Flipped modular flow and swing surfaces beyond flat holography

Frontier physics working month @ USTC

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AdS/CFT correspondence

Holographic principle states that gravity in a given spacetime region can be encoded on a lower-dimensional boundary of that region ['t Hooft '93; Susskind '94]. AdS/CFT correspondence is one of the most famous examples: quantum gravity in (d+1)-dimensional asymptotically Anti-de Sitter spacetime is equivalently described by a d-dimensional conformal field theory living on its boundary [Maldacena '97].

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- Top-down approach: start with a UV complete string theory. Example: type IIB string theory with string length l_s and coupling constant g_s on $AdS_5 \times S_5$ with curvature radius L is dual to 4D $\mathcal{N} = 4$ SU(N) Yang-Mills with the dictionary

$$g_{YM}^2 = 2\pi g_s \, , 2 g_{YM}^2 N = L^4 / l_s^4$$

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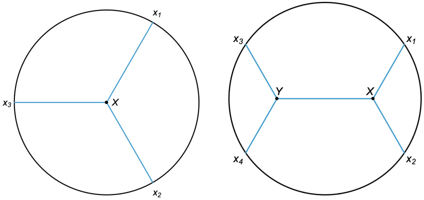

$$\langle O(x_1)O(x_2)O(x_3)\rangle_{CFT} = \int_{AdS} dX G_{b\partial}(x_1, X) G_{b\partial}(x_2, X) G_{b\partial}(x_3, X)$$

$$\begin{aligned}&\langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle_{CFT} \\ &= \int_{AdS} dXdY G_{b\partial}(x_1, X) G_{b\partial}(x_2, X) G_{bb}(X, Y) G_{b\partial}(x_3, Y) G_{b\partial}(x_4, Y)\end{aligned} \quad (1)$$

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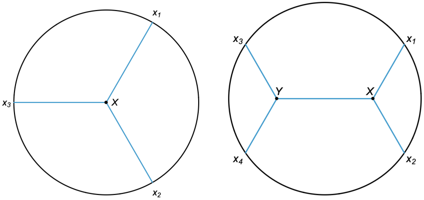

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  - Entanglement entropy: Ryu-Takayanagi proposal [Ryu-Takayanagi '06]

AdS_3/CFT_2 is special since the symmetry algebra is enhanced from $so(2,2) \simeq sl(2,R) \times sl(2,R)$ to $vir \times vir$ [Brown-Henneaux '86]:

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m,-n}\,, \\ [\bar{L}_m, \bar{L}_n] &= (m-n)\bar{L}_{m+n} + \frac{\bar{c}}{12}(m^3 - m)\delta_{m,-n}\,. \end{aligned} \tag{2}$$

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We always work in highest-weight representation

$$L_0|h,\bar{h}\rangle = h|h,\bar{h}\rangle \,, \bar{L}_0|h,\bar{h}\rangle = \bar{h}|h,\bar{h}\rangle \,,$$
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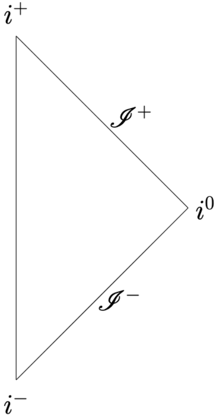
Lower point correlators are completely fixed by symmetry, e.g.

$$\langle O_{h_1,\bar{h}_1}(z_1) O_{h_2,\bar{h}_2}(z_2)\rangle = \frac{\delta_{h_1,h_2}\delta_{\bar{h}_1,\bar{h}_2}}{z_{12}^{2h_1}\,\bar{z}_{12}^{2\bar{h}_1}}\,. \tag{4}$$

Quantum gravity in asymptotically flat spacetime(AFS) is conjectured to be described by a Carrollian conformal field theory(CCFT) living at null infinity $\mathscr{I}^{\pm}$ [Barnich-Compere '06; Bagchi '10]. It can be viewed as a flat-space/ultra-relativistic(UR) limit of AdS/CFT, where the boundary CFT degenerates into a theory with conformal Carrollian symmetry [Bagchi-Fareghbal '12; Hijano '19].



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$Flat_4/CCFT_3$: Interesting soft physics; Real world.
$Flat_3/CCFT_2$: Simpler but fruitful.

The asymptotic symmetries of 3D flat space at null infinity are governed by the infinite-dimensional bms_3 algebra [Bondi-Metzner '62; Sachs '62]

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Induced(unitary) representation:

$$\begin{aligned}
&\mathcal{L}_0|\Delta,\xi\rangle = \Delta|\Delta,\xi\rangle\,, \quad M_0|\Delta,\xi\rangle = \xi|\Delta,\xi\rangle\,, \\
&M_n|\Delta,\xi\rangle = 0\,, \quad n \neq 0\,.
\end{aligned} \tag{6}$$

Highest weight(non-unitary) representation:

$$\begin{aligned}
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&\mathcal{L}_n|\Delta,\xi\rangle = M_n|\Delta,\xi\rangle = 0\,, \quad n > 0\,.
\end{aligned} \tag{7}$$

From AdS_3/CFT_2 to $Flat_3/CCFT_2$

Field theory side: UR contraction from $vir \times vir$ to bms_3:

$$M_n = \lim_{\epsilon \to 0} \epsilon(L_n + \bar{L}_{-n}), \mathcal{L}_n = \lim_{\epsilon \to 0} L_n - \bar{L}_{-n},$$
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$$HWR \to induced \;\; rep., flipped \;\; rep, \to HWR \,, \tag{9}$$

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Gravity side: Flat limit from AdS_3 to $Flat_3$:

- Poincare AdS_3 to $Minkowski_3$

$$ds^2 = -\frac{r^2}{L^2}du^2 - 2dudr + r^2dx^2$$

- BTZ black hole to flat space cosmology

$$ds^2 = -\frac{r^2 - (L\hat{r}_+)^2 - r_-^2}{L^2}du^2 - 2dudr - 2\hat{r}_+ r_- dud\phi + r^2 d\phi^2$$

Induced vs Highest-weight

" The relevance of the two types of representations — induced or highest-weight — of the dual CCFT2 for holography in 3d asymptotically flat spacetimes has been debated for a while in the literature. The most obvious argument is that since induced representations are explicitly unitary, while the highest-weight representations are not, it is the induced representation that should play more of a role in the construction of flat holography. Interestingly, though, most calculations for matching bulk and boundary observables had explicitly or implicitly used highest weights". [Aggarwal-Bagchi-Detournay-Grumiller-Riegler-Simon '25]

Induced vs Highest-weight

- Character and torus partition function: Depending on the regularization method, the result could agree with either induced rep.[Barnich- Gonzalez-Maloney-Oblak '15; Cotler-Jensen-Prohazka-Riegler-Salzer '24] or highest-weight rep. [Merbis-Riegler '19; Simon-Yu '24];

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The bulk action for 3d pure Einstein gravity is

$$S = \frac{1}{16\pi G} \int d^3x \sqrt{-g} R \,. \tag{10}$$

Asymptotically Minkowski metrics in 3d can be written as

$$ds^2 = M du^2 - 2du dr + 2N du d\varphi + r^2 d\varphi^2 \,, N = L + \frac{u}{2} \partial_\varphi M \,. \tag{11}$$

where $r \to \infty$ is the boundary $\mathscr{I}^+$. The phase space is preserved under the infinitesimal diffeomorphism generated by $\xi \in bms_3$

$$\xi = (\alpha + u\epsilon')\partial_u + \epsilon \partial_\varphi + (-r\epsilon' + \alpha'' + u\epsilon''')\partial_r \tag{12}$$

with $\alpha = \alpha(\varphi), \epsilon = \epsilon(\varphi)$.
The action of ξ induced a coadjoint action on M and N

$$\begin{aligned} \delta M &= \epsilon M' + 2\epsilon' M - 2\epsilon''' \\ \delta L &= \frac{1}{2} \alpha M' + \alpha' M - \alpha''' + \epsilon L + 2\epsilon' L. \end{aligned} \tag{13}$$

Using coadjoint orbit quantization method, 3d flat gravity is described by a two dimensional effective action in terms of a representative of the coadjoint orbit (M_0, L_0) and $(\alpha, f) \in BMS_3$

$$S = -\frac{k}{2\pi}\int dy d\varphi \left(i(L_0 f' + M_0\alpha')\partial_y f - i\frac{\alpha'(f'\partial_y f'' - f''\partial_y f')}{f'^3} - \frac{M_0}{2}f'^2 + \{f, \varphi\} \right) \tag{14}$$

with $k = \frac{1}{4G}, y = iu, \{f, \varphi\} = \frac{f'''}{f'} - \frac{3f''^2}{2f'^2}$.

We require the boundary to be a torus $(u, \varphi) \sim (u, \varphi + 2\pi) \sim (u + i\beta, \varphi + \beta\Omega)$, so that boundary conditions on the phase space are

$$\begin{aligned} f(y+\beta, \varphi+\beta\Omega) &= f(\varphi, y), f(y, \varphi + 2\pi) = f(y, \varphi) + 2\pi\,, \\ \alpha(y+\beta, \varphi+\beta\Omega) &= \alpha(\varphi, y), \alpha(y, \varphi+2\pi) = \alpha(y, \varphi)\,. \end{aligned} \tag{15}$$

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Task: Compute the partition function exactly

$$\mathcal{Z} = \int df d\alpha e^{-S} \quad (16)$$

Fermionic localization and one-loop exactness

One can expand f and α around saddles and compute $\mathcal{Z}$ order by order.
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- According to the Duistermaat-Heckman theorem,

$$\mathcal{Z} = \mathcal{Z}[s] := \int [dx^i][d\psi^i]\, e^{-(S' + sQF)} \tag{19}$$

where $Q^2 F = 0, (QF)_{\text{bosonic}} \geq 0$.

The $s \to \infty$ limit localizes the path integral to the localization manifold

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In the end, we find

$$\mathcal{Z} = e^{-S_0} \prod_{m,n\neq 0} (m - \frac{\beta\Omega n}{2\pi})^{-1} = \left\{ \begin{array}{c} e^{-S_0} \prod_{n=1}^{\infty} \frac{1}{|1-q^n|^2} \\ \text{or } e^{-S_0} q^{-\frac{1}{12}} \prod_{n=1}^{\infty} \frac{1}{(1-q^n)^2} \end{array} \right. \tag{22}$$

where $S_0 = \frac{k}{24}\beta(M_0 + 2i\Omega L_0)$ is the classical value of the Euclidean action and $q = e^{i\beta\Omega}$.

Let Σ be a Cauchy slice of a QFT and $A \subset \Sigma$ be a subregion, entanglement entropy S_A of A is defined as the von-Neumann entropy of the reduced density matrix ρ_A:

$$\rho_A = \mathrm{Tr}_{A^c}\rho\,, S_A = -\mathrm{Tr}\rho_A \ln \rho_A\,. \tag{23}$$

Within AdS/CFT, S_A is holographically dual to an extremal surface γ_A in the bulk homologous to A [Ryu-Takayanagi '06]:

$$S_A = \underset{B\sim A}{\mathrm{ext}} \frac{\mathrm{Area}(\gamma_A)}{4G}\,. \tag{24}$$

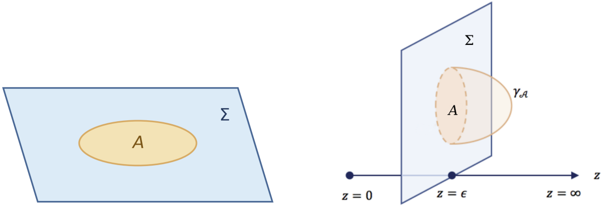

Figure 1: Left: Entanglement entropy S_A of boundary CFT. Right: Ryu-Takayanagi proposal.

In AdS_3/CFT_2, RT surface is given by the geodesic in the bulk connecting ∂A. If A is $(-l_{x^+}, -l_{x^-}) \to (l_{x^+}, l_{x^-})$, we have

$$S_A = \frac{c}{6} \log \frac{4 l_{x^+} l_{x^-}}{\epsilon^2} \,. \tag{25}$$

In $Flat_3/CCFT_2$, entanglement entropy is computed by a swing surface configuration.[Jiang-Song-Wen '17] The proposal is

$$S_A = \underset{\gamma_A \cup \gamma_+ \cup \gamma_- \sim A}{\text{ext}} \frac{Length(\gamma_A)}{4G}, \tag{26}$$

where $\gamma_\pm$ are two null geodesics emanating from ∂A.

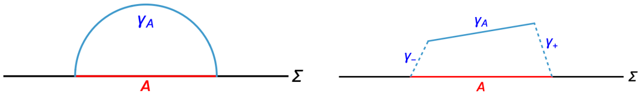

Figure 2: Left: RT surface in AdS_3. Right: swing surface in $Flat_3$.

In AdS_3/CFT_2, RT surface is given by the geodesic in the bulk connecting ∂A. If A is $(-l_{x^+}, -l_{x^-}) \to (l_{x^+}, l_{x^-})$, we have

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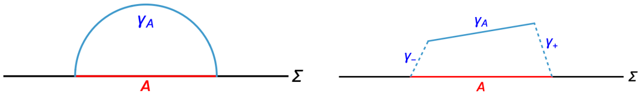


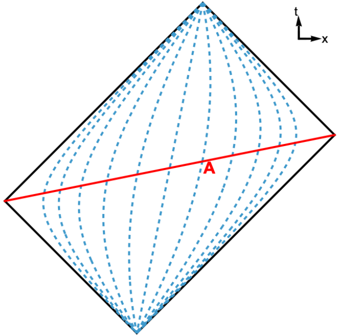
Figure 2: Left: RT surface in AdS_3. Right: swing surface in $Flat_3$.

Neither the swing surface configuration nor S_A can be obtained by a naive flat limit of the ordinary RT geodesic. Are swing surfaces intrinsically flat objects?

Modular flow in CFT_2

The reduced density can be written as $\rho_A = e^{-H}$ with H called the modular Hamiltonian.[Haag '92] If ρ is the vacuum state, H is local and generates a modular flow. [Bisognano-Wichmann '75; Casini-Huerta-Myers '11] Let ζ be the vector field tangent to the flow which should satisfy [Apolo-Jiang-Song-Zhong '20]

- ζ is a linear combination of global symmetry $sl(2,R) \times sl(2,R)$
- ζ leaves the causal domain $D[A]$ of A invariant;
- ∂A are fixed points of ζ;
- The flow $e^{s\zeta}$ has imaginary periodicity $s \sim s + i$.
- If $x \in D[A]$, then $e^{i\zeta/2}x \in D[A^c]$.



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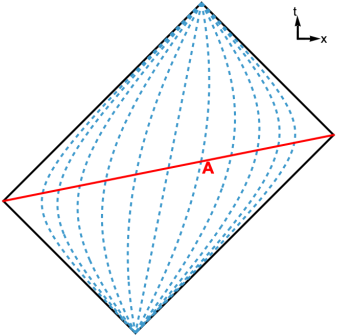

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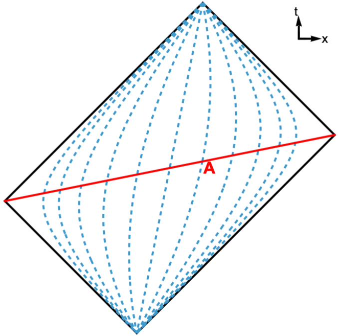
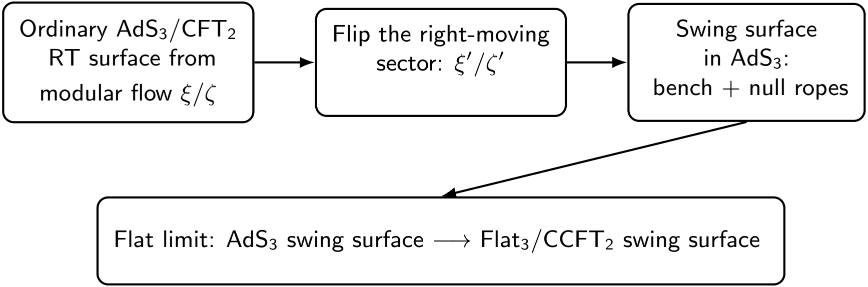


ζ can be uniquely uplifted to a bulk isometry generator ξ and RT surface γ_A is given by the bifurcating surface of ξ.However, flat limit of ζ or ξ is singular.

Roadmap: from RT surfaces to swing surfaces



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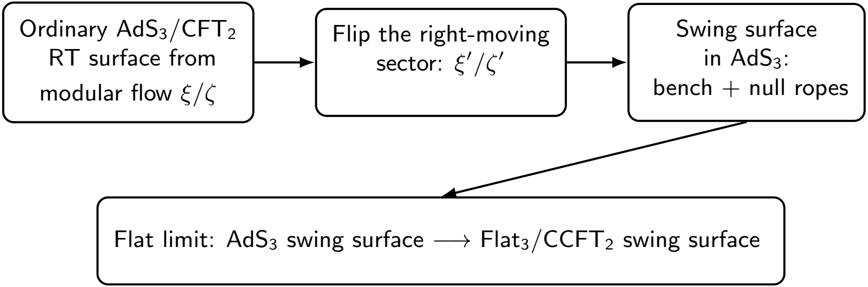

Main message

Swing surfaces are not intrinsically flat objects. They can be understood as the geometric fixed locus of a *flipped modular flow* already in AdS_3, whose flat limit reproduces the $Flat_3/CCFT_2$ entanglement proposal.

Flipped modular flow

In general, we have

$$\zeta = \sum_{i=-1}^{1} a_i L_i + \bar{a}_i \bar{L}_i$$

We define the flipped modular flow to be

$$\zeta' = \sum_{i=-1}^{1} a_i L_i - \bar{a}_i \bar{L}_i$$

It can be checked directly that ζ' satisfies the same properties as ζ except for the last one.

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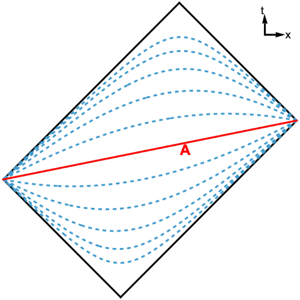

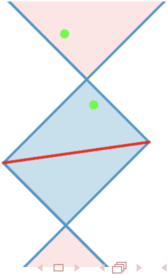

Using the flipped modular flow, we can construct a swing surface in AdS_3 as follows.

- Extending ζ' into a bulk isometry generator ξ';
- Solve for the bifurcating surface $\gamma_{\xi'}$ of ξ'.
- ∂A are no longer fixed points of ξ', but move along null directions. Solve null geodesics $\gamma_\pm$ emanating from ∂A and tangent to ξ'.
- Let $p_\pm = \gamma_\pm \cap \gamma_{\xi'}$. The bench γ_A is the piece of $\gamma_{\xi'}$ truncated by $p_\pm$.

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Example: BTZ geometry

In Bondi gauge, the metric is presented as

$$ds^2 = -\frac{r^2 - r_+^2 - r_-^2}{L^2}du^2 - 2dudr - \frac{2r_+r_-}{L}dud\theta + r^2 d\theta^2\,. \tag{27}$$

The six isometry generators are

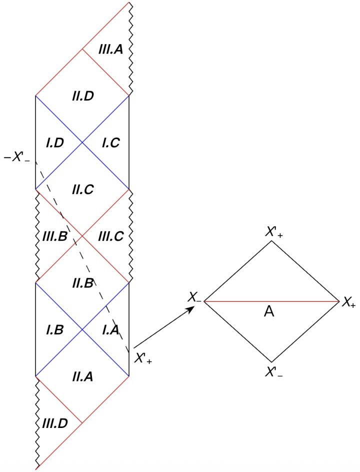
$$\begin{aligned}
L_{\pm 1} &= -\frac{L^2 e^{\frac{\pm(r_+ - r_-)(L\theta + u)}{L^2}}}{2(r_+ - r_-)}\left(\partial_u \mp \frac{(r_+ - r_-)(r - r_\pm)(r + r_\mp)}{L^2 r}\partial_r + \frac{r \mp (r_+ - r_-)}{Lr}\partial_\theta\right),\\
L_0 &= -\frac{L^2}{2(r_+ - r_-)}\left(\partial_u + \frac{1}{L}\partial_\theta\right),\\
\bar{L}_{\pm 1} &= -\frac{L^2 e^{\frac{\pm(r_+ + r_-)(L\theta - u)}{L^2}}}{2(r_+ + r_-)}\left(\partial_u \pm \frac{(r_+ + r_-)(r \pm r_+)(r \pm r_-)}{L^2 r}\partial_r - \frac{r \pm (r_+ + r_-)}{Lr}\partial_\theta\right),\\
\bar{L}_0 &= \frac{L^2}{2(r_+ + r_-)}\left(\partial_u - \frac{1}{L}\partial_\theta\right).
\end{aligned} \tag{28}$$

Let $\mathcal{A} : (-u_0, -\theta_0) \to (u_0, \theta_0)$ and the flipped bulk modular flow is found to be

$$\xi' = \sum_{i=-1}^{1} a_i L_i - \bar{a}_i \bar{L}_i \tag{29}$$

with

$$\begin{aligned}
(a_{-1}, a_0, a_1) &= \frac{\pi}{\sinh \frac{(r_+ - r_-)(L\theta_0 + u_0)}{L^2}} \left(1, -2\cosh \frac{(r_+ - r_-)(L\theta_0 + u_0)}{L^2}, 1\right) , \\
(\bar{a}_{-1}, \bar{a}_0, \bar{a}_1) &= -\frac{\pi}{\sinh \frac{(r_+ + r_-)(L\theta_0 - u_0)}{L^2}} \left(1, 2\cosh \frac{(r_+ + r_-)(L\theta_0 - u_0)}{L^2}, 1\right) .
\end{aligned} \tag{30}$$



Patch	X_1^+	X_1^-	X_2^+	X_2^-
I.A	$+$	$+$	$+$	$-$
II.A	$-$	$+$	$+$	$-$
III.A	$+$	$+$	$+$	$+$
I.B	$-$	$-$	$+$	$-$
II.B	$+$	$-$	$+$	$-$
III.B	$+$	$-$	$-$	$-$
I.C	$+$	$+$	$-$	$+$
II.C	$+$	$-$	$-$	$+$
III.C	$+$	$-$	$+$	$+$
I.D	$-$	$-$	$-$	$+$
II.D	$-$	$+$	$-$	$+$
III.D	$-$	$+$	$-$	$-$

Figure 3: Left: Penrose diagram for the maximally extended BTZ with angular momentum. Blue and red edges are outer and inner horizons respectively. The diagram is repeated infinitely in the vertical direction. Right: Sign of lightcone coordinates in each region. Dashed line is the trajectory of $\gamma^+_{\xi'}$.

Denoting $P_\pm = \gamma_{\xi'} \cap \gamma_\pm$, it can be shown

$$L_{P_+P_-} = L\left(\log \frac{\sinh \frac{2\pi}{\beta_+}(\theta_0 + u_0/L)}{\sinh \frac{2\pi}{\beta_-}(\theta_0 - u_0/L)} + \alpha\right) , \tag{31}$$

where $\beta_\pm = \frac{2\pi L}{r_+ \mp r_-}$ and α is related to the UV cutoff via

$$\frac{\epsilon_u}{\epsilon_\theta} = \frac{L(r_- \cosh\frac{\alpha}{2} - r_+ \sinh\frac{\alpha}{2})}{r_+ \cosh\frac{\alpha}{2} - r_- \sinh\frac{\alpha}{2}} . \tag{32}$$

Flat limit: letting $r_+ = \sqrt{M}L, r_- = -\frac{J}{2\sqrt{M}}, \alpha = L^{-1}\hat{\alpha}$ and taking $L \to \infty$, we have

$$L_{P_+P_-} = \sqrt{M}(2u_0 + \frac{J}{M}\theta_0) \coth \sqrt{M}\theta_0 - \frac{J}{M} - \frac{2\epsilon_u}{\epsilon_\theta} \tag{33}$$

in precise agreement with HEE in FSC.

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Remark:

- $L_{P_+P_-}$ can also be computed by extremizing the distance between two ropes $\gamma_\pm$;
- When $L \to \infty$, $P_\pm$ are located inside the outer horizon.

Bench as inner horizon

In CFT$_2$, there exists a unitary conformal transformation U such that $U\rho_A U^{-1} = \rho_{thermal} = e^{-\beta_L H_L - \beta_R H_R}$. From gravity side, the bulk conformal transformation maps AdS vacuum to a BTZ black hole with $D[A]$ being mapped to its entire boundary.[Casini-Huerta-Myers '11]

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$$ds^2 = \frac{L^2 dZ^2}{4Z^2} + \frac{dUdV}{Z}, \tag{34}$$

the boundary conformal transformation $U = f(u), V = \bar{f}(v)$ can be extended to the bulk via the following coordinate transformation

$$U = f - \frac{2L^2 z f'^2 \bar{f}''}{4f'\bar{f}' + L^2 z f'' \bar{f}''}, V = \bar{f} - \frac{2L^2 z \bar{f}'^2 f''}{4f'\bar{f}' + L^2 z f'' \bar{f}''}, Z = z\frac{16 f'^3 \bar{f}'^3}{(4f'\bar{f}' + L^2 z f''\bar{f}'')^2}. \tag{35}$$

For $\mathcal{A} : (-\frac{l_U}{2}, -\frac{l_V}{2}) \to (\frac{l_U}{2}, \frac{l_V}{2})$, the Rindler transformation corresponds to choosing

$$f = l_U \tanh\frac{\pi u}{b_1}, \quad \bar{f} = l_V \tanh\frac{\pi v}{b_2}. \tag{36}$$

The resulting metric becomes

$$ds^2 = \frac{L^2 dz^2}{4z^2} + \frac{(du + \frac{\pi^2 L^2 z}{b_1^2} dv)(dv + \frac{\pi^2 L^2 z}{b_2^2} du)}{z}, \tag{37}$$

which is a black hole metric. The bulk modular flow ξ and the corresponding flipped vector field ξ' become

$$\xi = -b_1 \partial_u + b_2 \partial_v, \quad \xi' = -b_1 \partial_u - b_2 \partial_v. \tag{38}$$

It can be shown that the Killing horizons of ξ and ξ' are $z = \frac{b_1 b_2}{\pi^2 L^2}$ and $z = -\frac{b_1 b_2}{\pi^2 L^2}$, which are the outer and inner horizons of the black hole respectively. In other words, the RT surface is mapped to the outer horizon of the black hole, and the bench is mapped to the inner horizon. The length of the bench is given by the inner horizon area. This picture is compatible with the flat limit.

Relation to flipped AdS_3/CFT_2

Let L_{γ_A} be the length of the bench in AdS_3. We find

$$e^{-mL_{\gamma_A}} = \frac{l_{x^-}^{mL}}{l_{x^+}^{mL}} \tag{39}$$

This is the correlator of two scalar primary operators with flipped weights $h = -\bar{h} = \frac{mL}{2}$ under the flipped vacuum.

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Generalization: Analytic continuation to timelike interval; Spinning operators; Conformal blocks and entanglement wedge cross section along the lines of in flat holography [Hijano '19].

Conclusions

- Regarding the torus partition function, we prove it is one-loop exact using fermionic localization method.
- We find a new modular flow satisfying almost all conditions required by an ordinary modular flow. In particular, it leaves the causal domain invariant;
- This flipped modular flow generates a swing surface configuration in AdS_3;
- The length of the bench agrees with the EE of CFT_2 in flipped vacuum;
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Outlook

- Higher dimensional generalization;
- Explicit construction of flipped modular Hamiltonian in toy models of CFT_2;
- Understand correlators of stress tensors within flipped AdS_3/CFT_2;
- Understand the role of flipped modular Hamiltonian in bulk reconstruction.

Thanks for your attention!