Study of $\gamma^*\gamma^* \to p\bar{p}$

双光子过程中的方位角调制与MC产生

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outline

□双光子过程中的ϕ调制

1. 传统分析基本假设
2. 准时光子的线偏振
3. ϕ如何进入振幅
4. 实验上如何观察ϕ调制
5. 从$\gamma^*\gamma^* \to \pi\pi$到$\gamma^*\gamma^* \to p\bar{p}$

Novel Azimuthal Observables from Two-Photon Collision at e^+e^- Colliders

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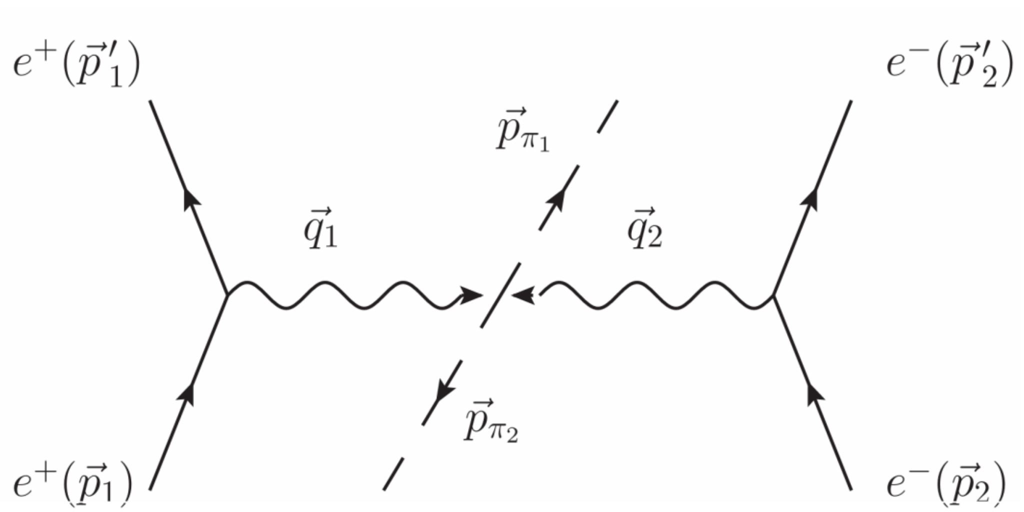
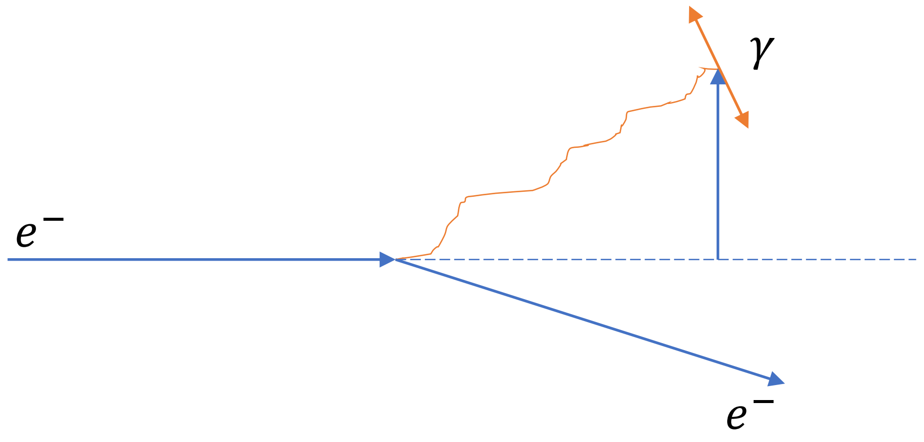
双光子过程中的ϕ调制：传统分析基本假设



假设1：Equivalent photon approximation
可以把产生截面表示为电子内两个光子部分子分布函数的积和$\gamma^*\gamma^* \to X$过程的截面的卷积

假设2：QED collinear factorization
光子主要沿z轴方向，其横动量和线偏振的方向信息被忽略/积分掉了。（仅适用于untag）

双光子过程中的ϕ调制：准时光子的线偏振



Transverse-momentum-dependent factorization framework:
准实光子的线偏振方向和横动量方向平行。两个光子分别引入ϕ_1和ϕ_2。

一个ϕ方向线性偏振准时光子可以表示为:

$$|\gamma(\phi)\rangle = \frac{1}{\sqrt{2}}(e^{-i\phi}|+\rangle + e^{i\phi}|-\rangle)$$

两光子耦合振幅具有类似如下的形式:

$$\mathcal{M}(\phi_1,\phi_2) = \frac{1}{2}[e^{-i(\phi_1+\phi_2)}M_{++} + e^{-i(\phi_1-\phi_2)}M_{+-} + e^{i(\phi_1-\phi_2)}M_{-+} + e^{i(\phi_1+\phi_2)}M_{--}]$$

双光子过程中的ϕ调制：ϕ如何进入振幅

In TMD factorization, the azimuthal-dependent cross section can be expressed as the convolution of the short-distance part and photon TMD parton distributions of the electron and positron.

$$\frac{d\sigma}{d^2\mathbf{p}_{1\perp}d^2\mathbf{p}_{2\perp}dy_1dy_2} = \frac{1}{16\pi^2Q^4}\int d^2\mathbf{k}_{1\perp}d^2\mathbf{k}_{2\perp}\times\delta^2(\mathbf{q}_\perp-\mathbf{k}_{1\perp}-\mathbf{k}_{2\perp})x_1x_2$$
$$\times\left\{\frac{1}{2}(|M_{+-}|^2+|M_{++}|^2)f(x_1,k_{1\perp}^2)f(x_2,k_{2\perp}^2)-\cos(2\phi_1)\mathrm{Re}[M_{++}M_{+-}^*]f(x_2,k_{2\perp}^2)h_1^\perp(x_1,k_{1\perp}^2)\right.$$
$$-\cos(2\phi_2)\mathrm{Re}[M_{++}M_{+-}^*]f(x_1,k_{1\perp}^2)h_1^\perp(x_2,k_{2\perp}^2)$$
$$\left.+\frac{1}{2}[\cos2(\phi_1-\phi_2)|M_{++}|^2+\cos2(\phi_1+\phi_2)|M_{+-}|^2]h_1^\perp(x_1,k_{1\perp}^2)h_1^\perp(x_2,k_{2\perp}^2)\right\},$$

第一项：只有这一项对总的积分截面有贡献。
第二三项：$\cos2\phi_i$项与不同helicity态之间的干涉相关$Re[M_{++}M_{+-}^*]$，对$k_{i\perp}$积分后留下$cos2\phi$调制项。
第四项：类似的，积分后留下$cos4\phi$调制项。

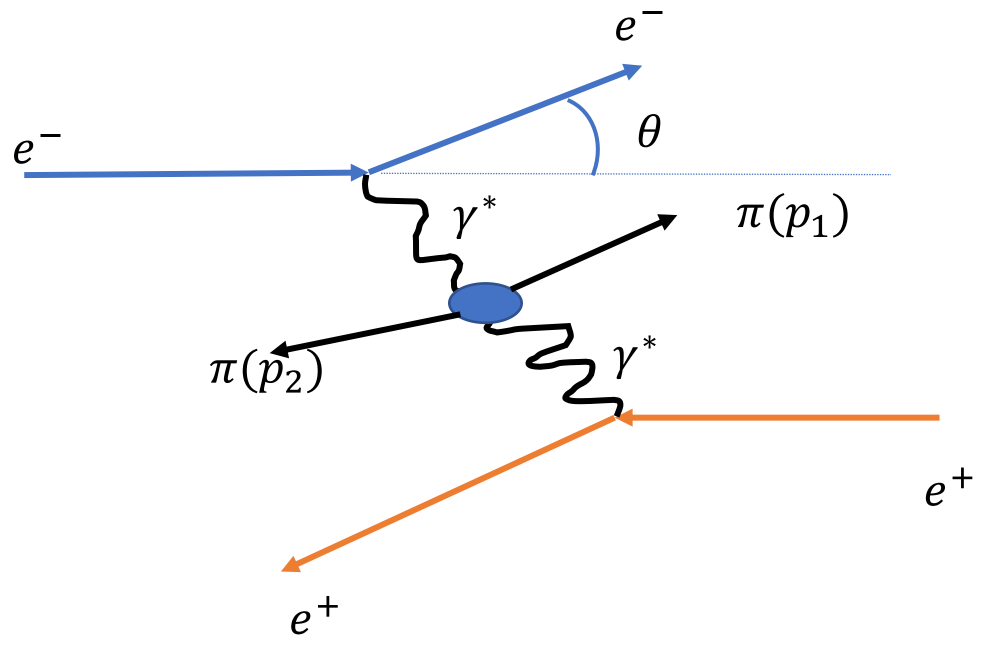
双光子过程中的ϕ调制：实验上如何观测ϕ调制

ϕ角定义:

$$P_\perp \equiv (p_{1\perp} - p_{2\perp})/2$$

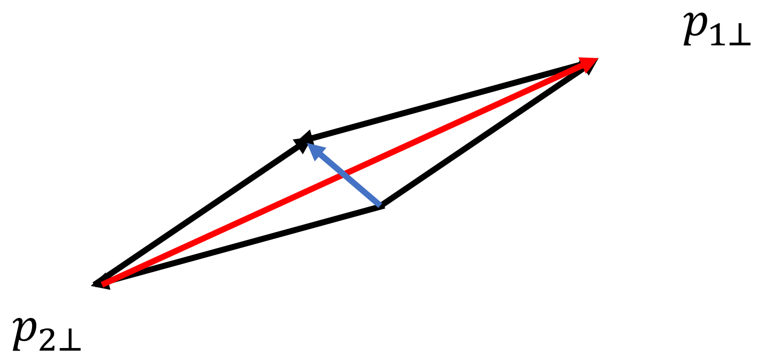
$$q_\perp \equiv p_{1\perp} - p_{2\perp}$$

$$cos\phi \equiv \hat{P}_\perp \cdot \hat{q}_\perp$$



引入观测量:

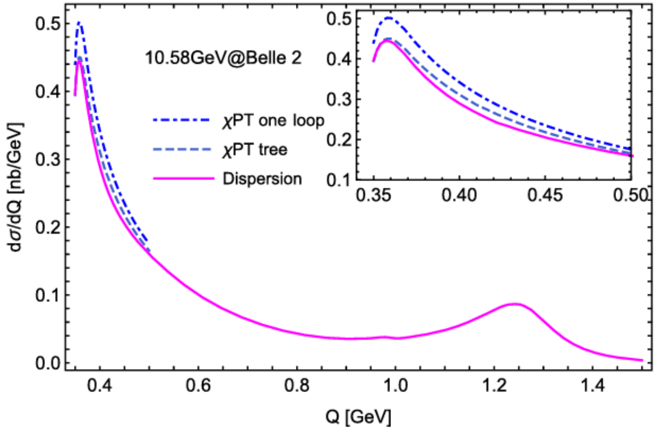
$$\langle\cos(n\phi)\rangle \equiv \frac{\int d\sigma \cos n\phi}{\int d\sigma}$$



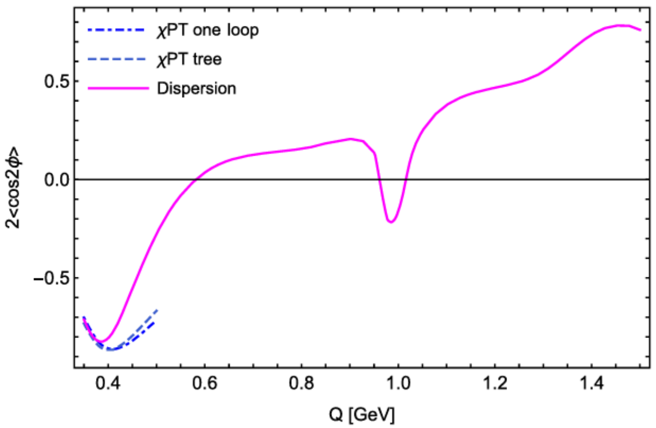
双光子过程中的ϕ调制：实验上如何观测ϕ调制

对$\gamma\gamma \to \pi\pi$过程的理论预言：横坐标Q表示$\pi\pi$不变质量。

微分截面　　　$\langle\cos(2\phi)\rangle$　　　$\langle\cos(4\phi)\rangle$



(a)



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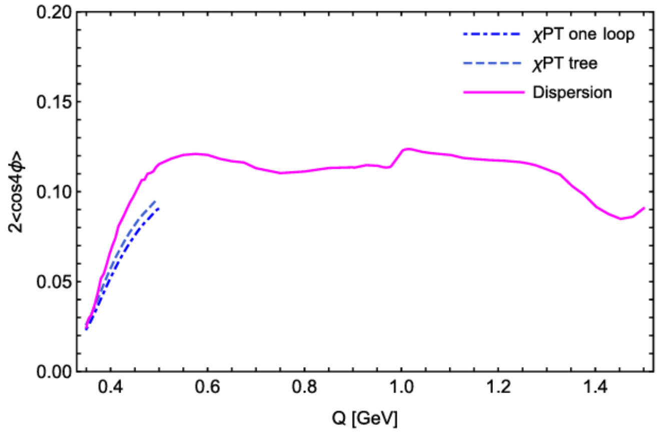

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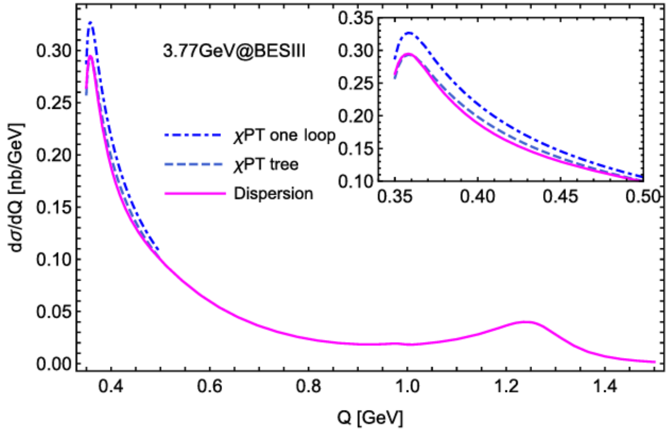

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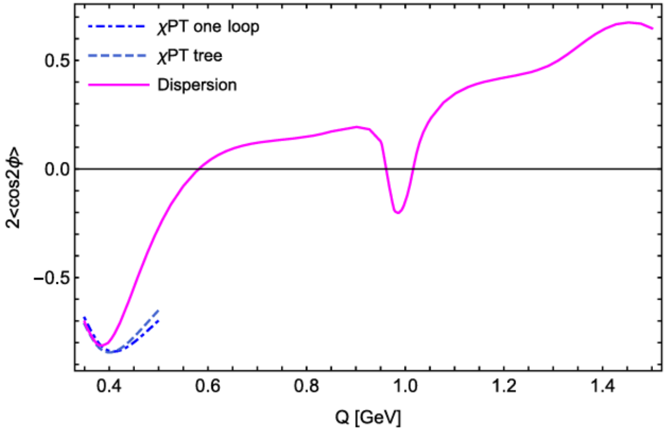

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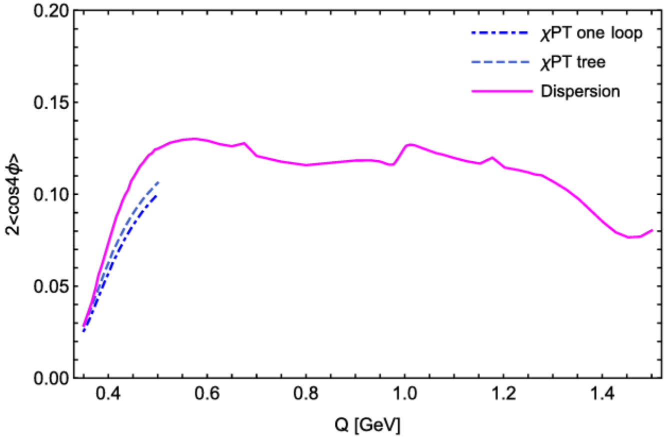
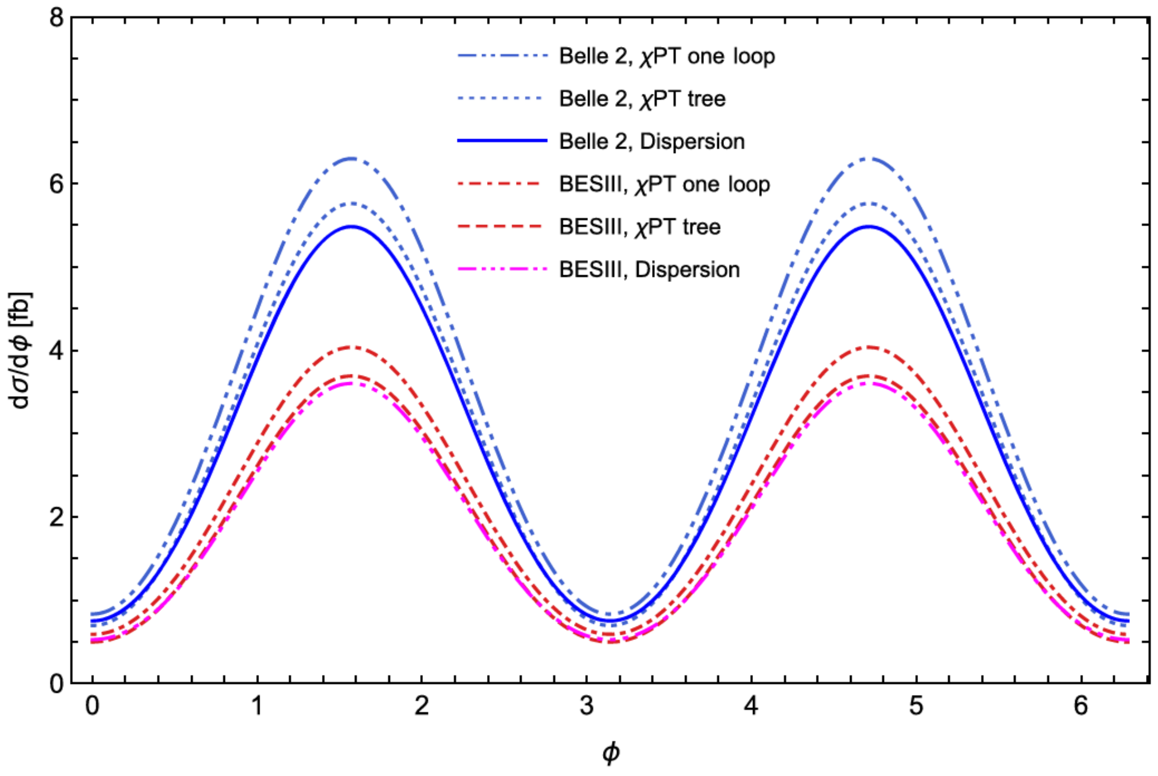


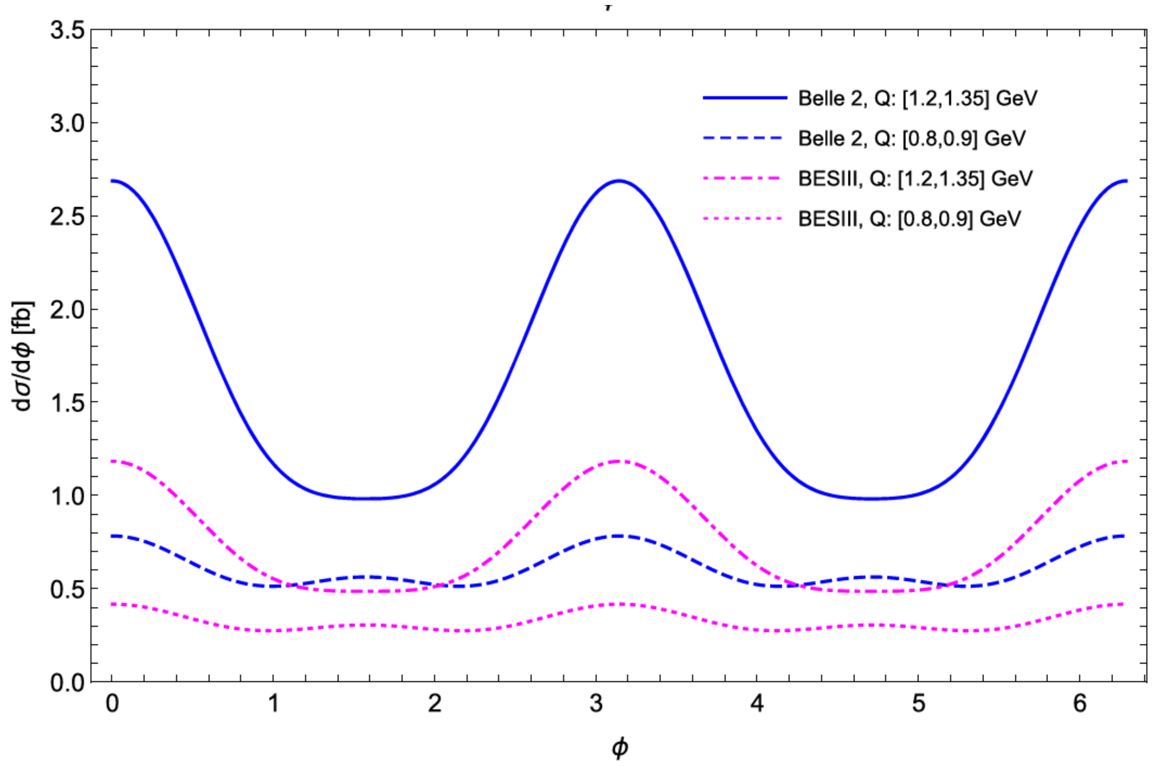
(f)

双光子过程中的ϕ调制：实验上如何观测ϕ调制

对$\gamma\gamma \to \pi\pi$过程的理论预言：

$0.35 < Q < 0.4 GeV$





双光子过程中的ϕ调制：从$\pi\pi$到$p\bar{p}$

光子偏振部分二者完全相同:

$$|\gamma(\phi)\rangle = \frac{1}{\sqrt{2}}(e^{-i\phi}|+\rangle + e^{i\phi}|-\rangle)$$

但是由于二者自旋不同$J(\pi) = 0, \mathrm{J(p)} = 1/2$，振幅不再是简单的$M_{\lambda_1\lambda_2}$，要拓展为:

$$M_{\lambda_1\lambda_2,\lambda_p\lambda\bar{p}}$$

涉及到的干涉可能会更复杂。

HadroTOPS：

HadroTOPS MC产生：

1. 基础理论
2. $\gamma^*\gamma^* \to \pi\pi$振幅
1. 双光子过程亮度函数

HADROTOPS: A Monte Carlo Event Generator For Hadron Production In Two-Photon Scattering In Electron Positron Collisions

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HadroTOPS：基础理论

从最一般的情况开始:

$$e^+(p_1)+e^-(p_2)\rightarrow e^+(p_1')+e^-(p_2')+X(p_x)$$

用helicity可以将振幅写成:

$$\sum_{h_1',h_2'}|\mathcal{M}|^2=\frac{e^4}{Q_1^2Q_2^2}\sum_{\lambda_1,\lambda_2,\lambda_1',\lambda_2'}\rho_{h_1}^{\lambda_1\lambda_1'}\rho_{h_2}^{\lambda_2\lambda_2'}H_{\lambda_1\lambda_2}H^*_{\lambda_1'\lambda_2'}$$

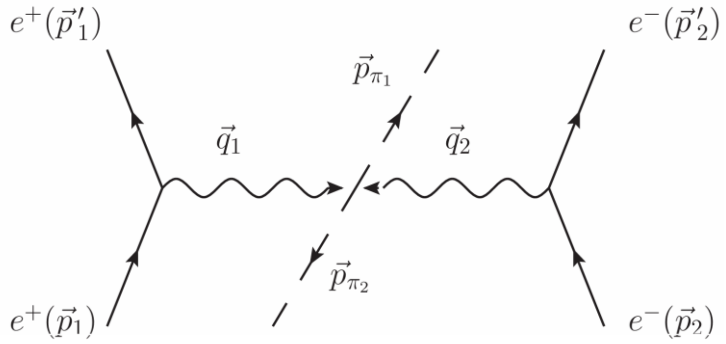
这里ρ是光子密度矩阵，描述电子辐射光子部分；H是表示$\gamma^*\gamma^*\rightarrow X$部分的张量。h是初态电子的helicity，做求和处理；λ是光子的helicity，$\lambda=0,\pm1$。
进一步可以得到:

$$\begin{aligned}
d\sigma_{h_1,h_2} \;=\; & \frac{\alpha^2}{8\pi^4\,Q_1^2\,Q_2^2}\,\frac{\sqrt{X}}{s(1-4m^2/s)^{1/2}}\,\frac{d^3\vec{p}_1'}{E_1'}\,\frac{d^3\vec{p}_2'}{E_2'}\Big\{4\rho_1^{++}\rho_2^{++}\,\frac{1}{2}(\sigma_0+\sigma_2)+\rho_1^{00}\rho_2^{00}\,\sigma_{LL}\\
&+2\rho_1^{++}\rho_2^{00}\,\sigma_{TL}+2\rho_1^{00}\rho_2^{++}\,\sigma_{LT}+2\,(\rho_1^{++}-1)\,(\rho_2^{++}-1)\,\cos\big(2\tilde{\phi}\big)\,\tau_{TT}\\
&+8\left[\big(\rho_1^{00}+1\big)\,\big(\rho_2^{00}+1\big)(\rho_1^{++}-1)\,(\rho_2^{++}-1)\right]^{1/2}\cos\tilde{\phi}\,\frac{1}{2}(\tau_0+\tau_1)\\
&+h_1h_2\,4\left[\big(\rho_1^{00}+1\big)\,\big(\rho_2^{00}+1\big)\right]^{1/2}\frac{1}{2}(\sigma_0-\sigma_2)\\
&+h_1h_2\,8\left[(\rho_1^{++}-1)\,(\rho_2^{++}-1)\right]^{1/2}\cos\tilde{\phi}\,\frac{1}{2}(\tau_0-\tau_1)\Big\},
\end{aligned}$$

$\tilde{\phi}$表示两个轻子截面的ϕ角差。

HadroTOPS：$\gamma^*\gamma^* \to \pi\pi$

在双光子参考系中定义$\phi_\pi = 0$，$\tilde{\phi}_1, \tilde{\phi}_2$为$e^+, e^-$平面的方位角，$\tilde{\phi}$为二者相角差。



$ee \to ee\pi\pi$过程完整的微分界面可以表示为：

$$d\sigma_{h_1,h_2} = d\sigma^{(0)} + h_1\, d\sigma^{(1)} + h_2\, d\sigma^{(2)} + h_1\, h_2\, d\sigma^{(12)}$$

$d\sigma^{(0)}$是我们关心的与入射轻子极化无关的微分界面：

$$\begin{aligned}
d\sigma^{(0)} = {} & \frac{\alpha^2}{8\pi^4 Q_1^2 Q_2^2} \frac{\sqrt{X}}{s(1-4m^2/s)^{1/2}} \frac{d^3\vec{p}_1'}{E_1'} \frac{d^3\vec{p}_2'}{E_2'} d\cos\theta_\pi \frac{4}{(1-\varepsilon_1)(1-\varepsilon_2)} \\
& \times \left\{ \frac{1}{2}\left(\frac{d\sigma_0}{d\cos\theta_\pi} + \frac{d\sigma_2}{d\cos\theta_\pi}\right) + \left[\varepsilon_1 + \frac{2m^2}{Q_1^2}(1-\varepsilon_1)\right]\left[\varepsilon_2 + \frac{2m^2}{Q_2^2}(1-\varepsilon_2)\right] \frac{d\sigma_{LL}}{d\cos\theta_\pi} \right. \\
& + \left[\varepsilon_2 + \frac{2m^2}{Q_2^2}(1-\varepsilon_2)\right]\left(1+\varepsilon_1\cos\left(2\tilde{\phi}_1\right)\right) \frac{d\sigma_{TL}}{d\cos\theta_\pi} + \left[\varepsilon_1 + \frac{2m^2}{Q_1^2}(1-\varepsilon_1)\right]\left(1+\varepsilon_2\cos\left(2\tilde{\phi}_2\right)\right) \frac{d\sigma_{LT}}{d\cos\theta_\pi} \\
& + \frac{1}{2}\varepsilon_1\varepsilon_2 \left[\cos 2(\tilde{\phi}_2 - \tilde{\phi}_1)\frac{d\sigma_0}{d\cos\theta_\pi} + \cos 2(\tilde{\phi}_1 + \tilde{\phi}_2)\frac{d\sigma_2}{d\cos\theta_\pi}\right] - \left[\varepsilon_1\cos\left(2\tilde{\phi}_1\right) + \varepsilon_2\cos\left(2\tilde{\phi}_2\right)\right] \frac{d\tau_{T2}}{d\cos\theta_\pi} \\
& + \left[\varepsilon_1(1+\varepsilon_1) + \frac{4m^2}{Q_1^2}\varepsilon_1(1-\varepsilon_1)\right]^{1/2} \left[\varepsilon_2(1+\varepsilon_2) + \frac{4m^2}{Q_2^2}\varepsilon_2(1-\varepsilon_2)\right]^{1/2} \\
& \times \left[\cos\left(\tilde{\phi}_2 - \tilde{\phi}_1\right)\left(\frac{d\tau_0}{d\cos\theta_\pi} + \frac{d\tau_1}{d\cos\theta_\pi}\right) + \cos\left(\tilde{\phi}_1 + \tilde{\phi}_2\right)\left(\frac{d\tau_1}{d\cos\theta_\pi} - \frac{d\tau_{L2}}{d\cos\theta_\pi}\right)\right] \\
& + \left[\varepsilon_1(1+\varepsilon_1) + \frac{4m^2}{Q_1^2}\varepsilon_1(1-\varepsilon_1)\right]^{1/2} \left[\cos\tilde{\phi}_1\left(\frac{d\tau_{-12}}{d\cos\theta_\pi} - \frac{d\tau_{-1T}}{d\cos\theta_\pi}\right)\right. \\
& \qquad - 2\left[\varepsilon_2 + \frac{2m^2}{Q_2^2}(1-\varepsilon_2)\right]\cos\tilde{\phi}_1 \frac{d\tau_{1L}}{d\cos\theta_\pi} \\
& \qquad \left. + \varepsilon_2\cos\left(\tilde{\phi}_1 + 2\tilde{\phi}_2\right)\frac{d\tau_{-12}}{d\cos\theta_\pi} - \varepsilon_2\cos\left(2\tilde{\phi}_2 - \tilde{\phi}_1\right)\frac{d\tau_{-1T}}{d\cos\theta_\pi}\right] \\
& + \left[\varepsilon_2(1+\varepsilon_2) + \frac{4m^2}{Q_2^2}\varepsilon_2(1-\varepsilon_2)\right]^{1/2} \left[\cos\tilde{\phi}_2\left(\frac{d\tau_{12}}{d\cos\theta_\pi} - \frac{d\tau_{1T}}{d\cos\theta_\pi}\right)\right. \\
& \qquad - 2\left[\varepsilon_1 + \frac{2m^2}{Q_1^2}(1-\varepsilon_1)\right]\cos\tilde{\phi}_2 \frac{d\tau_{-1L}}{d\cos\theta_\pi} \\
& \qquad \left.\left. + \varepsilon_1\cos\left(2\tilde{\phi}_1 + \tilde{\phi}_2\right)\frac{d\tau_{12}}{d\cos\theta_\pi} - \varepsilon_1\cos\left(2\tilde{\phi}_1 - \tilde{\phi}_2\right)\frac{d\tau_{1T}}{d\cos\theta_\pi}\right]\right\}.
\end{aligned}$$

还有在single tag情况下（limit $Q_2^2 \to 0$）

$$\begin{aligned}
d\sigma^{(0)}|_{Q_2^2\to 0} = {} & \frac{\alpha^2}{8\pi^4 Q_1^2 Q_2^2} \frac{\sqrt{X}}{s(1-4m^2/s)^{1/2}} \frac{d^3\vec{p}_1'}{E_1'} \frac{d^3\vec{p}_2'}{E_2'} d\cos\theta_\pi \frac{4}{(1-\varepsilon_1)(1-\varepsilon_2)} \\
& \times \left\{ \frac{1}{2}\left(\frac{d\sigma_0}{d\cos\theta_\pi} + \frac{d\sigma_2}{d\cos\theta_\pi}\right) + \left[\varepsilon_1 + \frac{2m^2}{Q_1^2}(1-\varepsilon_1)\right] \frac{d\sigma_{LT}}{d\cos\theta_\pi} - \varepsilon_1\cos\left(2\tilde{\phi}_1\right)\frac{d\tau_{T2}}{d\cos\theta_\pi} \right. \\
& \left. + \left[\varepsilon_1(1+\varepsilon_1) + \frac{4m^2}{Q_1^2}\varepsilon_1(1-\varepsilon_1)\right]^{1/2} \cos\tilde{\phi}_1\left(\frac{d\tau_{-12}}{d\cos\theta_\pi} - \frac{d\tau_{-1T}}{d\cos\theta_\pi}\right)\right\}
\end{aligned}$$

HadroTOPS：双光子亮度公式

HadroTOPS将总微分截面根据极化类型分成不同极化情况下，亮度和光光作用截面乘积的和：

$$\frac{d^3\sigma}{dW\,dQ_1^2\,dQ_2^2} = \frac{d^3L_{\mathrm{TT}}}{dW\,dQ_1^2\,dQ_2^2}\sigma_{\mathrm{TT}} + \frac{d^3L_{\mathrm{LL}}}{dW\,dQ_1^2\,dQ_2^2}\sigma_{\mathrm{LL}} + \frac{d^3L_{\mathrm{TL}}}{dW\,dQ_1^2\,dQ_2^2}\sigma_{\mathrm{TL}} + \frac{d^3L_{\mathrm{LT}}}{dW\,dQ_1^2\,dQ_2^2}\sigma_{\mathrm{LT}} + \frac{d^3\tilde{L}_{\mathrm{TT}}}{dW\,dQ_1^2\,dQ_2^2}\tau_{\mathrm{TT}} + \frac{d^3\tilde{L}_{\mathrm{TL}}}{dW\,dQ_1^2\,dQ_2^2}\frac{1}{2}(\tau_0 + \tau_1)$$