 random density matrices verify the bound

- **Depends on both** $\|\vec{B}^{+}\|$ and $\|\vec{B}^{-}\|$
- **Saturated by pure states** $(\vec{B}^{+} = \vec{B}^{-})$

Physical Interpretation:

- Polarization stores spin information **locally**
- Reduces room for **nonlocal** quantum correlation
- For a family of ρ with fixed B, the largest $\mathcal{C}$ achievable within that family.

Application to $e^{+}e^{-} \to Z^{0} \to q\bar{q}$:

- **Parity violation** generates polarization
- **Up:** $\mathcal{C}_{\max} \simeq 0.744$; **Down:** $\mathcal{C}_{\max} \simeq 0.353$
- Much smaller than unpolarized case $(\mathcal{C}_{\max} = 1)$

Concurrence of $\Lambda\bar{\Lambda}$ Pairs: e^+e^- vs pp vs pA

e^+e^- annihilation (cleanest):

- Single channel: $\gamma^*/Z \to s\bar{s}$
- At Z pole: parity violation polarizes $s\bar{s} \Rightarrow \mathcal{C}_{\max} \simeq 0.35$ (down-type bound)

pp collisions (diluted):

- Channel mixture $(gg, q\bar{q}) \Rightarrow$ mixed ρ
- STAR: short-range $P_{\Lambda\bar{\Lambda}} \approx 18\%$;

pA collisions (decohered):

- Final-state rescattering in nuclear matter
- Medium acts as "measurement" $\Rightarrow$ decoherence

System	Concurrence
$e^+e^- \to \gamma^*$	Largest ($\mathcal{C} \lesssim 1$)
e^+e^- at Z pole	Bounded ($\mathcal{C} \lesssim 0.35$)
pp (short-range pairs)	Small but nonzero
pA	Smallest, A-dependent

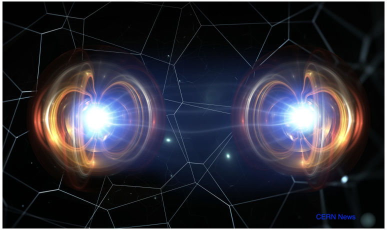
The suppression pattern

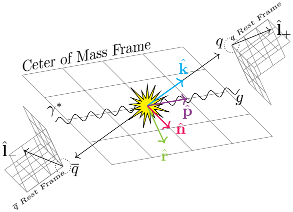
$$\mathcal{C}_{e^+e^-} > \mathcal{C}_{pp} > \mathcal{C}_{pA}$$

- e^+e^- / EIC: calibrate "vacuum" entanglement from hadronization
- $pp \to pA$: isolate channel mixing and medium-induced decoherence

"Quantum decoherence tomography of the nuclear medium"

Summary and Outlook





- Entanglement and Bell Nonlocality are measurable at high energy collisions.
- EIC offers a unique and clean experimental environment for measuring maximal entanglement and Bell Nonlocality.
- Using entanglement as a tool to probe **nuclear environment** and other QCD effects.
- **New bound:** final-state polarization sets a tighter upper limit on the concurrence — the more spin information stored locally, the less entanglement is possible.