深度非弹散射与核子结构的高精度理论研究

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based on works of the NPC collaboration, and works in collaboration with L. Dong, S. Fang, HT Li, DY Shao, HX Zhu, YJ Zhu

中国科学技术大学 合肥 2026/5/22

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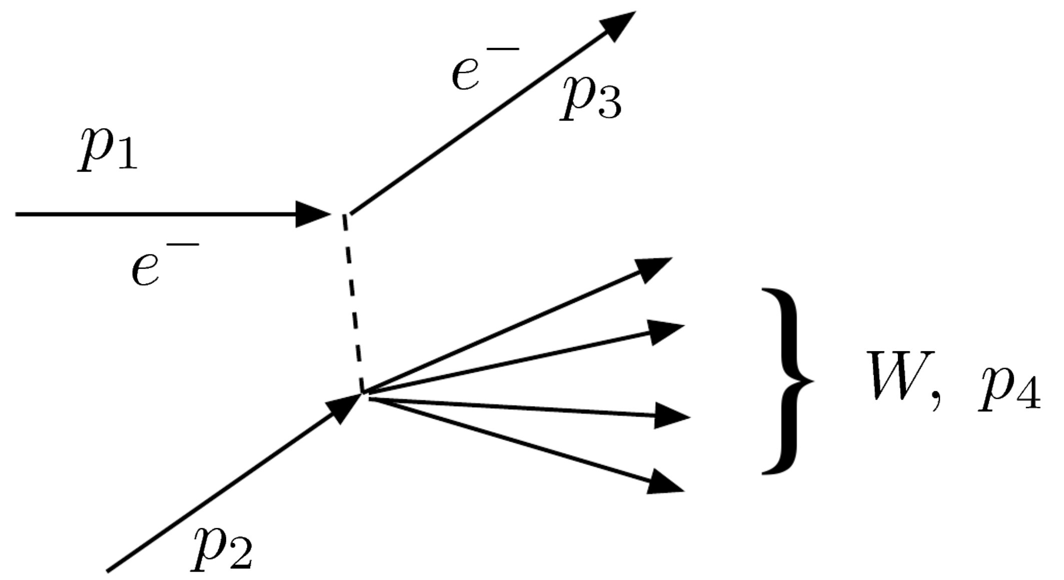
Outline

- 1. Deep inelastic scattering and nucleon structures
- 2. NPC23 analysis on FFs of light charged hadrons
- 3. Precision calculations for SIDIS up to N3LO
- 4. Summary

Deep inelastic scattering

- Deep inelastic scattering of high-energy leptons (~ 20 GeV) on fixed-target reveals the internal structure of nucleons, consisting of quarks and gluons (QPM) described by parton distribution functions (PDFs), and lead to the establishment of QCD

DIS kinematics



$$W^2 = M^2 + 2M\nu + q^2 = M^2 + Q^2(1/x - 1)$$

W=M → elastic scattering

1 < W < 2 GeV → inelastic scattering (exitation of resonances)

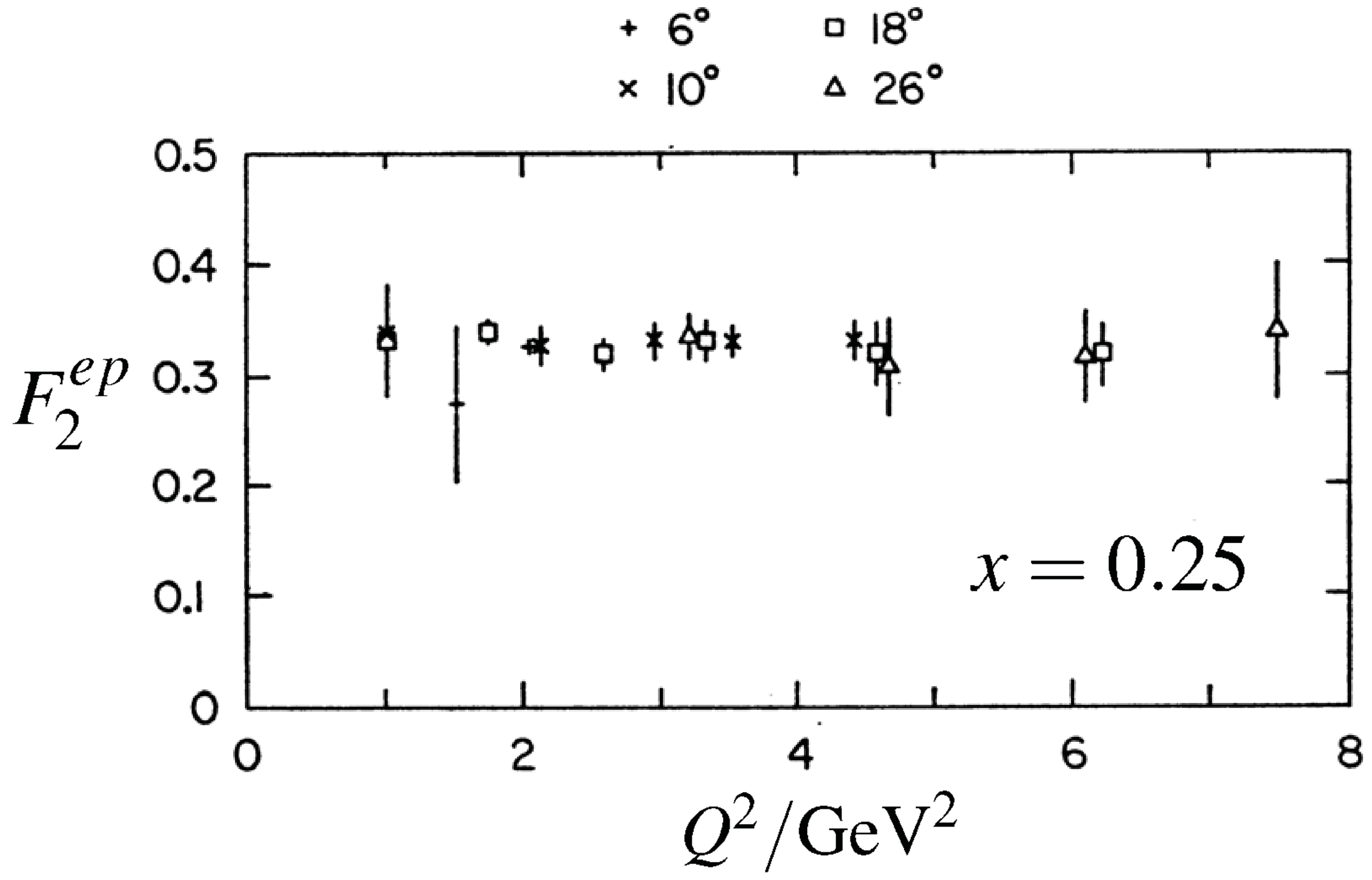
W > 2 GeV → deep inelastic scattering

	lab frame - proton at rest before collision: $p_2 = (M, 0, 0, 0)$		lorentz invariant form
energy loss of incoming particle	$\nu = E_1 - E_3$		$\nu = \frac{p_2 q}{M}$
Bjorken x	$x = \frac{Q^2}{2M\nu}$	x in [0,1]	$x = \frac{Q^2}{2p_2 q}$
fractional energy loss of incoming particle	$y = 1 - \frac{E_3}{E_1}$	y in [0,1]	$y = \frac{p_2 q}{p_2 p_1}$
4-momentum transver	$q^2 = (p_1 - p_3)^2$	$Q^2 = -q^2$	$q^2 = (p_1 - p_3)^2$

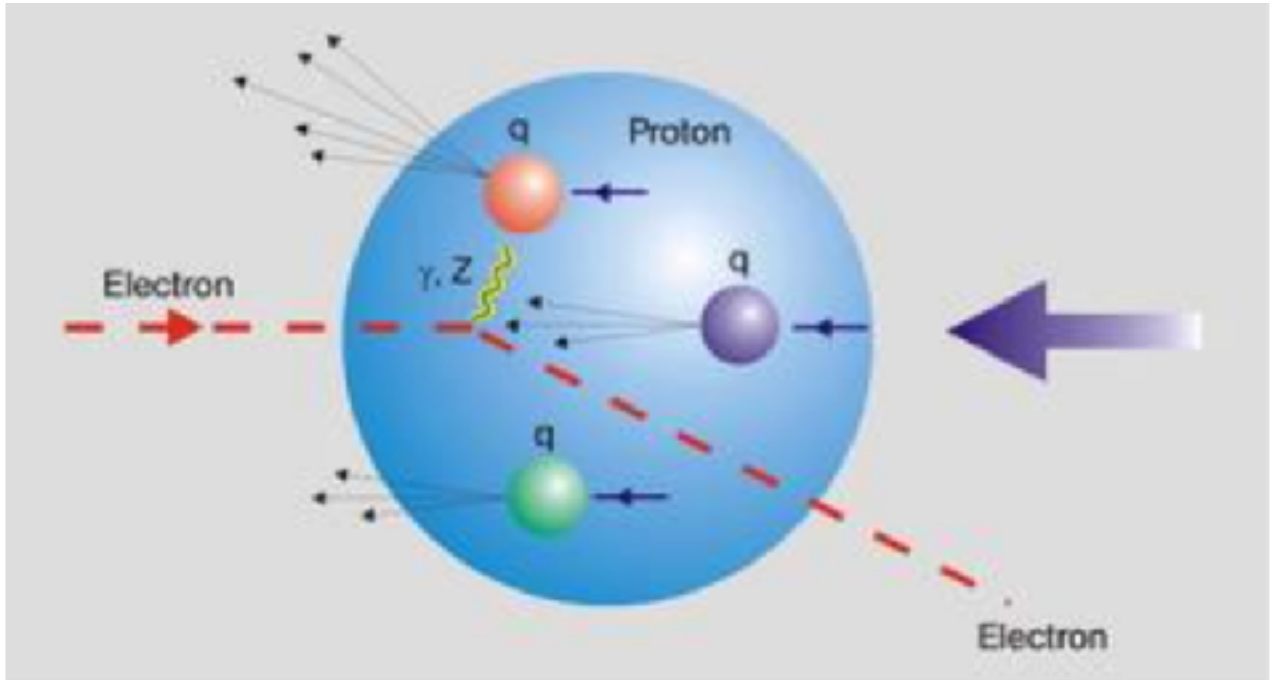
DIS cross sections and structure functions:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

Bjorken scaling of SFs and QPM



p_q=xp, with probability density ~ f(x)

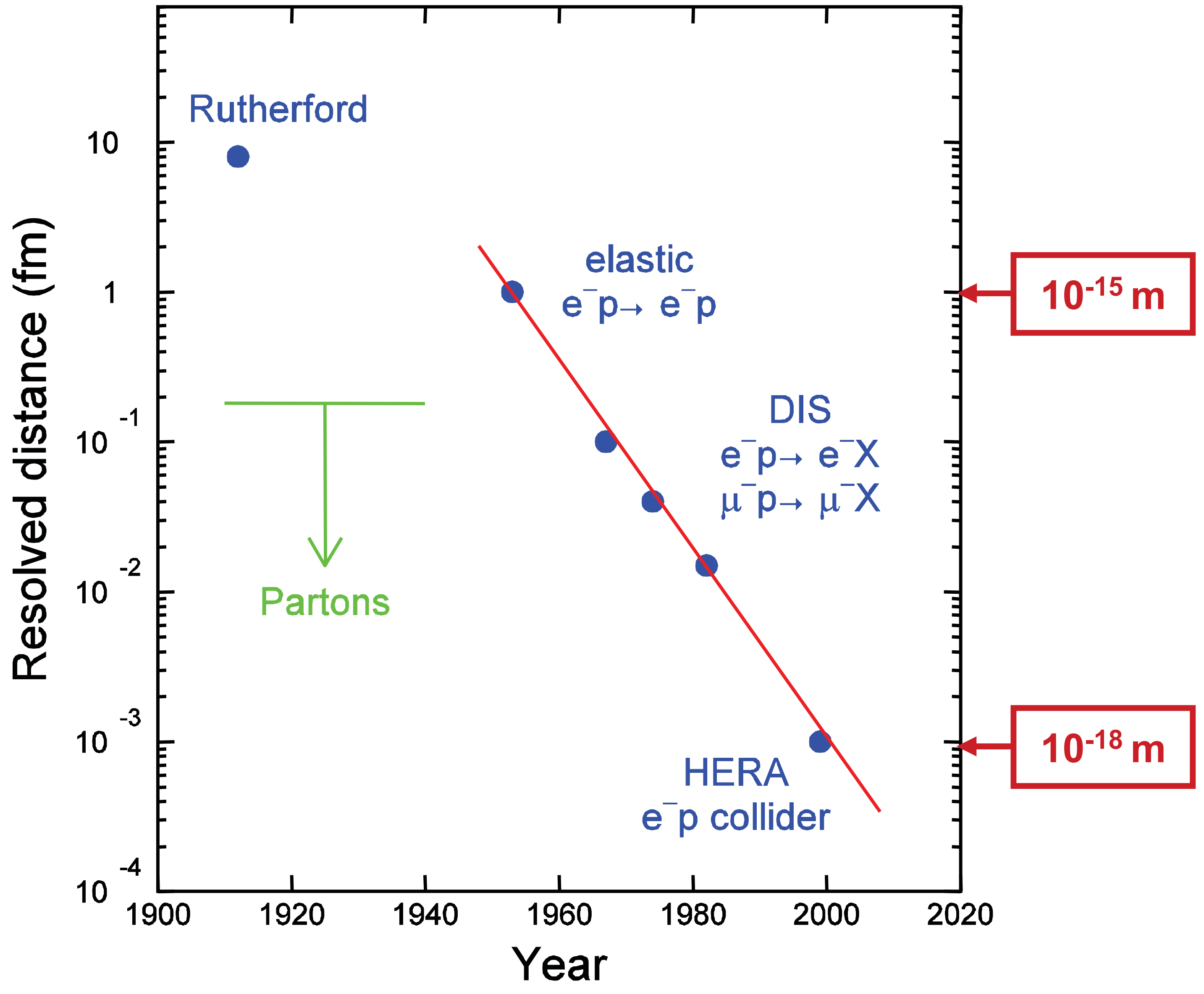


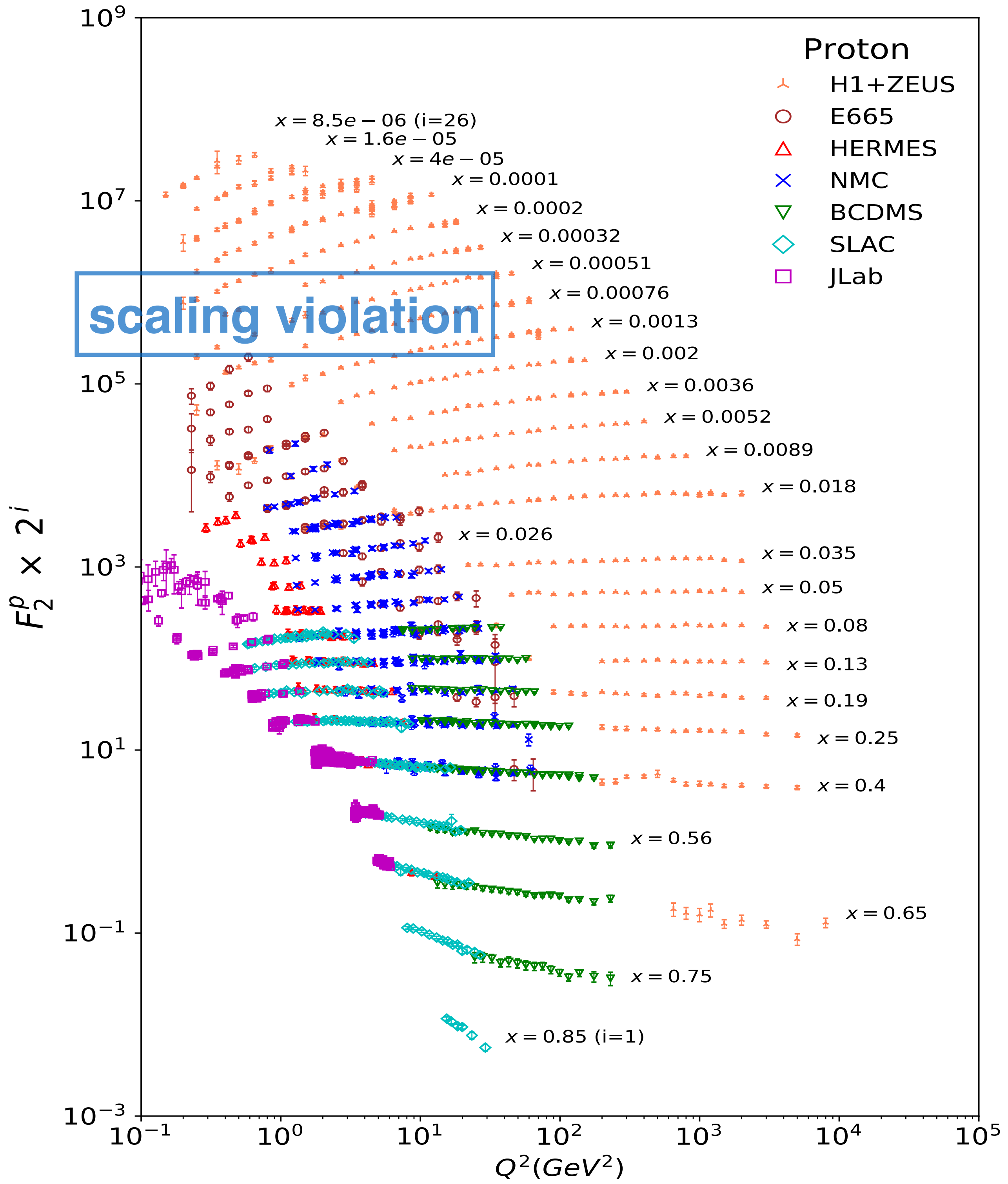
Parton distribution functions (PDFs)

Deep inelastic scattering

- The HERA collider together with fixed-target DIS experiments provide a complete and precise scan on the nucleon structure functions or PDFs, which are successfully described by DGLAP evolution equation from perturbative QCD due to asymptotic freedoms at high-energies

Legacy of HERA ep DIS





Evolution of DIS experiments and a summary of measured F_2

factorization of structure functions in QCD

$$F_2(x,Q^2) = \sum_{i=q,\bar{q},g} \int_0^1 d\xi\, C_2^i(x/\xi, Q^2/\mu_r^2, {\mu_f}^2/\mu_r^2, \alpha_s(\mu_r^2)) \times f_{i/h}(\xi, \mu_f)$$

[Collins, Soper, Sterman]

prediction of perturbative QCD

$$\frac{d}{d\ln Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

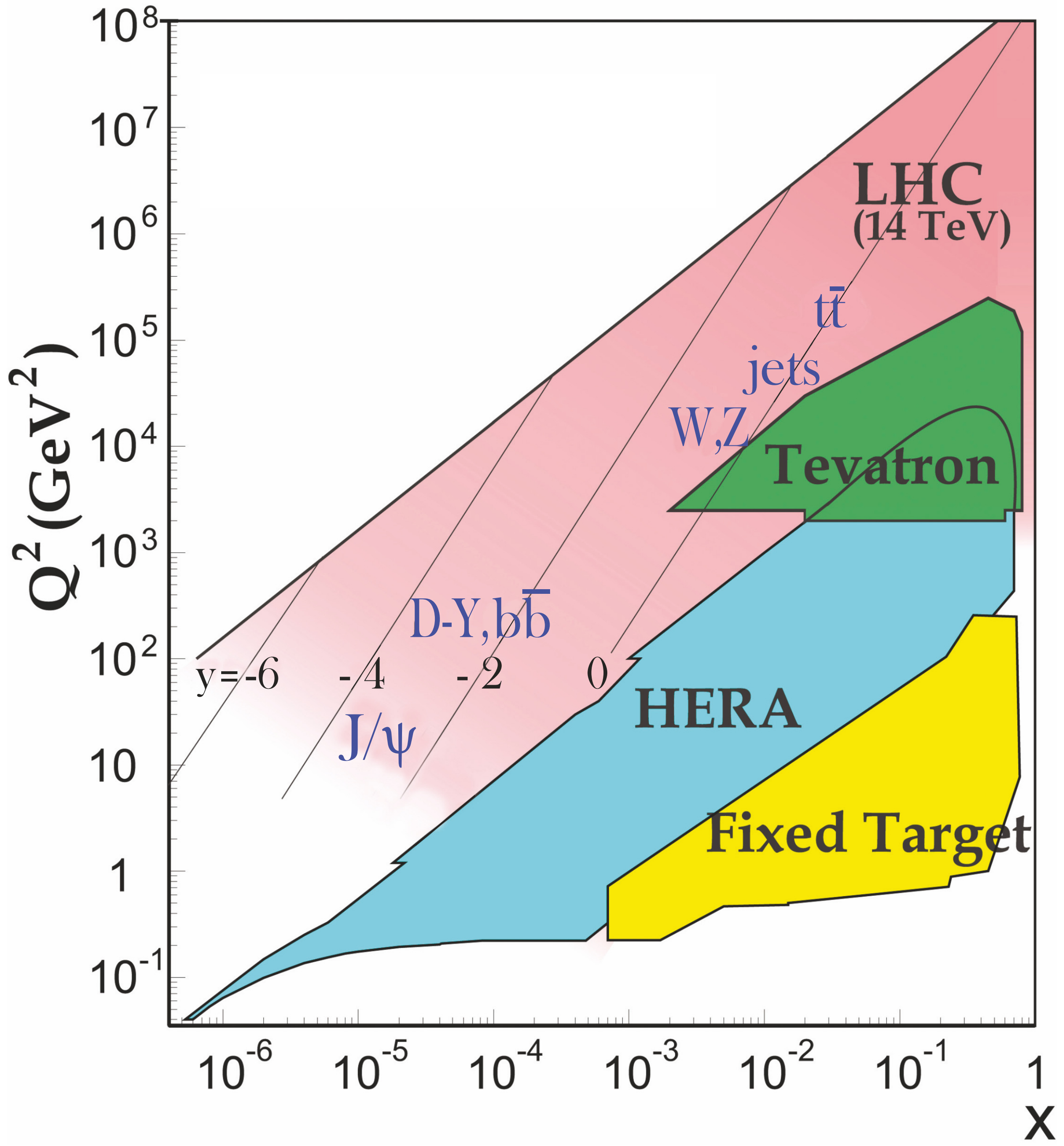
DGLAP evolution

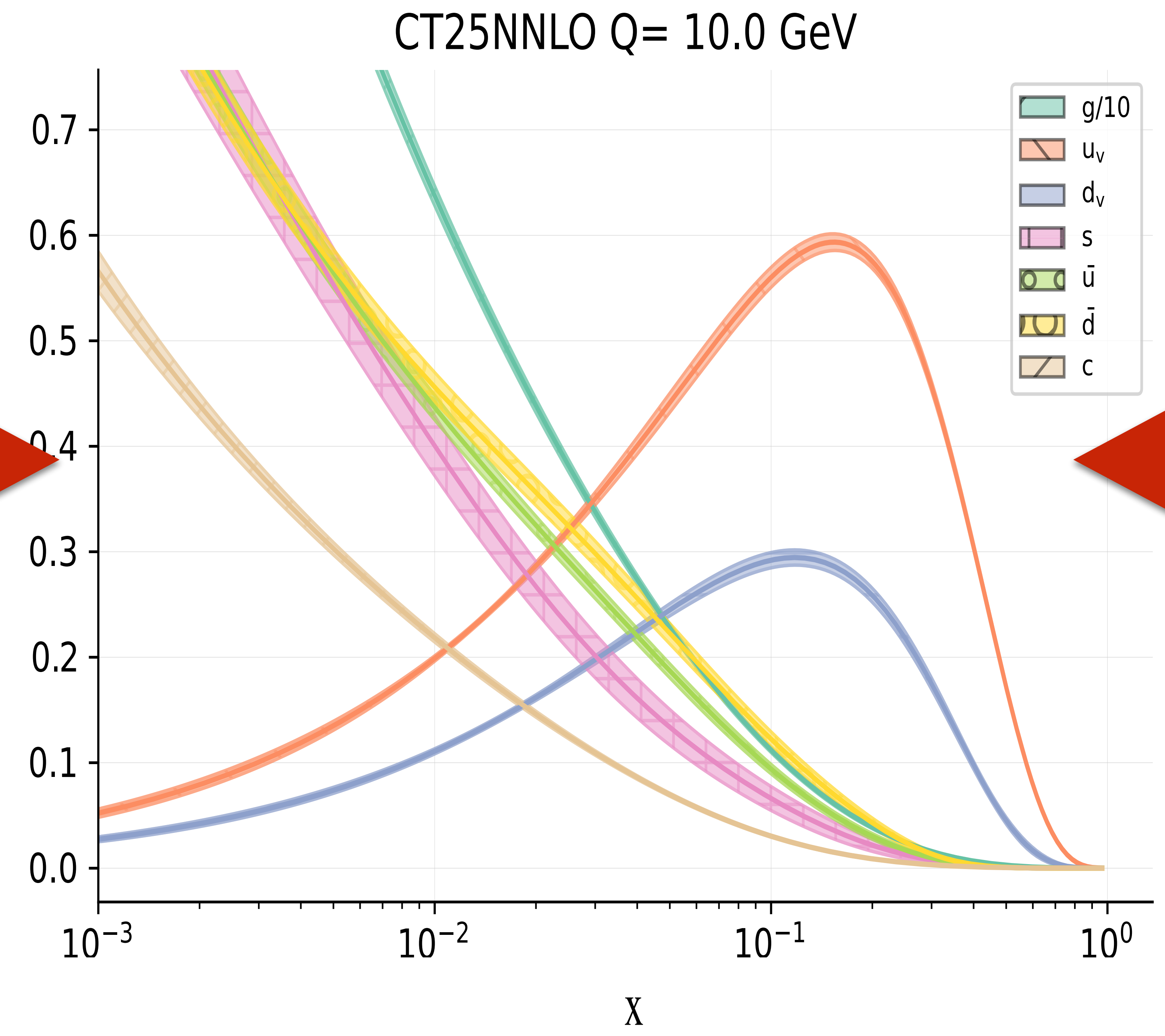
splitting kernels

Global analysis of PDFs

✦ Morden PDFs are usually extracted from global analysis on variety of data, e.g., DIS, Drell-Yan, jets and top quark productions at fixed-target and collider experiments, with increasing weight from LHC

[see 1709.04922, 1905.06957 for recent review articles]





parameter variations

α_S(Mz)

Mc, Mb, Mt

QCD/EW corrections

nuclear corrections

EW parameters

New Physics

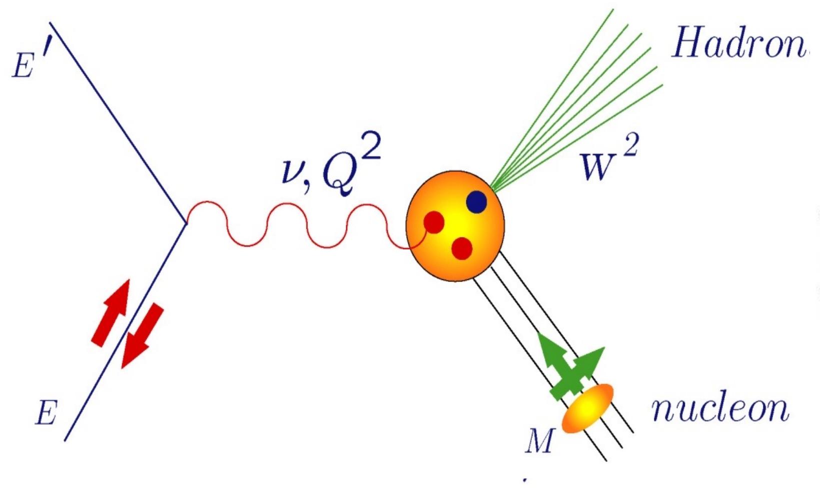
❖ diversity of the analysed data are important to ensure flavor separation and to avoid theoretical/experimental bias; possible extensions to include EW parameters and possible new physics for a self-consistent determination

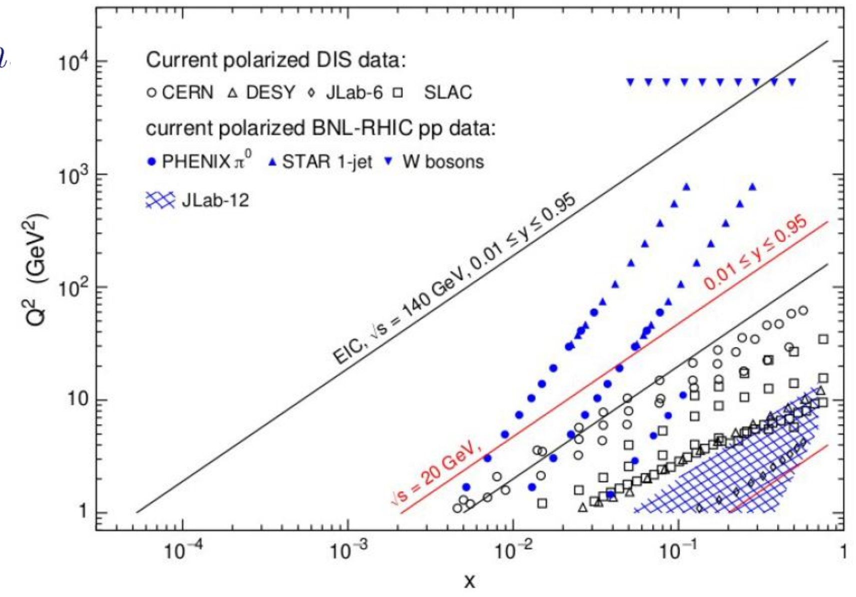
❖ alternative approach from lattice QCD simulations, for various PDF moments or PDFs directly calculated in x-space with large momentum effective theory or pseudo-PDFs [2004.03543]

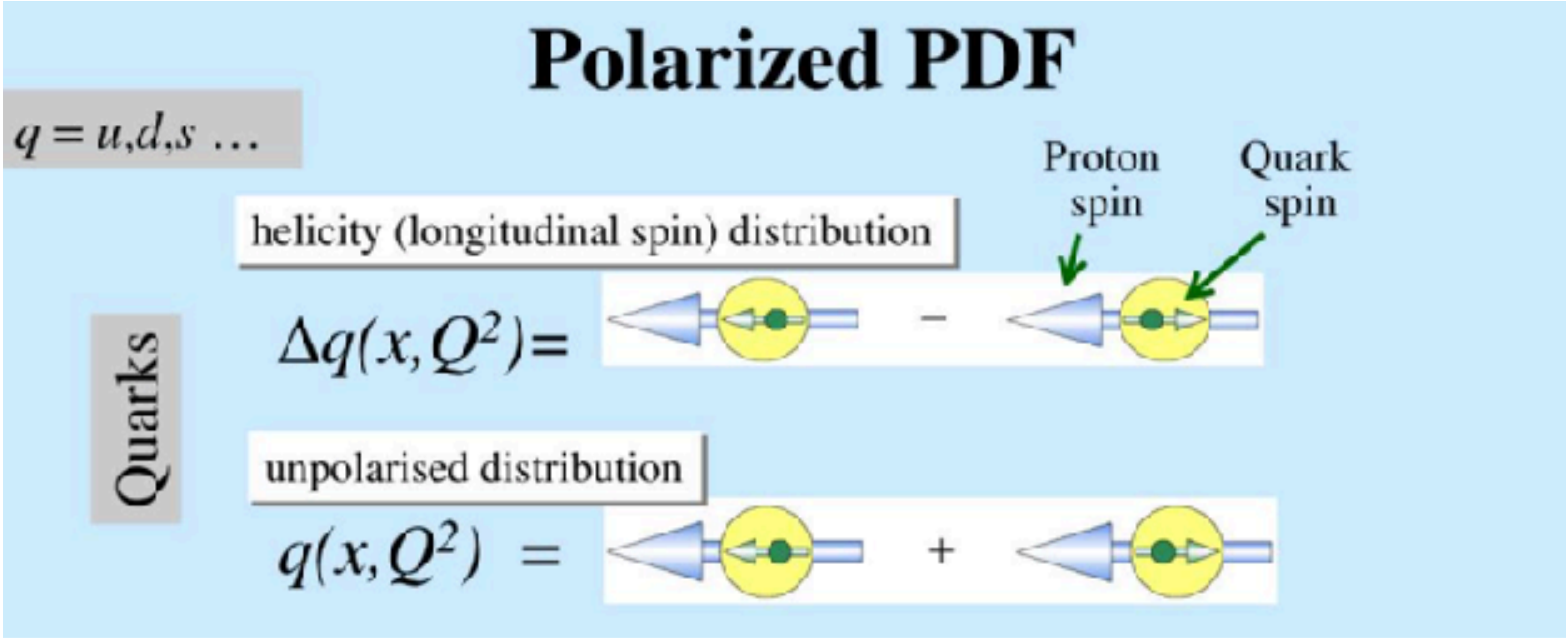
Polarized deep inelastic scattering

- Asymmetric cross section or structure functions measured in polarized deep inelastic scattering can access the PDFs of polarized parton inside a polarized nucleon (pPDFs), especially the longitudinal/ helicity PDFs are directly related to understanding of nucleon spin

Polarized Deep Inelastic Electron Scattering







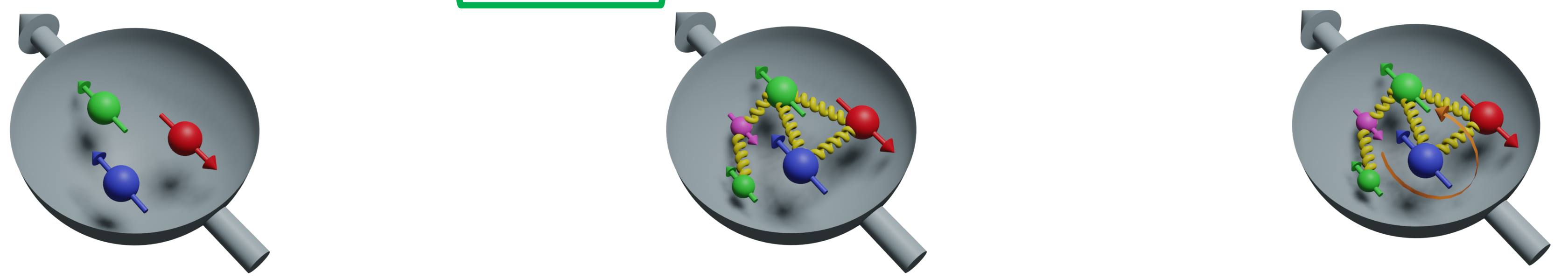
double-spin asymmetry

$$A_{LL}^h \equiv \frac{\sigma^h_{+-} - \sigma^h_{++} + \sigma^h_{-+} - \sigma^h_{--}}{\sigma^h_{+-} + \sigma^h_{++} + \sigma^h_{-+} + \sigma^h_{--}} \sim \frac{\Delta q(x)}{q(x)}$$

The incomplete nucleon: spin puzzle

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

Ji, 96



Quark helicity
Best known

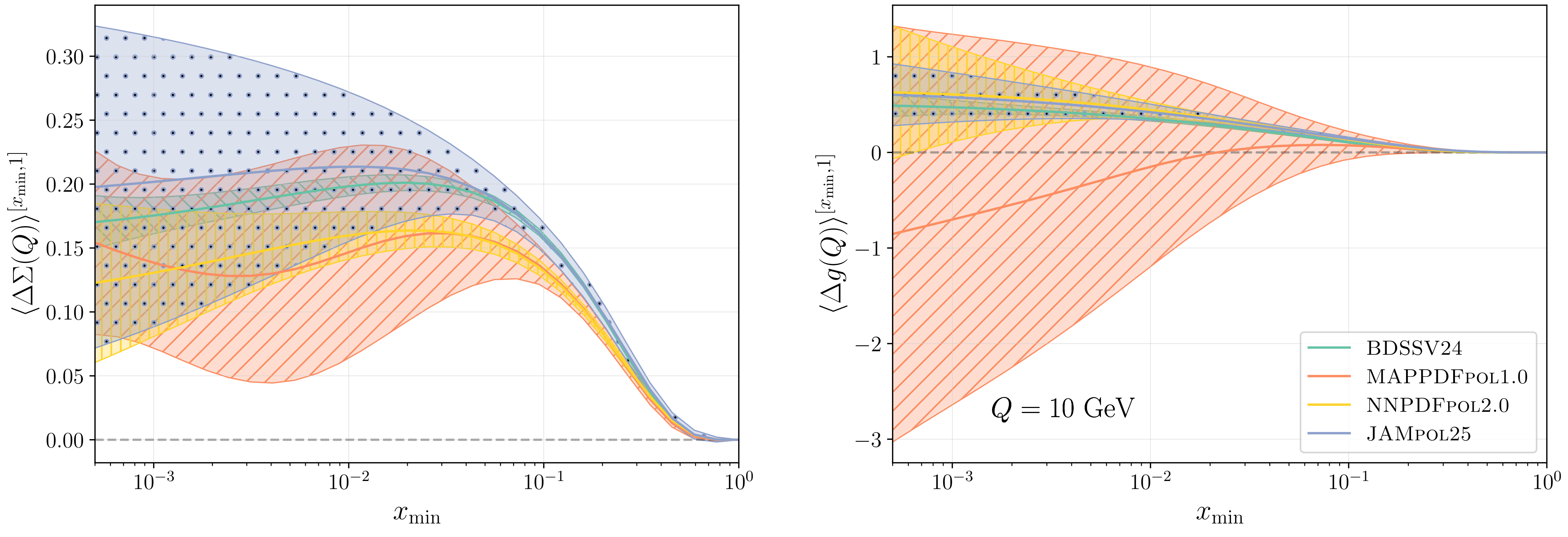
$$\frac{1}{2}\int dx\,(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s})$$

Gluon helicity
Start to know

$$\Delta G = \int dx \Delta g(x)$$

Orbital Angular Momentum of quarks and gluons
Little known

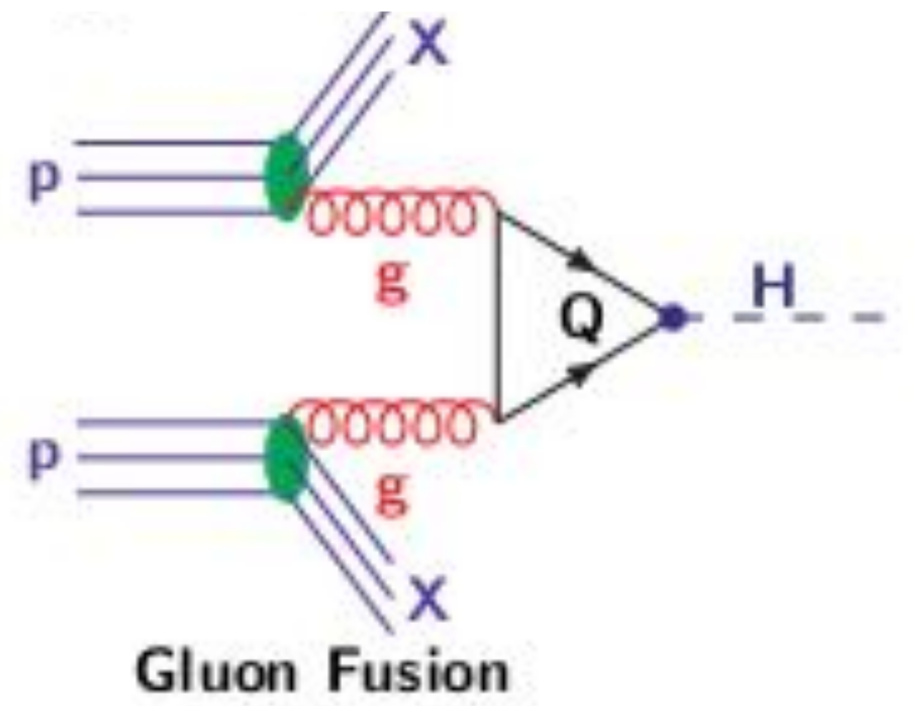
first moments of helicity PDFs **[2604.04765]**

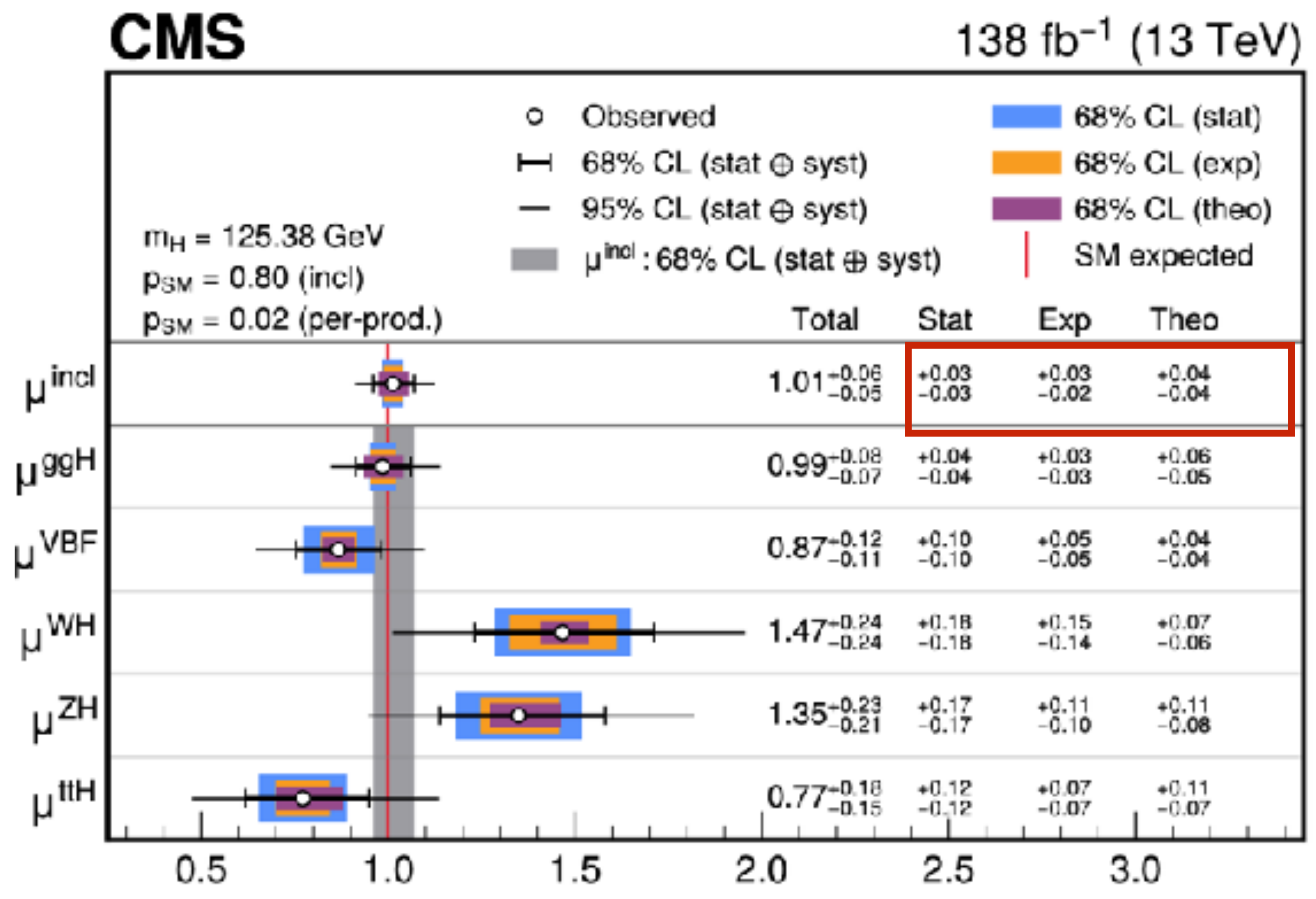


Nucleon structure and Higgs couplings

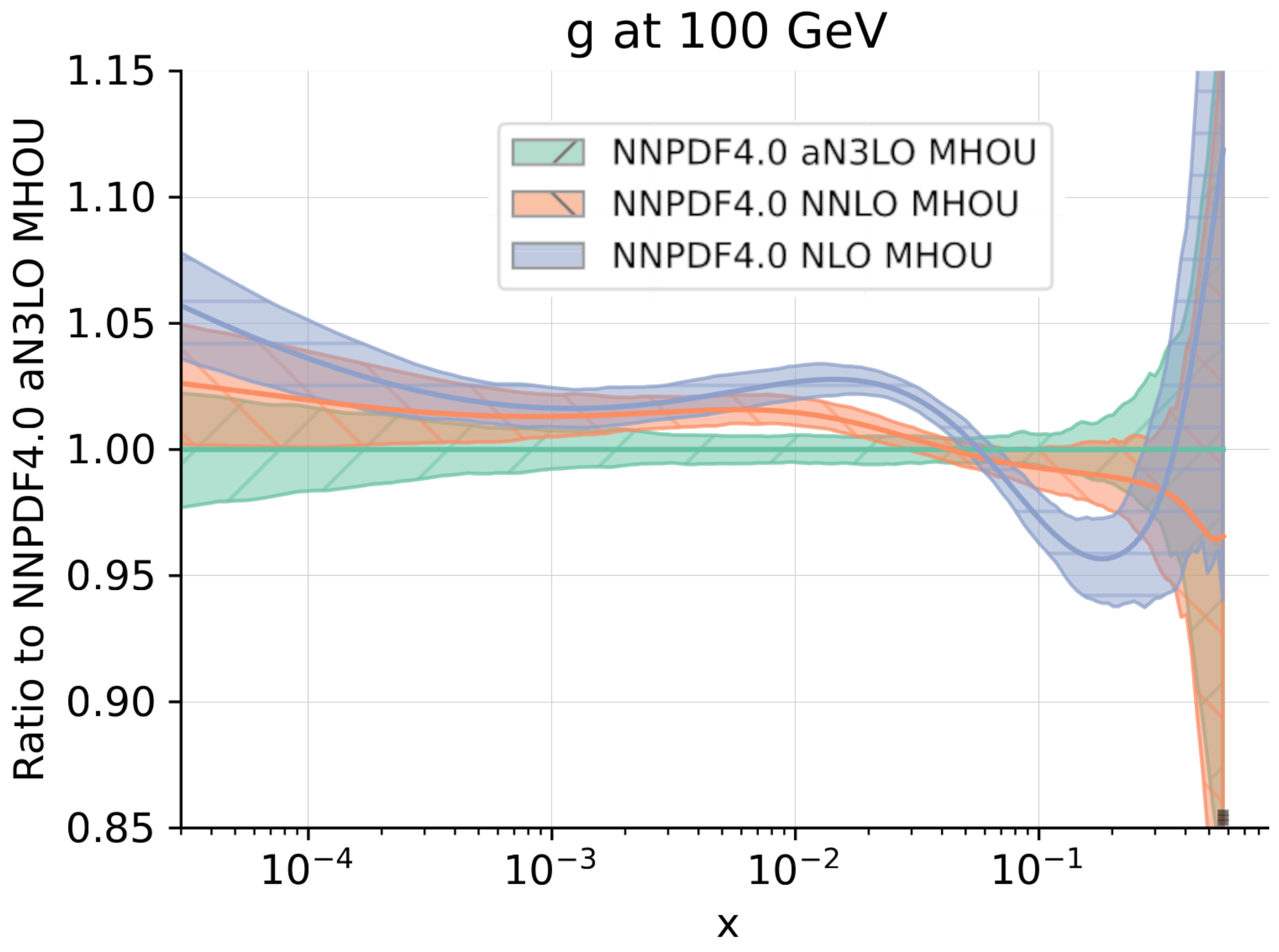
- Theoretical predictions on Higgs boson production cross sections at the LHC depends strongly on the gluon PDFs in proton that are determined mostly by the HERA data; theoretical uncertainty from PDFs and QCD scales are now dominant in the measurement of Higgs couplings

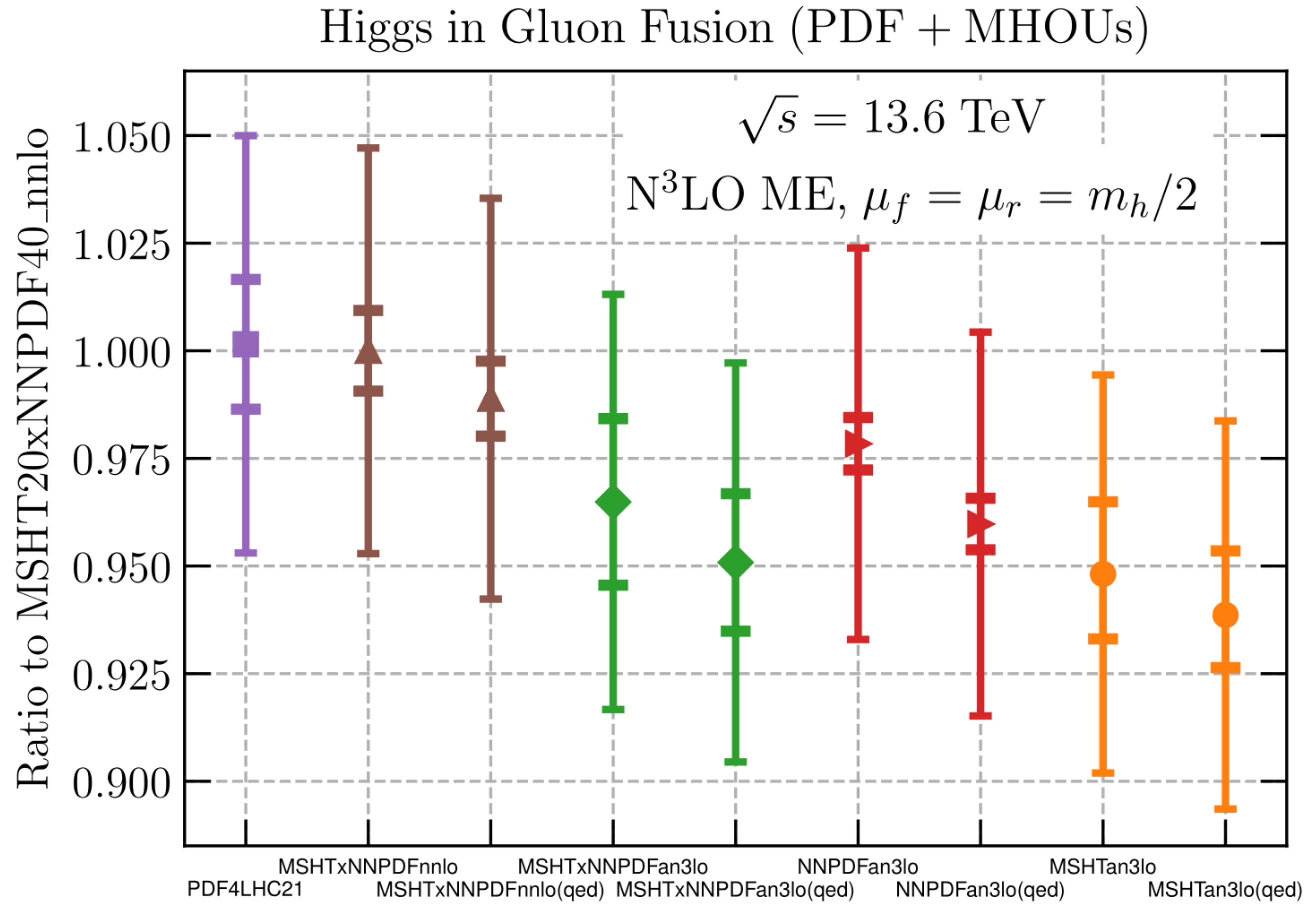
Higgs production and couplings from the LHC





gluon PDFs and Higgs cross section





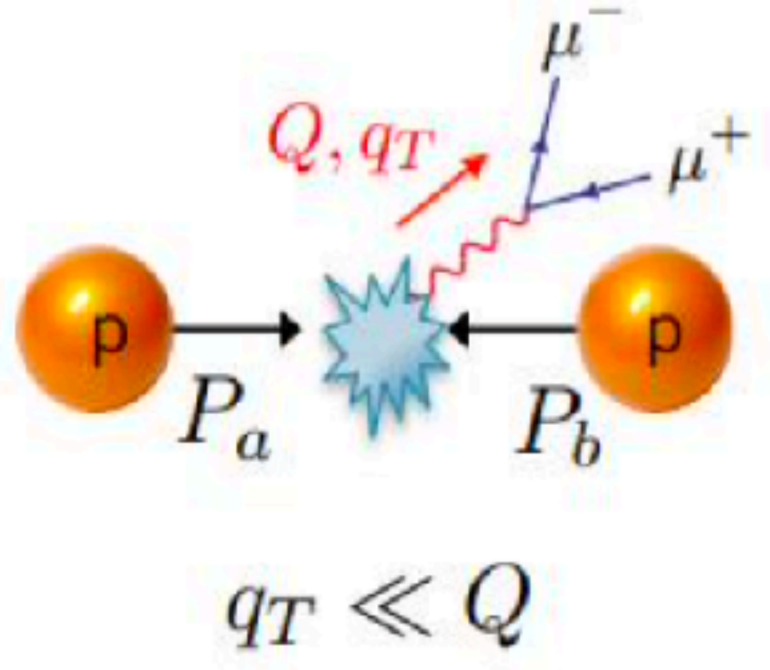
- The HERA data and pQCD did an amazing job on predicting the Higgs cross sections to N3LO, with a precision of ~4%

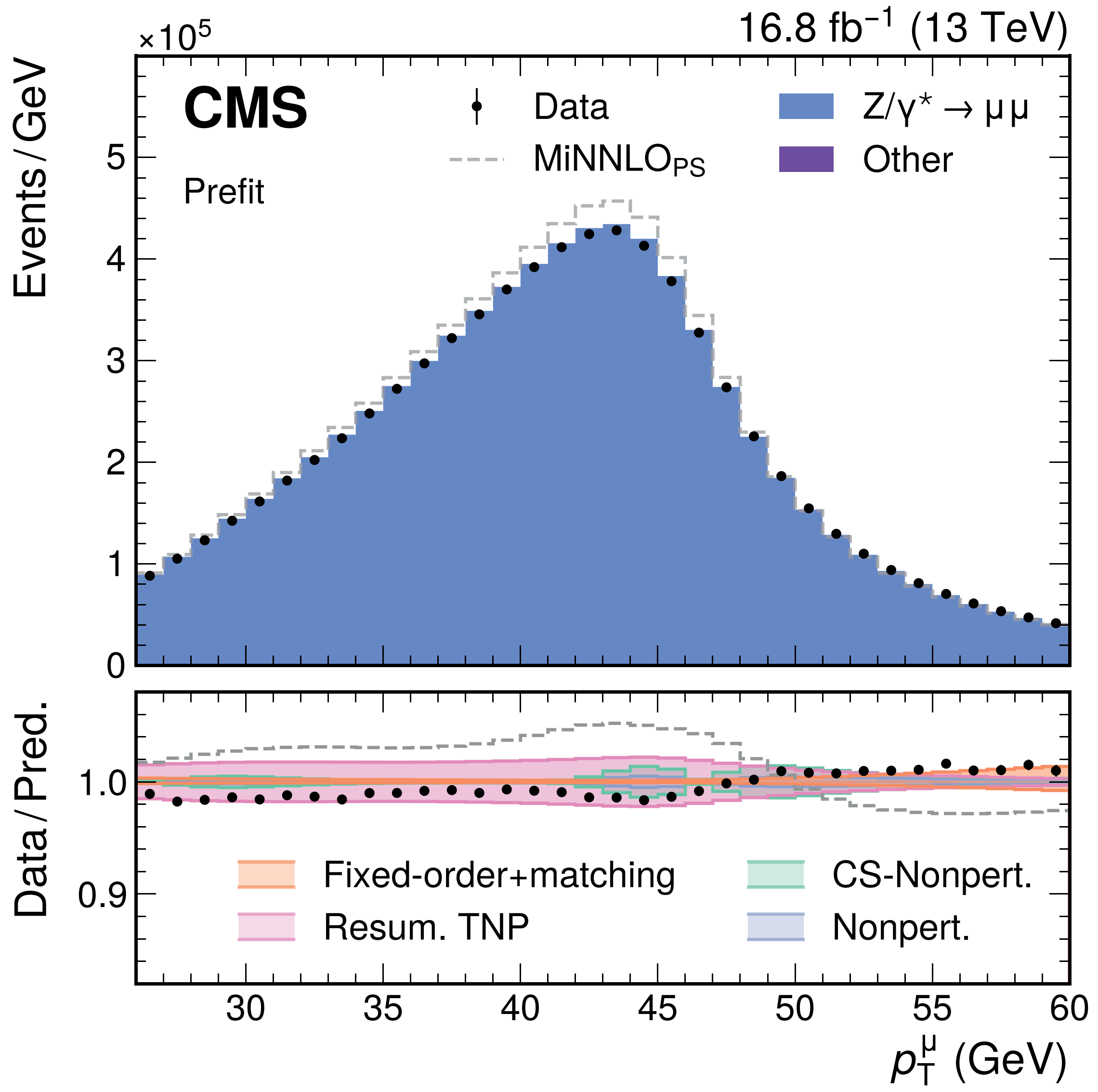
- but turns out not sufficient to match the LHC precision, calling for future improvements on both pQCD and PDF fit

Nucleon structure and W-mass

- At hadron colliders, modeling on the transverse momentum of the decayed lepton, thus extraction of the W-boson mass are sensitive to both PDFs (~4 MeV) and transverse-momentum-dependent (TMD) PDFs; improved nucleon structure especially on the TMD PDFs are important to scrutinize the existing tensions

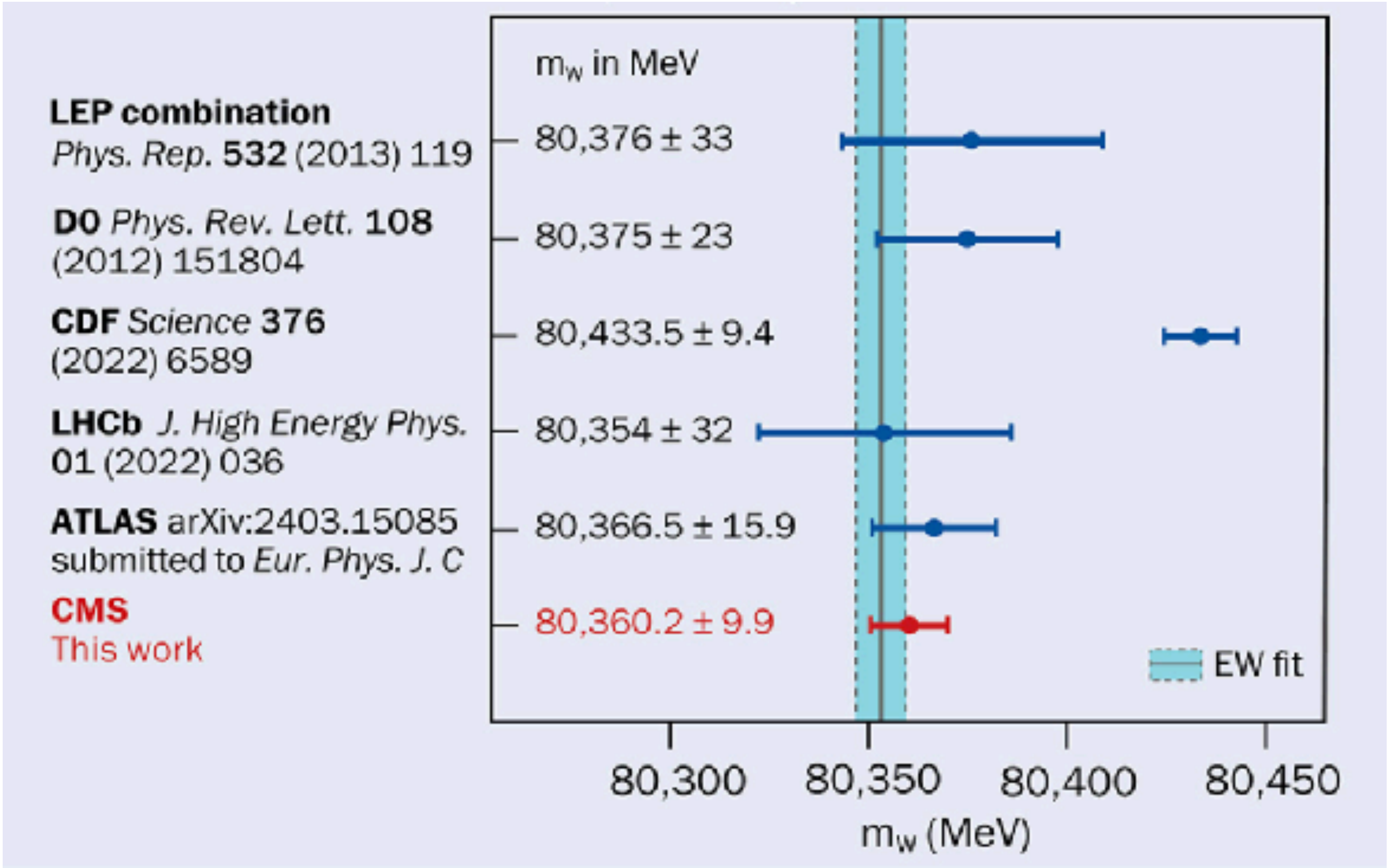
transverse motion of the initial parton



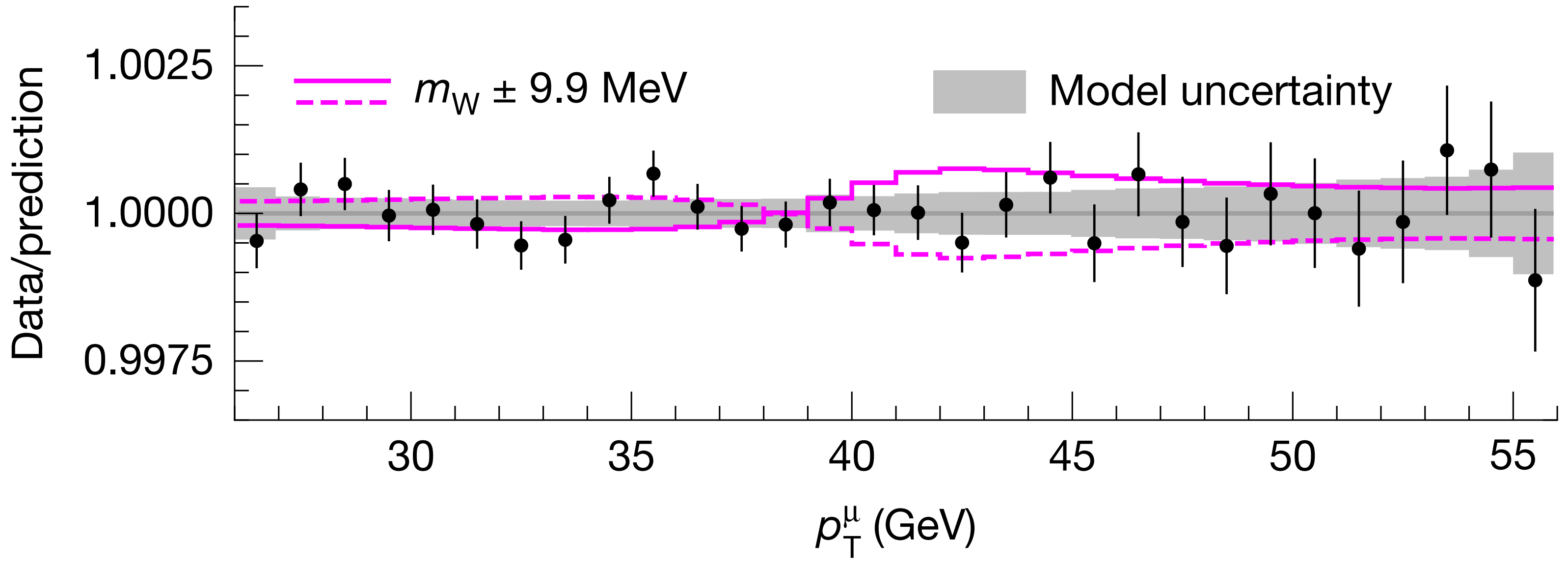


impact on the lepton p_T

world results on W-boson mass



CMS results

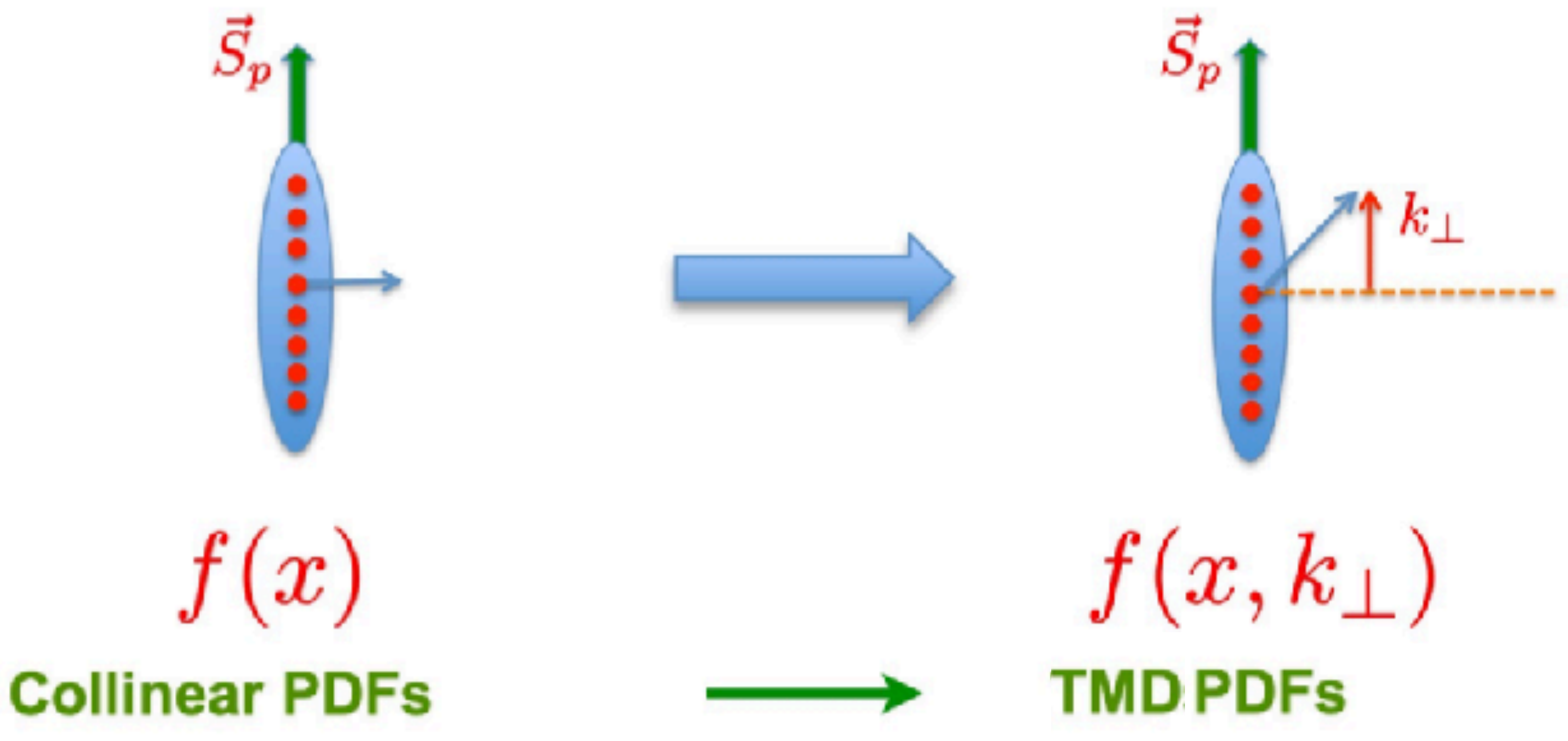


- The high precision CMS result rely on a sophisticated fit (self-data driven) to the transverse dynamics of initial state partons
- Precision TMD PDFs can provide independent input to the lepton p_T modeling

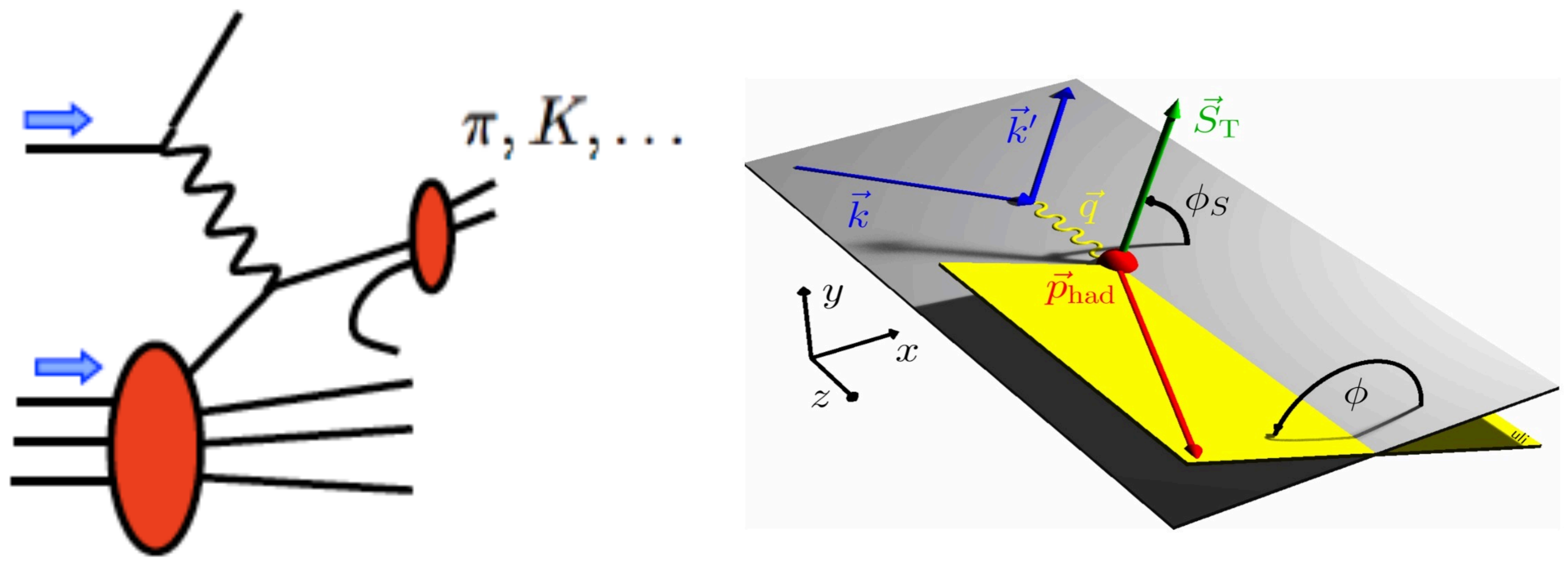
Semi-inclusive deep inelastic scattering

- Semi-inclusive DIS (SIDIS) play an essential role in study of multi-dimensional structure of nucleon and dynamics of hadronization which rely on complementary information from additional hadrons or jets apart from the scattered lepton

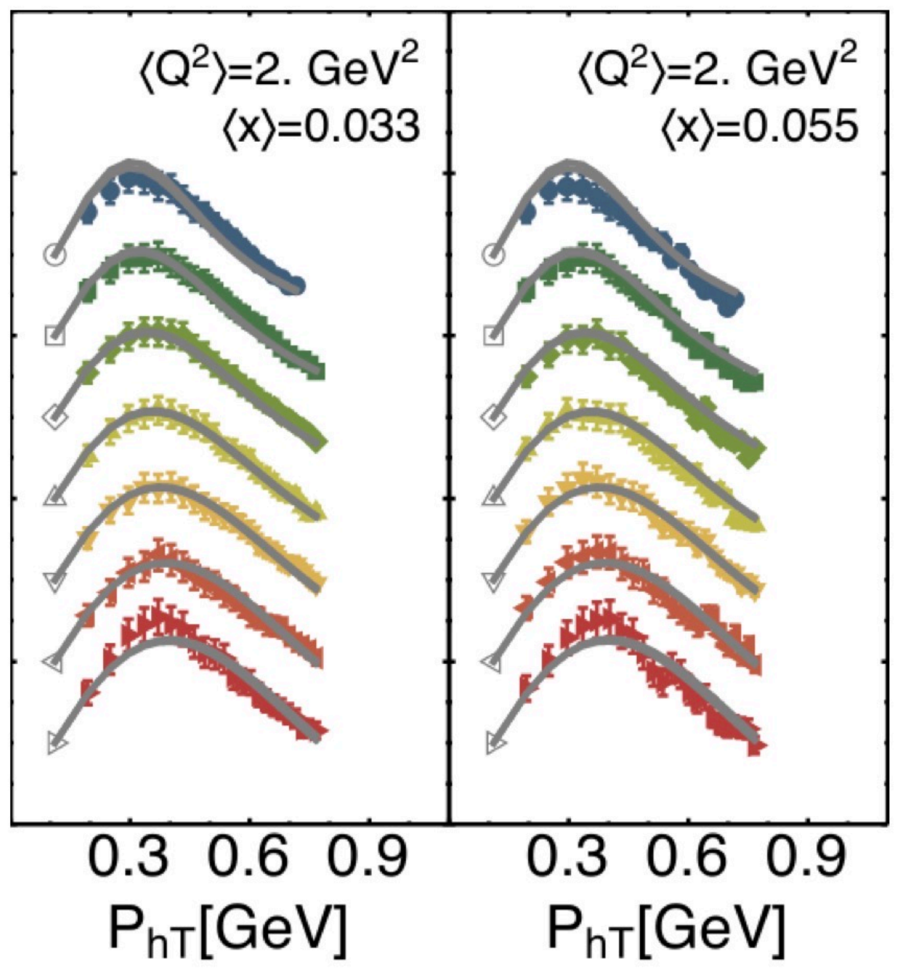
Transverse momentum dependent PDFs

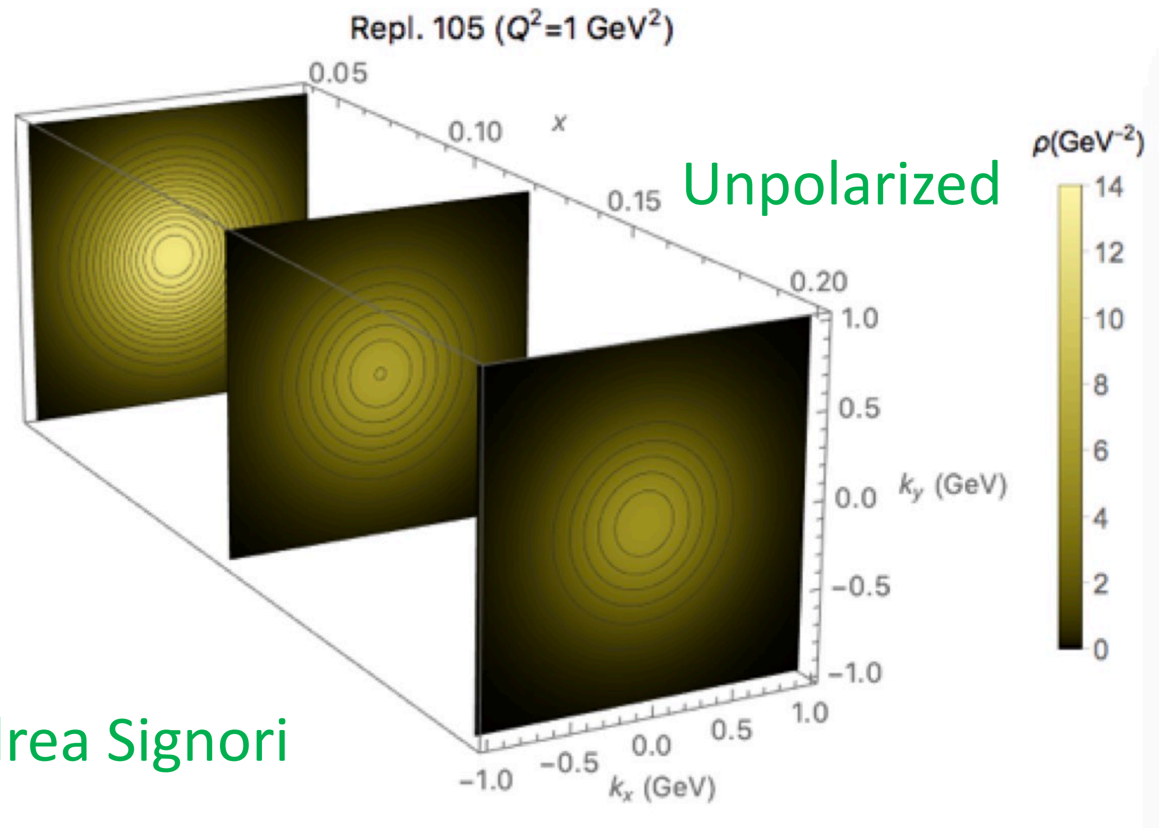


semi-inclusive DIS



extraction of TMD PDFs

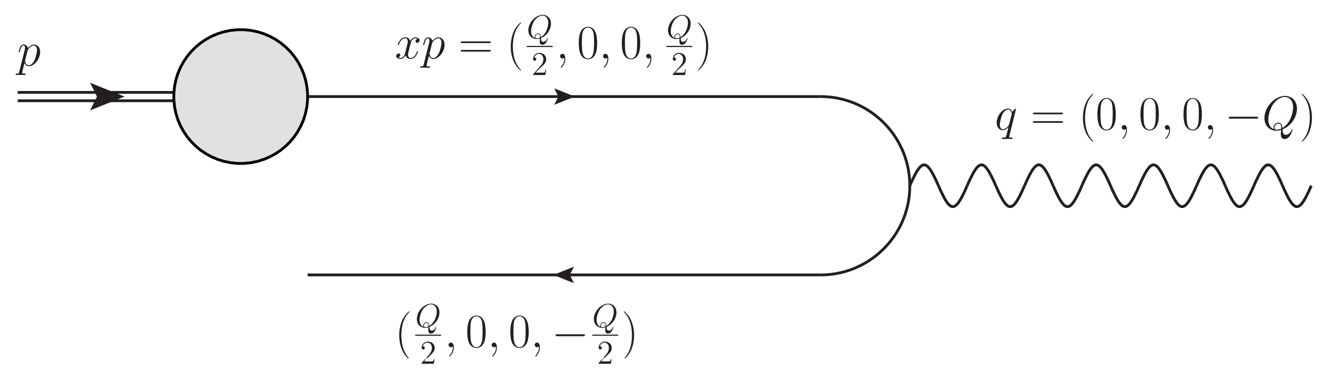




By Andrea Signori

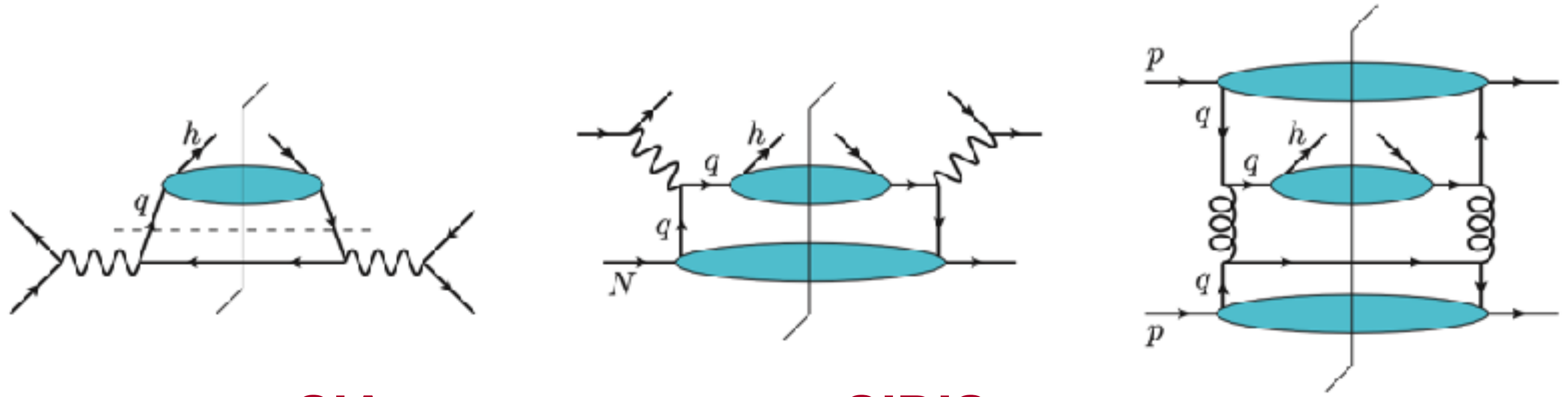
- Transverse momentum of final state hadrons provide the access to the TMD PDFs of initial partons

- tagged hadron also help with flavor separation of initial partons



QCD collinear factorization

✦ QCD collinear factorization ensures universal separation of long-distance and short-distance contributions in high energy scatterings involving initial/final state hadrons, and enables predictions on cross sections



(a) **SIA** (b) **SIDIS** (c) **pp**

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{e^+e^-\to hX}}{dz} = \frac{1}{\sum_q e_q^2}\left(2F_1^h(z,Q^2)+F_L^h(z,Q^2)\right)$$

$$2F_1^h(z,Q^2) = \sum_q e_q^2\left(D_1^{h/q}(z,Q^2)+\frac{\alpha_s(Q^2)}{2\pi}\left(C_1^q\otimes D_1^{h/q}+C_1^g\otimes D_1^{h/g}\right)(z,Q^2)\right)$$

$$\frac{d^3\sigma^{\ell p\to \ell hX}}{dx\,dy\,dz} = \frac{2\pi\alpha_{\rm em}^2}{Q^2}\left(\frac{1+(1-y)^2}{y}\,2F_1^h(x,z,Q^2)+\frac{2(1-y)}{y}\,F_L^h(x,z,Q^2)\right)$$

$$2F_1^h(x,z,Q^2) = \sum_q e_q^2\Big(f_1^{q/p}D_1^{h/q}+\frac{\alpha_s(Q^2)}{2\pi}\Big(\boxed{f_1^{q/p}}\otimes\boxed{C_1^{qq}\otimes D_1^{h/q}}$$
$$+f_1^{q/p}\otimes C_1^{gq}\otimes D_1^{h/g}+f_1^{g/p}\otimes C_1^{qg}\otimes D_1^{h/q}\Big)\Big],$$

The focus of this talk!

❖ coefficient functions, hard scattering; infrared (IR) safe, calculable in pQCD, independent of the hadron

❖ FFs/PDFs, reveal inner structure of hadrons or parton-hadorn transition; NP origin, universal, e.g. DIS vs. pp collisions; fitted from data

❖ runnings of FFs/PDFs with scales μ_D/μ_f are governed by the DGLAP equation

unpolarized collinear FFs, operator definition

$$D_1^{h/q}(z) = \frac{z}{4}\sum\!\!\!\!\!\!\int_X\int\frac{d\xi^+}{2\pi}e^{ik^-\xi^+}\,\mathrm{Tr}\Big[\langle 0|\,\mathcal{W}(\infty^+,\xi^+)\,\psi_q(\xi^+,0^-,\vec{0}_T)\,|P_h,S_h;X\rangle$$
$$\times\,\langle P_h,S_h;X|\,\bar\psi_q(0^+,0^-,\vec{0}_T)\,\mathcal{W}(0^+,\infty^+)\,|0\rangle\,\gamma^-\Big].$$

$$\frac{d}{d\ln\mu^2}D_1^{h/i}(z,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi}\sum_j\int_z^1\frac{du}{u}P_{ji}(u,\alpha_s(\mu^2))\,D_1^{h/j}\left(\frac{z}{u},\mu^2\right)$$

[Collins, Soper, Sterman]

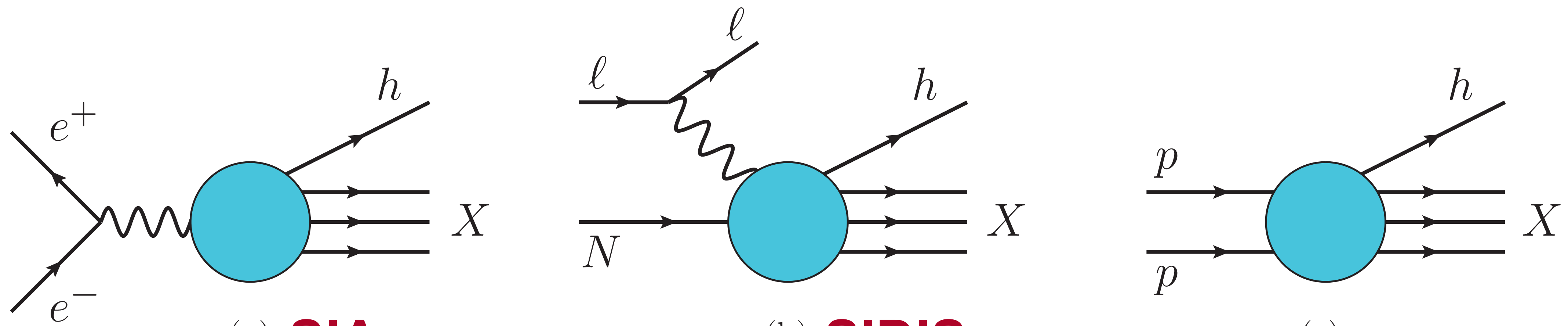
- 2. NPC23 analysis on FFs of light charged hadrons

Fragmentation functions

- In its simplest form, fragmentation functions (FFs) describe number density of the identified hadron wrt the fraction of momentum **(z)** of the initial parton it carries, as measured in single inclusive hadron production, e.g., from single-inclusive annihilation (SIA), SIDIS, and pp collisions

single inclusive hadron production/observable

[1607.02521]



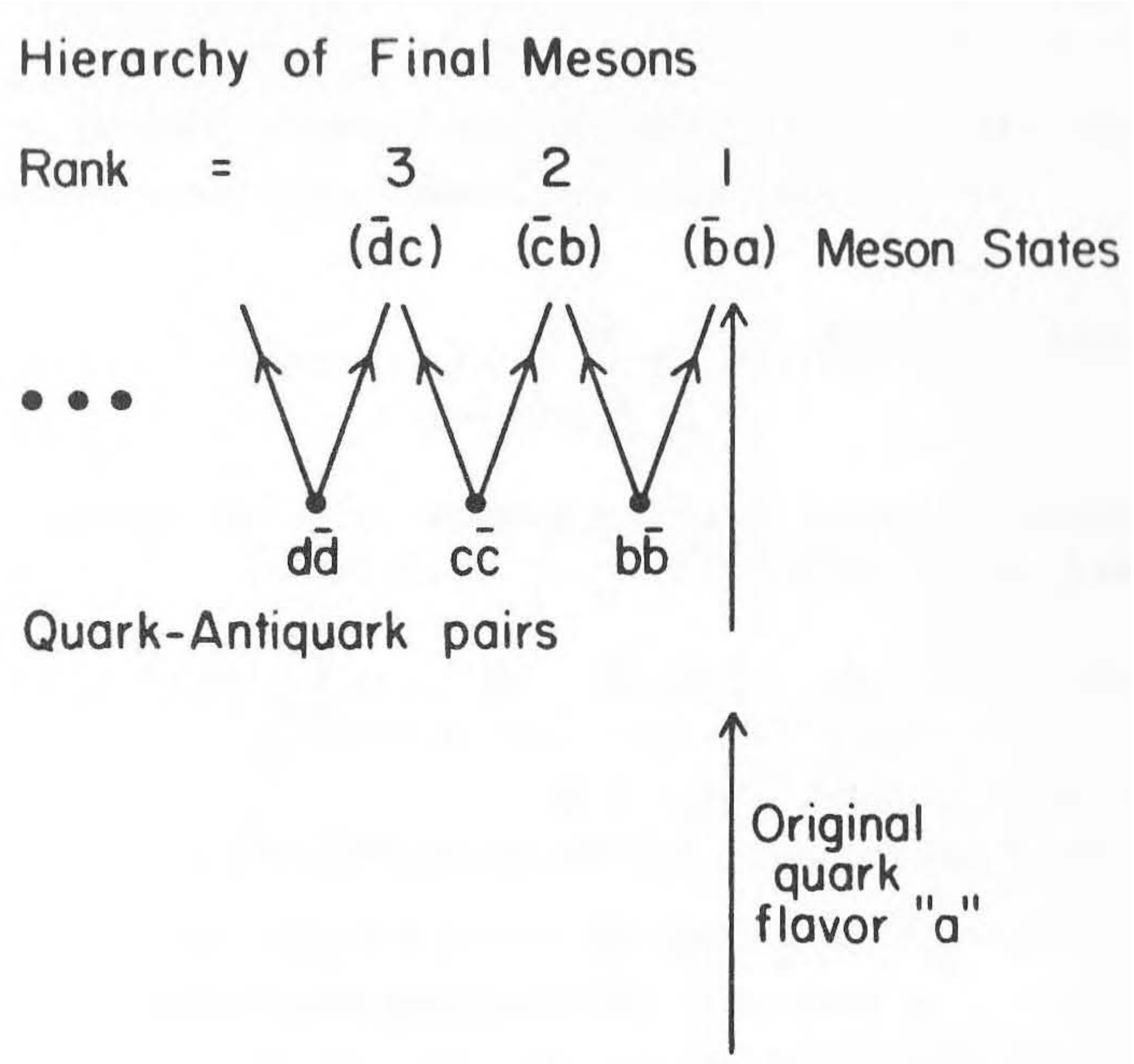
(a) **SIA** (b) **SIDIS** (c) **pp**

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma^{e^+e^-\to hX}}{dz} = F^h(z,Q^2)\,, \qquad z = \frac{2\,E_h}{\sqrt{s}}$$

$$\frac{d^3\sigma^h(x,z,Q^2)/dxdQ^2dz}{d^2\sigma^{DIS}(x,Q^2)/dxdQ^2} = D^h(z,Q^2), \quad z = \frac{2E_h^{BF}}{Q}$$

exp. definition of Fragmentation Functions (FFs)

parton model **[Field&Feynman]**

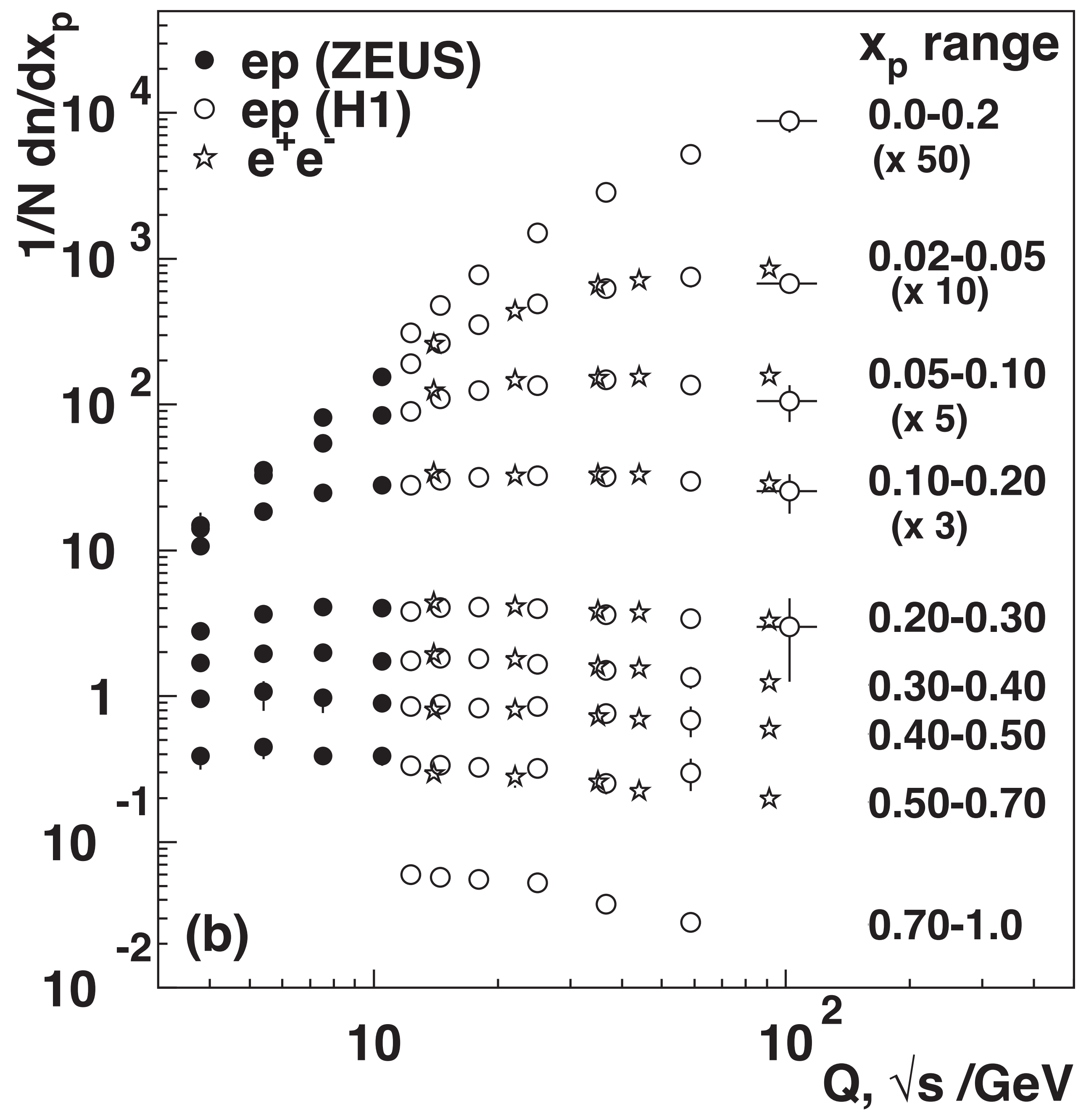


quark decaying functions to mesons via creation of quark-antiquark pairs in cascade

Global analysis of FFs

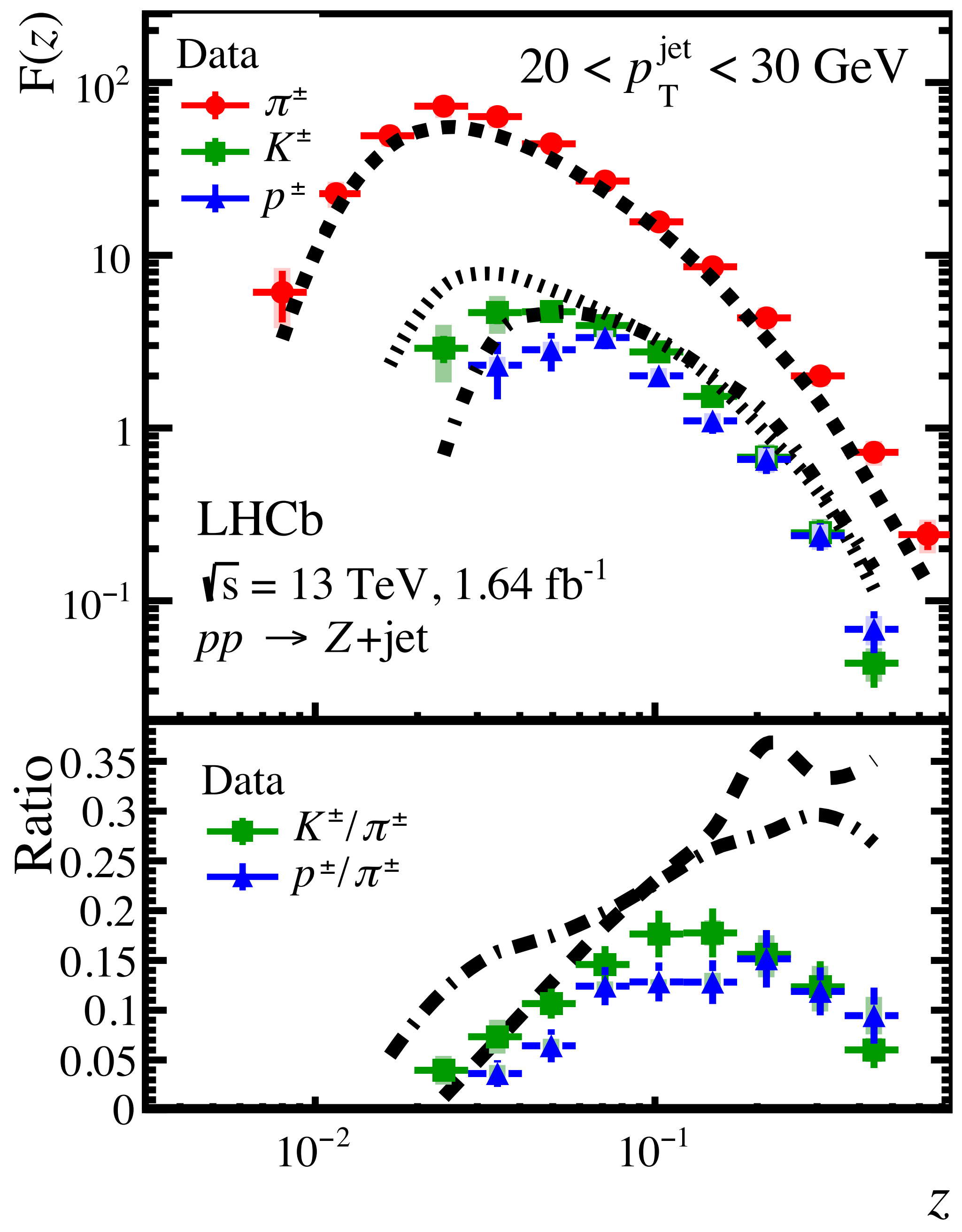
✦ Measurements are available from colliders SLAC, LEP, HERA, RHIC, LHC and fixed-target HERMES, COMPASS experiments for various light charged hadrons as well as neutral hadrons; many groups provide phenomenological FFs from global analysis at NLO/aNNLO in QCD

single incl. production of unidentified charged hadrons (SIA & SIDIS)



[Particle data group]

jet fragmentation to light charged hadrons (LHCb)



[2208.11691]

$$z = \frac{\mathbf{p}_{\rm had} \cdot \mathbf{p}_{\rm jet}}{|\mathbf{p}_{\rm jet}|^2}, \quad F(z) = \frac{1}{N_{Z+{\rm jet}}} \frac{dN_{\rm had}(z)}{dz}$$

global analysis

[1607.02521 for a review]

- major groups/families include BKK, AKK, HKNS, DSS, NNFF, MAPFF, JAM, SAK etc.
- mostly done at NLO in QCD since exact NNLO coefficient functions only known recently for SIDIS
- different determination can be quite different due to selection of data sets as well as theory treatments, not converge as well as the case of PDF fits

pinning down uncertainties of FFs is crucial for EIC(c) physics !

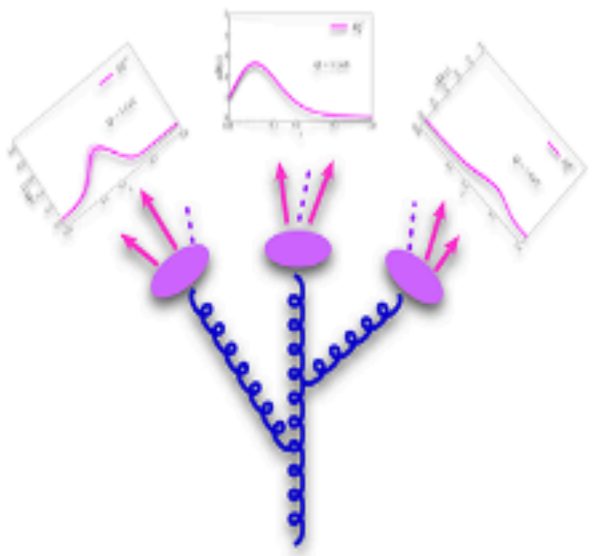
The first NPC23 analysis of FFs

✦ Establishing a new framework on global analysis of fragmentation functions to identified charged hadrons, including charged pion, kaon and proton, using most recent data from SIA, SIDIS, and pp collisions

parametrization of FFs to charged pion/kaon/proton at an initial scale (Q=5 GeV):

$$zD_i^h(z, Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^{m} a_{i,n}^h (\sqrt{z})^n\right)$$

parton-to-π^+	favored	α	β	a_0	a_1	a_2	d.o.f.
u	Y						5
$d \simeq u$	Y	-	-		-	-	1
$\bar{u} = d$	N					x	4
$s = \bar{s} \simeq \bar{u}$	N	-				x	3
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F				4



EDITORS' SUGGESTIONGlobal analysis of fragmentation functions to charged hadrons with high-precision data from the LHC

12 December, 2024

This paper presents a global analysis of fragmentation functions for light charged hadrons, which describe the production of these states from partons. The fit includes for the first time jet data from proton-proton collisions at the LHC. The authors find good agreement with data, but note significant differences with previous work and the need for careful experimental definitions for future efforts.

Jun Gao *et al.*

Phys. Rev. D **110**, 114019 (2024)

❖ a joint determination of FFs to charged pion, kaon and proton (via ratios or sum) at NLO in QCD (63 parameters) including estimation of uncertainties with Hessian sets

❖ apply a strong selection criteria on the kinematics of fragmentation processes to ensure validity of LT factorization and perturbative calculations ($E_h/p_{T,h}$>4 GeV, and z>0.01)

❖ including theory uncertainties (residual scale variations) into the covariance matrix

❖ use fast interpolation techniques as in FMNLO for calculations of cross sections which largely increase efficiency of the global fit

[JG, Liu, Shen, Xing, Zhao, 2401.02781 (PRL), 2407.04424 (PRD Editors' suggestion)]

Impact of the LHC data

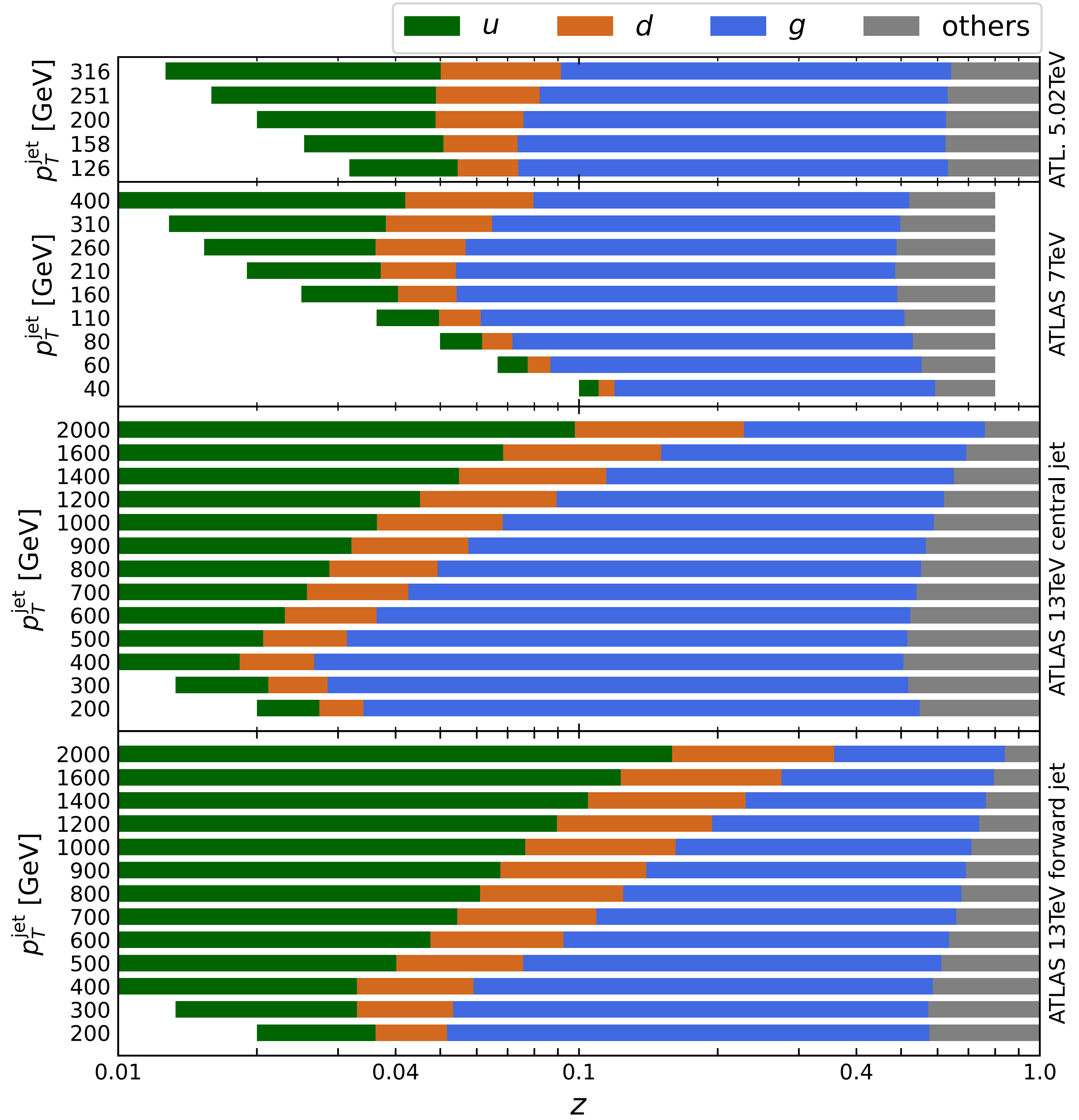
- For the first time the jet fragmentation data from LHC have been incorporated into the global analysis of FFs to light charged hadrons, including from processes of incl. jet, dijet, Z or photon tagged jet productions, due to the development of FMNLO

LHC measurements for hadron inside jet measurements (jet fragmentation)

exp.	$\sqrt{s}$(TeV)	luminosity	hadrons	final states	R_j	cuts for jets/hadron	observable	N_{pt}
ATLAS[60]	5.02	25 pb^{-1}	$h^\pm$	$\gamma + j$	0.4	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}$	$\frac{1}{N_{\rm jet}}\frac{dN_{\rm ch}}{dp_{T,h}}$	6
CMS[61]	5.02	27.4 pb^{-1}	$h^\pm$	$\gamma + j$	0.3	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}, \Delta R_{h,j} < R_j$	$\frac{1}{N_{\rm jet}}\frac{dN_{\rm ch}}{d\xi}$	4
ATLAS[62]	5.02	260 pb^{-1}	$h^\pm$	$Z + h$	no jet	$\Delta\phi_{h,Z} > \frac{3}{4}\pi$	$\frac{1}{n_Z}\frac{dN_{\rm ch}}{dp_{T,h}}$	9
CMS[63]	5.02	320 pb^{-1}	$h^\pm$	$Z + h$	no jet	$\Delta\phi_{h,Z} > \frac{7}{8}\pi$	$\frac{1}{n_Z}\frac{dN_{\rm ch}}{dp_{T,h}}$	11
LHCb[64]	13	1.64 fb^{-1}	$\pi^\pm, K^\pm, p/\bar{p}$	$Z + j$	0.5	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}, \Delta R_{h,j} < R_j$	$\frac{1}{n_Z}\frac{dN_{\rm ch}}{d\zeta}$	20
ATLAS[65]	5.02	25 pb^{-1}	$h^\pm$	inc. jet	0.4	-	$\frac{1}{N_{\rm jet}}\frac{dN_{\rm ch}}{d\zeta}$	63
ATLAS[66]	7	36 pb^{-1}	$h^\pm$	inc. jet	0.6	$\Delta R_{h,j} < R_j$	$\frac{1}{N_{\rm jet}}\frac{dN_{\rm ch}}{d\zeta}$	103
ATLAS[67]	13	33 fb^{-1}	$h^\pm$	dijet	0.4	$p_T^{\rm lead}/p_T^{\rm sublead} < 1.5$	$\frac{1}{N_{\rm jet}}\frac{dN_{\rm ch}}{d\zeta}$	280

- LHC measurements on hadron inside jet provide essential inputs for u/d/g flavor separation with wide kinematic coverages, both in energy scale Q and in momentum fraction z
- A best-fit with good agreements to the global data sets (1370 points in total) are found, $\chi^2/N \sim 0.9$, proven universality of FFs in different collisions at highest energies

kinematic/flavor coverage (LO) for ATLAS jet fragmentation



direct probe of gluon FFs!

Quality of the fit

- A best-fit with good agreements to the global data sets (1370 points in total) are found, χ^2/N well below 1; individual agreements to the 138 sub-datasets are also tested, motivating usage of a tolerance $\Delta\chi^2 \sim 2$ in determination of Hessian uncertainties

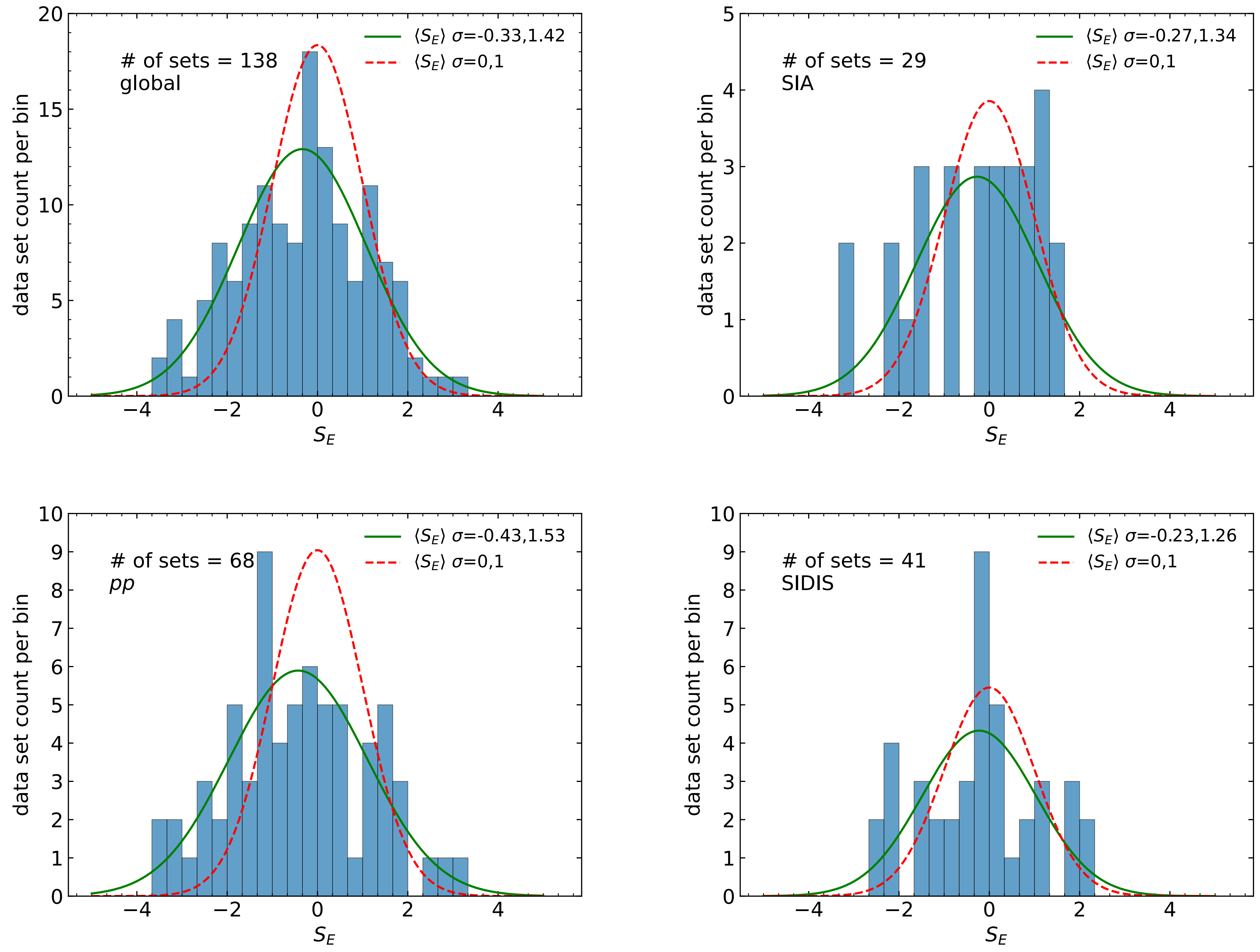
[CTEQ-TEA]

overall agreement: χ^2 breakdown to sub-groups for the best-fit

Experiments	N_{pt}	χ^2	χ^2/N_{pt}
ATLAS jets †	446	350.8	0.79
ATLAS Z/γ+jet †	15	31.8	2.12
CMS Z/γ+jet †	15	17.3	1.15
LHCb Z+jet	20	30.6	1.53
ALICE inc. hadron	147	150.6	1.02
STAR inc. hadron	60	42.2	0.70
pp sum	703	623.3	0.89
TASSO	8	7.0	0.88
TPC	12	11.6	0.97
OPAL	20	16.3	0.81
OPAL (202 GeV) †	17	24.2	1.42
ALEPH	42	31.4	0.75
DELPHI	78	36.4	0.47
DELPHI (189 GeV)	9	15.3	1.70
SLD	198	211.6	1.07
SIA sum	384	353.8	0.92
H1 †	16	12.5	0.78
H1 (asy.) †	14	12.2	0.87
ZEUS †	32	65.5	2.05
COMPASS (06I)	124	107.3	0.87
COMPASS (16p)	97	56.8	0.59
SIDIS sum	283	254.4	0.90
Global total	1370	1231.5	0.90

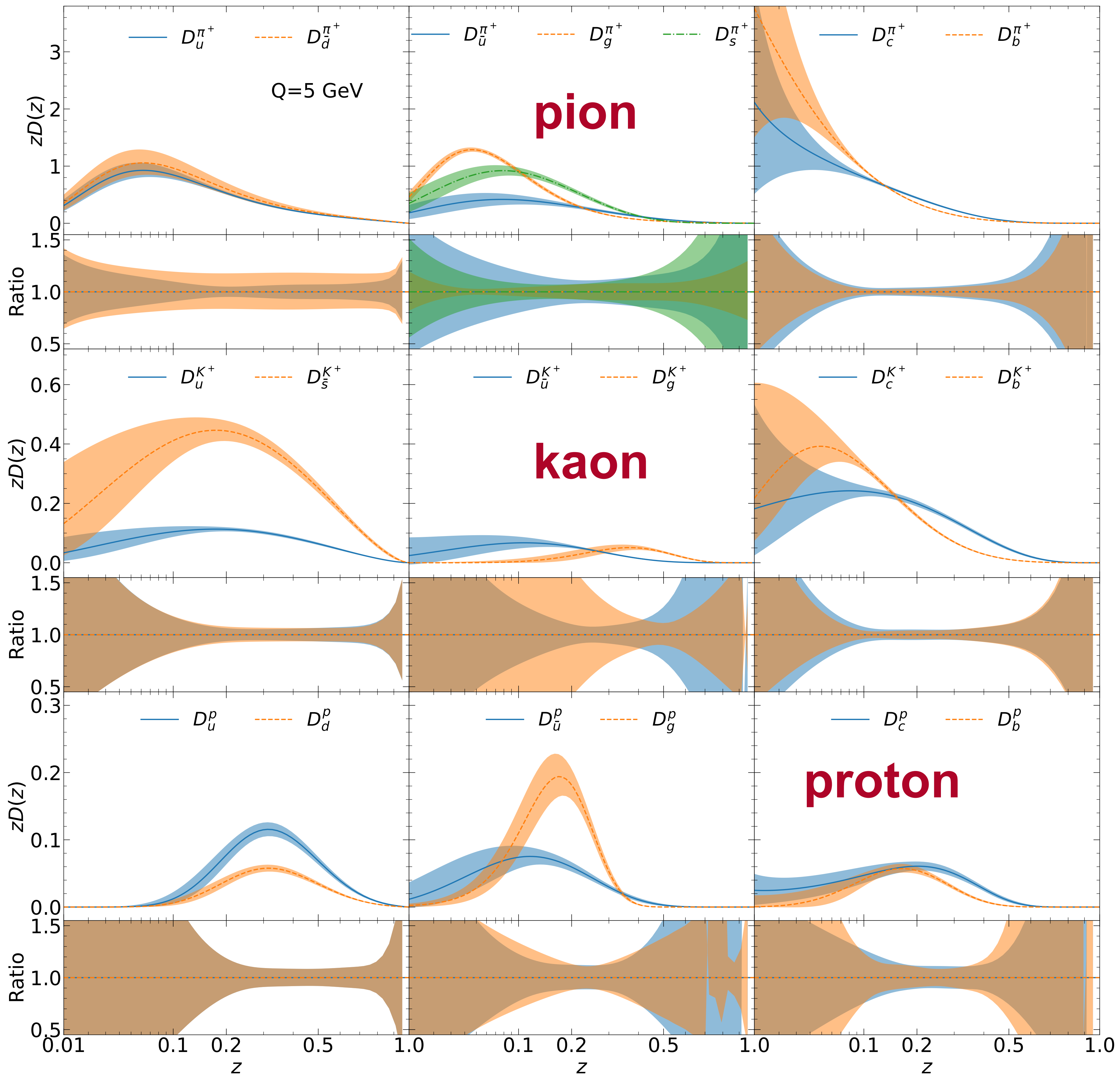
individual agreement: distributions of the effective Gaussian variable

$$S_E = \frac{(18N_{pt})^{3/2}}{18N_{pt}+1}\left\{\frac{6}{6-\ln(\chi^2/N_{pt})} - \frac{9N_{pt}-1}{9N_{pt}}\right\}$$



NPC23 FFs to charged hadrons

- We arrive at a best-fit of the charged pion, kaon and proton FFs together with 126 Hessian error FFs, two for each of the eigenvector direction; FFs are generally well constrained in the region with z~0.1-0.7



FFs (positively charged) vs. momentum fraction

- our results show an uncertainty of 3%, 4% and 8% for FFs of gluon to pion at z=0.05, 0.1 and 0.3, respectively
- similarly an uncertainty of 4%, 4% and 7% for FFs of u-quark to pion, kaon and proton at z=0.3, respectively
- FFs of heavy-quarks are well constrained for z between 0.1~0.5 due to the tagged SIA events of Z-pole measurements
- high precision of gluon FFs is mostly due to the data of jet fragmentation from the LHC
- a preference for larger FFs of s quark to pion due to pulls from SIA data

NPC23 analysis at NNLO

✦ Global analysis of FFs to light charged hadrons at full NNLO with SIA and SIDIS data, especially with the recent BESIII measurements, focusing on low momentum transfer region and test of collinear factorization

parametrization form (pi+/K+)

$$zD_i^h(z,Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h}\exp\left(\sum_{n=0}^{m} a_{i,n}^h z^{n/2}\right)$$

flavor	favored	a_0	α	β	a_1	a_2
$u=\bar{d}$	✓	✓	✓	✓	✓	✓
$d=\bar{u}$	✗	✓	✓	✓	✓	✓
$s=\bar{s}$	✗	✓	$=\alpha_d$	✓	✓	✓
$c=\bar{c}$	✗	✓	✓	✓	✓	✓
$b=\bar{b}$	✗	✓	✓	✓	✓	✓
g	✗	✓	✓	✓	✓	✗

flavor	favored	a_0	α	β	a_1	a_2
u	✓	✓	✓	✓	✓	✓
$\bar{s}$	✓	✓	$=\alpha_u$	$=\beta_u$	✓	✓
$s=\bar{u}=d=\bar{d}$	✗	✓	✓	✓	✓	✗
$c=\bar{c}$	✗	✓	✓	✓	✓	✓
$b=\bar{b}$	✗	✓	✓	✓	✓	✓
g	✗	✓	✓	✓	✓	✗

BESIII charged pion/kaon **[2502.16084]**

$\sqrt{s}$ (GeV)	$\mathcal{L}$ (pb^{-1})	$N_{\rm had}^{\rm tot}$	$N_{\rm bkg}$
2.0000	10.074	350298 ± 592	8722 ± 94
2.2000	13.699	445019 ± 668	10737 ± 104
2.3960	66.869	1869906 ± 1368	47550 ± 219
2.6444	33.722	817528 ± 905	21042 ± 146
2.9000	105.253	2197328 ± 1483	56841 ± 239
3.0500	14.893	283822 ± 533	7719 ± 88
3.5000	3.633	62670 ± 251	1691 ± 42
3.6710	4.628	75253 ± 275	6461 ± 81

[JG, Shen, Xing, Zhao, Zhou, 2502.17837 (PRL)]

- ❖ a joint determination of FFs to charged pion, kaon at NNLO in QCD (54 parameters) with Hessian uncertainties
- ❖ parametrization at Q_0=1.4 GeV for light flavors and at mass threshold for heavy quarks; 3-loop DGLAP evolution from **[Mitov, Moch, Vogt, Almasy; Chen, Yang, Zhu, Zhu]** implemented in HOPPET
- ❖ SIDIS coefficient functions from **[Bonino, Gehrmann, Stagnitto; Goyal, Moch, Pathak, Rana, Ravindran];** SIA with hadron mass corrections; include residual scale variations into the covariance matrix
- ❖ select SIA and SIDIS data with z>0.01, Q>2 GeV, and $E_h > E_{h,min}$ (0.8 GeV default) with later varied

first FFs of light hadrons at full NNLO ensuring a successful test of factorization at low Q2!

NPC23 analysis at NNLO

- A first Global analysis of FFs to light charged hadrons at full NNLO with SIA and SIDIS data, especially with recent BESIII data, focusing on low momentum transfer region and test of collinear factorization

quality of NNLO fit with varying hadron energy cut

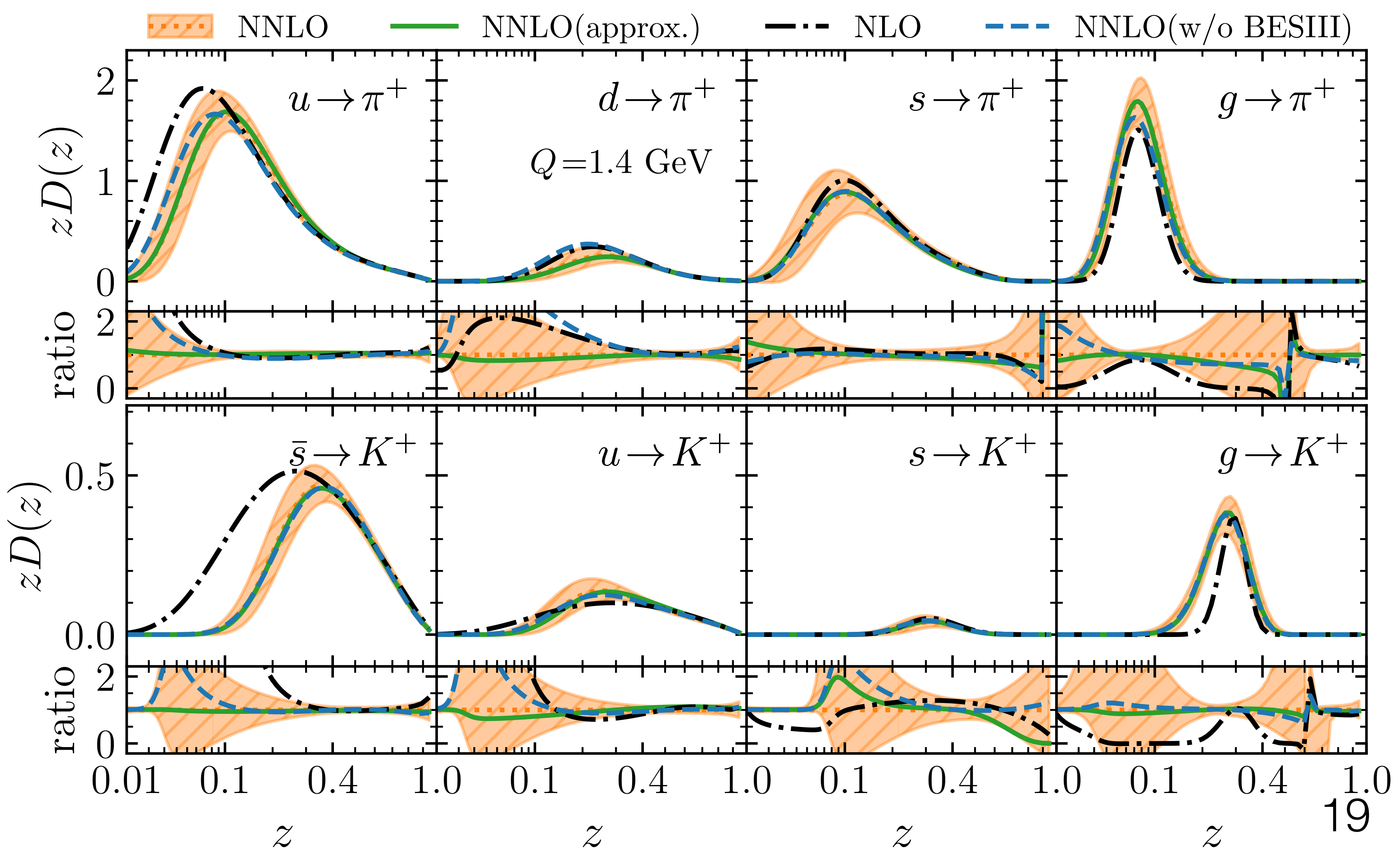
[JG, Shen, Xing, Zhao, Zhou, 2502.17837 (PRL)]

[BES III & NPC collaboration, 2502.16084 (PRL)]

	$E_{h,\min}$[GeV]	BESIII		COMPASS		B-factories		HE-SIA		global		
		$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	χ^2	$\chi^2/N_{\rm pt}$
NNLO	0.5	242	1.26	358	1.65	233	1.06	426	1.19	1259	1650.2	1.31
	0.6	212	1.21	290	1.59	228	0.92	423	0.97	1153	1338.8	1.16
	0.7	182	1.11	214	1.47	223	0.61	413	0.84	1032	997.2	0.97
	0.8	152	0.98	142	1.30	218	0.53	407	0.82	919	781.8	0.85
	0.9	122	1.05	94	1.29	213	0.52	407	0.80	836	687.1	0.82
	1.0	98	1.14	54	0.97	209	0.49	403	0.80	764	587.2	0.77
NLO	0.8	152	1.03	142	1.26	218	0.54	407	0.85	919	801.6	0.87
NNLO(approx.)	0.8	152	0.96	142	1.40	218	0.53	407	0.81	919	791.5	0.86
NNLO(w/o BES)	0.8	-	-	142	1.23	218	0.52	407	0.81	767	620.2	0.81

satisfactory agreements between NNLO theory and data are found for both the SIA and SIDIS data with Q~2-3 GeV

FFs to charged pion/kaon at Q=1.4 GeV



- proven factorization/universality of FFs in SIA and SIDIS at lowest Q values
- significantly worsen of all chi2 for hadron energy below 0.8 GeV; slightly larger global chi2 for NLO or aNNLO fits compared with NNLO fit

NPC23 analysis at NNLO

- We can explore complementary constraints from SIDIS data on the proton PDFs at NNLO in QCD, especially for the sea quark separations and the strangeness asymmetry in PDFs

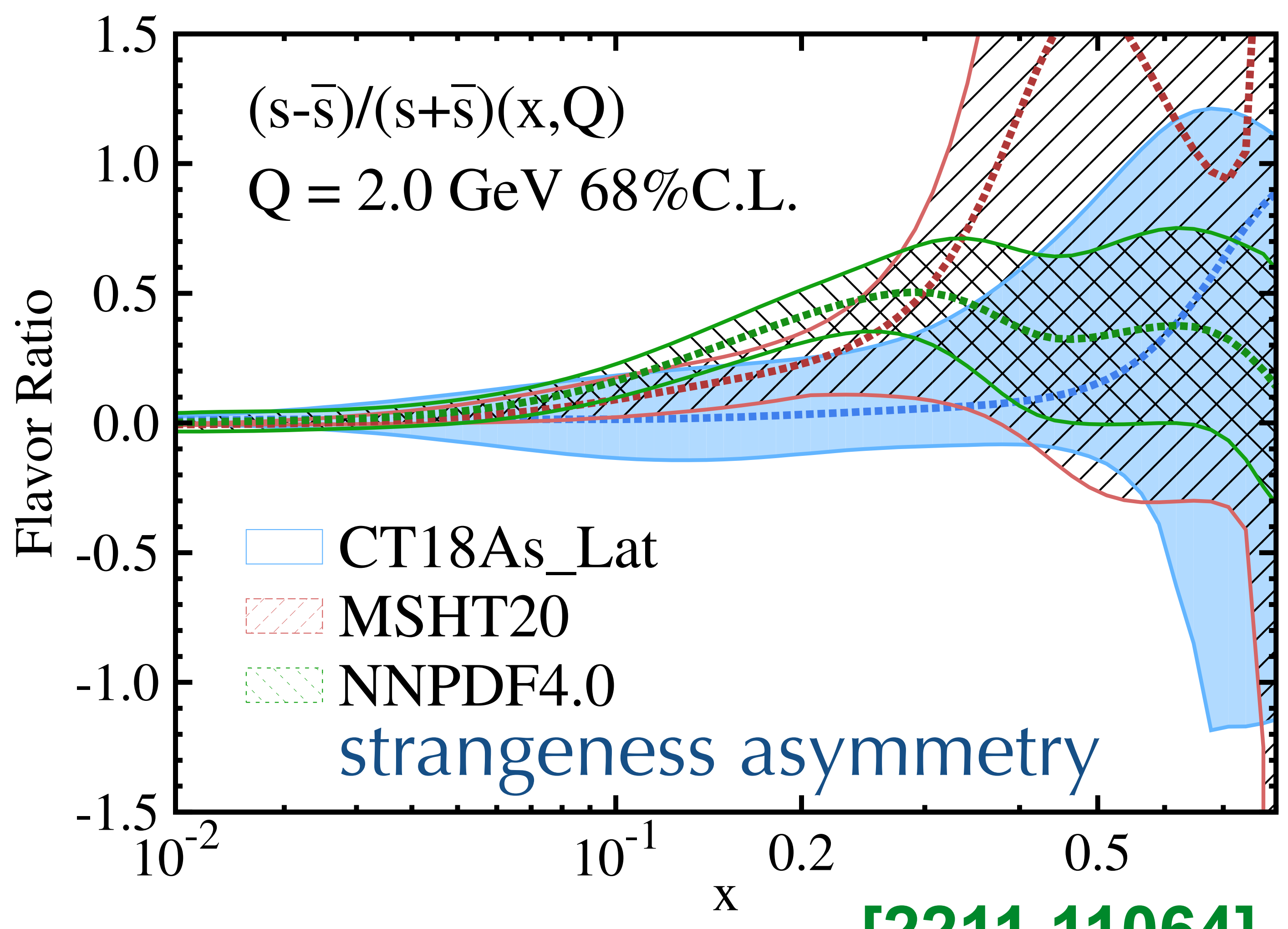
cross section difference of charged kaons with iso-scalar target

$$\frac{\mathrm{d}^3\sigma^{K^+-K^-}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} \propto 2(u_v(x)+d_v(x))(D_u^{K^+}(z)-D_{\bar{u}}^{K^+}(z)) + s_v(x)(D_s^{K^+}(z)-D_{\bar{s}}^{K^+}(z))$$

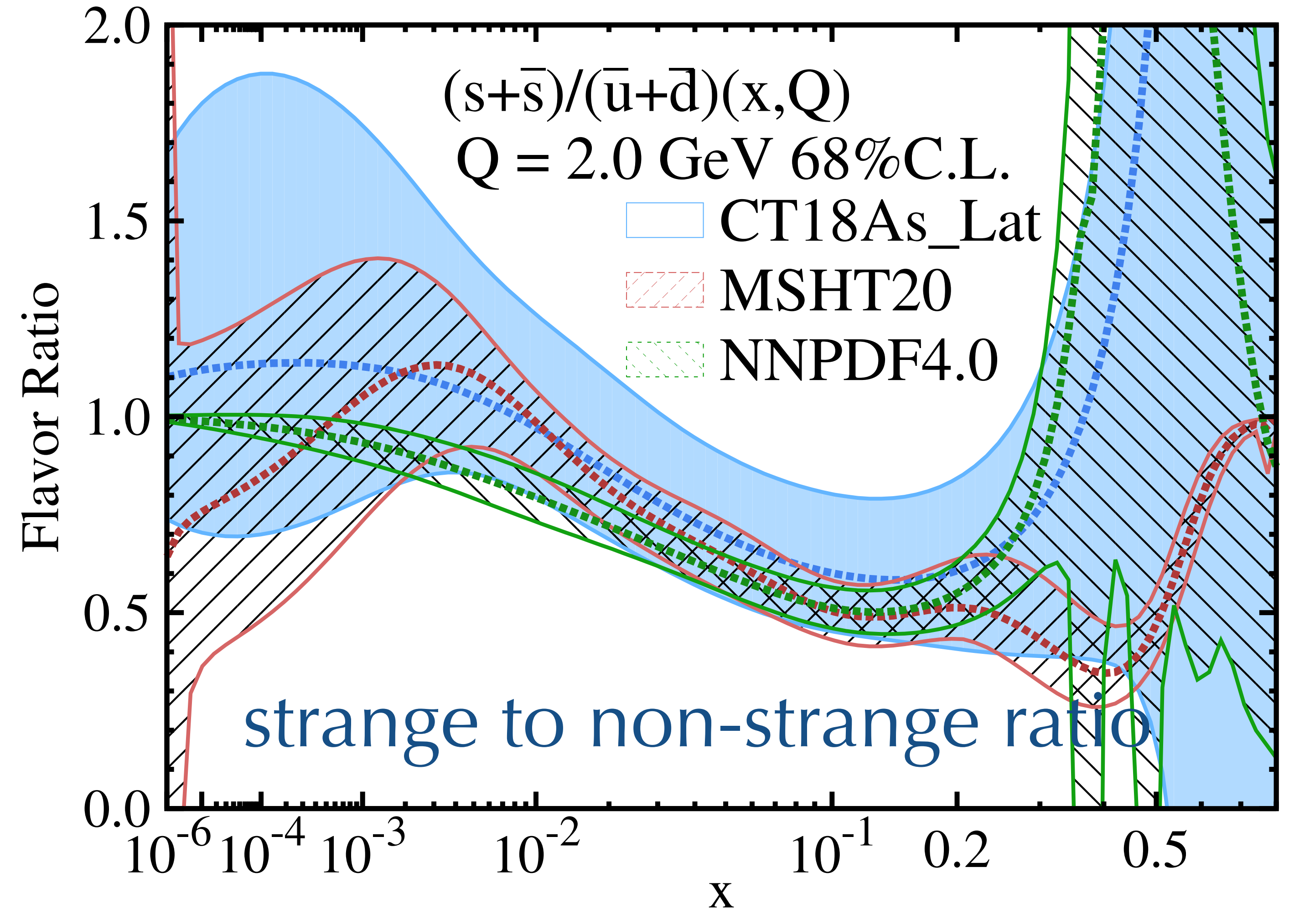
$$d_v \equiv d-\bar{d}, \quad r_s \equiv \frac{s+\bar{s}}{\bar{u}+\bar{d}}, \quad r_a \equiv \frac{s-\bar{s}}{s+\bar{s}}$$

- CT18, ABMP16 and ATLAS21assume zero strangeness asymmetry at initial scale; MSHT20 and NNPDF4.0 find a preference of positive asymmetry (at large-x) from inclusive data

- The chi2 of best-fit to SIDIS and SIA data varies by ~40 units with the choice of PDFs and show a preference of reduced strangeness asymmetry, as well as further strange-quark suppression at x=0.2



[2211.11064]



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$$\frac{d^3\sigma^{K^+-K^-}}{dxdydz} \propto 2(u_v(x)+d_v(x))(D_u^{K^+}(z)-D_{\bar{u}}^{K^+}(z)) + s_v(x)(D_s^{K^+}(z)-D_{\bar{s}}^{K^+}(z))$$

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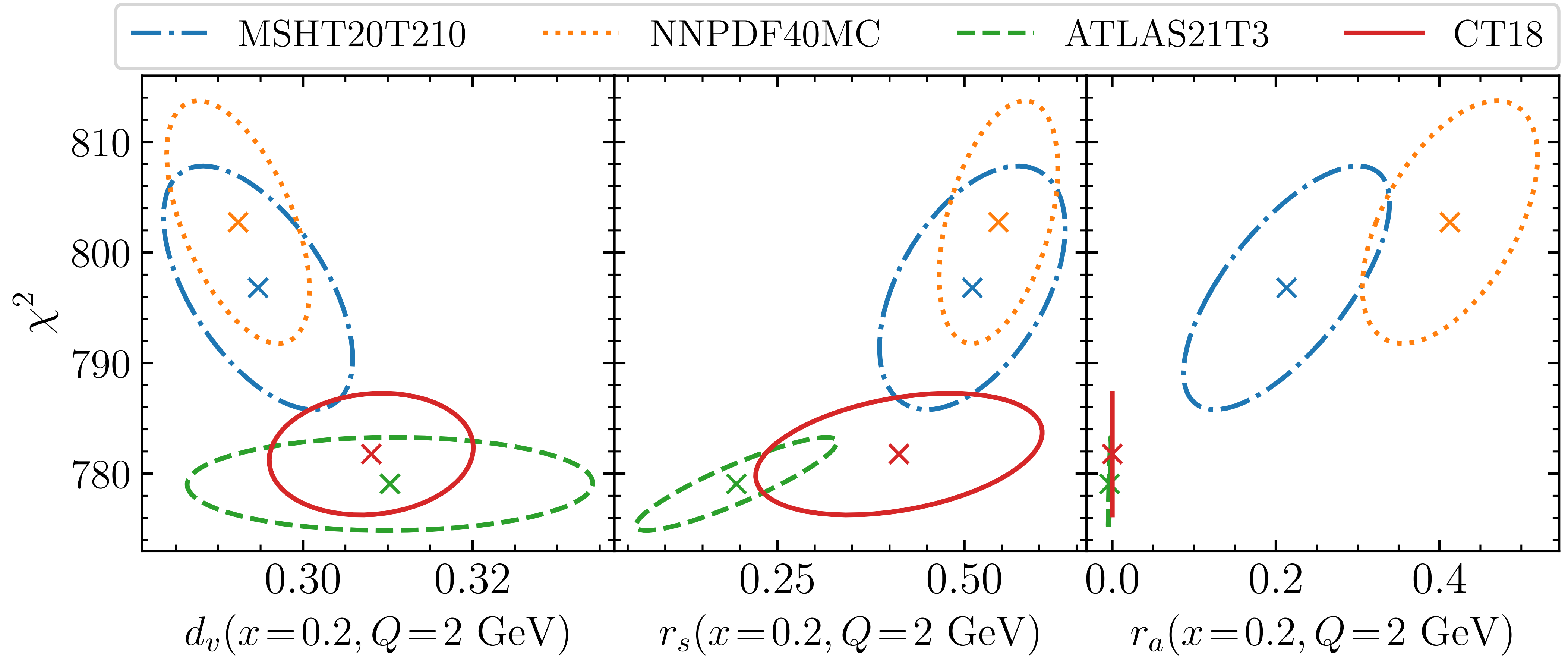
correlation of FF best-fit chi2 and PDF variables



PDF variables before/after reweighting/profiling

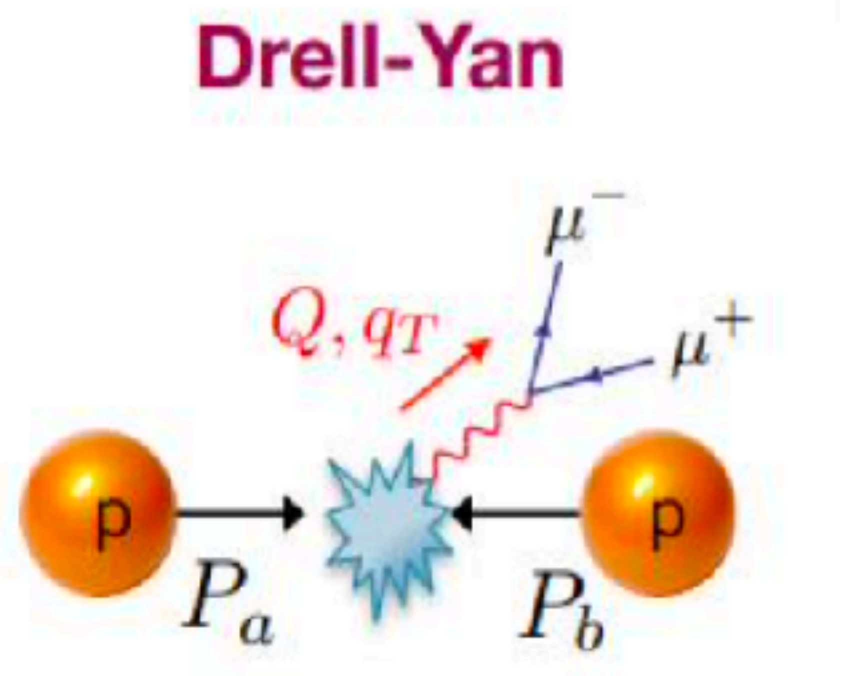
	$d_v(x=0.2, Q=2\text{GeV})$	$r_s(x=0.2, Q=2\text{GeV})$	$r_a(x=0.2, Q=2\text{GeV})$
NNPDF4.0	0.2924 ± 0.0084	0.547 ± 0.079	0.408 ± 0.107
NNPDF4.0(reweighting)	0.3021 ± 0.0069	0.438 ± 0.066	0.281 ± 0.086
MSHT20	0.295 ± 0.011	0.511 ± 0.124	0.213 ± 0.126
MSHT20(profiling)	0.298 ± 0.011	0.481 ± 0.121	0.167 ± 0.136

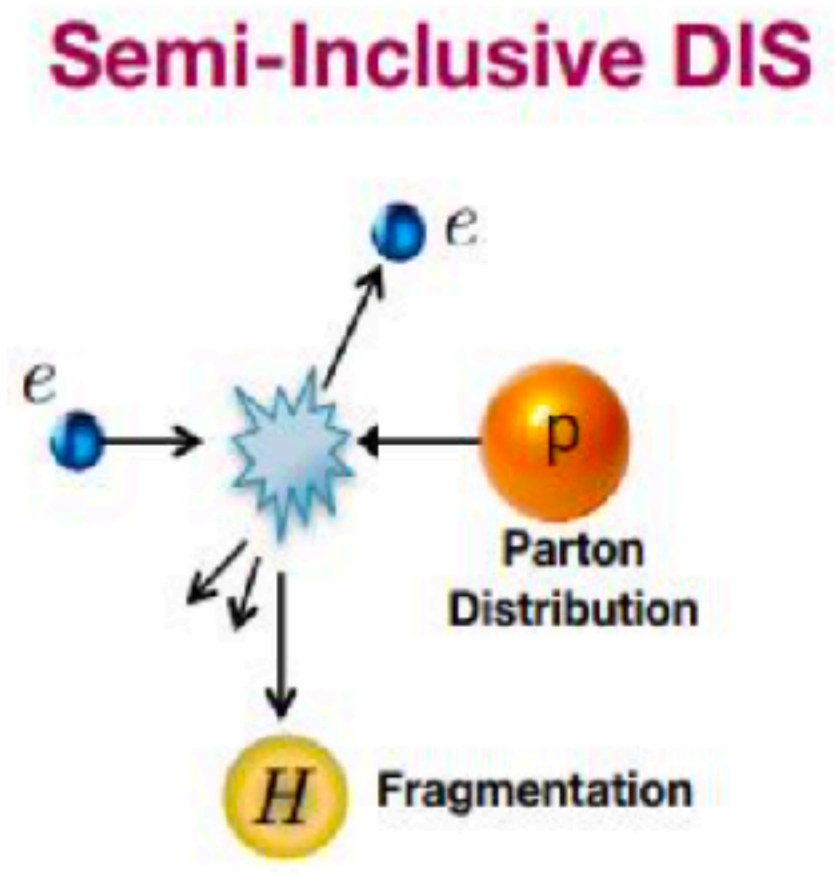
preference of reduced strangeness asymmetry

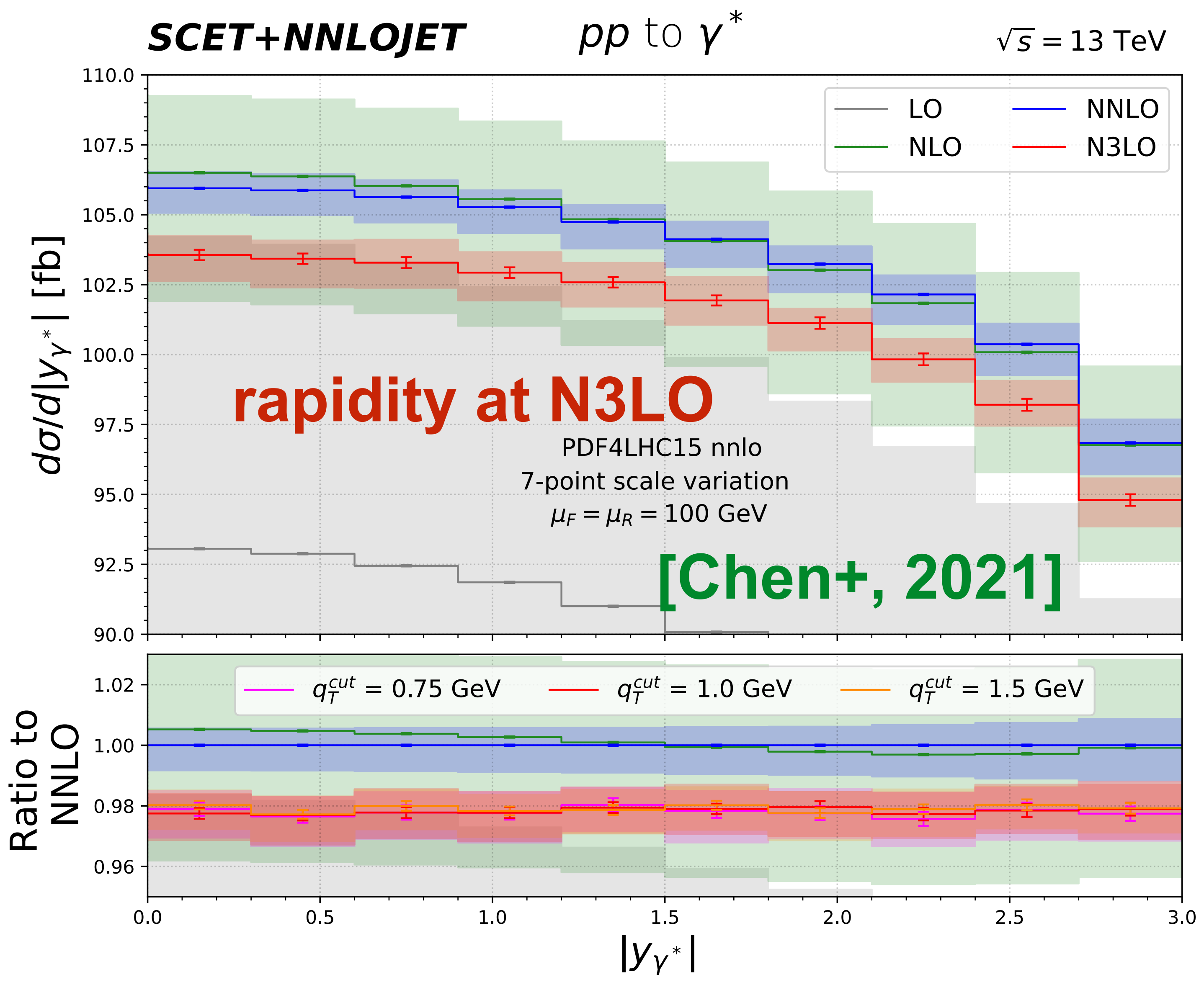
- 3. Precision calculations for SIDIS up to N3LO

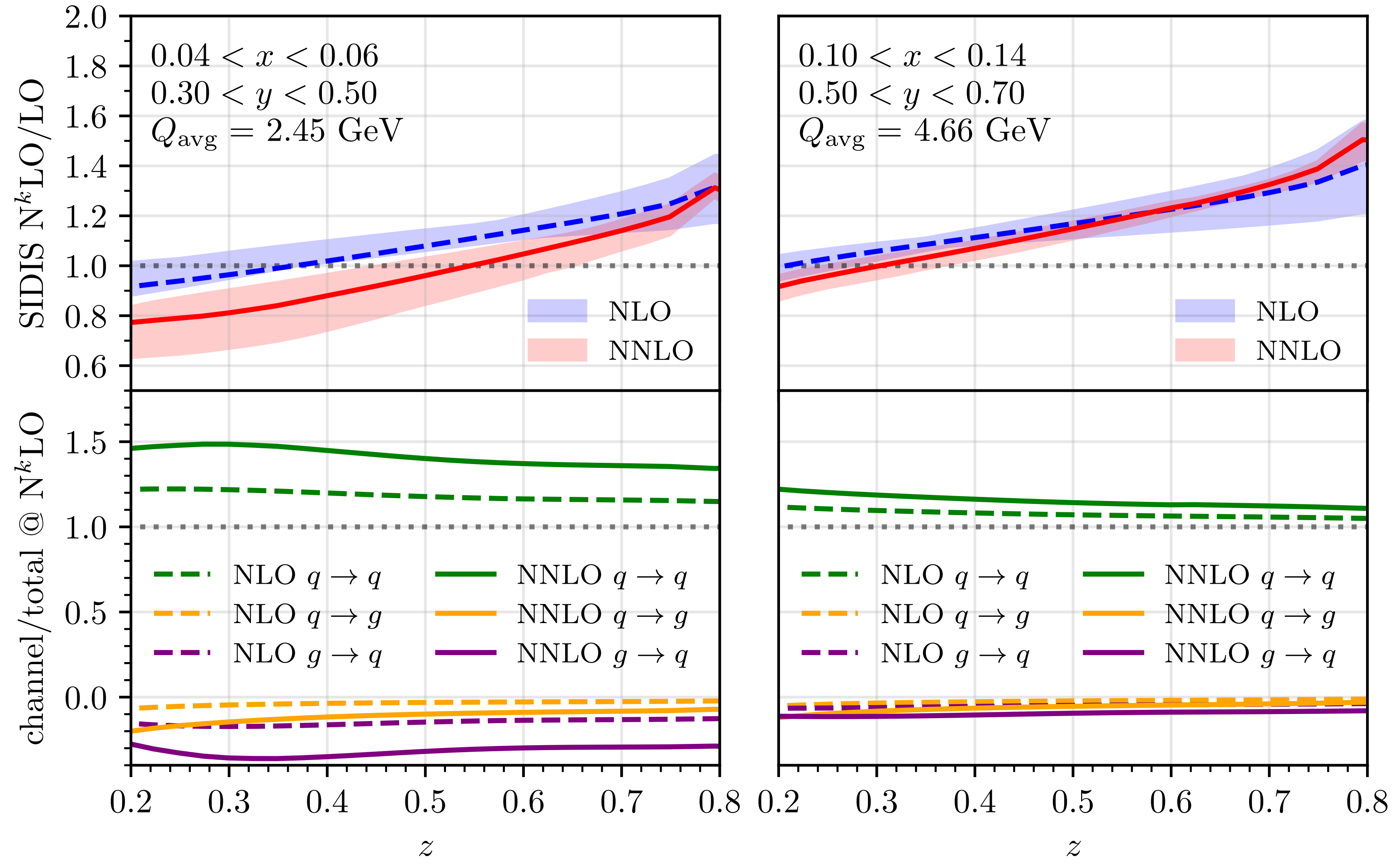
Frontier of the perturbative QCD calculations

- The LHC has pushed the pQCD precision on calculations of hard cross sections/coefficient functions to NNLO as a standard and to N3LO for a few processes, e.g. Drell-Yan production; while it is less matured for processes in ep collisions









[Bonino+; Goyal+, 2024]

- N3LO corrections in Drell-Yan show unexpected behavior of being outside the NNLO uncertainty band
- NNLO corrections to SIDIS are recently calculated in an analytic form; extension to N3LO is challenging due to complications from fragmentations

The story of higher-order calculations for SIDIS

- In early **2023**, I collaborated with HuaXing on a NLO calculation for hard functions of Energy-energy correlations. Soon we realized that the same method can be applied to hadron production as well.

- Around late **2023**, we started working on the NNLO QCD calculations for SIDIS with subtraction method. Somehow we got stuck with unreasonable numerical results…

- In the middle of **2024**, two other groups finished the analytical calculation of SIDIS at NNLO both published as Editors' Suggestion in PRL

PHYSICAL REVIEW LETTERS **132,** 251901 (2024)

Editors' Suggestion

Semi-Inclusive Deep-Inelastic Scattering at Next-to-Next-to-Leading Order in QCD

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(Received 31 January 2024; accepted 15 May 2024; published 18 June 2024)

PHYSICAL REVIEW LETTERS **132,** 251902 (2024)

Editors' Suggestion

Next-to-Next-to-Leading Order QCD Corrections to Semi-Inclusive Deep-Inelastic Scattering

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(Received 20 March 2024; accepted 15 May 2024; published 18 June 2024)

- In late **2025**, after a long cooling down, we finally find a bug in our numerical code and immediately we are able to reproduce the known NNLO corrections

- Running to **2026**, through a series of works, we now made a step forward of leveraging the perturbative precision to N3LO in QCD for SIDIS

Hadron production cross sections at NLO

- Generic algorithms on NLO calculations of jet production cross sections have been developed for long times, based on local subtraction or phase-space slicing method; especially automation of NLO jet cross sections exists, e.g., in MG5 and Sherpa

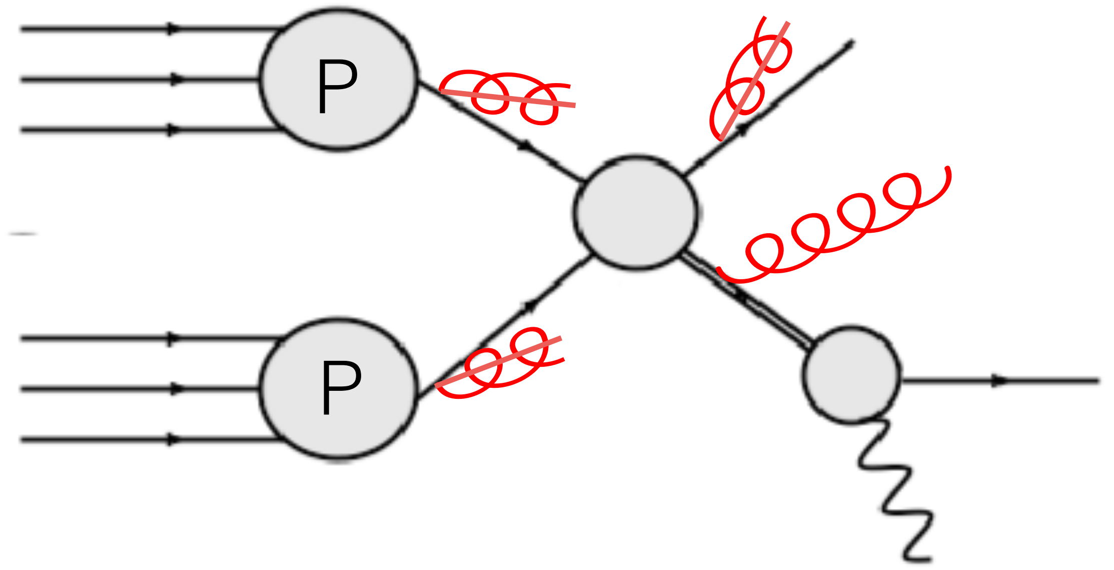
local subtraction

$$\frac{d\sigma}{dF} = \int dPS_m \Big[|M|^2_{B,m} + |M|^2_{V,m} + |\tilde{\mathcal{I}}|^2_m\Big]\delta(\hat{F}(p_m; f_m) - F)$$
$$+ \int dPS_{m+1}\Big[|M|^2_{R,m+1}\delta(\hat{F}(p_{m+1}; f_{m+1}) - F) - |\mathcal{I}|^2_{m+1}\delta(\hat{F}(\tilde{p}_m; \tilde{f}_m) - F)\Big]$$

phase-space slicing

$$\frac{d\sigma}{dF} = \int dPS_m \Big[|M|^2_{B,m} + |M|^2_{V,m}\Big]\delta(\hat{F}(p_m; f_m) - F)$$
$$+ \int dPS_{m+1}(\Theta(C-\lambda) + \Theta(\lambda - C))\Big[|M|^2_{R,m+1}\delta(\hat{F}(p_{m+1}; f_{m+1}) - F)\Big]$$

QCD radiations



- FKS subtraction (jet), **[Frixione, Kunszt, Signer]**, as implemented in Madgraph5
- Dipole subtraction (jet & hadron), **[Catani, Seymour]**, as implemented in MCFM
- Two-cutoff slicing (jet & hadron), **[Harris, Owens]**
- Antenna subtraction (jet & hadron), **[2406.09925]**; alternative subtraction **[2403.14574]**; semi-analytical calculations, **[1903.01529, BigkT]**
- STRIPPER (jet & hadron at NNLO), **[2503.11489]**

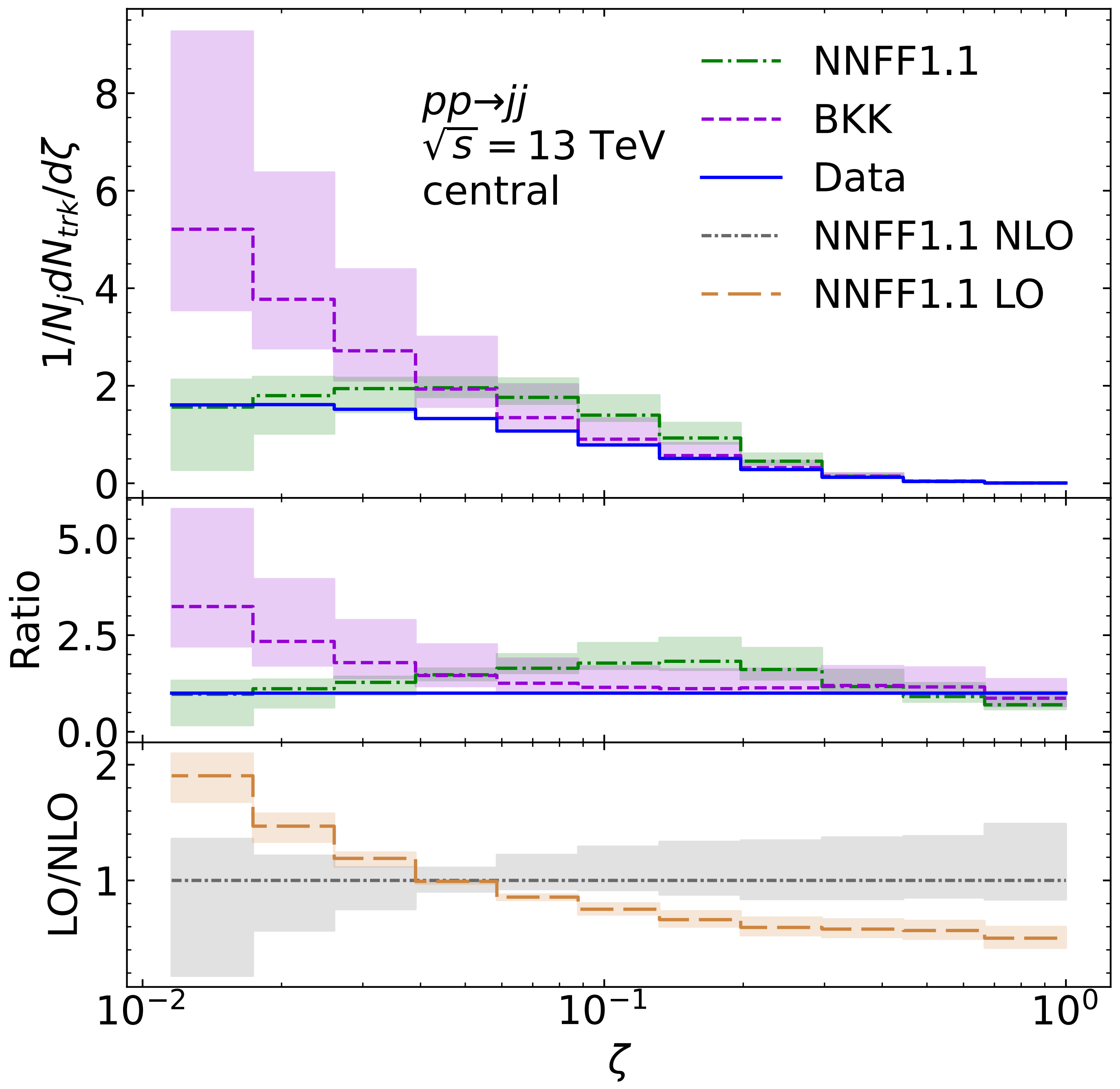
public tools and automation on the hadron production cross sections are very much limited!

FMNLO (fragmentation at NLO in QCD)

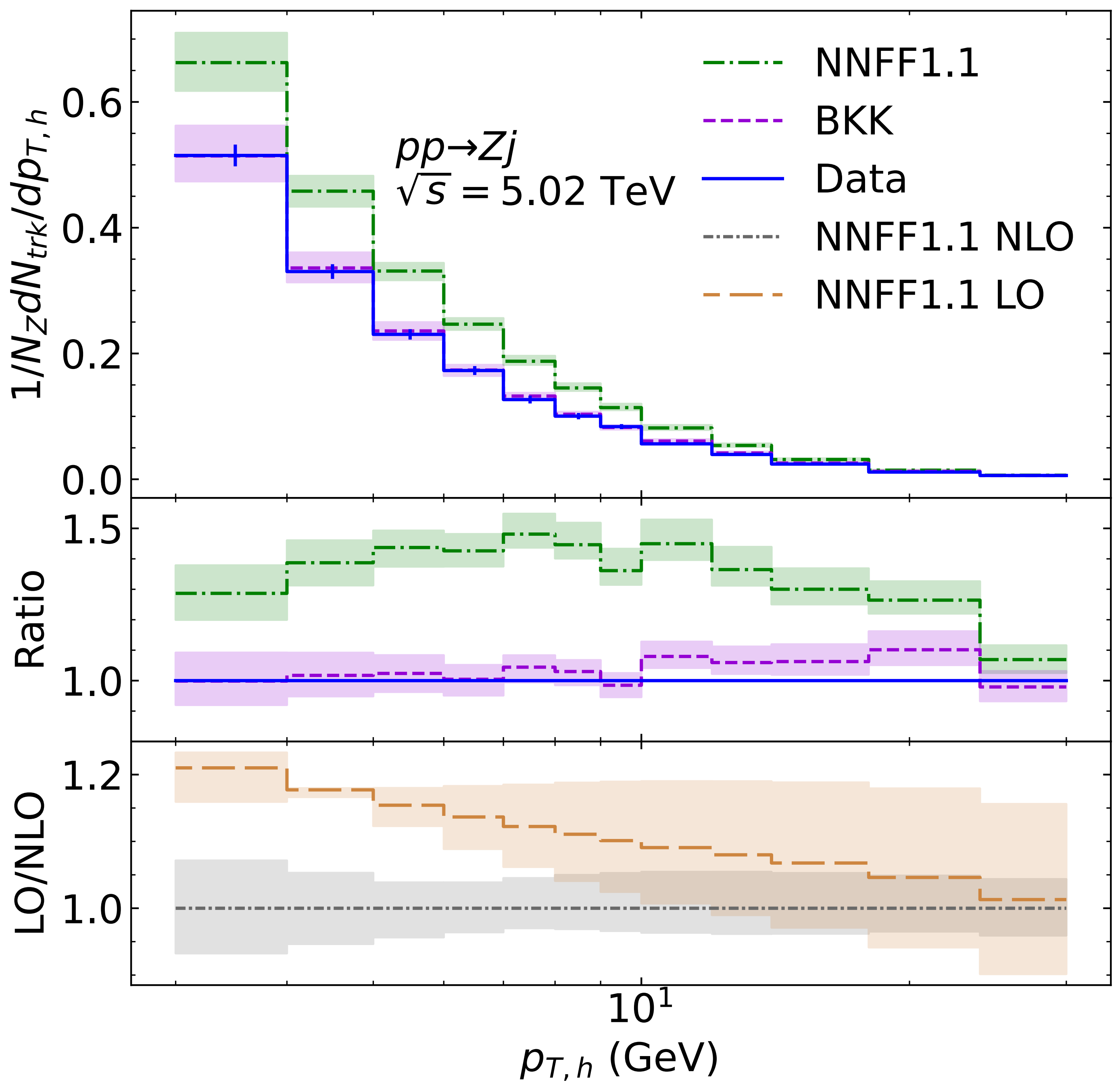
- FMNLO is a program for automated and fast calculations of fragmentation cross sections of arbitrary hard processes. It is based on a hybrid scheme of phase-space slicing method and local subtraction method, accurate to NLO in QCD

  - automation of fragmentation calculations for arbitrary hard processes up to NLO, within SM and BSMs via MG5_aMC@NLO
  - fast convolution of partonic cross sections with FFs without repeating the time consuming MC integrations using interpolation grids
  - generalizations: transverse observables, **NNLO corrections**

QCD inclusive dijets at LHC



Z-boson tagged jet



News

2024.07: FMNLOv2.1 NNLO calculations are available for limited cases, SIA, decay of the Higgs boson to gluons, and SIDIS.

2024.07: FMNLOv2.0 include a SIDIS module for calculations of SIDIS at NLO.

2023.05: FMNLOv1.0 first release of FMNLO interfaced with MG5_aMC@NLO.

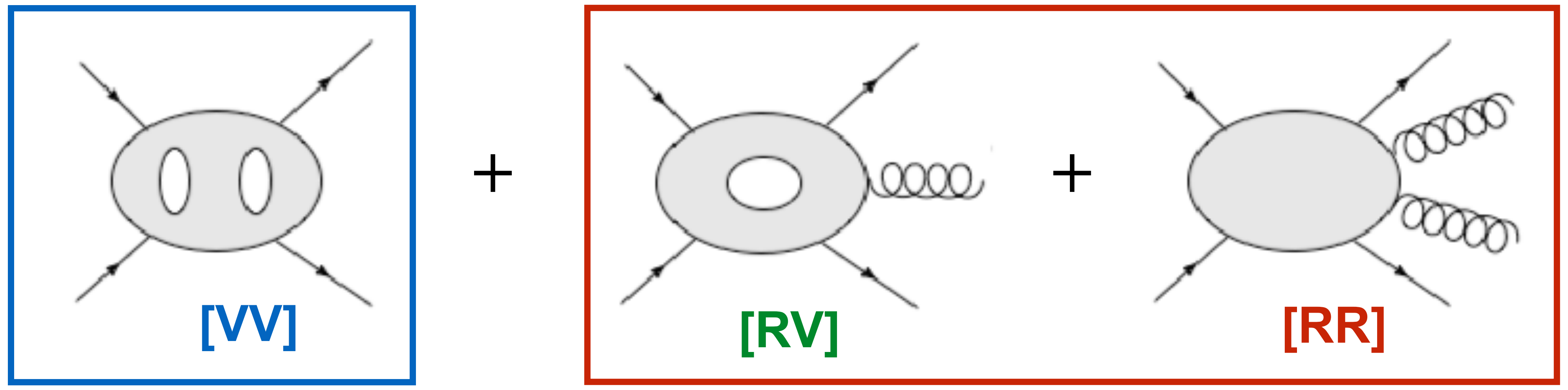
https://fmnlo.sjtu.edu.cn/~fmnlo/

[JG, Liu, Shen, Zhou, 2305.14620 (JHEP)]

Transverse momentum subtraction for SIDIS

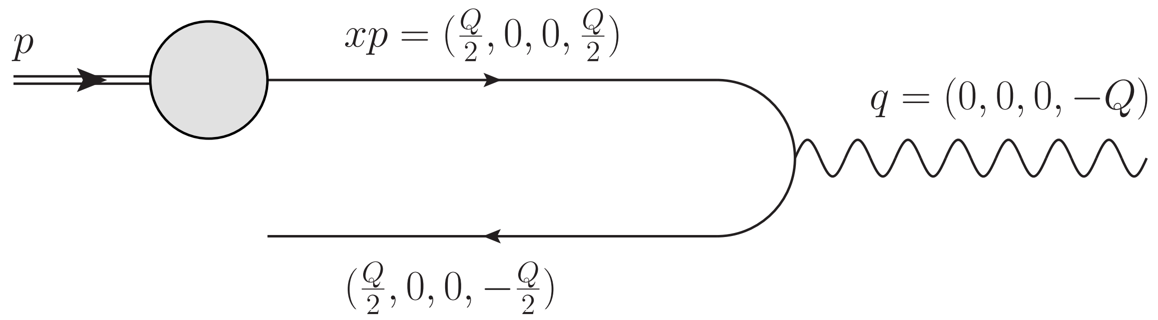
- Generalizing the qT-subtraction formalism (Catani–Grazzini) from Drell-Yan to SIDIS using **hadron transverse momentum qT (Breit frame)** as the slicing variable, validated against existing analytic NNLO results

components of NNLO corrections



qT subtraction/slicing method **NNLO → NLO**

$$\frac{d\sigma}{d\mathcal{O}} = \int_{q_{T,\mathrm{cut}}}^{q_{T,\max}} dq_T \frac{d\sigma}{dq_T d\mathcal{O}} + \int_0^{q_{T,\mathrm{cut}}} dq_T \frac{d\sigma}{dq_T d\mathcal{O}}$$

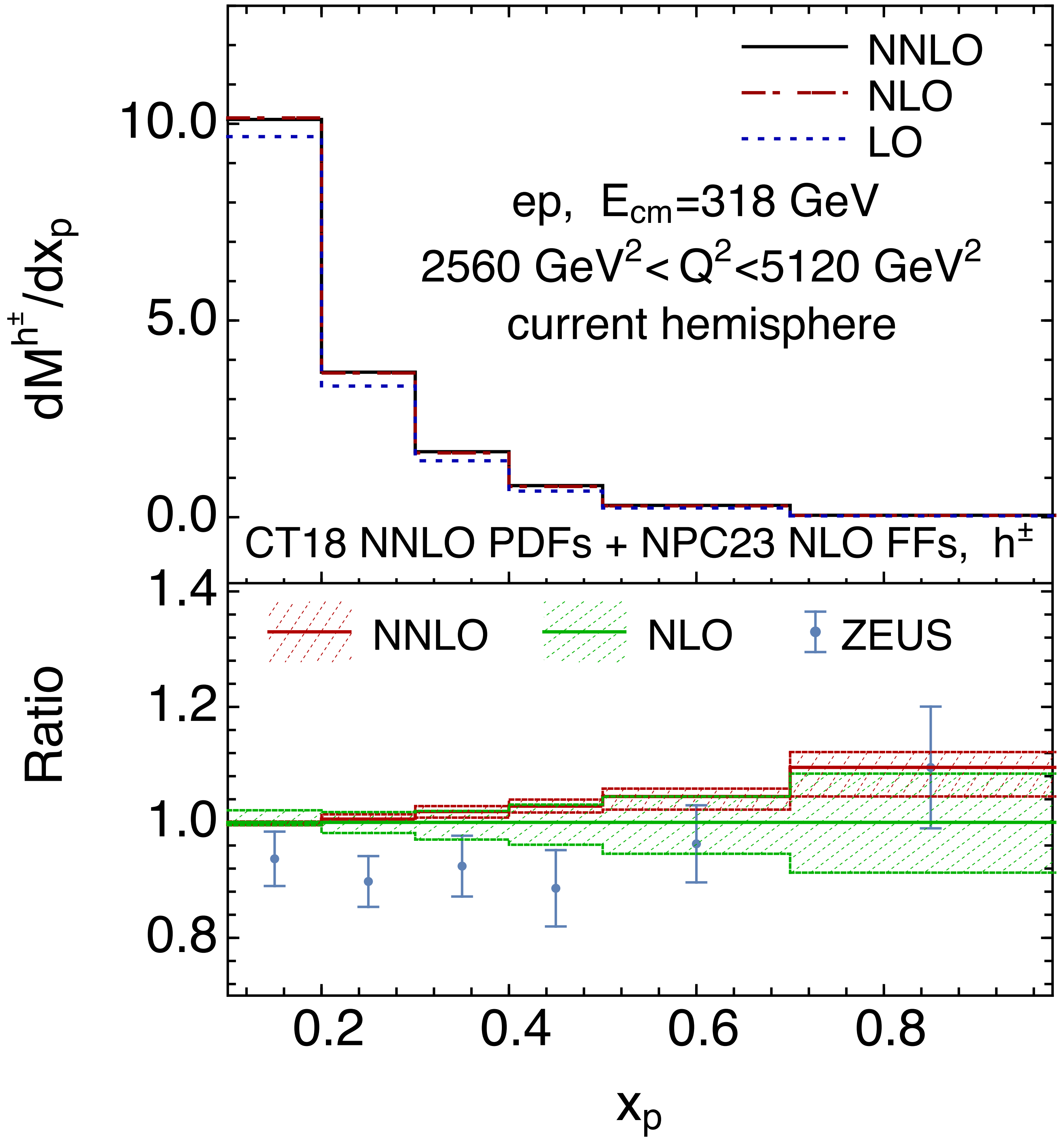


TMD factorization of the unresolved part

$$\frac{1}{\sigma_0}\frac{d\sigma_{\ell+N\to\ell'+h+X}}{d^2\vec{q}_\perp dx dy dz} \simeq \sum_q H_q(Q^2,\mu) \times \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{q}_\perp} f_1^q(x, b_\perp, \xi_0^n, \mu_0^n) \times D_1^q\left(z, \frac{b_\perp}{z}, \xi_0^{\bar{n}}, \mu_0^{\bar{n}}\right)$$
$$\times \prod_i e^{-2K^i_{\mathrm{cusp}}(\mu_0^i,\mu)+A_H^i(\mu_0^i,\mu)} \times \left(\frac{\xi^i}{{\mu_0^i}^2}\right)^{A^i_{\mathrm{cusp}}(\mu_0^i,\mu)} \times \left(\frac{\sqrt{\xi^i}}{\sqrt{\xi_0^i}}\right)^{K^i(b_\perp,\mu_0^i)},$$

[JG, Li, Zhu, Zhu, 2602.06364]

charged hadron multiplicity at HERA

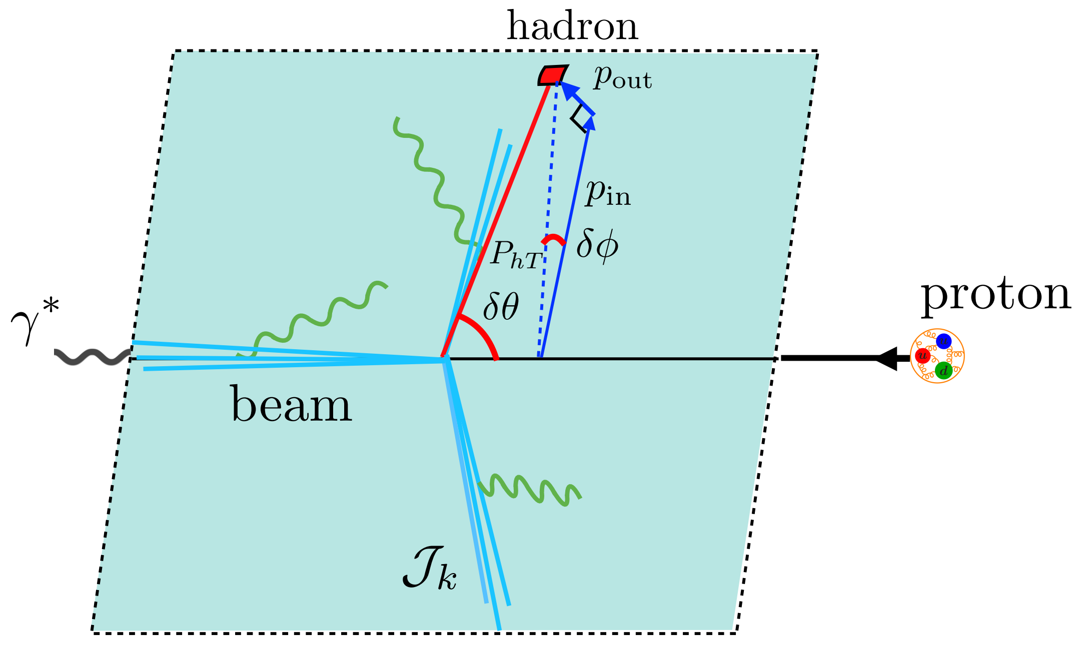


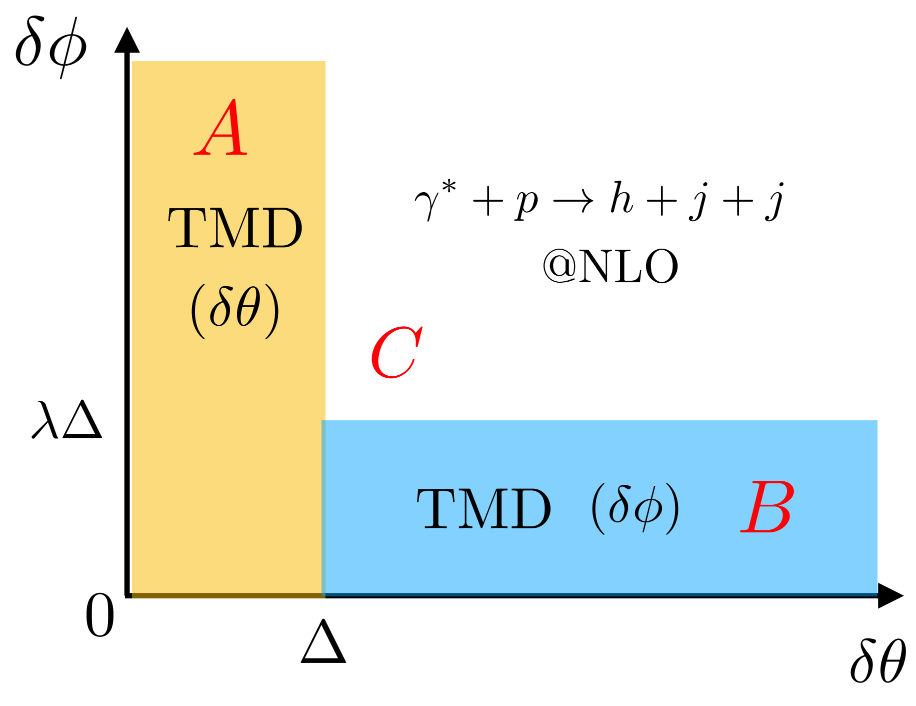
- excellent agreements with NNLO analytical results for all partonic channels; our calculations are fully differential and can be extended to N3LO

Two-dimensional transverse momentum subtraction

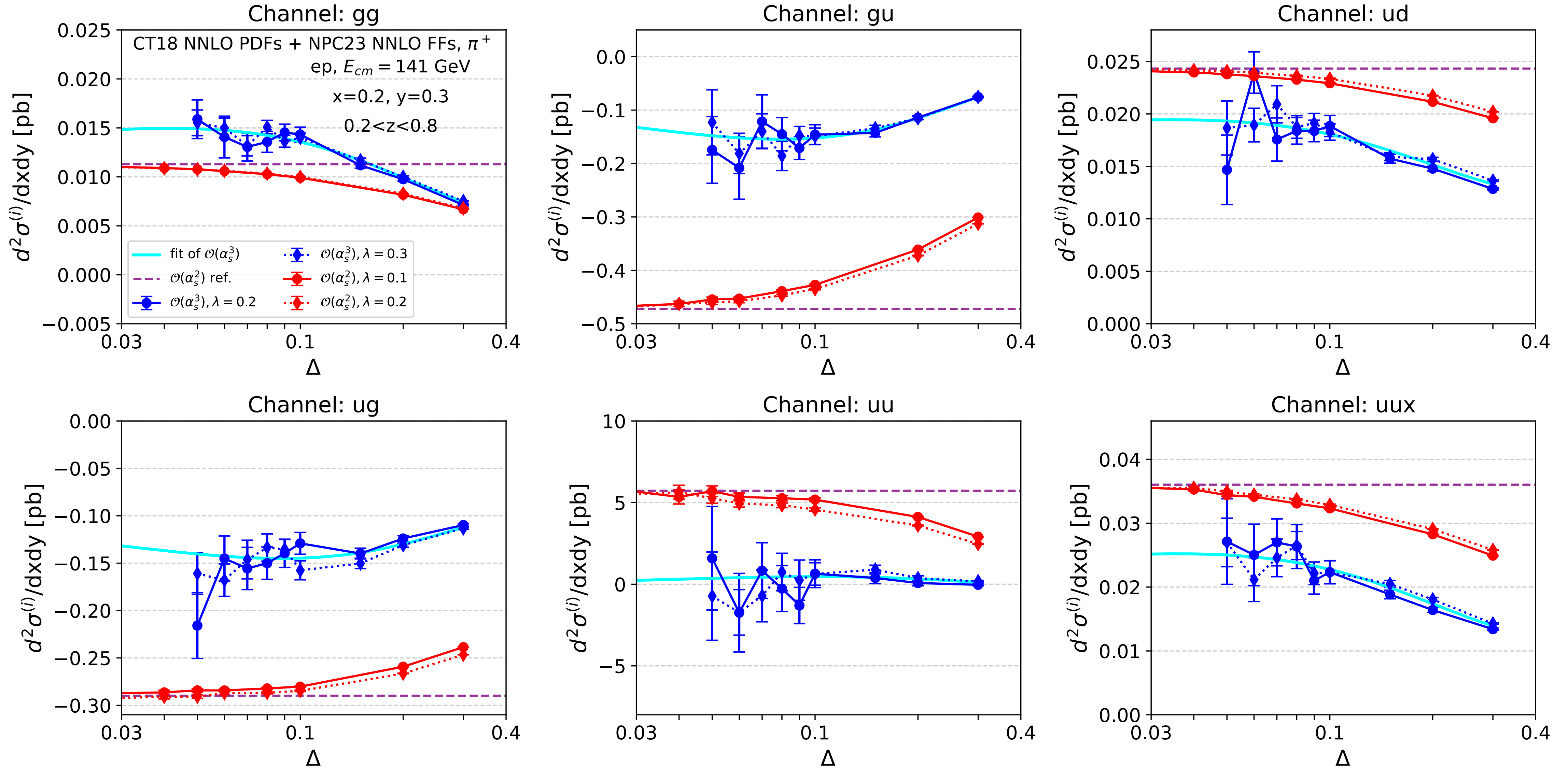
- Introduction of the two-dimensional transverse momentum subtraction formalism for identified hadron production, using both **conventional and out-of-plane qT** as slicing variables, resulting a first calculation of SIDIS at N3LO in QCD

hadron kinematics and slicing variables





pion cross sections vs. resolution parameter



double slicing and two cutoffs **N3LO → NLO**

$$\frac{d\sigma}{d\mathcal{O}} = \int_0^{\Delta} d\delta\theta \frac{d\sigma^A}{d\delta\theta d\mathcal{O}} + \int_{\Delta}^{\delta\theta^{\max}} d\delta\theta$$
$$\times \left(\int_0^{\lambda\Delta} d\delta\phi \frac{d\sigma^B}{d\delta\theta d\delta\phi d\mathcal{O}} + \int_{\lambda\Delta}^{\delta\phi^{\max}} d\delta\phi \frac{d\sigma^C}{d\delta\theta d\delta\phi d\mathcal{O}} \right)$$

[Dong, Fang, JG, Li, Shao, Zhu, Zhu, 2602.22972, 2603.29673]

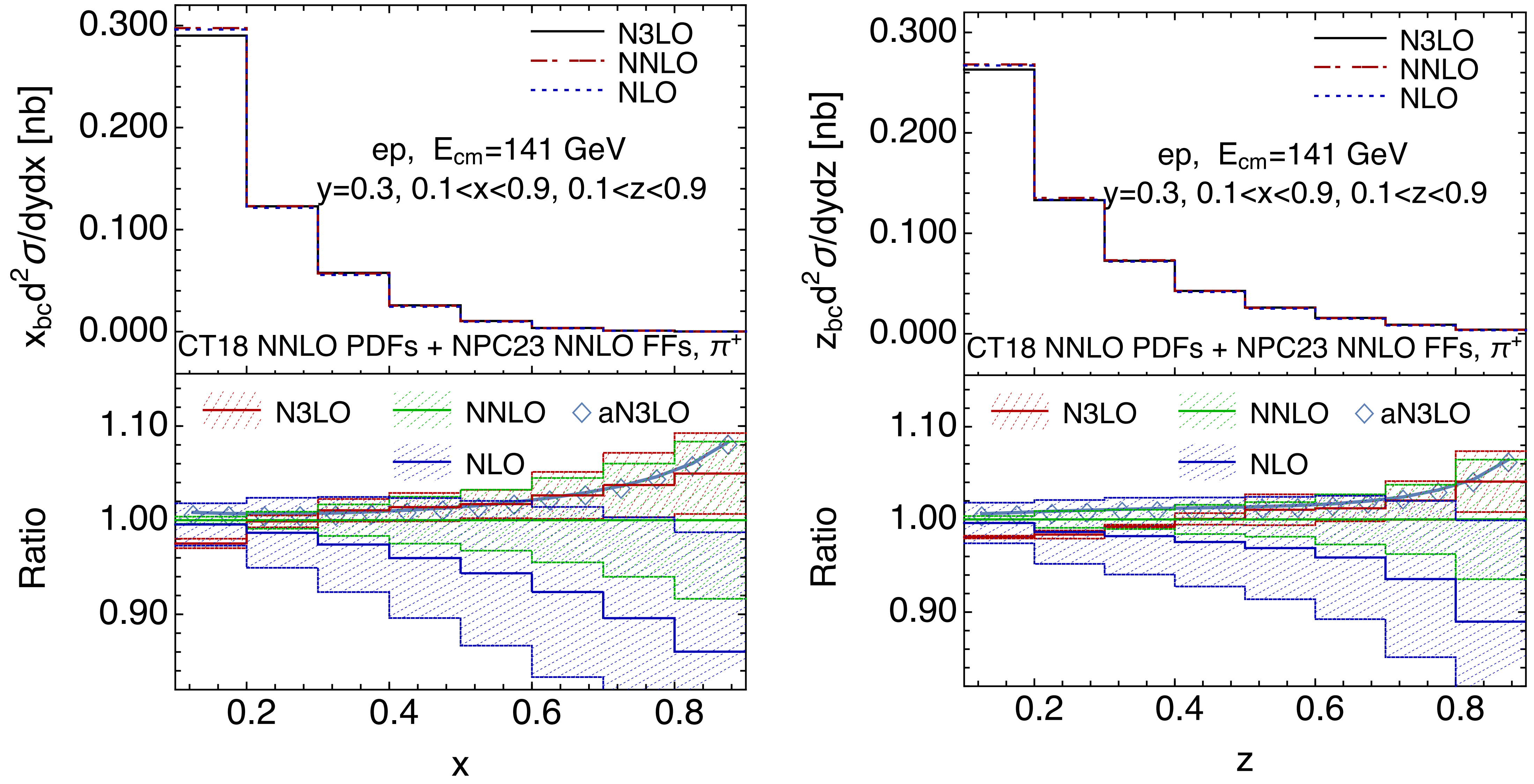
- good convergence of the QCD corrections wrt. both cutoffs for all partonic channels, numerically challenging for small cutoffs

- N3LO corrections (blue) are much smaller than NNLO ones (red) for three major channels uu, gu and ug

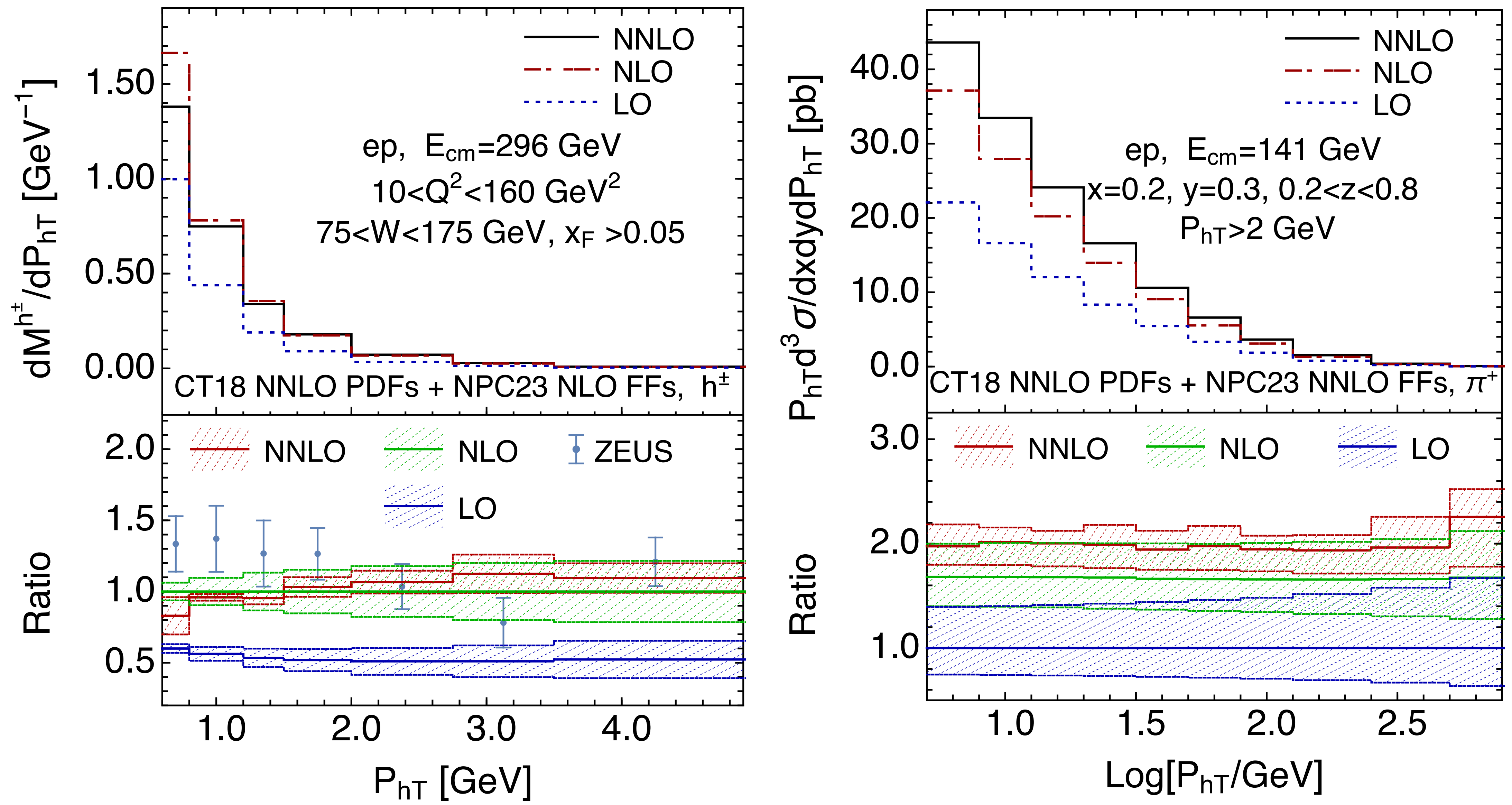
Implication for Electron-ion colliders

- New perturbative QCD corrections for SIDIS exhibit good convergence and reduced scale variations for differential cross sections at future electron-ion collider, providing state-of-the-art predictions for study of nucleon structure and hadronizations at EIC

distribution of Bjorken-x and z for SIDIS at EIC upto N3LO



hadron transverse momentum spectrum at HERA and EIC upto NNLO



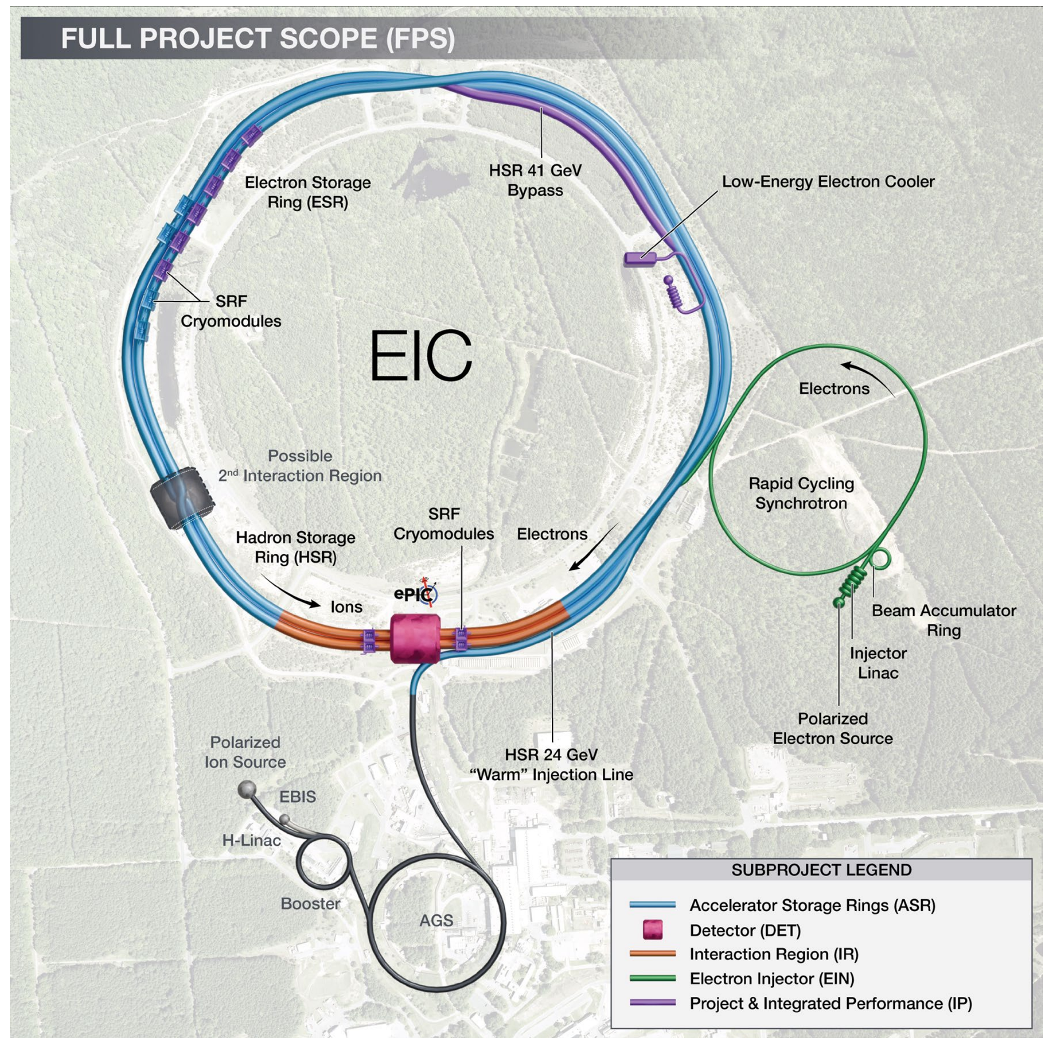
[Dong, Fang, JG, Li, Shao, Zhu, Zhu, 2602.22972, 2603.29673]

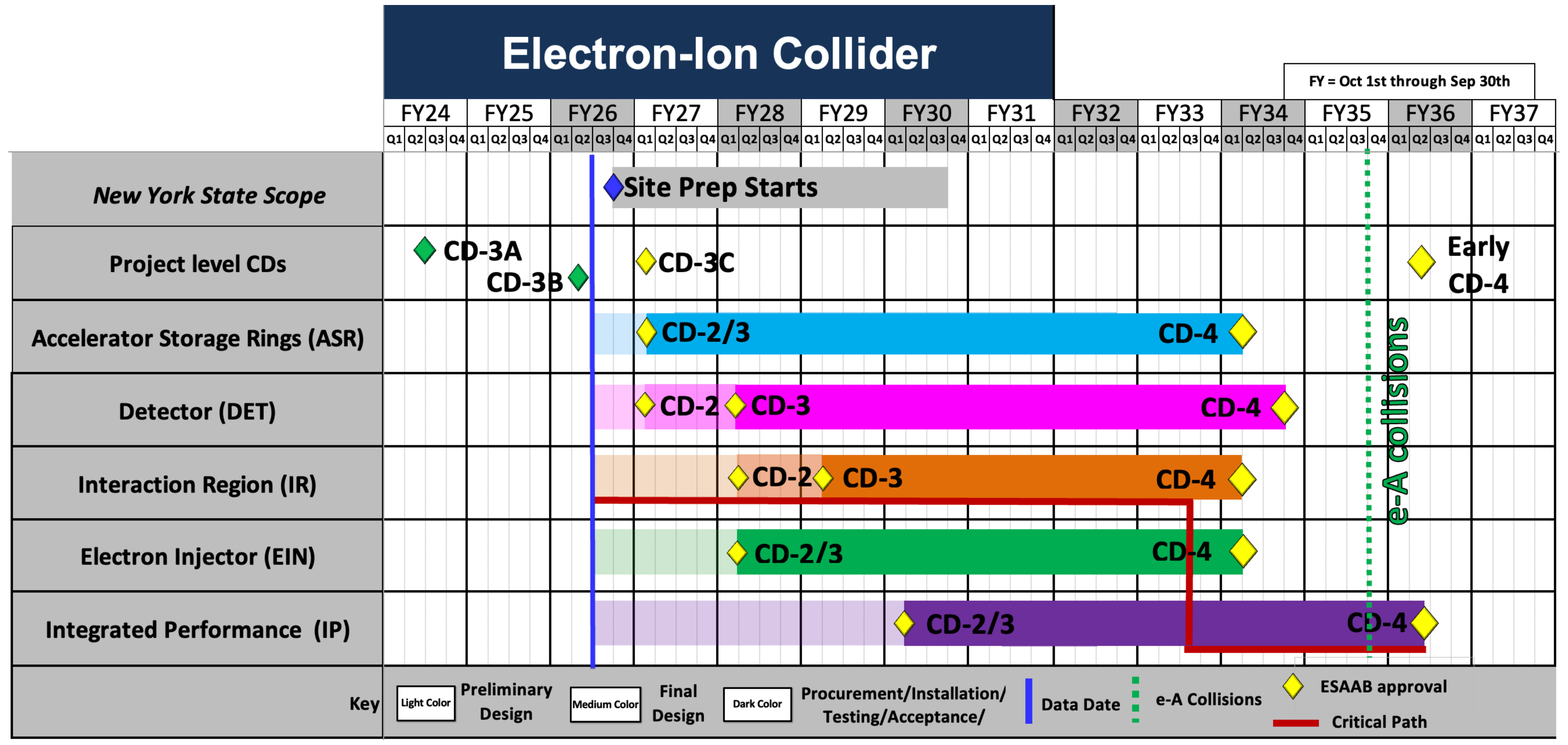
approximated N3LO from **[Goyal, Moch, Pathak, Rana, Ravindran 2025]**

Electron Ion collides

- The EIC will be the only operating particle collider in the US and will bring in precision (polarized) DIS data at center of mass energies of 20~140 GeV and luminosity of 10~100 fb^{-1} per year; a similar collider in China (EICs) runs at center of mass energy of ~16 GeV is also planned

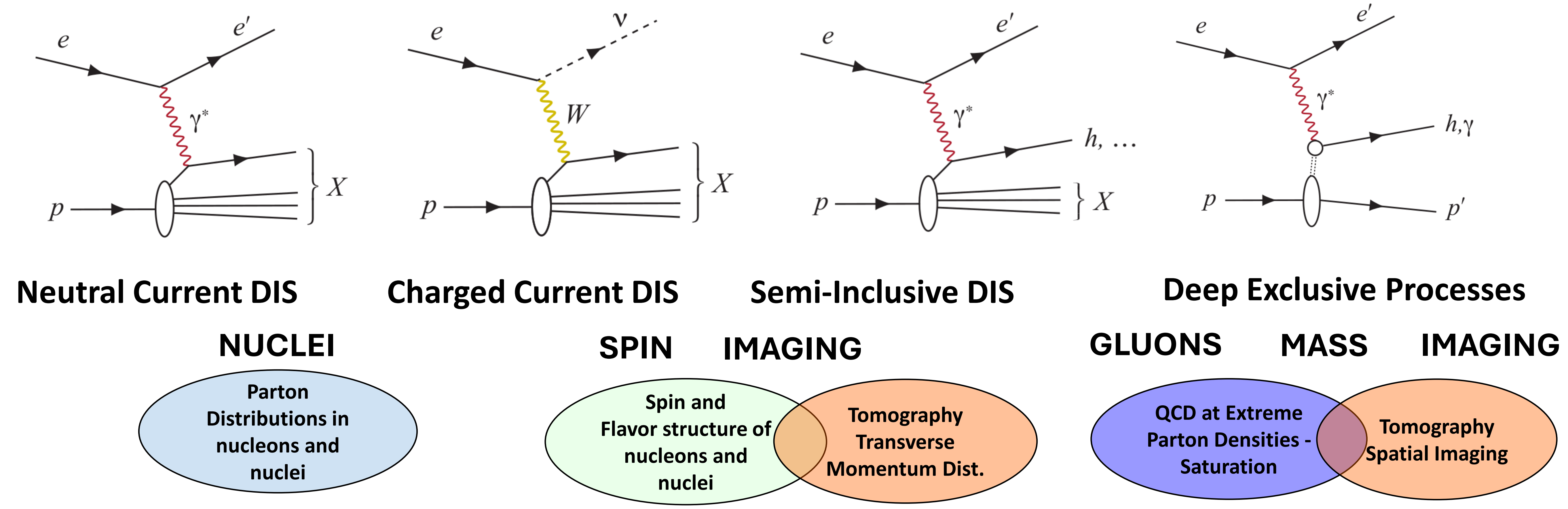
Preliminary Project Schedule





Experimental Processes to Access EIC Physics

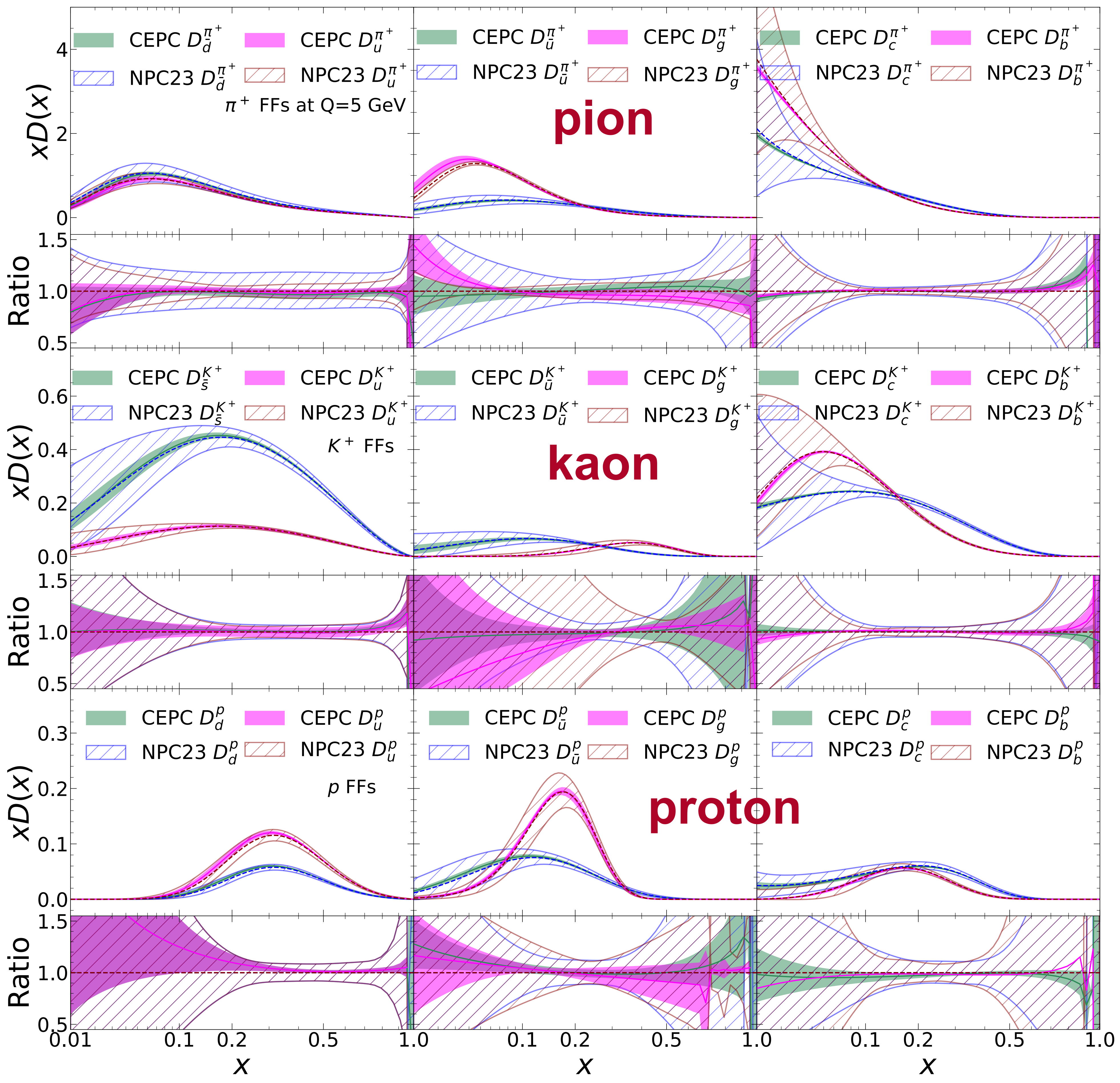
DIS event kinematics - scattered electron or final state particles (CC DIS, low y)



- High precision of the EIC calls for a timely improvements on various theoretical ingredients, especially for SIDIS

Higgs factories

- High luminosity and high energies of future lepton colliders open new opportunities for precision determination of FFs, especially with production of the W boson pairs and the Higgs boson with hadronic decays **[Zhou, JG, 2407.10059 (JHEP)]**



FFs (positively charged) vs. momentum fraction

- fits of FFs at NLO in QCD are carried out with pseudo-data solely from future CEPC
- (anti-)quark flavor separation from different energies, angular distributions, and heavy-flavor tagging
- d/s quark separation from W boson decays; probe of gluon fragmentation from Higgs boson decays
- ILC, FCC-ee and CEPC give quite similar results except in regions statistics are limited

Higgs factory alone are ideal machine for fragmentation functions!

Summary

- Deep inelastic scatterings are essential for refined study of nucleon structure, including PDFs, helicity PDFs, and TMD PDFs, with implications not only for understanding of QCD but also for precision test of the SM

- We developed a systematic routine of subtraction/slicing methods for calculations of single inclusive hadron production, and leverage perturbative precision of SIDIS to N3LO in QCD

- The NPC collaboration also provides most comprehensive/advanced global analyses on FFs of light hadrons, including on hadron species, data selections, and theoretical precisions (to NNLO)

- With the advancements on ingredients of both non-perturbative and perturbative QCD we are on the way of building a precision framework for theoretical predictions of SIDIS at future electron-ion colliders

Summary

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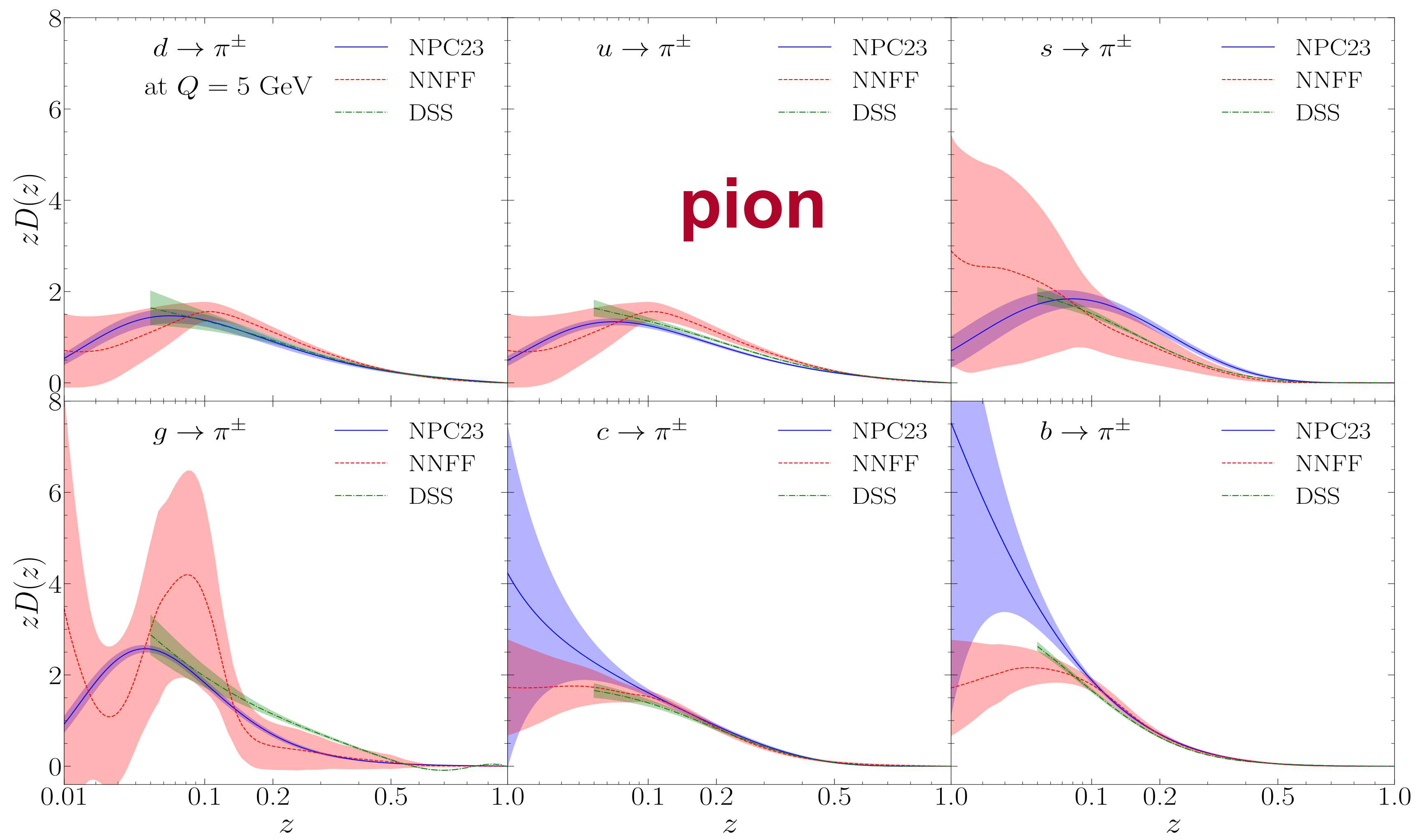
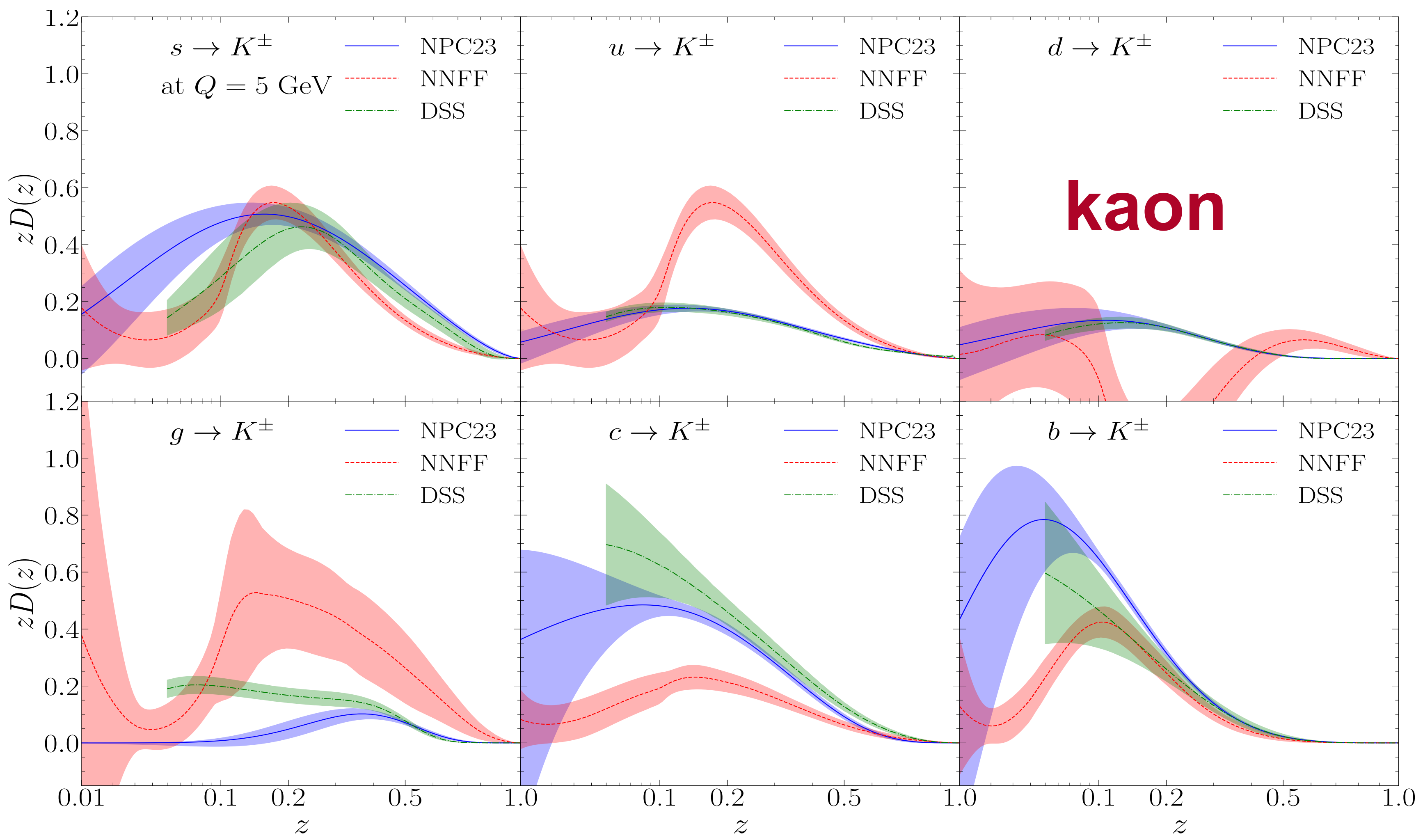
Thank you for your attention!

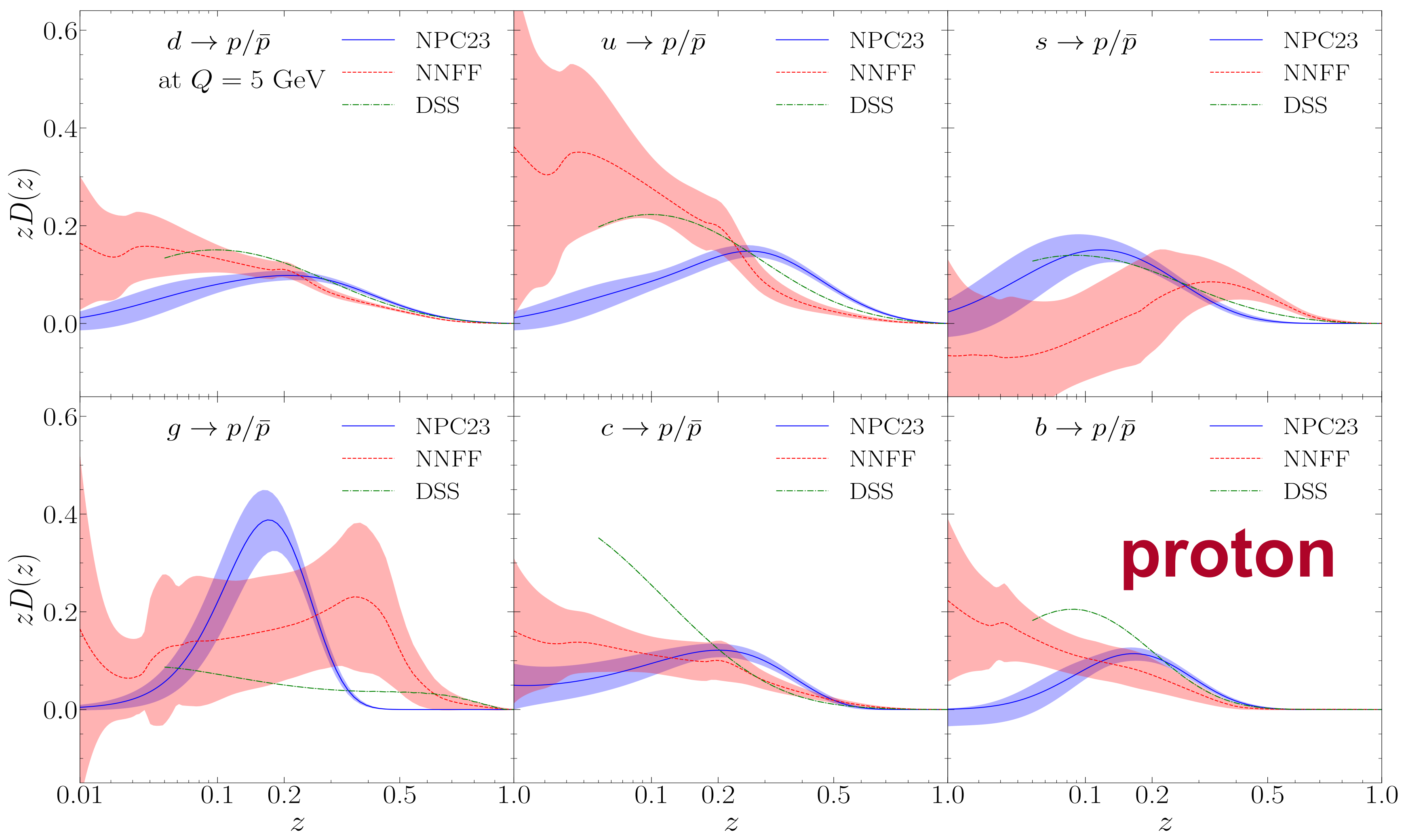
Comparison to other determinations

- Our new extractions on FFs are compared to previous determinations from other groups (e.g., DSS and NNFF) for the charge-summed pion, kaon and proton; discrepancies are found and further investigations will be needed

FFs (charge-summed) vs. momentum fraction **[BDSSV21, DSS17, DSS07, NNFF1.0]**







- We find general agreement between ours with DSS for FFs of u and d quarks to pion, and of u, d and s quark to kaon
- however, discrepancies are found for FFs to protons and for FFs of gluon to all three charged hadrons
- NNFF1.0 show larger uncertainties in general since only SIA data were used
- future benchmark works involving different groups will be needed for investigation on discrepancies

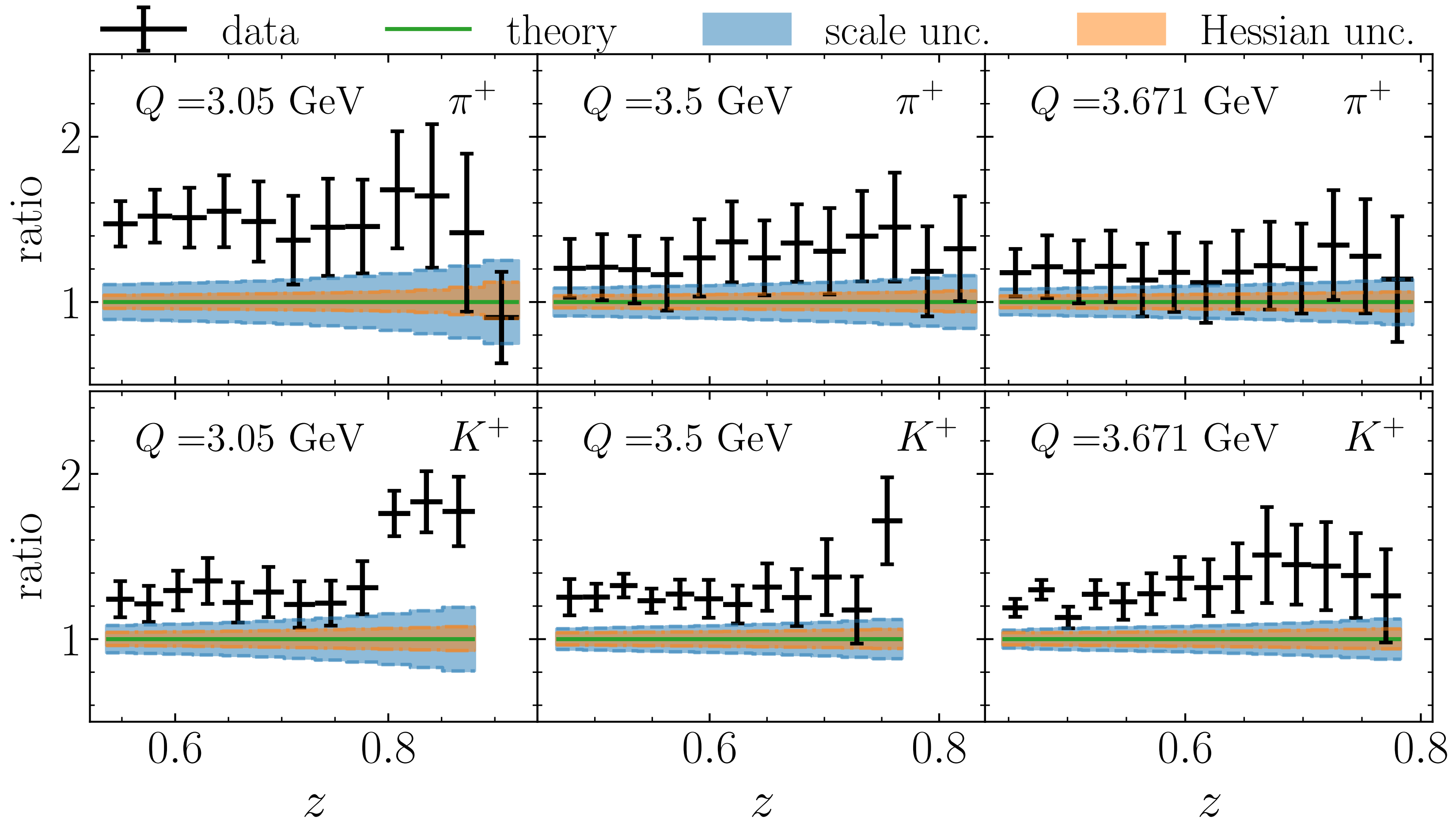
NPC23 analysis at NNLO

- Satisfactory agreements between NNLO theory and data are found for both the SIA and SIDIS data with Q~2-3 GeV, though the two seem to pull the FFs in different directions; note exp. uncertainties are mostly correlated systematic uncertainties

BESIII charged pion/kaon **[2502.16084]**

$\sqrt{s}$ (GeV)	$\mathcal{L}$ (pb^{-1})	$N_{\rm had}^{\rm tot}$	$N_{\rm bkg}$
2.0000	10.074	350298 ± 592	8722 ± 94
2.2000	13.699	445019 ± 668	10737 ± 104
2.3960	66.869	1869906 ± 1368	47550 ± 219
2.6444	33.722	817528 ± 905	21042 ± 146
2.9000	105.253	2197328 ± 1483	56841 ± 239
3.0500	14.893	283822 ± 533	7719 ± 88
3.5000	3.633	62670 ± 251	1691 ± 42
3.6710	4.628	75253 ± 275	6461 ± 81

theory vs data for the NNLO fit (BESIII)



theory vs data for the NNLO fit (COMPASS06)