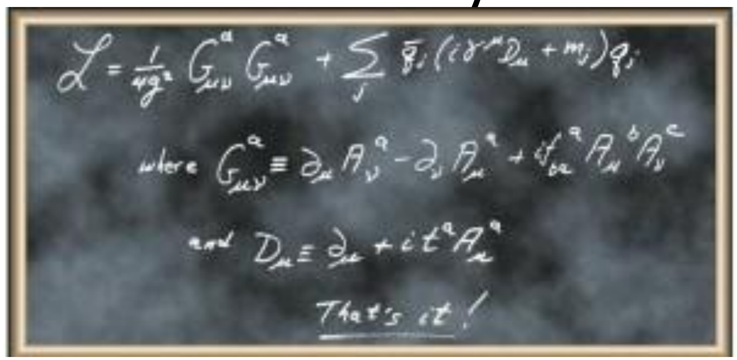
Amplitude analysis

Quantum Chromo Dynamics

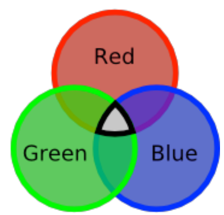
F. Wilczek, QCD Made Simple
Physics Today **53N8** 22-28, (2000)

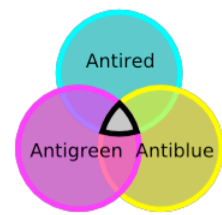


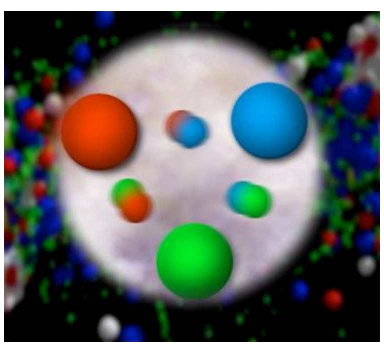
The rules that govern how the quarks froze out into hadrons are given by QCD.

Quarks have color charge: red, blue and green.
Antiquarks have anticolors: cyan, yellow and magenta.

Bound states of quarks are color neutral, ``white''.

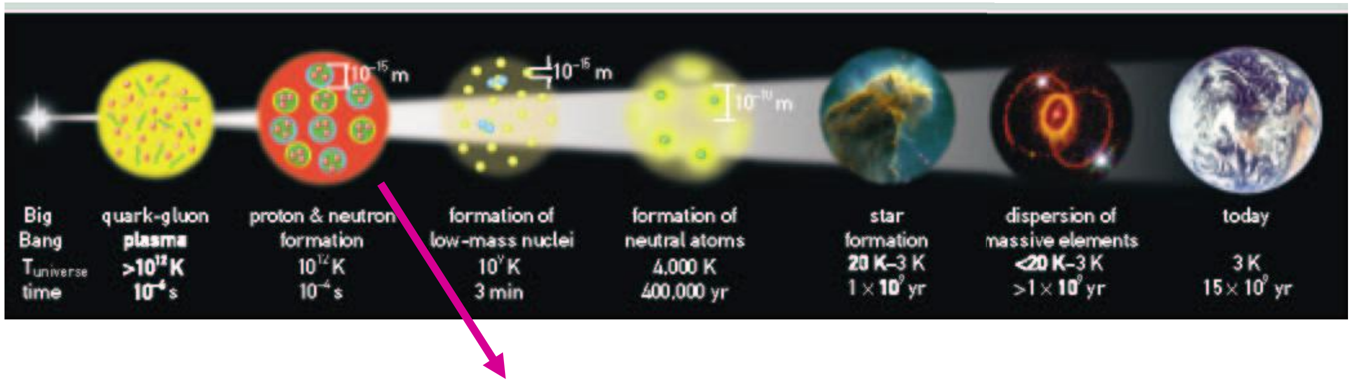




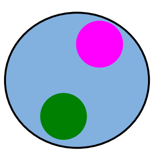


``White'' can be one of each color:
red-blue-green, cyan-yellow-magenta
or a color and an anticolor:
red-cyan, blue-yellow, green-magenta

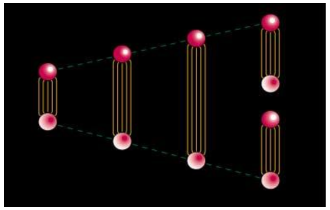
Confinement



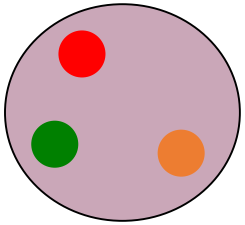
From about 10^{-6} s on, the quark and anti quarks became confined inside of Hadronic matter. At the age of 1s, only protons and neutrons remained.



Mesons



The gluons produce the 16 ton force that binds the quarks.



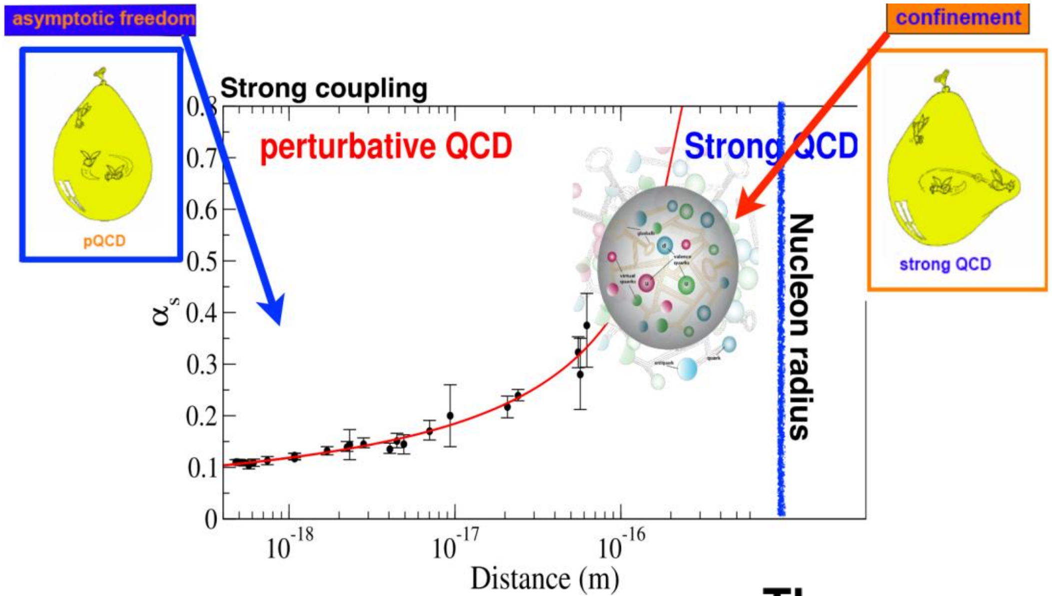
Baryons

Quarks can never be isolated

How does QCD give rise to excited hadrons?

- What is the origin of confinement?
- How are confinement and chiral symmetry breaking connected?
- How is the mass generated in QCD?

Hadron spectroscopy



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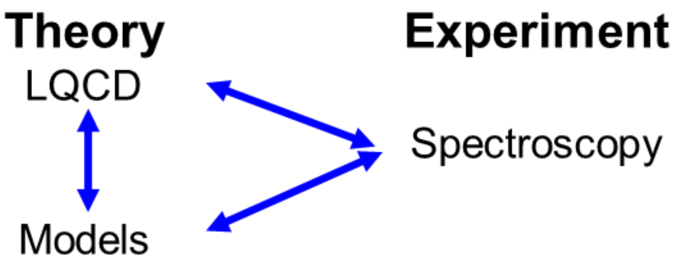
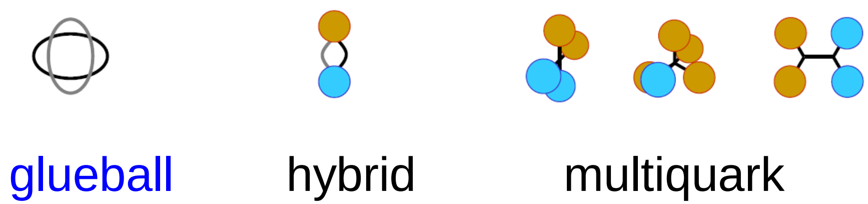


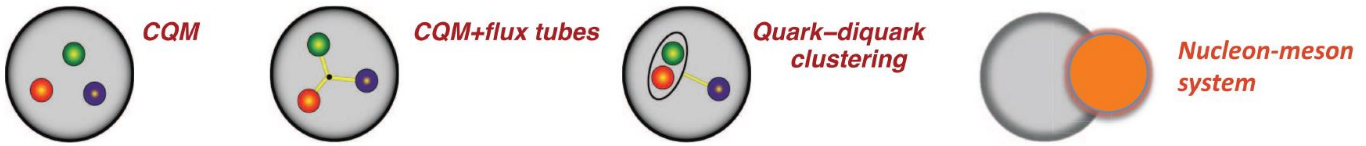
- Testing QCD in the confinement regime
- Revealing the fundamental degrees of freedom

Light hadron spectroscopy at BESIII

- Meson spectroscopy: What are the nature of QCD exotics? What's the role of gluonic excitation and how does it connect to the confinement?

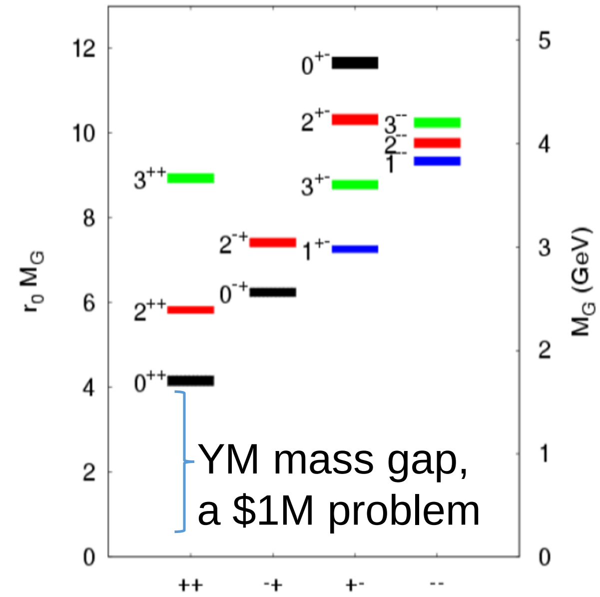


- Baryons spectroscopy: What are the fundamental degrees of freedom inside a nucleon? How do the degrees change with varying quark masses?



Glueballs

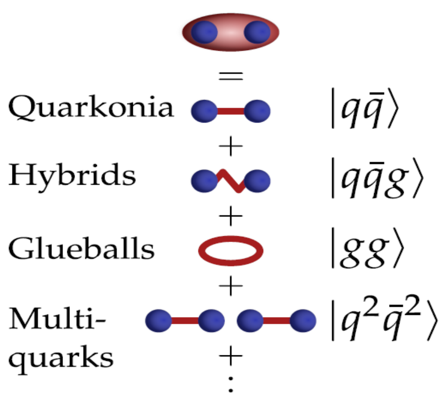
PRD 73 014516 (2006)

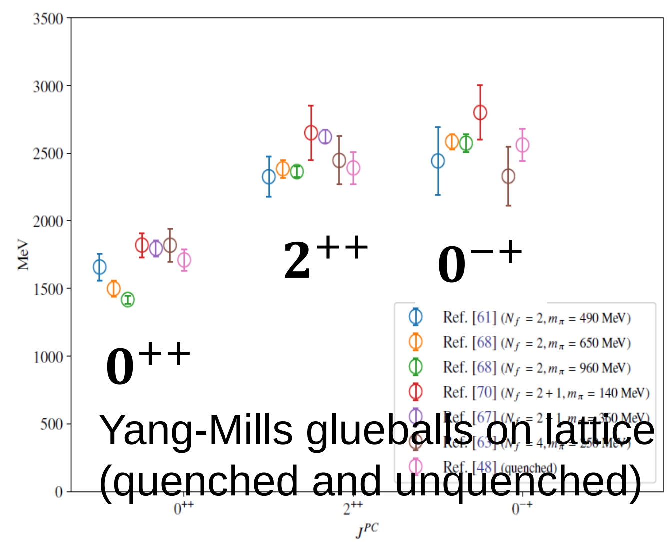


- Glueballs are the most direct prediction of QCD
  - Color singlets emerge as a consequence of the gluon self-interactions
  - Unique particles formed by gauge bosons (force)
- Essential for understanding of confinement and mass dynamical generation
- Theoretical predictions from lattice QCD and QCD-inspired models mostly consistent
  - Light-mass glueballs: $J^{PC} = 0^{++}, 2^{++}, \ 0^{-+}$

arXiv:2305.04869

non-$\mathbf{q\bar{q}}$ nature with ordinary quantum numbers is difficult to establish



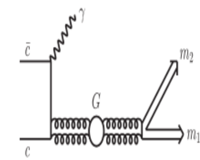


Glueball hunting for over 40 years

- **Supernumerary states** w.r.t. quark model
  - A priori, mixed with nearby $\mathbf{q\bar{q}}$
  - Assignment of some $q\bar{q}$ multiplets is difficult
- Detailed and accurate information about couplings to production and decay channels is required
- Strongly produced in **gluon-rich processes**
- Decay: **gluon is flavor-blind**
  - $\mathrm{SU(3)_{flavor}}$ symmetry expected, but differing quark masses leads exceptions
  - No rigorous predictions on decay patterns
  - Could be analogy to **OZI suppressed** decays of charmonium, as they all decay via gluons [PLB 380 189(1996), Commu. Theor. Phys. 24.373(1995)]

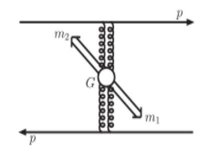
gluon-rich processes

[Phys. Rept. 454 1]



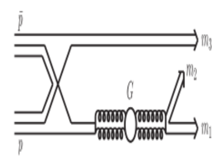
Charmonium decays:

BESIII, MRKIII…



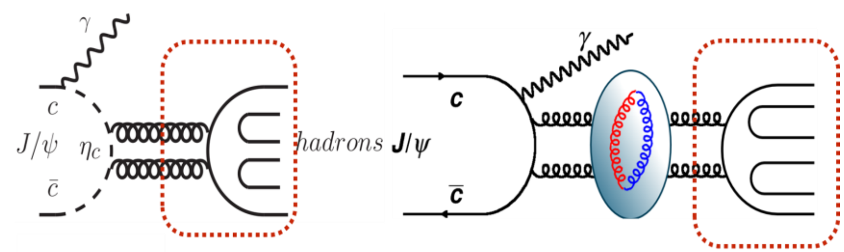
pp double-Pomeron exchange:

WA102, GAMS…



$p\bar{p}$ annihilation:

Crystal barrel, OBELIX…

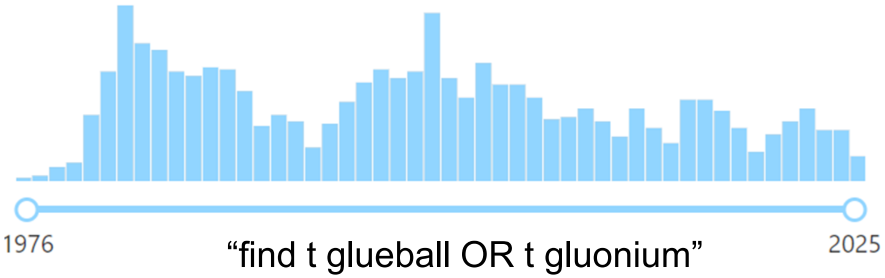


Some glueball candidates in the past

- The first glueball candidate, $\iota(1440)$, observed in J/ψ radiative decays in 1980s
- Scalar candidates $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ (MarkII in1980s, Crystal Barrel in 1990s)
- Narrow tensor glueball candidate $\xi(2230)$ (MarkIII in 1980s/BESI in 1990s)
  - Not confirmed by CLEO, BESII nor BESIII with much higher statistics

And,

- Odderon (odd C-parity) from D0 and TOTEM (2021)

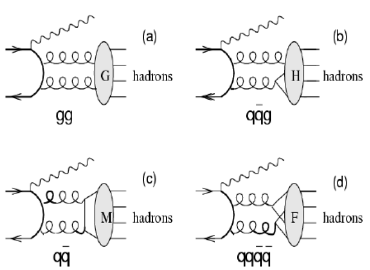


“find t glueball OR t gluonium”

“The Physics of Glueballs”Mathieu, Kochelev, and Vento, 2009
“The Status of Glueballs” Ochs, 2013
“Glueballs as the Ithaca of meson spectroscopy: From simple theory to challenging detection” Llanes-Estrada, 2021
“The Experimental Status of Glueballs” Crede and C. A.Meyer, 2009
…

Charmonium decays provide an ideal lab for light hadron physics

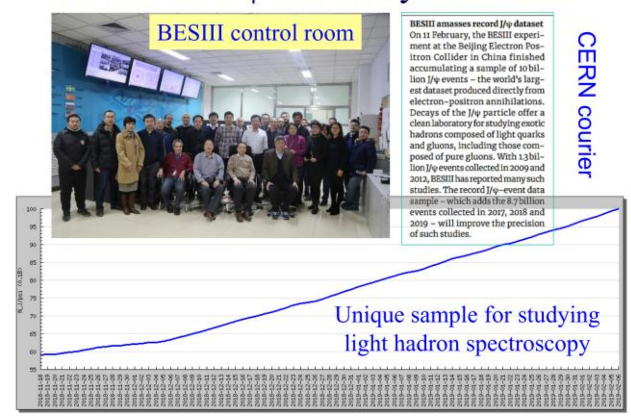
- Clean high statistics data samples
  - High cross sections of $e^+e^- \to J/\psi,\ \psi'$
  - Low background
- Well defined initial and final states
  - Kinematic constraints
  - $I(J^{PC})$ filter
- "Gluon-rich" process



$\Gamma(J/\psi \to \gamma G) \sim O(\alpha\alpha_s^2), \Gamma(J/\psi \to \gamma H) \sim O(\alpha\alpha_s^3),$

$\Gamma(J/\psi \to \gamma M) \sim O(\alpha\alpha_s^4), \Gamma(J/\psi \to \gamma F) \sim O(\alpha\alpha_s^4)$

10 Billion J/ψ events by Feb. 2019

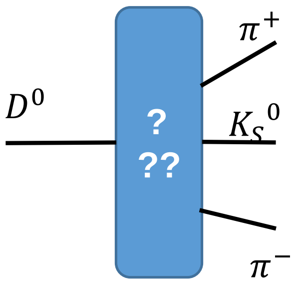


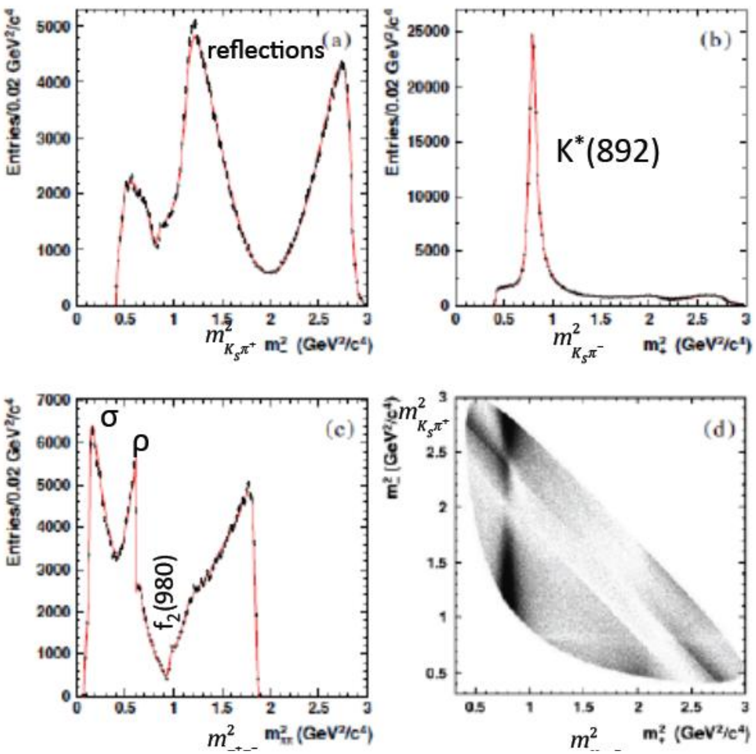
BESIII designed J/ψ stat. has been achieved

——A legacy data set

BESIII remains unique for Light hadron physics

Amplitude analysis

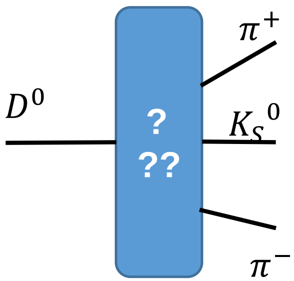


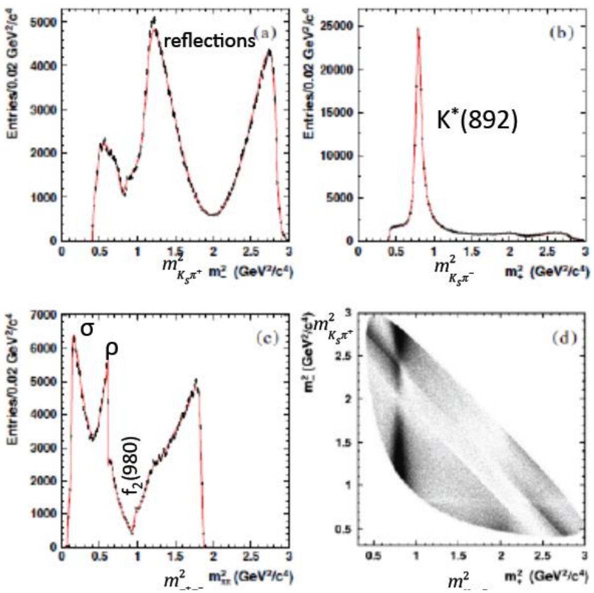


Tasks:

- ☐ **Map out the resonances**
- ☐ **Systematic determination of resonance properties:**
  - **spin-parity,**
  - **resonance parameters,**
  - **production properties,**
  - **decay properties, …**
  - ◆ resonances tend to be broad and plentiful, leading to intricate interference patterns, or buried under a background in the same and in other waves.

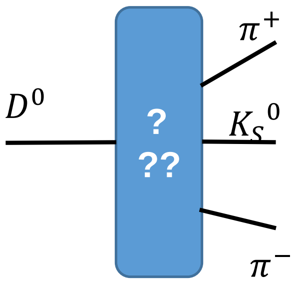
Amplitude Analysis

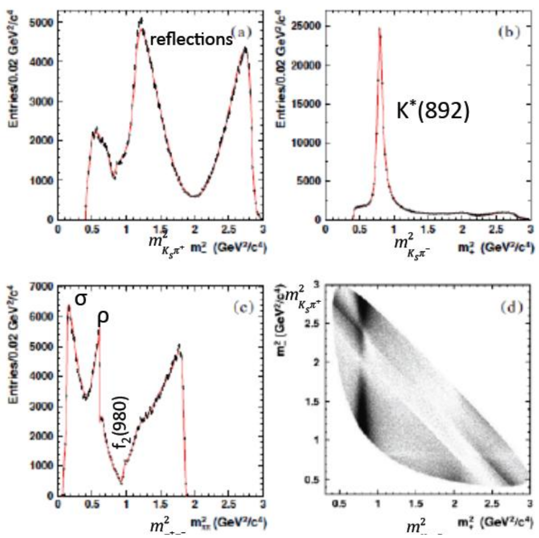




- **Production Amplitude** produces a state **X** with **J^PC** quantum numbers
- **Decay Amplitude** describes the decay of **X** to final state particles
- **Observables** are the four-momenta of the final-state particles

Amplitude Analysis





Several different states, all decaying to the same final particles are produced, and they interfere (complex amplitudes)

The probability to observe the event characterized by the measurement ξ

$$P(\xi:\alpha) = \frac{\omega(\xi,\alpha)\epsilon(\xi)}{\int d\xi\omega(\xi,\alpha)\epsilon(\xi)}$$

Differential cross section

$$\omega(\xi,\alpha) = \frac{d\sigma}{d\Phi} = |\sum_i A_i|^2$$

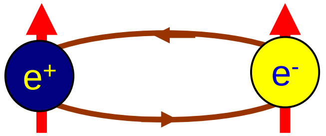
Standard likelihood

$$L = \prod_{i=1}^{N} P(\xi:\alpha)$$

Perform an un-binned log-likelihood fit (fit the data event-by event to high-dimensional distributions using complex weights) to make our model for ω agree with the experimental distribution for ω by varying the α.

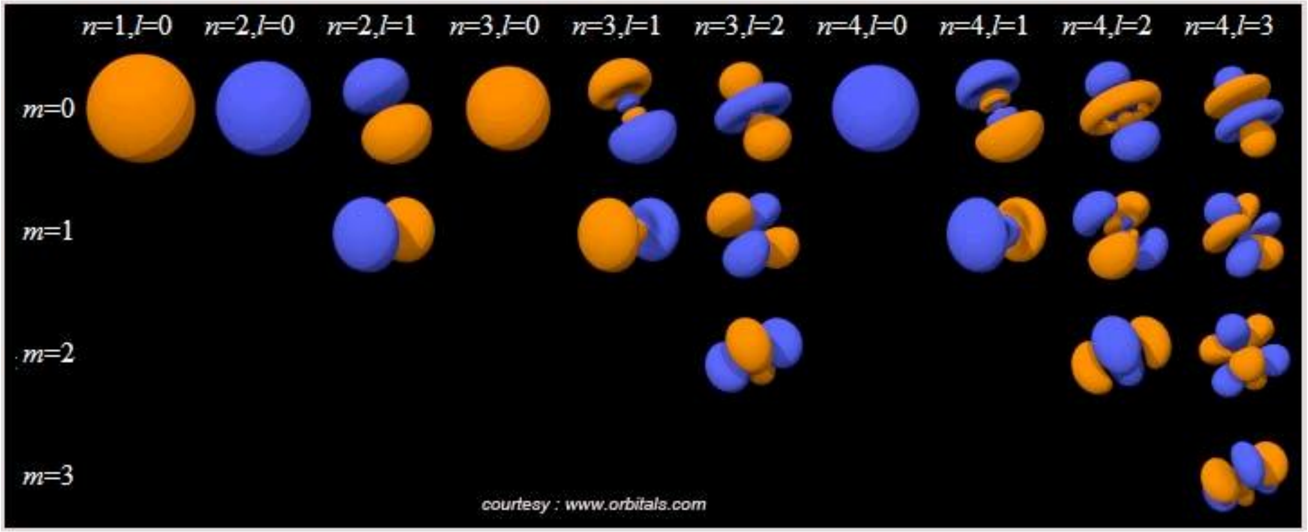
Partial Wave Amplitude

Positronium



$$\psi(\vec{r}, S) = R_{nl}(r)\, Y_{LM}(\theta, \phi)\, \chi(S)$$

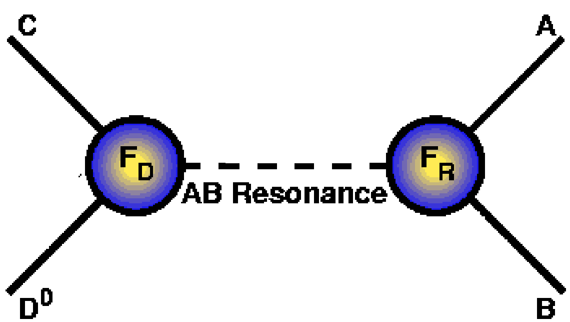
Angular distributions of reactions let you determine the spin and parity of intermediate resonances



Include spin, S, total angular momentum J: J = L + S: $^{(2S+1)}L_J$

Isobar model formalism

D^0 three-body decay $D^0 \rightarrow ABC$ decaying through an r=[AB] resonance



D^0 three-body amplitude

$$\mathcal{A}_D(s_{12}, s_{13}) = a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta_r} \mathcal{A}_r(s_{12}, s_{13})$$

NR term (direct 3 body decay)

a_0, δ_0, a_r, δ_r : Free parameters of fit

$$\mathcal{A}_r(s_{12}, s_{13}) = F_D^J F_r^J \times M_r^J \times BW_r^J$$

Dynamical function, e.g. Breit-Wigner (BW) propagator

Angular distribution: spin formalism

Blatt-Weisskopf centrifugal barrier factor for the D (resonance) decay vertex with radius R,

$J \rightarrow L + l$	Angular distribution
$0 \rightarrow 0 + 0$	uniform
$0 \rightarrow 1 + 1$	$(1+\zeta^2)\cos^2\theta$
$0 \rightarrow 2 + 2$	$\left(\zeta^2 + \frac{3}{2}\right)^2 (\cos^2\theta - 1/3)^2$

Recap

- Amplitude analysis is a key tool of hadron spectroscopy: A state-of-the-art way to disentangle contributions from individual, and even small, resonances and to extract the resonance's spin-parity, mass, width and decay properties with high sensitivity and accuracy
- Event-based fits allow one to take into account the full correlation between final-state particles
- Amplitude analysis remains challenging in models and techniques

Challenge: Computing

- PWA is time consuming
  - Large amount of fits (hypothesis tests, systematics)
  - Large amount of data (unbinned likelihood fit)

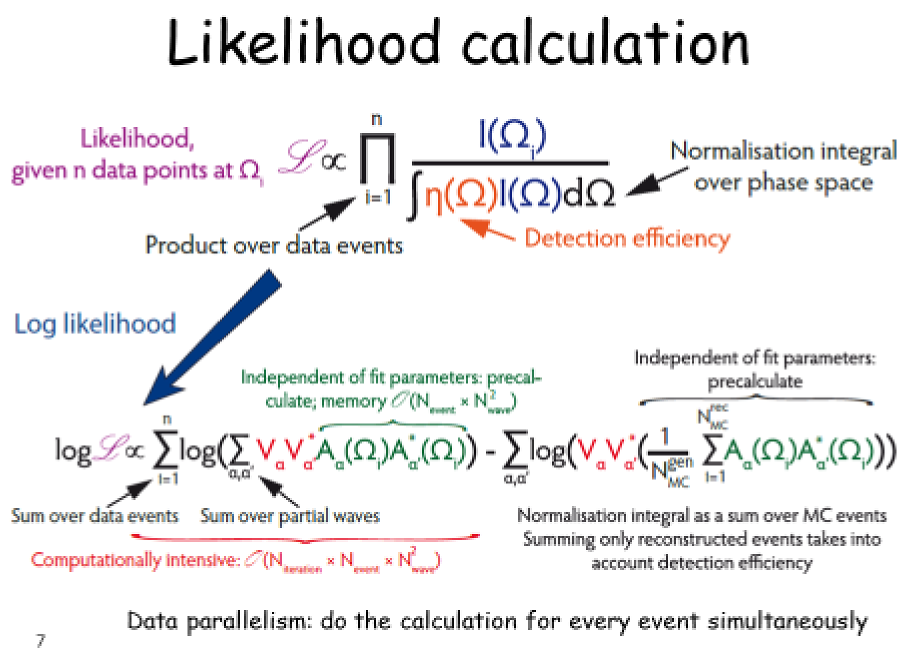
Event-wise ML fit to all observables simultaneously

dynamic angular

$$\omega(\xi) \equiv \frac{d\sigma}{d\Phi} = \left|\sum_i c_i R_i B(p,q) Z(L)\right|^2$$

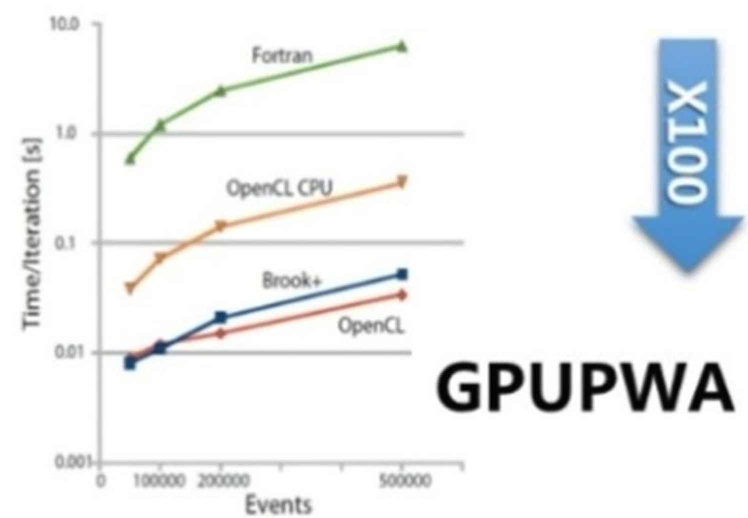
Event-wise efficiency correction

$$P(\xi) = \frac{\omega(\xi)\epsilon(\xi)}{\int \omega(\xi)\epsilon(\xi)}$$



Challenge: Computing

- High performance computing with GPU, pioneered by BESIII



http://sourceforge.net/projects/gpupwa

N. Berger, B. J. Liu, and J. K. Wang, J. Phys. Conf. Ser. 219, 042031(2010)

ATHOS White paper: Analysis Tools for Next-Generation Hadron Spectroscopy Experiments [Acta. Phys.Polon. B46 (2015) 257]

The most promising avenue for PWA is general purpose graphical processor unit (GPGPU) programming. Making use of the many cores on a GPU, likelihood calculations can be performed on many chunks of data at the same time. The pioneer approach of harnessing GPU parallel acceleration in PWA was performed in the framework of BESS-III [171]. Presently there are several hardware-specific programming models (CUDA, OpenCL), but the field is in a state of rapid

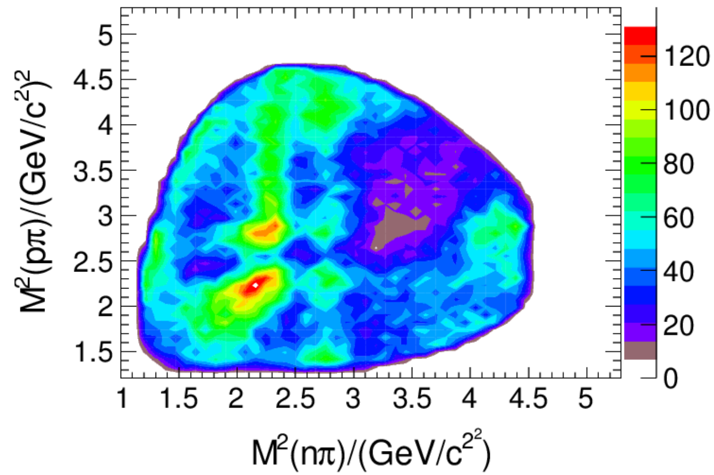
Challenge: Background

- Background modeling

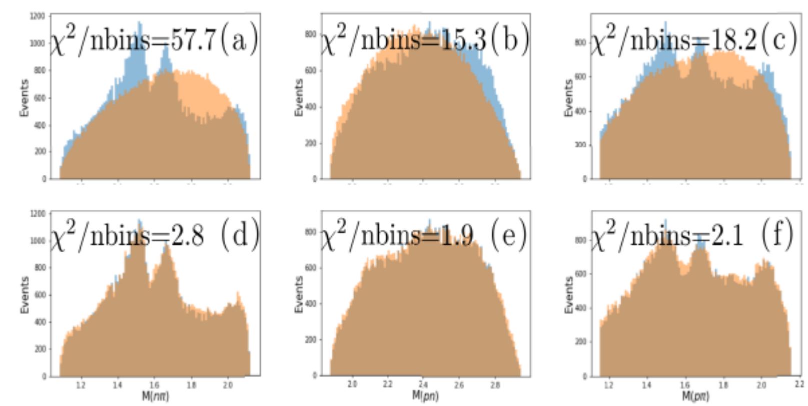
  L_{S+B}

- Background subtraction

  $L_S(data) - L_S(background)$



- Recent progress: ML based multi-dimensional reweighting
  - to obtain data-like MC for dominate background



EPJ Web Conf.CHEP 2018

Challenge: Resolution

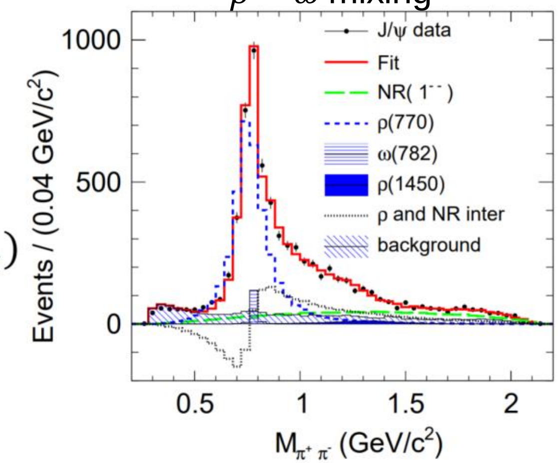
$$\frac{d\sigma}{d\Phi} = |\sum F_i A_i|^2 = \sum F_i F_j^* A_i A_j^* = \sum F_i F_j^* T_i T_j P_i P_j^*$$

Propagator
where mass resolution matters

Magnitude/Phase
independent of mass

Orbital Tensor
weak dependency on mass

$J/\psi \to \pi^+\pi^-\eta'$
Phys. Rev. D 96, 112012
$\rho - \omega$ mixing

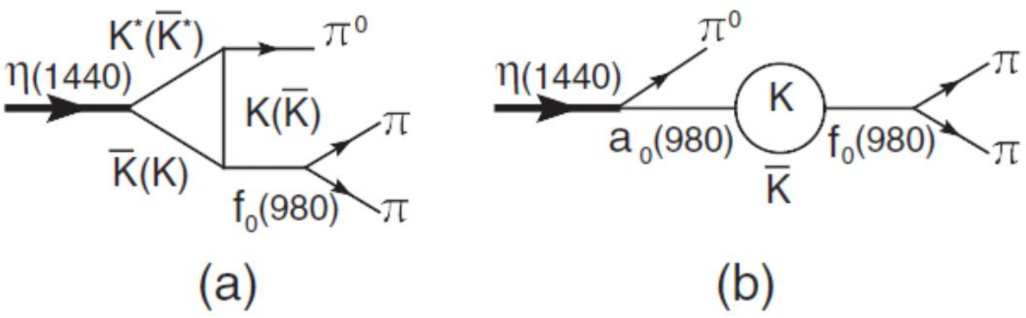


w/o mass resolution $P_i P_j^* = f_i(x) f_j^*(x) \cdot f_i(y) f_j^*(y) \cdot f_i(z) f_j^*(z) \cdot \ldots$

w/ mass resolution $P_i P_j^* = f_i(x) f_j^*(x) \otimes g(x) \cdot f_i(y) f_j^*(y) \otimes g(y) \cdot f_i(z) f_j^*(z) \otimes g(z) \cdot \ldots$
$\equiv h_{ij}(x) \cdot h_{ij}(y) \cdot h_{ij}(z) \cdot \ldots$

$$h_{ij}(x) = \int f_i(x-m) f_j^*(x-m) g(m) \mathrm{d}m \simeq \sum_k w(m_k) f_i(x-m_k) f_j^*(x-m_k)$$

Challenge: Dynamic models
——Experiment-theory cooperation

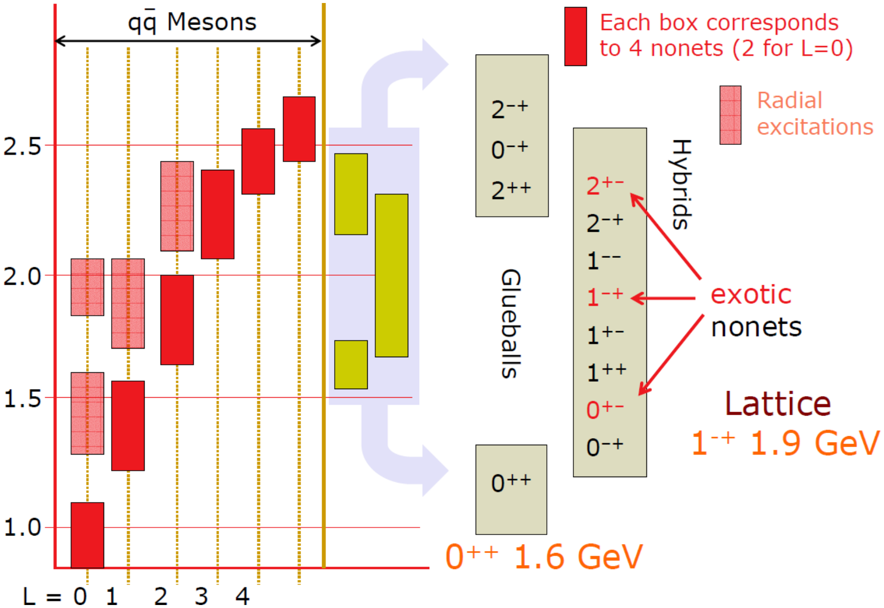


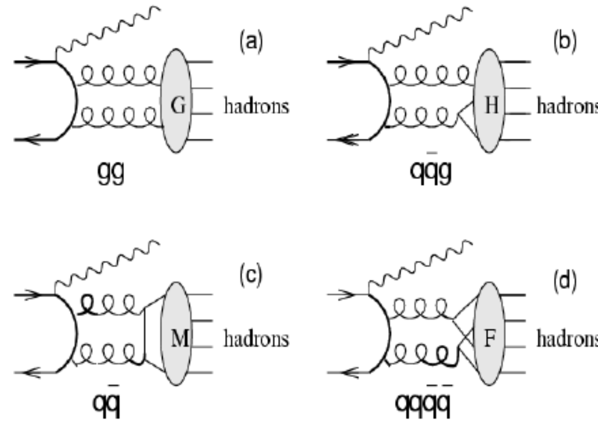
Triangle singularity

Original : $A_{total} = \epsilon_{\psi_\mu}\epsilon_{\gamma_\nu}\Lambda U^{\mu\nu}$

Modified: $A_{total} = \epsilon_{\psi_\mu}\epsilon_{\gamma_\nu}(\Lambda U^{\mu\nu} + \Lambda' \; I \; U^{\mu\nu})$

coefficient to be fitted   Loop integral of coupling





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$\Gamma(J/\psi \to \gamma M) \sim O(\alpha\alpha_s^4), \Gamma(J/\psi \to \gamma F) \sim O(\alpha\alpha_s^4)$

Highlights: Light meson spectroscopy

- Search for glueballs and hybrids

Scalar glueball candidate

- **Supernumerary scalars** suggest additional degrees of freedom
  - However, mixing scenarios are controversial
- Measured $\mathrm{B}(\mathrm{J}/\psi \to \gamma f_0(1710))$ is **x10 larger** than $f_0(1500)$

  BESIII [PRD 87 092009, PRD 92 052003, PRD 98 072003]

  - LQCD: $\Gamma(\mathrm{J}/\psi \to \gamma G_{0+})/\Gamma_{\mathrm{total}} = 3.8(9) \times 10^{-3}$ [PRL 110, 091601(2013)]

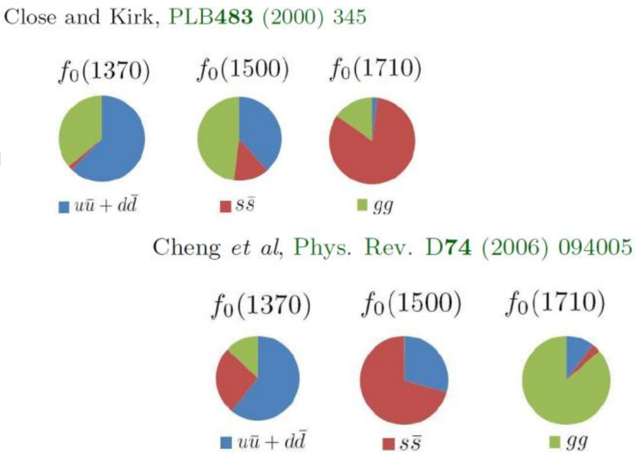
➢**BESIII: $f_0(1710)$ largely overlays with the scalar glueball**

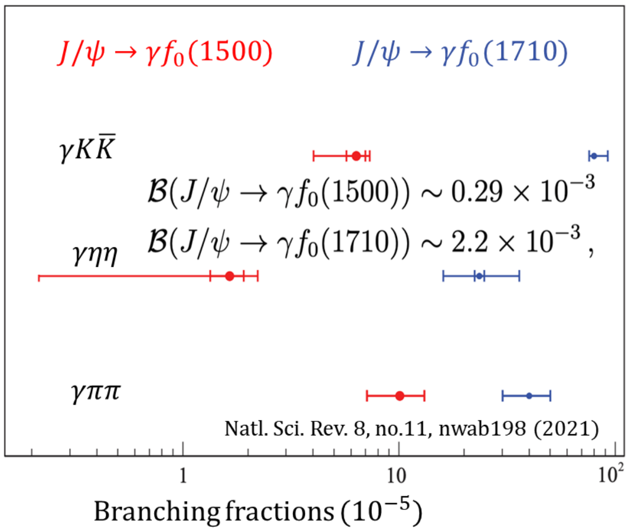
➢**Identification of scalar glueball with coupled-channel analyses based on BESIII data**

[PLB 816, 136227 (2021), EPJC 82, 80 (2022), PLB 826, 136906 (2022)]

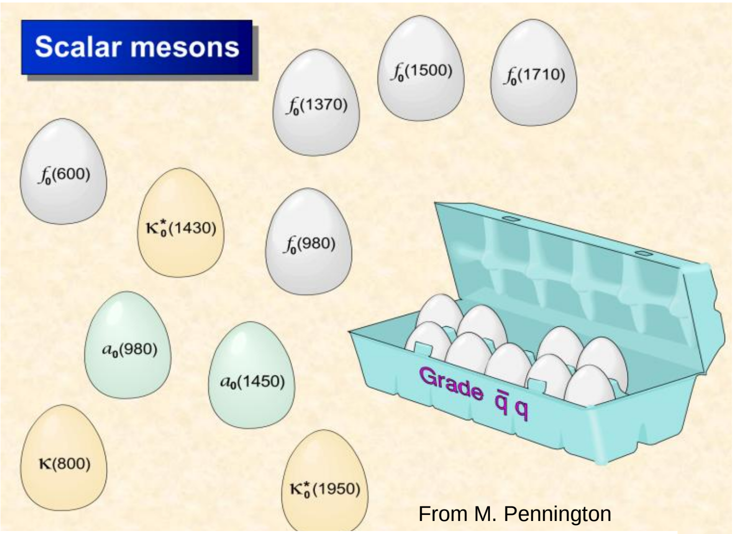
- **Further more, suppression of $f_0(1710) \to \eta\eta'$ supports $f_0(1710)$ has a large overlap with glueball**

BESIII [PRD 106 072012(2022)]

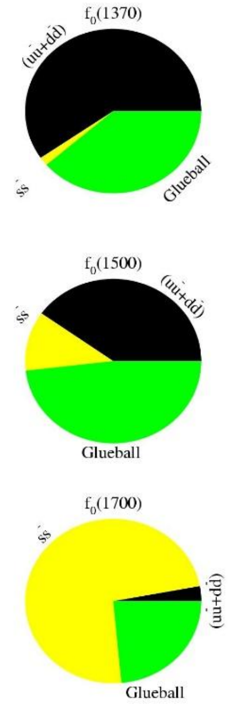




Overpopulated scalar mesons



Name	Mass [MeV/c^2]	Width [MeV/c^2]
$f_0(600)$ *	400 – 1200	600 – 1000
$f_0(980)$ *	980 ± 10	40 – 100
$f_0(1370)$ *	1200 – 1500	200 – 500
$f_0(1500)$ *	1507 ± 5	109 ± 7
$f_0(1710)$ *	1718 ± 6	137 ± 8
$f_0(1790)$		
$f_0(2020)$	1992 ± 16	442 ± 60
$f_0(2100)$	2103 ± 7	206 ± 15
$f_0(2200)$	2189 ± 13	238 ± 50



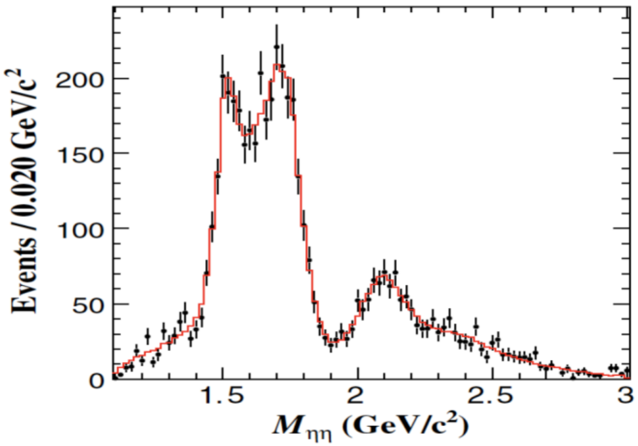
Mixing scheme：

very controversial and model dependent

$f_0(1500)$, $f_0(1710)$, which one has more gluonic component?

Amplitude analysis of $J/\psi \to \gamma\eta\eta/K_S^0K_S^0$

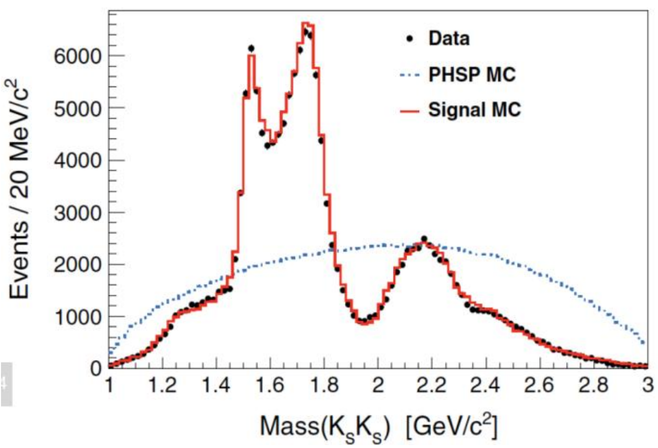
BESIII PRD 87, 092009 (2013)



Resonance	Mass (MeV/c^2)	Width (MeV/c^2)	$\mathcal{B}(J/\psi \to \gamma X \to \gamma\eta\eta)$	Significance
$f_0(1500)$	1468^{+14+23}_{-15-74}	$136^{+41+28}_{-26-100}$	$(1.65^{+0.26+0.51}_{-0.31-1.40}) \times 10^{-5}$	8.2σ
$f_0(1710)$	$1759 \pm 6^{+14}_{-25}$	$172 \pm 10^{+32}_{-16}$	$(2.35^{+0.13+1.24}_{-0.11-0.74}) \times 10^{-4}$	25.0σ
$f_0(2100)$	$2081 \pm 13^{+24}_{-36}$	273^{+27+70}_{-24-23}	$(1.13^{+0.09+0.64}_{-0.10-0.28}) \times 10^{-4}$	13.9σ
$f_2'(1525)$	$1513 \pm 5^{+4}_{-10}$	75^{+12+16}_{-10-8}	$(3.42^{+0.43+1.37}_{-0.51-1.30}) \times 10^{-5}$	11.0σ
$f_2(1810)$	1822^{+29+66}_{-24-57}	$229^{+52+88}_{-42-155}$	$(5.40^{+0.60+3.42}_{-0.67-2.35}) \times 10^{-5}$	6.4σ
$f_2(2340)$	$2362^{+31+140}_{-30-63}$	$334^{+62+165}_{-54-100}$	$(5.60^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$	7.6σ

Br of $f_0(1710)$ ~10x larger than $f_0(1500)$

BESIII PRD 98, 072003 (2018)



Resonance	M (MeV/c^2)	M_{PDG} (MeV/c^2)	Γ (MeV/c^2)	Γ_{PDG} (MeV/c^2)	Branching fraction	Significance
$K^*(892)$	896	895.81 ± 0.19	48	47.4 ± 0.6	$(6.28^{+0.16+0.59}_{-0.17-0.52}) \times 10^{-6}$	35σ
$K_1(1270)$	1272	1272 ± 7	90	90 ± 20	$(8.54^{+1.07+2.35}_{-1.20-2.13}) \times 10^{-7}$	16σ
$f_0(1370)$	$1350 \pm 9^{+12}_{-2}$	1200 to 1500	$231 \pm 21^{+28}_{-48}$	200 to 500	$(1.07^{+0.08+0.36}_{-0.07-0.34}) \times 10^{-5}$	25σ
$f_0(1500)$	1505	1504 ± 6	109	109 ± 7	$(1.59^{+0.16+0.18}_{-0.16-0.56}) \times 10^{-5}$	23σ
$f_0(1710)$	$1765 \pm 2^{+1}_{-1}$	1723^{+6}_{-5}	$146 \pm 3^{+7}_{-1}$	139 ± 8	$(2.00^{+0.03+0.31}_{-0.02-0.10}) \times 10^{-4}$	$\gg 35\sigma$
$f_0(1790)$	$1870 \pm 7^{+2}_{-3}$	...	$146 \pm 14^{+7}_{-15}$	...	$(1.11^{[illegible]}_{-0.06-0.32}) \times 10^{-5}$	24σ
$f_0(2200)$	$2184 \pm 5^{+4}_{-2}$	2189 ± 13	$364 \pm 9^{+4}_{-7}$	238 ± 50	$(2.72^{+0.08+0.17}_{-0.06-0.47}) \times 10^{-4}$	$\gg 35\sigma$
$f_0(2330)$	$2411 \pm 10 \pm 7$	...	$349 \pm 18^{+23}_{-1}$	...	$(4.95^{+0.21+0.66}_{-0.21-0.72}) \times 10^{-5}$	35σ
$f_2(1270)$	1275	1275.5 ± 0.8	185	$186.7^{+2.2}_{-2.5}$	$(2.58^{+0.08+0.59}_{-0.09-0.20}) \times 10^{-5}$	33σ
$f_2'(1525)$	1516 ± 1	1525 ± 5	$75 \pm 1 \pm 1$	73^{+6}_{-5}	$(7.99^{+0.03+0.69}_{-0.04-0.50}) \times 10^{-5}$	$\gg 35\sigma$
$f_2(2340)$	$2233 \pm 34^{+9}_{-25}$	2345^{+50}_{-40}	$507 \pm 37^{+18}_{-21}$	322^{+70}_{-60}	$(5.54^{+0.34+3.82}_{-0.40-1.49}) \times 10^{-5}$	26σ
0^{++} PHSP	...	...	...	...	$(1.85^{+0.05+0.68}_{-0.05-0.26}) \times 10^{-5}$	26σ
2^{++} PHSP	...	...	...	...	$(5.73^{+0.99+4.18}_{-1.00-3.74}) \times 10^{-5}$	13σ

Scalar glueball candidate

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}|E_1(0)|^2 = 0.35(8) keV$$
$$\Gamma/\Gamma_{tot} = 0.33(7)/93.2 = 3.8(9) \times 10^{-3}$$

CLQCD, Phys. Rev. Lett. 110, 021601 (2013)

Experimental results

- $B(J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K}) = (8.5^{+1.2}_{-0.9}) \times 10^{-4}$
- $B(J/\psi \to \gamma f_0(1710) \to \gamma\pi\pi) = (4.0 \pm 1.0) \times 10^{-4}$
- $B(J/\psi \to \gamma f_0(1710) \to \gamma\omega\omega) = (3.1 \pm 1.0) \times 10^{-4}$
- $B(J/\psi \to \gamma f_0(1710) \to \gamma\eta\eta) = (2.35^{+0.13+1.24}_{-0.11-0.74}) \times 10^{-4}$

$\Rightarrow$ $B(J/\psi \to \gamma f_0(1710)) > 1.7 \times 10^{-3}$

Flavor-blindness of glueball decays

$$\frac{1}{P.S.}\Gamma(G \to \pi\pi : K\overline{K} : \eta\eta : \eta\eta' : \eta'\eta') = 3:4:1:0:1$$

*with chiral suppression

PRL 98 149103

$$\Gamma(G \to \pi\pi)/\Gamma(G \to K\bar{K}) \approx \frac{f_\pi^4}{f_K^4} \approx 0.48$$

$$\frac{1}{P.S.}\Gamma(G \to \pi\pi : K\bar{K} : \eta\eta) \approx 1.3 : 3.16 : 1$$

$f_0(1710)$ largely overlapped with scalar glueball

Other information

Two photon couplings

"Stickness"

PDG2018

Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)

$f_0(1710)$ $\Gamma(i)\Gamma(\gamma\gamma)/\Gamma(\text{total})$

$\Gamma(K\overline{K}) \times \Gamma(\gamma\gamma)/\Gamma_{\text{total}}$ **$\Gamma_1\Gamma_4/\Gamma$**

VALUE (eV)	CL%	DOCUMENT ID		TECN	COMMENT
$12^{+3}_{-2}{}^{+227}_{-\ 8}$		UEHARA	13	BELL	$\gamma\gamma \to K^0_S K^0_S$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
<480	95	ALBRECHT	90G	ARG	$\gamma\gamma \to K^+K^-$
<110	95	[1] BEHREND	89C	CELL	$\gamma\gamma \to K^0_S K^0_S$
<280	95	[1] ALTHOFF	85B	TASS	$\gamma\gamma \to K\overline{K}\pi$

However, a scalar in $\gamma\gamma \to \pi^0\pi^0$

Belle PRD 78 052004

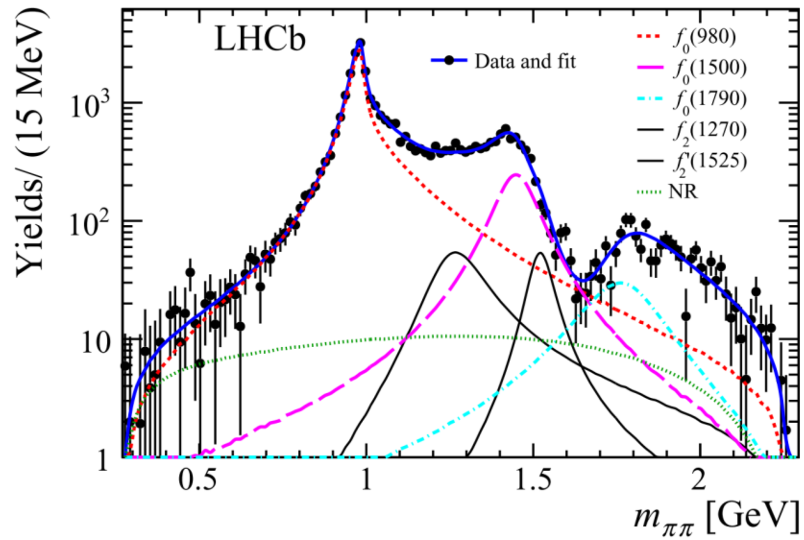
TABLE VI: Fitted parameters of the $f_0(Y)$

Parameter	Belle($\pi^0\pi^0$)	Crystal Ball	$f_0(1370)$(PDG)	$f_0(1500)$(PDG)	Unit
Mass	$1470\ ^{+6}_{-7}\ ^{+72}_{-255}$	1250	1200 - 1500	1507 ± 5	MeV/c^2
Γ_{tot}	$90\ ^{+2}_{-1}\ ^{+50}_{-22}$	268 ± 70	150 - 200	109 ± 7	MeV
$\Gamma_{\gamma\gamma}\mathcal{B}(\pi^0\pi^0)$	$11\ ^{+4}_{-2}\ ^{+603}_{-7}$	430 ± 80	Unknown	Not seen	eV

$f_0(1370)$? $f_0(1500)$?

$B_s \to J/\psi f_0$
is selective for $s\bar{s}$

PLB 797 (2019) 134789



observation of $f_0(1500)$,
non-observation of $f_0(1710)$

Assignment requires further study with more sophisticated model

Tensor glueball candidate

$$\Gamma(J/\psi \rightarrow \gamma G_{2^+}) = 1.01(22)keV$$

$$\Gamma(J/\psi \rightarrow \gamma G_{2^+})/\Gamma_{tot} = 1.1 \times 10^{-2}$$

CLQCD, Phys. Rev. Lett. 111, 091601 (2013)

Experimental results

$$Br(J/\psi \rightarrow \gamma f_2(2340) \rightarrow \gamma\eta\eta) = (3.8^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$$

Phys.Rev. D87, 092009 (2013)

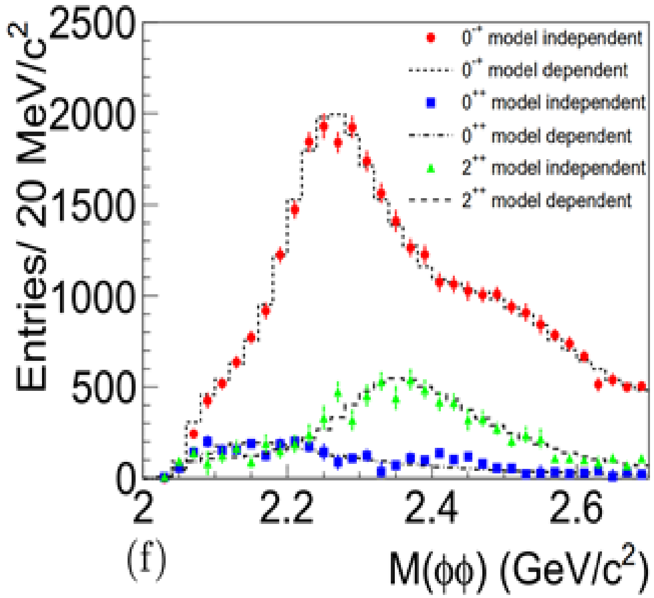
$$Br(J/\psi \rightarrow f_2(2340) \rightarrow \gamma\phi\phi) = (1.91 \pm 0.14^{+0.72}_{-0.73}) \times 10^{-4}$$

Phys.Rev. D93, 112011 (2016)

$$Br(J/\psi \rightarrow \gamma f_2(2340) \rightarrow \gamma K_S K_S) = (5.54^{+0.34+3.82}_{-0.40-1.49}) \times 10^{-5}$$

Phys.Rev. D98, 072003 (2018)

BESIII $J/\psi \rightarrow \gamma\phi\phi$

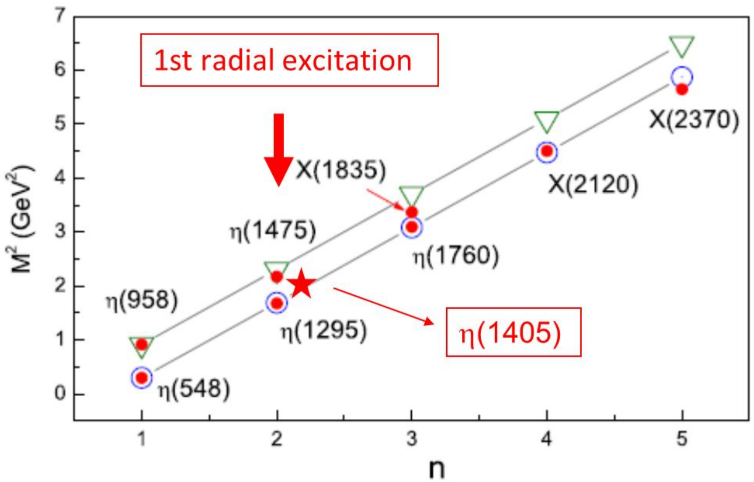


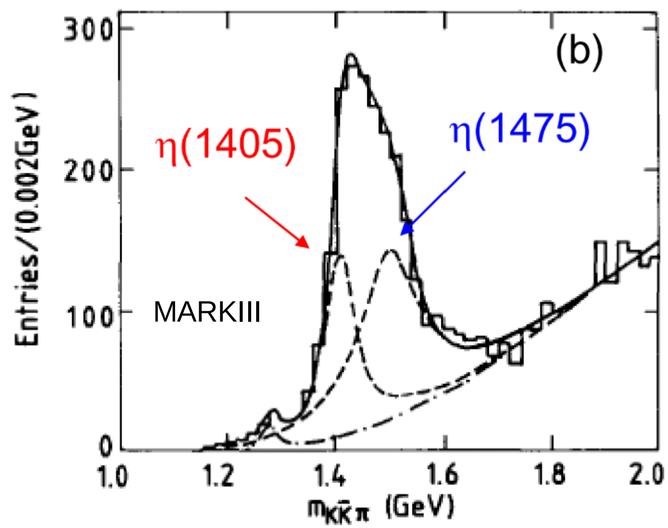
$f_2(2010)$, $f_2(2300)$ and $f_2(2340)$ stated in $\pi^- p$ reactions are observed with a strong production of $f_2(2340)$ Consist with central exclusion production in WA102

It is desirable to search for more decay modes

Pseudoscalar glueball

The small number of expected pseudoscalars in the quark model provide a clean and promising environment for the search of glueballs





Where is the 0^{-+} glueball

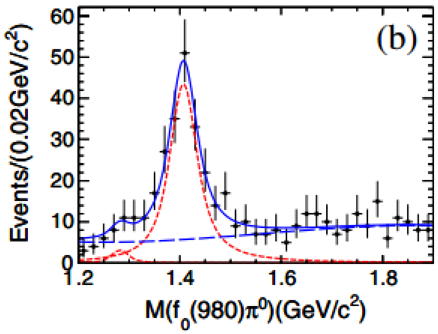
- LQCD: 0^{-+}(2.3~2.6 GeV)
- Does $\eta(1295)$ exist?
- What' s the nature of the outnumbered $\eta(1405)$?

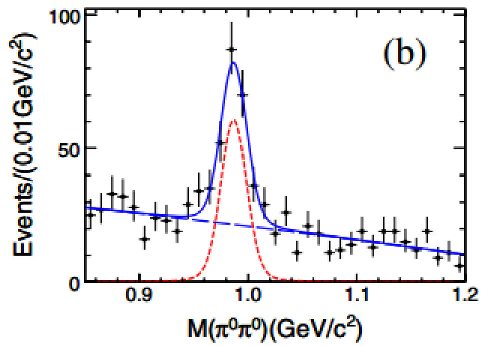
Long standing E-ι puzzle

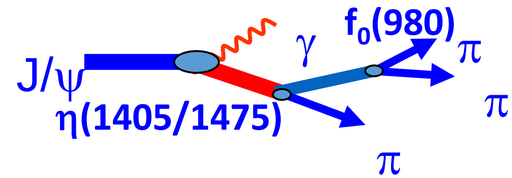
$$M = 1416 \pm 8^{+7}_{-5};\ \Gamma = 91^{+67}_{-31-38}{}^{+15}\ \mathrm{MeV}/c^2$$

$$M = 1490^{+14+3}_{-8-6};\ \Gamma = 54^{+37+13}_{-21-24}\ \mathrm{MeV}/c^2$$

Isospin-violating decay of $\eta(1405) \to f_0(980)\pi^0$







BESIII PRL 108 182001

f0(980) is extremely narrow: $\Gamma \cong 10$ MeV.

PDG: Γ(f0(980)) $\cong$ 40~100 MeV.

Anomalously large isospin violation:

$$\frac{Br(\eta(1405) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0)}{Br(\eta(1405) \to a_0^0(980)\pi^0 \to \eta\pi^0\pi^0)} \cong (17.9 \pm 4.2)\%$$

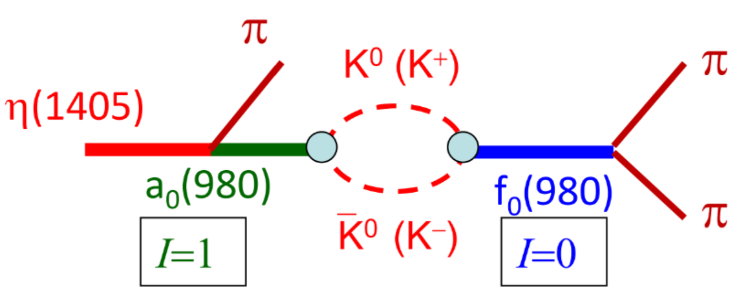
$$\xi_{af} = \frac{Br(\chi_{c1} \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0)}{Br(\chi_{c1} \to a_0(980)\pi^0 \to \eta\pi^0\pi^0)} < 1\%(90\%\ C.L.)$$

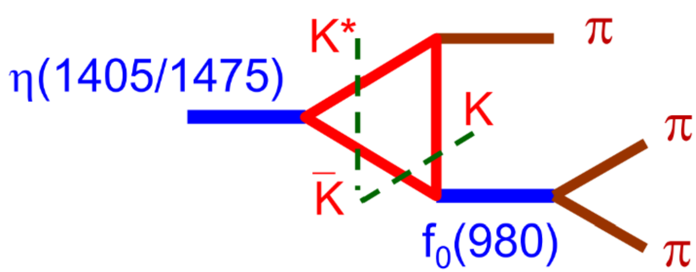
PRD, 83(2100)032003

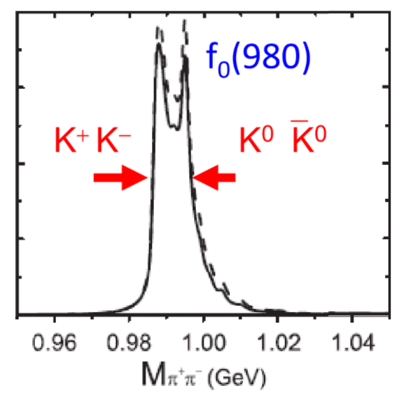
Isospin-violating decay of $\eta(1405) \to f_0(980)\pi^0$

PDG2012

However, the issue remains controversial as to whether two pseudoscalar mesons really exist. According to Ref. [18] the splitting of a single state could be due to nodes in the decay amplitudes which differ in $\eta\pi\pi$ and $K^*(892)\overline{K}$. Based on the isospin violating decay $J/\psi(1S) \to \gamma 3\pi$ observed by BES [19] the splitting could also be due to a triangular singularity mixing $\eta\pi\pi$ and $K^*(892)\overline{K}$ [20].







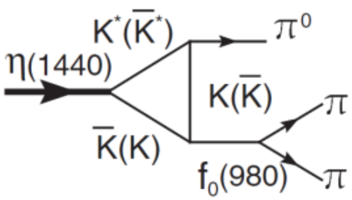
→No need for two pseudoscalars around 1.4 GeV

→Look for pseudoscalar glueball in higher mass region

Isospin-violating decay of $\eta(1405) \to f_0(980)\pi^0$

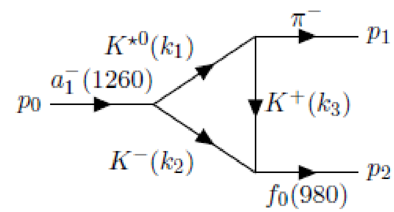
- Inspired by BESIII's observation, the triangle singularity mechanism plays an important role in the study of threshold phenomena

BESIII



COMPASS@CERN

$a_1(1420)$



Hadronic molecules

Rev.Mod.Phys. 90 (2018), 015004

et al., 2016). However, two recent experimental observations expose novel features in their decay mechanisms which illustrate the relevance of their couplings to the two-meson continua. The BESIII Collaboration observed an anomalously large isospin symmetry breaking in $\eta(1405)/\eta(1475) \to 3\pi$ (Ablikim *et al.*, 2012), which could be accounted for by the so-called triangle singularity (TS) mechanism as studied in Ref. (Aceti *et al.*, 2012; Wu *et al.*, 2012). This special threshold phenomenon arises in triangle (three-point loop) diagrams

Manifestations of TS in various processes

Phys.Rev.Lett. 108 (2012) 081803
Phys.Rev. D86 (2012) 114007
Phys.Rev. D88 (2013) 014045
Phys.Rev. D87 (2013) 014023
Phys.Rev. D89 (2014), 054038
Phys.Rev. D92 (2015) 034010
Phys.Rev. D91 (2015) 094022
Phys.Rev. D92 (2015) 036003
Phys.Lett. B753 (2016) 297
Phys.Rev. D93 (2016) 114027
Phys.Rev. D95 (2017) 034015
Phys.Rev. D97 (2018) 096002

Shed new lights on the $\eta(1405)/\eta(1475)$ puzzle

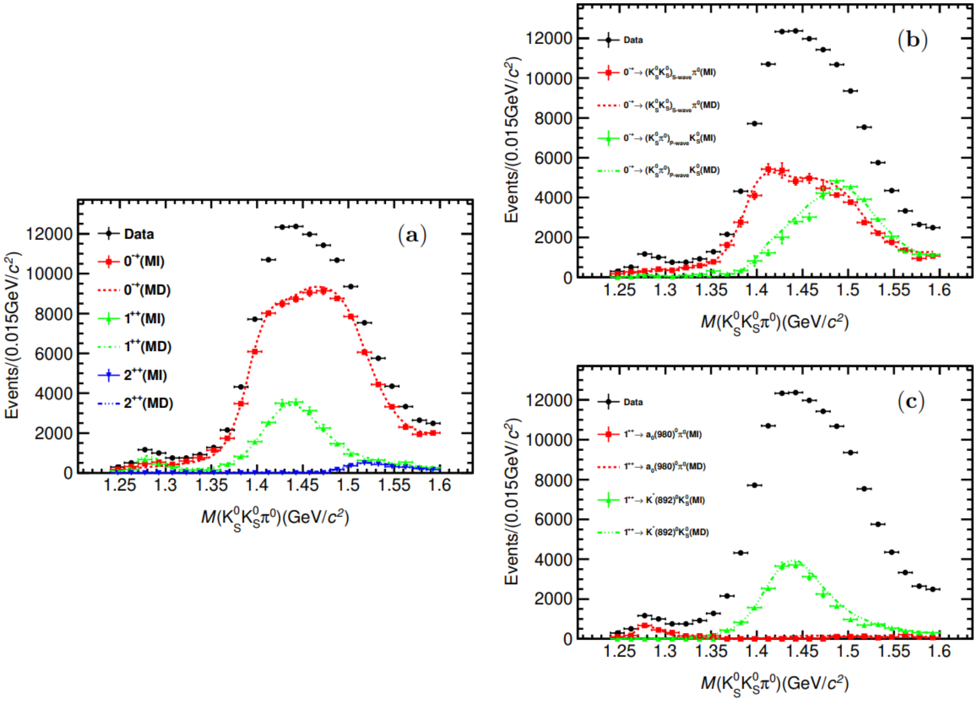
$J/\psi \to \gamma K_S K_S \pi^0$

BESIII JHEP 03 121(2023)

- **M**ass **I**ndependent PWA in bins of $M(K_S K_S \pi^0)$ to detangle J^{PC} components
  - **Valuable inputs to develop models**
- **M**ass **D**ependent PWA with BW to extract resonances
- **Consistency between MI and MD results**

- **Dominated by 0^{-+}**
  - **Two BWs around 1.4 GeV is needed**

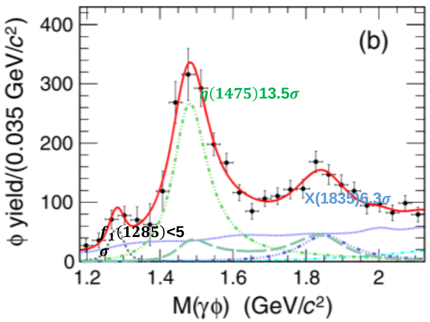
- Coupled-channel analysis
  - PRD 107, L091505 (2023) ; PRD 109, 014021 (2024); arXiv:2407.10234



$J/\psi \rightarrow \gamma\gamma\phi$, a $s\bar{s}$ flavor filter

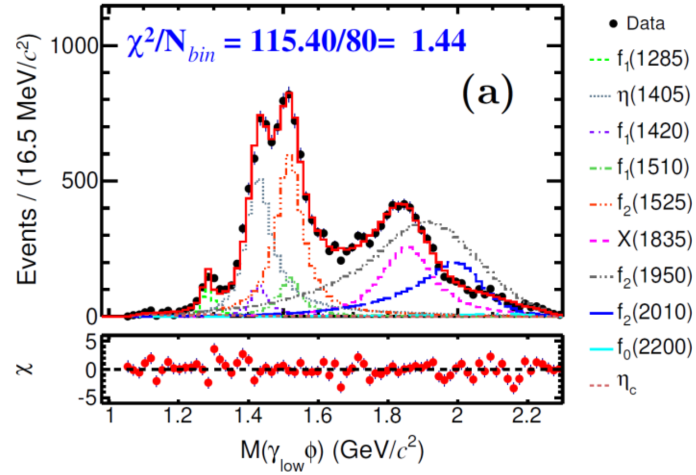
BESIII PhysRevD.111.052011(2025)

PR D97 051101 (2018)



Amplitude analysis with ML techniques for background subtraction

←Fit to mass spectrum



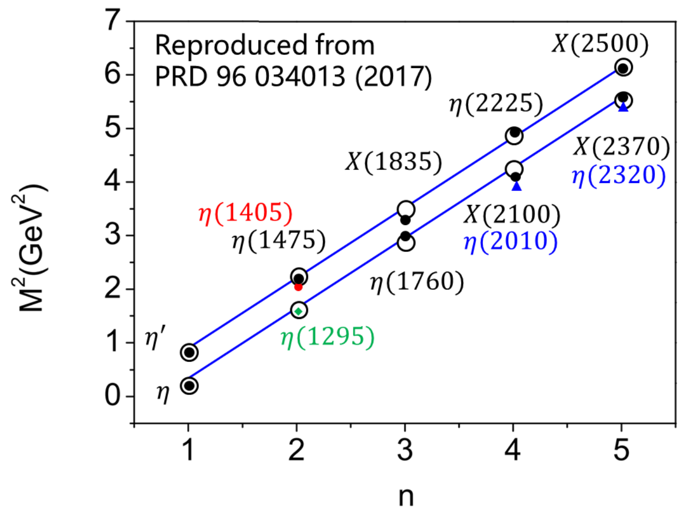
From the amplitude analysis,

- $\eta(1405)$ is observed, while $\eta(1475)$ can not be excluded
- $X(1835) \rightarrow \gamma\phi$ suggests its assignment of η' excitation
- $\eta_c \rightarrow \gamma\phi$ are observed. The very first radiative decay mode of η_c
- Observation of $f_2(1950)$ and $f_0(2200) \rightarrow \gamma\phi$ unfavored their glueball interpretations
  [PRD 108, 014023, Sci.China Phys.Mech.Astron. 67 (2024) 11, 111012]

Where is the 0^{-+} glueball

- Pseudoscalar sector, a promising window
  - Only η, η′ (& radial excitations) from quark model
- Mass
  - LQCD： 0^{-+} glueball (2.3~2.6 GeV)
- Production
  - LQCD: $\Gamma(J/\psi \to \gamma G_{0-})/\Gamma_{\rm total} = 2.31(80) \times 10^{-4}$, at the same level as 0^{-+} mesons

    [PRD.100.054511(2019)]
- **Decays**
  - **Possible guidance: OZI suppressed decays of η_c**
  - **3 pseudoscalar final state is a good place to look for** ($0^{-+} \to 2P$ is forbidden)



$\eta_c \to 3\,P$ in PDG

Decays involving hadronic resonances

Γ_1	$\eta'(958)\pi\pi$	(1.87 ± 0.26) %
Γ_2	$\eta'(958)K\overline{K}$	(1.61 ± 0.25) %

Decays into stable hadrons

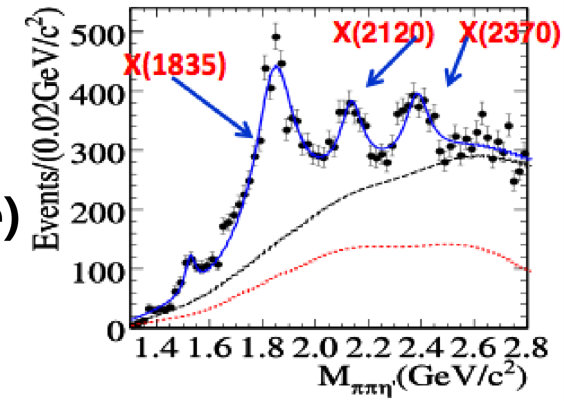
Γ_{34}	$K\overline{K}\pi$	$(7.0\ \pm0.4\ )$ %
Γ_{35}	$K\overline{K}\eta$	(1.32 ± 0.15) %
Γ_{36}	$\eta\pi^+\pi^-$	$(1.7\ \pm0.5\ )$ %

- No dominant decay
- Flavor symmetric

A glueball-like state $X(2370)$

- **Discovered by BESIII in $J/\psi \to \gamma\eta'\pi\pi$ in 2011**
- Confirmed by BESIII in $J/\psi \to \gamma\eta'\pi\pi$, $\boldsymbol{\gamma\eta' KK}$ **(new mode)**
  - Not seen in $J/\psi \to \gamma\eta'\eta\eta$ [BESIII PRD 103 012009 (2021)], $J/\psi \to \gamma\gamma\phi$ [BESIII PRD.111.052011(2025)]. Upper limits of BF are well consistent with predictions of 0^{-+}glueball
- **Mass consistent with LQCD prediction for 0^{-+} glueball**

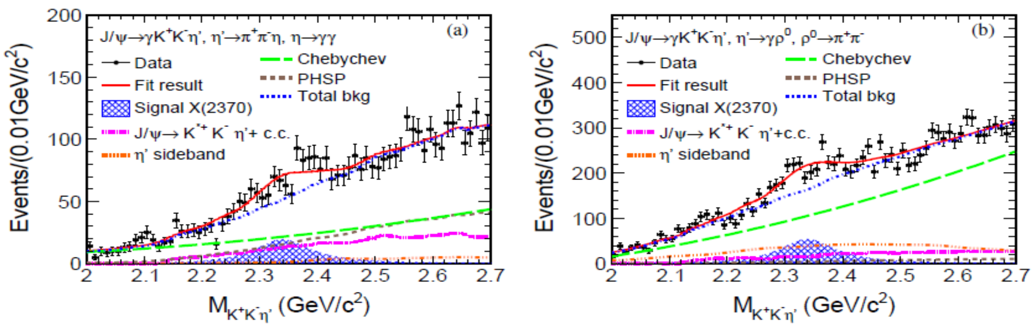
$J/\psi \to \gamma\eta'\pi\pi$

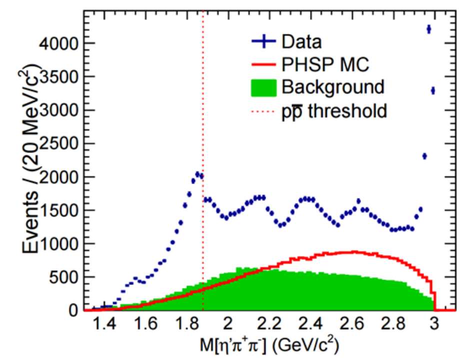


BESIII PRL 106, 072002(2011)

$J/\psi \to \gamma\eta' KK$

BESIII EPJC 80 746(2020)



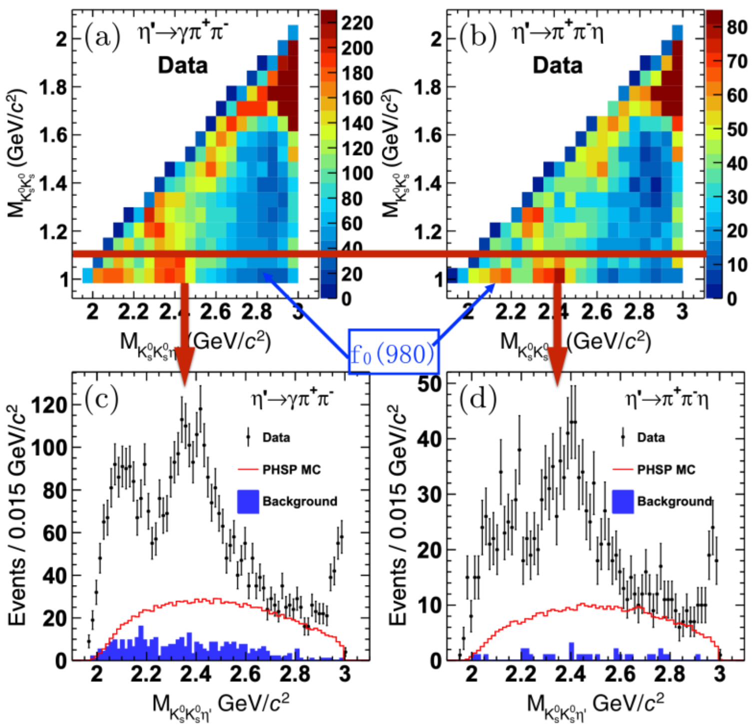


BESIII PRL 117, 042002 (2016)

A glueball-like state $X(2370)$

- **Spin-parity determined to be 0^{-+}**

$J/\psi \to \gamma\eta' K^0_S K^0_S$ **BESIII PRL 132, 181901(2024)**



- Almost background free channel
- Amplitude analysis in $f_0(980)$ region

$J^{PC} = 0^{-+}$ with significance >9.8σ

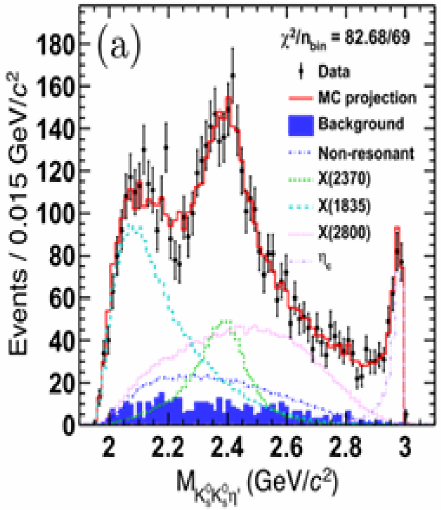
$M = 2395 \pm 11^{+26}_{-94}$ MeV

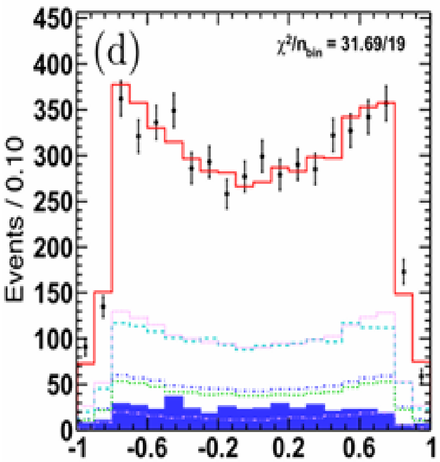
$\Gamma = 188^{+18}_{-17}{}^{+124}_{-33}$ MeV

$B(J/\psi \to \gamma X(2370))B(X(2370) \to f_0(980)\eta')B(f_0(980) \to K^0_S K^0_S)$
$= 1.31 \pm 0.22^{+2.85}_{-0.84} \times 10^{-5}$

Highlights of ICHEP2024

Spin-parity Determination of $X(2370)$ in $J/\psi \to \gamma\eta' K_S^0 K_S^0$





BESIII PRL 132 181901(2024)

Nominal fit solution

state	J^{PC}	Decay mode	Mass (MeV/c^2)	Width (MeV/c^2)	Significance
X(2370)	0^{-+}	$f_0(980)\eta'$	2395^{+11}_{-11}	188^{+18}_{-17}	14.9σ
X(1835)	0^{-+}	$f_0(980)\eta'$	1844	192	22.0σ
X(2800)	0^{-+}	$f_0(980)\eta'$	2799^{+52}_{-48}	660^{+180}_{-116}	16.4σ
η_c	0^{-+}	$f_0(980)\eta'$	2983.9	32.0	$> 20.0\sigma$
PHSP	0^{-+}	$\eta'(K_S^0K_S^0)_{S-wave}$	– – –	– – –	9.0σ
		$\eta'(K_S^0K_S^0)_{D-wave}$	– – –	– – –	16.3σ

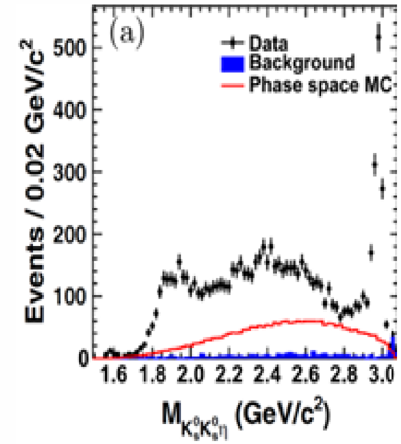
- **X(2370)′s $J^{PC} = 0^{-+}$ with 9.8 σ**
- Product branching fraction:

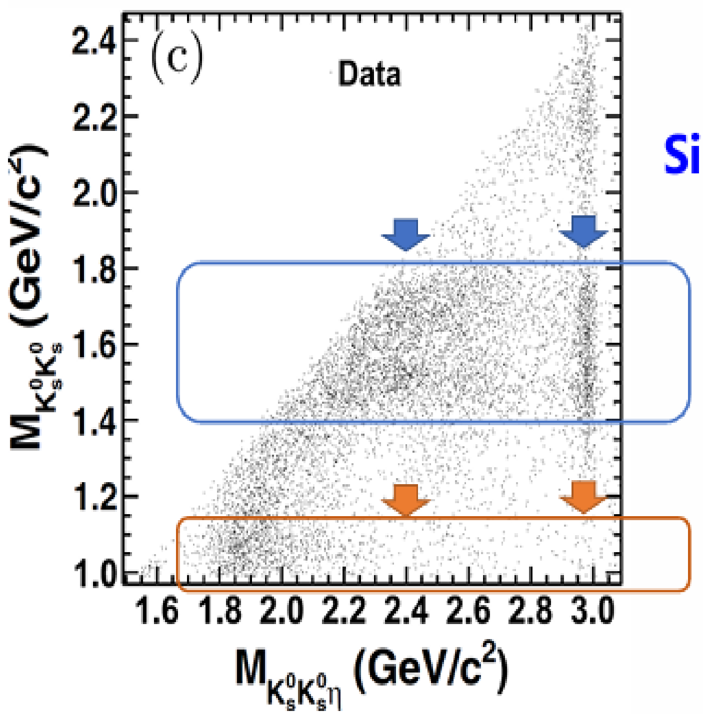
$$B(J/\psi \to \gamma X(2370)) B(X(2370) \to \eta' K_S^0 K_S^0) B(f_0(980) \to K_S^0 K_S^0) = (1.31 \pm 0.22^{+2.85}_{-0.84}) \times 10^{-5}$$

$\mathbf{X}(2370)$ seen in $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta$

Observation and Spin-Parity Determination of the $X(1835)$ in $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta$

BESIII PRL 115 091803(2015)



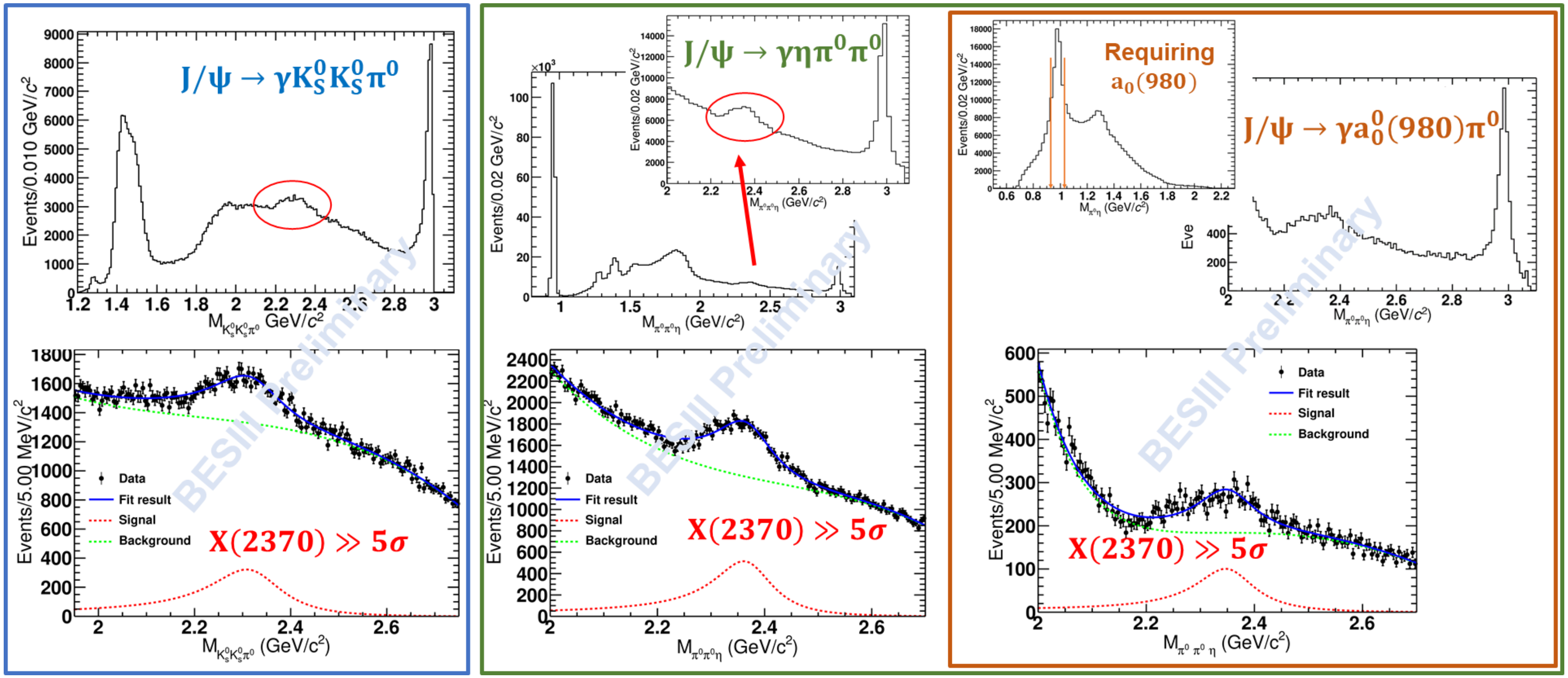


Similar decay patterns of the $X(2370)$ and η_c

clear $X(2370)$ AND η_c signals

no $X(2370)$ OR η_c signal

12



New

X(2370) observed in the gluon-rich J/ψ radiative decays

- **A first-time determination of $J^{PC}=0^{-+}$**
- **Mass and production rate consistent with LQCD**
- **Decay modes $X(2370) \rightarrow \eta'\pi\pi, \eta' KK, K^0_S K^0_S \eta, K^0_S K^0_S \pi^0, \eta\pi^0\pi^0, a^0_0(980)\pi^0$, in analog to η_c** ($\eta(2320) \rightarrow \eta\eta\eta, \eta\pi\pi$ [PL B496 145(2000)] could be the same state)

Consistent with 0^{-+} glueball

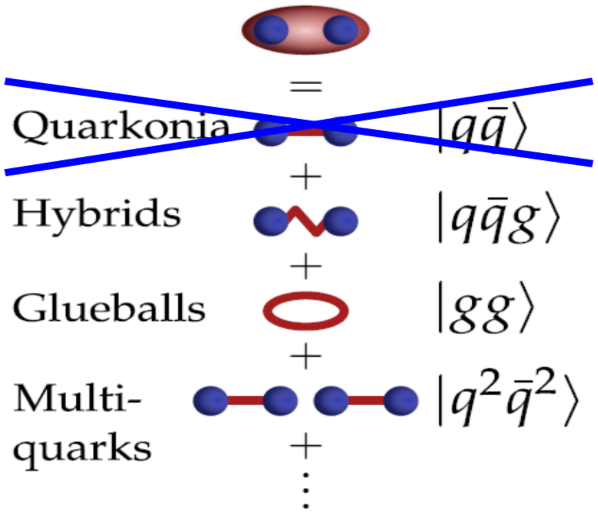
Light hadrons with exotic quantum numbers

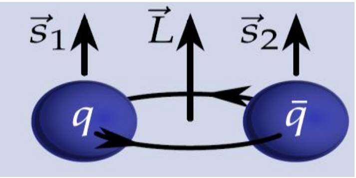
- **Finding unambiguous signature for exotics**
  - **Efforts concentrate on Spin-exotic**
    - **Forbidden for $\mathbf{q\bar{q}}$:**

$\mathbf{J^{PC} = 0^{--}, even^{+-}, odd^{-+}}$

Experiments:

- **Hadroproduction:** GAMS, VES, E852, COMPASS
- **$\mathbf{p\bar{p}}$ annihilation:** Crystal Barrel, OBELIX, PANDA(under construction)
- **Photoproduction:** GlueX(2017-), CLAS

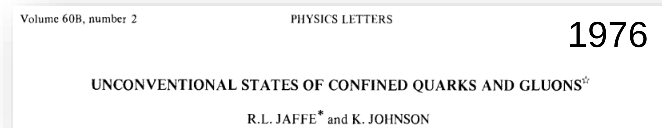




$\vec{\mathbf{J}} = \vec{\mathbf{L}} + \vec{\mathbf{S}}$ $\quad \mathbf{P = (-1)^{L+1}}$ $\quad \mathbf{C = (-1)^{L+S}}$

Allowed $\mathbf{J^{PC}}$: $\mathbf{0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 2^{++}, \ldots}$

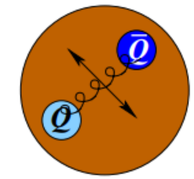
Predictions


Volume 60B, number 2 PHYSICS LETTERS

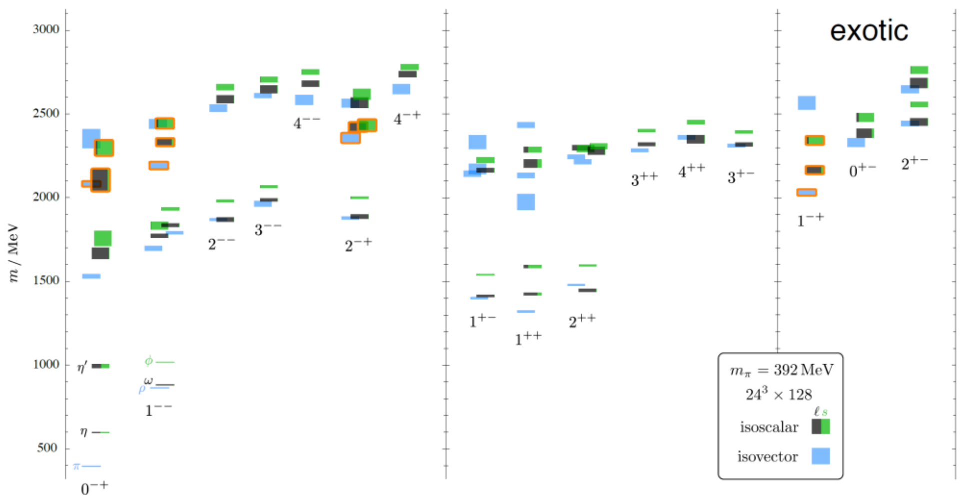
UNCONVENTIONAL STATES OF CONFINED QUARKS AND GLUONS☆

R.L. JAFFE* and K. JOHNSON

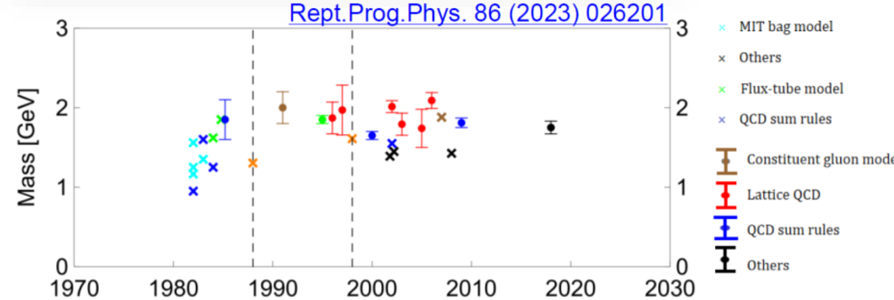
1976



PRD 88 094505(2013)

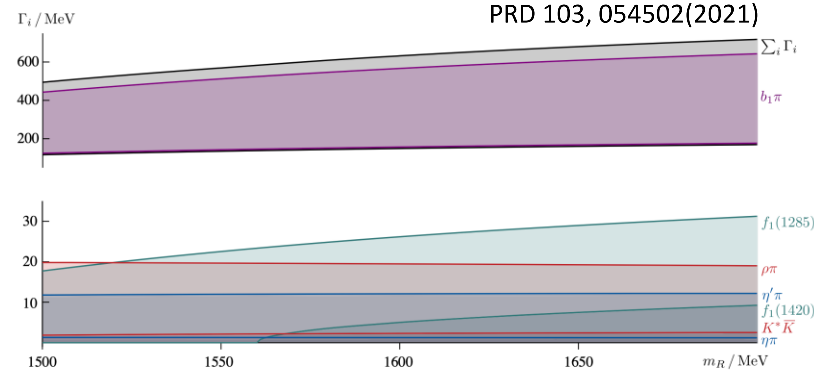


Rept.Prog.Phys. 86 (2023) 026201



Mass of $\mathbf{1^{-+}}$ hybrid

PRD 103, 054502(2021)



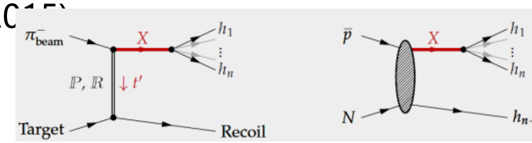
Decay width of $\mathbf{1^{-+}}$ hybrid π_1

On lattice,

- Meson excitations similar to quark model
- Hybrid supermultiplet: $\mathbf{0^{-+}}$, $\mathbf{1^{--}}$, $\mathbf{2^{-+}}$, $\mathbf{1^{-+}}$
- Lightest spin-exotic state in LQCD: $\mathbf{1^{-+}}$ hybrid

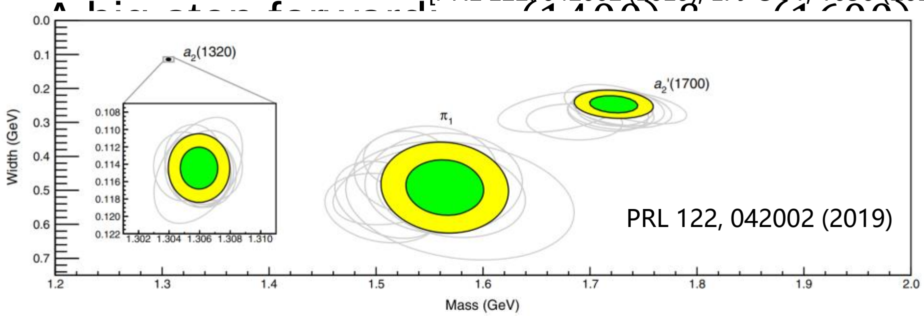
Spin-exotic mesons

Detailed reviews:
PRC 82, 025208 (2010), PPNP 82, 21 (20



- **Over 3 decades, only 3 candidates so far: All 1^{-+} isovectors**
  - $\pi_1(1400)$: mostly in $\eta\pi$
  - $\pi_1(1600)$: seen in $\rho\pi$, $\eta'\pi$, $b_1\pi$, $f_1\pi$, but not $\eta\pi$
  - $\pi_1(2015)$: needs confirmation

[PRL 122, 042002 (2019), EPJ C 81, 1056 (2021)] can be

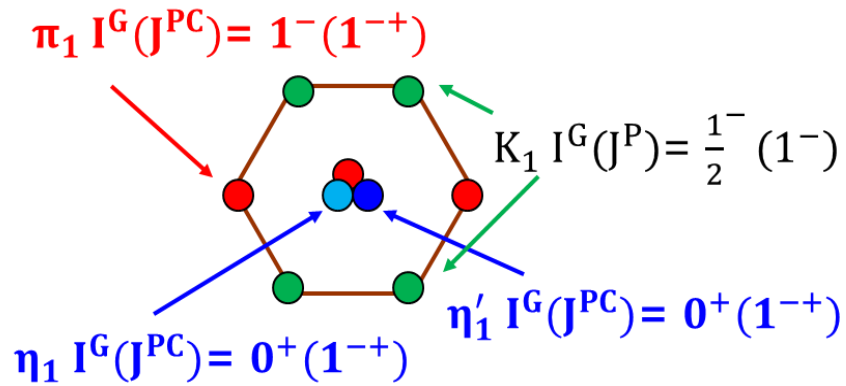


Coupled-channel analysis of COMPASS data

	Decay mode	Reaction	Experiment
$\pi_1(1400)$	$\eta\pi$	$\pi^-p \to \pi^-\eta p$ $\pi^-p \to \pi^0\eta n$ $\pi^-p \to \pi^-\eta p$ $\pi^-p \to \pi^0\eta n$ $\bar{p}n \to \pi^-\pi^0\eta$ $\bar{p}p \to \pi^0\pi^0\eta$	GAMS KEK E852 E852 CBAR CBAR
	$\rho\pi$	$\bar{p}p \to 2\pi^+2\pi^-$	Obelix
$\pi_1(1600)$	$\eta'\pi$	$\pi^-Be \to \eta'\pi^-\pi^0 Be$ $\pi^-p \to \pi^-\eta' p$	VES E852
	$b_1\pi$	$\pi^-Be \to \omega\pi^-\pi^0 Be$ $\bar{p}p \to \omega\pi^+\pi^-\pi^0$ $\pi^-p \to \omega\pi^-\pi^0 p$	VES CBAR E852
	$\rho\pi$	$\pi^-Pb \to \pi^+\pi^-\pi^- X$ $\pi^-p \to \pi^+\pi^-\pi^- p$	COMPASS E852
	$f_1\pi$	$\pi^-p \to p\eta\pi^+\pi^-\pi^-$ $\pi^-A \to \eta\pi^+\pi^-\pi^- A$	E852 VES
$\pi_1(2015)$	$f_1\pi$	$\pi^-p \to \omega\pi^-\pi^0 p$	E852
	$b_1\pi$	$\pi^-p \to p\eta\pi^+\pi^-\pi^-$	

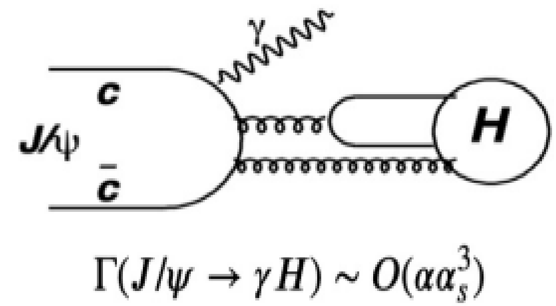
1^{-+} Hybrids

- **Isoscalar 1^{-+} is critical to establish the hybrid nonet**
  - **Can be produced in the gluon-rich charmonium decays**
  - **Can decay to $\eta\eta'$ in P-wave**



PRD 83,014021 (2011), PRD 83,014006 (2011), EPJ P135, 945(2020)

→ Search for $\eta_1\ (1^{-+})$ in $J/\psi \to \gamma\eta\eta'$



Observation of An Exotic 1^{-+} Isoscalar State $\eta_1(1855)$

PRL 129 192002(2022) , PRD 106 072012(2022)

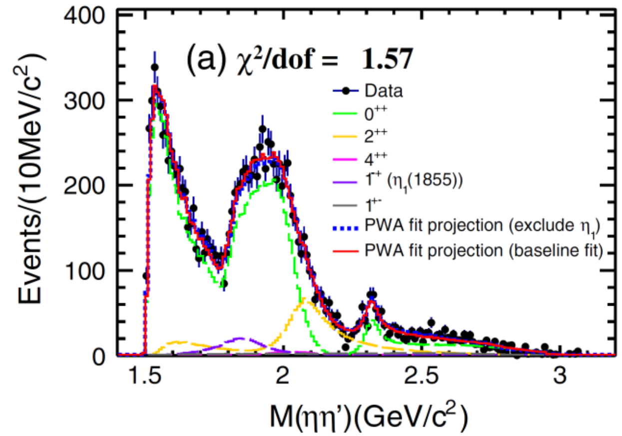
- **An isoscalar 1^{-+} , $\eta_1(1855)$, has been observed in $J/\psi \to \gamma\eta\eta'$ (>19σ)**

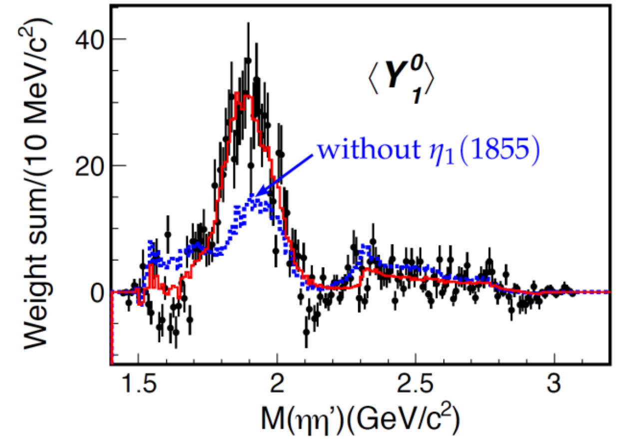
  $M = (1855 \pm 9^{+6}_{-1})$ MeV/c^2, $\Gamma = (188 \pm 18^{+3}_{-8})$ MeV/c^2

  $B(J/\psi \to \gamma\eta_1(1855) \to \gamma\eta\eta') = (2.70 \pm 0.41^{+0.16}_{-0.35}) \times 10^{-6}$

  - **Mass consistent with hybrid on LQCD**

- **$\eta\eta'$ in P-waves uniquely indicates 1^{-+} exotic quantum numbers**



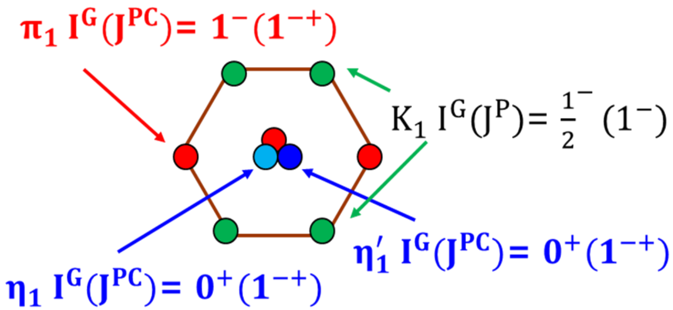


Observation of An Exotic 1^{-+} Isoscalar State $\eta_1(1855)$

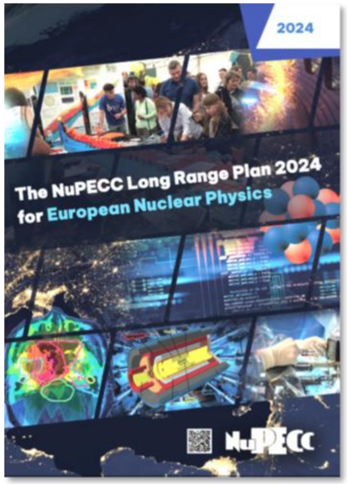
PRL 129 192002(2022) , PRD 106 072012(2022)

- Inspired many interpretations:
  Hybrid/$K\bar{K}_1$Molecule/Tetraquark?

  NPA 1047 122874(2024); Rept.Prog.Phys. 86 (2023) 026201; PRD 107 (2023) 7, 074028; SCPMA 65 (2022) 6, 261011; CPC 46 , 051001(2022); CPL 39, 051201 (2022); PLB 834, 137478(2022); PRD 106 , 074003(2022); PRD 106, 036005(2022) ;...



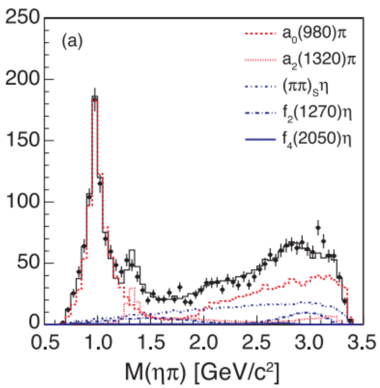
- Opens a new direction to completing the picture of spin-exotics
  - As a "recent achievements and highlights" in hadron spectroscopy in the NuPECC LRP
  - 50 years of QCD: Exotic mesons, "observation of an $\eta_1(1855)$ state could be a breakthrough" [EPJ.C 83 (2023) 1125]



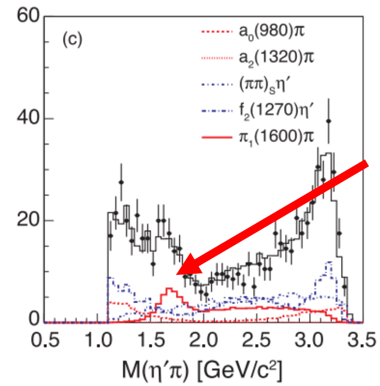
Studies of π_1 in $\chi_{c1} \to \pi^+\pi^-\eta^{(\prime)}$

PR D84 112009 (2011)

$\mathbf{2.6 \times 10^7}$ $\psi(3686)$@CLEO − c



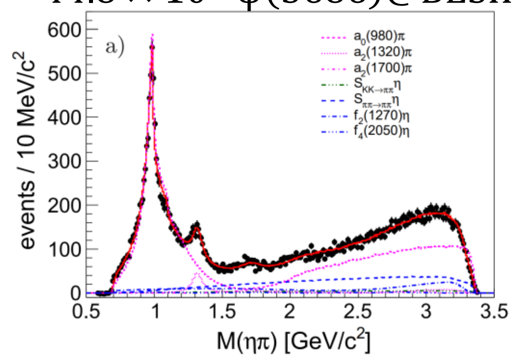
No evidence of $\pi_1 \to \eta\pi$



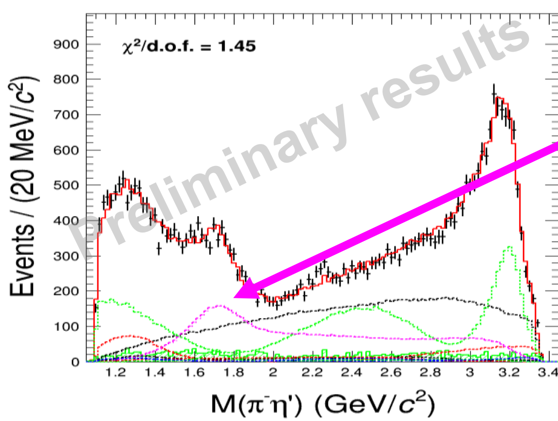
Evidence of $\pi_1 \to \eta'\pi$ (without significant BW phase motion)

PR D95 032002(2017)

44.8×10^7 $\psi(3686)$@BESIII

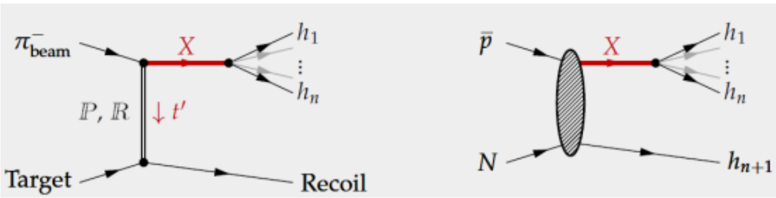


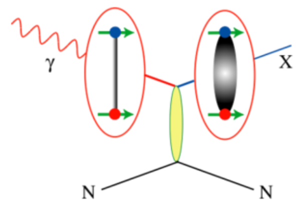
$\mathbf{2.7 \times 10^9}$ $\psi(3686)$@BESIII [New]

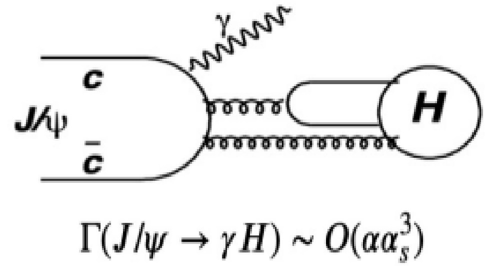


- $\boldsymbol{\pi_1(1600)}$ **observed>10σ**
- **with a significant BW phase motion**
- $\mathbf{J^{PC}}$**=** $\mathbf{1^{-+}}$**, better than other assignments well over 10σ**

Observations of π_1 and η_1 in charmonium decays provide a new path to study 1^{-+}







- **Measure the properties of $\pi_1(1600)$**
- **Confirm $\eta_1(1855)$ and measure its properties in more processes**
- **Identify the expected $\eta_1^{(\prime)}$ partner**
- **Other exotic quantum numbers: 0^{--} , 2^{+-} , ……**
- **Analog in $\bar{c}c$**

Landscape of light glueball has changed

Scalar: Overpopulation

- LQCD : ground state 0^{+} glueball ~1.7 GeV, first excitation ~2.1 GeV

✓ **Strong production of $f_0(1710)/f_0(2100)$ in $J/\psi \to \gamma\, \eta\eta/KK/\pi\pi$**, the pattern consists with LQCD' s prediction

Tensor: large uncertainty

- LQCD: 2^{++}(2.3~2.4 GeV)

✓ **Strong production of $f_2(2340)$ in $J/\psi \to \gamma\eta\eta/KK/\pi\pi/\phi\phi$**; consists with LQCD' s prediction

Pseudoscalar: very little known above 2 GeV, puzzles in low mass region

- LQCD: 0^{-+}(2.3~2.6 GeV)

✓ **Trajectory**:

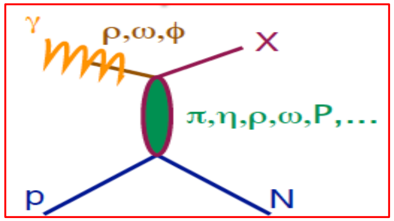
- **$f_1(1285)$, no $\eta(1295)$**
- **$\eta(1405)$ /$\eta(1475)$ can be one resonance**

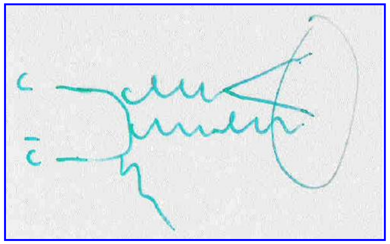
☐ **Above 2 GeV: X(2370)?**

Hybrids

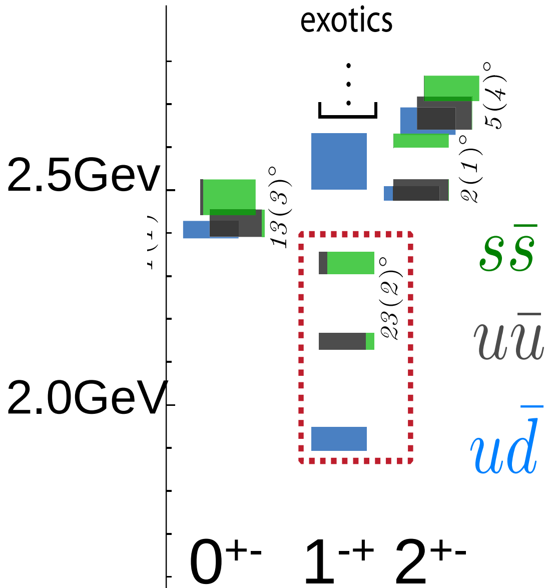
GlueX@JLab

BESIII

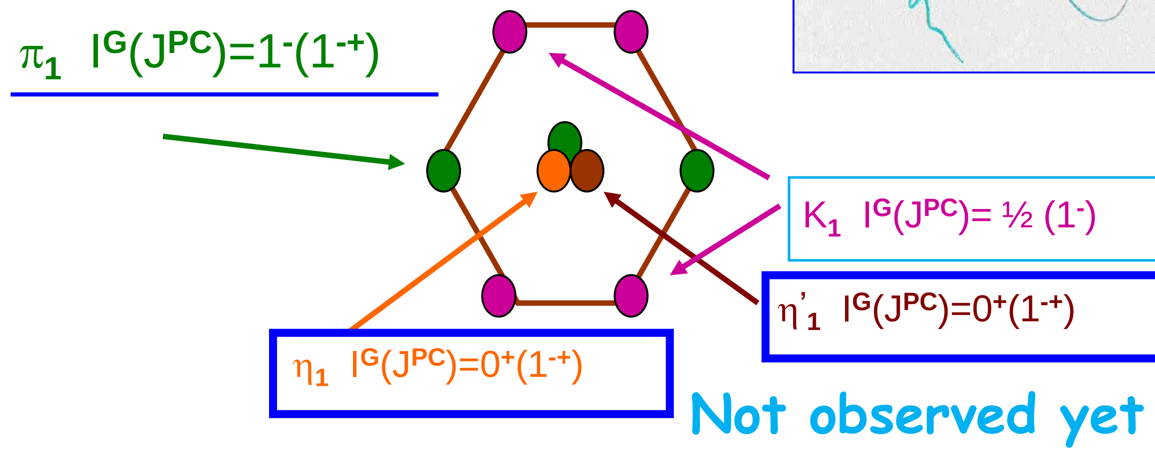




Lattice QCD Predictions:



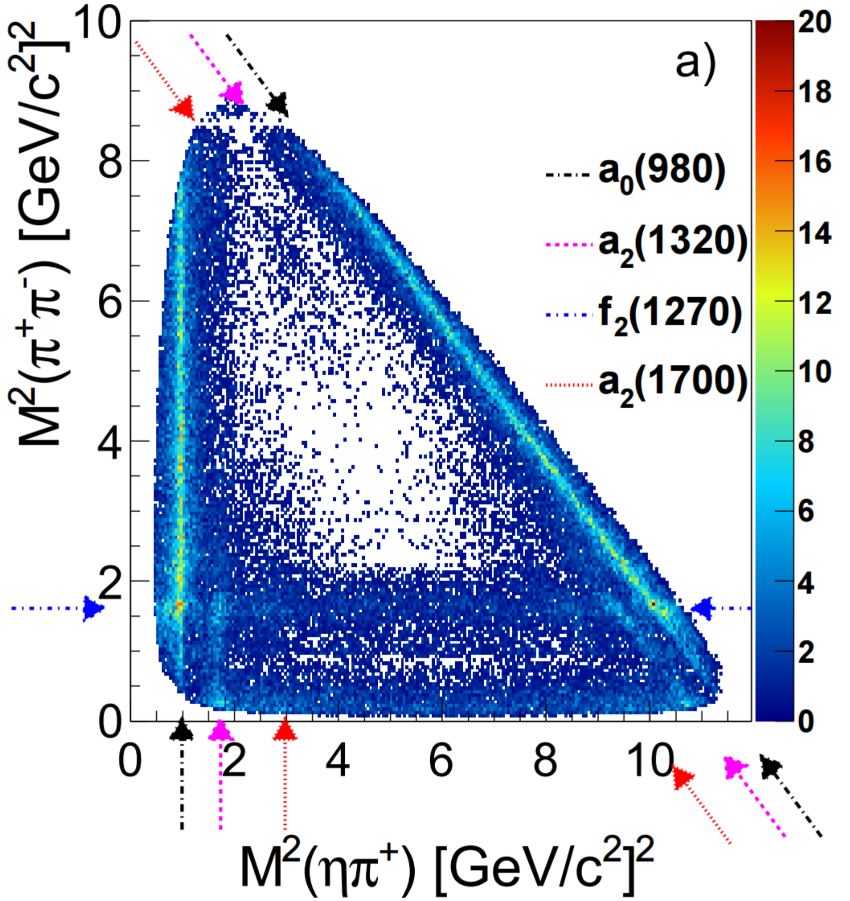
PRD 83 111502 (2011)

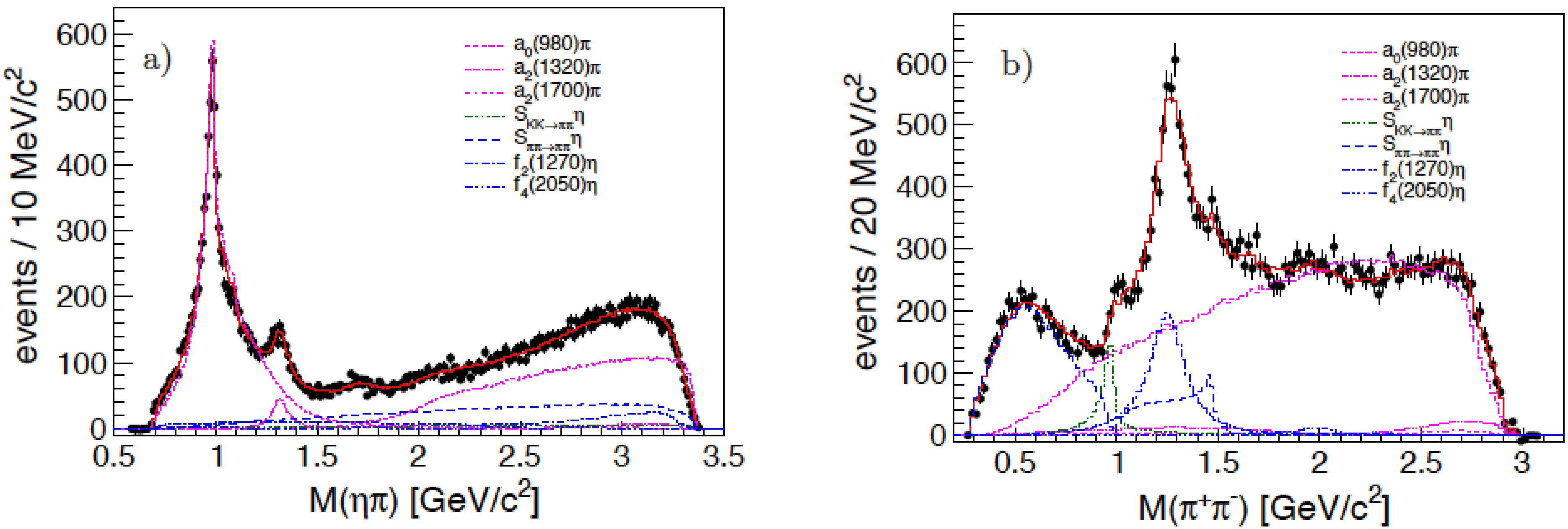


- Lightest hybrid is with exotic $J^{PC}= 1^{-+}$
- Isoscalar 1^{-+} is critical to establish the nonect
- $J/\psi \rightarrow \gamma + X$, synergies between other experiments with different production mechanism
- χ_{c1} decays can be another source

Amplitude analysis of $\chi_{c1} \to \eta\pi^+\pi^-$

- χ_{c1} **provides another suitable environment to look for 1^{-+}**
  - π_1**(1600) studied in χ_{c1} decays by CLEO-c**
  - **only π_1(1400) has been reported decays to $\eta\pi$**
- **Properties of a_0 and a_2 still need further studies**





Most of resonances are BW's
Dispersion integrals for
π π S-wave: N/D approach PRD84 112009
a_0(980): PRD78,74023

- **Clear evidence for a_2(1700) in χ_{c1}decays**
- **First measurement of $g'_{\eta'\pi} \neq 0$ using $a_0(980) \rightarrow \eta\pi$ line shape**
- **Measured upper limits for $\pi_1(1^{-+})$ in 1.4 - 2.0 GeV/c^2 region**

Summary and outlook

- Light hadron spectroscopy: Map out light hadrons as complete/precise as possible

→ Provide critical information on the quantitative understanding of confinement

- Amplitude analysis is a key tool. Experiment-theory cooperation is important

- BESIII collected 10 billions of J/ψ and will continue to run for more years. Data with unprecedented statistical accuracy provides great opportunities to map out light meson spectroscopy and study QCD exotics

Thank you