Proton Structure from Basis Light-front Quantization

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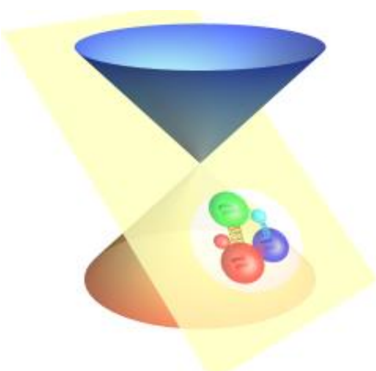
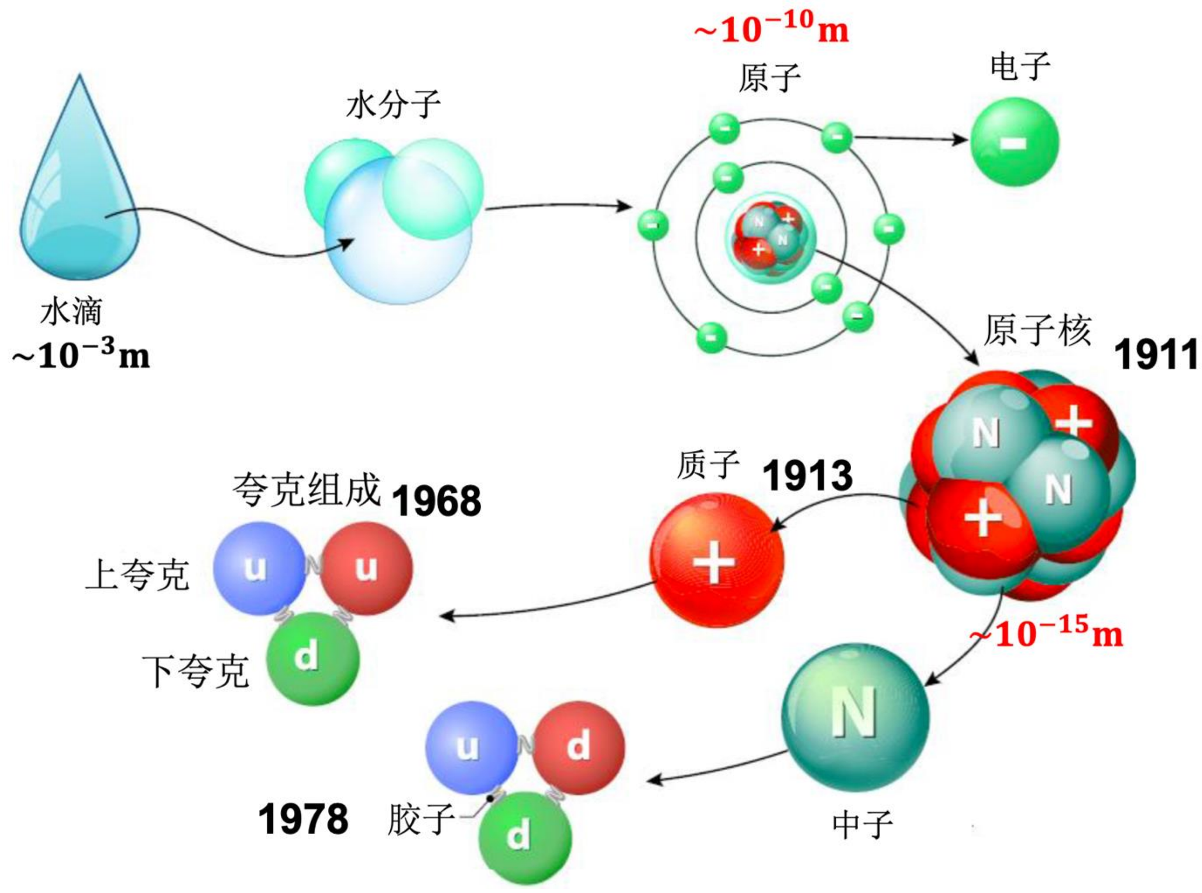


University of Science and Technology of China, Hefei, 04/16/2026

Outline

- Proton structure:
  - history and challenges
- Light-front Quantization:
  - connection with experiments
  - advantages and challenges
- Basis Light-front Quantization:
  - application to the proton

Fundamental Component of Matter



- Proton: “first” building block of all elements, present in every atom

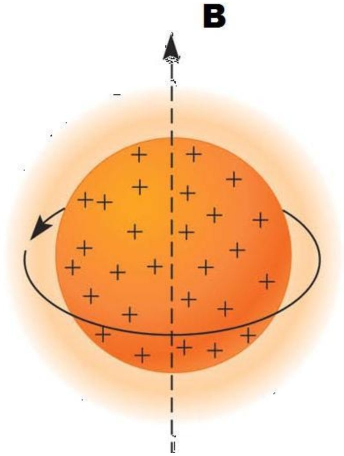
Discovery of Proton

- Proton was discovered by Rutherford in 1919

$$^{14}N + \alpha \rightarrow {}^{17}O + p$$

- Anomalous magnetic moment → not point-like

[Otto Stern, 1933]



If proton were point-like

$$\mu_p = \frac{e\hbar}{2m_N c} \equiv \mu_N$$

→

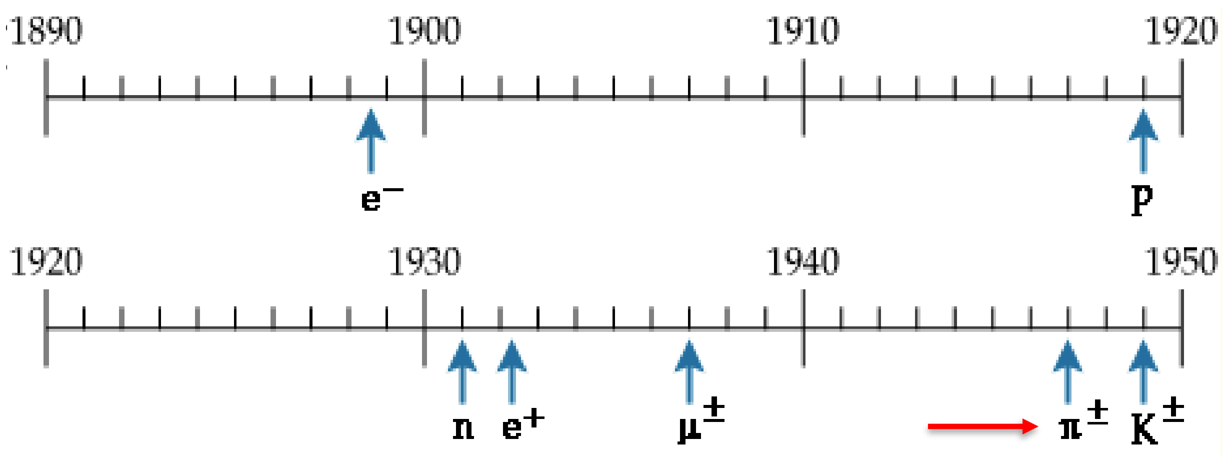
Experimental value

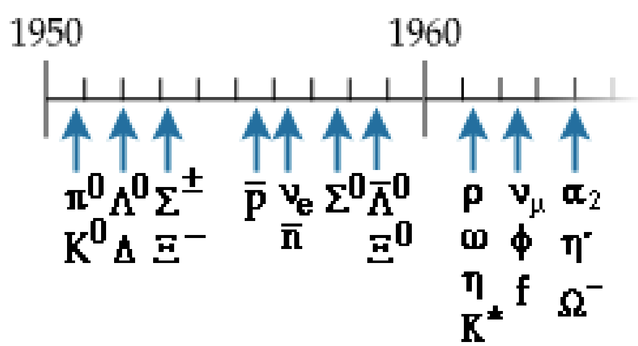
$$\mu_p = 2.5\mu_N$$

Quark Model

- Many hadrons were discovered in 1950s

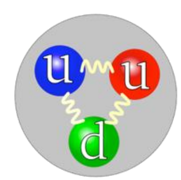
Gell-Mann



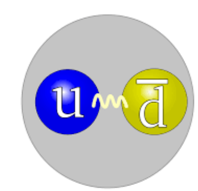


1969

质子：



π介子：



质量：～940MeV

质量：～140MeV

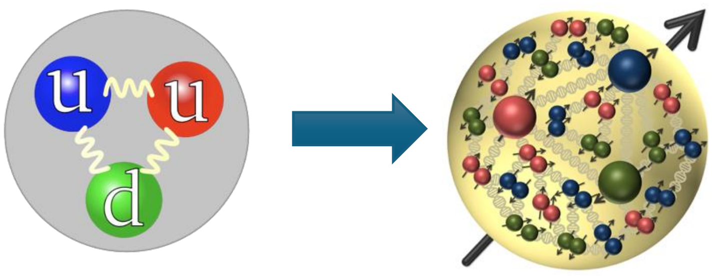
Theory of Strong Interaction

- Quantum Chromodynamics (QCD): fundamental theory for strong interaction (1973)

$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \bar{\Psi}(i\not{D} - m)\Psi$$

Nobel Prize, 2004

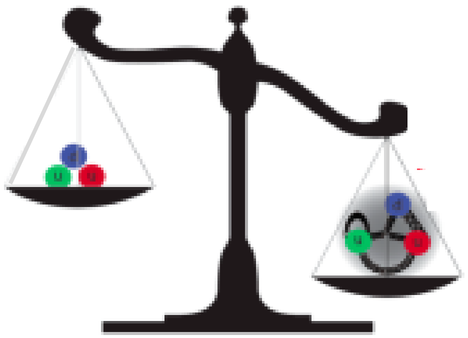
- Quark and gluons are bound together by “color” charges



Valence quarks → gluons → sea quarks

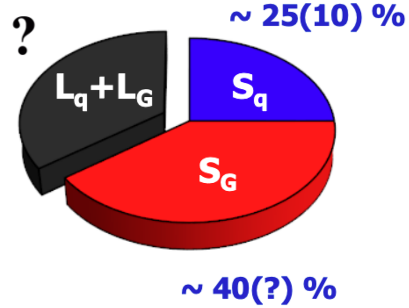
Major Questions in Nuclear Physics

Origin of mass

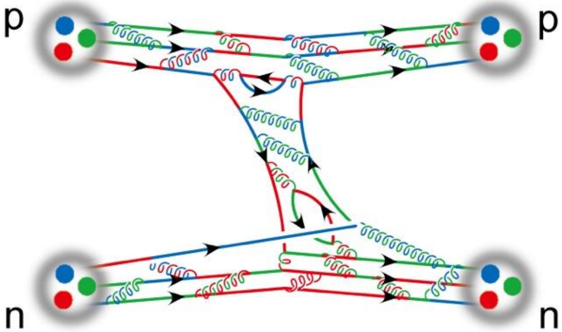


Spin puzzle

Ortibal angular momentum
$\vec{L} = \vec{r} \times \vec{p}$



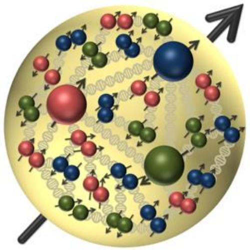
Nuclear force



Need to know 3D tomography of nucleon from QCD

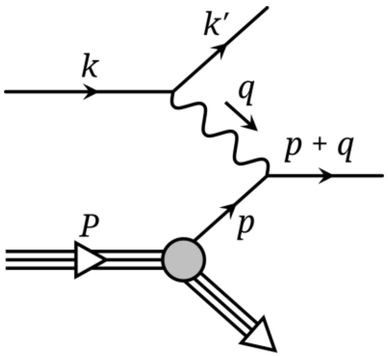
$$\mathcal{L}_{QCD} = \left(\bar{\psi}_q(i\not{D} - m_q)\psi_q\right) - \frac{1}{4}G^{\alpha}_{\mu\nu}G^{\mu\nu}_{\alpha}$$

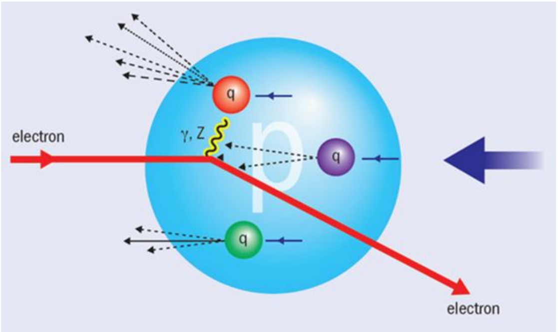
?



Deep Inelastic Scattering

- Modern Rutherford Experiment – DIS (SLAC 1968)





$Q^2 = -q^2 > 0$: photon virtuality

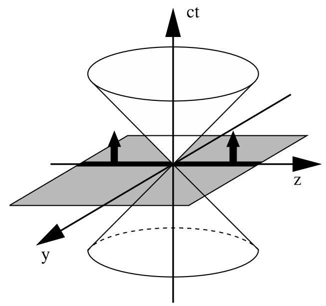
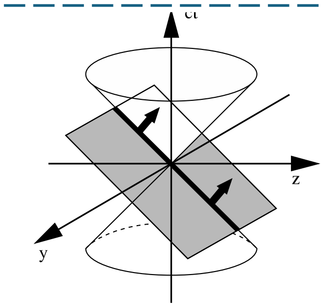
$x_B = \frac{Q^2}{2P\cdot q}$: Bjorken scaling variable

- Bjorken limit: $Q^2 \to \infty$ and x_B fixed ➡ large q^+ direction
- Select $x^+ = 0$ (light front) ➡ probes physics on the light front

$$\sigma_{\rm DIS} \sim \int d^4x\, e^{iq\cdot x}\langle P|J(x)J(0)|P\rangle = \int d^4y\, e^{i(q^+x^- + q^-x^+)}\langle P|J(x)J(0)|P\rangle$$
$$\sim \int d^3x\, e^{iq^+x^-}\langle P|J(x)J(0)|P\rangle\Big|_{x^+=0}$$

Light-front Quantization

[Dirac, 1949]

	Equal time quantization	Light-front quantization
Time variable	$t \Box x^0$	$t \Box x^+ = x^0 + x^3$
Quantization surface		
Coordinate space	x^1, x^2, x^3	$x^- = x^0 - x^3,$ $x^\perp = x^{1,2}$
Momentum Space	$P^0, \vec{P}$	$P^- = P^0 - P^3,$ $P^+ = P^0 + P^3, P^\perp = P^{1,2}$
	$i\frac{\Box}{\Box t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$	$i\frac{\Box}{\Box x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
Dispersion relation	$P^0 = \sqrt{m^2 + P^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

Hamiltonian Formalism

- Hamiltonian formalism describes bound-state structure

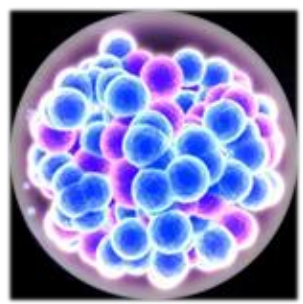
$$H|\psi\rangle = E|\psi\rangle$$

- Eigenstates $|\psi\rangle$ encode full information of the system

Nonrelativistic

atom

Nonrelativistic



nucleus

Relativistic

nucleon

- Relativity → retardation effect

Retardation Effect

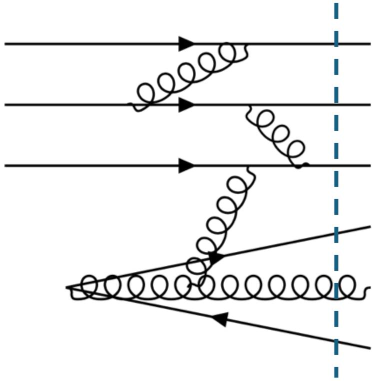
- Equal-time vs. light-front quantization

Proton wave function in Fock space:

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$$

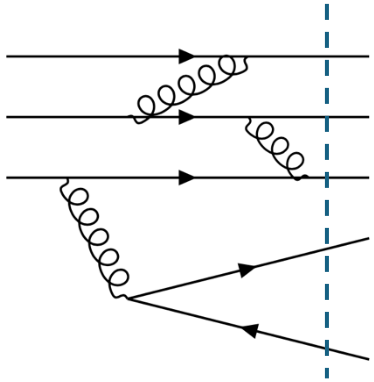
Equal-time quantization

$x^0 = \text{const}$



Light-front quantization

$x^+ = \text{const}$



	Equal-time quantization	Light-front quantization
Vacuum mom:	$k_{\text{vac}} = 0$	$k^+_{\text{vac}} = 0$
Particle mom:	$k^3 \geq 0$ or $k^3 < 0$	$k^+ = k^0 + k^3 > 0$
Higher Fock sectors:	vacuum bubbles	retardation effect
Wave function:	snapshots	hologram of hadron structure

Basis Light-Front Quantization

$$P^{-}|\psi\rangle = P_{\psi}^{-}|\psi\rangle$$

[Vary, et.al, 2010]

- Guiding principle: preserve symmetries in Hamiltonian
  - rotational symmetry in transverse directions
  - translational symmetry in 3D-space
  - exchange symmetry among identical particles

- Basis construction:
  - Fock sector expansion: $|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$
  - single particle basis for noncolor quantum numbers:

$$|qqq\rangle = |n_1, m_1, n_2, m_2, n_3, m_3\rangle \otimes |k_1^+, k_2^+, k_3^+\rangle \otimes |\lambda_1, \lambda_2, \lambda_3, C\rangle$$

2-d harmonic oscillator (2DHO)	Discretized longitudinal momentum	Helicity and color
$\sum_i (2n_i + \lvert m_i\rvert + 1) \le N_{\max}$	$\sum_i k_i^+ = K_{\max}$	$\mathrm{m}_J = \sum_i (\lambda_i + m_i)$

Color Structure

$$|N\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle$$
$$+ |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

$|qqq\rangle \sim$ 1 color singlet state

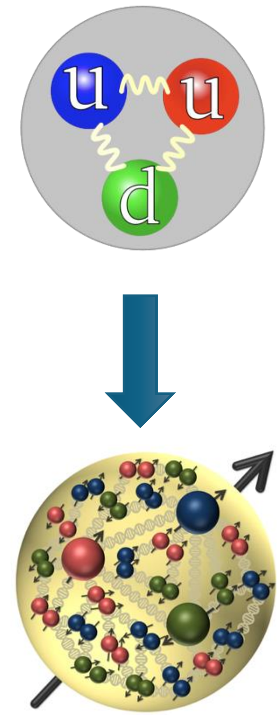
$|qqq\ g\rangle \sim$ 2 color singlet states

$|qqq\ q\bar{q}\rangle \sim$ 3 color singlet states

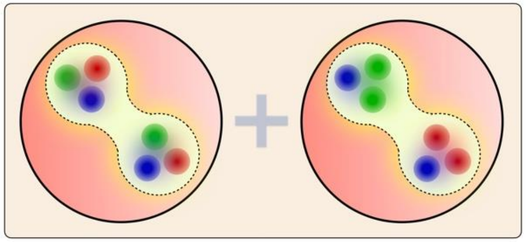
$|qqq\ gg\rangle \sim$ 6 color singlet states

$|qqq\ q\bar{q}g\rangle \sim$ 8 color singlet states

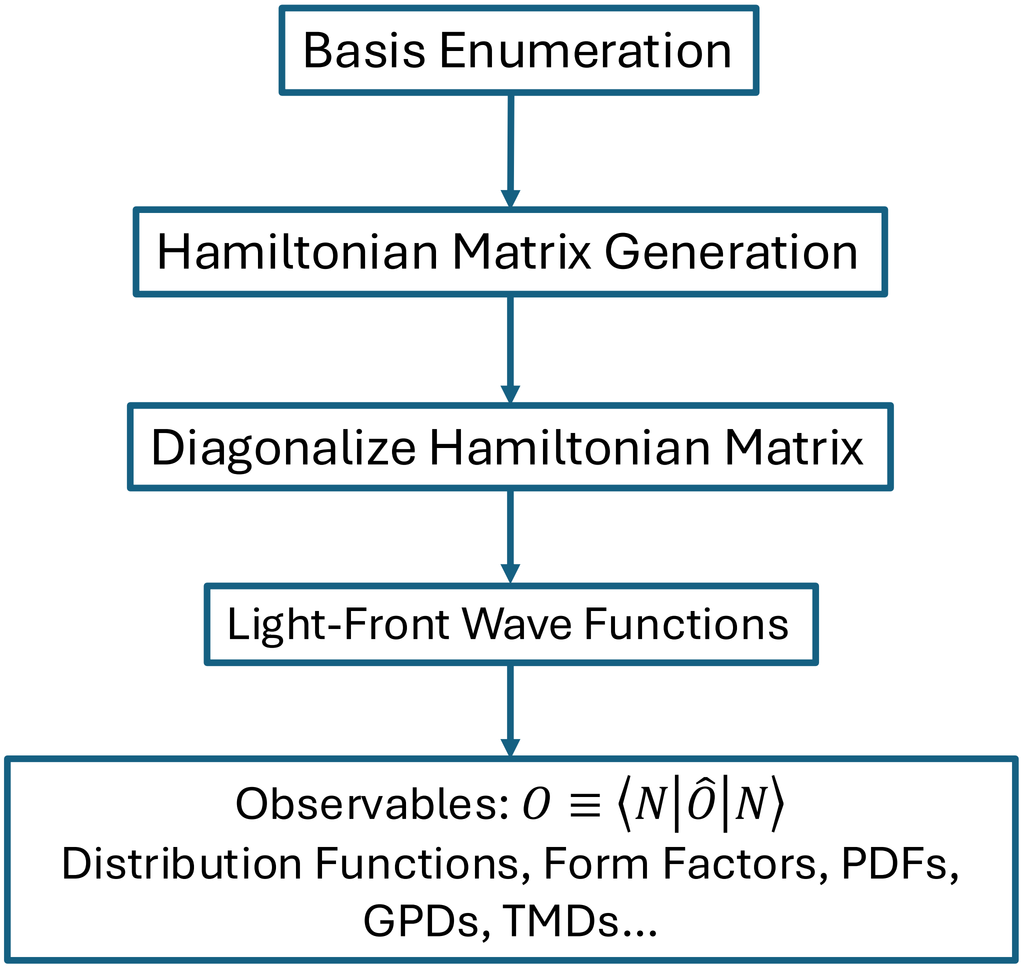
$|qqq\ ggg\rangle \sim$ 22 color singlet states



- Complicated color structure in higher Fock sectors
- Hidden color structure for nuclei



BLFQ Algorithm Flowchart



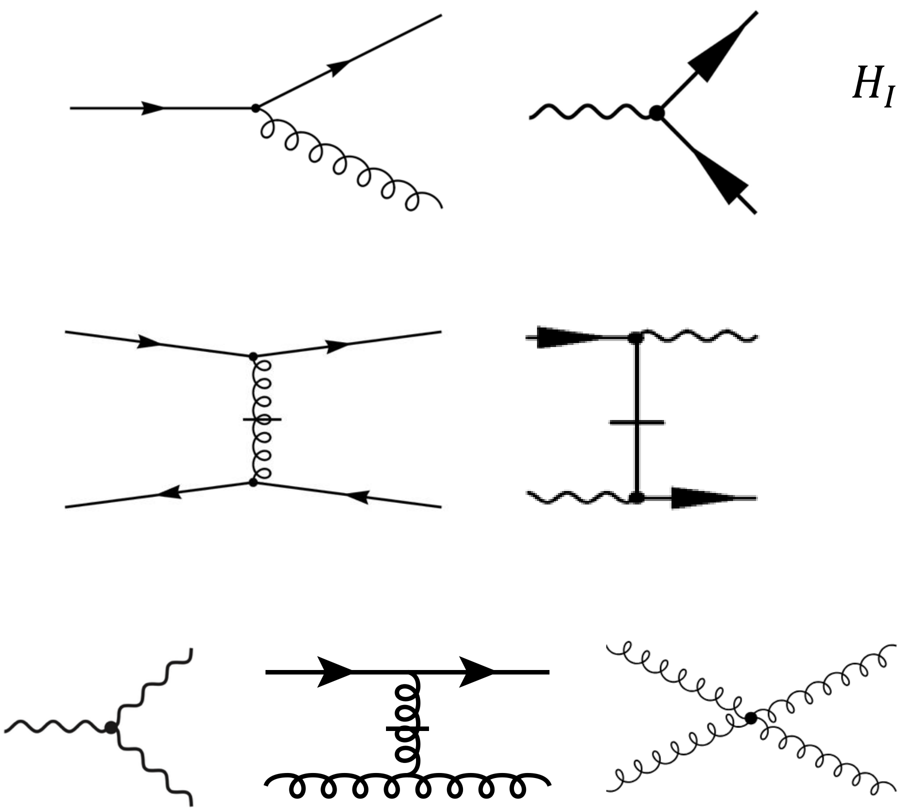
QCD Light-front Hamiltonian

[S. Brodsky, H-C Pauli, S. Pinsky, '97]

- QCD light-front Hamiltonian from QCD Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\mu\nu}_{\alpha} \quad \Longrightarrow \quad P^{-}_{QCD} = H_K + H_I$$

$$A^+ = 0$$



$$H_K = \tfrac{1}{2}\int \mathrm{d}^3x\, \bar{\psi}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\psi - \tfrac{1}{2}\int \mathrm{d}^3x\, A^i_a (\mathrm{i}\partial^\perp)^2 A^i{}_a$$

$$\begin{aligned}
H_I = &+ g\int \mathrm{d}^3x\, \bar{\psi}\gamma_\mu A^\mu \psi \\
&+ \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, \bar{\psi}\gamma_\mu A^\mu \frac{\gamma^+}{\mathrm{i}\partial^+}\gamma_\nu A^\nu \psi \\
&- \mathrm{i}g^2 \int \mathrm{d}^3x\, f^{abc}\bar{\psi}\gamma^+ T^c \psi \frac{1}{(\mathrm{i}\partial^+)^2}\left(\mathrm{i}\partial^+ A^\mu_a A_{\mu b}\right) \\
&+ \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, \bar{\psi}\gamma^+ T^a \psi \frac{1}{(\mathrm{i}\partial^+)^2}\bar{\psi}\gamma^+ T^a \psi \\
&+ \mathrm{i}g \int \mathrm{d}^3x\, f^{abc}\mathrm{i}\partial^\mu A^{\nu a} A^b_\mu A^c_\nu \\
&- \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, f^{abc} f^{ade}\mathrm{i}\partial^+ A^\mu_b A_{\mu c}\frac{1}{(\mathrm{i}\partial^+)^2}\left(\mathrm{i}\partial^+ A^\nu_d A_{\nu e}\right) \\
&+ \tfrac{1}{4}g^2 \int \mathrm{d}^3x\, f^{abc} f^{ade} A^\mu_b A^\nu_c A_{\mu d} A_{\nu e}.
\end{aligned}$$

ψ: quark field operator
A^a_μ: gluon field operator

7 terms in H_I

Dimension of Basis Space

- Expansion in BLFQ basis

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,q\bar{q}\rangle + |qqq\,gg\rangle + |qqq\,ggg\rangle + |qqq\,q\bar{q}\,g\rangle$$

$N_{max} = 7, K_{max} = 16$

	$\|qqq\rangle$	$\|qqqg\rangle$	$\|qqq\,q\bar{q}\rangle$	$\|qqq\,gg\rangle$	$\|qqq\,ggg\rangle$	$\|qqq\,q\bar{q}\,g\rangle$
dimension	35,088	592,960	3,901,500	5,169,360	19,603,584	7,128,576
color config	1	2	3	6	22	8

$$|N\rangle = |qqq\rangle$$

Basis Dimension= 35, 088

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,gg\rangle + |qqq\,ggg\rangle$$

Basis Dimension= 25,400,992

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,q\bar{q}\rangle + |qqq\,gg\rangle + |qqq\,ggg\rangle + |qqq\,q\bar{q}\,g\rangle$$

Basis Dimension= 58,491,220

Progress toward First Principles

$$|\mathrm{N}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, u\bar{u}\rangle + |qqq\, d\bar{d}\rangle + |qqq\, s\bar{s}\rangle + \cdots$$

- **Wave Functions:**

  [PRD,102,016008] (2019) [PRD,108 9, 094002] (2023) [PLB, 867,139599] (2025)

  [Eur. Phys. J. Spec. Top.] (2025)

- GPDs:

  [PRD,104,094036] (2021) [PLB,847,138305] (2023)

  [PRD,105,094018] (2022) [PRD,110.056027] (2024)

  [PRD,109,014015] (2024) [PLB,860,139153] (2025)

  [PLB,855,138809] (2024)

- **TMDs:**

  [PLB,833,137360] (2022) [PLB,855 138831] (2024)

  [PRD,108,036009] (2023)

- Higher-twist Distribution (GPD,TMD,DPD):

  [PRD,109,034031] (2024) [PLB,855 138829] (2024)

  [PRD,112,014018] (2025) [PRD] (2026)

- Gravitational Form Factors:

  [PRD,110,056027] (2024) [PRD,112,114001] (2025)

Input Parameters

- Truncation Parameters: Fock sectors up to 6 partons with $N_{\text{max}} = 7$ & $K = 6$

$$|\mathrm{N}\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + \left|qqqd\bar{d}\right\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle$$
$$+ \left|qqqd\bar{d}g\right\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

- Model Parameters:

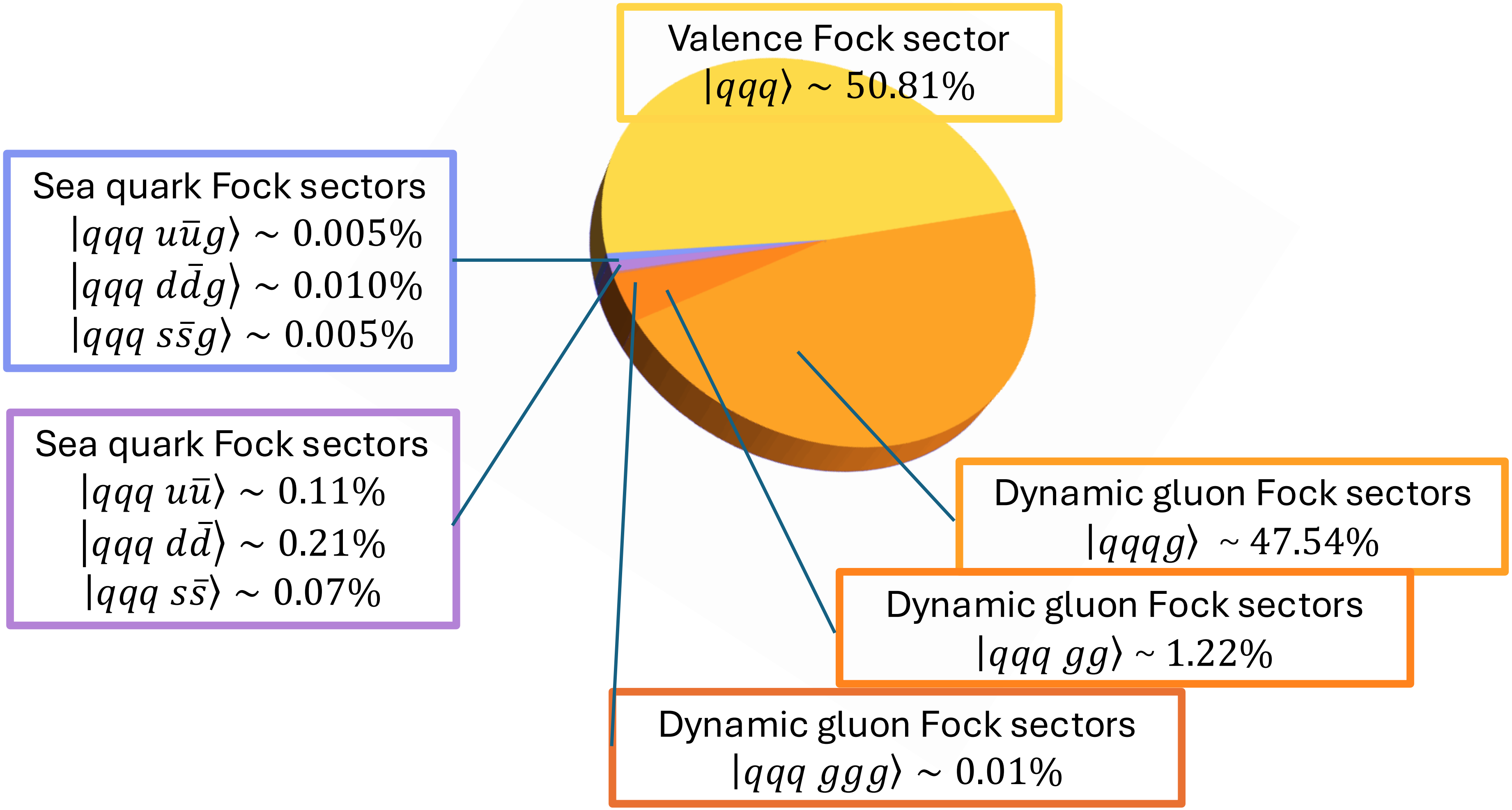
m_u	m_d	m_s	m_f	g	b
0.3 GeV	0.299 GeV	0.5 GeV	0.9 GeV	2.2	0.8 GeV

- determined by fitting electromagnetic form factors and mass of proton
- 1MeV mass difference between u and d quark to lift doublet degeneracy

$M_p = 1\ \text{GeV}$

Fock Sector Contribution

$$|N\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

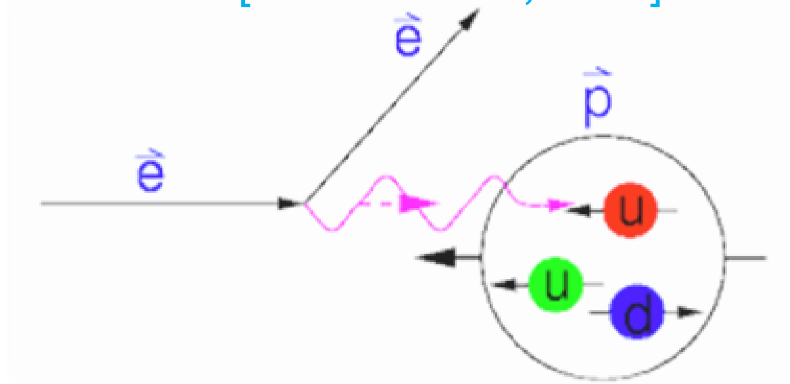


Electromagnetic Form Factor

- Elastic scattering of proton

[R. Hofstadter, 1961]

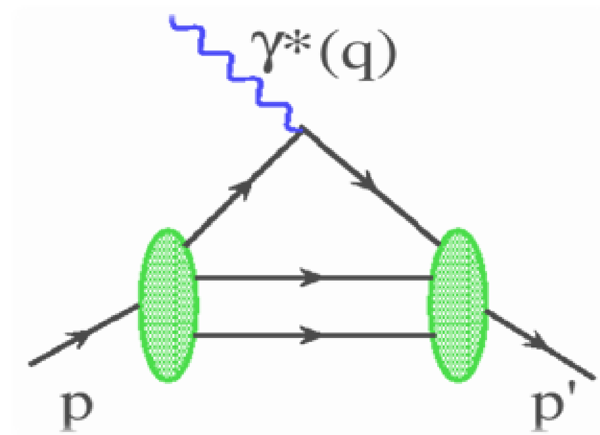
$$e(p) + h(P) \rightarrow e(p') + h(P')$$



- Elastic electron scattering established the extended nature of the proton (proton radius)

$$\langle N(p')|J^{\mu}(0)|N(p)\rangle = \bar{u}(p')\left[\gamma^{\mu} \underbrace{F_1(q^2)} + \frac{i\sigma^{\mu\nu}}{2m_N} q_{\nu} \underbrace{F_2(q^2)}\right] u(p)$$

Dirac Form Factor   Pauli Form Factor

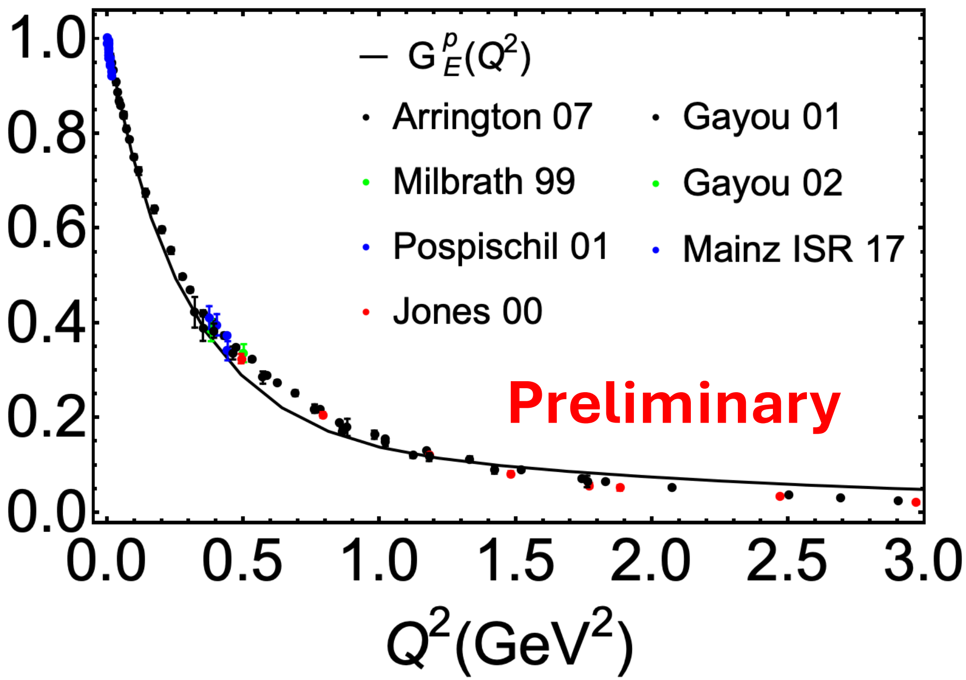


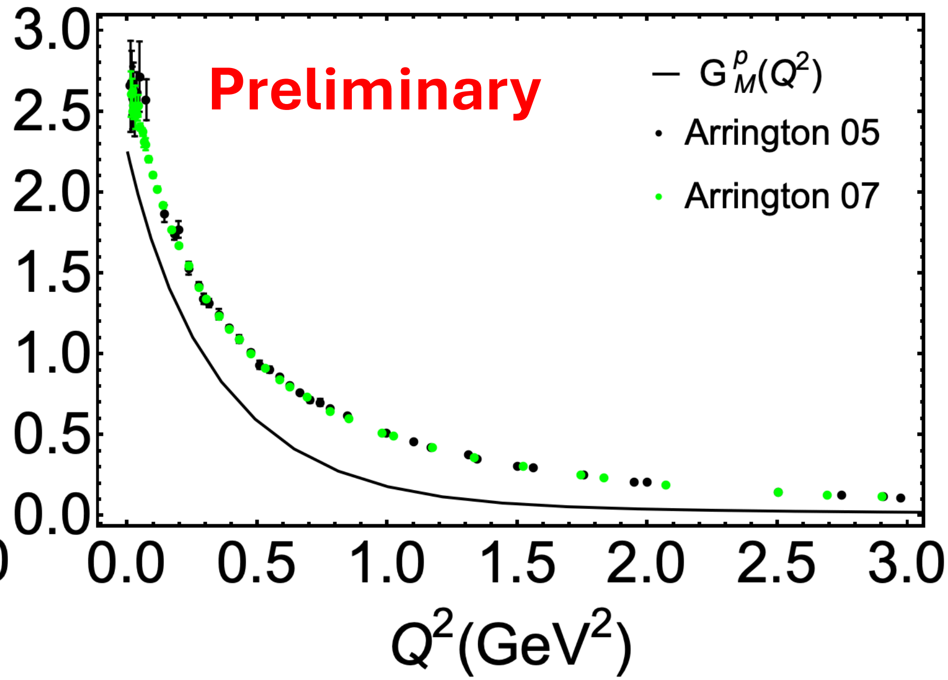
- The Fourier transformation of these form factors provide spatial distributions (charge and magnetization distributions)

Proton Form Factors

- Sachs form factors

$$G_E(Q^2) = \sum_q e_q F_1^q(Q^2) - \frac{Q^2}{4M_N^2} \sum_q e_q F_2^q(Q^2) \qquad G_M(Q^2) = \sum_q e_q F_1^q(Q^2) + \sum_q e_q F_2^q(Q^2)$$



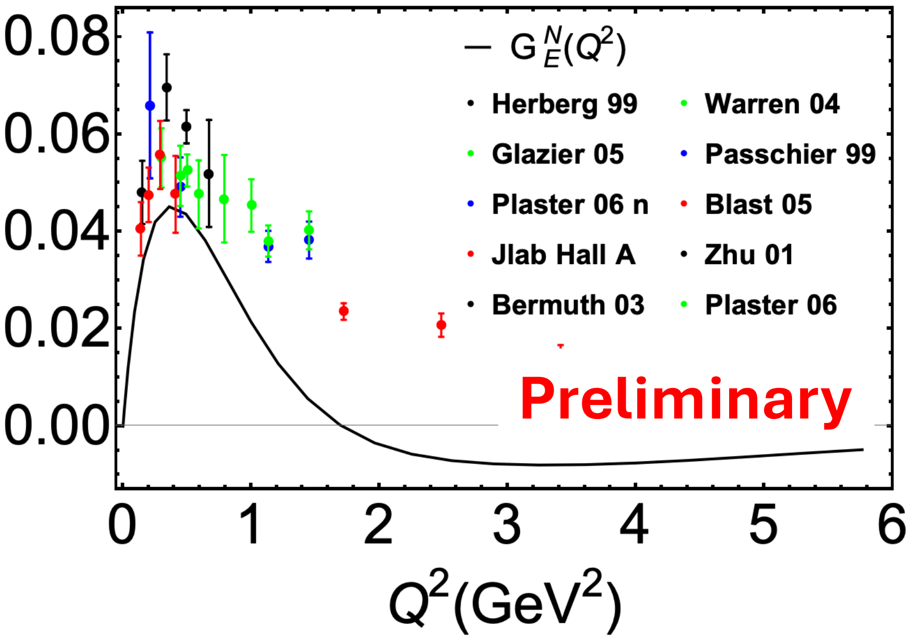


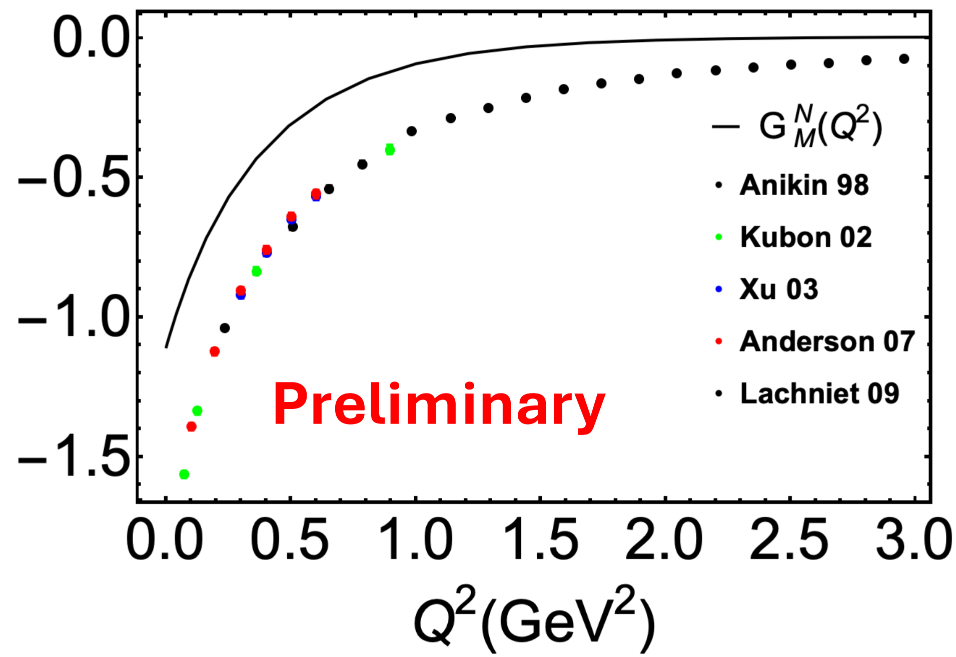
- Qualitative agreement in general trends between BLFQ results and exp data
- Pauli form factor underestimated

Neutron Form Factors

- Sachs form factors

$$G_E(Q^2) = \sum_q e_q F_1^q(Q^2) - \frac{Q^2}{4M_N^2} \sum_q e_q F_2^q(Q^2) \qquad G_M(Q^2) = \sum_q e_q F_1^q(Q^2) + \sum_q e_q F_2^q(Q^2)$$





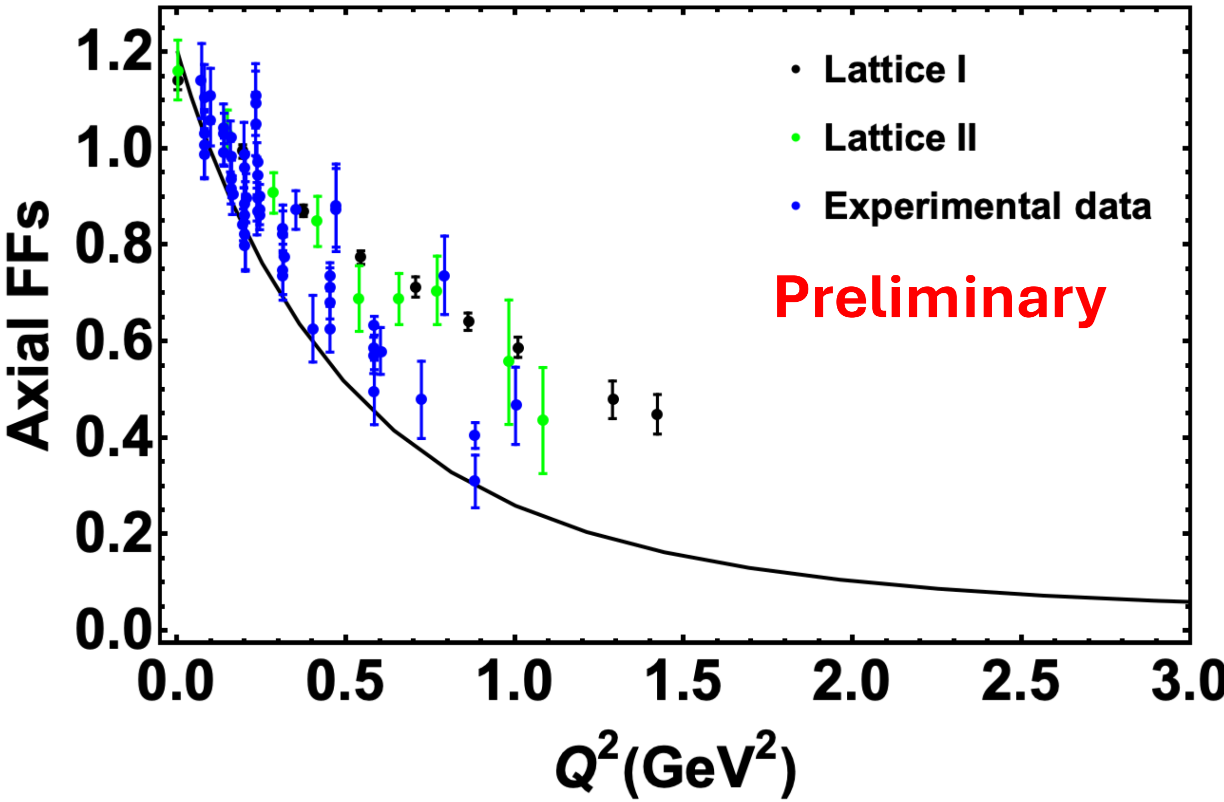
- Neutron Electric form factor qualitative agree with experimental data
- Negative charge radius!
- Need more work and Larger Kmax and Nmax to improve the results

Axial Form Factors

- Provide information on axial charge distributions

$$\langle N(p')|A_\mu^a|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu G_A(t) + \frac{(p'-p)_\mu}{2m} G_P(t)\right]\gamma_5 \frac{\tau^a}{2} u(p)$$

$$A_\mu^a = \bar{q}\gamma_\mu\gamma_5 T^a q \qquad G_A(Q^2) = G_u(Q^2) - G_d(Q^2)$$



- Black line: **valence quark**

$G_u(0) = 0.95$

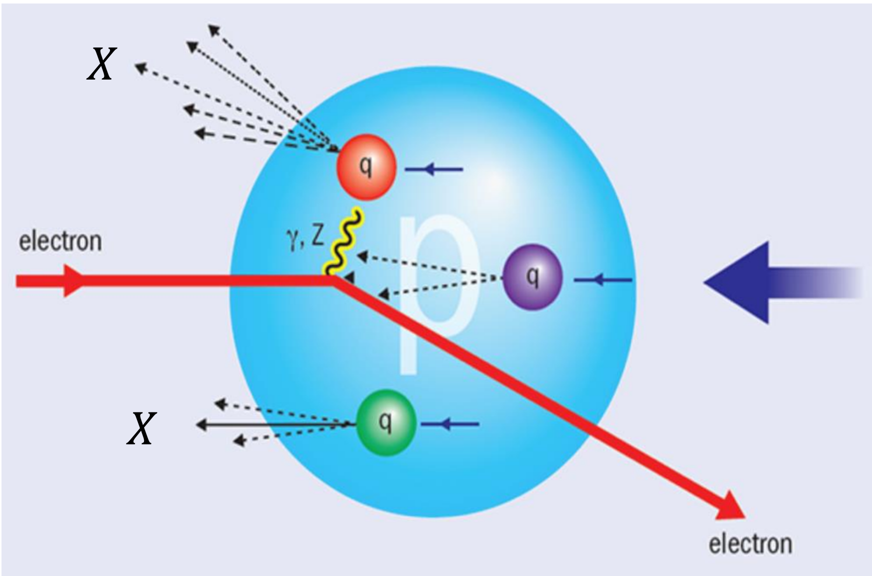
$G_d(0) = -0.25$

$G_A(0) = 1.20$

$G_g(0) = 0.11$

Parton Distribution Functions (PDF)

- **Deep Inelastic Scattering (SLAC 1968)**



$$e(p) + h(P) = e'(p') + X(P')$$

✧ **Localized probe:**

$$Q^2 = -(p - p')^2 \gg 1 \text{ fm}^{-2}$$

$$\Rightarrow \frac{1}{Q} \ll 1 \text{ fm}$$

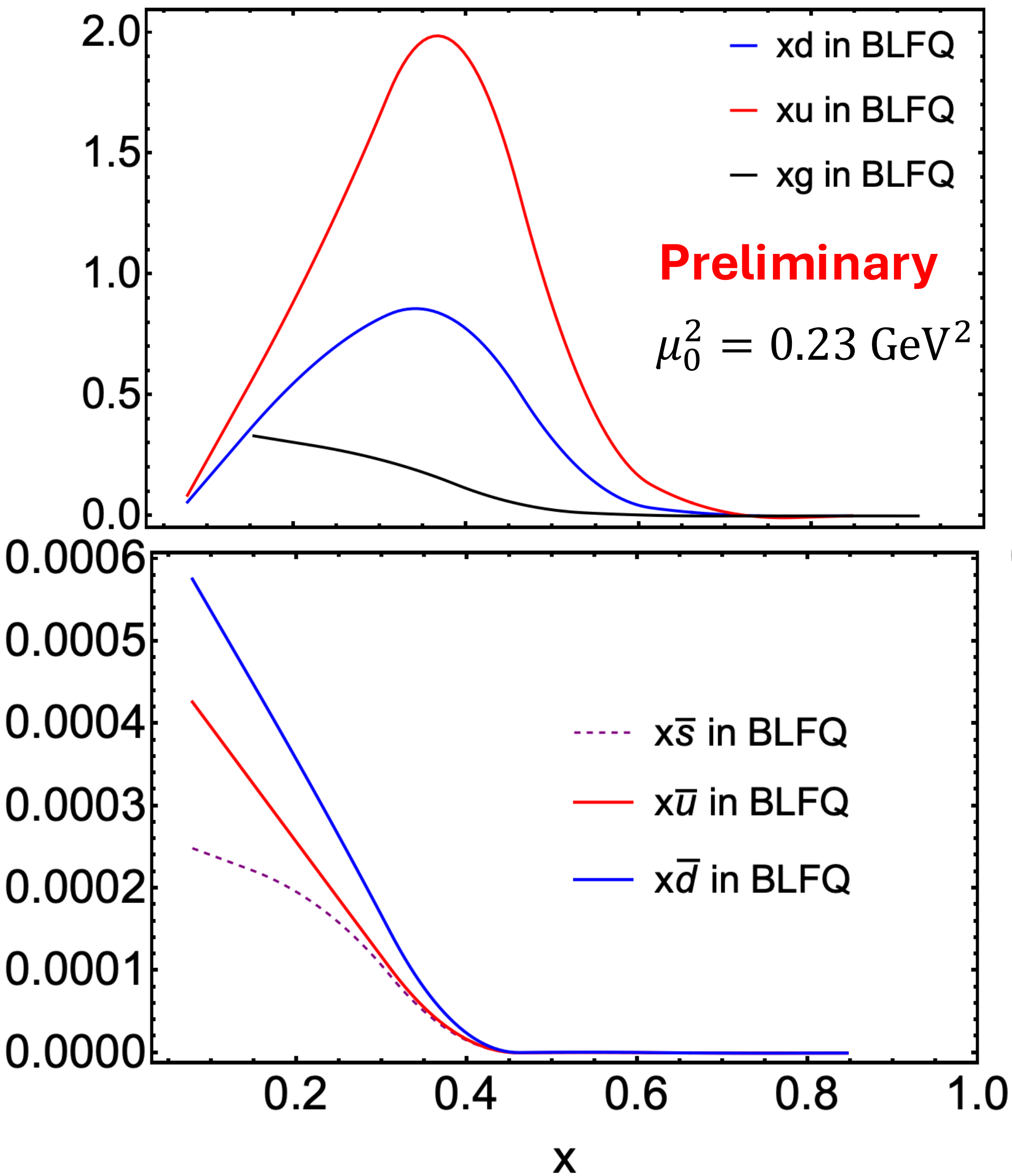
Discovery of spin ½ quarks and partonic structure

- **Parton distribution functions** (PDFs) are extracted from DIS processes.

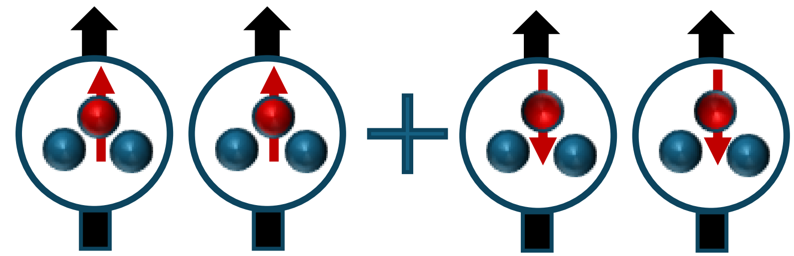
$$\Phi^{[\gamma^+]}(x, Q^2) = \int \frac{dz^-}{8\pi} e^{ixP^+z^-/2} \langle P, \Lambda | \bar{\psi}(z) \gamma^+ \psi(0) | P, \Lambda \rangle$$

PDFs encode the distribution of longitudinal momentum and polarization carried by the constituents

Unpolarized PDF



Unpolarized PDFs:

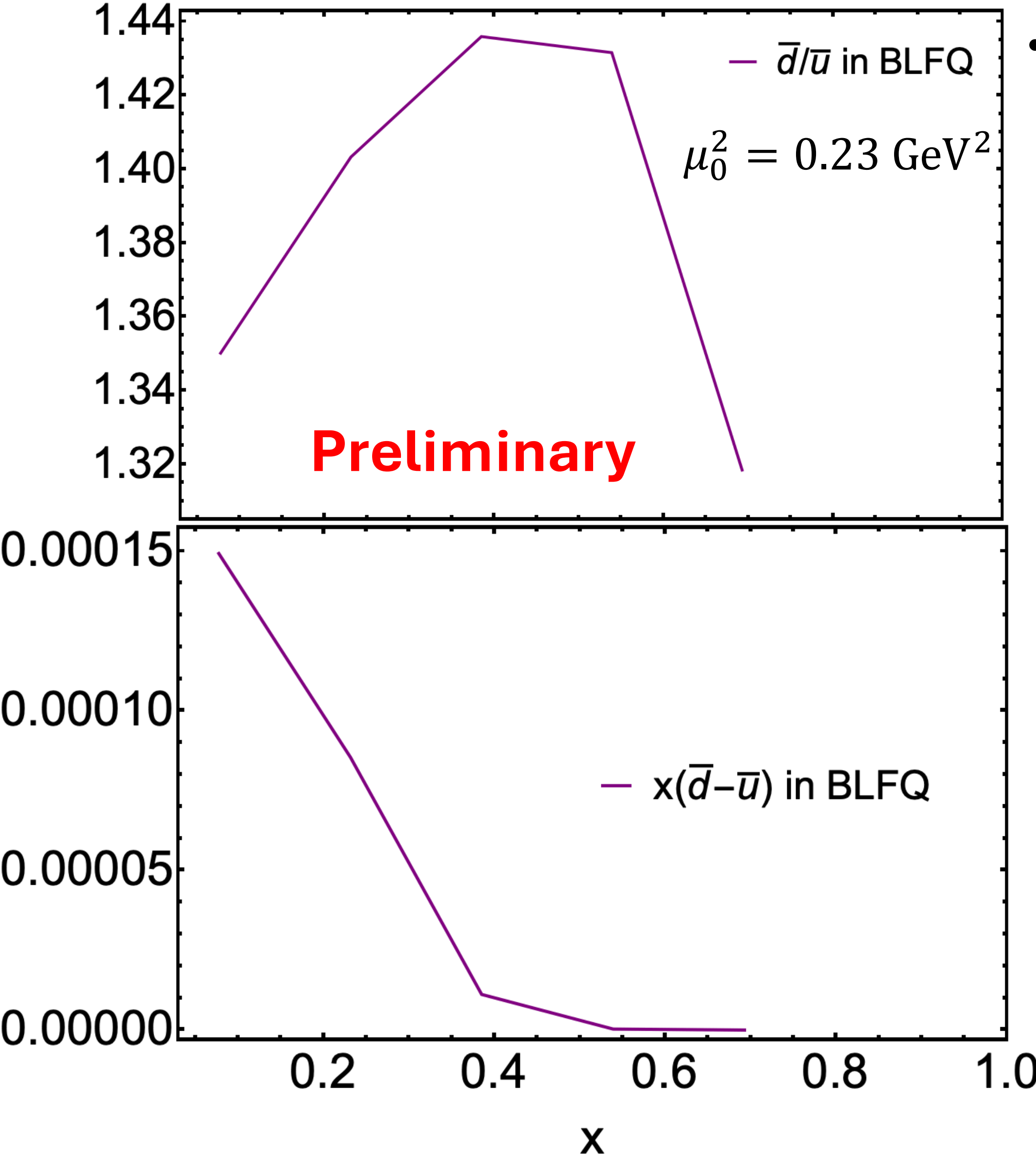


- Parton number density distribution in longitudinal direction

All results are at the initial scale

- Valence quarks contribute mainly at $x \sim 0.3$
- Gluon distributions are larger than those of sea quark
- Gluon and sea quark dominate in small x region

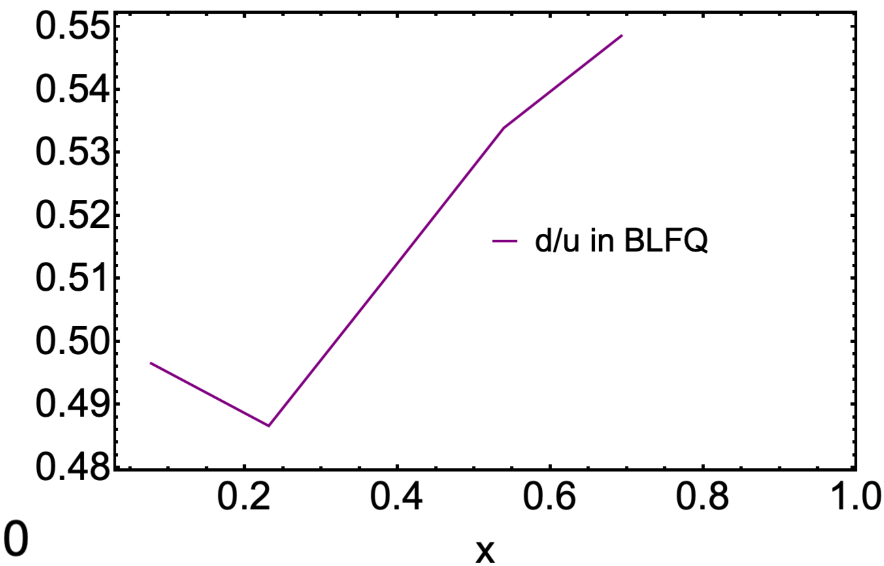
Unpolarized PDF



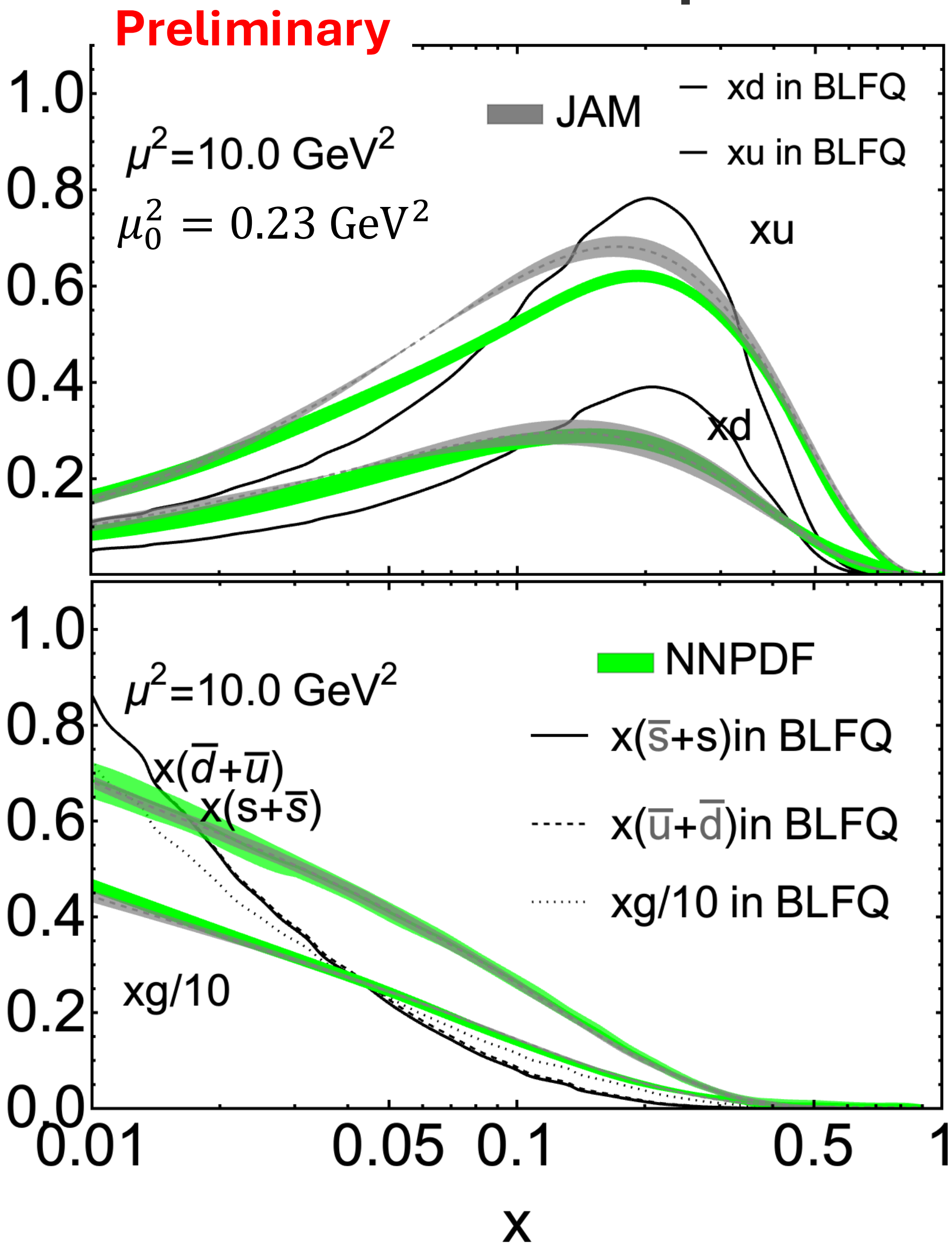
- Parton number density distribution in longitudinal direction

  All results are at the initial scale

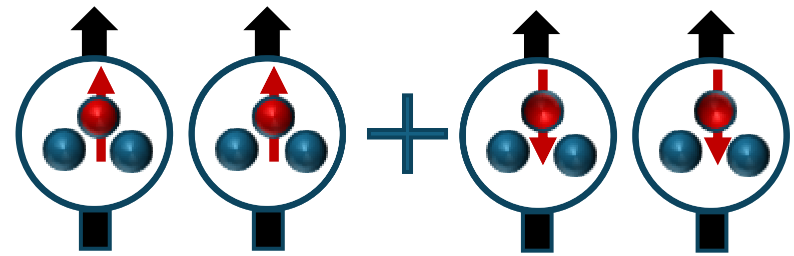
- $\bar{d} > \bar{u}$ over entire range of x
- $\bar{d}/\bar{u}$ first increase and then decrease



Unpolarized PDF



Unpolarized PDFs:



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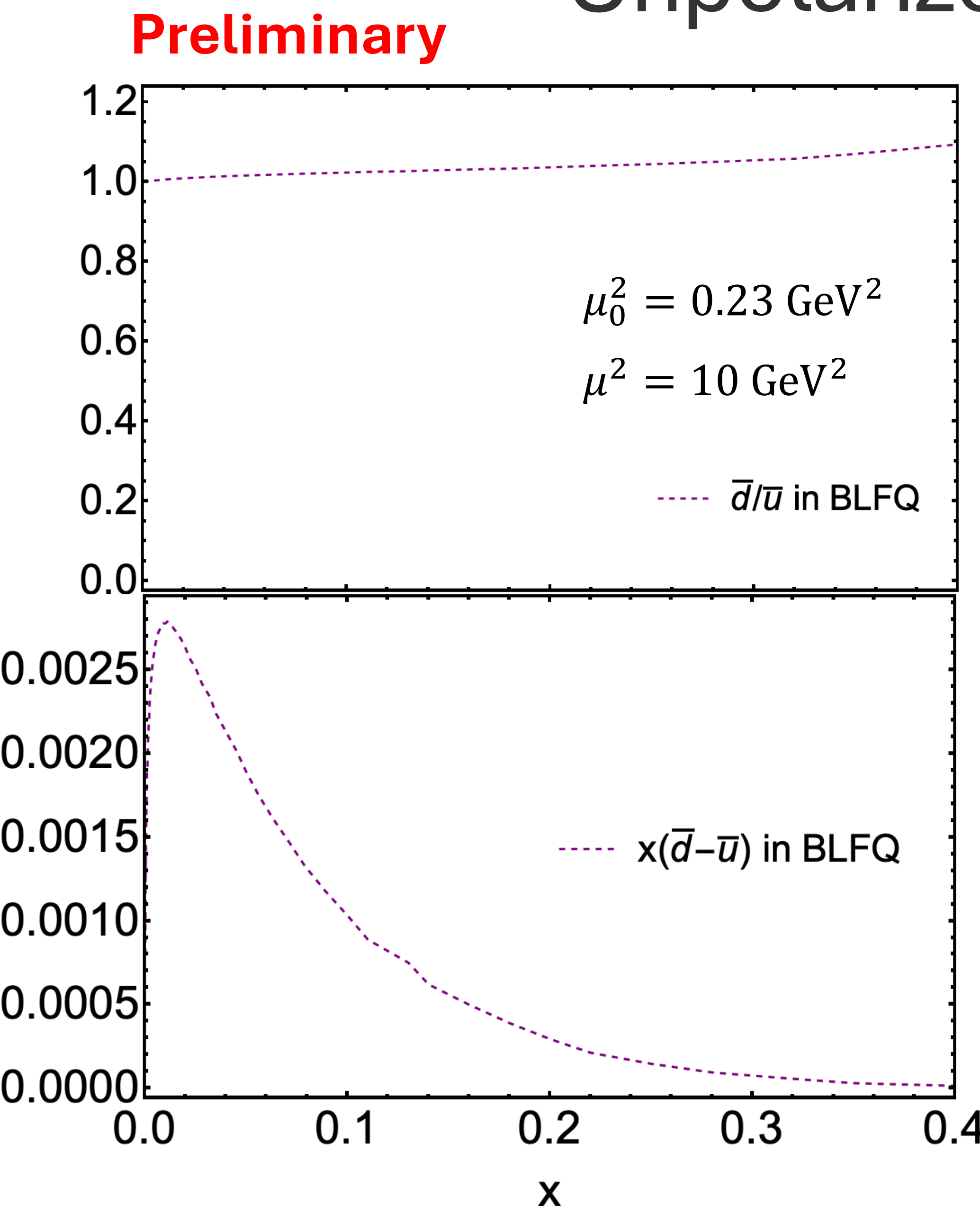
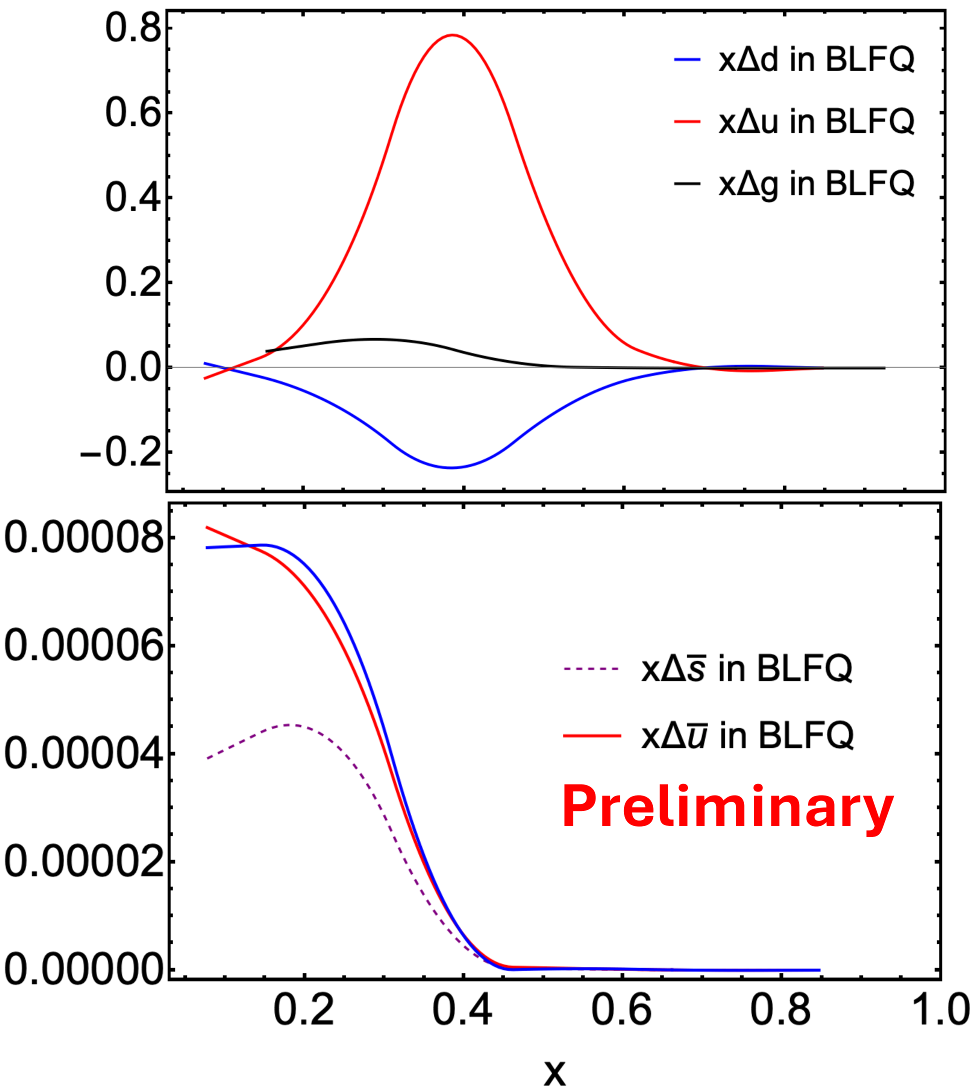


- Parton number density distribution in longitudinal direction

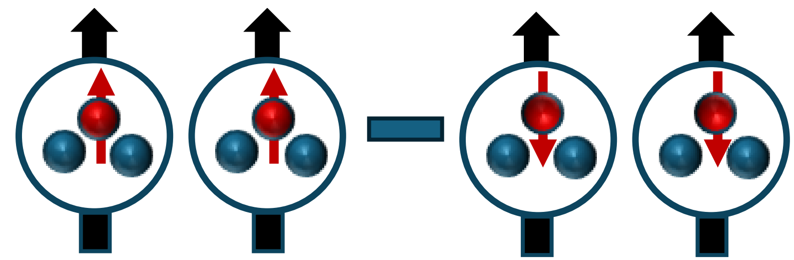
All results are at the experimental scale

- $\bar{d} > \bar{u}$ over the range of $x \sim [0.0, 0.4]$
- Maximum value of $\frac{\bar{d}}{\bar{u}} \approx 1.18$
- $x(\bar{d} - \bar{u})$ Qualitative agree with global fitting

Helicity Parton Distribution Functions



Helicity PDFs:



- Opposite signs for u and d quark
- Gluon contribution is positive
- Qualitatively agree with expectation

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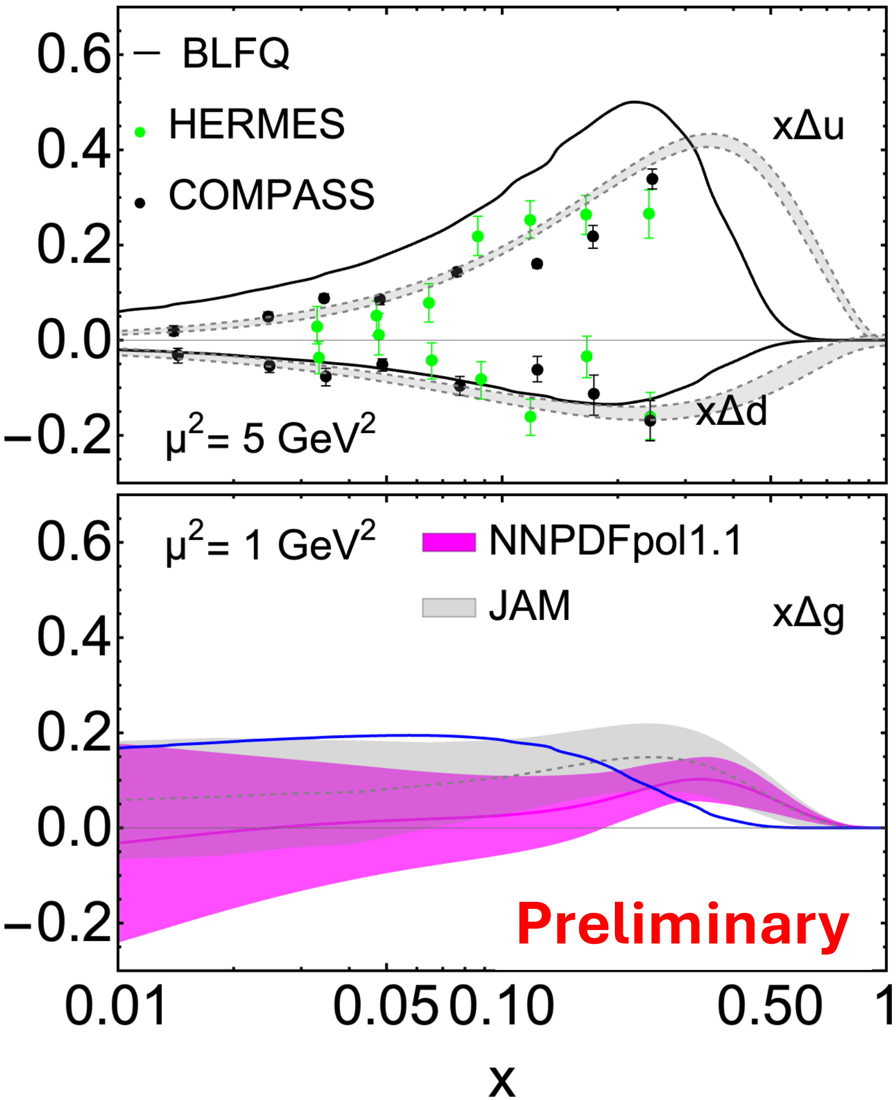
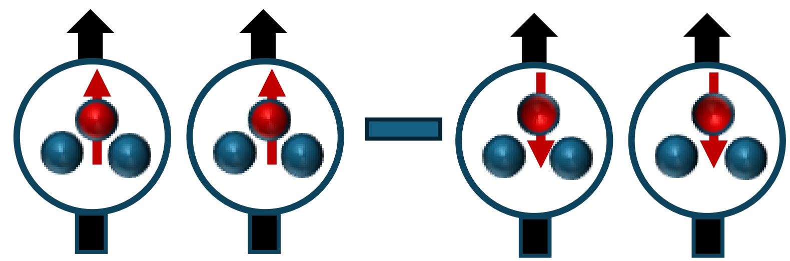


Helicity PDFs:



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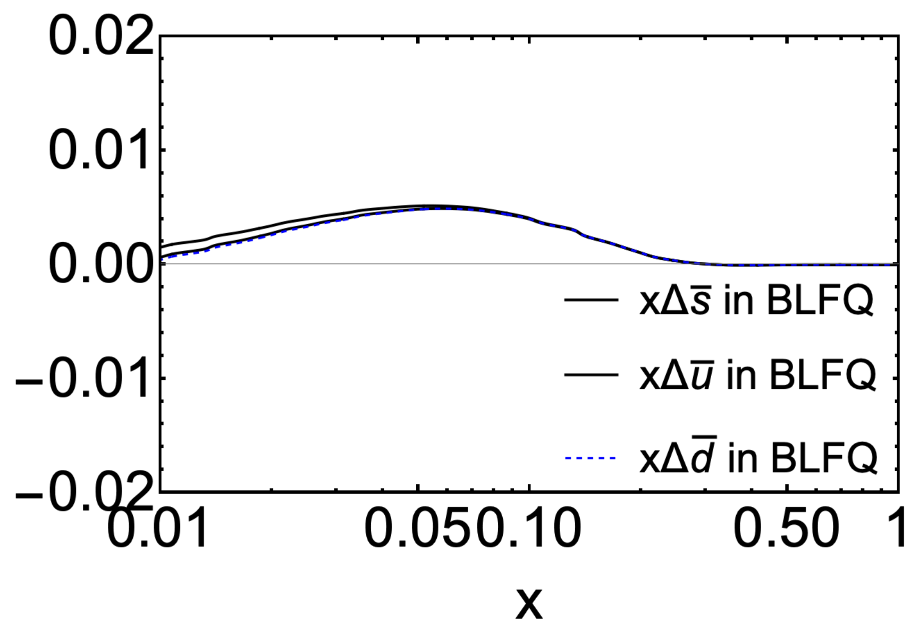


Conclusions

- Light-front quantization
  - probed by DIS kinematics in the Bjorken limit
  - offer static, frame-independent, and probabilistic description of relativistic bound states
  - Hamiltonian formalism for quantum field theory

- Basis Light-front Quantization
  - nonperturbative approach in Minkowski spacetime
  - describing hadron structure from QCD
  - keep full quantum correlation between partons

Outlook

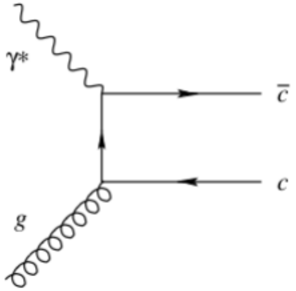
Current status

→ Full QCD interaction
→ Deutron calculation
$|qqq\ qqq\rangle + |qqq\ qqq\ g\rangle$

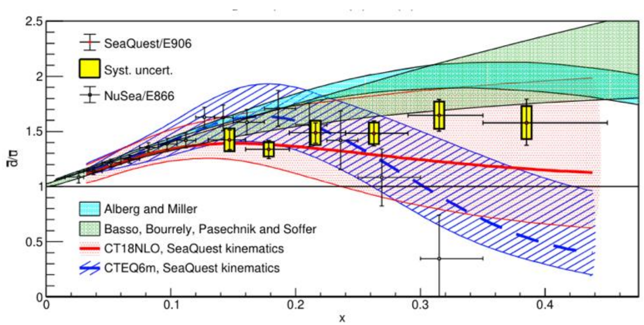
Deutron calculation → EMC effect

Full QCD interaction → Intrinsic charm, Sea asymmetry, Origin of spin and mass

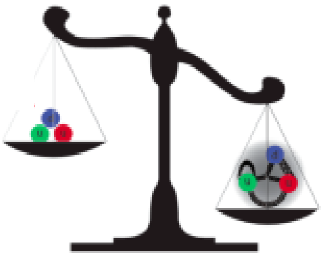
Intrinsic charm

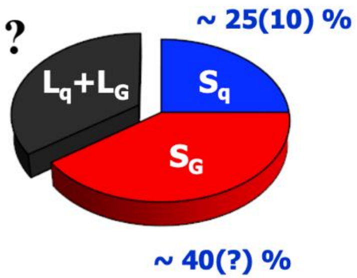


Sea asymmetry



Origin of spin and mass





Thank you!