Global Methods of Pattern Recognition

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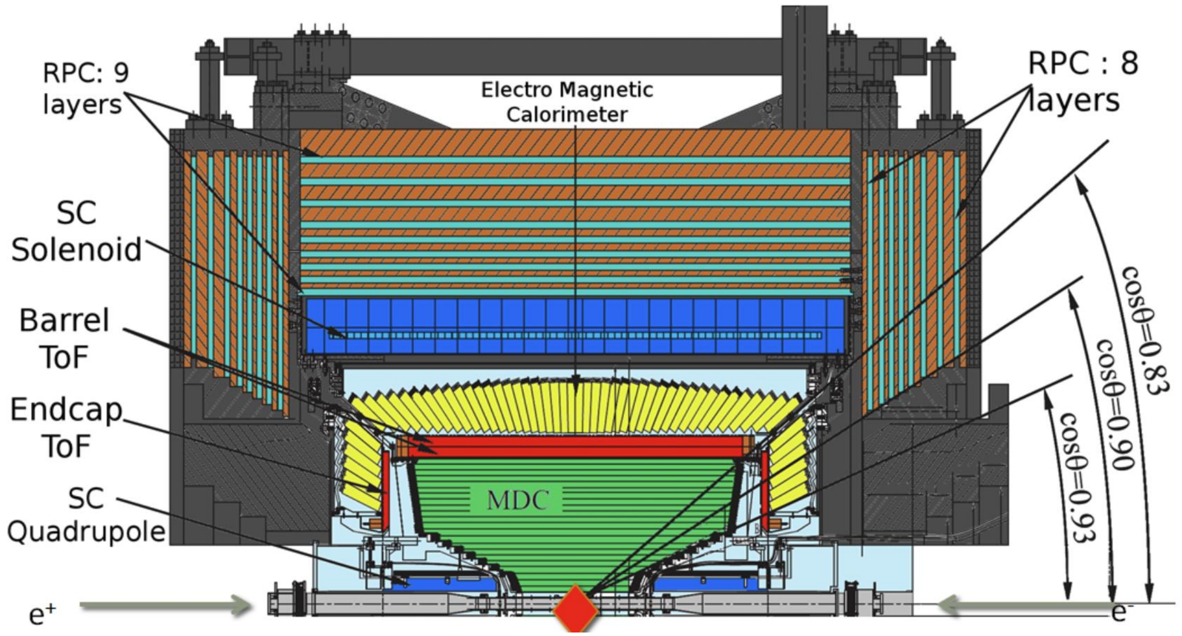
Global Method

- The global method means all the points are considered together to perform a track finding rather starting from a seed
  - **Advantage**
    - The characteristics of global methods give them the advantage of noise resistance
    - Don't need a high quality seed
    - When some points in the track are not detected, the global method can still give a good result
  - **Disadvantage**
    - Computationally intensive
    - Resolution is limited by parameter space discretization

- We will introduce some global method and their application in experiments
  - Template matching
  - Hough transform and Legendre transform

Template matching

Template matching in BESIII



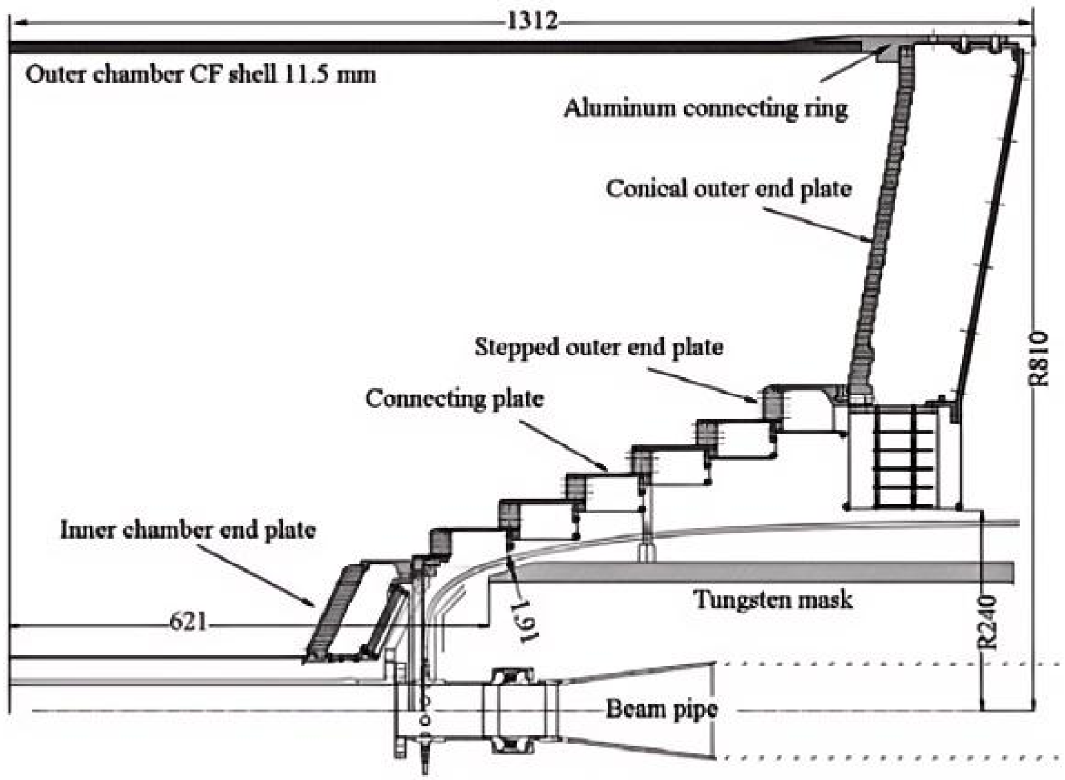


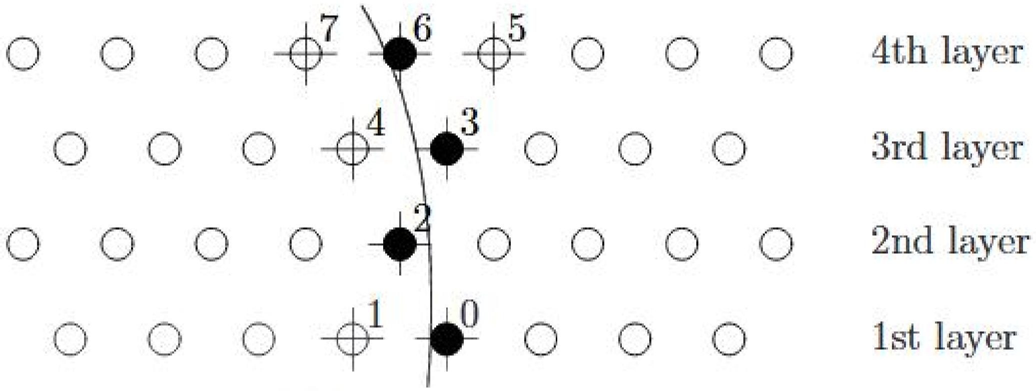
Fig. 14. The MDC mechanical structure.

Template matching in BESIII

- **Segment**.
  - In a super–layer, a series of hits of a charged particle are defined as a segment.

- **Segment pattern and segment pattern dictionary**.
  - The fired wire distribution of a segment in a super–layer is defined as a segment pattern.

- **8 wires - template matching**
  - Segment finder uses the hits in the 2nd layer of a super layer as seeds to search for segments in every group of 8 signal wires



a segment and an 8-wire group

Fig. 1. A schematic plot for illustrating a segment and an 8-wire group. The 4 layers of circles denote the signal wires in one super-layer. The arc shows a track, and the black solid circles are the fired wires of the particle. The 8 adjacent signal wires marked with a cross compose an 8-wire group used to match the segment pattern in segment finding.

Template matching in BESIII

1. Hit information →
2. matching pattern dictionary →
3. Combine super layer trajectories to form circular trajectories and fit →
4. Combine stereo super-layers to get helix tracks and apply helix fit →
5. Tracks are stored for Kalman fit, tracks extrapolation, physics analysis

- Combine and fitting
  - Use least squares to combine different super layers' track and fit the helix parameters

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{\mathrm{drift}_i - \mathrm{doca}_i}{\sigma_i} \right)^2 ,$$

Template matching in BESIII

- **Limitation of 8-wire Groups: Low-pT Efficiency Loss**

$$\Delta\phi = \left|\phi_{L-L(L-R)} - \phi_{U-R(U-L)}\right|,$$

  - Original 8-wire azimuthal coverage: Δϕ.
  - Track span in one super-layer: δϕ.
  - Low-pT particles bend more, so δϕ becomes larger.
  - If δϕ > Δϕ, hits can fall outside the local group, so the segment cannot be fully represented.
  - Asymmetric wire groups in SL-1 to SL-5 further reduce the minimum coverage.

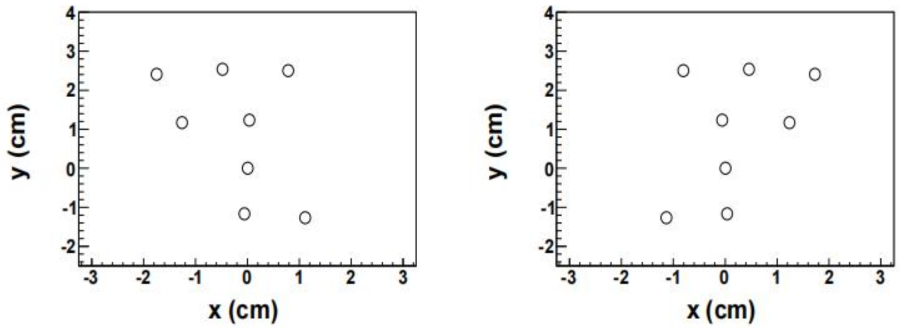
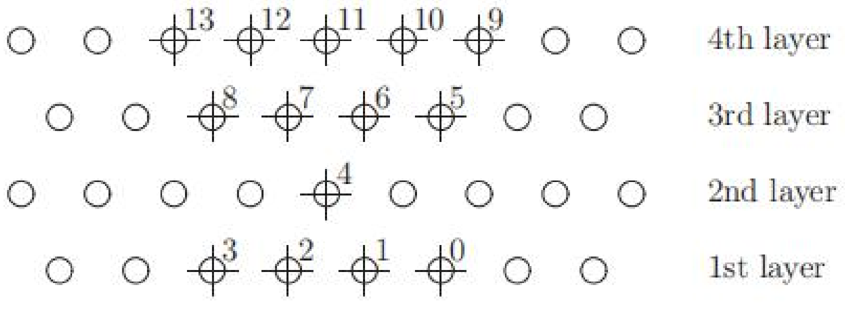


Fig. 2. The relative position of signal wires of 2 asymmetric 8-wire groups in SL-1. The origin of coordinate in each plot is set at the position of the signal wire in the second layer.

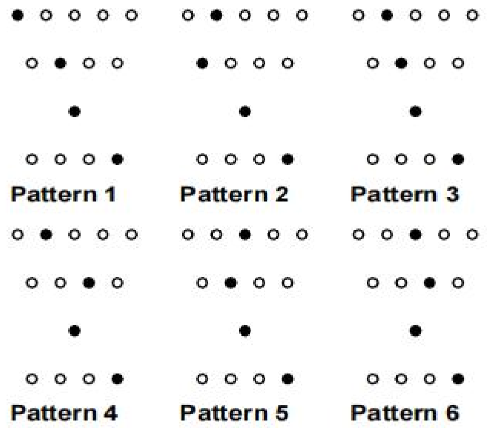
Template matching in BESIII

- **Key Improvement: Expand 8-wire Groups to 14-wire Groups**

    - 14-wire group numbering: 4 wires in layer 1, 1 wire in layer 2, 4 wires in layer 3, and 5 wires in layer 4.



a 14-wire group



pattern	decimal	binary	pattern	decimal	binary	pattern	decimal	binary
1	8337	10000010010001	2	4369	01000100010001	3	4241	01000010010001
4	4177	01000001010001	5	2193	00100010010001	6	2129	00100001010001

Template matching in BESIII

- **8–wire–group vs 14–wire–group**

  - 14–wire groups have a clear advantage in azimuthal coverage.

  - For low–pT particles, 14–wire groups can better cover the possible δϕ.

Dictionary	SL-1	SL-2	SL-3	SL-4	SL-5	SL-6	SL-7	SL-8	SL-9	SL-10	SL-11
8-wire	18.6	12.4	10.9	8.4	6.7	5.6	5.1	4.3	3.8	3.5	2.5
8-wire min	10.9	7.9	6.8	5.2	4.1	—	—	—	—	—	—
14-wire	34.1	22.5	19.7	15.2	12.1	10.1	9.2	7.8	6.8	6.3	5.0
14-wire min	26.4	18.0	15.6	12.0	9.5	—	—	—	—	—	—

p_t (MeV/c)	SL-1	SL-2	SL-3	SL-4	SL-5	SL-6	SL-7	SL-8	SL-9	SL-10	SL-11
50	6.7	6.6	11.2	16.9							
70	4.7	4.5	6.8	7.5	8.9	14.0					
90	3.6	3.4	5.0	5.3	5.6	6.4	8.0	11.9			
110	2.9	2.8	4.0	4.1	4.2	4.5	5.1	5.7	6.7	12.0	
130	2.5	2.3	3.3	3.4	3.4	3.6	3.9	4.1	4.4	5.4	4.1
150	2.1	2.0	2.9	2.9	2.9	3.0	3.2	3.3	3.4	3.9	2.7
170	1.9	1.8	2.5	2.5	2.5	2.6	2.8	2.8	2.8	3.1	2.1

Table 1 / Table 2: coverage ranges of 8–wire and 14–wire groups, and δϕ for tracks with different pT values.

Template matching in BESIII

- **14–wire–group Performance**

- The 14–wire dictionary retains more low–pT physics events, but costs more CPU time.

- For ψ(3770) data, reconstructed tracks with pT < 130 MeV/c increase by about 15%.

- CPU cost: segment finder +2 ms/event; MdcPatRec +30%; total reconstruction +10%.

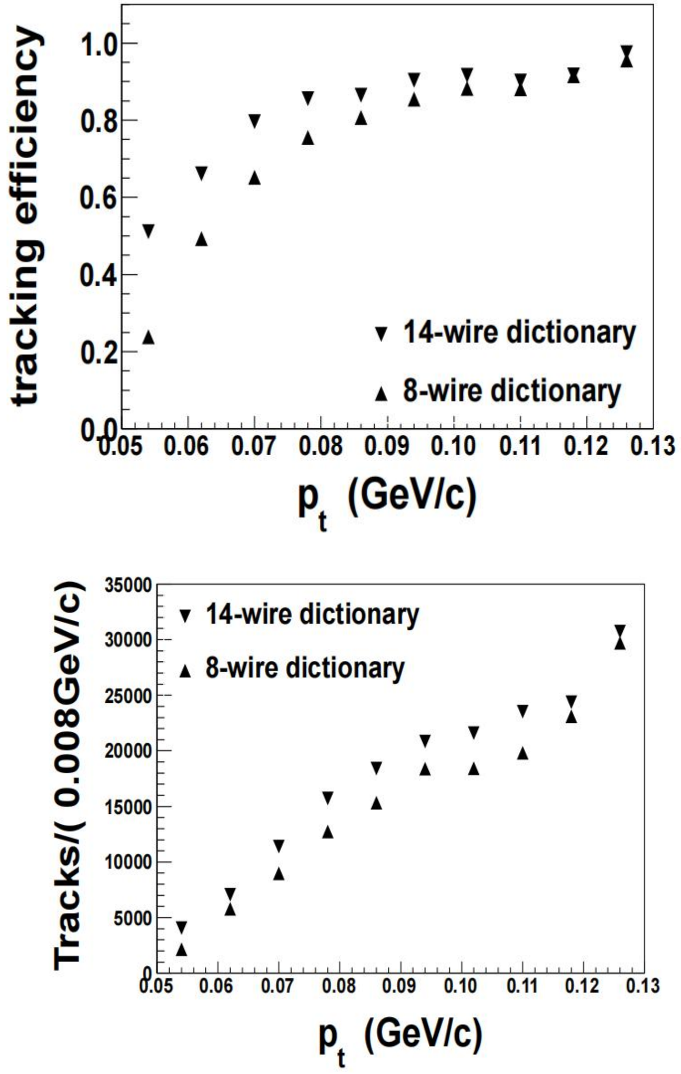
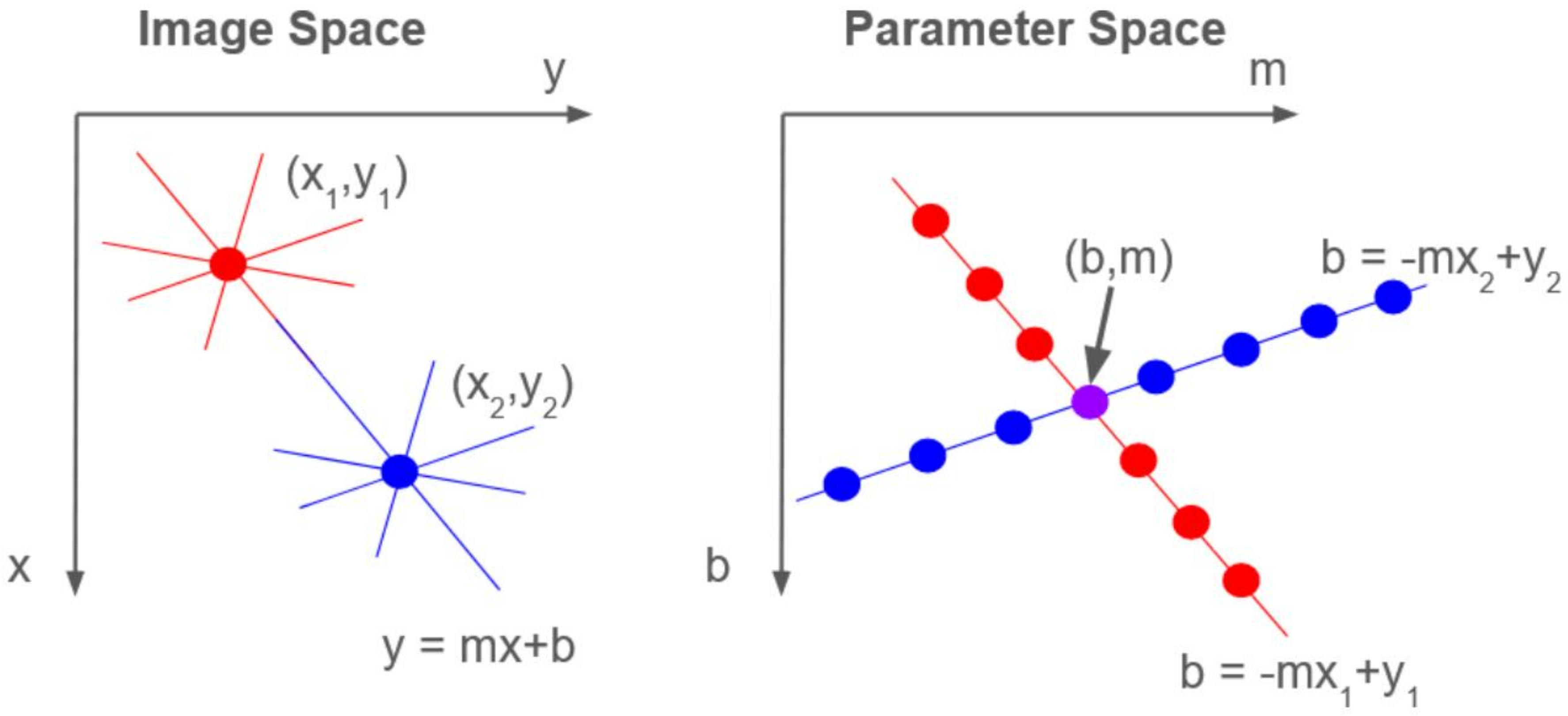


Fig. 11. The reconstructed tracks numbers from $\psi(3770)$ experimental data.

Hough transform

Hough transform

- The Hough Transform (HT) is a versatile tracking algorithm which applies a change in the coordinate system of the track under test, creating a new parameter space



Legendre transform

- The Legendre transform is a well-known mathematical tool in Thermodynamics and Analytical Mechanics

$$f(x) \overset{\mathscr{L}}{\longleftrightarrow} F(p)$$

$$F(p) = \sup_x [px - f(x)] = -\inf_x [f(x) - px]$$

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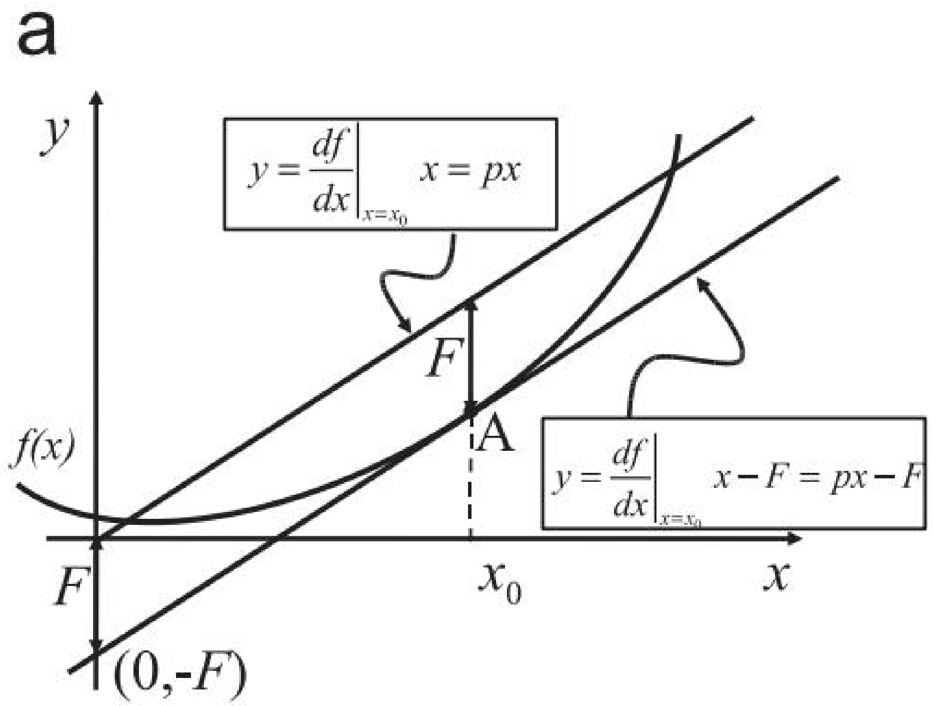
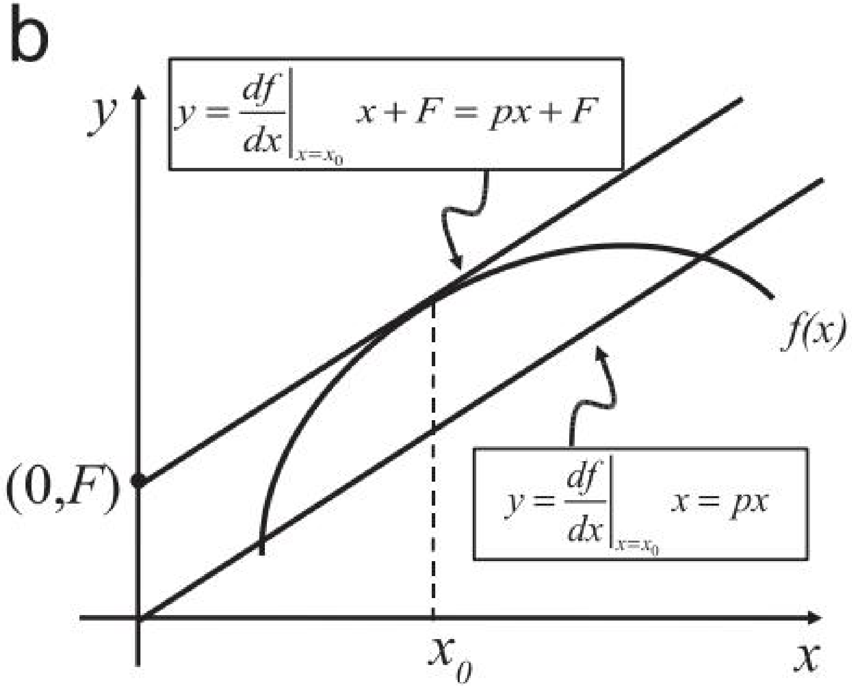




Legendre transform

- The Legendre transform of some common functions and properties of the Legendre transform

$$p = \left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x=x_0}$$

$$F(p) = px_0 - f(x_0)$$

Original function	Legendre transform
$x^2/2$	$p^2/2$
e^x	$p(\ln p - 1)$
$\ln x$	$-\ln(ep)$
x^a/a	p^b/b where $1/b + 1/a = 1$
$x^3/3$	$(2/3)p^{3/2}$
$-1/x$	$-2p^{1/2}$

Property	Result
Scaling	$af(x) \overset{\mathscr{L}}{\longleftrightarrow} aF(p/a)$
Stretching	$f(ax) \overset{\mathscr{L}}{\longleftrightarrow} F(p/a)$
Translation	$f(x-a) \overset{\mathscr{L}}{\longleftrightarrow} F(p) + a$
Linear addition	$f(x) + ax + b \overset{\mathscr{L}}{\longleftrightarrow} F(p-a) + b$
Young's inequality	$px \leqslant f(x) + F(p)$
If $f(0) = \mathrm{d}f/\mathrm{d}x\vert_{x=0} = 0$	$F(p) = \int_0^p (\mathrm{d}f/\mathrm{d}x)^{-1}\,\mathrm{d}x$
Infimal convolution $(f \oplus g)(x) = \inf_y \{f(x-y) + g(y)\}$	$(f \oplus g)(x) \overset{\mathscr{L}}{\longleftrightarrow} F(p) + G(p)$

Legendre transform

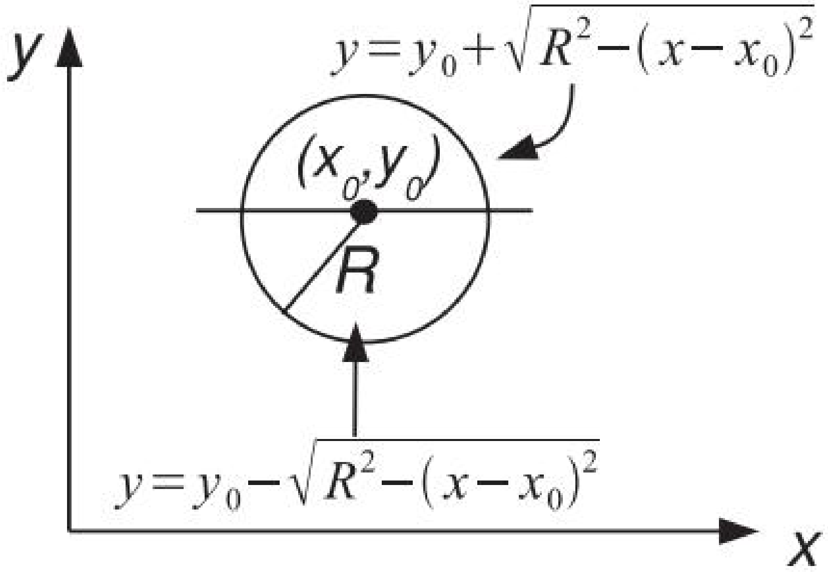
- The hit in the drift tube chambers is represented by drift circle. So we need to deal with the Legendre transform of a circle

$$f(x) = \begin{cases} f_1(x) = y_0 + \sqrt{R^2 - (x - x_0)^2} \\ f_2(x) = y_0 - \sqrt{R^2 - (x - x_0)^2} \end{cases}$$

- With some simple mathematical calculations, we can get the Legendre transform of a circle is

$$F_1(p) = f_1(x) - px = y_0 - x_0 p + R\sqrt{p^2 + 1}$$

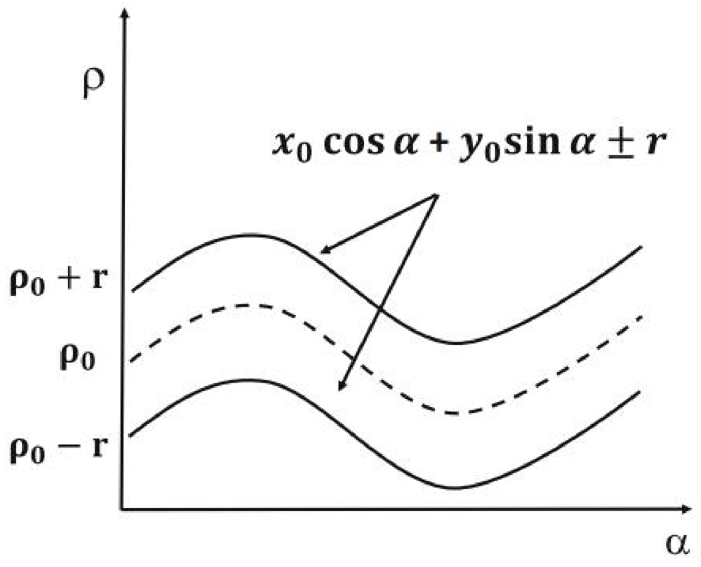
$$F_2(p) = x_0 p - y_0 + R\sqrt{p^2 + 1}$$

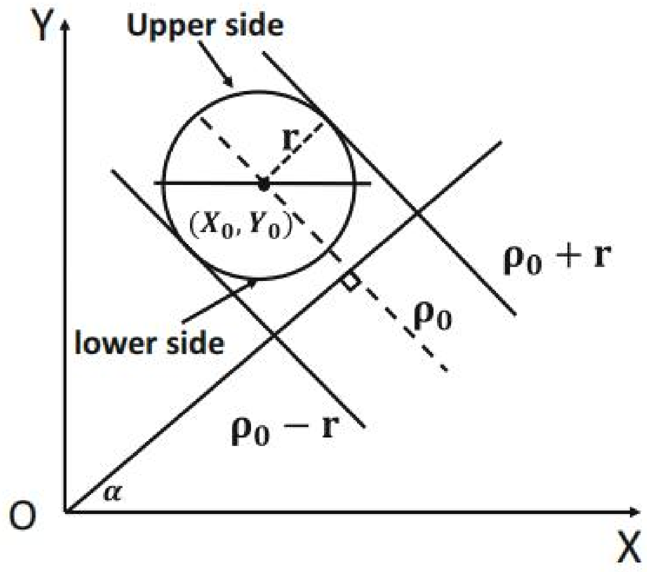


Legendre transform

- As mentioned before, each pair (p, F(p)) defines a tangent to the circle. For normal behavior at large values of p, it is more suitable to express the line equation by its canonical form

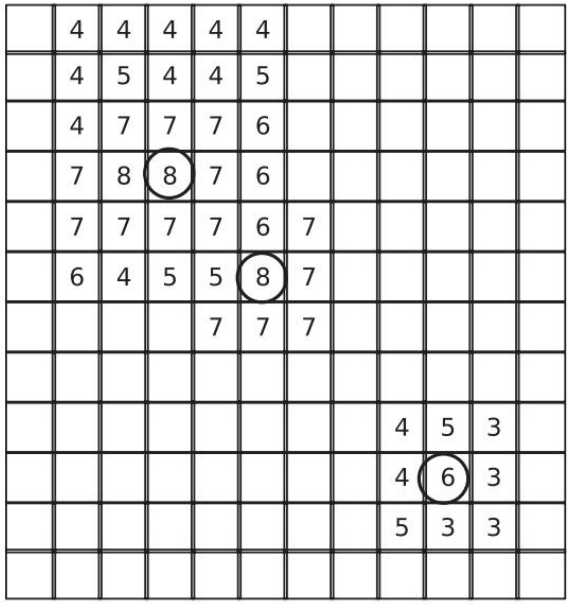
$$f(x) \overset{\mathscr{L}}{\longleftrightarrow} \begin{cases} r = x_0 \cos\theta + y_0 \sin\theta + R & \text{for concave} \\ r = x_0 \cos\theta + y_0 \sin\theta - R & \text{for convex} \end{cases}$$

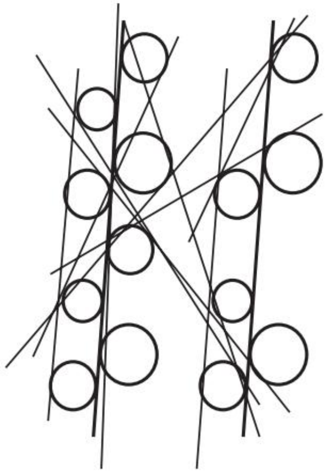


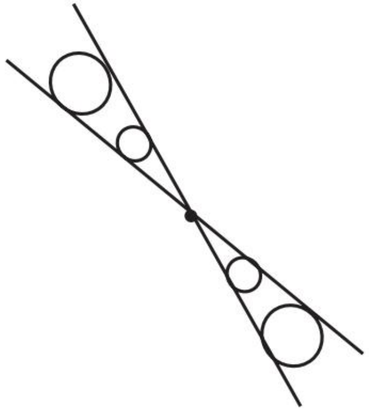


Legendre transform

- Finally, the question is transformed to find the collinear point in Legendre space
  - The height of a neighboring bin is lower than the height of the central bin.
  - The height of a neighboring bin is equal to the height of the central bin and the bins are adjacent to each other.
  - The height of a neighboring bin is equal to the height of the central bin but there are smaller peaks between them







$$\delta r = \sqrt{\left(\frac{\partial r}{\partial R}\delta R\right)^2 + \left(\frac{\partial r}{\partial \theta}\delta \theta\right)^2}$$

$$\Rightarrow \delta r = \sqrt{\delta R^2 + (x_0 \sin\theta - y_0 \cos\theta)^2 \delta\theta^2}$$

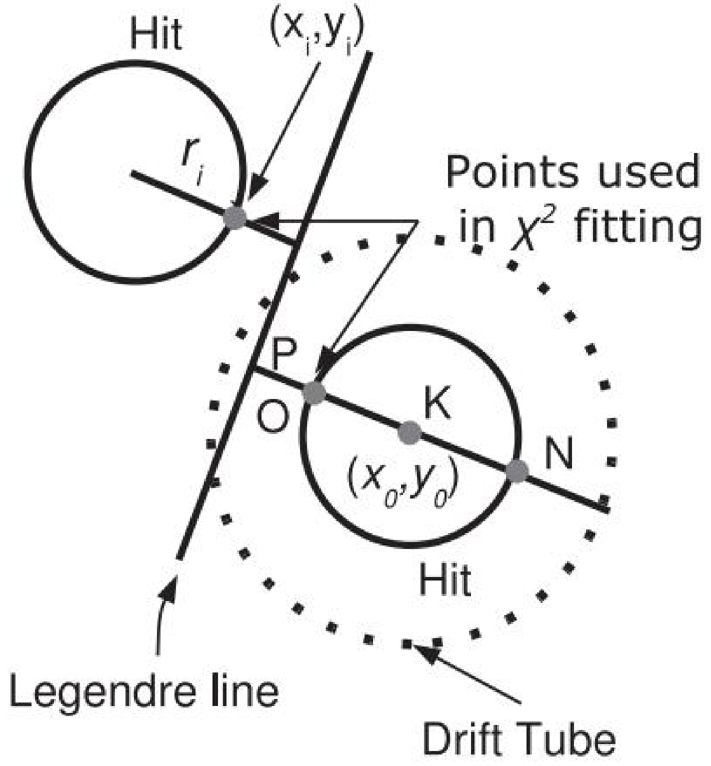
Legendre transform

• After identifying the lines in the Legendre histogram, the various drift circles are been associated with the lines. The criterion used for associating a circle with a line is

$$|d_i - r_i| < 5\sigma$$

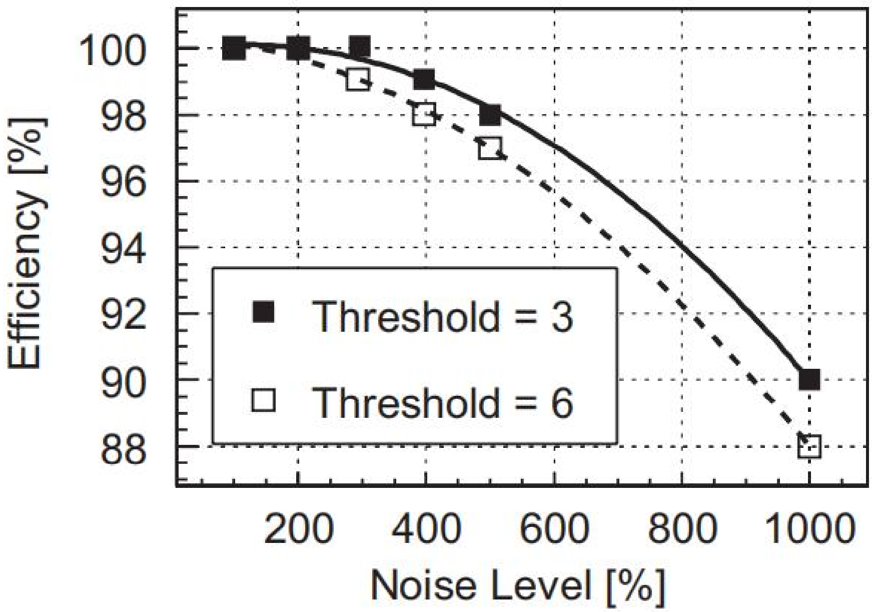
• Use least squares to fit the track parameters

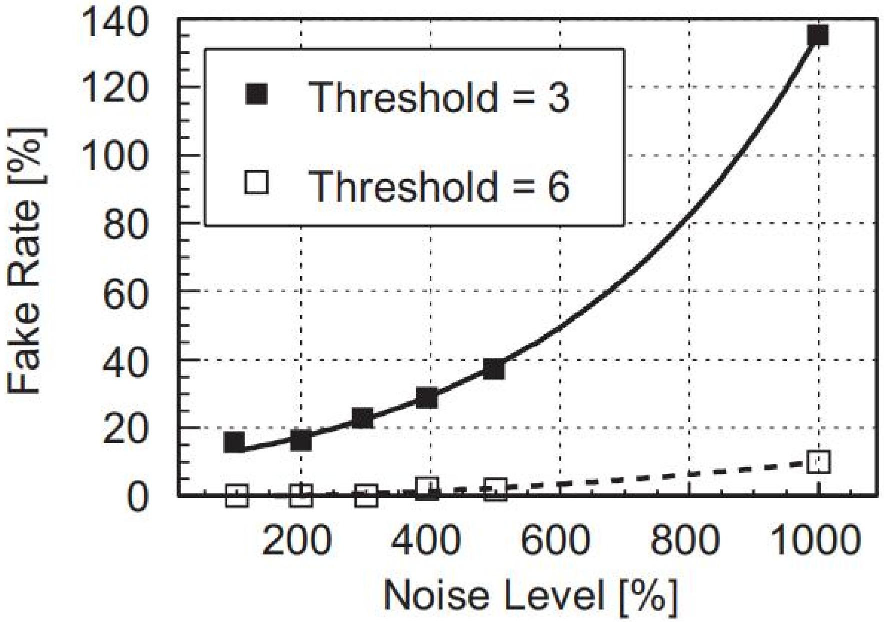
$$\chi^2 = \sum_{i=1}^{n} \left(\frac{\text{drift}_i - \text{doca}_i}{\sigma_i} \right)^2 ,$$



Legendre transform

- **Performance**





Hough transform in BESIII

- We have introduced the template matching algorithm in BESIII. It has a good performance for high P_T tracks, but for low P_T tracks it's efficiency drops significantly

- Difficulties for low P_T tracks
    - The track becomes curling with multiturn loops. As a result, the hits from different turns may leave adjacent hits or overlapped hits in one super-layer
    - Due to the energy loss, the tracks deviate more from a perfect helix

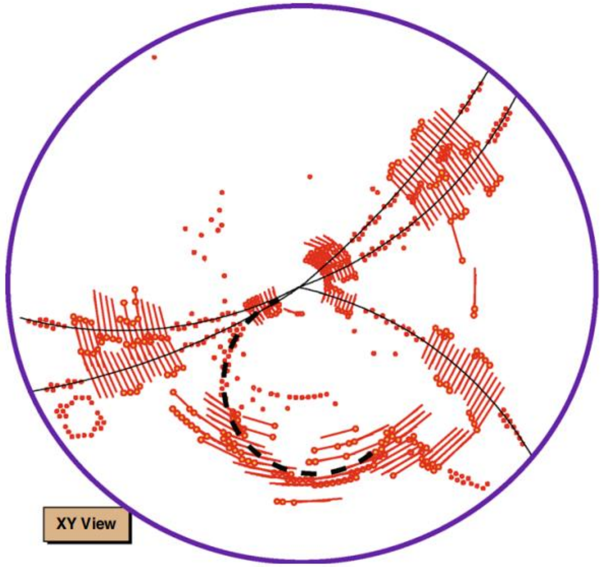
- Adopt the hough transform for low P_T tracking. It is effective for high-energy physics for the convenience to detect the linear and circular tracks as a global track finding method

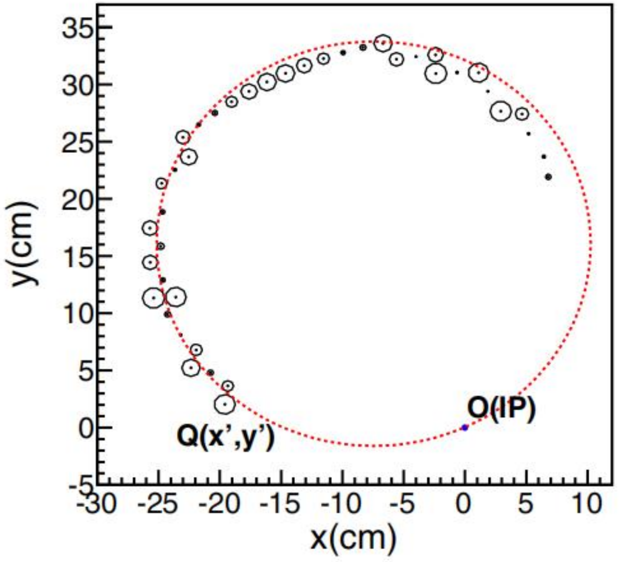
Hough transform in BESIII

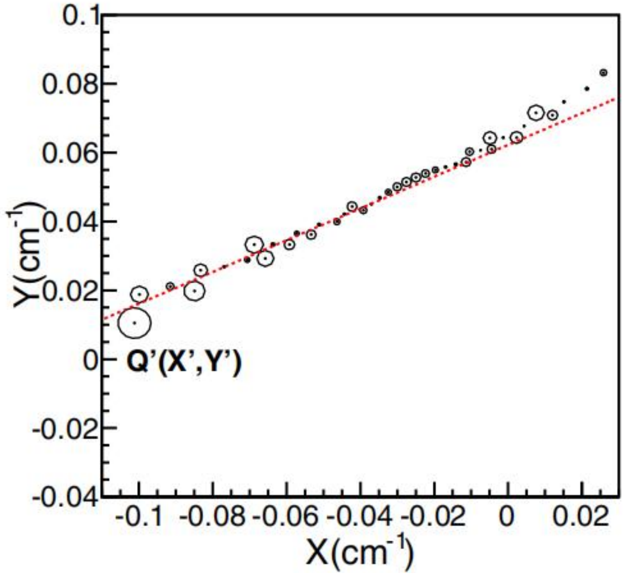
- Conformal mapping
    - For the circle track in x–y plane, map it to a straight line through conformal transformation

$$X = \frac{2x}{x^2 + y^2}, \quad Y = \frac{2y}{x^2 + y^2}.$$

    - Owing to energy loss. In the track finding stage on the x—y plane, hits on the first and second halves of the circle, respectively, can be treated as two tracks with different charges.

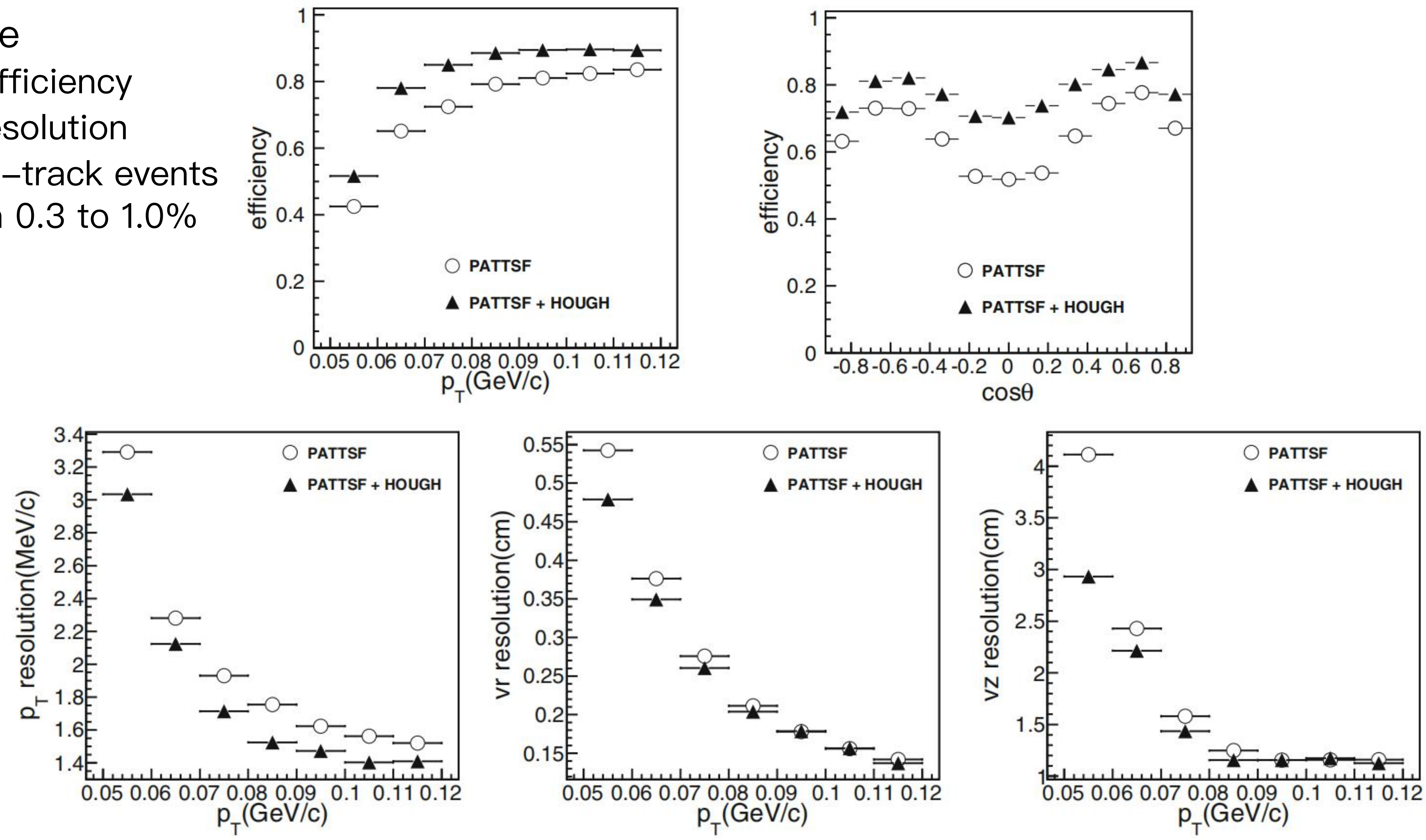






Hough transform in BESIII

- Performance
  - Higher efficiency
  - Better resolution
  - The fake-track events increases from 0.3 to 1.0%



Summary

- The global method of pattern recognition is widely used in high energy physics and have a good performance in track finding
- The track finding for low pT particles is a difficult task. But we can use more wire group or adopt hough transform to improve its performance.

Reference

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