Outline

- **General discussion**

  -SM Higgs mechanism -Custodial symmetry

  -Hierarchy problem -Higgs and BAU

  -Higgs and dark matter -Higgs and neutrino

- **BSM Higgs – EFT**

  -Intro to EFT -Hierarchy Problem revisit

  -SMEFT -HEFT

- **BSM Higgs – UV**

  -Reals Singlet Model

  -2HDM

General Discussion

Review of SM Higgs

- **Standard Model Lagrangian involving Higgs**

$$\mathcal{L}_{\text{Yukawa}} = -Y^e \bar{L}_L H e_R - Y^u \bar{Q}_L \tilde{H} u_R - Y^d \bar{Q}_L H d_R + h.c.,$$
$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D^\mu H) - V(H),$$
$$V(H) = -\mu^2 (H^\dagger H) + \frac{\lambda}{4}(H^\dagger H)^2,$$

- **For $\mu^2>0$ the ground state is not spontaneously break**

$SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$

$$\langle H^\dagger H \rangle = v^2/2 \qquad v = 2\sqrt{\frac{\mu^2}{\lambda}}$$

- **In the unitary gauge** $V(H) \supset \frac{\lambda v^2}{4} h^2 + \frac{\lambda v}{4} h^2 + \frac{\lambda}{4} h^4$

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

- **Higgs self-interaction**

$$V(H) \supset \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2v} h^3 + \frac{\lambda}{4} h^4 \qquad m_h^2 = \frac{\lambda v^2}{2}$$

- **Higgs-Gauge bosons interaction**

$$|D_\mu H|^2 \supset \frac{1}{2} m_Z^2 Z^\mu Z_\mu + m_W^2 W_\mu^+ W^{-\mu}$$

$$\frac{2m_W^2}{v}\left(hW_\mu^+ W^{-\mu} + \frac{1}{2v} h^2 W_\mu^+ W^{-\mu}\right)$$

$$\frac{m_Z^2}{v}\left(hZ_\mu Z^\mu + \frac{1}{2v} h^2 Z_\mu Z^\mu\right)$$

$$m_W = gv/2$$

$$m_Z = \frac{m_W^2}{\cos^2\theta_W}$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$$

- **Higgs-Fermion interactions**

$$\mathcal{L}_{\text{Yukawa}} = -m_f \bar{f} f - \frac{m_f}{v} h \bar{f} f \qquad m_f = \frac{y_f v}{\sqrt{2}}$$

Custodial Symmetry and Electroweak Precisoin Test

- **Global Symmetry of Higgs Potential and vev**

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \qquad \Phi = (\tilde{H}, H) = \begin{pmatrix} H^{0*} & H^+ \\ -H^- & H^0 \end{pmatrix}$$

- **The Higgs potential in terms of Φ**

$$V(\Phi) = \frac{\mu^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\lambda}{4}\left[\mathrm{Tr}(\Phi^\dagger\Phi)\right]$$

Invariant under $\Phi \to L\Phi R^\dagger$, where $L \in SU(2)_L$ and $R \in SU(2)_R$

- **Its vev gives**

$$\langle\Phi\rangle = \begin{pmatrix} v/\sqrt{2} & 0 \\ 0 & v/\sqrt{2} \end{pmatrix}$$

which is invariant under $L=R$, that is $SU(2)_V$,

This global symmetry is called Custodial Symmetry

- **Custodial Symmetry is not exact in the SM**

  - **Breaking Source 1:** $U(1)_Y$ **gauge interaction**

$$D_\mu \Phi = \partial_\mu \Phi - igW_\mu \Phi + ig' B_\mu \Phi T^3, \quad W_\mu = W_\mu^a T^a$$

$$\partial_\mu L \Phi R^\dagger \to L \partial_\mu \Phi R^\dagger \quad ✓$$

$$W_\mu L \Phi R^\dagger = L W'_\mu \Phi R^\dagger \qquad W'_\mu = L^\dagger W_\mu L \to \text{a gauge transformation} \quad ✓$$

$$B_\mu L \Phi R^\dagger T^3 = L B_\mu \Phi T^3 R^\dagger + B_\mu L \Phi [R^\dagger, T^3] \quad ✗$$

$$D_\mu \Phi \to L D_\mu \Phi R^\dagger \text{ if } g' = 0$$

  - **Breaking Source 2: Yukawa**

$$Q_R \equiv \begin{pmatrix} u_R \\ d_R \end{pmatrix} \qquad \mathcal{L}_{\text{Yukawa}} = -\overline{Q}_L \Phi \mathcal{Y} Q_R + h.c. \qquad \mathcal{Y} = \text{diag}(Y^u, Y^d)$$

invariant under $SU(2)_L \times SU(2)_R$ **if** $Y^u = Y^d$ **, and** $Q_R \to R\, Q_R$ **,** $Q_L \to L\, Q_L$

Custodial Symmetry and Electroweak Precisoin Test

- **The consequence of Custodial Symmetry**

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \text{ (tree level)}$$

- **Coming from the fact that** (W^1, W^2, W^3) **forming triplet of** $SU(2)_V$

  Therefore they have the same mass (if $g'=0$ **)**

$$\mathcal{L}_W = \frac{g^2 v^2}{4}\left[\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2 + \left(W_\mu^3\right)^2\right]$$

- $g' \neq 0$ **mix the** W_3 **and** B **at tree level:**

$$Z_\mu = \cos\theta_W\, W_\mu^3 - \sin\theta_W\, B_\mu$$

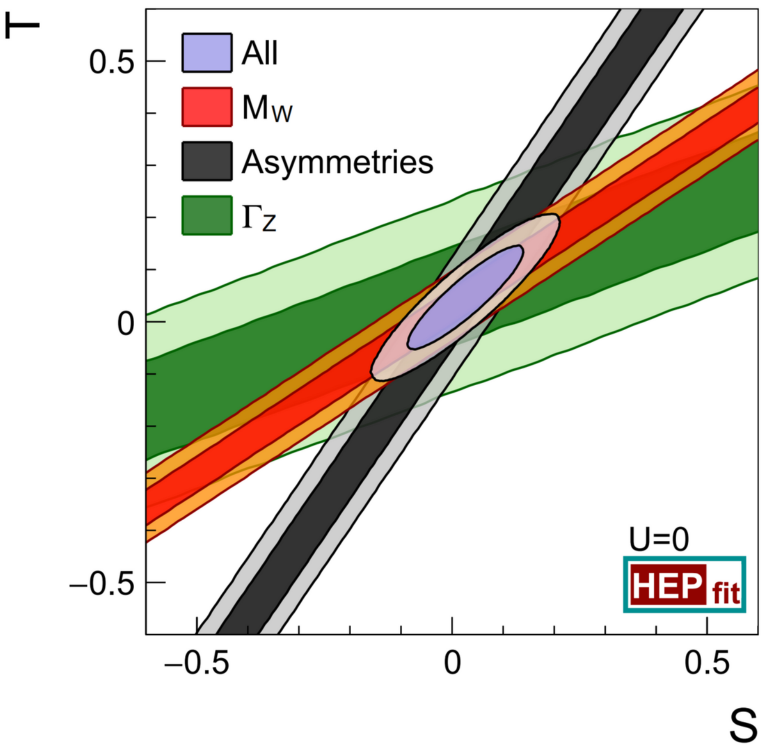
- **Heuristically,** $m_Z \cos\theta_W$ **project out the mass of** W^3 **in** Z

- **Breaking is measured by the deviation** ρ **(*T*) from 1**

$$\Delta\rho^{SM} = \rho - 1 = \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2} \qquad \rho \to \rho_0 - \Delta\rho^{SM}$$

Custodial Symmetry and Electroweak Precisoin Test

- **Strong Constraint from Electroweak Precison Observable**



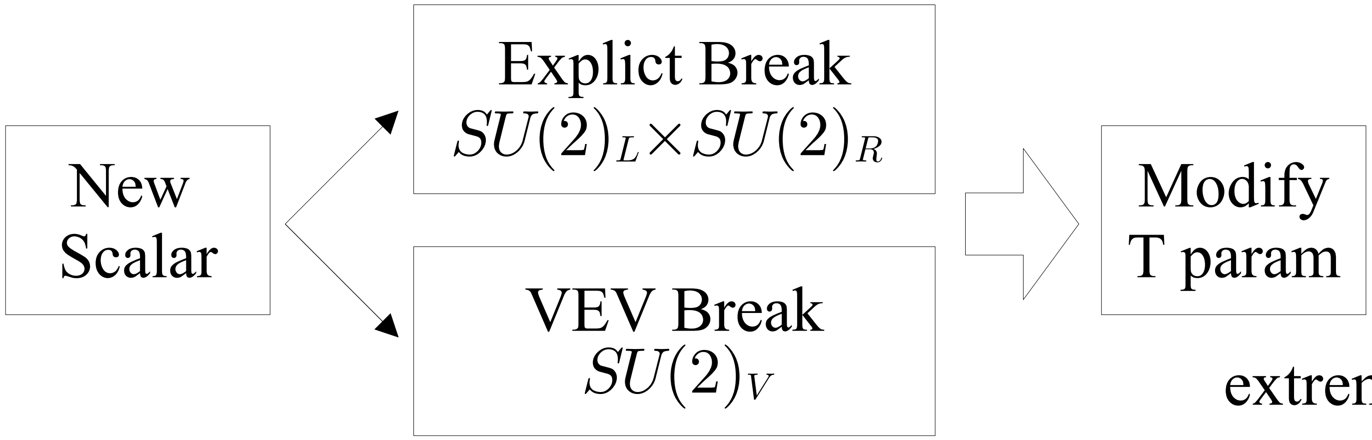
$$T = \frac{\rho - 1}{\alpha_e}$$ Measure Custodial symmetry breaking

$$S \equiv \frac{4c^2 s^2}{\alpha_e}\left[\frac{\Pi^{new}_{ZZ}(m_Z^2)}{m_Z^2} - \frac{c^2 - s^2}{cs}\frac{\Pi^{new}_{Z\gamma}(m_Z^2)}{m_Z^2} - \frac{\Pi^{new}(m_Z^2)}{m_Z^2}\right]$$

Measure new generation of fermions

U is usually less constraint as it's a dim-8 effect in decoupled scenario

	Result	Correlation		Result	Correlation		
S	0.026 ± 0.075	1.00		0.021 ± 0.096	1.00		
T	0.047 ± 0.066	0.90	1.00	0.04 ± 0.12	0.91	1.00	
U	0	–	–	0.008 ± 0.092	−0.62	−0.83	1.00



extremely constraint

- **A Real Triplet example**

$$\xi^a \sim \mathbf{3}_0 \qquad \Xi = \frac{1}{2}\sigma^a \xi^a \qquad \Xi \to L\Xi L^\dagger,\ L \in SU(2)_L$$

$$\Xi \to \Xi,\ R \in SU(2)_R$$

- **SM gauge symmetry allow the following interaction**

$$\kappa H^\dagger \sigma^a H \xi^a \sim \kappa \mathrm{Tr}(\Phi^\dagger \Xi \Phi \sigma^3)$$ Not invariant under $SU(2)_R$

- **It may generate vev for the ξ^3**

$$\langle \xi^3 \rangle \equiv v_\xi \propto \kappa$$

- **The Kinetic term contribution to the mass of gauge boson**

$$\mathcal{L}_{\mathrm{kin}}^{\xi} = \frac{1}{2}(D_\mu \xi^a)(D^\mu \xi^a) \supset \frac{g^2 v_\xi^2}{2}(W_1^2 + W_2^2)$$

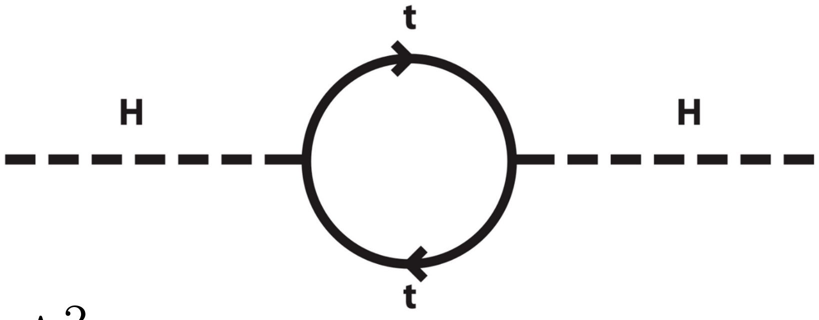
$$m_{W_1} = m_{W_2} \neq m_{W_3} \qquad \langle \xi^a \rangle = (0, 0, v_\xi)$$

$$\rho = 1 + \frac{4v_\xi^2}{v^2}$$

$$\rho = \frac{\sum_{i=1}^{n}\left[I_i(I_i+1) - \frac{1}{4}Y_i^2\right]v_i}{\sum_{i=1}^{n}\frac{1}{2}Y_i^2 v_i}$$

- **Scalar mass are not protected by symmetry**

  With the cut-off regulator



$$\delta m_H^2 \sim -iy_t^2 \int \frac{d^4\ell}{(2\pi)^4} \frac{1}{\ell^2 - m_t^2}$$

$$= -\frac{y_t^2}{8\pi^2} \int_0^{\Lambda_{\rm UV}} d\ell \frac{\ell^3}{\ell^2 + m_t^2} = -\frac{y_t^2}{16\pi^2} \Lambda_{\rm UV}^2 + \cdots$$

$$\Lambda_{\rm UV} \sim M_{\rm SUSY},\ M_{\rm GUT},\ M_{\rm pl}$$

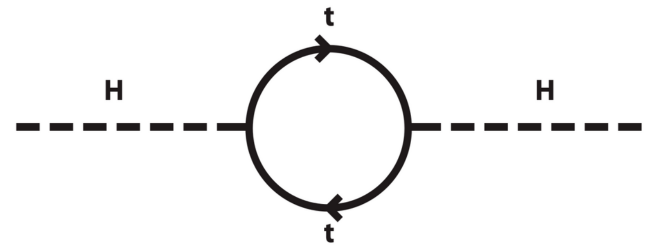
- **Fine cancellation needed to have a small Higgs mass**

  Aesthetically, we don't like fine tunning→Conjectured TeV scale physics

Hierarchy Problem

- **Solution to the Hierarchy problem**

- **Symmetry based solutions**



  - **Supersymmetry** [hep-ph/9709356]

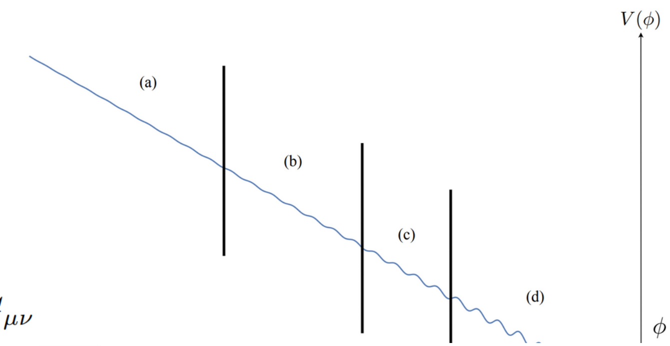
  - **Shift symmetry (Higgs as PNGB)**

    -Composite Higgs model [1506.01961]

    -Twin Higgs model [hep-ph/0506256]

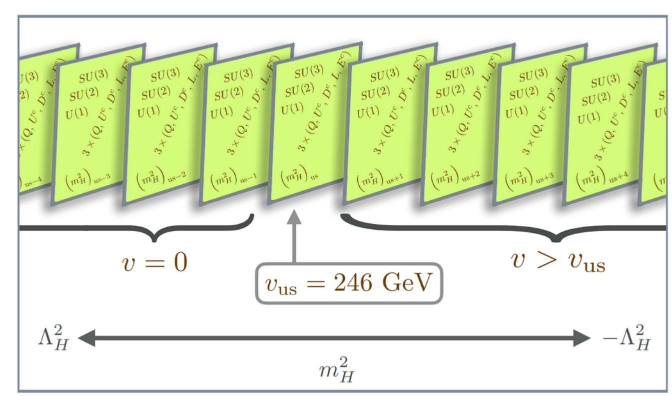
    -Little Higgs Model [hep-ph/0206021]

- **Cosmological solutions**



  - **Relaxion** [1504.07551] $(-M^2+g\phi)|h|^2+V(g\phi)+\frac{1}{32\pi^2}\frac{\phi}{f}\tilde{G}^{\mu\nu}G_{\mu\nu}$

  - **N-natrualness** [1607.06821] $\Gamma_{m_H^2<0}\sim\frac{1}{m_{h_i}^2},\ \Gamma_{m_H^2>0}\sim\frac{1}{m_{H_i}^4}.$



- **Non-decoupling UV-IR mixing** [1909.01365]

Higgs and Baryon asymmetric Universe

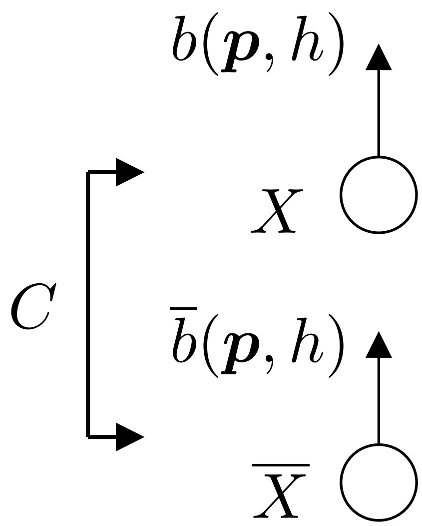
- **Our universe have a residual baryon asymmetry**

$$\eta = \frac{n_B - n_{\overline{B}}}{n_\gamma} \approx 6 \times 10^{-10} \ \text{(CMB)}$$

- **Successful Baryogenesis requires Sakharov Conditions:**

-B Violation

-C and CP violation



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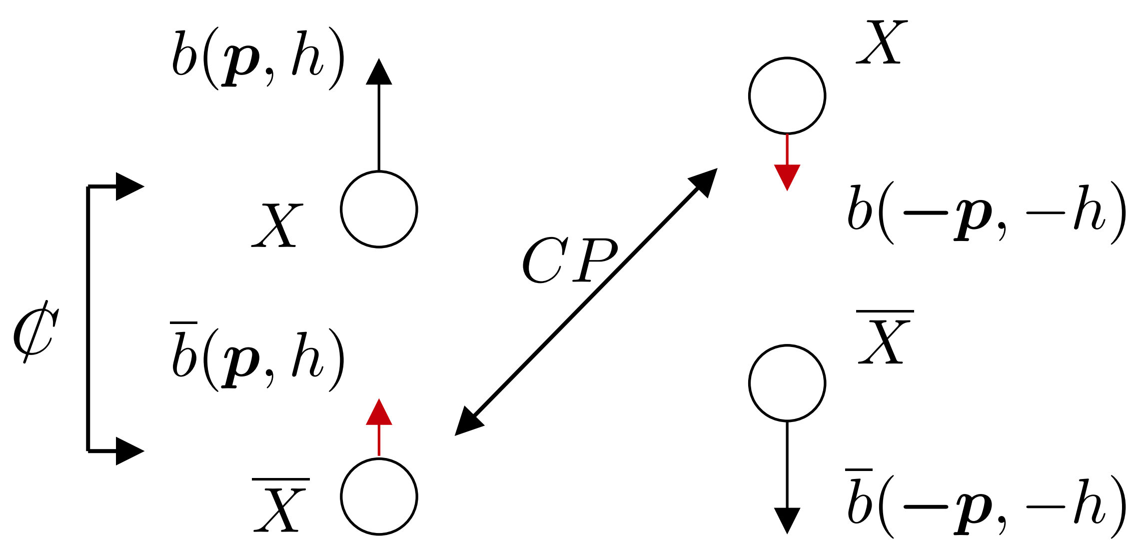


-Departure from Thermal equilibrium or CPT violation

$$\Gamma(X \to b) \neq \Gamma(b \to X)$$

- **One testable Baryogenesis mechanism—Electroweak Baryogenesis**

  \- B Violation through EW sphaleron effect in the SM

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = \frac{3g^2}{16\pi^2} W^a_{\mu\nu} W^{a\mu\nu}$$

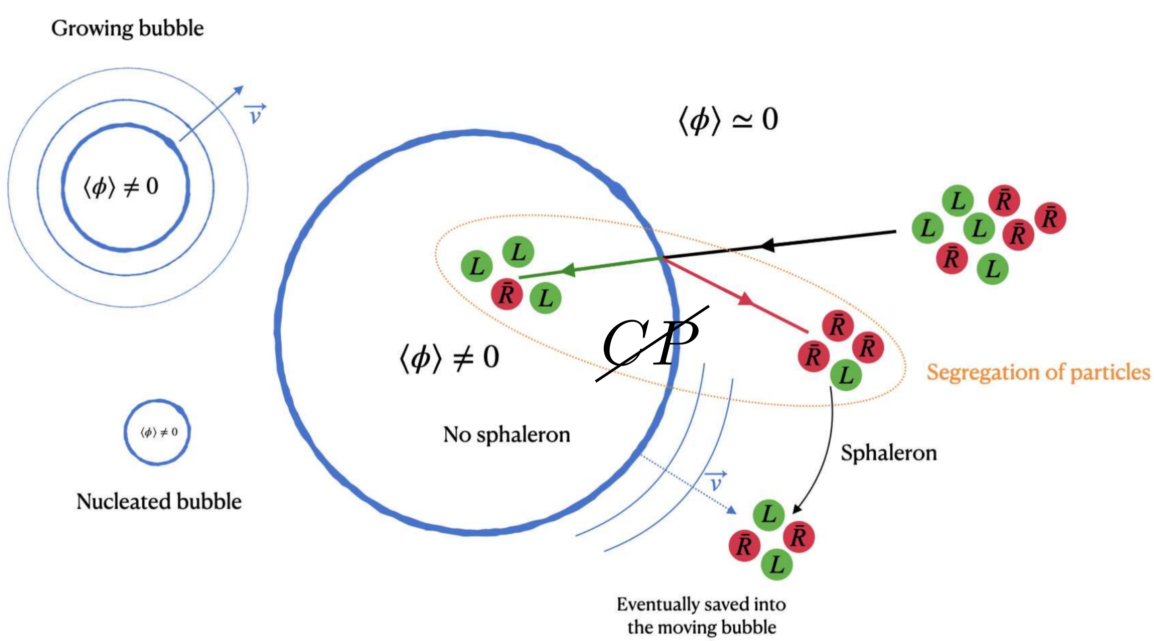
$$\bar{u} + \bar{d} + \bar{c} \longrightarrow d + 2s + 2b + t + \nu_e + \nu_\mu + \nu_\tau .$$

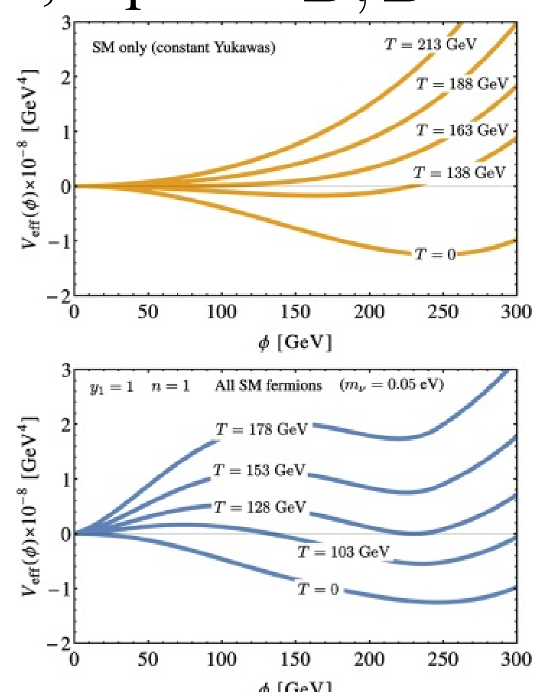
  \- Strong first-order EWPT

  fist-order: bubble nucleate and expand

  strong: suppress the sphaleron process in the broken phase

  \- CP Violation scattering with bubble wall during EWPT, separate $\overline{B}, B$





Higgs and Baryon asymmetric Universe

- **One testable Baryogenesis mechanism—Electroweak Baryogenesis**
  - B Violation through EW sphaleron effect in the SM ✓

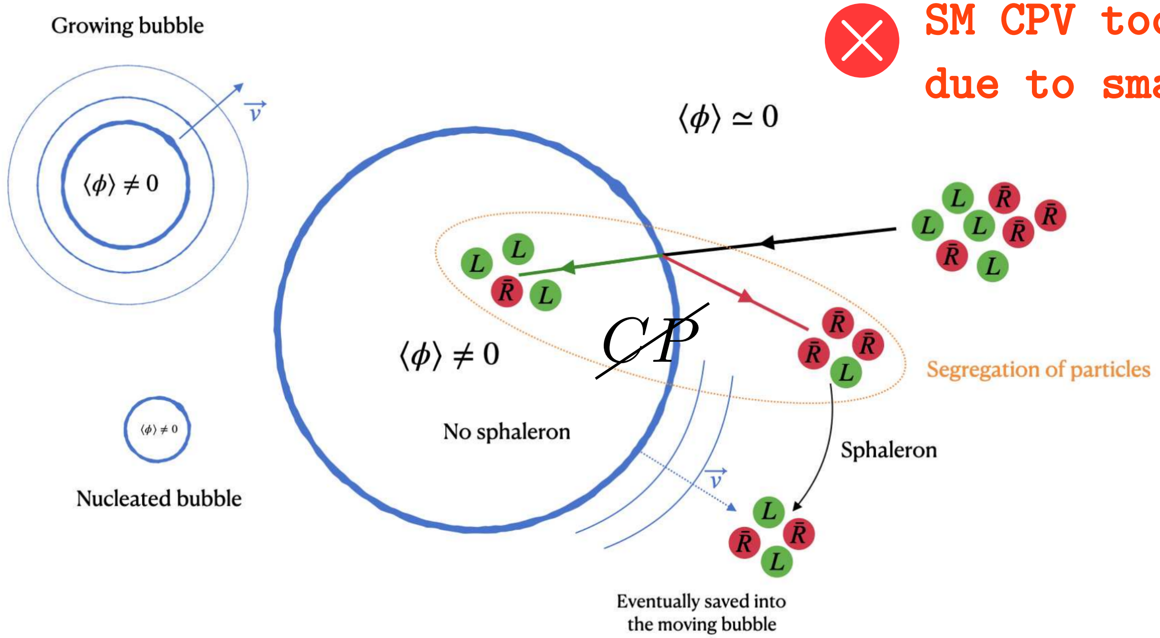
$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = \frac{3g^2}{16\pi^2} W^a_{\mu\nu} W^{a\mu\nu}$$

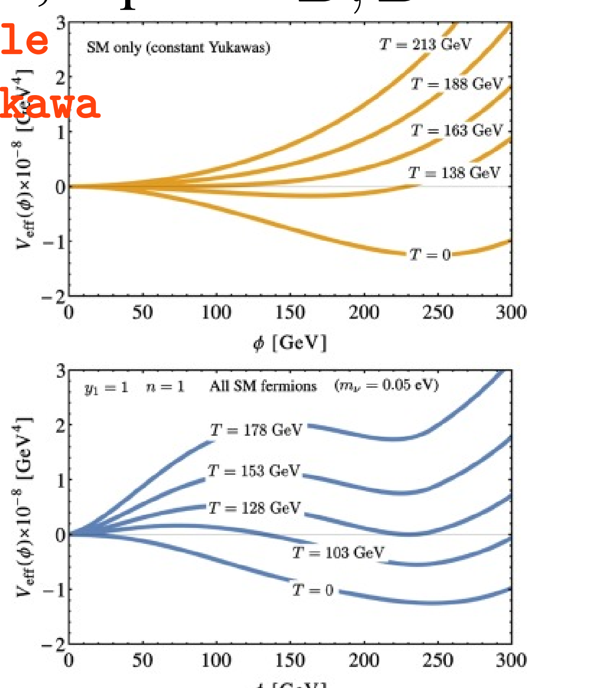
$$\bar{u} + \bar{d} + \bar{c} \longrightarrow d + 2s + 2b + t + \nu_e + \nu_\mu + \nu_\tau .$$

  - Strong first-order EWPT ✗ SM EWPT cross-over

    fist-order: bubble nucleate and expand

    strong: suppress the sphaleron process in the broken phase
  - CP Violation scattering with bubble wall during EWPT, separate $\overline{B}, B$ ✗ SM CPV too feeble due to small Yukawa





Higgs and Baryon asymmetric Universe

- **One testable Baryogenesis mechanism—Electroweak Baryogenesis**

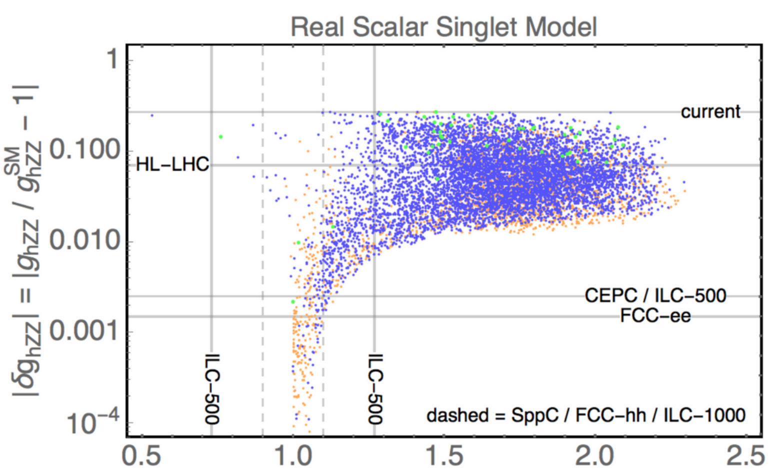
  - Why testable? New physics are conjectured to be around TeV scale

- New scalar are introduced to modify the evolution Higgs potential

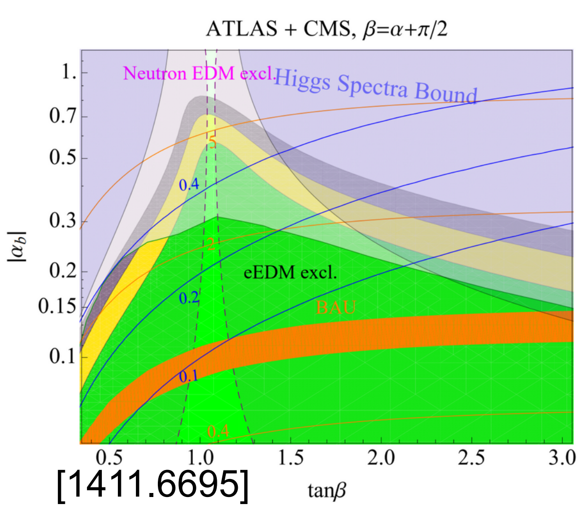
  - can be test at future collider [1912.07189]

  - new CP violation source can be test with EDM experiments [1710.02504]

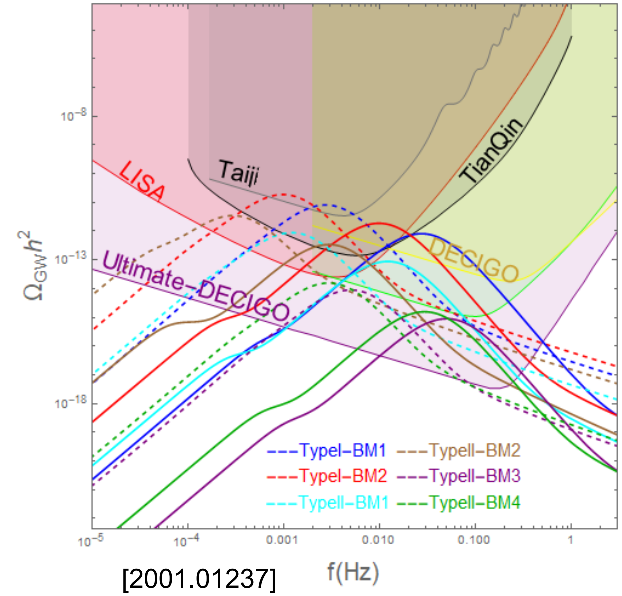
  - Bubble collision can produce primordial stochastic gravitational-wave with peak frequency~0.1-10 mHZ, can be test with space-based experiments [1705.01783]



[1608.06619]



[1411.6695]



[2001.01237]

- **Higgs portal dark-matter**

  Dark matter interact with SM through Higgs particle

  $$\bar{\chi}\chi H^\dagger H \qquad S^2 H^\dagger H$$

  Dark matter may get mass from the same Higgs mechanism

  -Higgs invisible decay $h \to \chi\chi$ [2301.10731]

  -Can be linked to EWPT [2307.02187]

- **Electroweak Multiplet dark matter**

  Accidental Z2 symmetry at renormalizable level [1812.07892]

  If $\Phi \simeq (1, 2j, Y)$ with j≥ 2 $\quad H^\dagger H \Phi^\dagger \Phi, \quad H^\dagger \sigma^a H \Phi^\dagger T^a \Phi$

  The neutral component Φ^0 usually is lightest, serves as a DM candidate

- **Neutrino Mass and Seesaw Mechanism**
- **Type-I seesaw** $y_\nu \overline{L}\tilde{H}N_R + \frac{M_R^2}{2}\overline{N_R^c}N_R$ $\begin{pmatrix} 0 & y_\nu/\sqrt{2} \\ y_\nu/\sqrt{2} & M_R \end{pmatrix}$ $m_\nu \sim \frac{y^2v^2}{2M_R}$

  for a O(1) y_ν **,** $M_R \simeq 10^{14} - 10^{15}\ GeV$**.**

- **Type-II seesaw, adding a complex SU(2) triplet**

$$\mathcal{L} \supset y_{ij}\, L_i^T C\, i\sigma_2\, \Delta\, L_j + \mu H^T i\sigma_1 \Delta^\dagger H$$

$$(m_\nu)_{ij} = y_{ij} v_\Delta \simeq y_{ij}\frac{\mu v^2}{\sqrt{2}M_\Delta^2}. \qquad v_\Delta \simeq \frac{\mu v^2}{\sqrt{2}M_\Delta^2}$$

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

Two smallness

- small μ **is technical natural as it breaks the lepton number**

- v/M_Δ **suppressed by heavy triplet mass**

- $y \simeq O(1), \mu \simeq O(eV)$ **results in** $M_\Delta \simeq O(TeV)$

rich collider phenomena

$$pp \to \Delta^{++}\Delta^{--},\ \Delta^{++}\Delta^{-},\ \Delta^{+}\Delta^{-},\ \Delta^0 h$$

see [1903.02493] for review its phenomenology

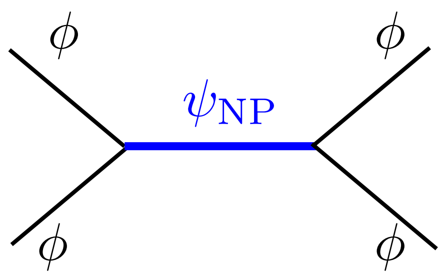
Effective Field Theory

A snapshot of EFT

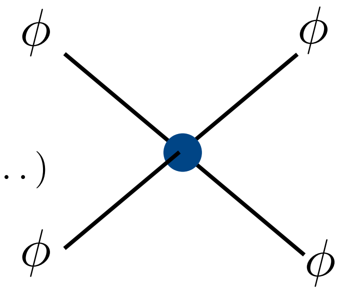
$$\mathcal{L}_{\mathrm{NP}}(\phi_{\mathrm{SM}}(x), \psi_{\mathrm{NP}}(x); g_i, y_i, \tilde{g}_i, \tilde{y}_i, \Lambda, \dots)$$

$$e^{i\int d^4x \mathcal{L}_{\mathrm{SMEFT}}} = \int \mathcal{D}[\psi] e^{i\int d^4x \mathcal{L}_{\mathrm{NP}}}$$

$E \ll \Lambda$



$$\frac{\tilde{g}^2}{p^2 - \Lambda^2} \sim -\frac{\tilde{g}^2}{\Lambda^2}\left(1 + \frac{p^2}{\Lambda^2} + \dots\right)$$



$+ \dots$

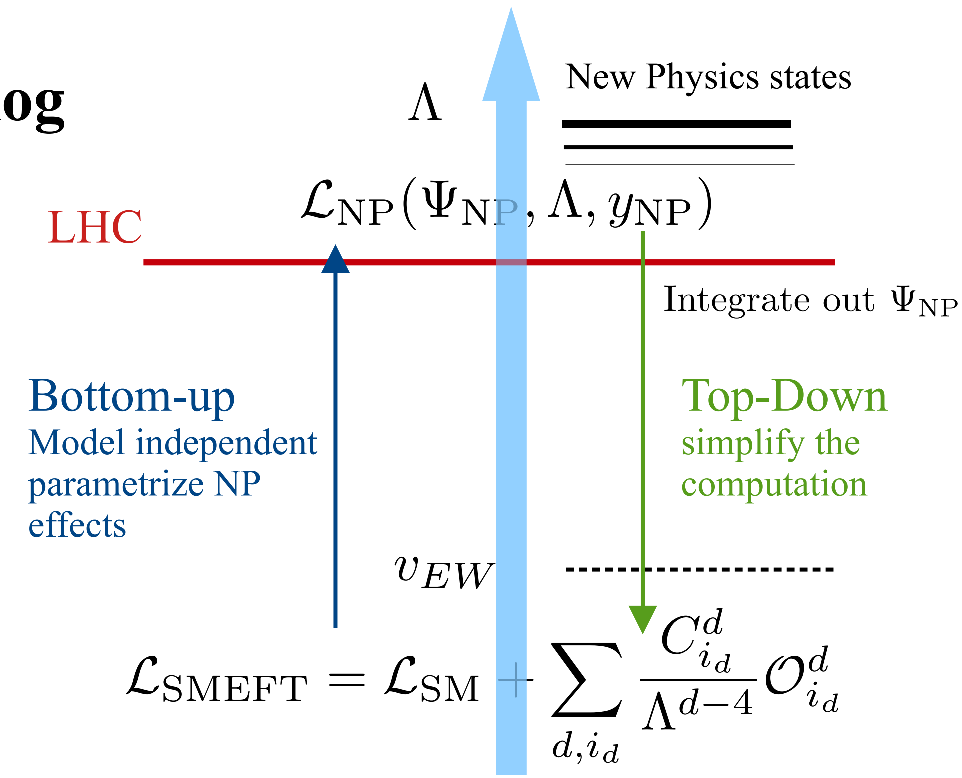
$\mathcal{A}_{NP}$ Non-Local amplitude

$\mathcal{A}_{SMEFT}$ Infinite series of local amplitude

$$\mathcal{L}_{\mathrm{SMEFT}}(\phi_{\mathrm{SM}}, \dots) = \mathcal{L}_{\mathrm{SM}}^{d\leq 4} + \frac{c_i^5}{\Lambda} O_i^{d=5} + \frac{c_i^6}{\Lambda^2} O_i^{d=6} + \frac{c_i^7}{\Lambda^3} O_i^{d=7} + \dots + \frac{c_i^n}{\Lambda^{n-4}} O_i^{d=n} + \dots$$

$\left(\frac{E}{\Lambda}\right)^{d-4}$ suppressed

- **Why we need EFT**
  - **No new physics has been observed at collider**
  - **NP scale might be high and decouple**
  - **Bottom-Up: EFT is universal**
  - **Top-Down: EFT helps resum large log**



$$(\overline{Q_L}\gamma^\mu Q_L)(e_R\gamma^\mu e_R)$$

$$H^\dagger \tau^k H W^{i\mu\nu} B_{i\mu\nu}$$

Introduction to effective field theory

- **Why we need EFT**
  - **No new physics has been observed at collider**
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  - **Bottom-Up: EFT is universal**
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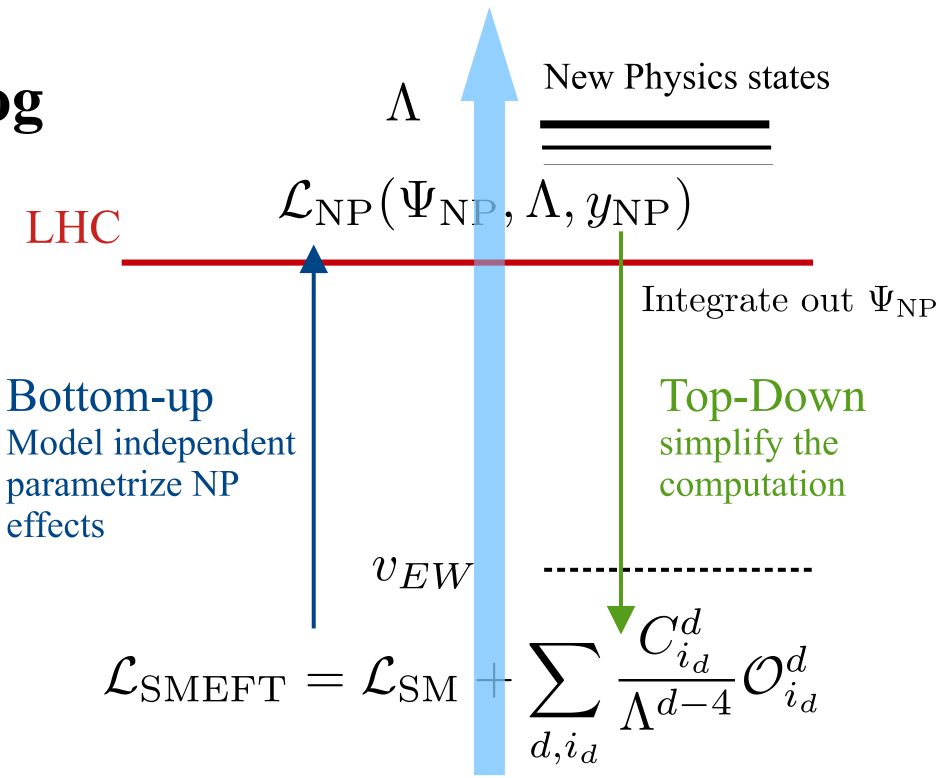
- **Principle of Using SMEFT**
  - **Power Counting: canonical dim**
  - **Symmetry** $SU(3)_C \times SU(2)_L \times U(1)_Y$



$$(\overline{Q_L}\gamma^\mu Q_L)(e_R\gamma^\mu e_R)$$

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- **Principle of Using SMEFT**
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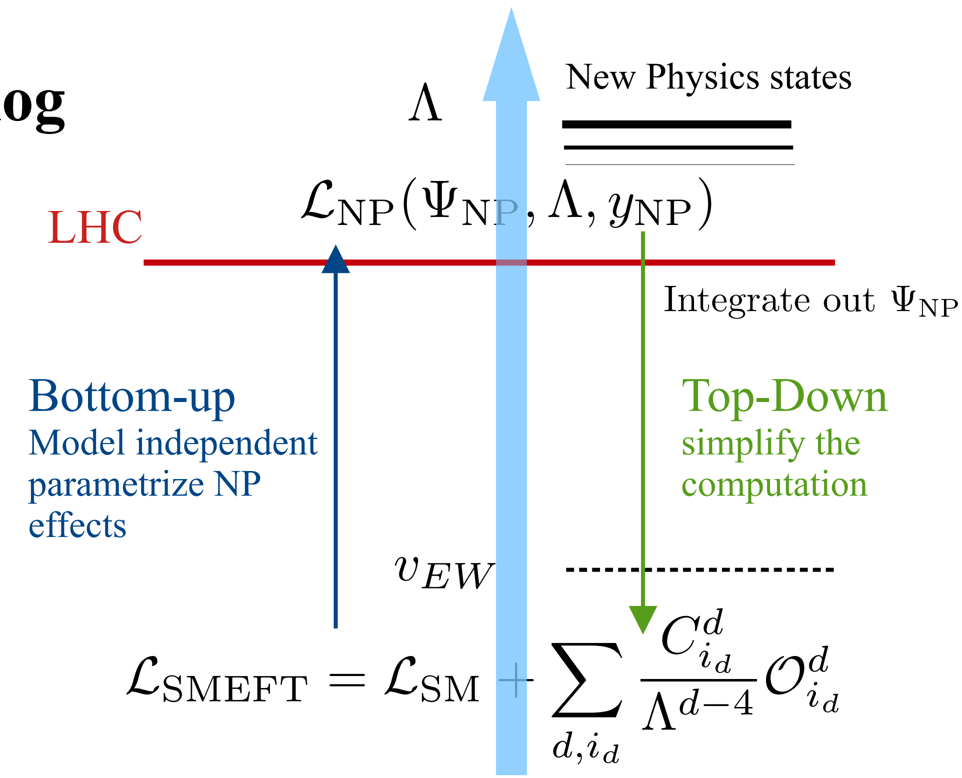
  - **Symmetry** $SU(3)_C \times SU(2)_L \times U(1)_Y$
- **How to systematically use EFT**
  - **Operator Basis construction**
  - **Matching between UV and EFT**
  - **RGE in EFT improvements**

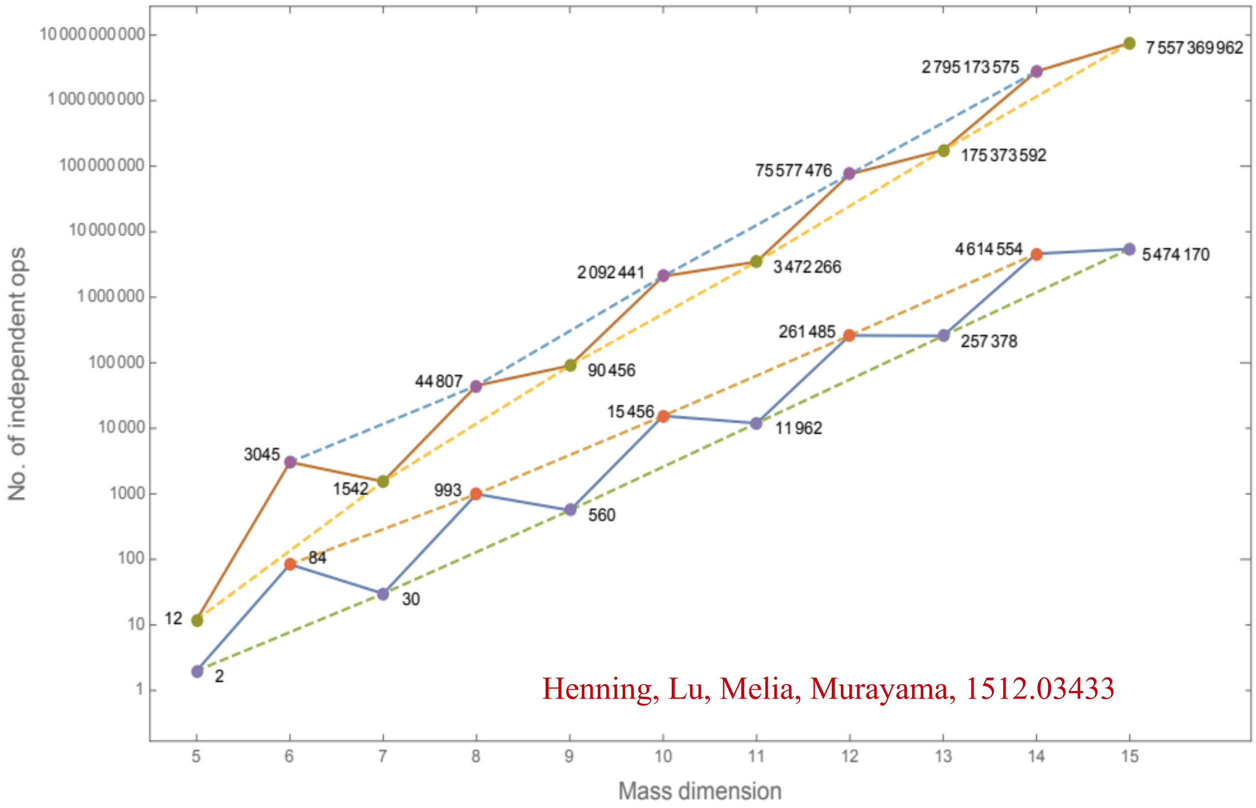


$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_{d,i_d} \frac{C^d_{i_d}}{\Lambda^{d-4}} \mathcal{O}^d_{i_d}$$

$$(\overline{Q_L}\gamma^\mu Q_L)(e_R\gamma^\mu e_R)$$

$$H^\dagger \tau^k H W^{i\mu\nu} B_{i\mu\nu}$$

- **Operator Basis construction**
- **Why need a complete and independent operator basis?**
  - **Need to find NDOF in the theory**
  - **Deriving RGE**
- **Counting by Hilbert Series technique**



- **Operator Basis construction**
- **Why need a complete and independent operator basis?**
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- **Counting by Hilbert Series technique**
- **Difficulties in basis construction**
  - **Integration by parts** $D_\mu(\phi_1\phi_2\phi_3)D^\mu\phi_4 = -\phi_1\phi_2\phi_3 D_\mu D^\mu \phi_4$
  - **Field redefinition** $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \frac{c_1}{\Lambda}\phi^3\Box\phi$

    $\Box\phi = -m^2\phi - \lambda\phi^3$ $\quad \frac{c_1}{\Lambda}\phi^3\Box\phi = -\frac{c_1}{\Lambda}\phi^3(m^2\phi + \lambda\phi^3)$ $\quad$ field redefinition: $\phi \to \phi + \frac{c_1}{\Lambda^2}\phi^3$
  - **Group identity**

    $\epsilon^{ij}\epsilon^{kl} + \epsilon^{il}\epsilon^{jk} - \epsilon^{ik}\epsilon^{jl} = 0, \ldots$
  - **Flavor relation** $O^{LLHH}_{f_1f_2} = \epsilon^{i_1j_1}\epsilon^{i_2j_2}\epsilon^{\alpha_1\alpha_2}L^{f_1}_{\alpha_1,i_1}L^{f_2}_{\alpha_2,i_2}H_{j_1}H_{j_2}$

    $O^{LLHH}_{f_1f_2} = O^{LLHH}_{f_2f_1}$

- **Traditional approach**

  1)Find an over complete set of operators:

$$\{O_1, O_2, \ldots, O_n\}$$

  2)Find an over complete set of redundancy relations:

$$C_{i1}O_1 + C_{i2}O_2 + \cdots + C_{in}O_n = 0$$

$$C = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{i1} & C_{i2} & \cdots & C_{in} \end{bmatrix}$$

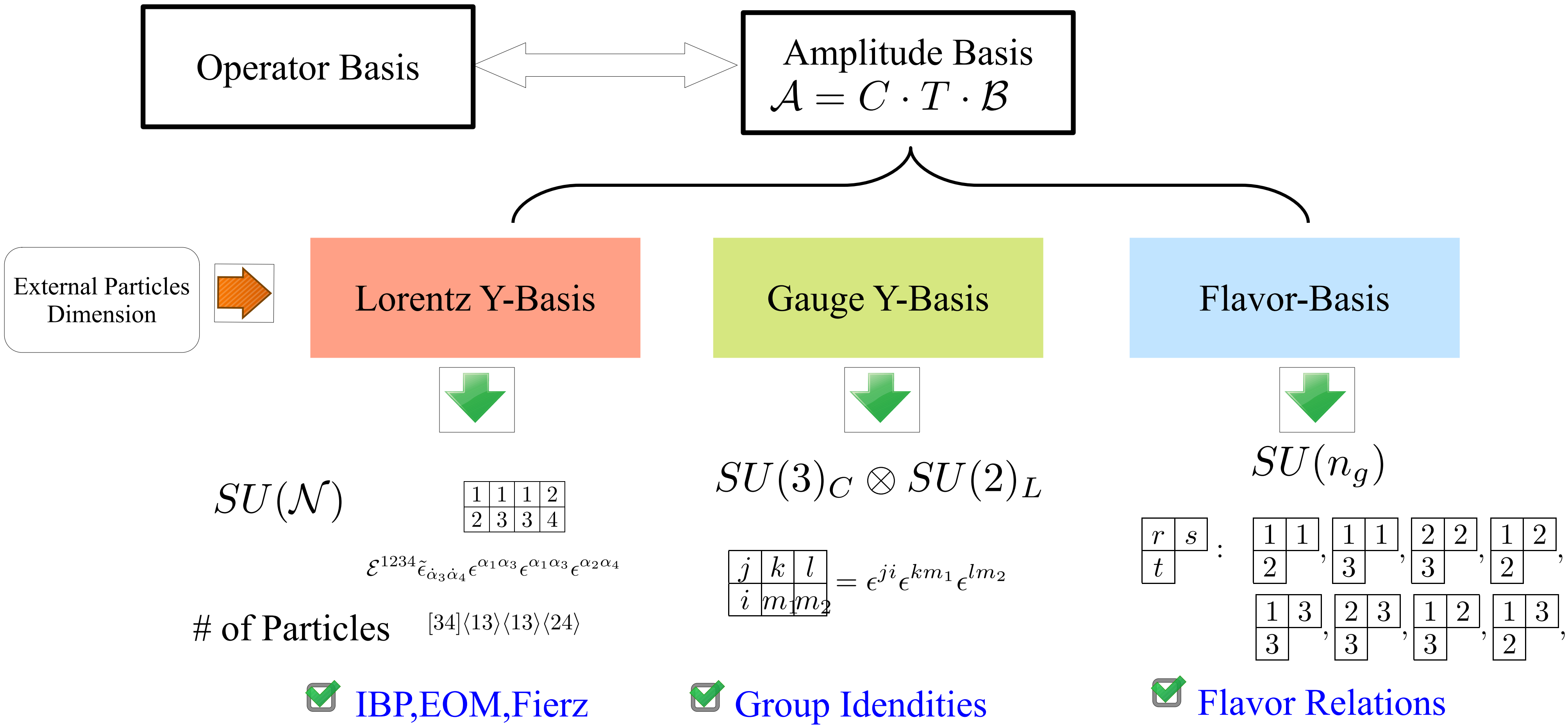
Gaussian eliminaiton

$$C' = \begin{bmatrix} C'_{11} & C'_{12} & \cdots & C'_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ C'_{k1} & C'_{k2} & \cdots & C'_{kn} \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

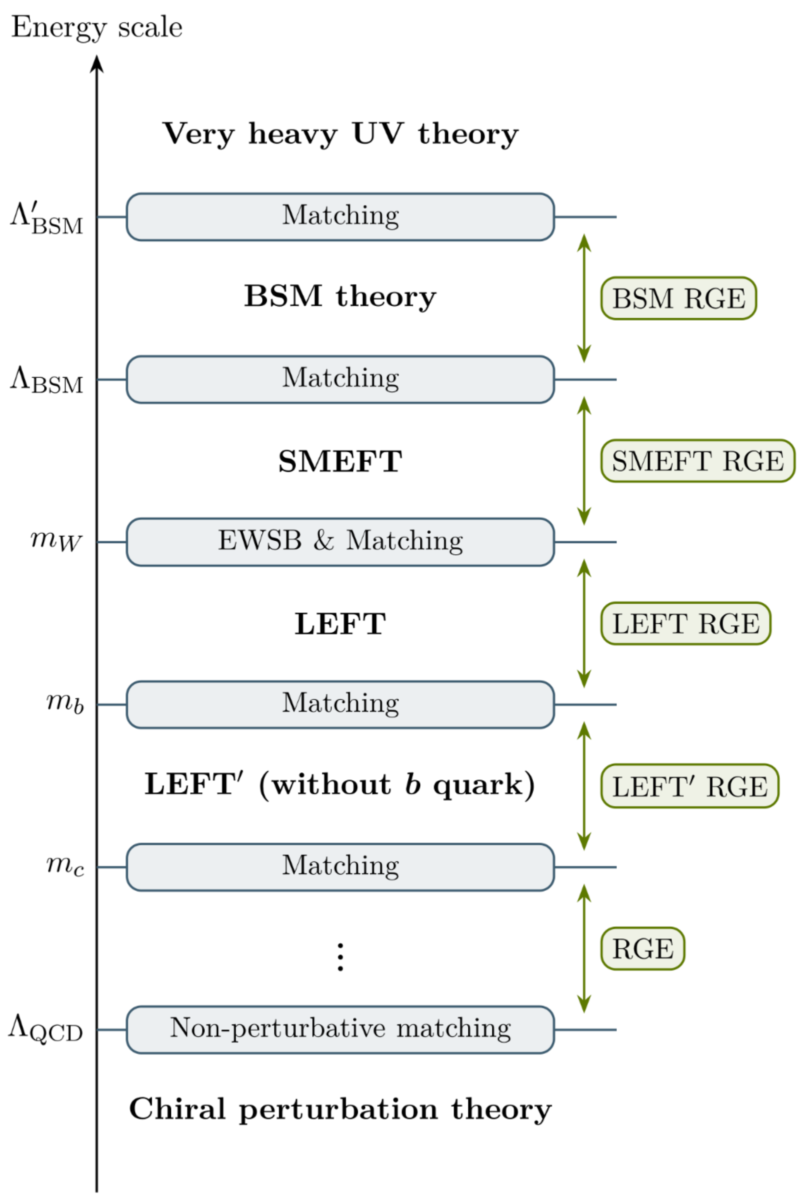
Introduction to effective field theory

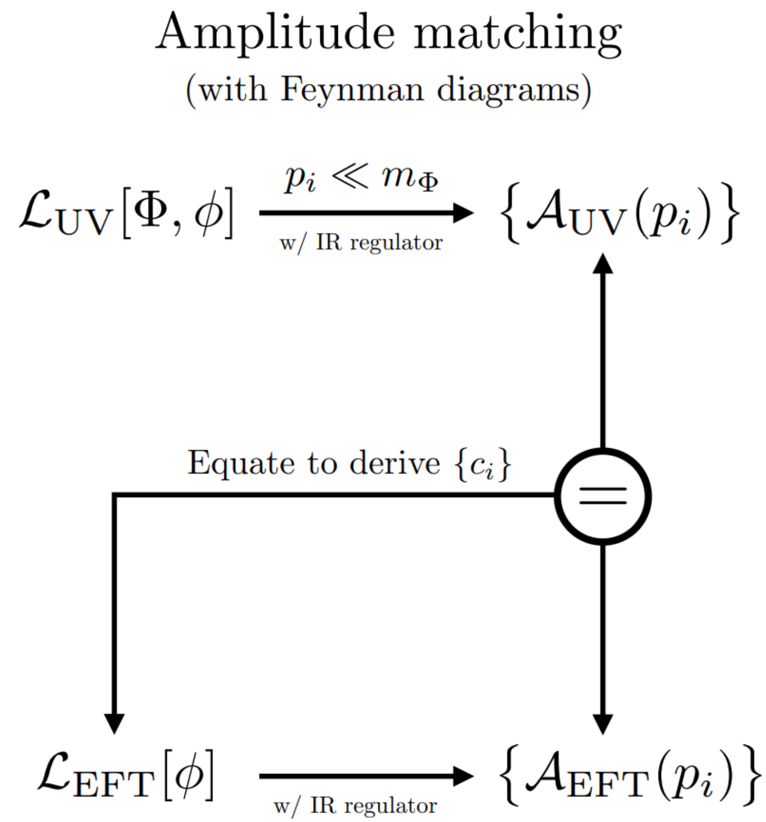
- **Young Tensor Method for basis construction**

[**HLL**, et.al. 2005.00008, 2201.04639]



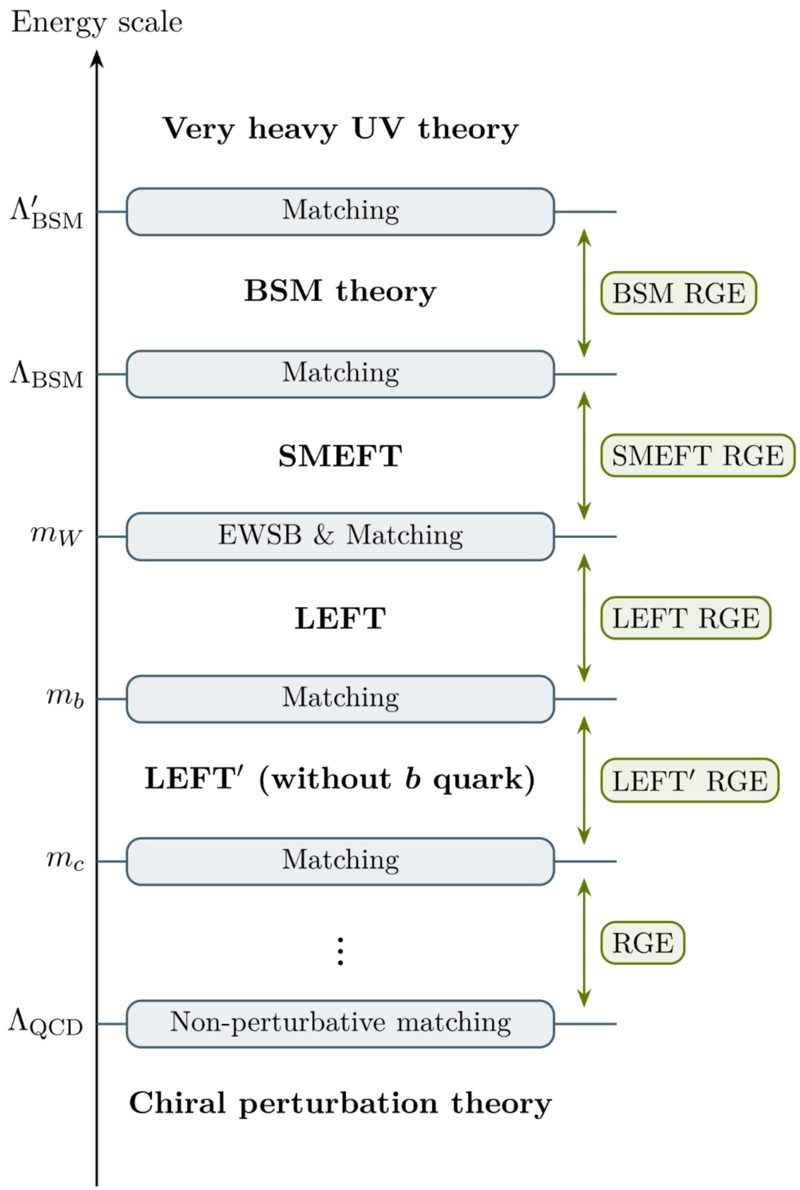
- **Matching—manually decouple heavy DoF**

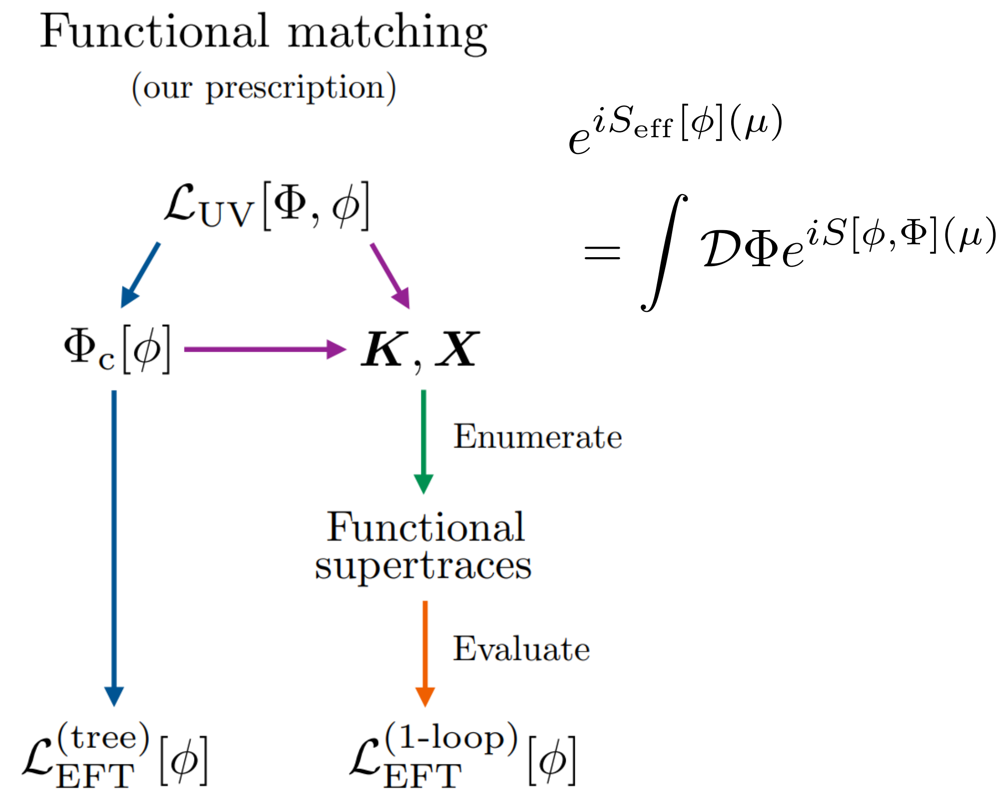




Pros:
-- physically intuitive,
-- on-shell matching free of operator conversion
Cons:
-- Can be inefficient if diagram is too many
-- Needs to know operator basis in advance
Code: Matchmakereft [2112.10787]

- **Matching—manually decouple heavy DoF**





Pros:
-- Very efficient, don't need operator basis in advance
-- Obtain directly the gauge invariant operators
Cons:
-- Must implement operator conversion
Code: `matchete` [2212.04510]

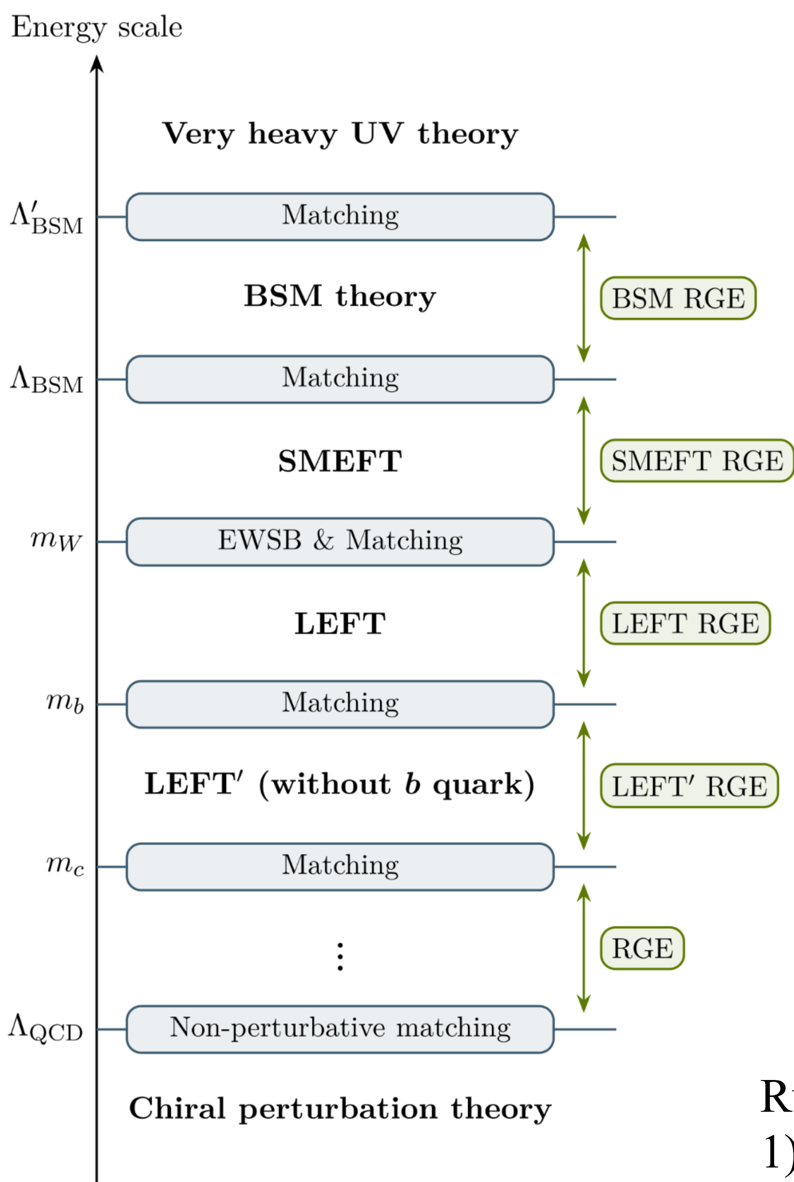
- **MatchingDB—A database to store the matching results**

```
from IPython.display import Math
Math(db.select_terms(fields=["Xi1"], output_format="latex")["ephi"])
```

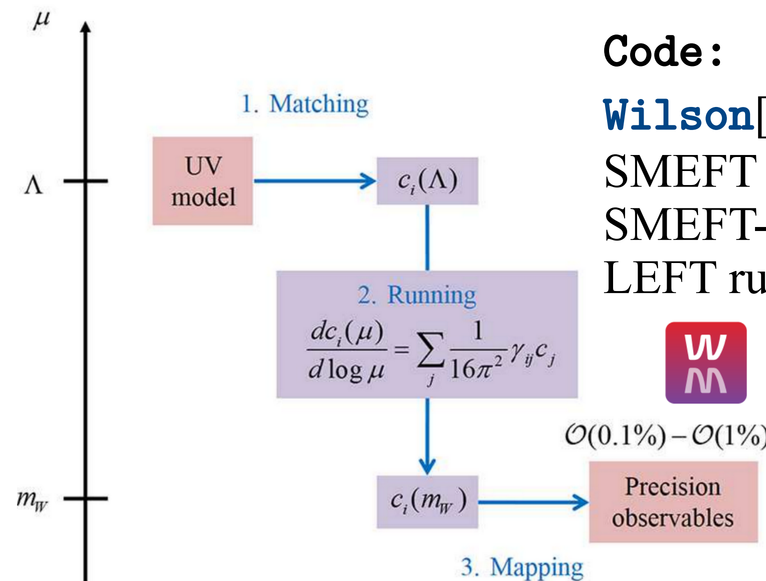
$$+\frac{(-1)i\left(\hat{y}^{e}\right)^{*}_{ci}\left(z_{\Delta_1\mathcal{L}_1}\right)^{*}_{abc}\left(\gamma_{\mathcal{L}_1}\right)^{*}_{b}\left(\lambda_{\Delta_1}\right)_{aj}}{2M^{2}_{\mathcal{L}_1,b}M_{\Delta_1,a}}+\frac{(-1)i\left(\hat{y}^{e}\right)^{*}_{ci}\left(z_{\Delta_1\mathcal{L}_1}\right)_{abj}\left(\lambda_{\Delta_1}\right)^{*}_{ac}\left(\gamma_{\mathcal{L}_1}\right)_{b}}{2M_{\Delta_1,a}M^{2}_{\mathcal{L}_1,b}}$$

https://gitlab.com/jccriado/matchingdb/-/tree/main/python

- **Running – RG imporved perturbation theory**



LO	1		
NLO	$\alpha\, L$	α	
NNLO	$\alpha^2\, L^2$	$\alpha^2\, L$	α^2
$\dots$	$\dots$	$\dots$	$\dots$
$\mathrm{N}^k\mathrm{LO}$	$\alpha^k\, L^k$	$\alpha^k\, L^{k-1}$	$\alpha^k\, L^{k-2}$



Code:

Wilson[1804.05033]
SMEFT running
SMEFT-LEFT matching
LEFT running

Running from c_j to c_i is important in SMEFT if:
1) c_i is not generated at tree-level
2) c_j is generated at tree-level

- **Question the following integral is in is not quadratic divergent in the Dim-Reg**

$$\delta m_H^2 \sim -iy_t^2 \int \frac{d^4\ell}{(2\pi)^4}\frac{1}{\ell^2 - m_t^2} \rightarrow \quad -iy_t^2\mu^\epsilon \int \frac{d^d\ell}{(2\pi)^4}\frac{1}{\ell^2 - m_t^2}$$

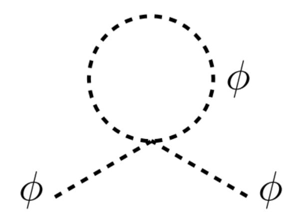
$$= \frac{y_t^2}{16\pi^2} m_t^2 \left(\frac{1}{\epsilon} + 1 + \log\frac{\tilde{\mu}^2}{m_t^2}\right)$$

- **In MS, divergence absorbed by counter term, correction** $\sim m_t^2$

- **EFT Matching point of view**

$$\mathcal{L}^{\text{Full}} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m_{\text{F}}^2\phi^2 + \frac{1}{2}(\partial_\mu\Phi)(\partial^\mu\Phi) - \frac{1}{2}M^2\Phi^2 - \frac{1}{4}\kappa\phi^2\Phi^2 - \frac{1}{4!}\eta\phi^4.$$

$$\mathcal{L}^{\text{EFT}} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m_{\text{E}}^2\phi^2 - \frac{C_4}{4!}\phi^4$$



- **If** $\kappa=0$ Tree-level $m_{E,0}^2 = m_{\text{F}}^2 \quad c_4^0 = \eta$

1-Loop-level $m_{E,1}^2 + \Gamma_{\text{EFT}}^{(1)}(p=0) = \Gamma_{\text{UV}}^{(1)}(p=0) \rightarrow m_{E,1}^2 = 0$

$$m_{E,1-\text{loop}}^2 = m_{E,0}^2 + m_{E,1}^2 = m_{E,0}^2 = m_{\text{F}}^2$$

- **Question the following integral is in is not quadratic divergent in the Dim-Reg**

$$\delta m_H^2 \sim -iy_t^2 \int \frac{d^4\ell}{(2\pi)^4}\frac{1}{\ell^2 - m_t^2} \to \quad -iy_t^2\mu^\epsilon \int \frac{d^d\ell}{(2\pi)^4}\frac{1}{\ell^2 - m_t^2}$$

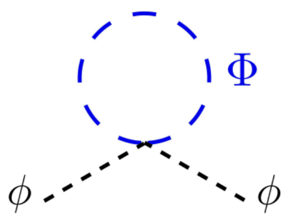
$$= \frac{y_t^2}{16\pi^2} m_t^2 \left(\frac{1}{\epsilon} + 1 + \log\frac{\tilde{\mu}^2}{m_t^2}\right)$$

- **In MS, divergence absorbed by counter term, correction** $\sim m_t^2$

- **EFT Matching point of view**

$$\mathcal{L}^{\rm Full} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m_{\rm F}^2\phi^2 + \frac{1}{2}(\partial_\mu\Phi)(\partial^\mu\Phi) - \frac{1}{2}M^2\Phi^2 - \frac{1}{4}\kappa\phi^2\Phi^2 - \frac{1}{4!}\eta\phi^4.$$

$$\mathcal{L}^{\rm EFT} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m_{\rm E}^2\phi^2 - \frac{C_4}{4!}\phi^4$$



- **If** $\kappa \neq 0$ Tree-level $m_{E,0}^2 = m_{\rm F}^2$ $\quad c_4^0 = \eta$

1-Loop-level

$$m_{E,1}^2 + \Gamma_{\rm EFT}^{(1)}(p=0) = \Gamma_{\rm UV}^{(1)}(p=0) \to m_{E,1}^2 = \frac{-\kappa}{32\pi^2} M^2 \left[\log\frac{\tilde{\mu}_M^2}{M^2} + 1\right]$$

$$m_{E,1-\rm loop}^2 = m_{E,0}^2 + m_{E,1}^2 \sim M^2$$

Hierarchy problem occurred

Hierarchy problem revisit

- **If you believe SM valid till arbitrary High scale, then no hierarchy problem**
- **If SM couple to new physics at UV, and no symmetry protects the mass, then there usually is a hierarchy problem**
- **Someone also thinks it's not a problem at all**

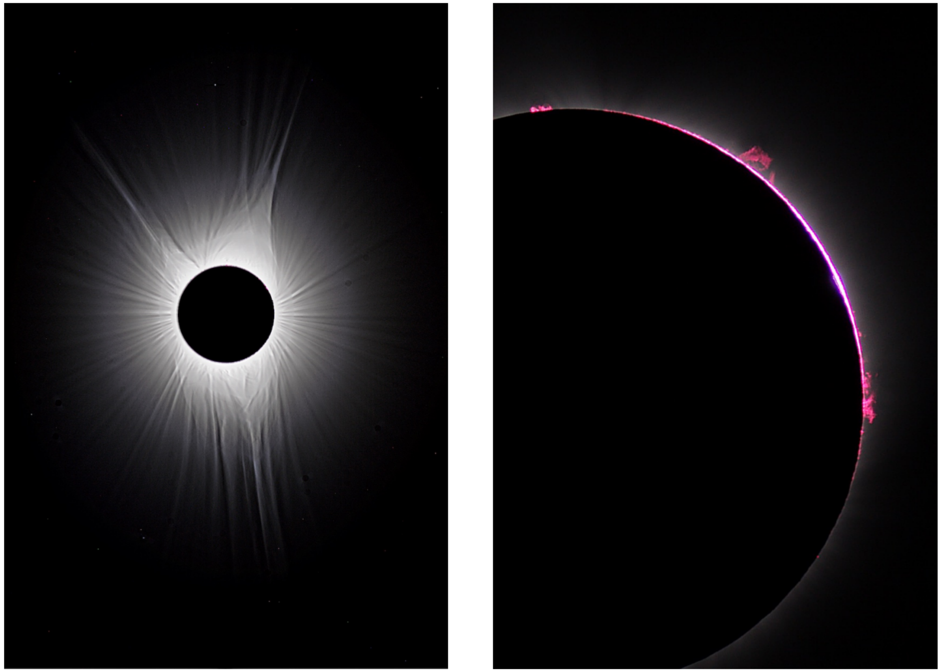


Fig. A.1 Photos of the 21 Aug 2017 solar eclipse. *[Credit: P. Stoffer]*

(see Fig. A.1). The angular diameters of the Sun and Moon are both experimentally measured (unlike in the Higgs problem where the Higgs mass parameter m at the high scale M_G is not measured) and the difference of angular diameters is much smaller than either.[1] Do you want to spend your life solving such problems?

A. Manohar [1804.05863]

SMEFT-dim6 Higgs Physics

- **Pure Higgs operators**

Operator	Definition	Main Higgs effect
Q_H	$(H^\dagger H)^3$	Modifies Higgs self-couplings hhh and $hhhh$
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	Higgs wave-function rescaling; shifts many hXX couplings
Q_{HD}	$(H^\dagger D_\mu H)^*(H^\dagger D^\mu H)$	Modifies neutral Higgs/gauge structure; affects hZZ and EW inputs

- **CP even Higgs-Field strength**

Operator	Definition	Main effect
Q_{HG}	$(H^\dagger H)G^A_{\mu\nu}G^{A\mu\nu}$	Contact hgg interaction; affects gluon fusion
Q_{HW}	$(H^\dagger H)W^I_{\mu\nu}W^{I\mu\nu}$	hWW, hZZ, $h\gamma\gamma$, $hZ\gamma$ interactions
Q_{HB}	$(H^\dagger H)B_{\mu\nu}B^{\mu\nu}$	hZZ, $h\gamma\gamma$, $hZ\gamma$ interactions
Q_{HWB}	$(H^\dagger \tau^I H)W^I_{\mu\nu}B^{\mu\nu}$	Mixed electroweak Higgs couplings; also affects electroweak precision observables

SMEFT-dim6 Higgs Physics

- **Higgs-Fermion Yukawa like**

Operator	Definition	Main effect
Q_{eH}^{pr}	$(H^\dagger H)(\bar{l}_p e_r H)$	Modifies charged-lepton Yukawa couplings
Q_{uH}^{pr}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$	Modifies up-type Yukawa couplings, especially htt
Q_{dH}^{pr}	$(H^\dagger H)(\bar{q}_p d_r H)$	Modifies down-type Yukawa couplings, especially hbb

- **Higgs-Fermion Current**

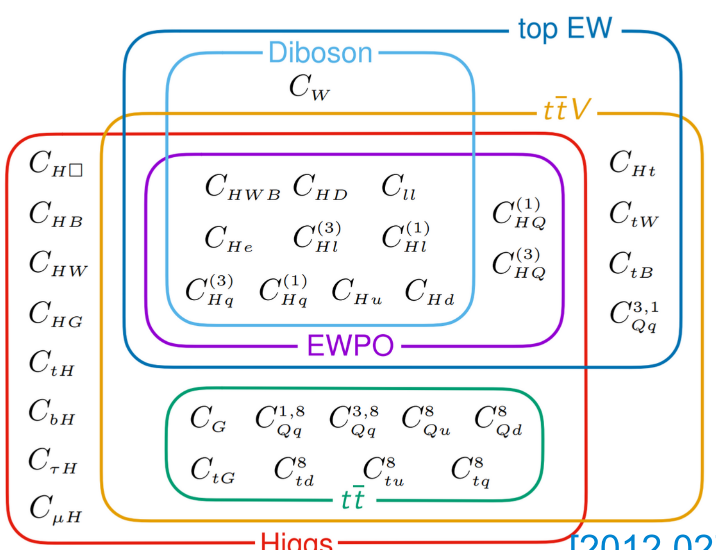
Operator	Definition	Main effect
$Q_{Hl}^{(1)pr}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p\gamma^\mu l_r)$	$hZ\ell\ell$, shifted $Z\ell\ell$ couplings
$Q_{Hl}^{(3)pr}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p\tau^I\gamma^\mu l_r)$	$hZ\ell\ell$, $hW\ell\nu$, shifted W/Z couplings
Q_{He}^{pr}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$	$hZee$, shifted Zee couplings
$Q_{Hq}^{(1)pr}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p\gamma^\mu q_r)$	$hZqq$, shifted Zqq couplings
$Q_{Hq}^{(3)pr}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p\tau^I\gamma^\mu q_r)$	$hZqq$, $hWqq'$, shifted W/Z quark couplings
Q_{Hu}^{pr}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$	$hZuu$, shifted Zuu couplings
Q_{Hd}^{pr}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$	$hZdd$, shifted Zdd couplings
Q_{Hud}^{pr}	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p\gamma^\mu d_r)$	Right-handed charged-current $hWud$ interaction

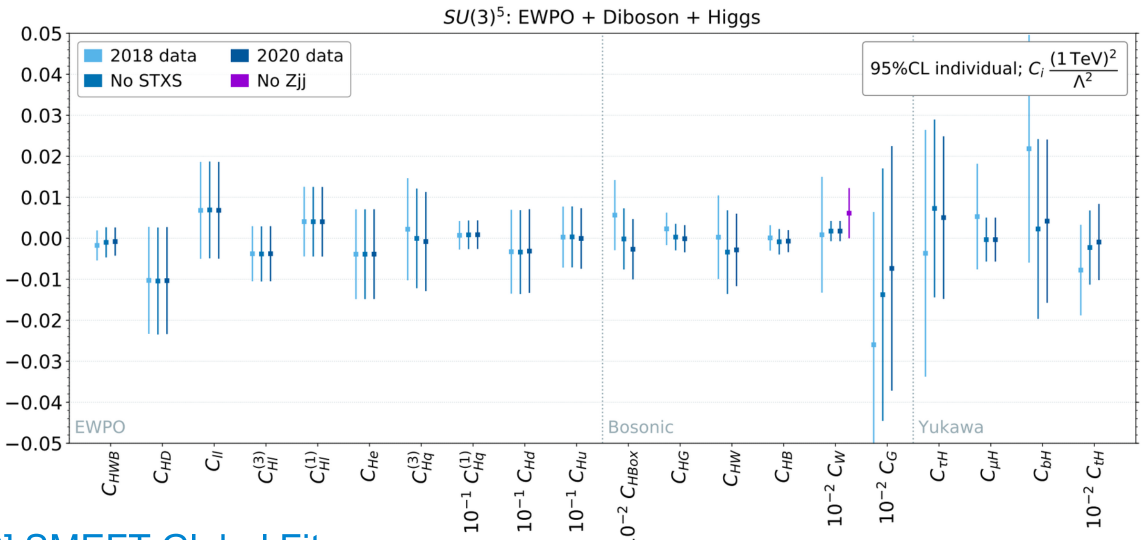
SMEFT-dim6 Higgs Physics

- **Dipole like operator**

Operator	Definition	Main Higgs effect
Q_{eW}^{pr}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H\, W_{\mu\nu}^I$	$h\ell\ell Z/\gamma$, weak dipoles
Q_{eB}^{pr}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H\, B_{\mu\nu}$	$h\ell\ell Z/\gamma$, electromagnetic dipoles
Q_{uG}^{pr}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$	Top/charm chromomagnetic Higgs-related couplings
Q_{uW}^{pr}	$(\bar{q}_p \sigma^{\mu\nu} \tau^I u_r)\widetilde{H}\, W_{\mu\nu}^I$	Top/up weak dipoles
Q_{uB}^{pr}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{H}\, B_{\mu\nu}$	Top/up neutral dipoles
Q_{dG}^{pr}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$	Bottom/down chromomagnetic Higgs-related couplings
Q_{dW}^{pr}	$(\bar{q}_p \sigma^{\mu\nu} \tau^I d_r) H\, W_{\mu\nu}^I$	Down-type weak dipoles
Q_{dB}^{pr}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$	Down-type neutral dipoles

-





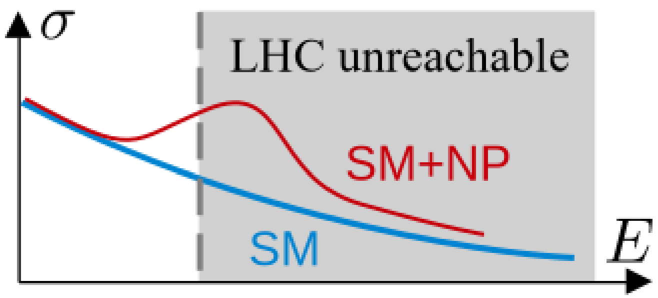
[2012.02779] SMEFT Global Fit

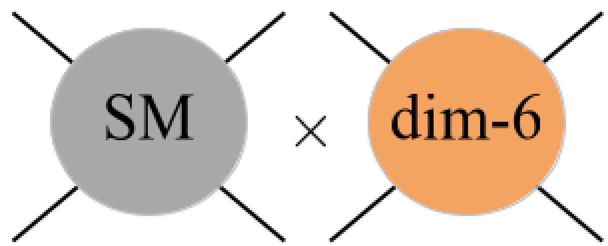
- **When taking high energy tail of LHC to probe SMEFT**

[1607.05236]

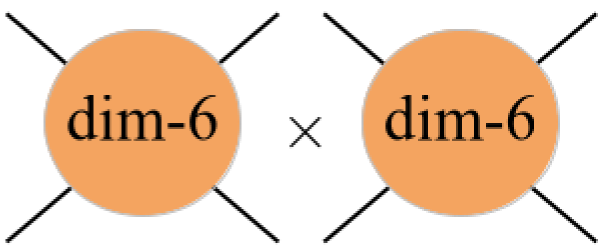
A_4	$\lvert h(A_4^{\rm SM})\rvert$	$\lvert h(A_4^{\rm BSM})\rvert$
$VVVV$	0	4,2
$VV\phi\phi$	0	2
$VV\psi\psi$	0	2
$V\psi\psi\phi$	0	2
$\psi\psi\psi\psi$	2,0	2,0
$\psi\psi\phi\phi$	0	0
$\phi\phi\phi\phi$	0	0

$$\sigma \sim |\mathcal{A}_{\rm SM} + \mathcal{A}_{{\rm dim}-6}|^2$$

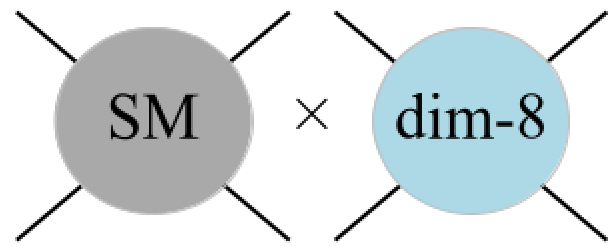




$$\sim \frac{m_{\rm SM}^2}{\Lambda^2}$$



$$\frac{E^4}{\Lambda^4}$$



$$\frac{E^4}{\Lambda^4}$$

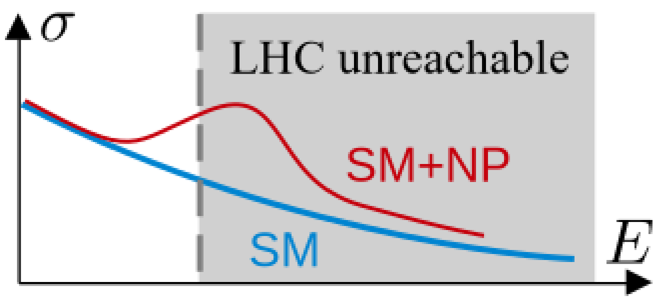
When Dim-8 is important

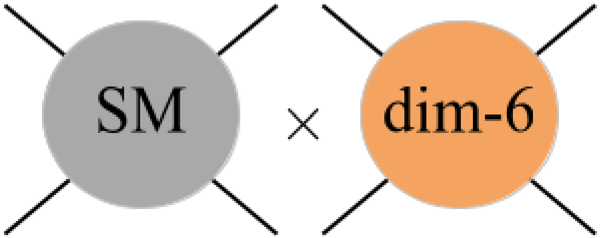
- **When taking high energy tail of LHC to probe SMEFT**

$$\sigma \sim |\mathcal{A}_{\mathrm{SM}} + \mathcal{A}_{\mathrm{dim-6}}|^2$$

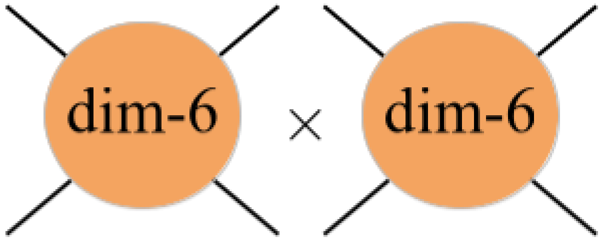
[1607.05236]

A_4	$\lvert h(A_4^{\mathrm{SM}})\rvert$	$\lvert h(A_4^{\mathrm{BSM}})\rvert$
$VVVV$	0	4,2
$VV\phi\phi$	0	2
$VV\psi\psi$	0	2
$V\psi\psi\phi$	0	2
$\psi\psi\psi\psi$	2,0	2,0
$\psi\psi\phi\phi$	0	0
$\phi\phi\phi\phi$	0	0



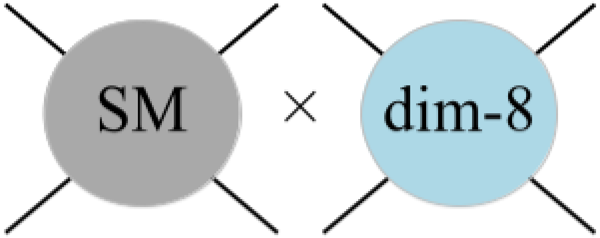


$$\sim \frac{m_{\mathrm{SM}}^2}{\Lambda^2}$$



$$\frac{1}{(4\pi)^4}\frac{E^4}{\Lambda^4}$$

Dim-8 may be more important if dim-6 are **loop generated**

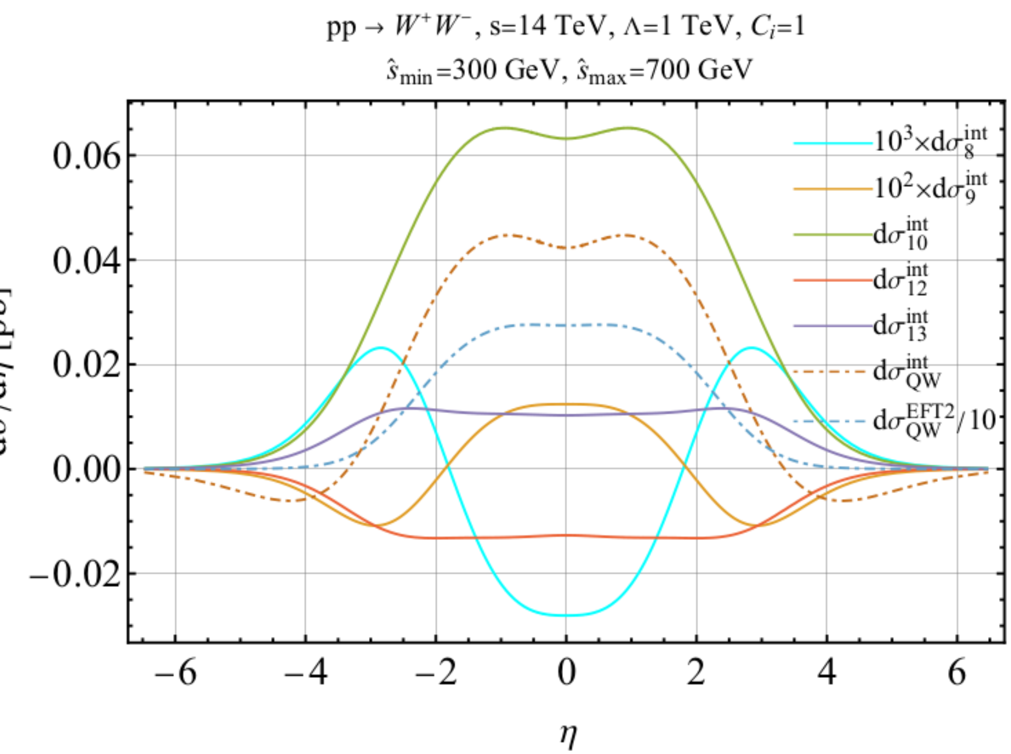


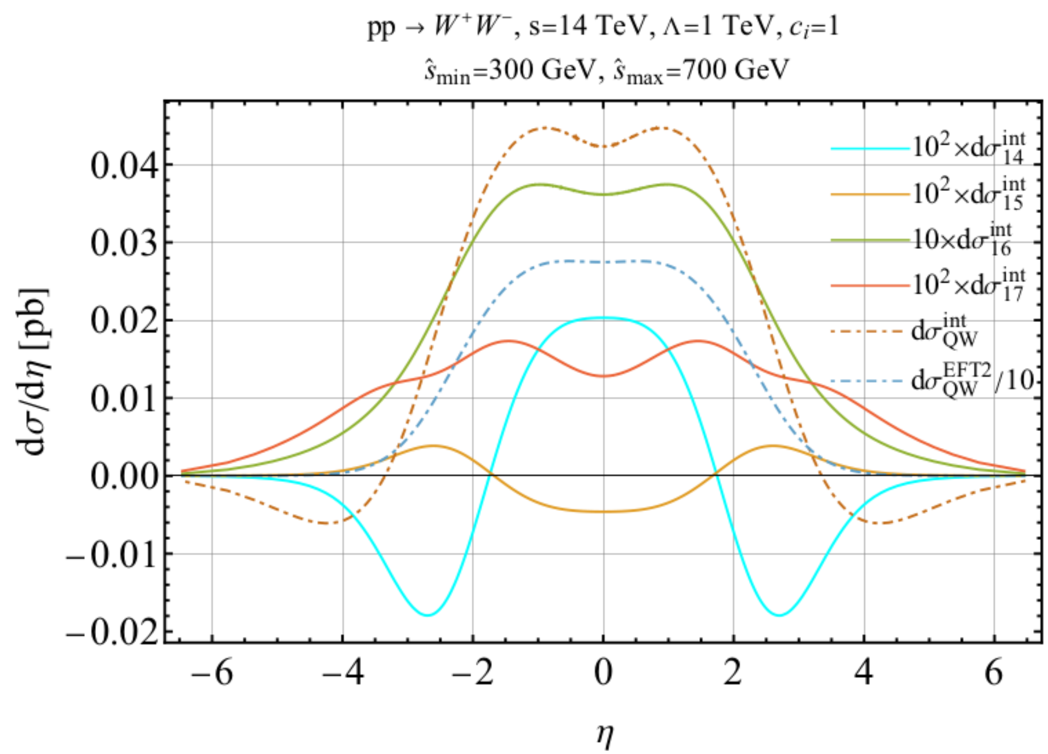
$$\frac{1}{4\pi}\frac{E^4}{\Lambda^4}$$

When Dim-8 is important

- **Example qq→WW at LHC**

[C. Degrande **HLL**, JHEP 06 (2023) 149]





$$O_W = \epsilon^{IJK} W_\mu^{I\nu} W_\nu^{JI\rho} W_\rho^{K\mu}$$

Dominant operators:

$$O_{10} = i W^{I\mu}{}_\nu W^{I\nu}{}_\lambda \left(\bar{q}_{Lr}\gamma^\lambda \overleftrightarrow{D}_\mu q_{Lp}\right),$$

$$O_{12} = i\epsilon^{IJK} \tilde{W}^{I\mu}{}_\nu W^{J\nu}{}_\lambda \left(\bar{q}^i_{Lp}\gamma^\lambda \left(\tau^K\right)_i{}^j \overleftrightarrow{D}_\mu q_{Lrj}\right), \qquad O_{16} = i\left(\bar{q}_{Lr}\gamma^\lambda \overleftrightarrow{D}_\mu q_{Lp}\right)\left(D_\lambda H^\dagger D^\mu H\right),$$

$$O_{13} = i\epsilon^{IJK} W^{I\mu}{}_\nu \tilde{W}^{J\nu}{}_\lambda \left(\bar{q}^i_{Lp}\gamma^\lambda \left(\tau^K\right)_i{}^j \overleftrightarrow{D}_\mu q_{Lrj}\right), \qquad O_{17} = i\left(\bar{q}_{Lp}\gamma^\lambda \tau^K \overleftrightarrow{D}_\mu q_{Lr}\right)\left(D_\lambda H^\dagger \tau^K D^\mu H\right),$$

When Dim-8 is important

- **[2602.12326] study h to 4 lepton decay upto** $1/\Lambda^4$

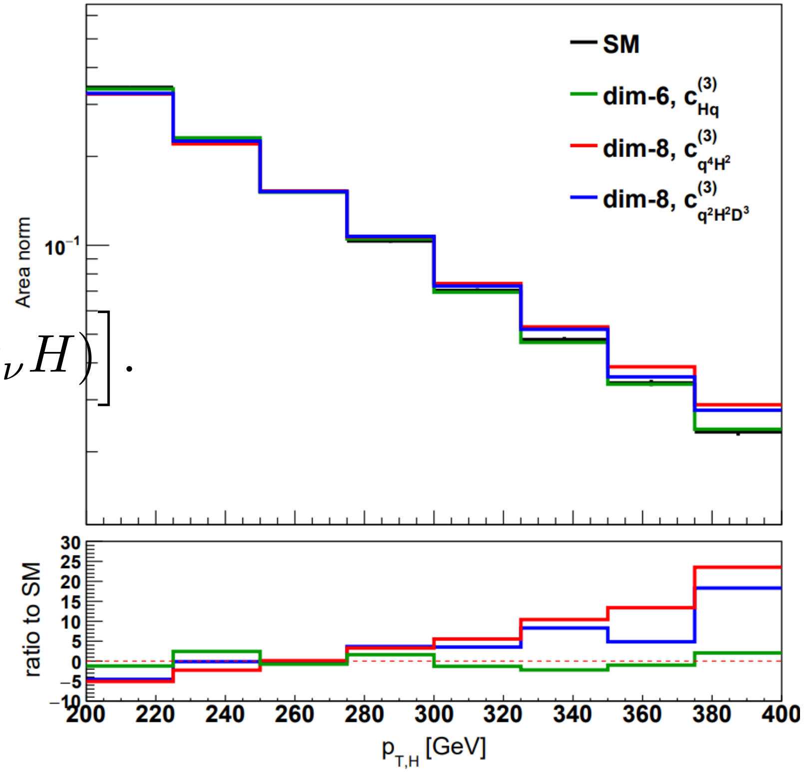
$$|\mathcal{A}|^2 = |A_{\rm SM}|^2 + 2\,{\rm Re}(A_{\rm SM}^* A_6) + |A_6|^2 + 2\,{\rm Re}(A_{\rm SM}^* A_8)$$

- **For** $h \to \ell\ell\, Z(\ell\ell)$ **,**
  $O_{HW} = |H^2| W_{\mu\nu}^2$ **dim-6 square gives dominant contribution**

- **[2410.21563] study the dim-8 effects on the VBF Higgs production process**

$$Q_{q^4H^2}^{(3)} \quad (\bar{q}_p \gamma^\mu \sigma^I q_r)(\bar{q}_p \gamma_\mu \sigma^I q_r)(H^\dagger H)$$

$$Q_{q^2H^2D^3} = i(\bar{\psi}_p \gamma^\mu \sigma^I \psi_r) \left[(D_\nu H)^\dagger \tau^I (D^2_{(\mu,\nu)} H) - (D^2_{(\mu,\nu)} H)^\dagger \sigma^I (D_\nu H) \right].$$



- **HEFT ⊃ SMEFT ⊃ SM**
- **Electroweak symmetry is nonlinearly realized in HEFT**

$$\Sigma(x) = \frac{v+\hat{h}(x)}{\sqrt{2}} U(x) \qquad U(x) = \exp\left(i \frac{\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)}{v} \right)$$

- **Under Custodial symmetry** $\hat{h} \to \hat{h} \qquad U \to L U R^{\dagger}$
- **Power counting: Chiral dimension**
  L **-loop renormalization need to find operator of** $d_\chi = 2L+2$

$$[\partial_\mu]_\chi = 1, \qquad [g]_\chi = [g']_\chi = [g_s]_\chi = 1, \qquad [y_f]_\chi = 1,$$

[1312.5624]

$$[\bar{\psi}\psi]_\chi = 1, \qquad [U]_\chi = [h]_\chi = [W_\mu]_\chi = [B_\mu]_\chi = [G_\mu]_\chi = 0.$$

$$\begin{aligned}\mathcal{L}_2 \;=\; & \frac{v^2}{4}\mathrm{Tr}[D_\mu U D^\mu U^\dagger] F(h/v) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{m_h^2}{2} h^2 - \mathcal{V}(h) \\ & + i\bar{\psi}\not{D}\psi - (v\ \bar{\psi}_L U \mathcal{Y}(h/v)\psi_R + \text{h.c.}) \\ & - \frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\end{aligned}$$

$$F(\frac{h}{v}) = 1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \cdots$$

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[1312.5624]

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$$\begin{aligned}\mathcal{L}_2 \; = & \; \frac{v^2}{4} \mathrm{Tr}[D_\mu U D^\mu U^\dagger] F(h/v) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{m_h^2}{2} h^2 - \mathcal{V}(h) \\ & + i \bar{\psi} \not{D} \psi - (v \; \bar{\psi}_L U \mathcal{Y}(h/v) \psi_R + \text{h.c.}) \\ & - \frac{1}{2}\langle G_{\mu\nu} G^{\mu\nu} \rangle - \frac{1}{2}\langle W_{\mu\nu} W^{\mu\nu} \rangle - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}\end{aligned}$$

$$F(\frac{h}{v}) = 1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \cdots$$

When can it go back to linearly realized with field redefinition?[1605.03602]

$$\exists h_* \; s.t. \; F(h_* = 0)$$

- **HEFT ⊃ SMEFT ⊃ SM**
- **Electroweak symmetry is nonlinearly realized in HEFT**

$$\Sigma(x) = \frac{v + \hat{h}(x)}{\sqrt{2}} U(x) \qquad U(x) = \exp\left(i \frac{\boldsymbol{\tau} \cdot \boldsymbol{\pi}(x)}{v} \right)$$

- **Under Custodial symmetry** $\hat{h} \to \hat{h} \qquad U \to L U R^\dagger$
- **Power counting: Chiral dimension**
  L **-loop renormalization need to find operator of** $d_\chi = 2L+2$

$$[\partial_\mu]_\chi = 1, \qquad [g]_\chi = [g']_\chi = [g_s]_\chi = 1, \qquad [y_f]_\chi = 1,$$

[1312.5624]

$$[\bar{\psi}\psi]_\chi = 1, \qquad [U]_\chi = [h]_\chi = [W_\mu]_\chi = [B_\mu]_\chi = [G_\mu]_\chi = 0.$$

$$\begin{aligned}\mathcal{L}_2 \;=\; & \frac{v^2}{4} \mathrm{Tr}[D_\mu U D^\mu U^\dagger] F(h/v) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{m_h^2}{2} h^2 - \mathcal{V}(h) \\ & + i \bar{\psi} \not{D} \psi - (v\ \bar{\psi}_L U \mathcal{Y}(h/v) \psi_R + \text{h.c.}) \\ & - \frac{1}{2}\langle G_{\mu\nu} G^{\mu\nu} \rangle - \frac{1}{2}\langle W_{\mu\nu} W^{\mu\nu} \rangle - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}\end{aligned}$$

$$F_{\mathrm{SM}}\left(\frac{h}{v}\right) = \left(1 + \frac{h}{v}\right)^2$$

$$h_* = -v$$

For general F it is hard to detect, since it's a non-perturbative probe

HEFT

- **HEFT ⊃ SMEFT ⊃ SM**
- **Electroweak symmetry is nonlinearly realized in HEFT**

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$$[\bar{\psi}\psi]_\chi = 1, \qquad [U]_\chi = [h]_\chi = [W_\mu]_\chi = [B_\mu]_\chi = [G_\mu]_\chi = 0.$$

- **In Unitary Gauge** $U = 1$

$$\mathcal{L}_2 \supset m_W^2 W_\mu^+ W^{-\mu}\left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \cdots\right)$$

If $\kappa_V \simeq 1$ **while** $\kappa_{2V} \neq 1$ **, then suggest using HEFT instead of EFT**

Real Singlet Model

- **Simplest extension to the standard model, can trigger SFOEWPT**
- **Adding only one real singlet S**

$$V_0(H,S) = -\mu^2(H^\dagger H) + \lambda(H^\dagger H)^2 + \frac{a_1}{2}(H^\dagger H)S + \frac{a_2}{2}(H^\dagger H)S^2$$
$$+ \frac{b_2}{2}S^2 + \frac{b_3}{3}S^3 + \frac{b_4}{4}S^4,$$

- **Parametrize the vevs**

$$S = x_0 + s,$$
$$H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v_0 + h + iG^0) \end{pmatrix}$$

- **Tadpole condition relates vev to potential parameters**

$$\mu^2 = \lambda v_0^2 + (a_1 + a_2 x_0)\frac{x_0}{2},$$
$$b_2 = -b_3 x_0 - b_4 x_0^2 - \frac{a_1 v_0^2}{4x_0} - \frac{a_2 v_0^2}{2}.$$

Real Singlet Model

- **The mass matrix**

$$\mathcal{M}^2 = \begin{pmatrix} m_s^2 & m_{hs}^2 \\ m_{hs}^2 & m_h^2 \end{pmatrix} = \begin{pmatrix} -\frac{a_1 v_0^2}{4x_0} + x_0\left(b_3 + 2b_4 x_0\right) & \frac{v_0}{2}\left(a_1 + 2a_2 x_0\right) \\ \frac{v_0}{2}\left(a_1 + 2a_2 x_0\right) & 2\lambda v_0^2 \end{pmatrix}$$

$$h_1 = h\cos\theta + s\sin\theta$$

$$h_2 = s\cos\theta - h\sin\theta$$

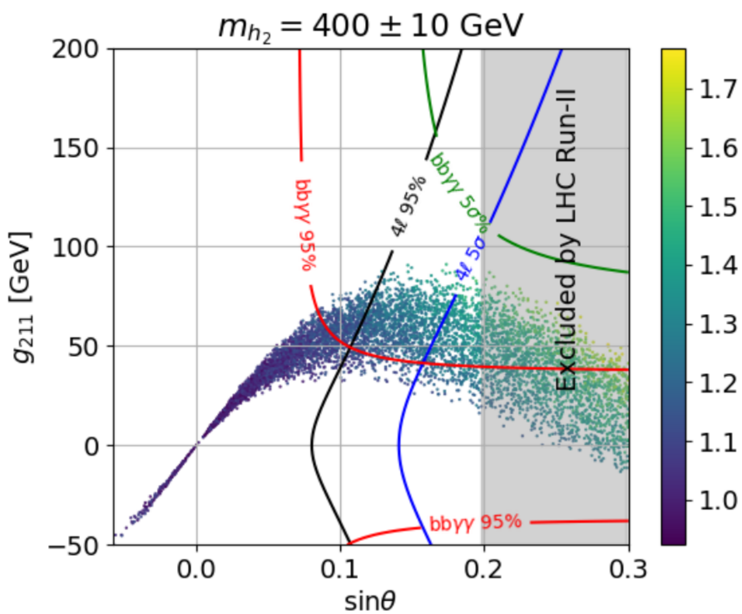
- **Potential Stability requires** $a_2 \geq -2\sqrt{\lambda b_4}$.

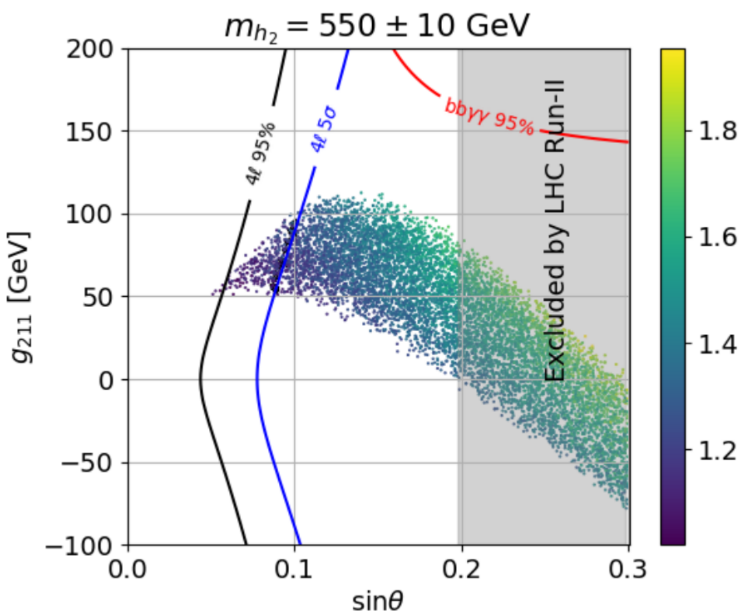
$$a_2 = \frac{\pm\sqrt{m_h^2 m_s^2 - m_{h_1}^2\left(m_h^2 + m_s^2 - m_{h_1}^2\right)} - a_1 v_0/2}{x_0 v_0}$$

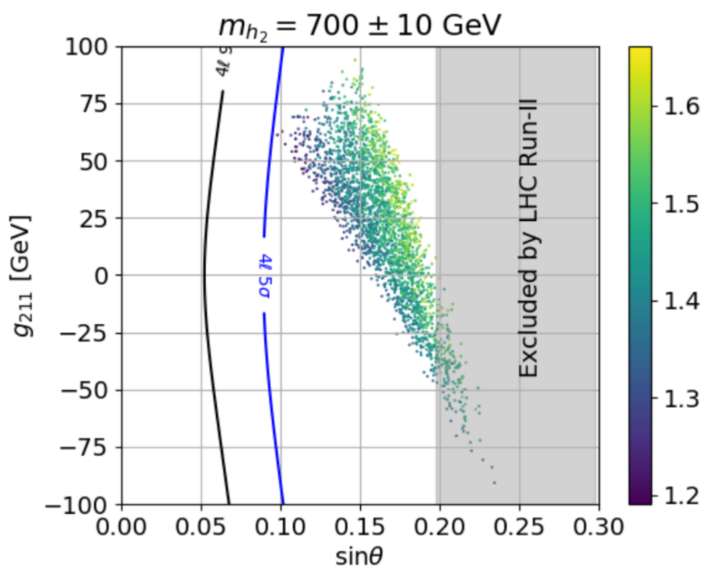
- **Free parameters are** $a_1, b_3, b_4, \lambda, x_0$ **, performing scan to select benchmark have SFOEWPT**

- **We can hardly find benchmark to have SFOEWPT with** $m_{h2} > 800\,GeV$

- **Combined analysis for** $h_2 \to ZZ \to 4\ell$ **and** $h_2 \to h_1 h_1 \to bb\gamma\gamma$







Real Singlet Model

- **The mass matrix**

$$\mathcal{M}^2 = \begin{pmatrix} m_s^2 & m_{hs}^2 \\ m_{hs}^2 & m_h^2 \end{pmatrix} = \begin{pmatrix} -\frac{a_1 v_0^2}{4x_0} + x_0\left(b_3 + 2b_4 x_0\right) & \frac{v_0}{2}\left(a_1 + 2a_2 x_0\right) \\ \frac{v_0}{2}\left(a_1 + 2a_2 x_0\right) & 2\lambda v_0^2 \end{pmatrix}$$

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$$h_2 = s\cos\theta - h\sin\theta$$

- **Potential Stability requires** $a_2 \geq -2\sqrt{\lambda b_4}$.

$$a_2 = \frac{\pm\sqrt{m_h^2 m_s^2 - m_{h_1}^2\left(m_h^2 + m_s^2 - m_{h_1}^2\right)} - a_1 v_0/2}{x_0 v_0}$$

- **Free parameters are** $a_1, b_3, b_4, \lambda, x_0$ **, performing scan to select benchmark have SFOEWPT**

- **We can hardly find benchmark to have SFOEWPT with** $m_{h2} > 800\,GeV$

2HDM

- **General CP violating 2HDM**

$$
\begin{aligned}
V(\phi_1,\phi_2) &= -\frac{1}{2}\left[m_{11}^2(\phi_1^\dagger\phi_1) + \left(m_{12}^2(\phi_1^\dagger\phi_2) + \text{h.c.}\right) + m_{22}^2(\phi_2^\dagger\phi_2)\right] \\
&+\frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
&+\frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_2)(\phi_1^\dagger\phi_1) + \lambda_7(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_2) + \text{h.c.}\right] .
\end{aligned}
$$

- **Soft breaking Z_2 only retain $m_{12}^2 \neq 0$ but setting $\lambda_6 = \lambda_7 = 0$**
- **Z_2 helps to prevent tightly constraint FCNC at tree-level**

$$Z_2 \text{ odd} : \phi_2$$

Model	u_R^i	d_R^i	e_R^i
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1
Lepton-specific	Φ_2	Φ_2	Φ_1
Flipped	Φ_2	Φ_1	Φ_2

- **Scalar mass spectrum**

$$\phi_1 = \begin{pmatrix} H_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + H_1^0 + iA_1^0) \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} H_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + H_2^0 + iA_2^0) \end{pmatrix} \qquad \tan\beta = \frac{v_2}{v_1}$$

Phase of vev can be absorbed with global U(2) that leave $\lambda_6, \lambda_7 = 0$

In Unitary Gauge

$$\phi_1 = \begin{pmatrix} -\sin\beta H^+ \\ \frac{1}{\sqrt{2}}(v\cos\beta + H_1^0 - i\sin\beta A^0) \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \cos\beta H^+ \\ \frac{1}{\sqrt{2}}(v\sin\beta + H_2^0 + i\cos\beta A^0) \end{pmatrix}$$

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \mathrm{Im}\lambda_5, \mathrm{Re}\lambda_5, \mathrm{Re}\, m_{12}^2, \mathrm{Im}\, m_{12}^2, m_{11}^2, m_{22}^2$$

Minimization condition (3)

Mass of charge Higgs (1)

Diagonalization of neutral Mass matrix (6)

$$v, \tan\beta, \nu, \alpha, \alpha_b, \alpha_c, m_{h_1}, m_{h_2}, m_{h_3}, m_{h_H^+}$$

$$\nu = \frac{\mathrm{Re} m_{12}^2}{v^2 \sin 2\beta}$$

- **Scalar mass spectrum**

$$RM_n^2R^{\mathrm{T}} = \mathrm{diag}(m_{h_1}^2, m_{h_2}^2, m_{h_3}^2) \quad (h_1, h_2, h_3) = (H_1^0, H_2^0, A^0)R$$

$$R = R_{23}(\alpha_c)R_{13}(\alpha_b)R_{12}(\alpha + \pi/2)$$

$$-\frac{\pi}{2} < \alpha_c,\ \alpha_b,\ \alpha \leq \frac{\pi}{2}$$

$$M_n^2 = v^2 \begin{pmatrix} \lambda_1 c_\beta^2 + \nu s_\beta^2 & (\lambda_{345} - \nu)c_\beta s_\beta & -\frac{1}{2}\,\mathrm{Im}\,\lambda_5 s_\beta \\ (\lambda_{345} - \nu)c_\beta s_\beta & \lambda_2 s_\beta^2 + \nu c_\beta^2 & -\frac{1}{2}\,\mathrm{Im}\,\lambda_5 c_\beta \\ -\frac{1}{2}\,\mathrm{Im}\,\lambda_5 s_\beta & -\frac{1}{2}\,\mathrm{Im}\,\lambda_5 c_\beta & -\mathrm{Re}\,\lambda_5 + \nu \end{pmatrix}$$

- **3 Neutral Scalar + 1 charged Scalar**

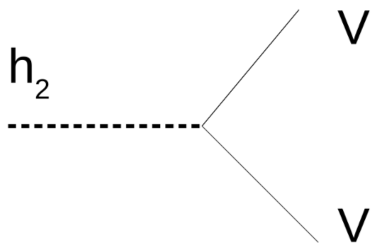
$$g_{hVV} = \sin(\beta - \alpha)\, g_{h_{\mathrm{SM}}VV}, \quad g_{HVV} = \cos(\beta - \alpha)\, g_{h_{\mathrm{SM}}VV}, \quad g_{AVV} = 0.$$

- **Boson couplings In CP conserving case**

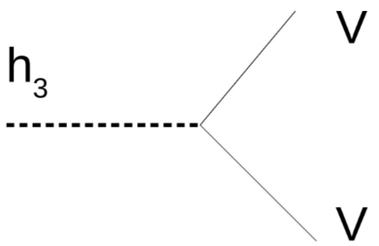
$$g_{hVV} = \sin(\beta - \alpha)\, g_{h_{\rm SM}VV}, \quad g_{HVV} = \cos(\beta - \alpha)\, g_{h_{\rm SM}VV}, \quad g_{AVV} = 0.$$

$$g_{ZAh} \propto \cos(\beta - \alpha), \quad g_{ZAH} \propto -\sin(\beta - \alpha), \quad g_{ZHH} = g_{ZAA} = 0$$

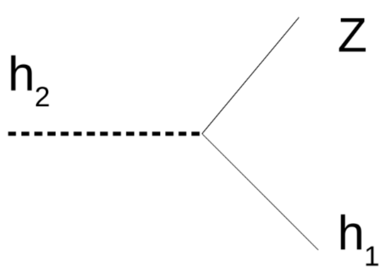
- **Turn on CP violation we have**



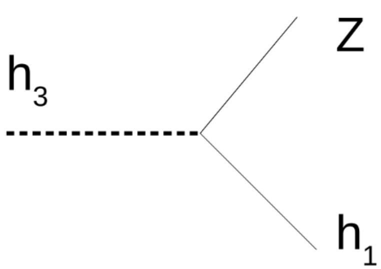
$$a_2 \propto \theta + \mathcal{O}(\alpha_b^2)$$



$$a_3 \propto -\alpha_b + \mathcal{O}(\theta\alpha_b)$$



$$g_{2z1} \propto -\alpha_b + \mathcal{O}(\theta\alpha_b)$$



$$g_{3z1} \propto -\theta + \mathcal{O}(\alpha_b^2)$$

- **Coupling to fermion CP conversing case**

$$\mathcal{L}_{\text{Yukawa}}^{\text{2HDM}} = - \sum_{f=u,d,\ell} \frac{m_f}{v} \left(\xi_h^f \overline{f} f h + \xi_H^f \overline{f} f H - i \xi_A^f \overline{f} \gamma_5 f A \right)$$
$$- \left\{ \frac{\sqrt{2} V_{ud}}{v} \overline{u} \left(m_u \xi_A^u \mathrm{P}_L + m_d \xi_A^d \mathrm{P}_R \right) d H^+ + \frac{\sqrt{2} m_\ell \xi_A^\ell}{v} \overline{\nu_L} \ell_R H^+ + \text{H.c.} \right\}$$

	Type I	Type II	Lepton-specific	Flipped
ξ_h^u	$\cos\alpha / \sin\beta$	$\cos\alpha / \sin\beta$	$\cos\alpha / \sin\beta$	$\cos\alpha / \sin\beta$
ξ_h^d	$\cos\alpha / \sin\beta$	$-\sin\alpha / \cos\beta$	$\cos\alpha / \sin\beta$	$-\sin\alpha / \cos\beta$
ξ_h^ℓ	$\cos\alpha / \sin\beta$	$-\sin\alpha / \cos\beta$	$-\sin\alpha / \cos\beta$	$\cos\alpha / \sin\beta$
ξ_H^u	$\sin\alpha / \sin\beta$	$\sin\alpha / \sin\beta$	$\sin\alpha / \sin\beta$	$\sin\alpha / \sin\beta$
ξ_H^d	$\sin\alpha / \sin\beta$	$\cos\alpha / \cos\beta$	$\sin\alpha / \sin\beta$	$\cos\alpha / \cos\beta$
ξ_H^ℓ	$\sin\alpha / \sin\beta$	$\cos\alpha / \cos\beta$	$\cos\alpha / \cos\beta$	$\sin\alpha / \sin\beta$
ξ_A^u	$\cot\beta$	$\cot\beta$	$\cot\beta$	$\cot\beta$
ξ_A^d	$-\cot\beta$	$\tan\beta$	$-\cot\beta$	$\tan\beta$
ξ_A^ℓ	$-\cot\beta$	$\tan\beta$	$\tan\beta$	$-\cot\beta$