

Study of $\gamma^* \gamma^* \rightarrow p \bar{p}$

Kedi Hao (USTC)

MC Generation

➤ Yield Estimation of $\gamma\gamma \rightarrow p\bar{p}$ at 3.773GeV

$$N_{ee \rightarrow p\bar{p}} = \int \sigma_{\gamma\gamma \rightarrow p\bar{p}}(W_{\gamma\gamma}) * L_{int} * \frac{dL_{\gamma\gamma}(W_{\gamma\gamma}, E_{cms})}{dW_{\gamma\gamma}} dW_{\gamma\gamma}$$

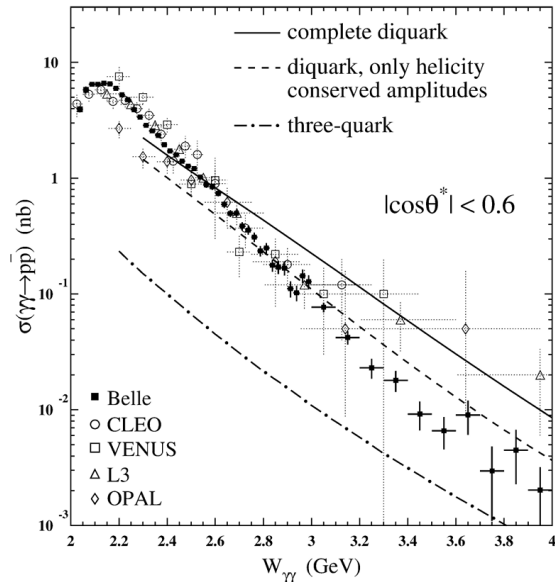


Fig. $\sigma_{\gamma\gamma \rightarrow p\bar{p}}(W_{\gamma\gamma})$ Belle Collaboration Physics Letters B 621 (2005) 41–55

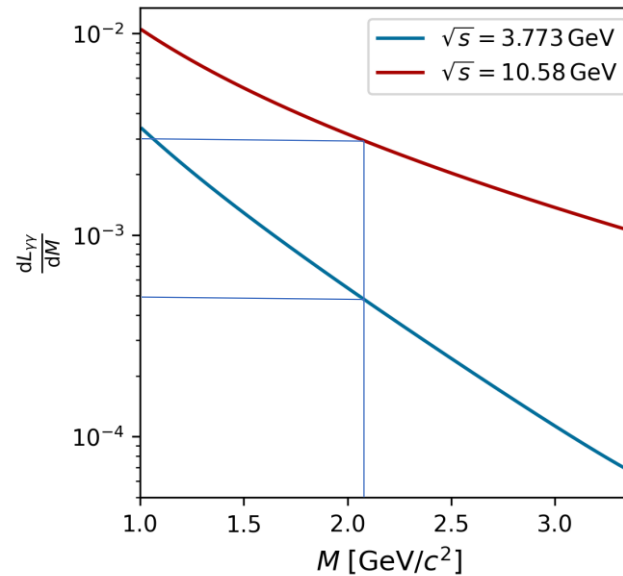


Fig. $\frac{dL_{\gamma\gamma}(W_{\gamma\gamma})}{dW_{\gamma\gamma}}$ calculation.

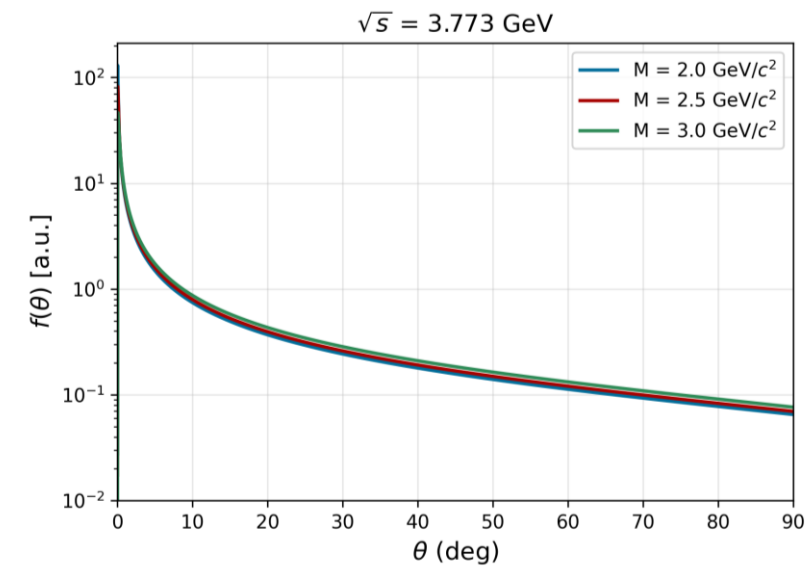
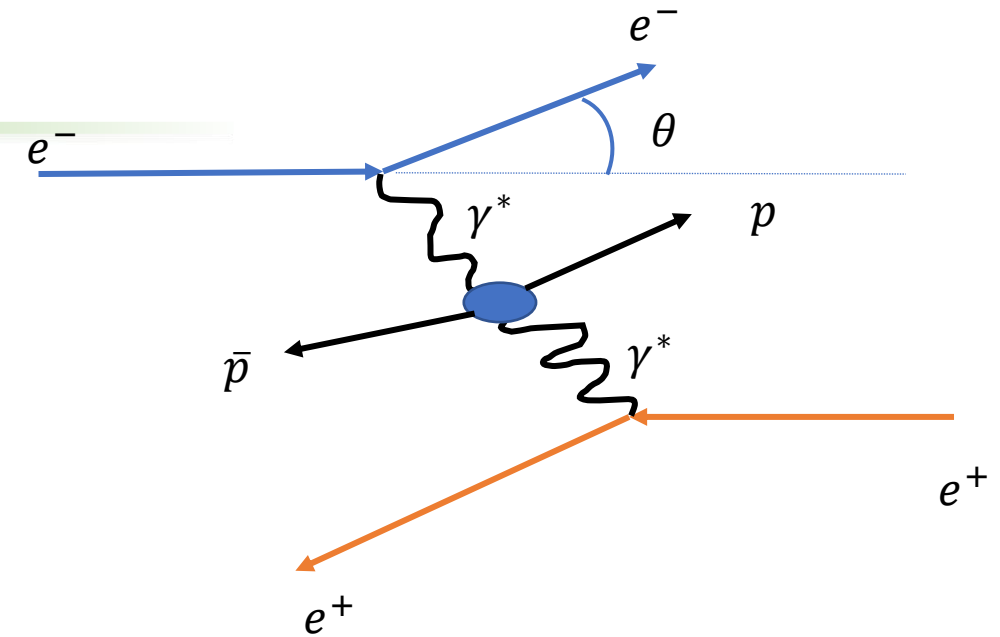


Fig. angular distribution of electron

$P(\cos\theta > 0.92)$:
0.84 for 2.0GeV
0.82 for 2.5GeV
0.81 for 3.0GeV

Untag:0.69
Single:0.28
Double:0.03

MC Generation

➤ Yield Estimation of $\gamma\gamma \rightarrow p\bar{p}$ at 3.773 GeV

Angle distribution of $p/\bar{p}$ in $\gamma\gamma \rightarrow p\bar{p}$

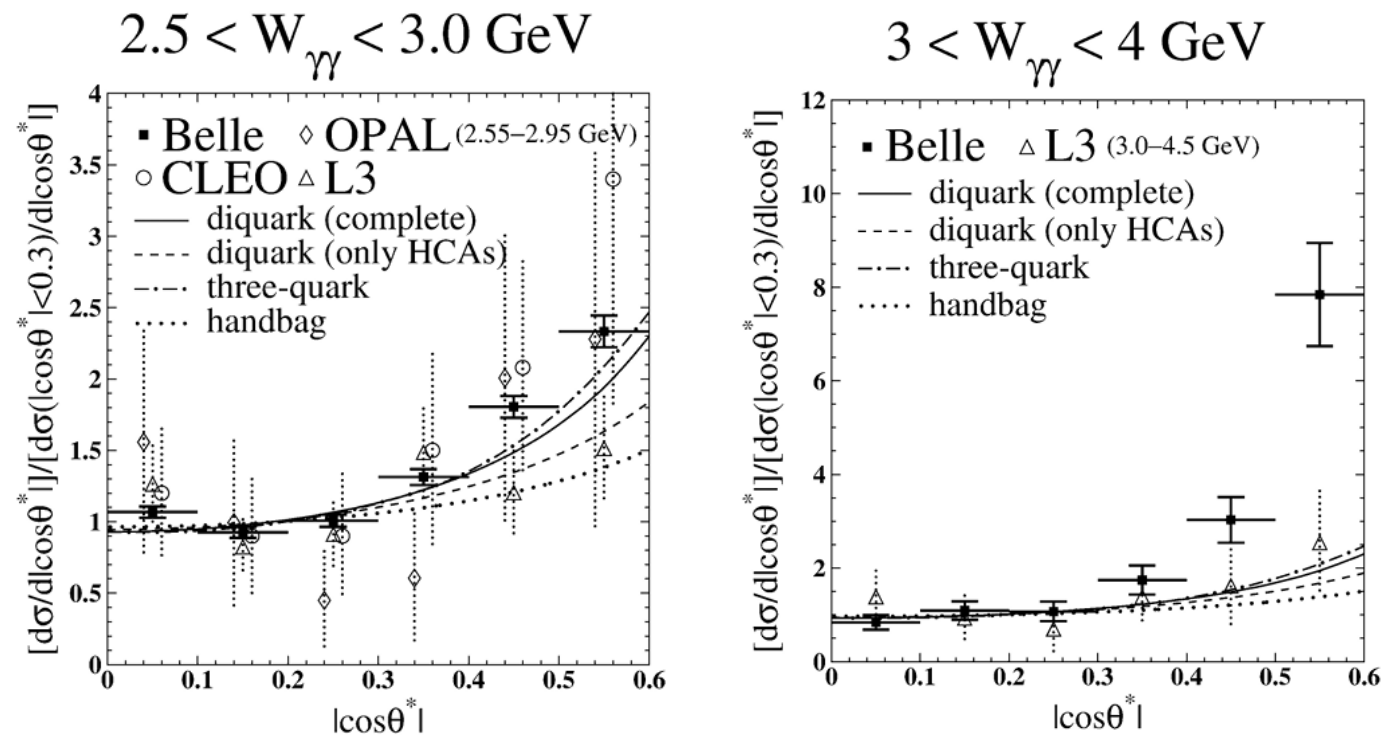


Fig. Differential cross section as a function of $|\cos\theta^*|$.

MC Generation

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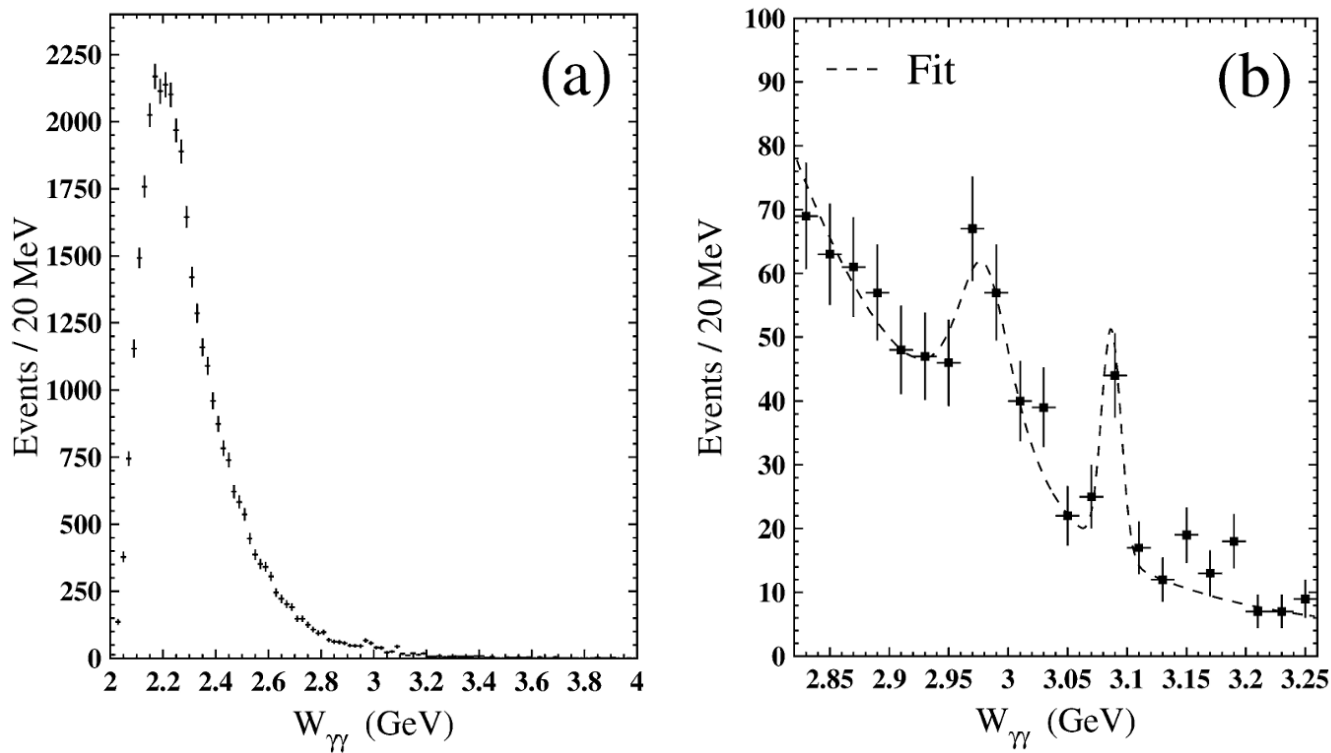


Fig. $W_{\gamma\gamma}$ distribution of events passing all the selection criteria.

Belle 89fb^{-1} at 10.58GeV; $W_{\gamma\gamma} = 2 - 4\text{GeV}$, 36094 events

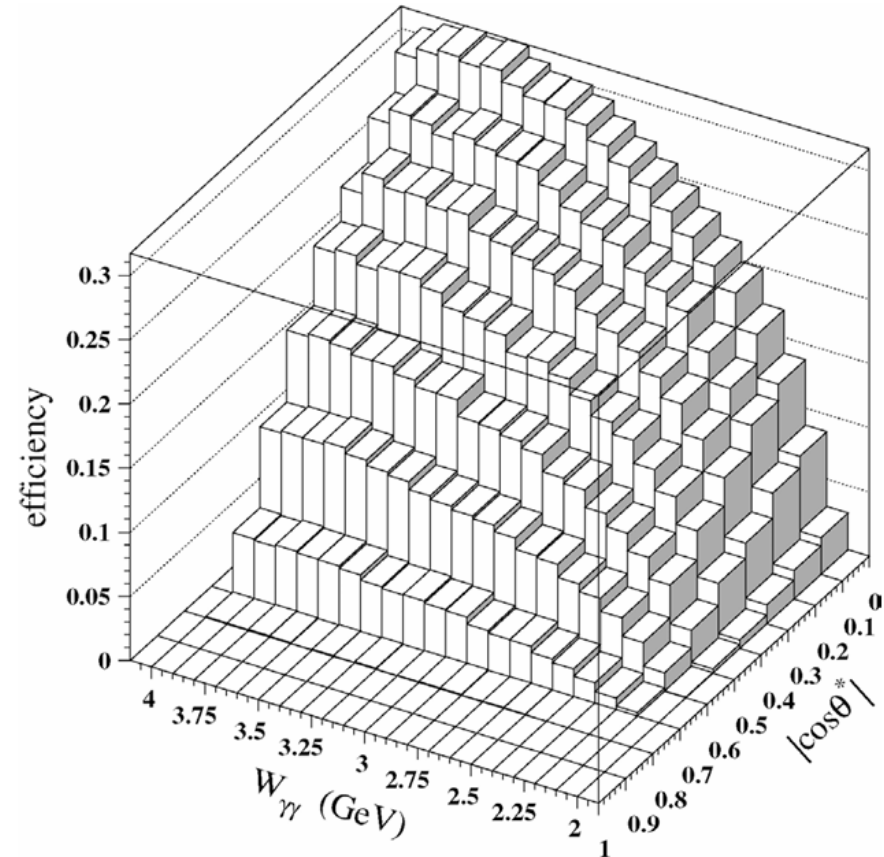


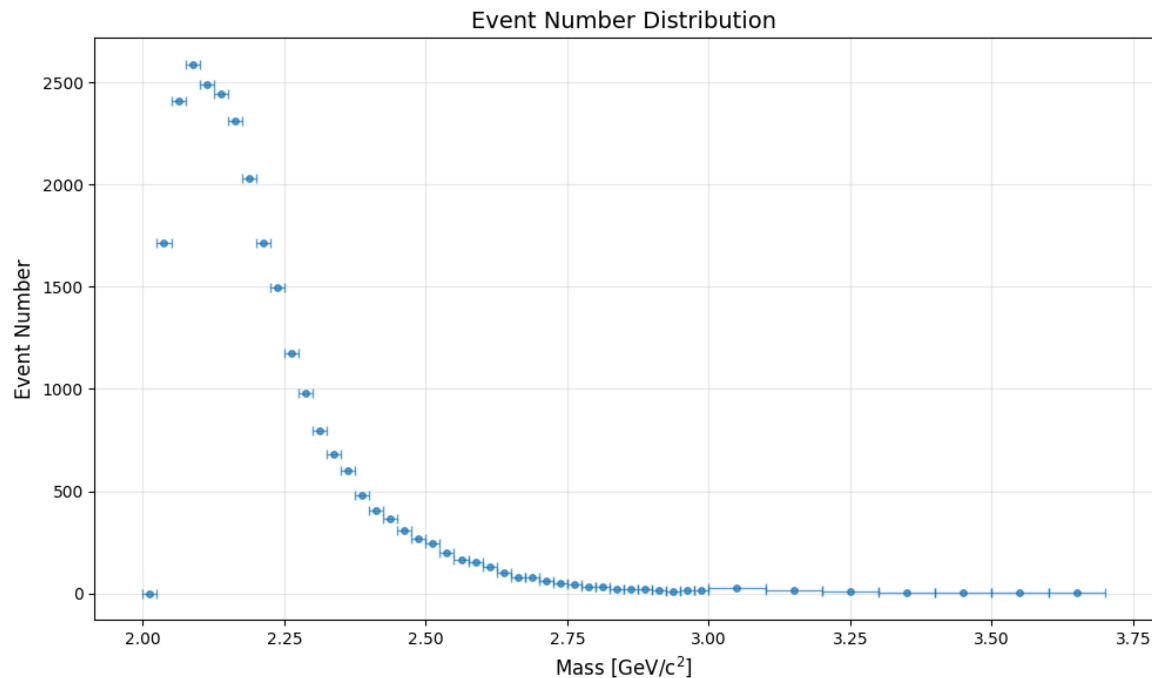
Fig. Overall detection efficiency of $\gamma\gamma \rightarrow p\bar{p}$ as a function of $W_{\gamma\gamma}$ and $|\cos\theta^*|$

MC Generation

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Based on the assumption that $\gamma\gamma \rightarrow p\bar{p}$ follows a phase space distribution.



$$\frac{L_{Belle} * \frac{dL}{dW}(Belle) * \epsilon_{Belle}}{L_{BES} * \frac{dL}{dW}(BES) * \epsilon_{BES}} = 4.5 * 5 * 0.2 = 4.5$$

预期:

Untag 10000 events

Stag 1000 events

MC Generation

➤ Two Generators

1. cppGamGam & HadroTOPS
2. use same $\sigma_{\gamma\gamma \rightarrow p\bar{p}}(W_{\gamma\gamma})$
3. Derive momentum of final particles

➤ Simulation: TrackGen

Cppgamgam:

权重=亮度x相空间因子x形状因子
x强子截面

HadroTOPS:

权重=亮度(TT)x相空间x截面($W, Q_1^2, Q_2^2, \cos\theta^*$))(TT)

+亮度(TL)x相空间x截面 (TL)

+亮度(LT)x相空间x截面 (LT)

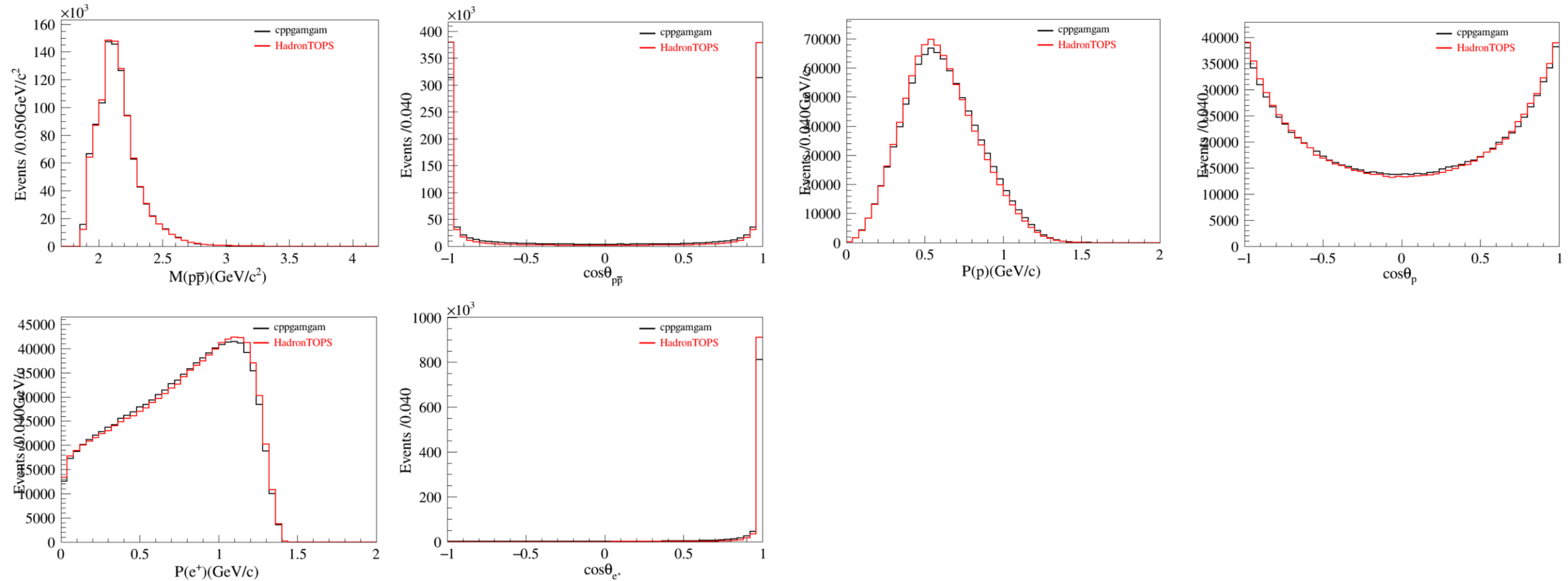
+亮度(LL)x相空间x截面 (LL)

+亮度(TT干涉)x相空间x截面 (TT干涉)

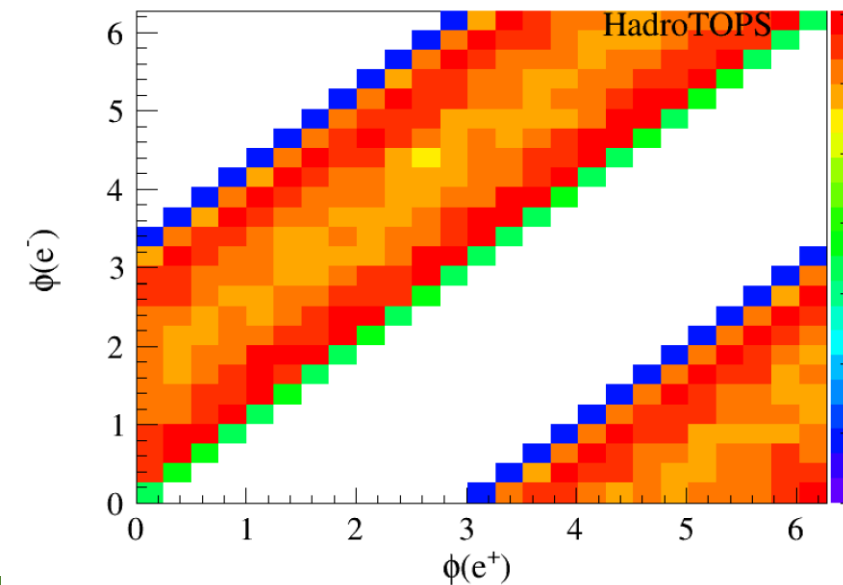
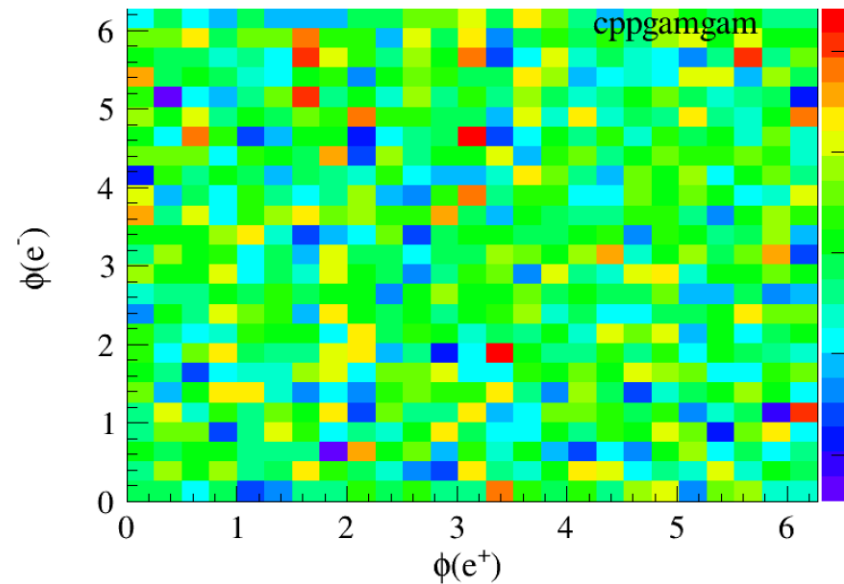
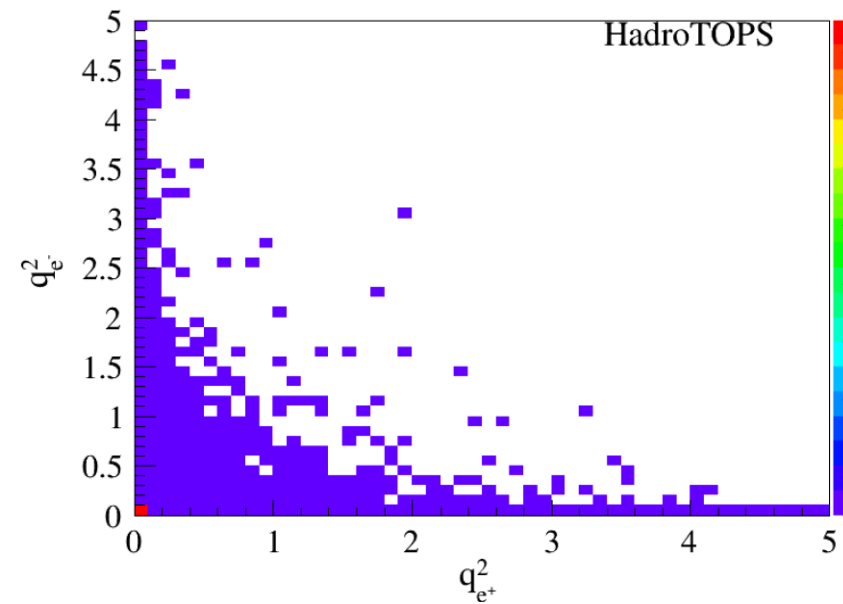
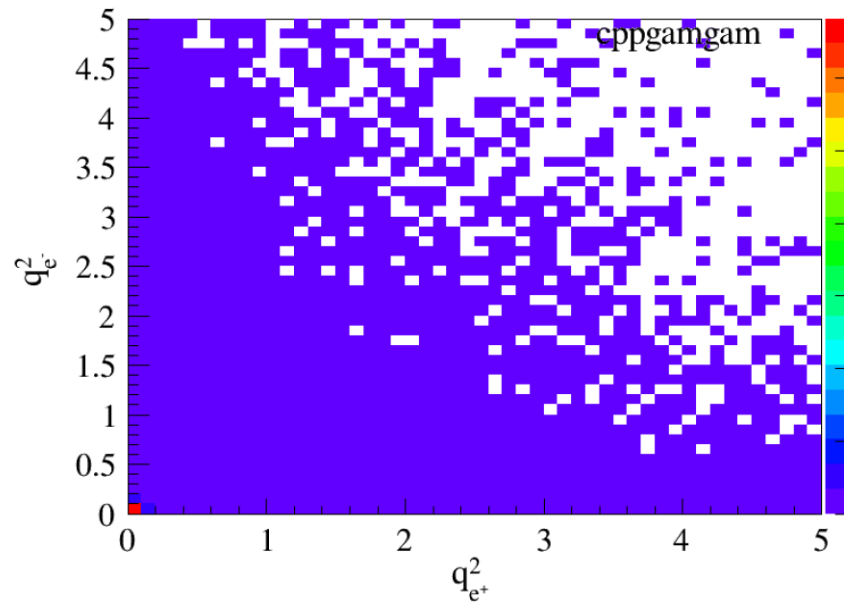
+亮度(TL干涉)x相空间x截面 (TL干涉)

Comparison of two Generators at truth level

Using the same input: Belle $\sigma_{\gamma\gamma\rightarrow p\bar{p}}(W_{\gamma\gamma})$ [PLB 621 (2005) 41], $(Q^2, \cos\theta^*)$ treated as phase space.



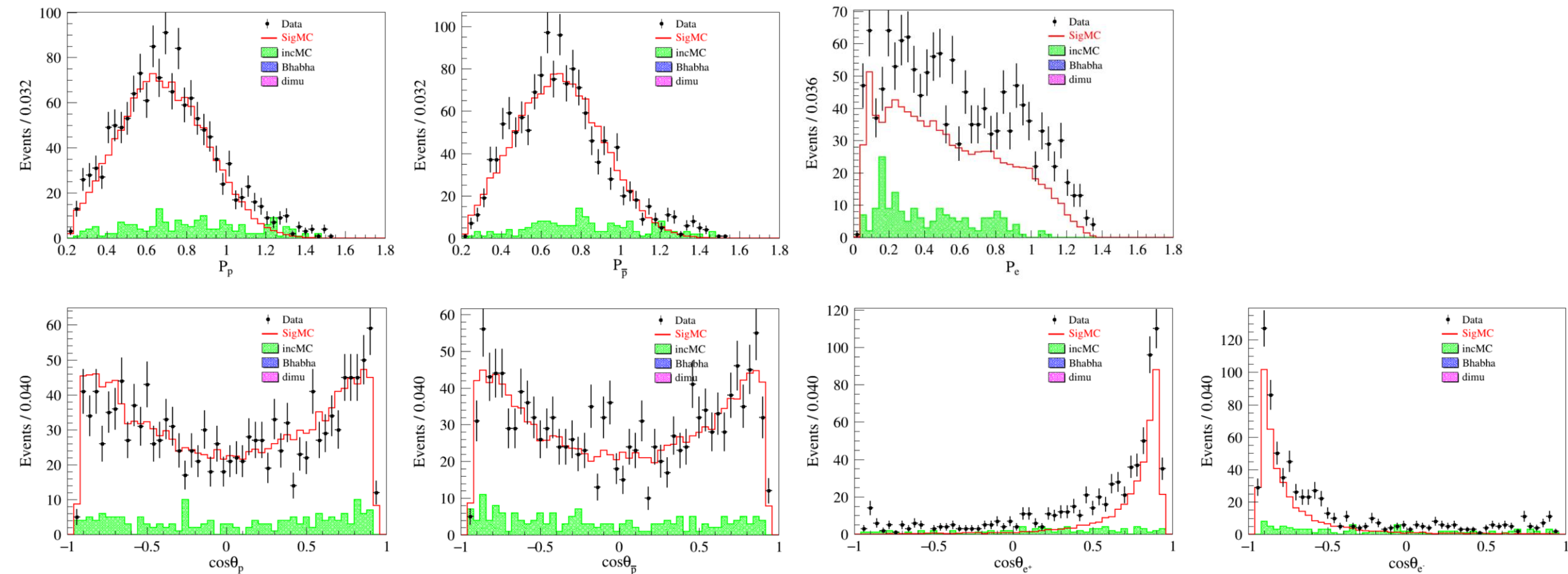
Comparison of two Generators at truth level



Comparison between data and cppgamgam MC.

$$\begin{aligned} \cos\theta_{miss} * \text{sign}(e) &< -0.99; \\ 20 < \theta(p, \bar{p}) &< 170; \\ 10 < \theta(p/\bar{p}, e) &< 170; \\ -0.1 < U_{miss} &< 0.1 \end{aligned}$$

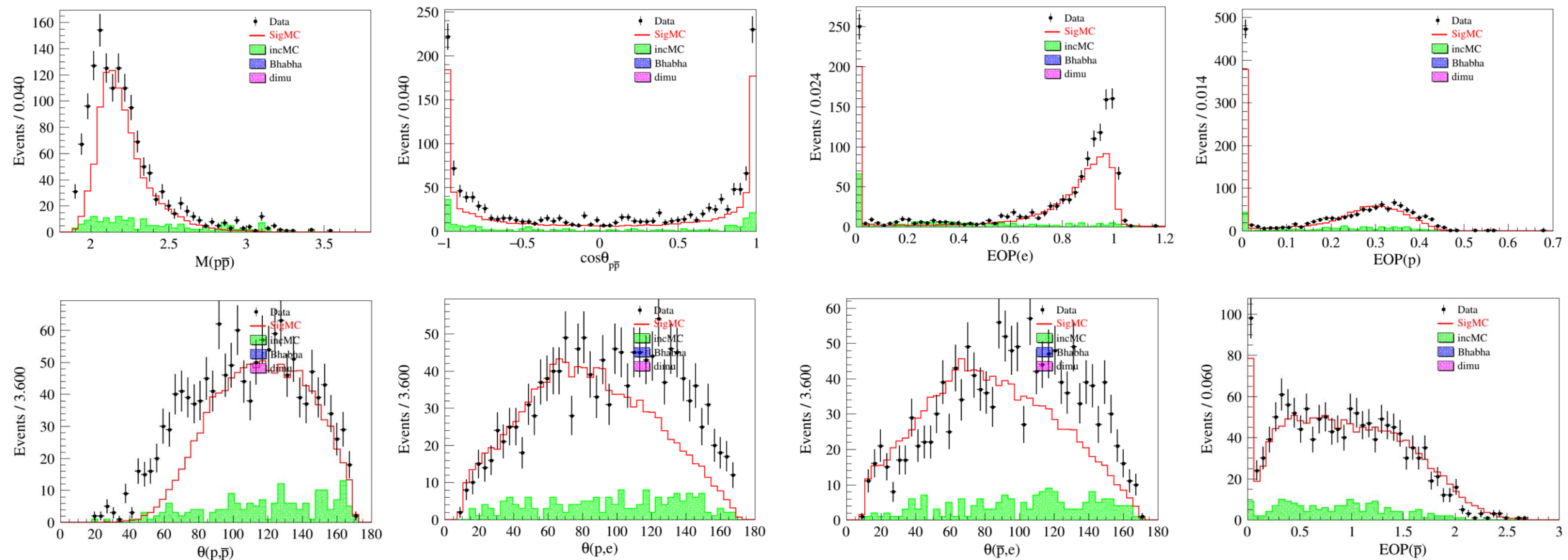
IncMC and Bhabha dimu were scaled by luminosity, sigMC was scaled to $0.8 * \text{data_max} / \text{sigMC_max}$.



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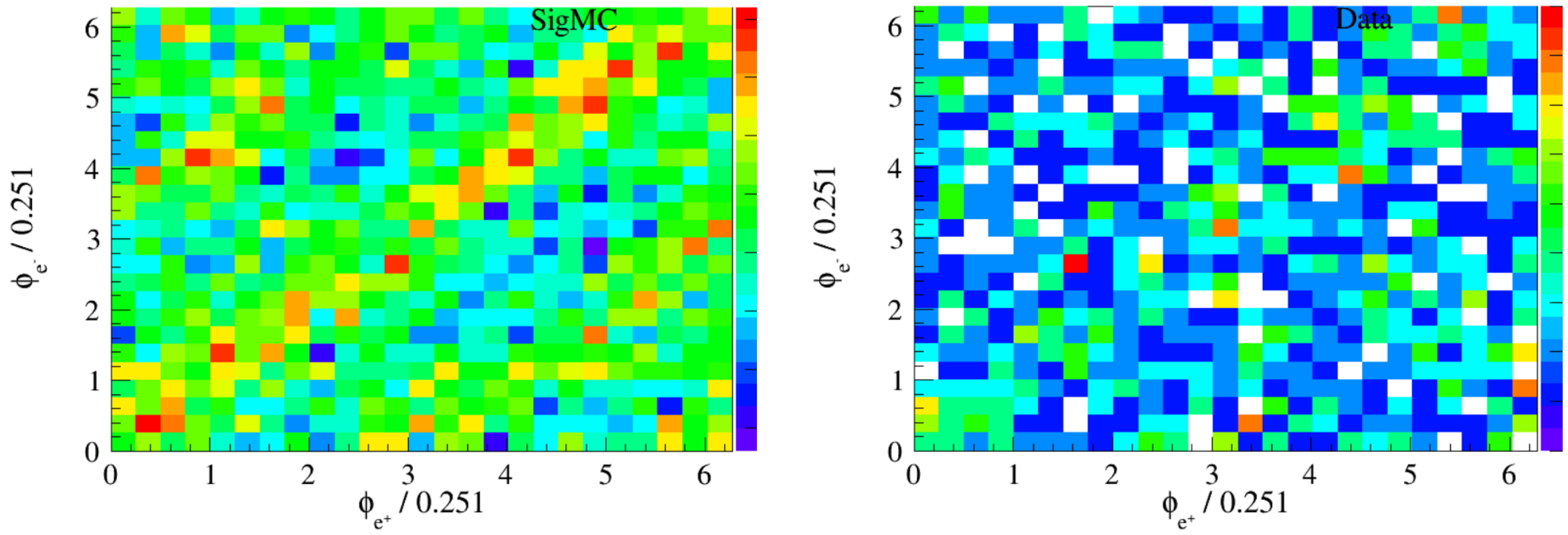
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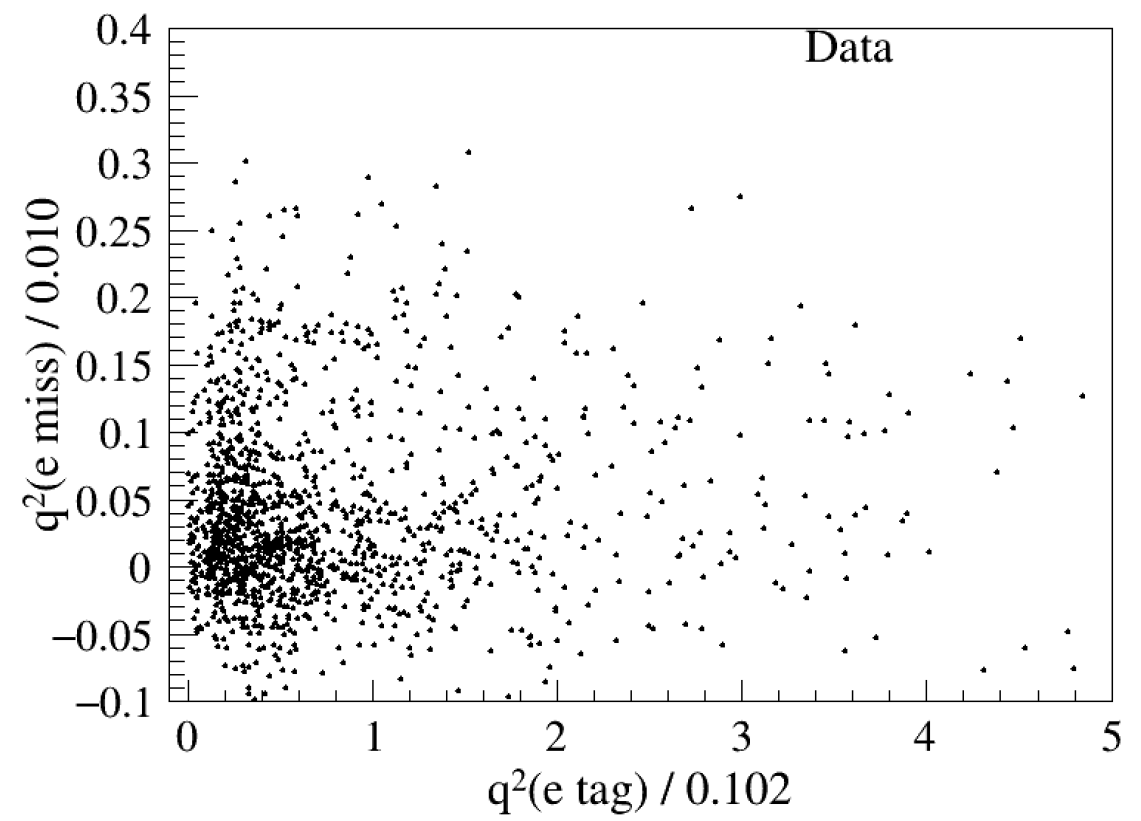
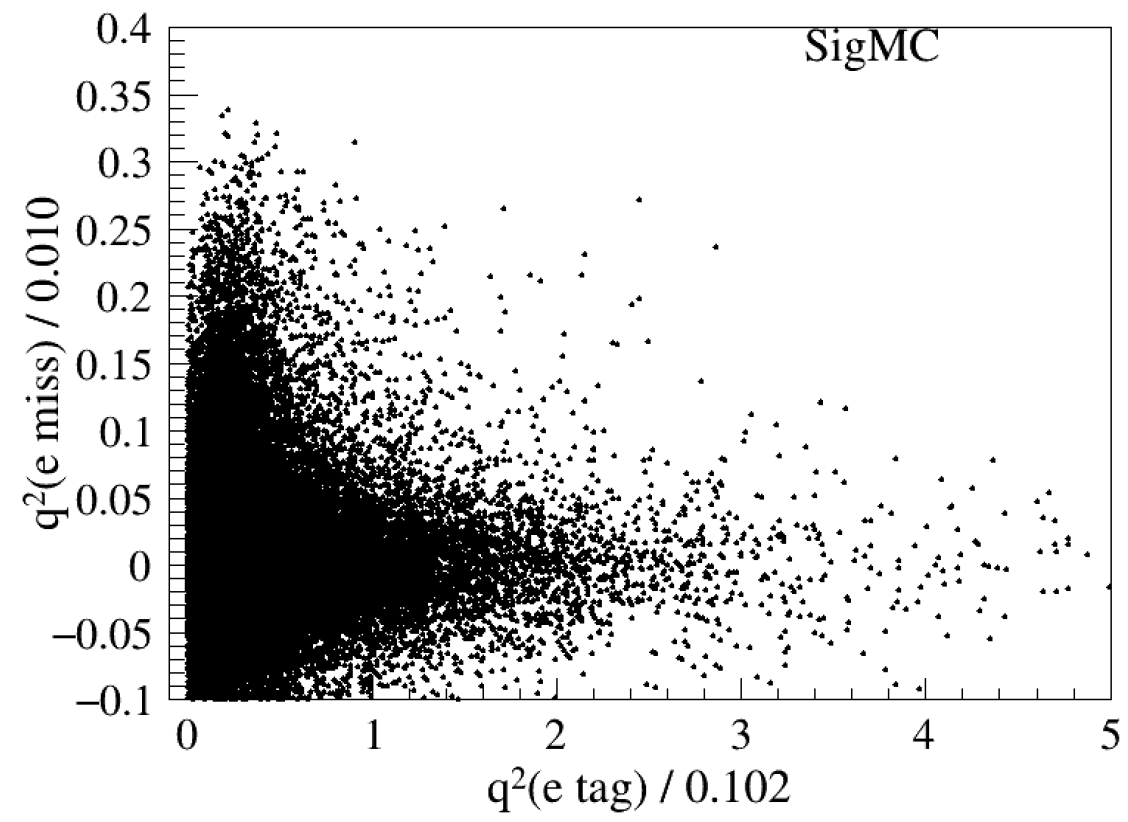


Faint structures can be observed. Since the distribution at the MC truth level (shown in page 6) is flat, this feature is likely caused by reconstruction effects.

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Faint structures can be observed. Since the distribution at the MC truth level (shown in page 6) is flat, this feature is likely caused by reconstruction effects.

Single tag

➤ Data and MC samples:

BOSS: 712

Data set: full data sat at 3.773GeV, 20.3fb⁻¹

IncMC sample: Hadron Hybrid; 0.25x Bhabha; 15x dimu; 1x $\gamma\gamma \rightarrow ee, \mu\mu, \pi\pi, KK, \eta, \eta'$

ExcMC sample: 1.5M $\gamma\gamma \rightarrow p\bar{p}$

➤ Event selection:

- Good charged track:

$$|V_{xy}| < 1cm, |V_z| < 10cm, |\cos\theta| < 0.93$$

$$N_{good} = 3, N_{charge} = \pm 1$$

- PID:

p: prob(p) > prob(e, μ , π , K), 0.001

e: prob(e) > prob(μ , π , K, p), 0.001

$$N_p = N_{\bar{p}} = N_e = 1$$

- Vertex Fit:

Vertex fit for $p\bar{p}e$, success;

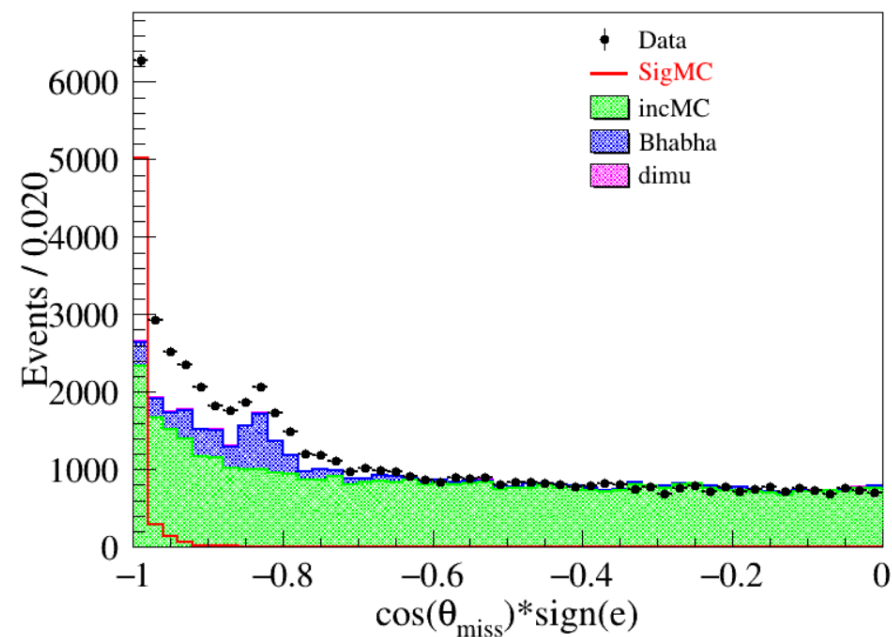
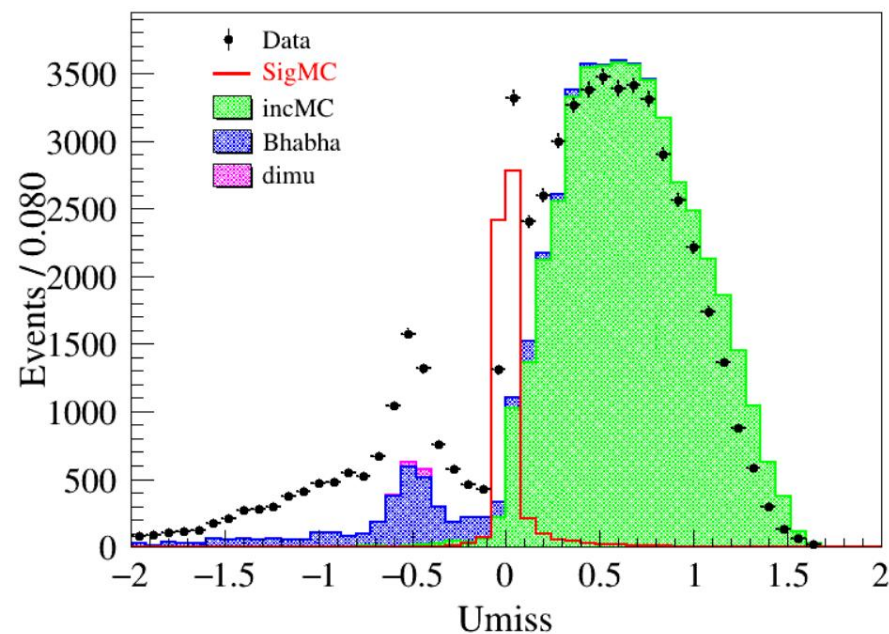
- Other:

$\cos\theta_{miss} \times \text{sign}(e) < 0$;

$$\cos\theta_{\text{miss}} * \text{sign}(e) < 0;$$

Further Event selection

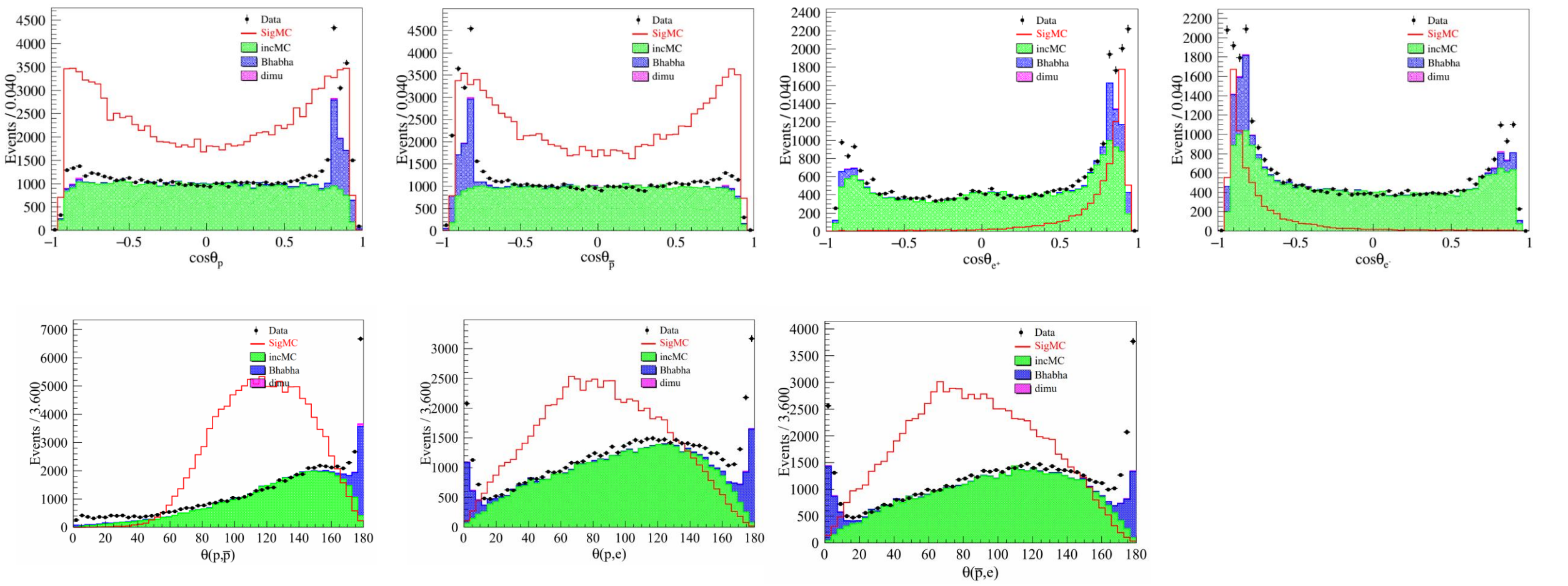
Miss 系统的质量和角度:



IncMC 主要成分: $p\bar{p} + n * \pi$

Further Event selection

$$\cos\theta_{miss} * \text{sign}(e) < 0;$$

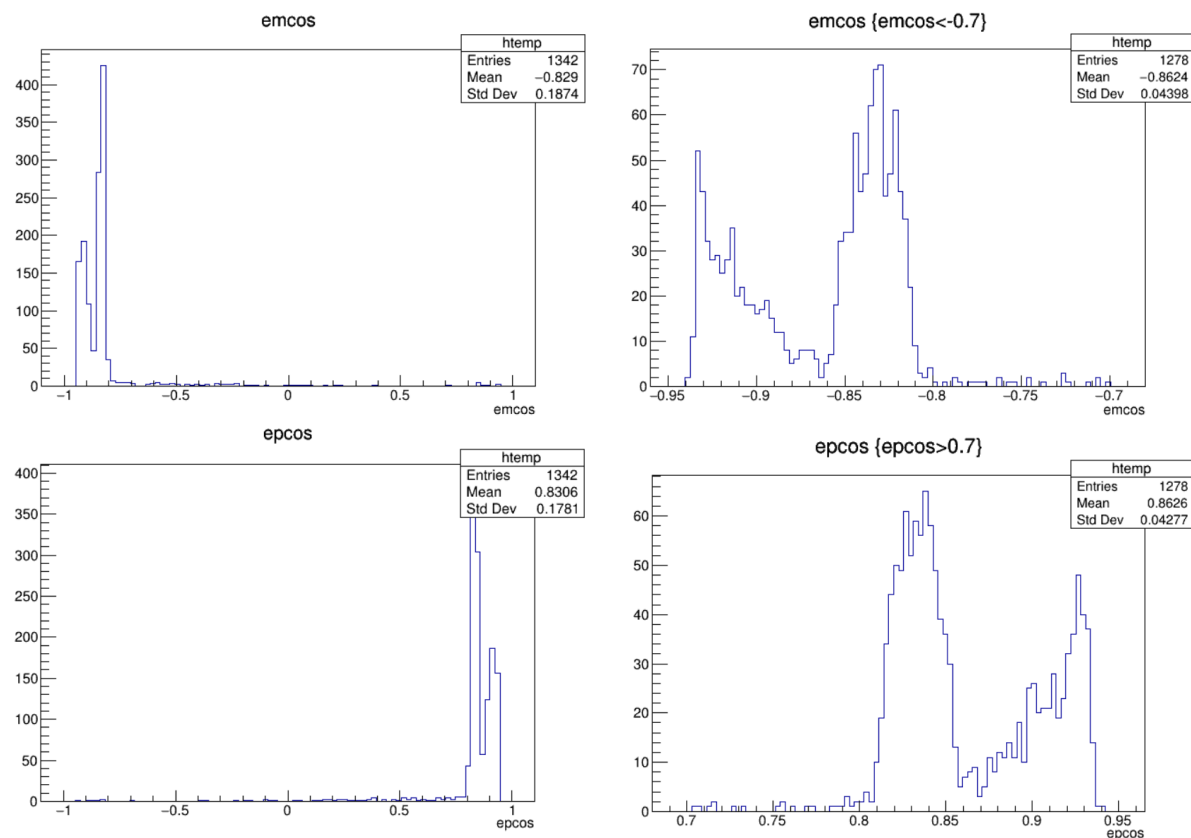


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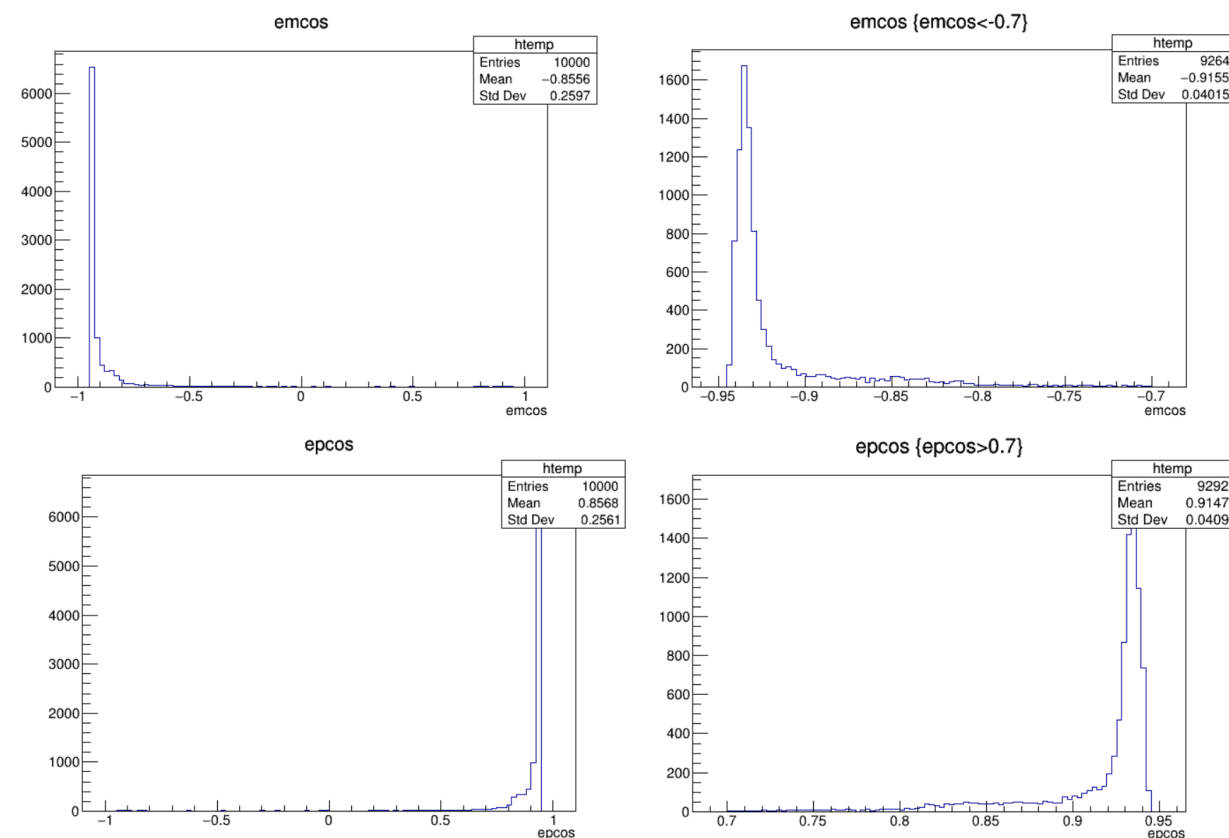
Further Event selection

为什么Bhabha MC描述不了数据：只产生了接收度范围的事例

事例挑选后Bhabha事例的Mctruth:



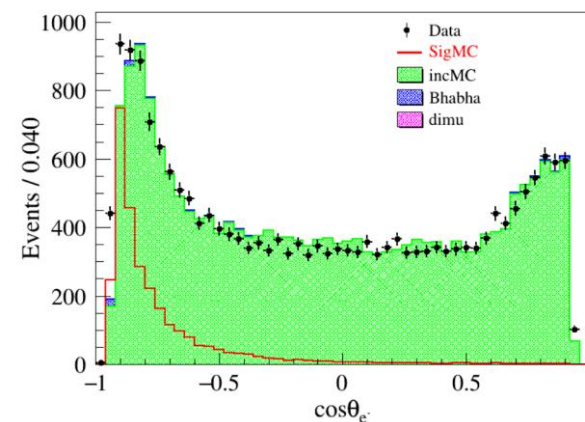
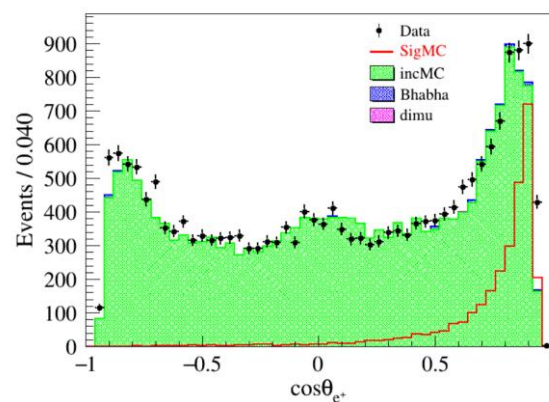
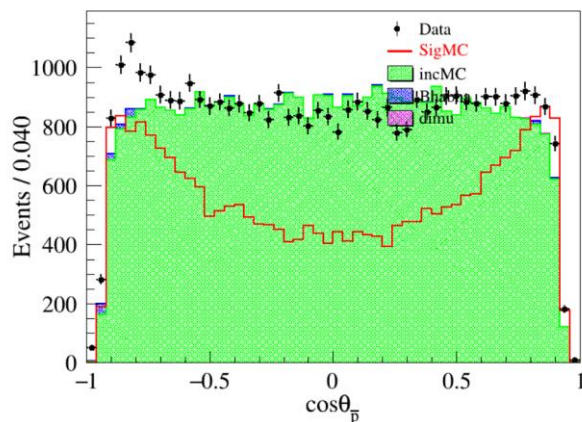
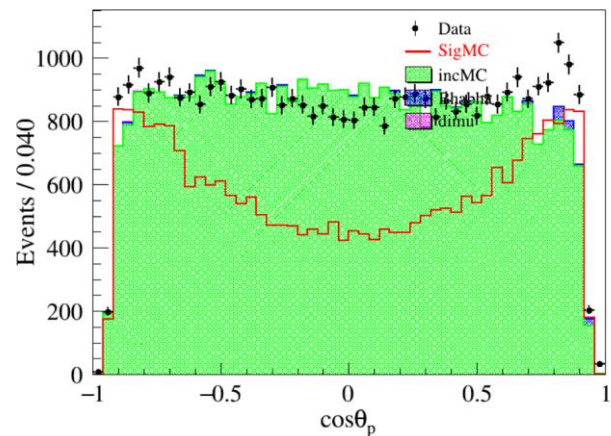
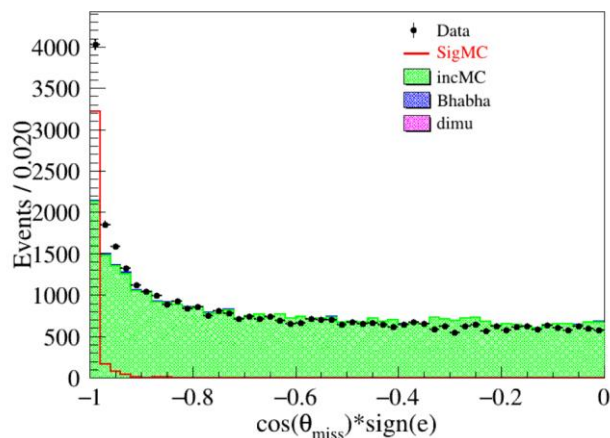
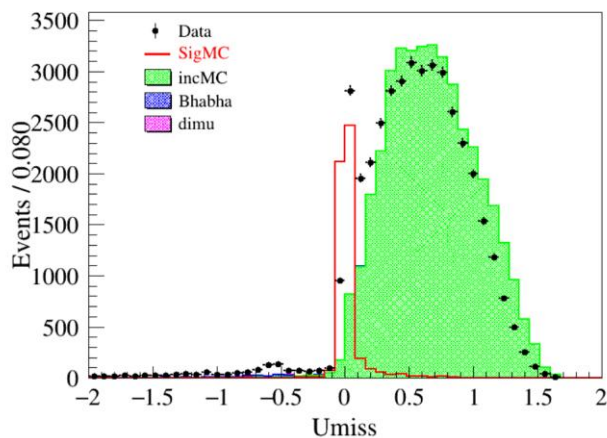
全部Bhabha事例的Mctruth:



Further Event selection

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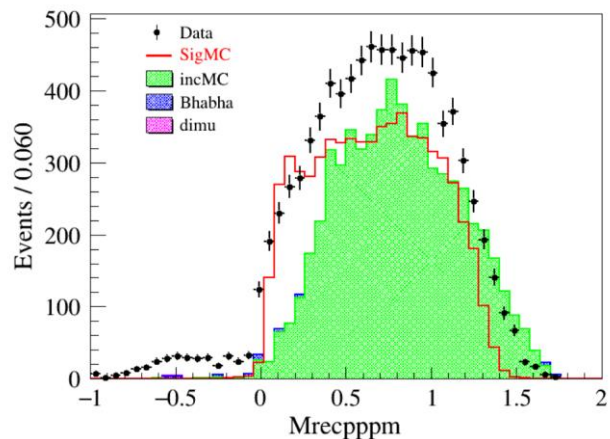
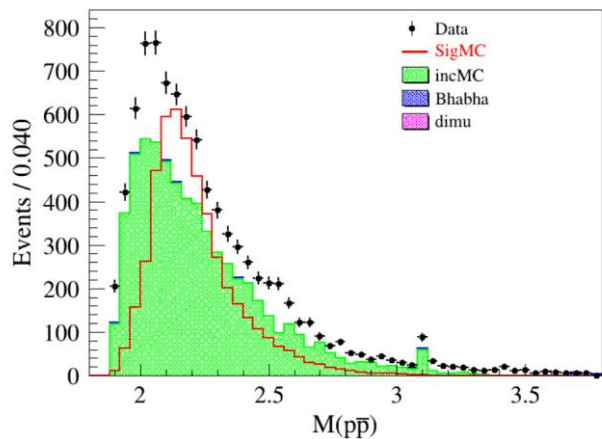
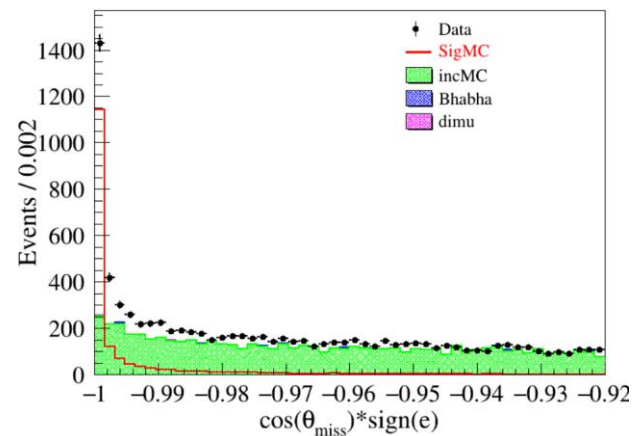
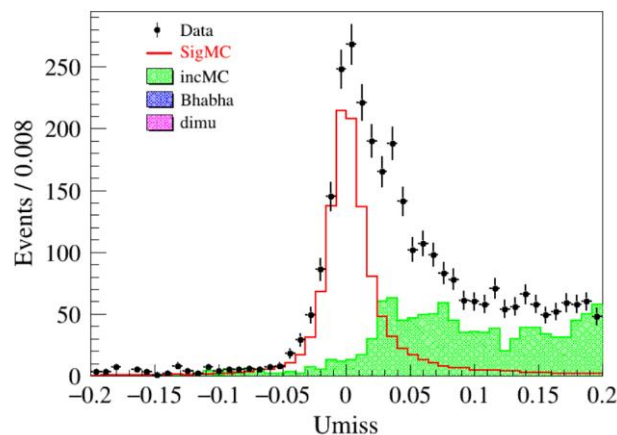
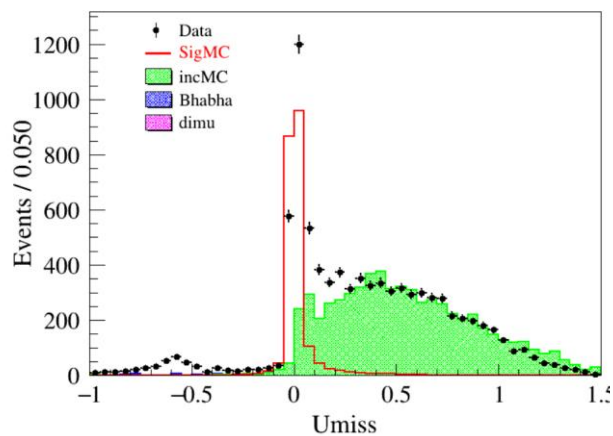
要求 $20 < \theta(p, \bar{p}) < 170$, $10 < \theta(p/\bar{p}, e) < 170$



Further Event selection

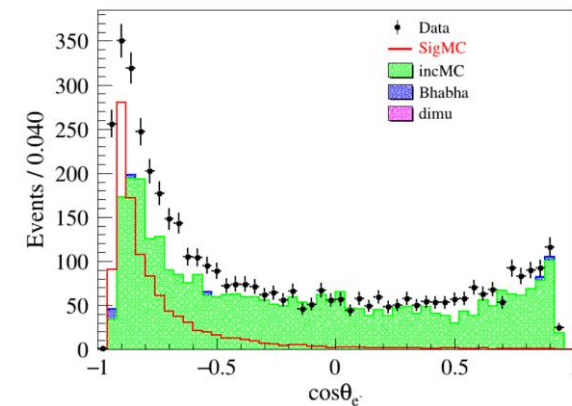
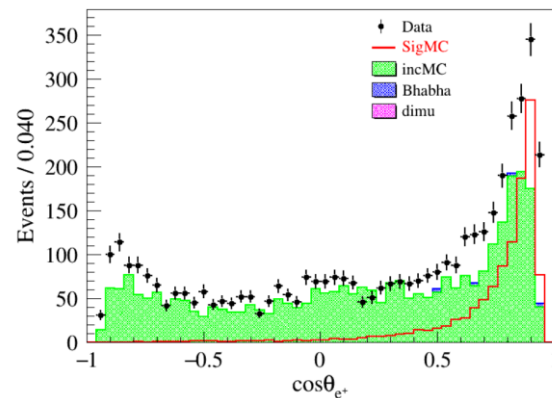
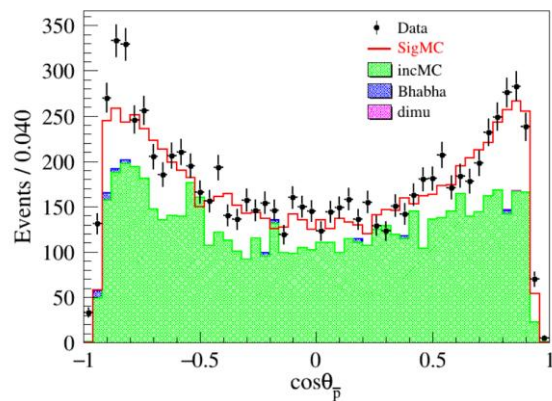
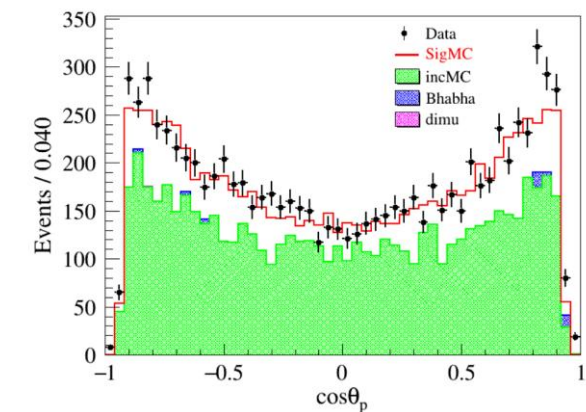
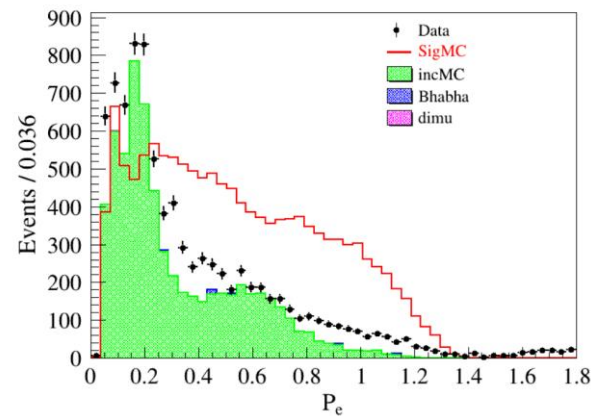
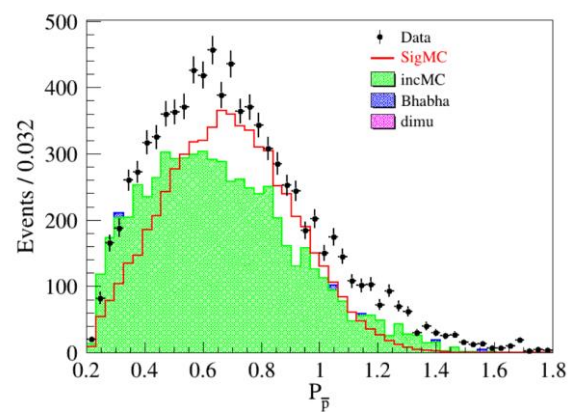
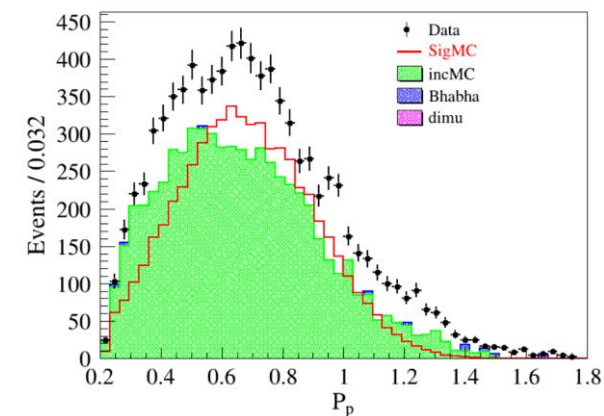
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要求 $\cos\theta_{\text{miss}} * \text{sign}(e) < -0.92$



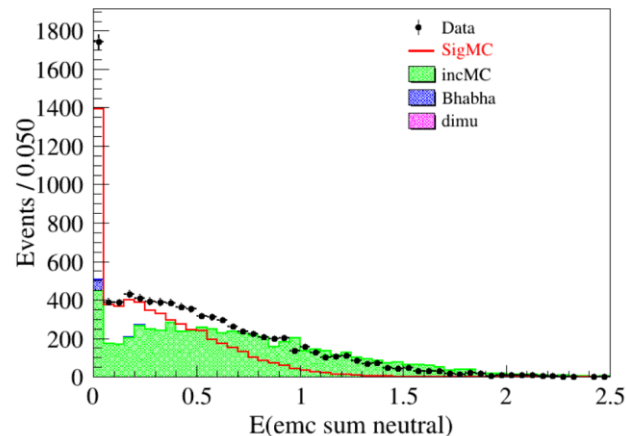
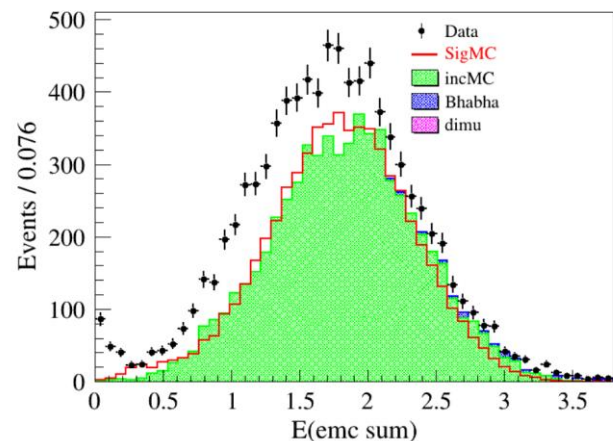
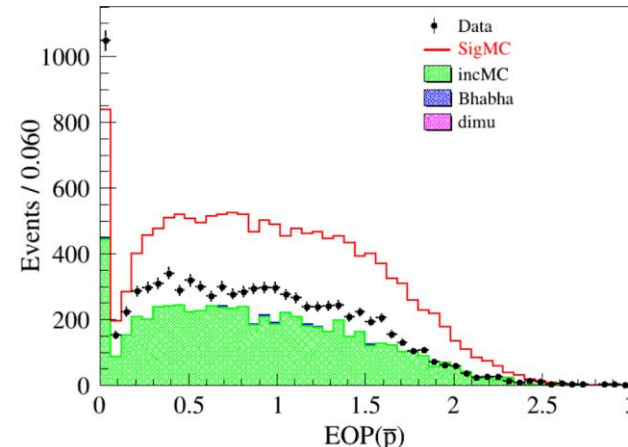
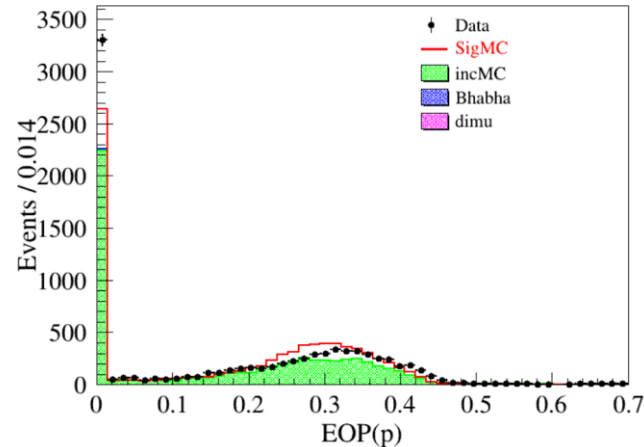
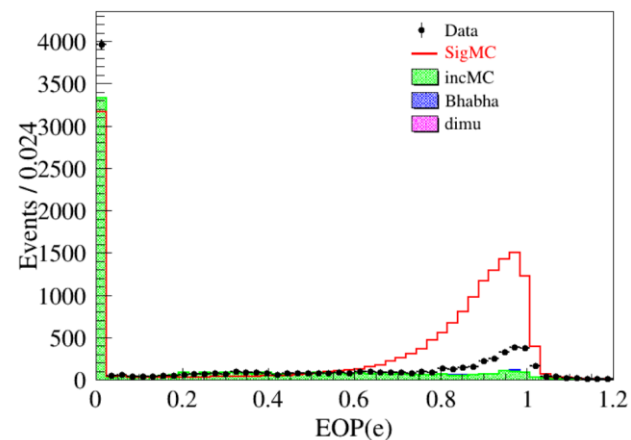
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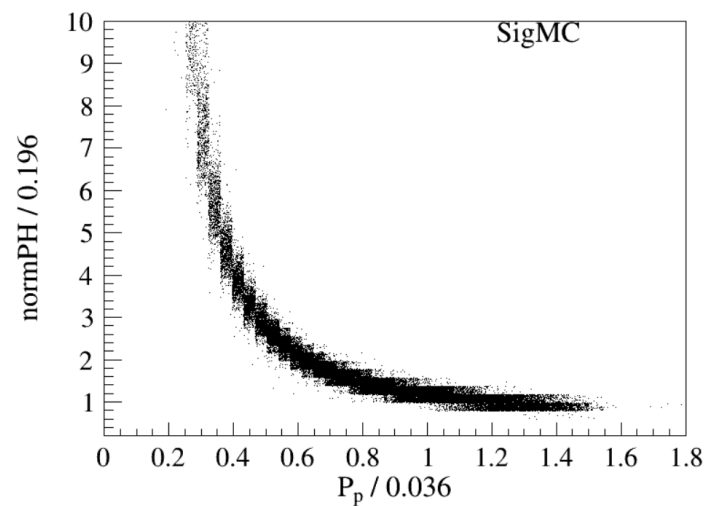
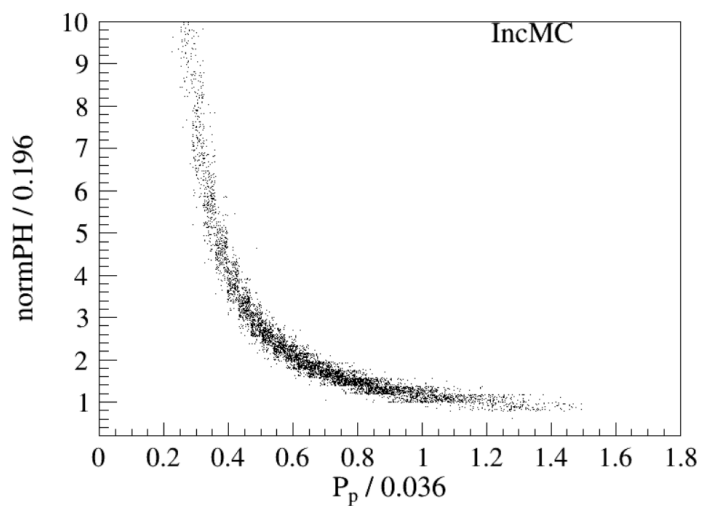
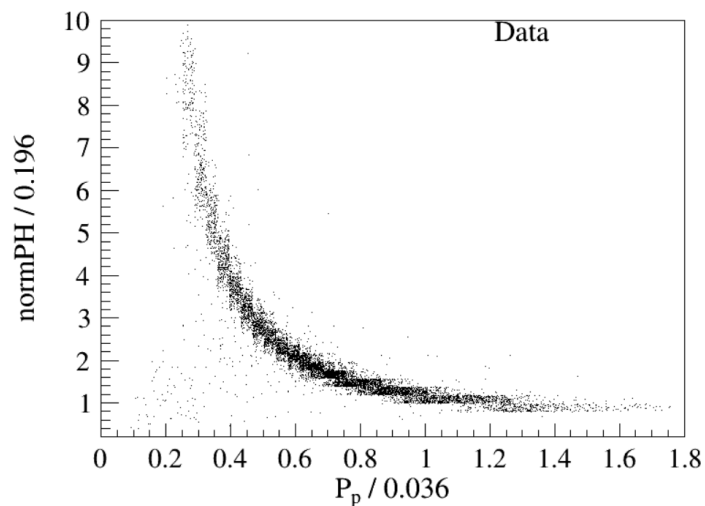
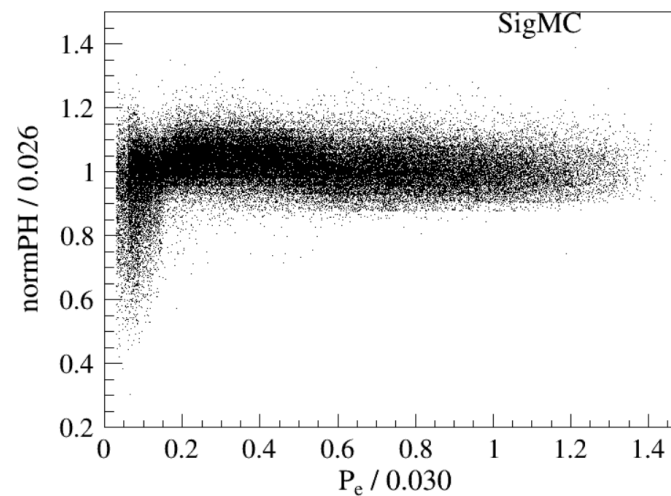
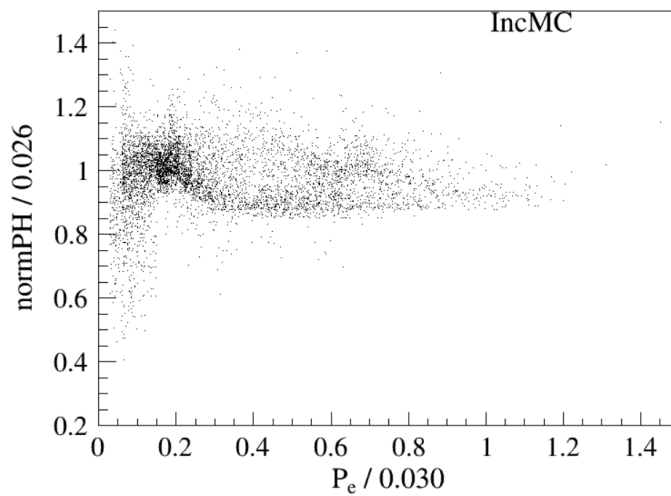
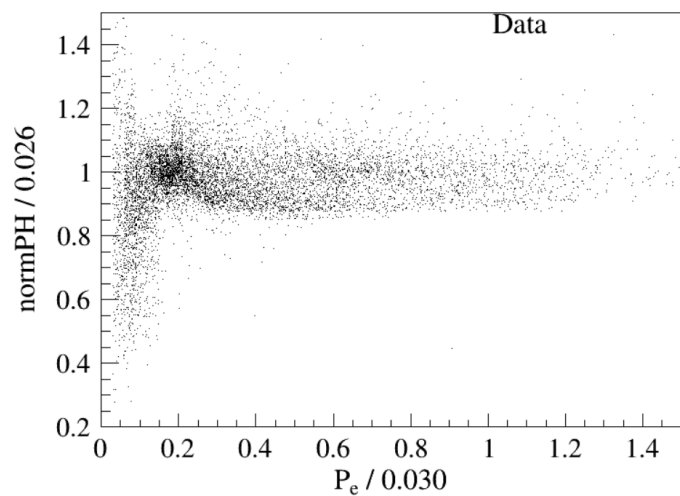
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rowNo	decay tree	decay final state	iDcyTr	nEtr	nCEtr
1	$x \rightarrow K^- p \bar{\Lambda} \gamma^f, \bar{\Lambda} \rightarrow \pi^+ \bar{p}$	$\pi^+ K^- p \bar{p} \gamma^f$	5	259	259
2	$x \rightarrow K^+ \bar{p} \Lambda \gamma^f, \Lambda \rightarrow \pi^- p$	$\pi^- K^+ p \bar{p} \gamma^f$	1	213	472
3	$x \rightarrow K^+ K^- p \bar{p} \gamma^f$	$K^+ K^- p \bar{p} \gamma^f$	3	194	666
4	$x \rightarrow \pi^+ \pi^- p \bar{p} \gamma^f$	$\pi^+ \pi^- p \bar{p} \gamma^f$	81	119	785
5	$x \rightarrow J/\psi \gamma, J/\psi \rightarrow \pi^+ \pi^- p \bar{p}$	$\pi^+ \pi^- p \bar{p} \gamma$	61	112	897
6	$x \rightarrow K^+ K^- p \bar{p}$	$K^+ K^- p \bar{p}$	67	87	984
7	$x \rightarrow J/\psi \gamma, J/\psi \rightarrow \pi^+ p \bar{\Delta}^{--}, \bar{\Delta}^{--} \rightarrow \pi^- \bar{p}$	$\pi^+ \pi^- p \bar{p} \gamma$	73	78	1062
8	$x \rightarrow p \bar{p} a_0^0 \gamma^f, a_0^0 \rightarrow \pi^0 \eta, \eta \rightarrow \pi^0 \pi^0 \pi^0$	$\pi^0 \pi^0 \pi^0 \pi^0 p \bar{p} \gamma^f$	22	76	1138
9	$x \rightarrow K^- p \bar{\Lambda}, \bar{\Lambda} \rightarrow \pi^+ \bar{p}$	$\pi^+ K^- p \bar{p}$	99	68	1206
10	$x \rightarrow p \bar{p} a_0^0 \gamma^f, a_0^0 \rightarrow \pi^0 \eta, \eta \rightarrow \gamma \gamma$	$\pi^0 p \bar{p} \gamma \gamma \gamma^f$	13	57	1263
11	$x \rightarrow K^+ \bar{p} \Lambda, \Lambda \rightarrow \pi^- p$	$\pi^- K^+ p \bar{p}$	23	53	1316
12	$x \rightarrow \Delta^{++} \bar{\Delta}^{--} \gamma^f, \Delta^{++} \rightarrow \pi^+ p, \bar{\Delta}^{--} \rightarrow \pi^- \bar{p}$	$\pi^+ \pi^- p \bar{p} \gamma^f$	59	51	1367
13	$x \rightarrow \pi^+ p \bar{\Delta}^{--} \gamma^f, \bar{\Delta}^{--} \rightarrow \pi^- \bar{p}$	$\pi^+ \pi^- p \bar{p} \gamma^f$	41	50	1417
14	$x \rightarrow J/\psi \gamma, J/\psi \rightarrow \pi^0 \Delta^{++} \bar{\Delta}^{--}, \Delta^{++} \rightarrow \pi^+ p, \bar{\Delta}^{--} \rightarrow \pi^- \bar{p}$	$\pi^0 \pi^+ \pi^- p \bar{p} \gamma$	71	47	1464
15	$x \rightarrow f_2(1270) p \bar{p} \gamma^f, f_2(1270) \rightarrow \pi^+ \pi^-$	$\pi^+ \pi^- p \bar{p} \gamma^f$	42	45	1509
16	$x \rightarrow \pi^+ \pi^- p \bar{p} \gamma^f$	$\pi^+ \pi^- p \bar{p} \gamma^f$	35	44	1553
17	$x \rightarrow p \bar{p} a_0^0, a_0^0 \rightarrow \pi^0 \eta, \eta \rightarrow \pi^0 \pi^0 \pi^0$	$\pi^0 \pi^0 \pi^0 \pi^0 p \bar{p}$	40	43	1596
18	$x \rightarrow p \bar{p} a_1^0 \gamma^f, a_1^0 \rightarrow \pi^- \rho^+, \rho^+ \rightarrow \pi^0 \pi^+$	$\pi^0 \pi^+ \pi^- p \bar{p} \gamma^f$	90	41	1637
19	$x \rightarrow \Delta^{++} \bar{\Delta}^{--}, \Delta^{++} \rightarrow \pi^+ p, \bar{\Delta}^{--} \rightarrow \pi^- \bar{p}$	$\pi^+ \pi^- p \bar{p}$	198	41	1678
20	$x \rightarrow J/\psi \gamma, J/\psi \rightarrow \pi^0 \Delta^+ \bar{\Delta}^-, \Delta^+ \rightarrow \pi^0 p, \bar{\Delta}^- \rightarrow \pi^0 \bar{p}$	$\pi^0 \pi^0 \pi^0 p \bar{p} \gamma$	14	40	1718
.....					
1727	$x \rightarrow \psi' \gamma, \psi' \rightarrow \pi^0 \pi^0 J/\psi, J/\psi \rightarrow \Xi^- \bar{\Xi}^{*+}, \Xi^- \rightarrow \pi^- \Lambda, \bar{\Xi}^{*+} \rightarrow \pi^+ \bar{\Xi}^0,$ $\Lambda \rightarrow \pi^- p, \bar{\Xi}^0 \rightarrow \pi^0 \bar{\Lambda}, \bar{\Lambda} \rightarrow \pi^+ \bar{p}$	$\pi^0 \pi^0 \pi^0 \pi^+ \pi^+ \pi^- \pi^- p \bar{p} \gamma$	1726	1	6225

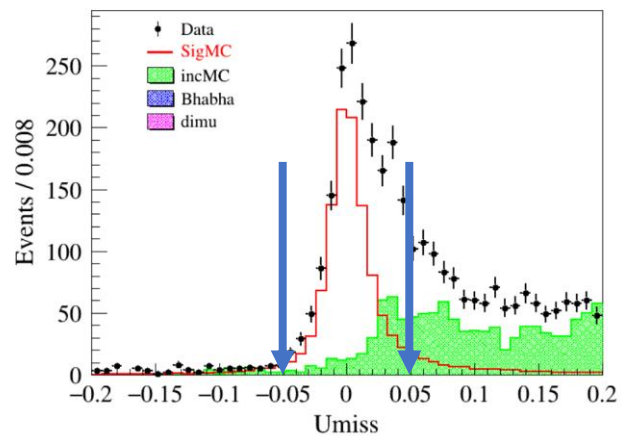
Further Event selection

$$\begin{aligned} \cos\theta_{miss} * \text{sign}(e) &< -0.92; \\ 20 &< \theta(p, \bar{p}) < 170; \\ 10 &< \theta(p/\bar{p}, e) < 170; \end{aligned}$$

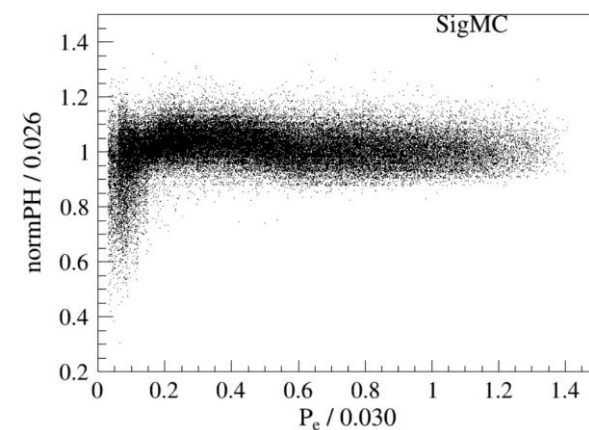
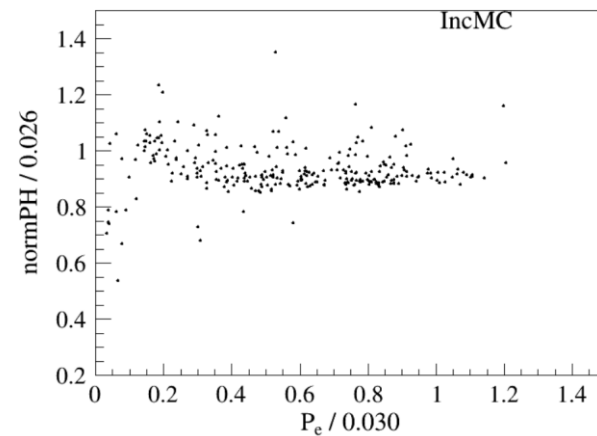
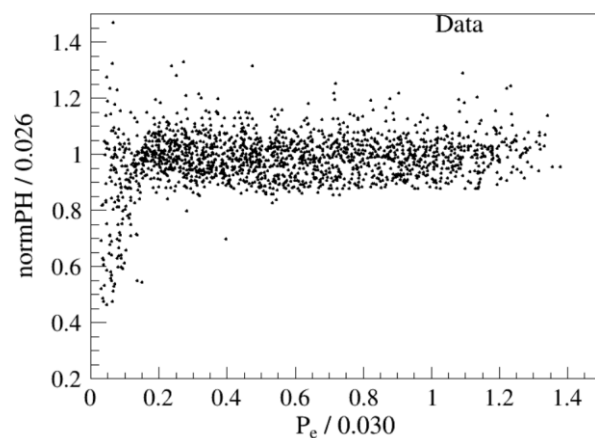


Further Event selection

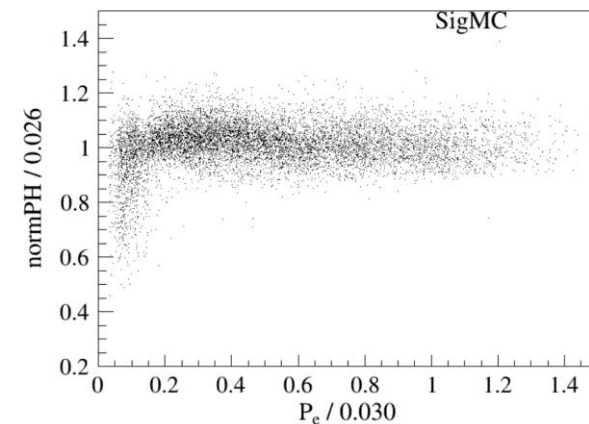
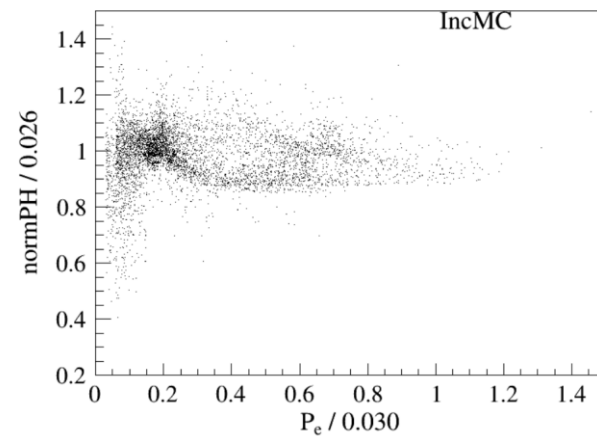
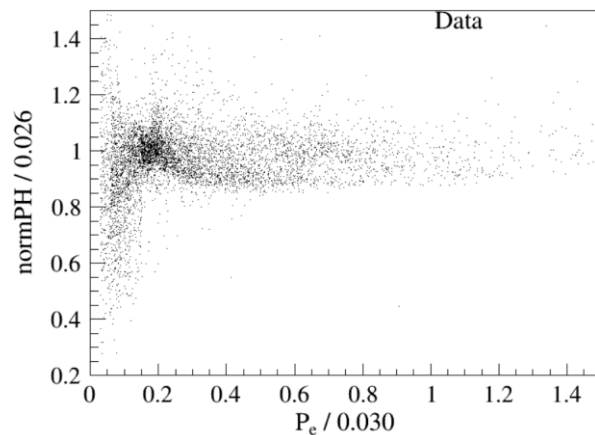
$$\begin{aligned} \cos\theta_{\text{miss}} * \text{sign}(e) &< -0.92; \\ 20 < \theta(p, \bar{p}) &< 170; \\ 10 < \theta(p/\bar{p}, e) &< 170; \end{aligned}$$



$$-0.05 < U_{\text{miss}} < 0.05$$



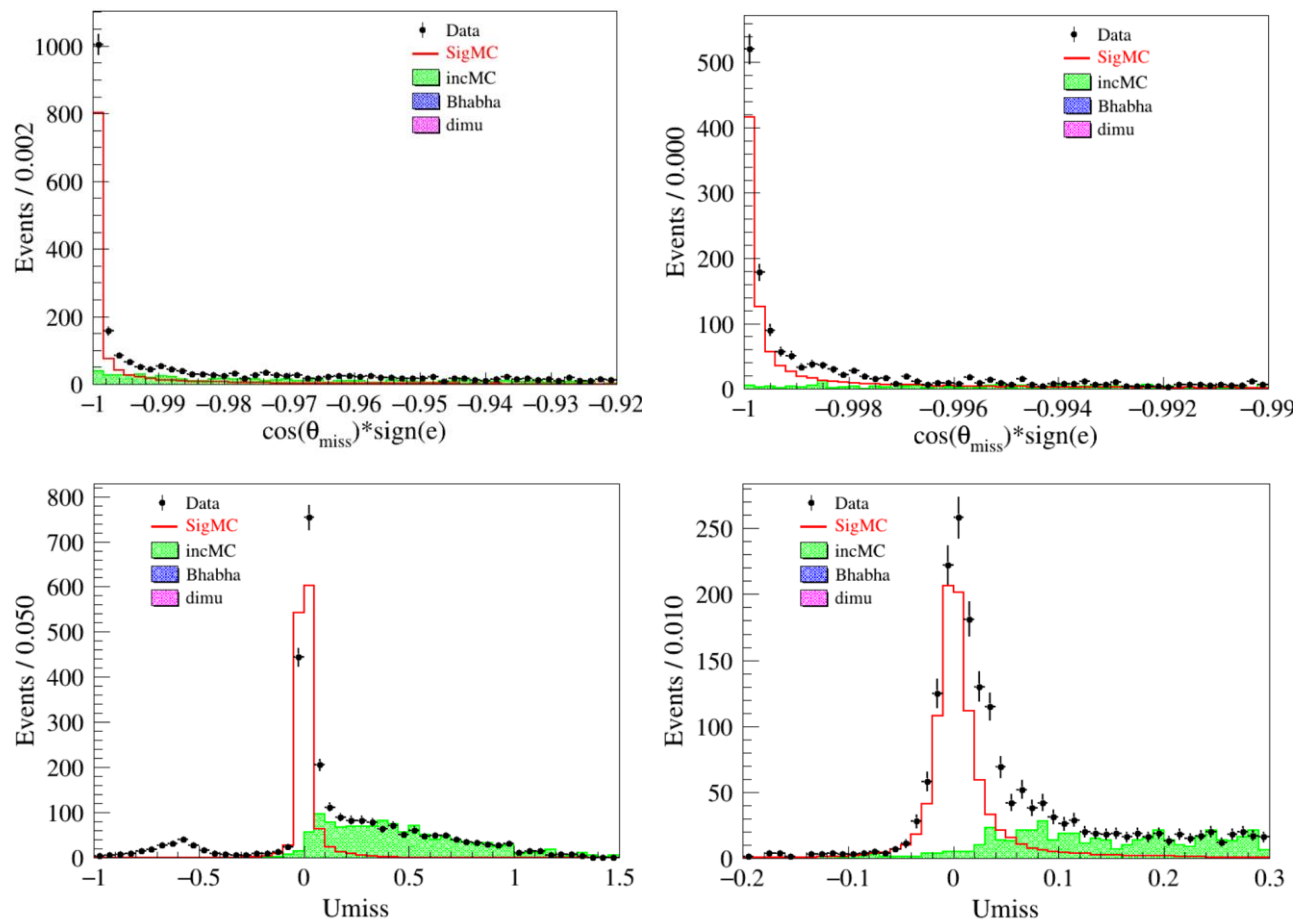
other



Further Event selection

要求 $\cos\theta_{miss} * \text{sign}(e) < -0.99$, $-0.1 < U_{miss} < 0.1$

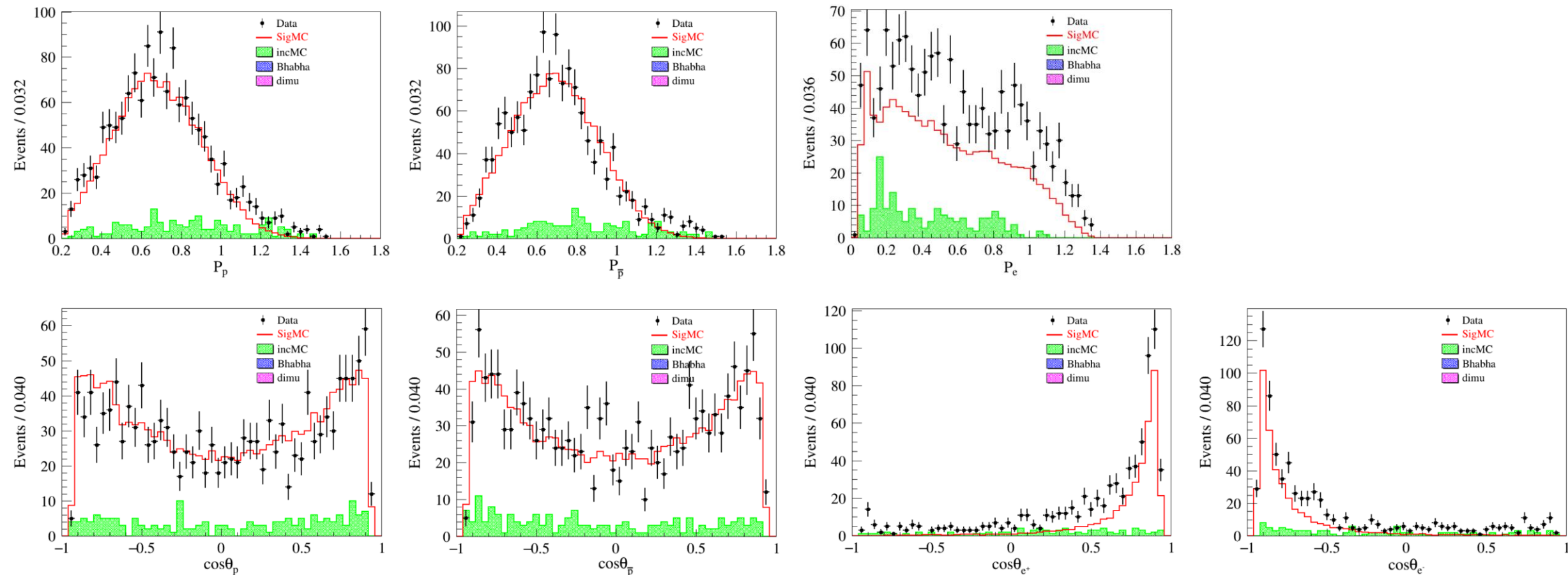
$$\begin{aligned} \cos\theta_{miss} * \text{sign}(e) &< -0.99; \\ 20 &< \theta(p, \bar{p}) < 170; \\ 10 &< \theta(p/\bar{p}, e) < 170; \\ -0.1 &< U_{miss} < 0.1 \end{aligned}$$



剩余的事例数: 1434
MC估计本底率: 13%

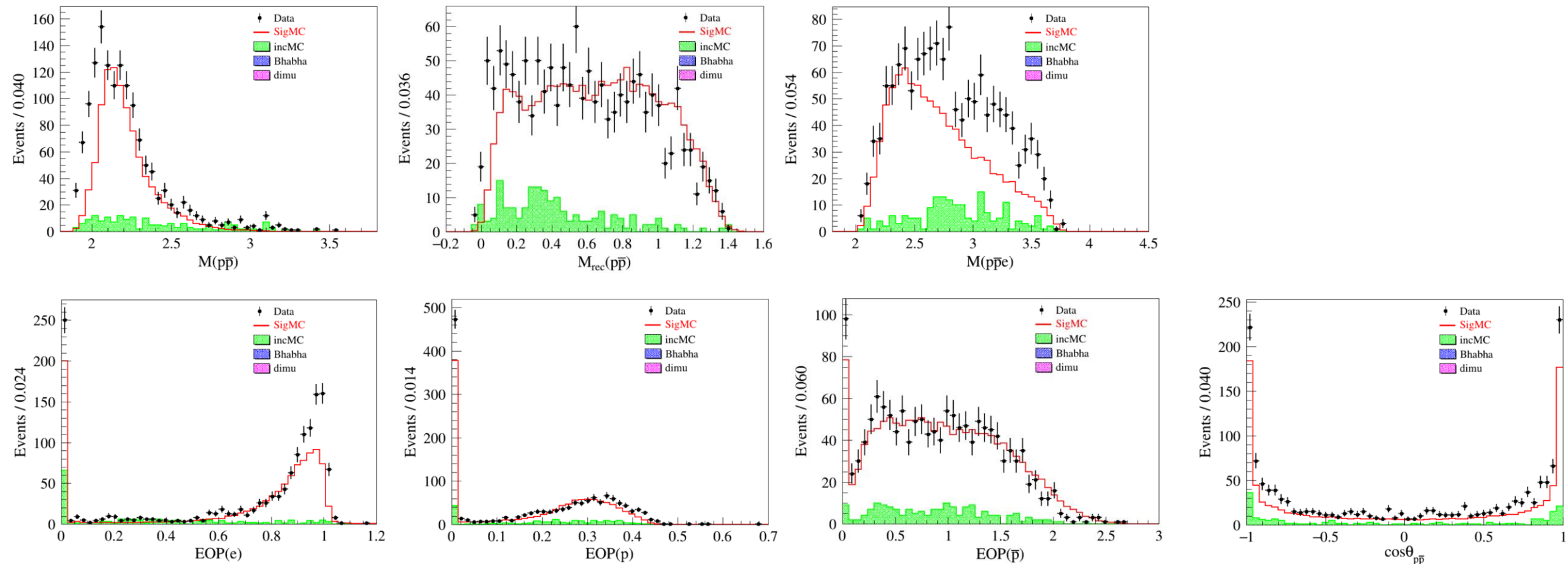
Further Event selection

$$\begin{aligned} \cos\theta_{\text{miss}} * \text{sign}(e) &< -0.99; \\ 20 < \theta(p, \bar{p}) &< 170; \\ 10 < \theta(p/\bar{p}, e) &< 170; \\ -0.1 < U_{\text{miss}} &< 0.1 \end{aligned}$$



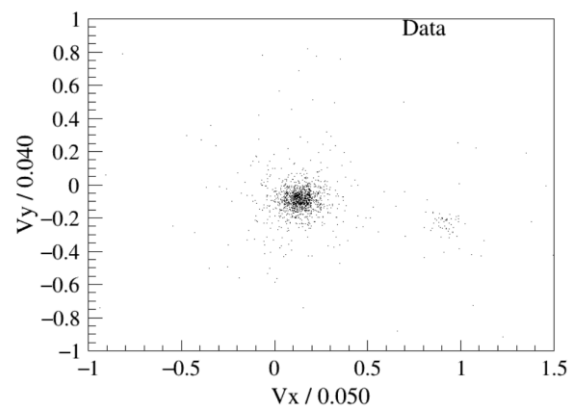
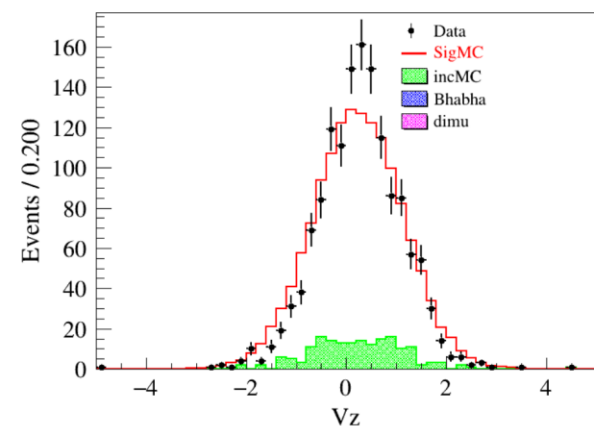
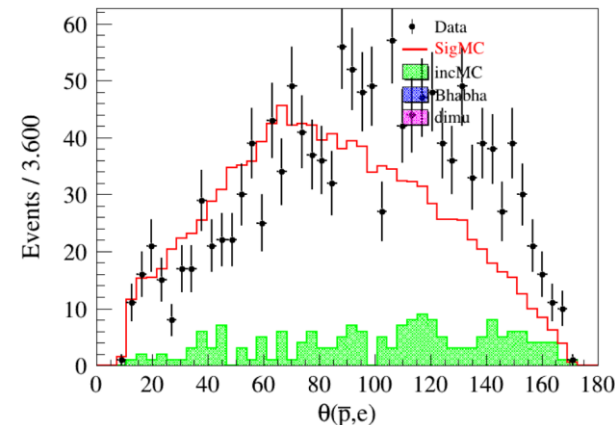
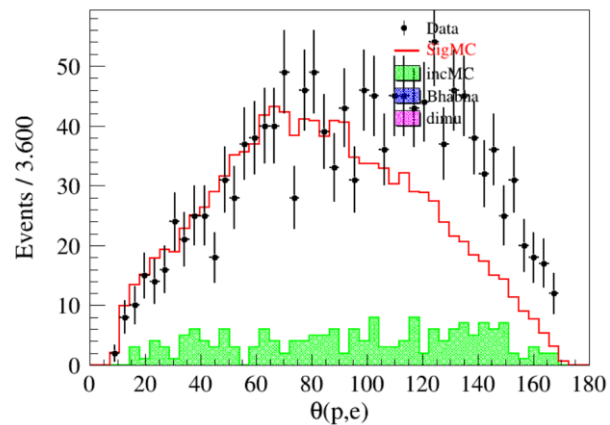
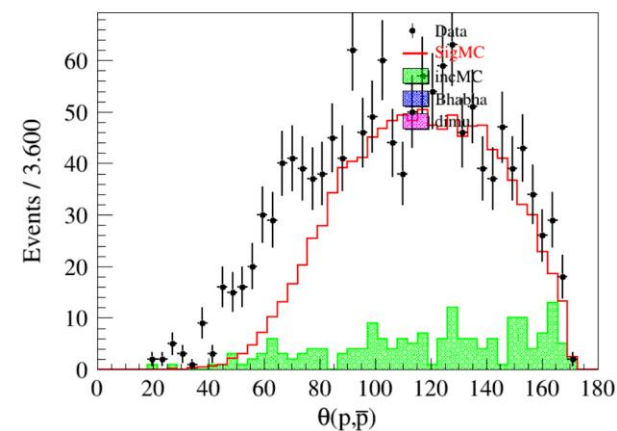
Further Event selection

$$\begin{aligned} \cos\theta_{miss} * \text{sign}(e) &< -0.99; \\ 20 < \theta(p, \bar{p}) &< 170; \\ 10 < \theta(p/\bar{p}, e) &< 170; \\ -0.1 < U_{miss} &< 0.1 \end{aligned}$$



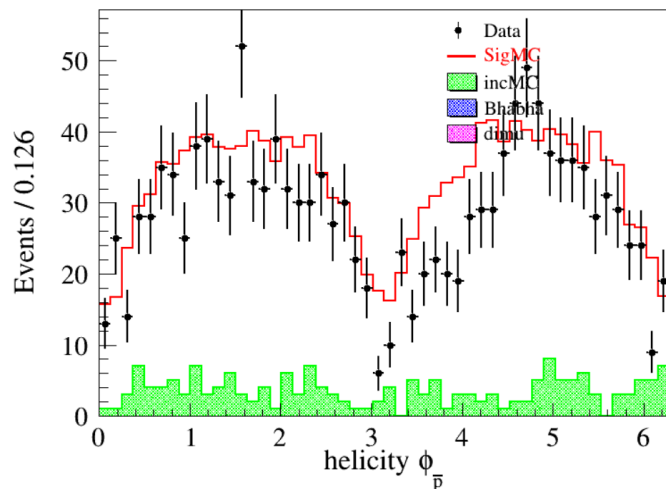
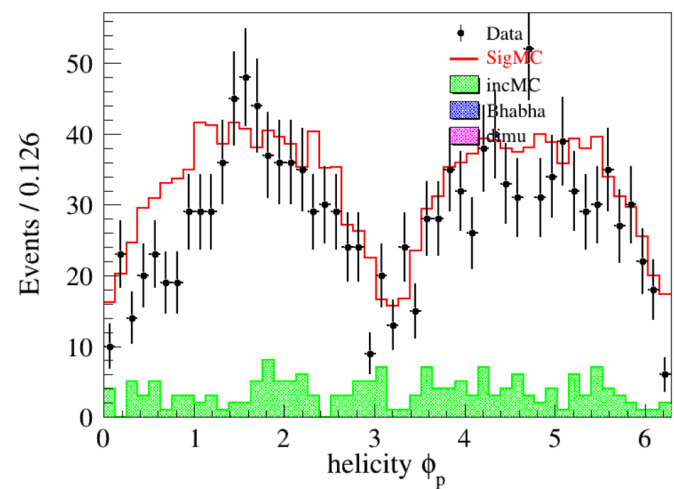
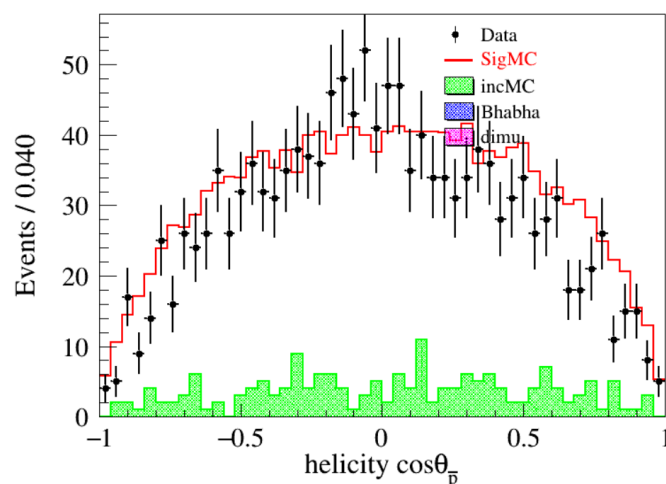
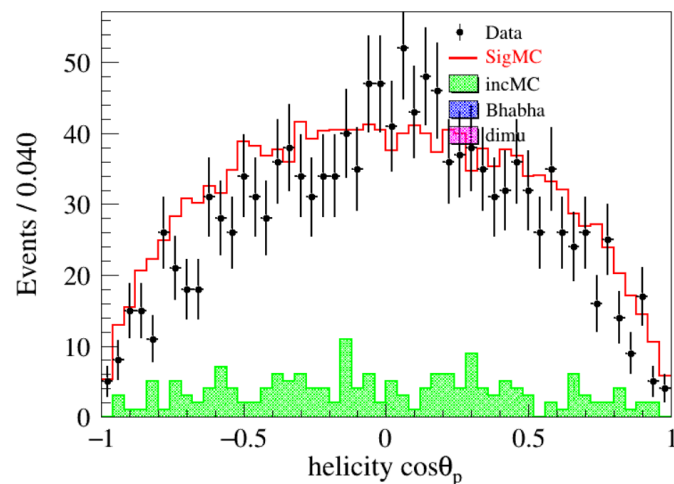
Further Event selection

$$\begin{aligned} \cos\theta_{miss} * \text{sign}(e) &< -0.99; \\ 20 < \theta(p, \bar{p}) &< 170; \\ 10 < \theta(p/\bar{p}, e) &< 170; \\ -0.1 < U_{miss} &< 0.1 \end{aligned}$$



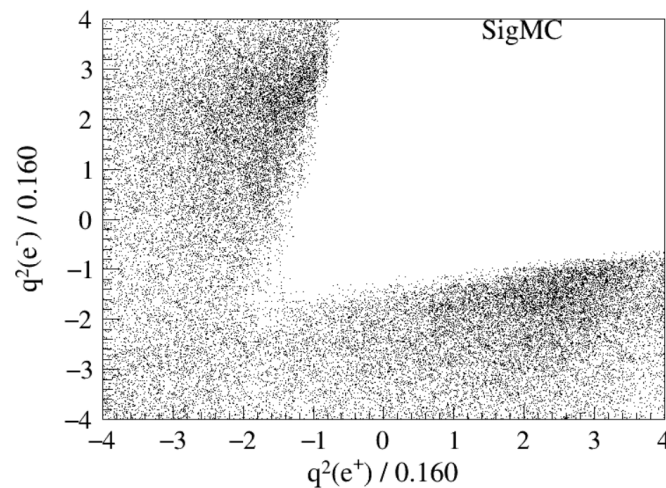
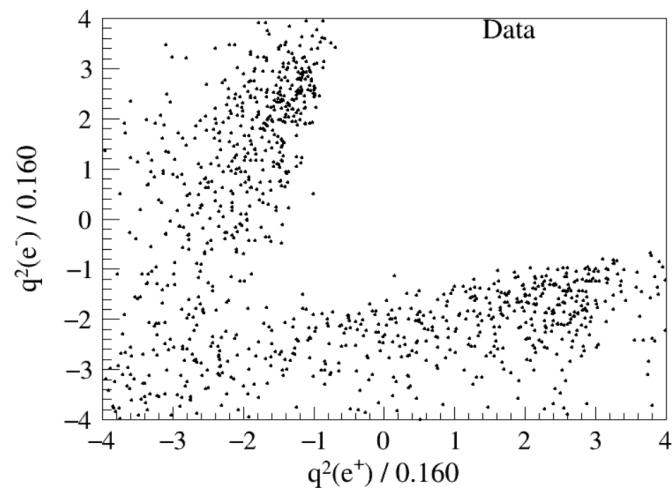
Further Event selection

$$\begin{aligned} \cos\theta_{miss} * \text{sign}(e) &< -0.99; \\ 20 &< \theta(p, \bar{p}) < 170; \\ 10 &< \theta(p/\bar{p}, e) < 170; \\ -0.1 &< U_{miss} < 0.1 \end{aligned}$$



Further Event selection

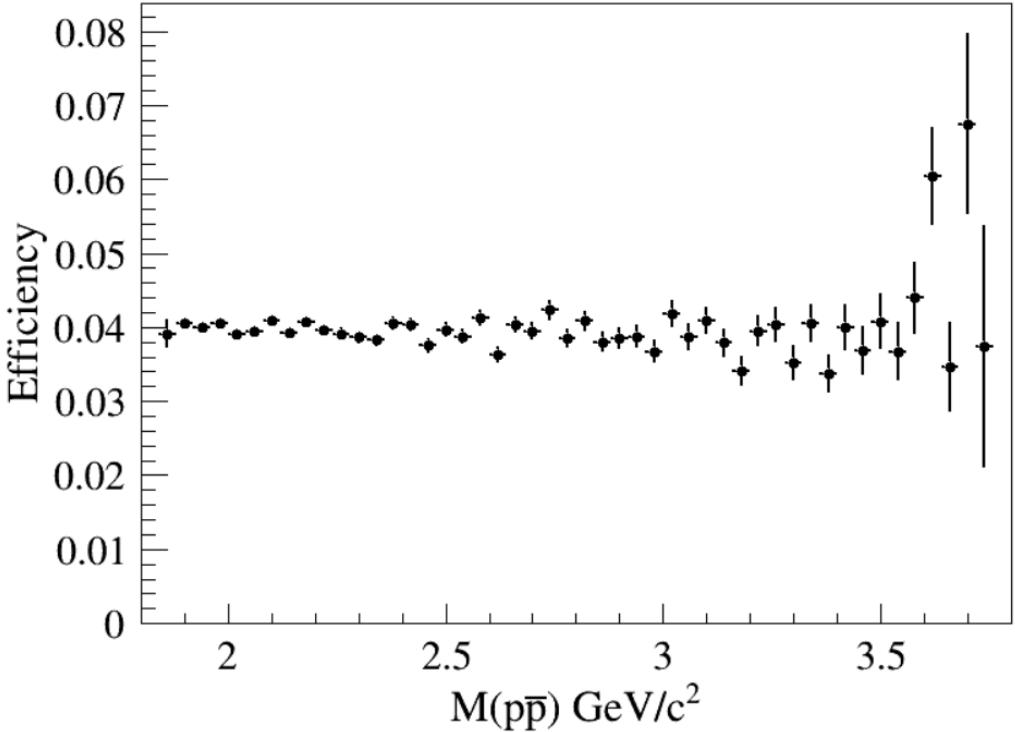
$$\begin{aligned} \cos\theta_{\text{miss}} * \text{sign}(e) &< -0.99; \\ 20 &< \theta(p, \bar{p}) < 170; \\ 10 &< \theta(p/\bar{p}, e) < 170; \\ -0.1 &< U_{\text{miss}} < 0.1 \end{aligned}$$



Efficiency

Cut flow 根据没有对 $M_{p\bar{p}}$ 质量reweight的MC给出

	Event	Rel eff (%)
total	1490000	\
$N_{good} = 3$	113512	7.6
Pass PID	88803	78.2
Pass Vtxfit	88391	99.5
$\cos\theta_{miss} * \text{sign}(e) < -0.99$	69708	78.8
$20 < \theta(p, \bar{p}) < 170$ $10 < \theta(p/\bar{p}, e) < 170$	62036	89.0
$-0.1 < U_{miss} < 0.1$	59008	95.1
eff	3.96%	



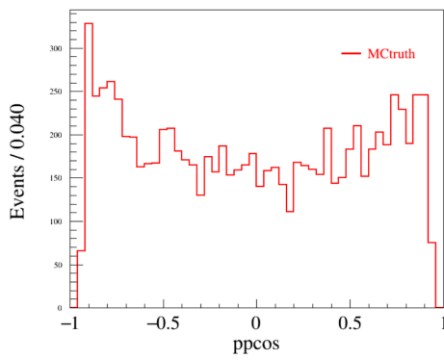
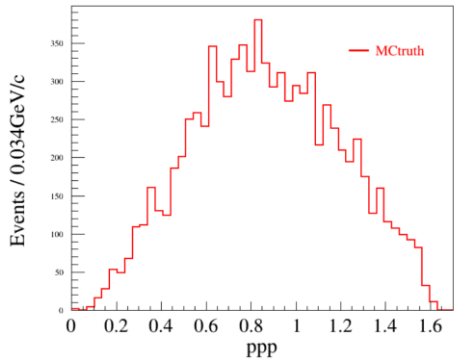
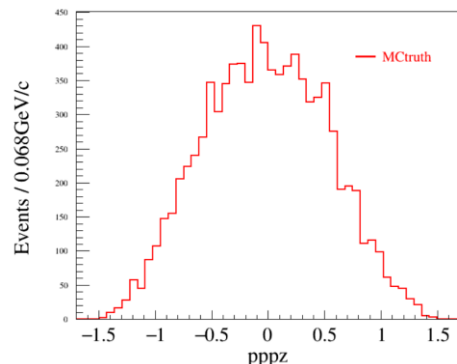
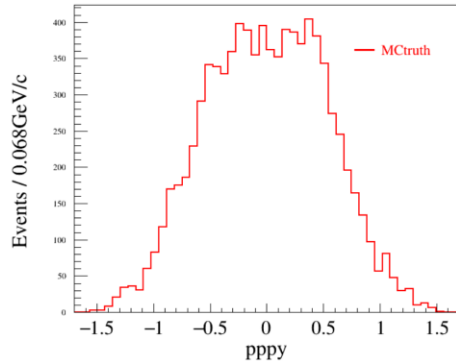
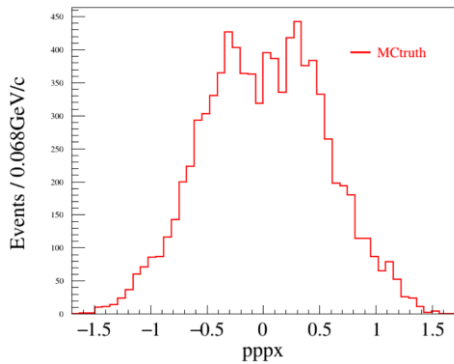
Summary

- 如何确认信号**MC**可靠性，之后如何调整信号**MC**？
- e, π **PID**研究。
- 本底如何估计与描述？

Back Up

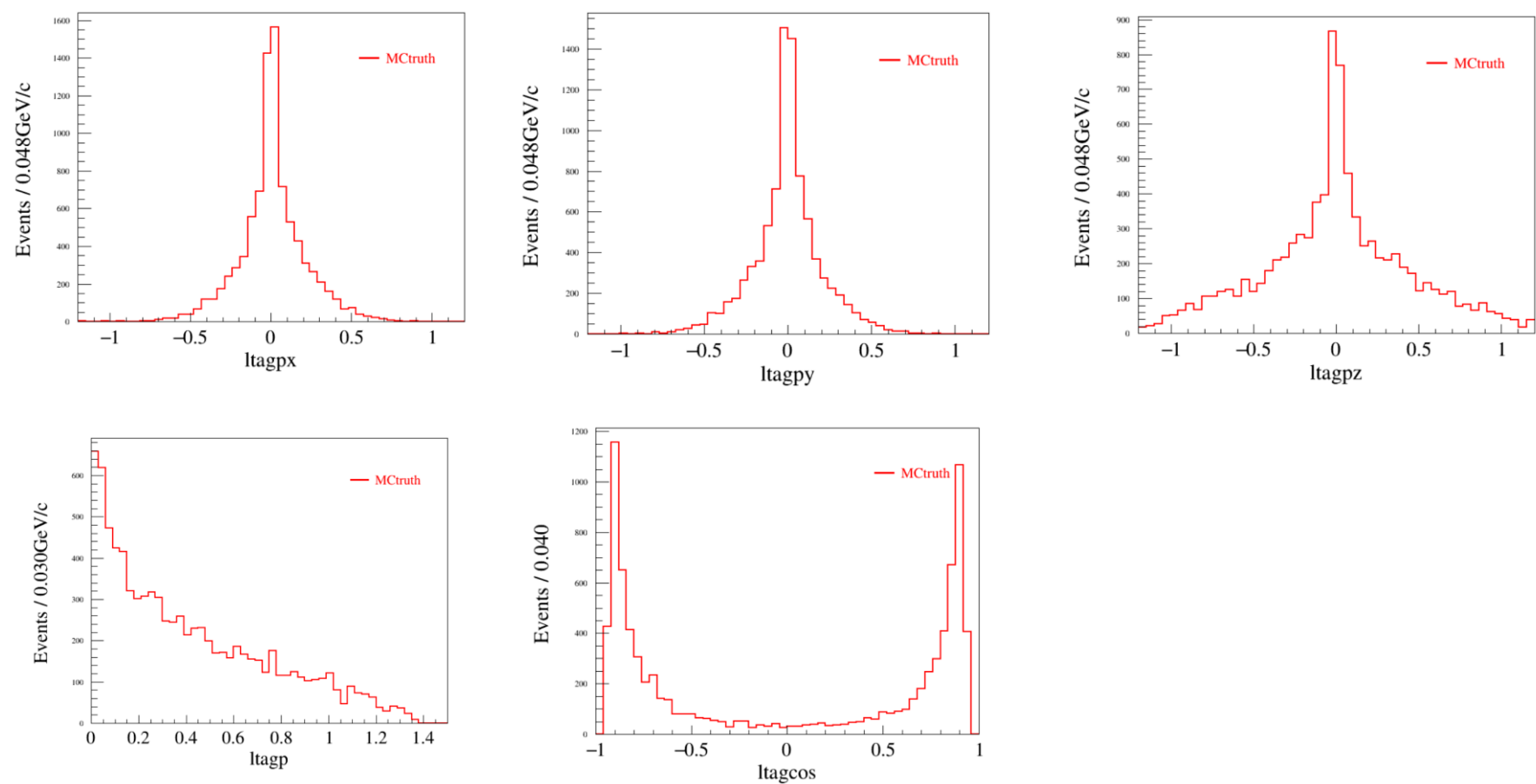
MCtruth information for single-tag acceptance

Mctruth information (before selection), $E_{\text{cms}} = 3.773\text{GeV}$, Generator: cpp_gamgam + TrackGen



Mctruth information for single-tag

Mctruth information (no selection), $E_{\text{cms}} = 3.773\text{GeV}$, Generator: cpp_gamgam + TrackGen



Outline

- Motivation
- MC Generation
- Single-tag method
- Un-tag method
- Summary and Next to do

Motivation

- A novel theoretical prediction ^[1] states that a $\cos(2\phi)$ azimuthal asymmetry exists in the $\gamma\gamma \rightarrow \pi\pi$ production process, and a similar phenomenon may also occur in $\gamma\gamma \rightarrow p\bar{p}$ production.

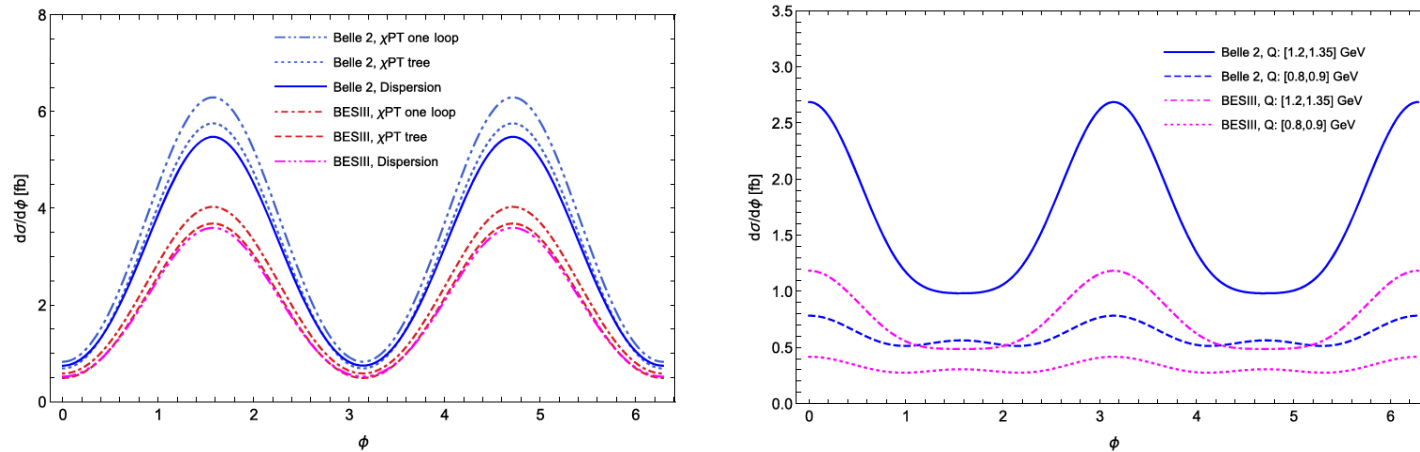


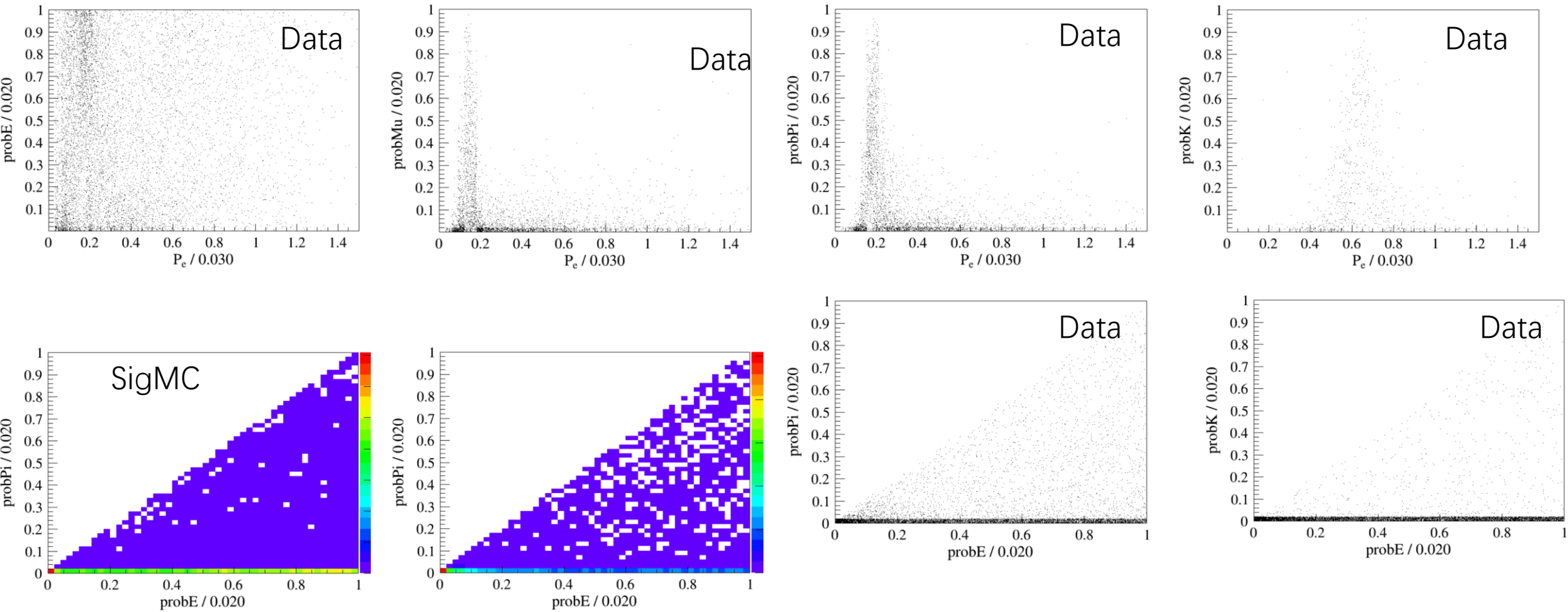
Fig. The differential cross section of $ee \rightarrow \gamma\gamma ee \rightarrow \pi\pi ee$ with respect to the azimuthal angle ϕ .

- BESIII has collected 41fb^{-1} data samples in the $\sqrt{s} = 3.510 - 4.946$ GeV.

[1] PHYSICAL REVIEW LETTERS 134, 141901 (2025)

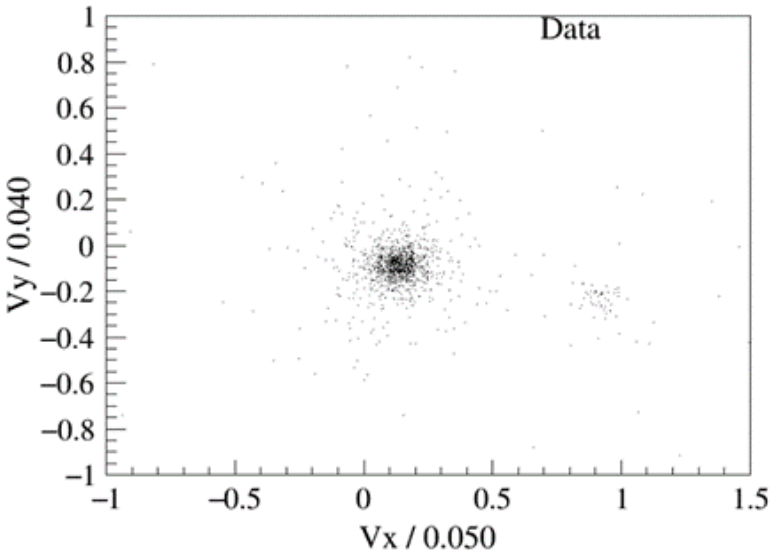
$\text{Cos}(p4\text{miss.theta()}) * \text{Itagsign} < -0.92;$
 $20 < \theta(p, \bar{p}) < 170,$
 $10 < \theta(p/\bar{p}, e) < 170$

- 还可以尝试:
- 1.继续收紧miss系统的角度约束
 - 2.提升pi, e, K鉴别

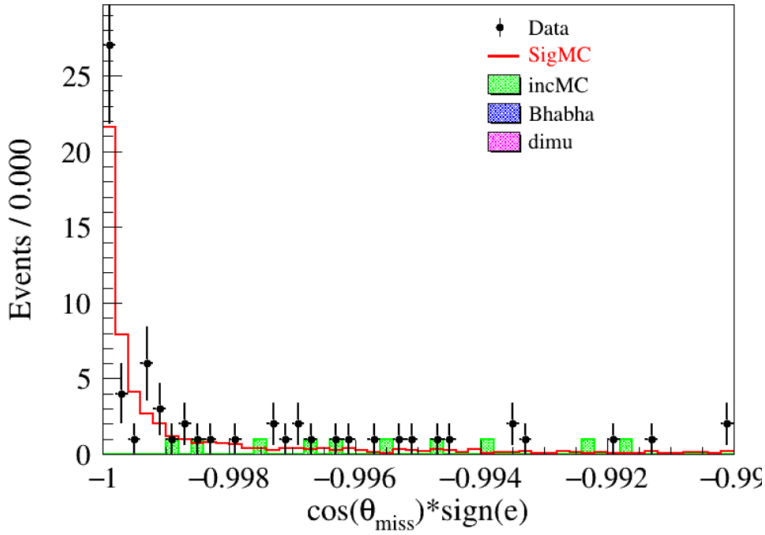
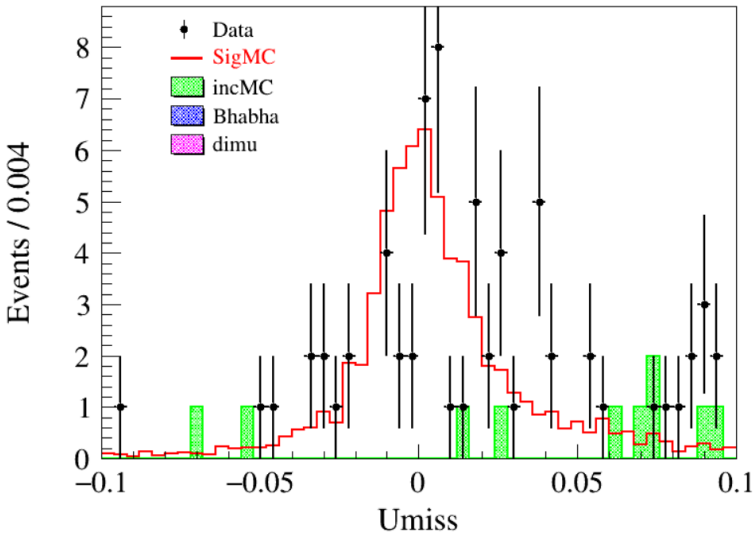


Futher event selection

$$\begin{aligned} \cos\theta_{miss} * \text{sign}(e) &< -0.99; \\ 20 &< \theta(p, \bar{p}) < 170; \\ 10 &< \theta(p/\bar{p}, e) < 170; \\ -0.1 &< U_{miss} < 0.1 \end{aligned}$$



要求 $V_x > 0.7$:



The Belle II Detector

A **multipurpose HEP spectrometer** with vertexing, PID, neutrals, electrons, muons and hermeticity.

EM Calorimeter:
CsI(Tl), waveform sampling (barrel+ endcap)

electrons (7 GeV)

Beryllium beam pipe
2cm diameter

Vertex Detector
2 layers DEPFET + 4 layers DSSD

Central Drift Chamber
He(50%):C₂H₆(50%), small cells, long lever arm, fast electronics (Core element), dE/dx

KLong and muon detector:
Resistive Plate Chambers (barrel outer layers)
Scintillator + WLSF + SiPM's (end-caps , inner 2 barrel layers)

Particle Identification
TOP detector system (barrel)
Prox. focusing Aerogel RICH (fwd)

positrons (4 GeV)

接收度(角度): BESIII: 23~157

KLM: 18~155

KLM: 24~161

ECL: 12.4~31.4, 32.2~128.7, 130.7~155.1

ECL: 16.4~40.8, 41.8~140.0, 141.7~161.0

ARICH: 14~30

ARICH: 18.5~39.0

TOP: 31~128

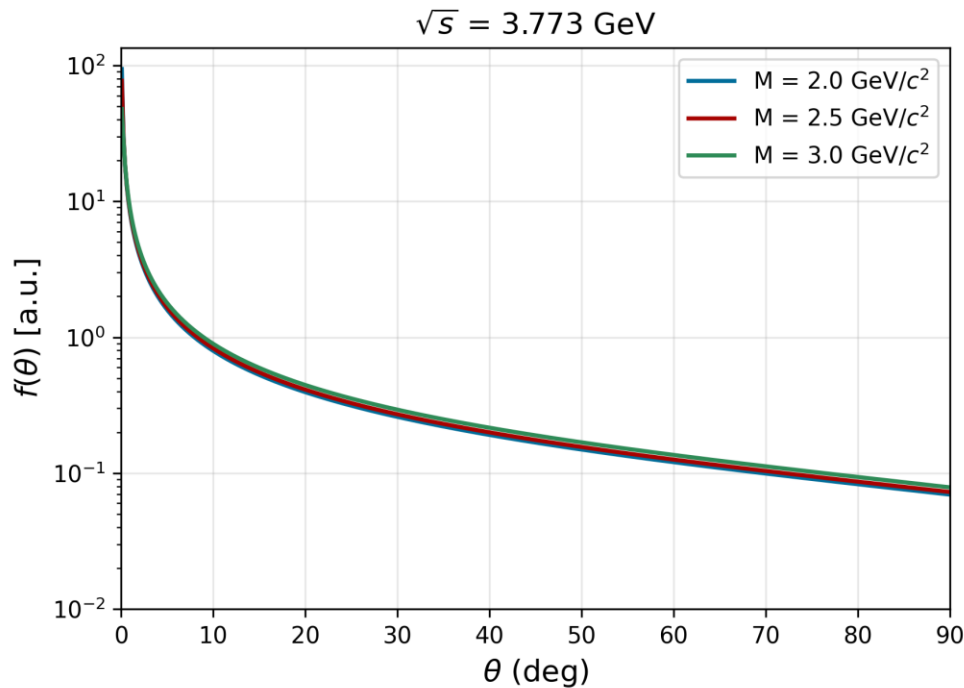
TOP: 40~140

CDC: 17~150

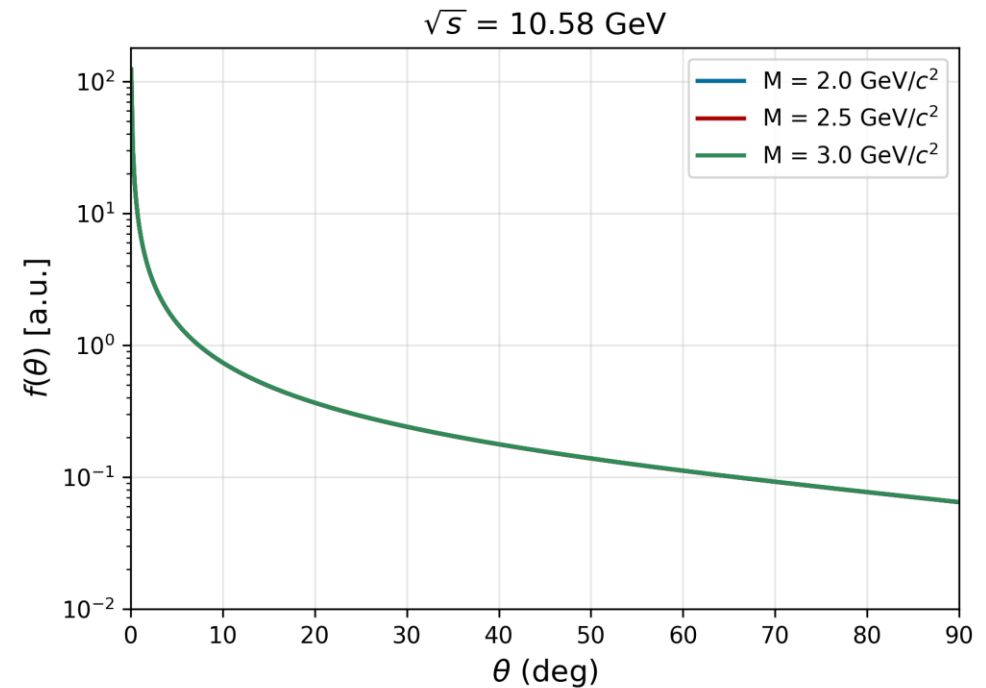
CDC: 22~157

BELLE: 22~140 (157)

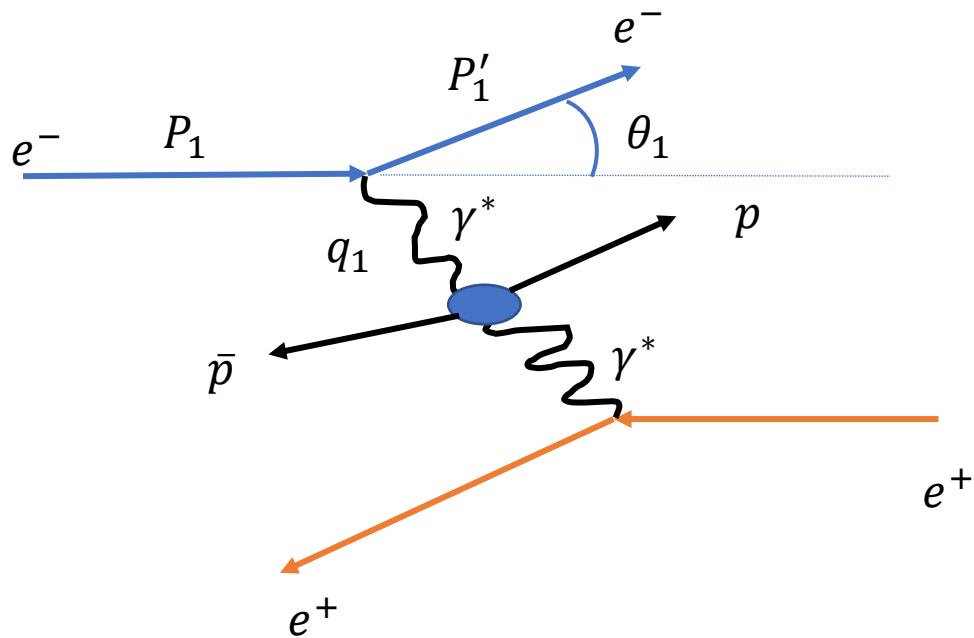
Scattered Electron Angular Distribution in Two-Photon Processes
(Equivalent Photon Approximation, Budnev 1975)



$P(\cos\theta > 0.92)$:
0.84 for 2.0GeV
0.82 for 2.5GeV
0.81 for 3.0GeV



$P(\cos\theta > 0.92)$:
0.88 for 2.0GeV
0.87 for 2.5GeV
0.87 for 3.0GeV



$$\mathbf{q}_1 = (\mathbf{E}_1 - \mathbf{E}'_1, \mathbf{P}_1 - \mathbf{p}'_1),$$

$$q_1^2 \approx -2\mathbf{P}_1\mathbf{P}'_1(1 - \cos\theta_1) + m^2(2 - \frac{P'_1}{P_1} - \frac{P_1}{P'_1});$$

新查表法 vs 原参数化公式对比

