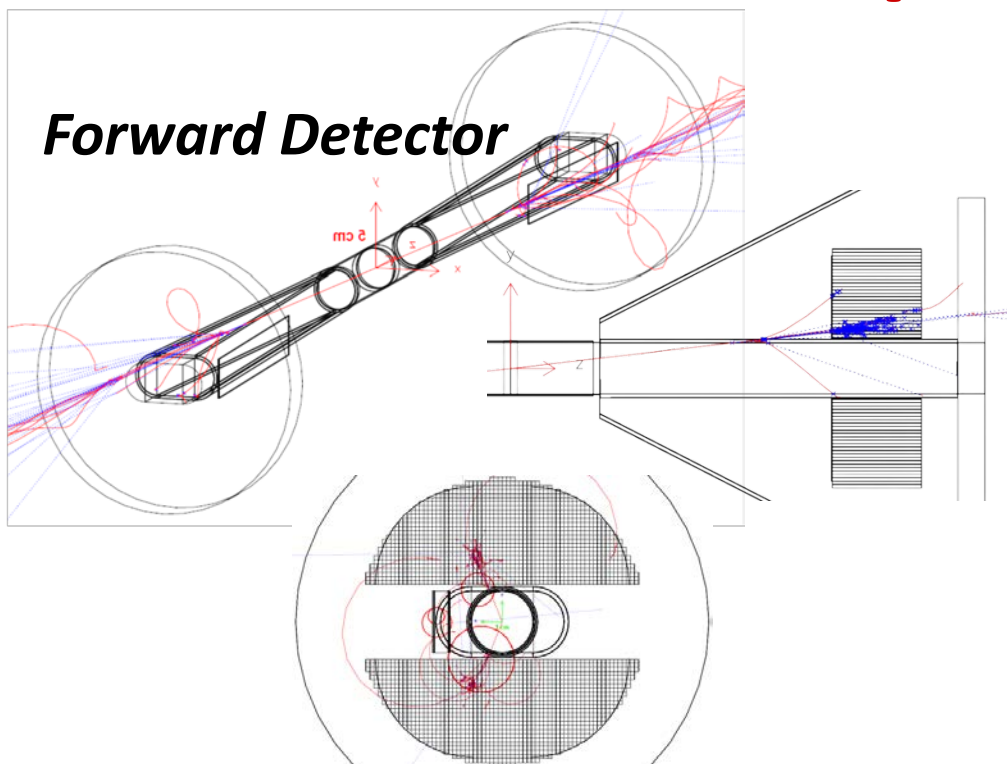


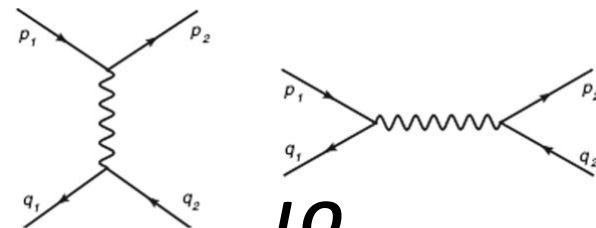


Precision test of QED on radiative Bhabha photons

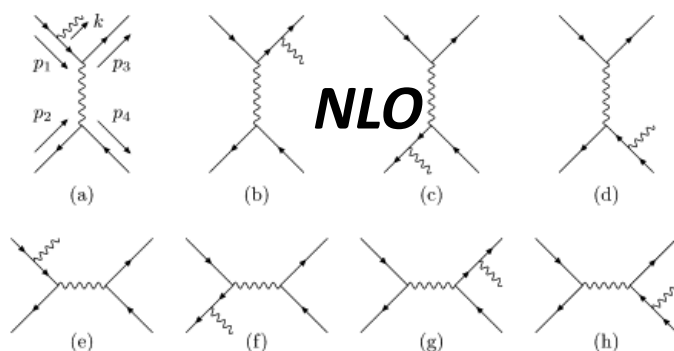
侯书云 Suen Hou
Academia Sinica
2026.07.02



Radiative Bhabha elastic scattering

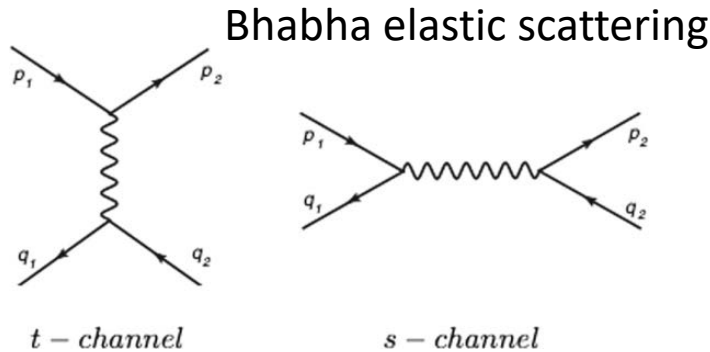


LO

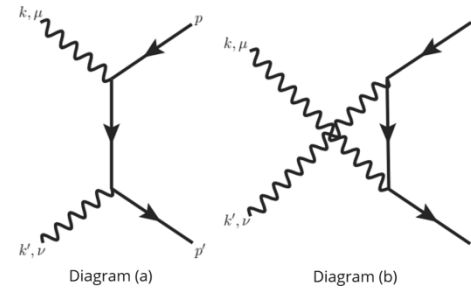


QED processes at e^+e^- colliders

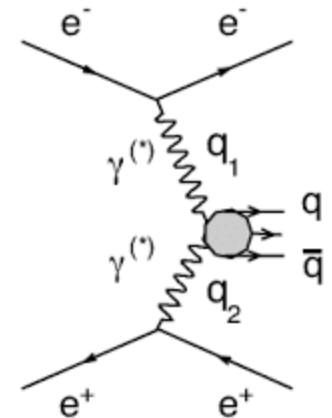
1. Bhabha elastic scattering
2. Two photon interaction



two-photon pair

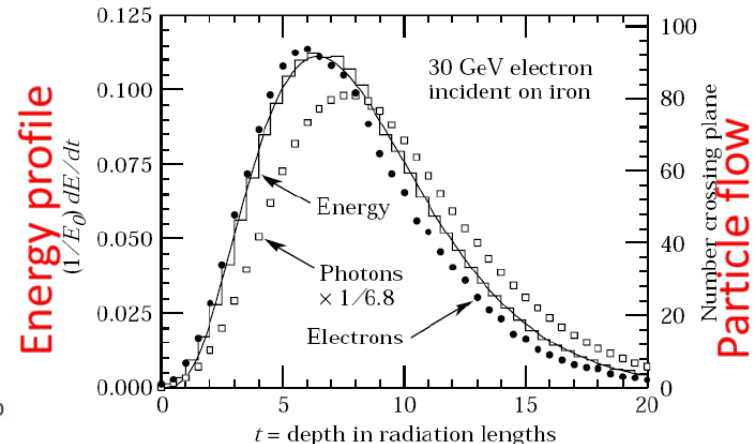
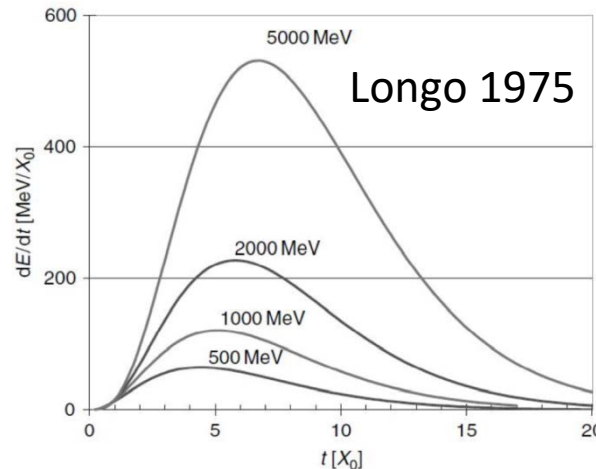
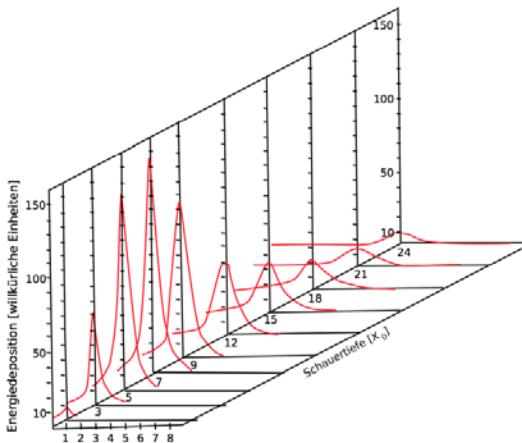


two-photon fusion



3. Beam background: Sync. Radiation, IP Bremsstrahlung
4. Electromagnetic Shower, Calorimetry

a 6 GeV electron in lead



Bhabha experimental results

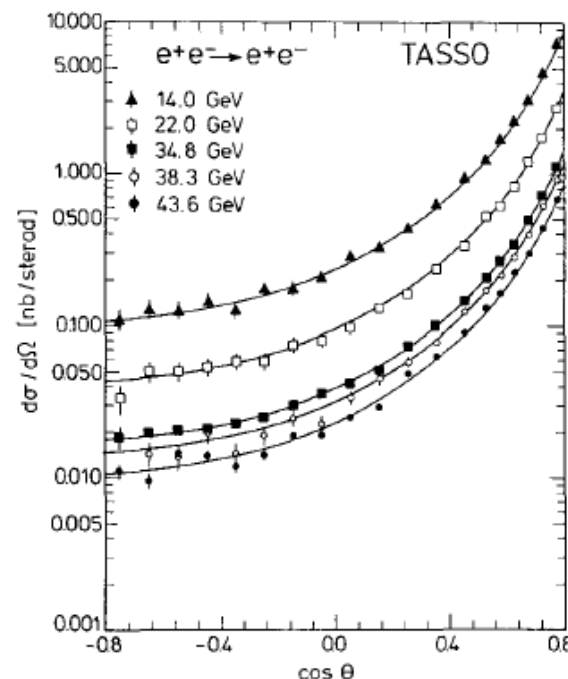
$$e^+e^- \rightarrow e^+e^- (\gamma)$$

- **TASSO** Bhabha [1988, ZPC 37, 171]
Systematic error **~3%**

$\sqrt{s} = 12 - 47 \text{ GeV}$

Table 1. Data samples used for the analysis $e^+e^- \rightarrow e^+e^-$

$\langle \sqrt{s} \rangle$ (GeV)	$\int \mathcal{L} dt$ (pb^{-1})	N_{Bhabha}
14.0	1.7	10730
22.0	2.7	7106
34.8	174.5	166348
38.3	8.9	6035
43.6	37.1	22951

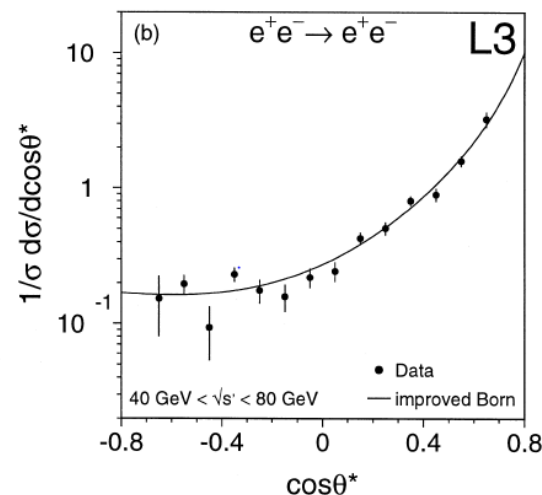
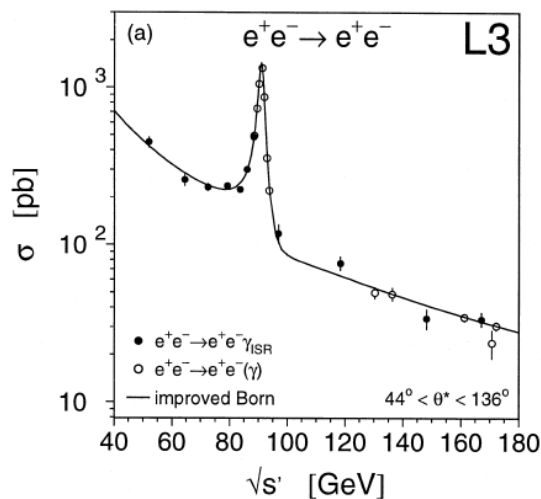


- **L3** radiative Bhabha with **ISR**

- Systematic error at **~1% level**

$\sqrt{s} = 50 \sim 170 \text{ GeV}$,
 232 pb^{-1} , 2856 event

[1998, PLB 439, 183]



Bhabha Luminosity for SM, R ratio

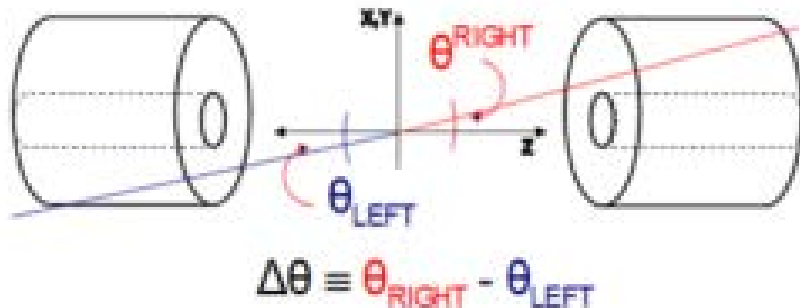
Luminosity \mathcal{L} is derived by Bhabha event counting

$$e^+e^- \rightarrow e^+e^-(n\gamma)$$

$$\mathcal{L} = \frac{1}{\epsilon} \frac{N_{\text{acc}}}{\sigma^{\text{vis}}} \quad \sigma = \frac{16\pi\alpha^2}{s} \left(\frac{1}{\theta_{\text{min}}^2} - \frac{1}{\theta_{\text{max}}^2} \right)$$

Bhabha detected for

- a pair of back-back electrons,
- precision ϑ of $e, e(\gamma)$ in fiducial region



$R(s)$ ratio for SM predictions

Requiring precision on

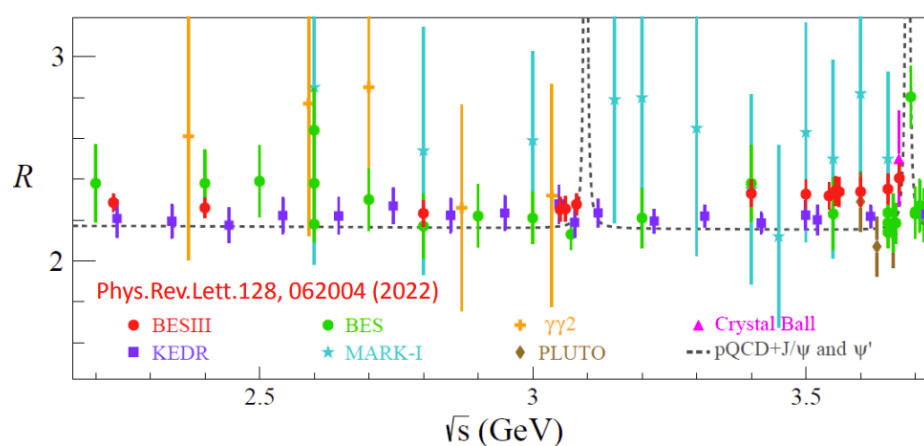
$$a_\mu = (g_\mu - 2)/2 \text{ and } \Delta\alpha_{\text{had}}(M_Z)$$

$$a_\mu = \frac{\alpha^2}{3\pi^2} \int_{m_\pi^2}^{\infty} ds K(s) \frac{R(s)}{s}$$

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} \text{Re} \int_{m_\pi^2}^{\infty} \frac{R(s) ds}{s(s - M_Z^2 - i\epsilon)}$$

BESII in 2–5 GeV, precision 6%

BESIII 2022 3%



Radiative Bhabha, NLO, NNLO $e^+e^- \rightarrow e^+e^- (n\gamma)$

Methods used for multiple photon corrections

1. SF: analytical collinear QED Structure Functions
2. YFS exponentiation Small angle, **0.054% BHLUMI (LEP)**
YFS exponentiation Large angle, **BHWIDE (LEP)**
3. PS: Parton Shower, Large angle **0.1% BabaYaga@NLO**

e^+e^- collision luminosity

$$\int \mathcal{L} dt = N_{\text{obs}}/\sigma_{\text{th}}$$

$$\frac{\delta \mathcal{L}}{\mathcal{L}} = \frac{\delta \mathcal{L}_{\text{exp}}}{\mathcal{L}_{\text{exp}}} \oplus \frac{\delta \sigma_{\text{th}}}{\sigma_{\text{th}}}$$

Luminosity errors:
Experiment
Theory

collinear log : $L \equiv \log \frac{s}{m_e^2}$

G. Montagna
Ustron, 2015

$L = \log(s/m_e^2) \simeq 15$	Large angle @ Flavor
$L = \log(t /m_e^2) \simeq 17$	Small angle @ LEP
$L = \log(t /m_e^2) \simeq 20$	Small angle @ $t\bar{t}$ thresh.

Flavor Factories

collinear log : $L \equiv \log \frac{s}{m_e^2}$

C.M. Carloni Calame
ECFA Higgs CERN 2021

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	\dots

Red: matched PS, SF + NLO

LO	90%		
NLO	10%	0.5%	
NNLO	0.5%	0.05%	0.01%
h.o.	0.01%	\dots	\dots

Typically at flavour factories (on integrated Bhabha σ)

Bhabha: $e^+e^- \rightarrow e^+e^-(n\gamma)$ vs dependency

BHWIDE center-of-mass

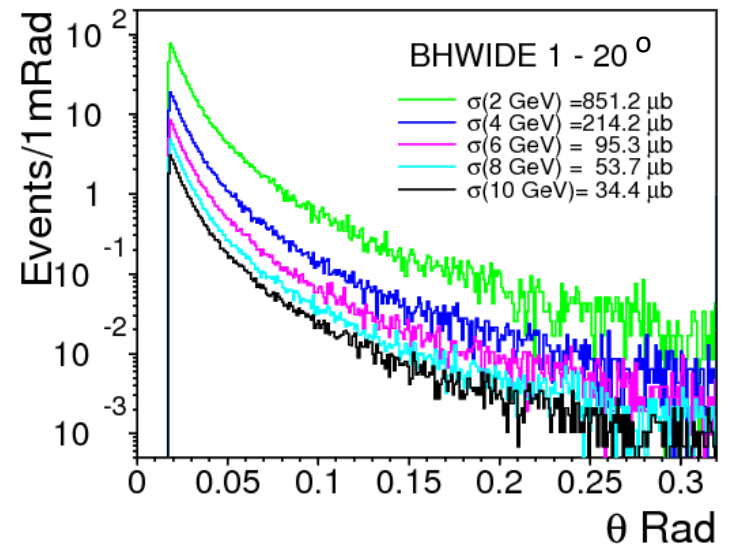
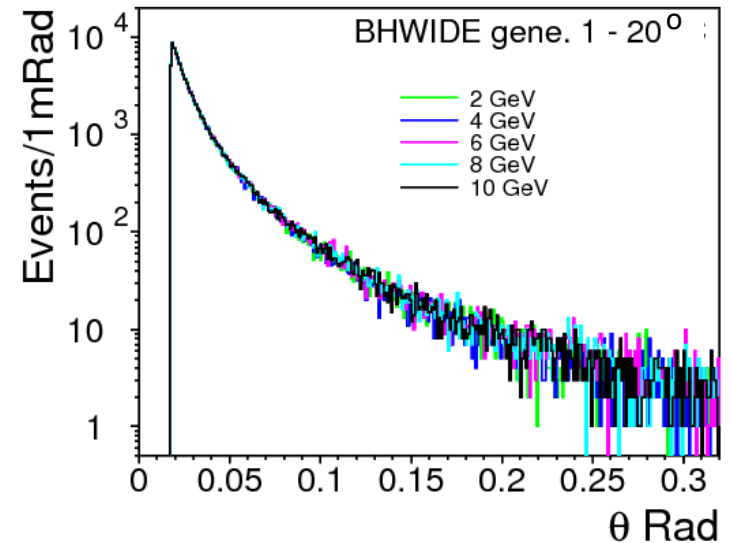
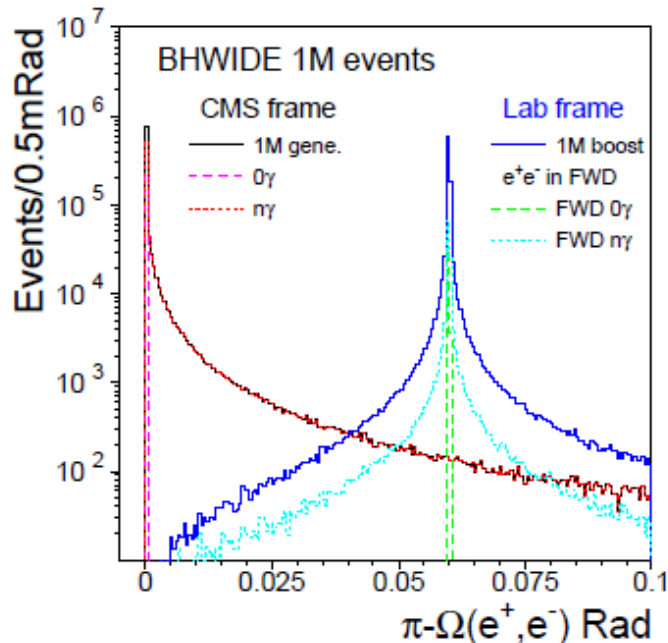
$\sqrt{s} = 2 - 10$ GeV, θ range $1 - 20^\circ$

- θ distribution: well overlapped
- Cross section much higher at lower \sqrt{s}

e^+e^- back-to-back

60 mRad beam-crossing boost

Delta/tails for $0\gamma/n\gamma$ events



Bhabha: $e^+e^- \rightarrow e^+e^-(n\gamma)$ cross section

BHLUMI/BHWIDE CM-frame

YFS algorithm for radiation

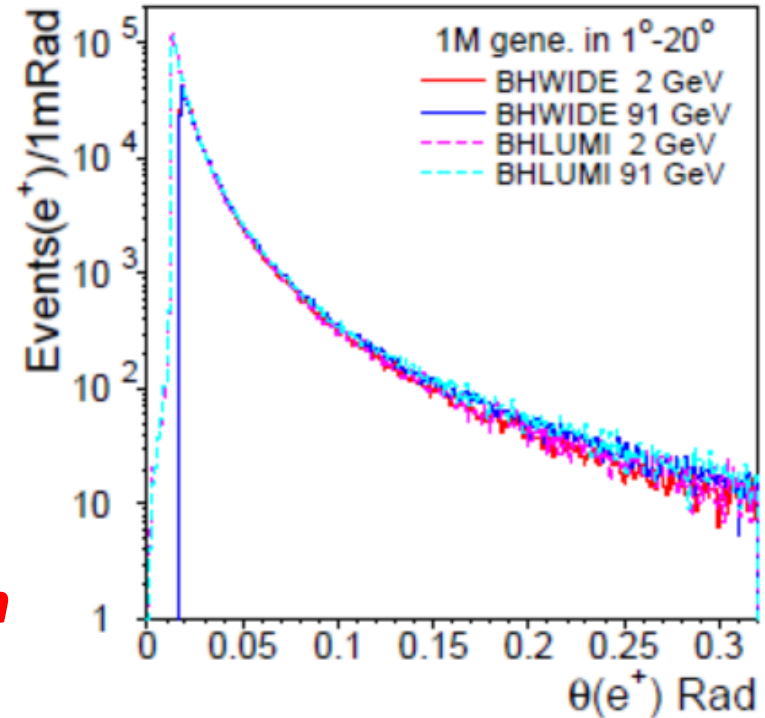
Compare cross section in $1^\circ - 20^\circ$

BHLUMI $E_\gamma > 10^{-4} E_{beam}$

BHWIDE $E_\gamma > 10^{-5} E_{beam}$

- BHWIDE is for “Large Angle” region $> 6^\circ$
t-channel effect contributes ?
- **CM-frame cross section agrees $\sim 0.1\%$**
- **FSTD Forward Det. $\rightarrow \sim 2\%$ deviation**

effect c



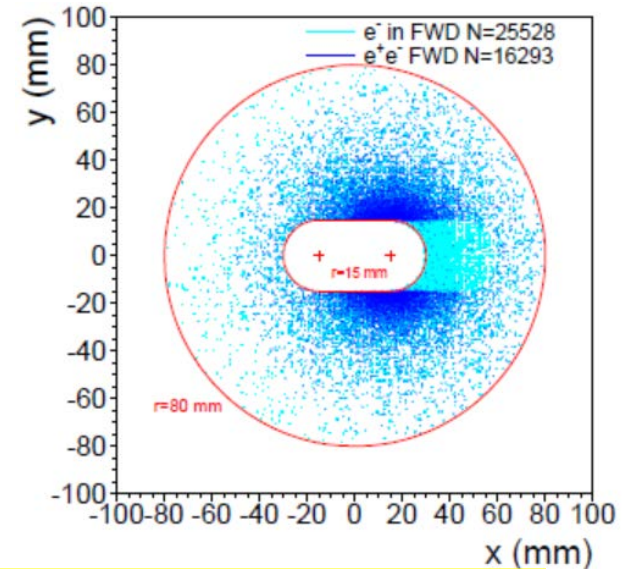
BHWIDE \sqrt{s} (GeV)	2	4	6	8	10	91.2	160	240	360
$\sigma(1^\circ-20^\circ)$ nb	854000	214600	95420	53780	34460	423.4	138.7	62.08	27.78
$\sigma(\text{FWD})$ nb	139900	35230	15710	8871	5692	71.93	23.68	10.65	4.788
$\epsilon(\text{FWD}/[1^\circ-20^\circ])$.1638	.1642	.1646	.1649	.1652	.1699	.1708	.1716	.1724
BHLUMI \sqrt{s} (GeV)	2	4	6	8	10	91.2	160	240	360
$\sigma(1^\circ-20^\circ)$ nb	857800	214300	95430	53830	34460	424.0	138.7	62.08	27.79
$\sigma(\text{FWD})$ nb	137900	34640	15450	8728	5587	69.00	22.50	10.16	4.567
$\epsilon(\text{FWD}/[1^\circ-20^\circ])$.1607	.1616	.1619	.1621	.1621	.1627	.1625	.1637	.1644

Bhabha STCF FWD acceptance

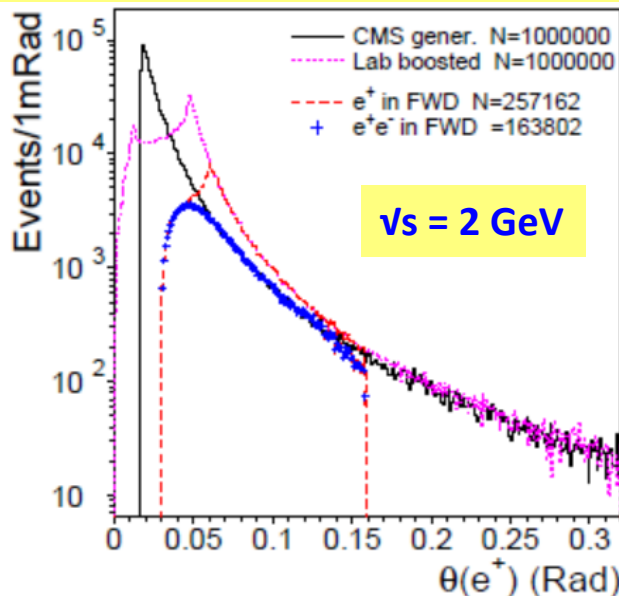
Forward Det. (FWD) acceptance
both $e^+ e^-$ detected in FWD

- @ $|z| = 500\text{mm}$
- off beampipe $\varnothing = 30\text{ mm}$; $r < 80\text{mm}$
- 60 mRad beam-crossing boost w.r.t. CM generated in $1\text{-}20^\circ$
acceptance: $\epsilon \sim 16.3\%$

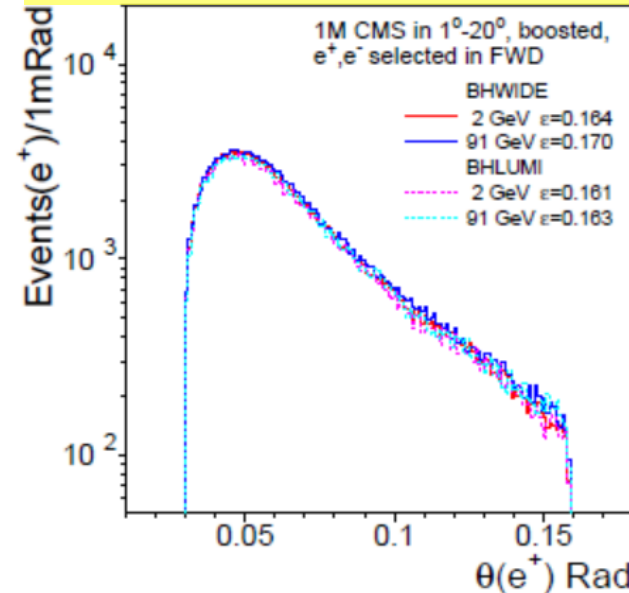
Accepted in FWD @ $z=500\text{mm}$



Theta of e^+ , gen. to FWD accepted



FWD accepted, BHWIDE/BHLUMI



Radiative Bhabha $e^+e^-(n\gamma)$

BHLUMI/BHWIDE radiative Bhabha

QED radiation calculated with **YFS exponentiation**

$n\gamma$ in **Poisson distribution**

BHLUMI E_γ cut = $1 \times 10^{-4} \times E_{\text{beam}}$ **BHWIDE** E_γ cut = $1 \times 10^{-5} \times E_{\text{beam}}$

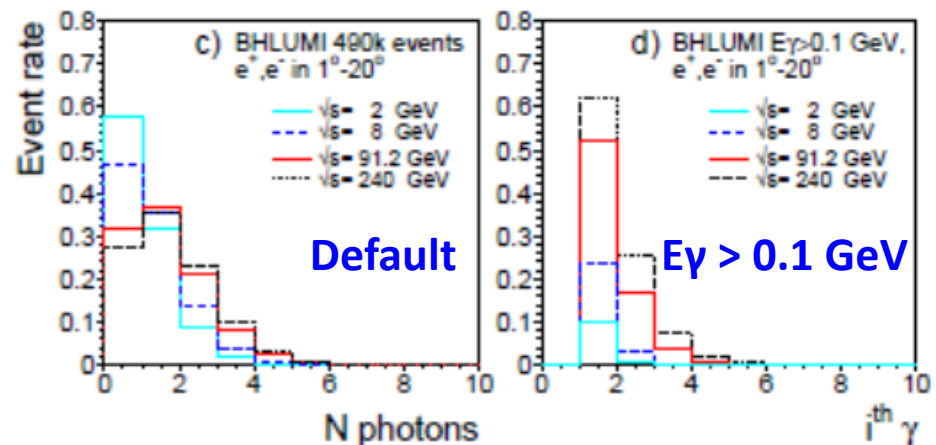
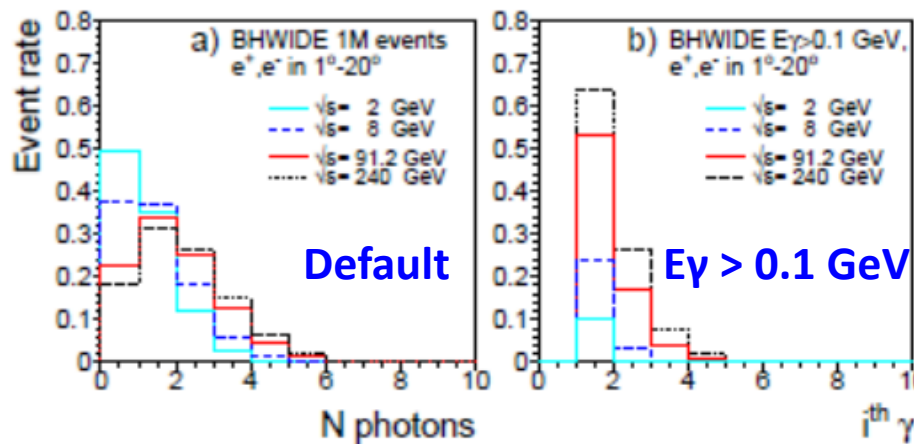
Count photons detectable $E_\gamma > 0.1$ GeV

→ Photons $n\gamma$ rates well consistent with BHLUMI/BHWIDE

$n\gamma$ event rates

BHWIDE

BHLUMI



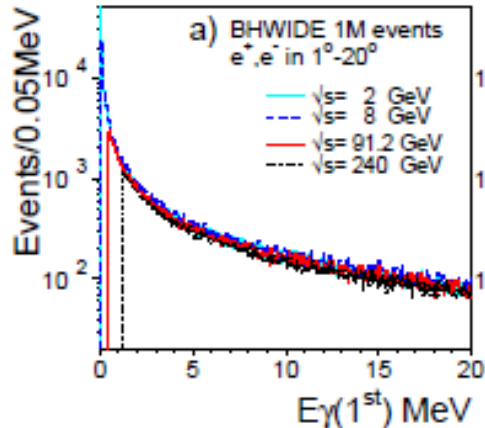
Radiative Bhabha $e^+e^-(n\gamma)$

BHLUMI/BHWIDE radiative Bhabha

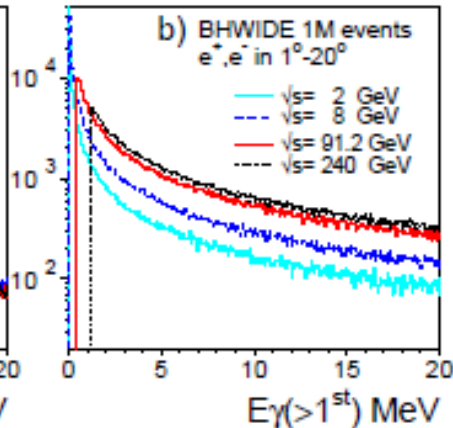
- **1st energetic photon, same rates, indep. of \sqrt{s}**
 event rates vs E_γ , near threshold ~ 20 MeV
 well consistent with BHLUMI/BHWIDE
- **2nd energetic photon, higher rates with \sqrt{s}**

BHWIDE E_γ

1st photon

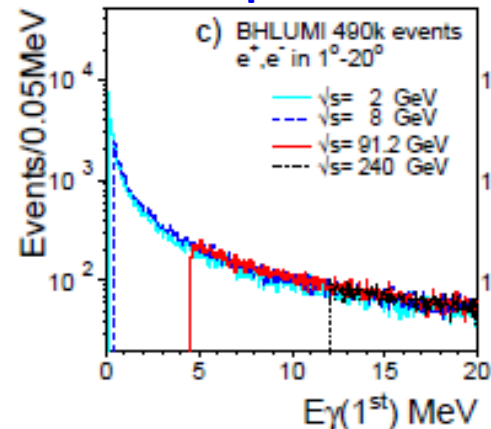


2nd photon

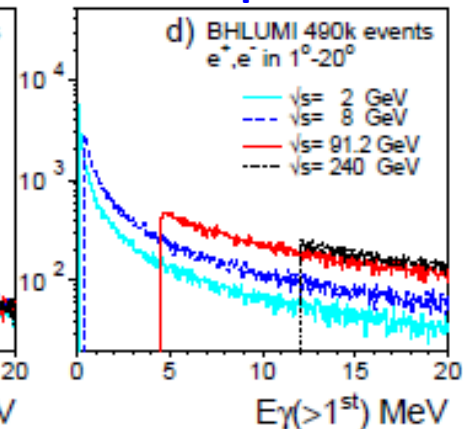


BHLUMI E_γ

1st photon



2nd photon



Radiative Bhabha $e^+e^-(n\gamma)$

BHLUMI/BHWIDE radiative Bhabha

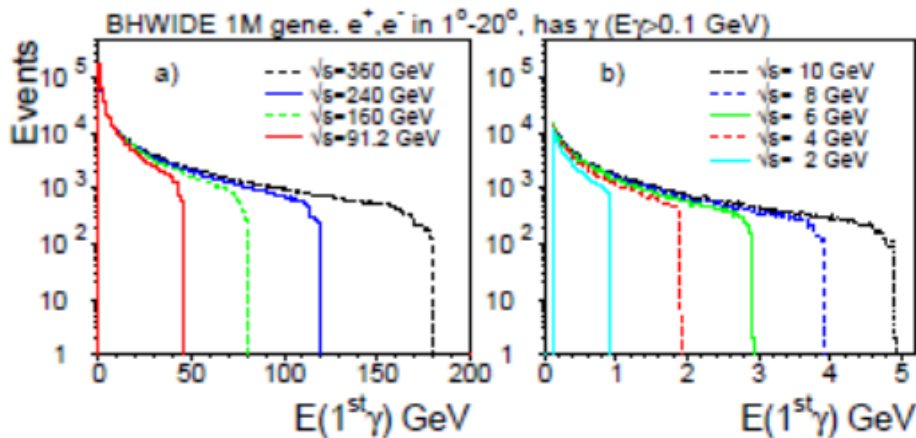
Most energetic energy event rates

→ events vs E_γ , at allowed range indept of \sqrt{s}

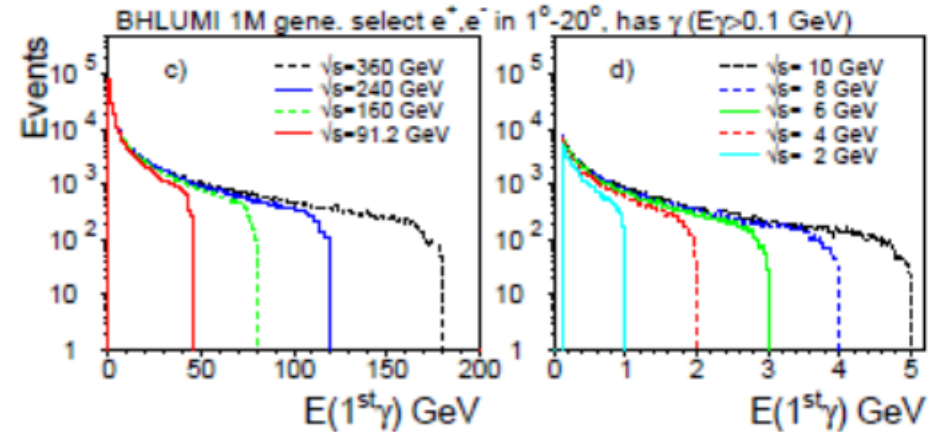
→ E_γ range reaching E_{beam}

E_γ , 1st photon in events > 0.1 GeV

BHWIDE



BHLUMI



Detecting radiative radiative photons

Bhabha scattering, highest event rates @ flavor, Higgs factories

BHLUMI .037% precision

QED basics

$$e^+e^- \rightarrow e^+e^-(n\gamma)$$

Radiation by YFS exponentiation

applied in all QED radiation \rightarrow photons look alike

\rightarrow *Beam background shall look like Bhabha radiation ??*

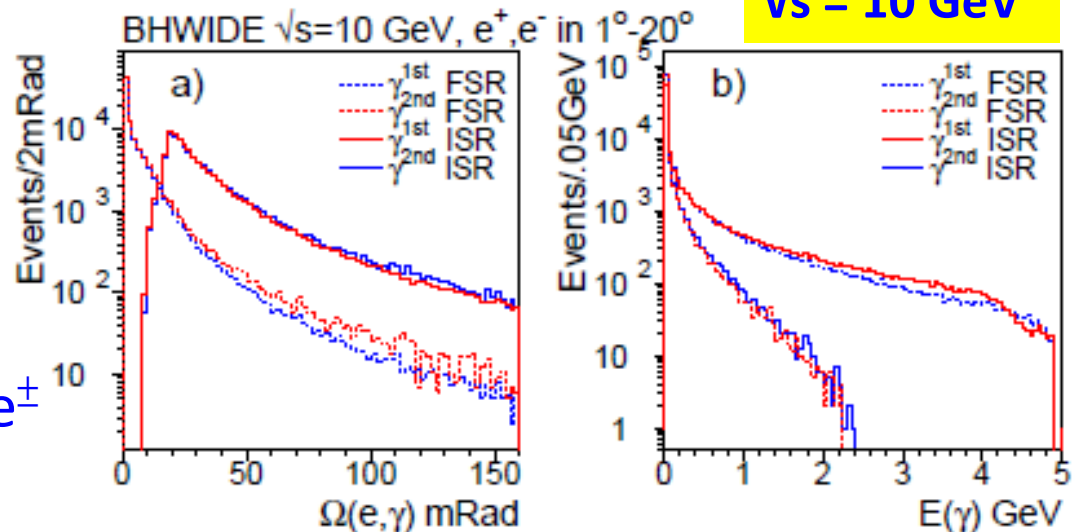
Photons

ordered by E_γ

ISR/FSR by

Open Angle $\Omega(e,\gamma)$

closer to incident/scatter e^\pm

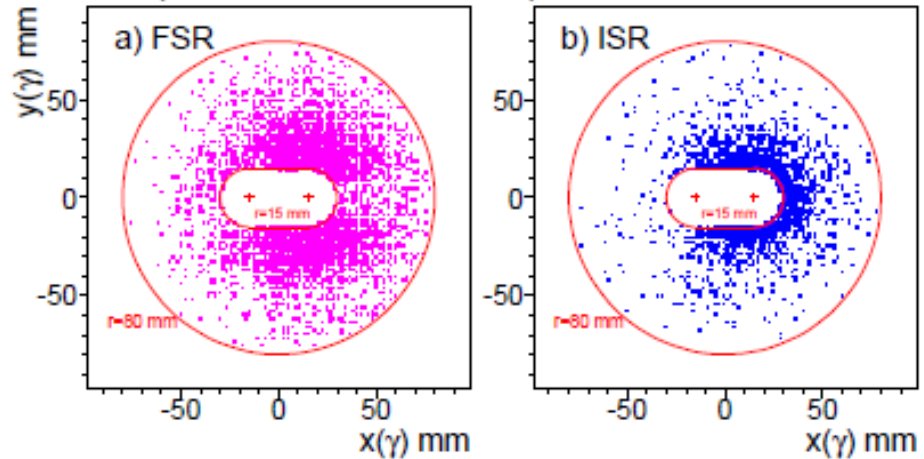


Rad-Bhabha photons $e^+e^- \rightarrow e^+e^-(n\gamma)$

- Detect rad-Bhabha $e^+, e^-, \gamma^{1st}, \gamma^{2nd}$
- FWD acceptance:
 - off-pipe $\phi 30\text{mm}$, $x_c = \pm 15\text{mm}$, $r < 80\text{mm}$
- γ selection:
 - $E_\gamma > 0.1\text{ GeV}$,
 - opening angle $\Omega(e^\pm, \gamma^{1st}) > 10\text{ mRad}$
- event rate in Bhabha
 - to both e^+, e^- detected in FWD
 - either z-side, e^\pm with $\gamma^{1st}, \gamma^{2nd}$ measured

$\sqrt{s} = 10\text{ GeV}$ $e^\pm \gamma^{1st}$ (FSR) $e^\pm \gamma^{1st}$ (ISR)

e^+, e^-, γ in FWD at $|z|=500\text{mm}$, $E_\gamma > 0.1\text{ GeV}$, $\Omega(e, \theta) > 10\text{mRad}$



BHWIDE	2	4	6	8	10	91.2	160	240	360
$R_{\text{FWD}} (e^+e^- \text{ in FWD} + n\gamma)$.563	.619	.649	.673	.684	.799	.821	.835	.850
R_{FWD} .and. $E_\gamma > 0.1\text{ GeV}$.119	.188	.232	.266	.291	.552	.615	.656	.695
$R_{\pm z}(\gamma^{1st}\text{ISR})$.0040	.0049	.0058	.0064	.0063	.0095	.0103	.0103	.0102
$R_{\pm z}(\gamma^{1st}\text{FSR})$ $\Omega(e\gamma) > 10\text{mR}$.0094	.0126	.0147	.0162	.0169	.0262	.0272	.0291	.0301
$R_{\pm z}(\gamma^{2nd}\text{ISR})$ $E_\gamma > 0.1\text{ GeV}$.000052	.000073	.00013	.00018	.00022	.00084	.00089	.0011	.0012
$R_{\pm z}(\gamma^{2nd}\text{FSR})$.000098	.00028	.00040	.00060	.00071	.0024	.0029	.0036	.0040
BHLUMI									
$R_{\text{FWD}} (e^+e^- \text{ in FWD} + n\gamma)$.480	.533	.561	.583	.599	.719	.743	.757	.774
R_{FWD} .and. $E_\gamma > 0.1\text{ GeV}$.120	.188	.233	.265	.293	.555	.614	.653	.693
$R_{\pm z}(\gamma^{1st}\text{ISR})$.0048	.0054	.0059	.0065	.0067	.0096	.0103	.0105	.0105
$R_{\pm z}(\gamma^{1st}\text{FSR})$ $\Omega(e\gamma) > 10\text{mR}$.0092	.0126	.0150	.0164	.0169	.0257	.0273	.0283	.0307
$R_{\pm z}(\gamma^{2nd}\text{ISR})$ $E_\gamma > 0.1\text{ GeV}$.000058	.000070	.00017	.00027	.00028	.00070	.00096	.0010	.0012
$R_{\pm z}(\gamma^{2nd}\text{FSR})$.000077	.00031	.00047	.00051	.00063	.0025	.0028	.0033	.0039

Perspective for measuring Bhabha @ STCF

BHWIDE $\sqrt{s} = 2 - 10$ GeV

scattered $e^\pm > 0.1$ GeV; $\Omega(e^+e^-) : 0-\pi$

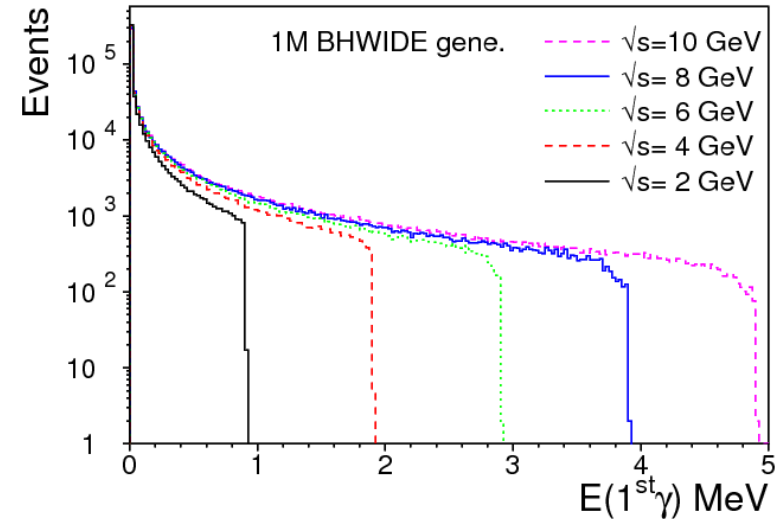
CMS generated for $\sigma(1-20^\circ)$

Acceptance e^+e^- in FWD = **16.5%**

Both e^+e^- fall in FWD

event ratio with

- **0/ $n\gamma$** generated
- **γ^{1st}** $E_\gamma > 0.1$ GeV, $\Omega(e^\pm, \gamma^{1st}) > 10$ mR

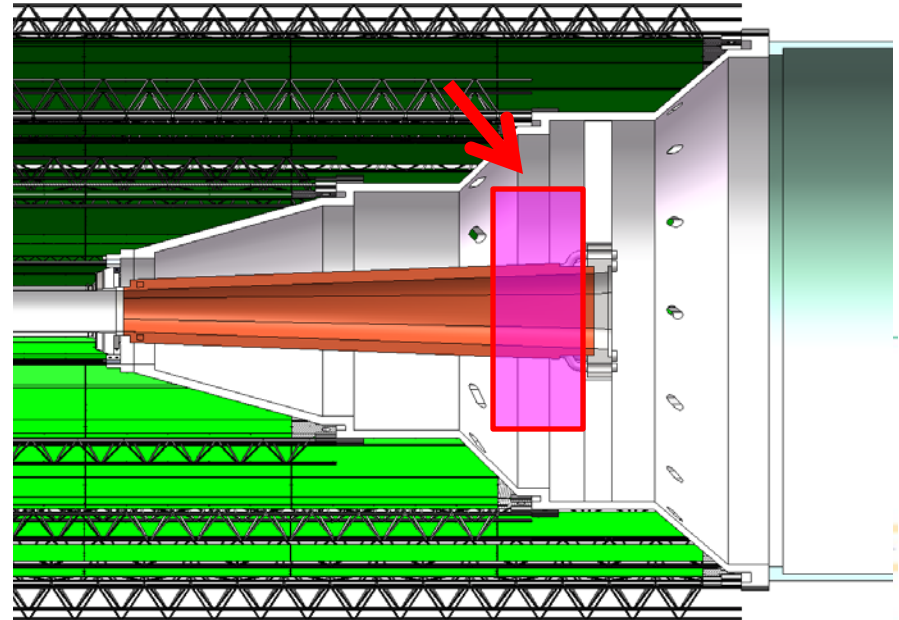
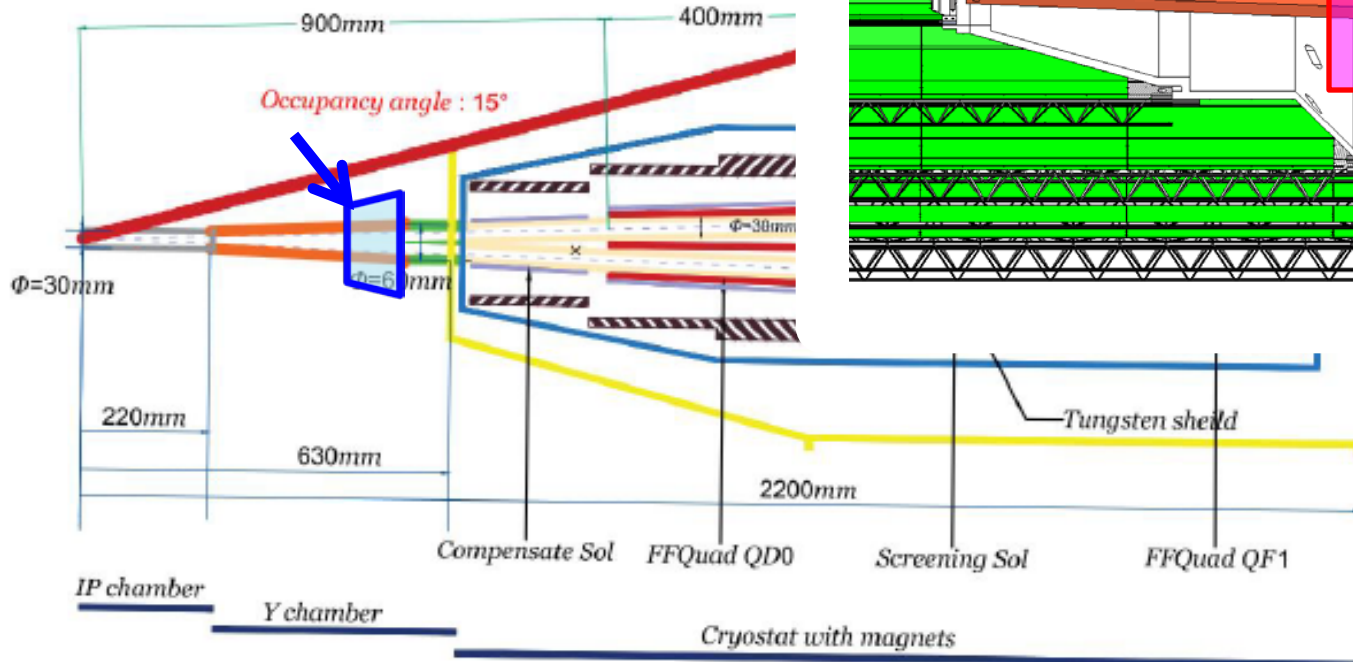


Detected in each z-side	$\sqrt{s} =$	10 GeV	8 GeV	6 GeV	4 GeV	2 GeV
both e^+e^- in FWD	$e^\pm, 0\gamma$	56.1 %	57.1 %	59.1 %	61.6 %	66.0 %
	$e^\pm, n\gamma$	43.9 %	42.9 %	40.9 %	38.4 %	34.0 %
e^\pm, γ^{1s} in FWD $E_\gamma > 0.1\text{GeV}, \Omega(e^\pm, \gamma^{1st}) > 10\text{mR}$	e^\pm, γ^{1s}	ISR 0.63 % FSR 1.69 %	ISR 0.64 % FSR 1.62 %	ISR 0.58 % FSR 1.47 %	ISR 0.50 % FSR 1.26 %	ISR 0.40 % FSR 0.94 %

Forward Calorimeter @ STCF

60 – 240 mRad (3.4°-14°), Z = 350 – 500 mm

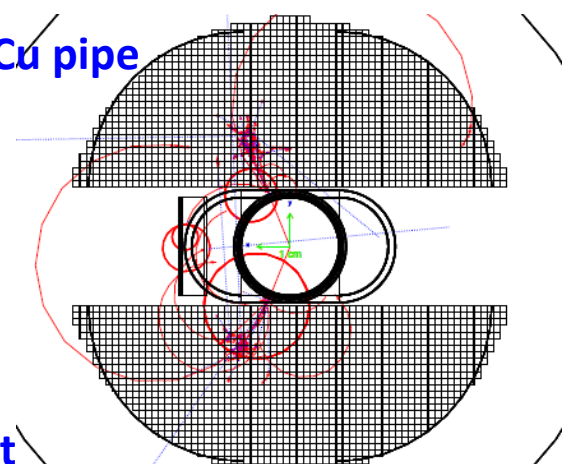
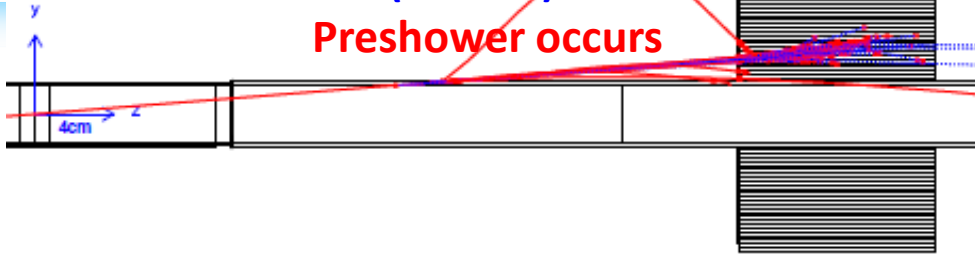
- Precision QED radiative Bhabha to better than 10^{-3}
- e/ γ by Si-det + Crystal
 - 2D, Si strip, 50 μm pitch
 - LYSO, 2x2 mm², 50 mm long



GEANT

3.5 GeV e^- 80 mrad passing 2 mm Cu pipe
(1.84 X⁰)

Preshower occurs

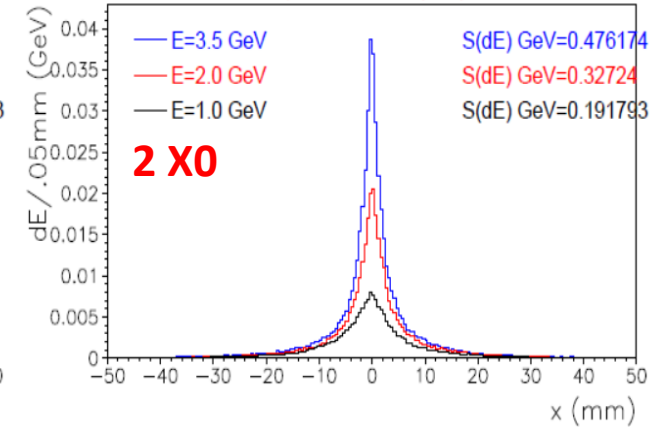
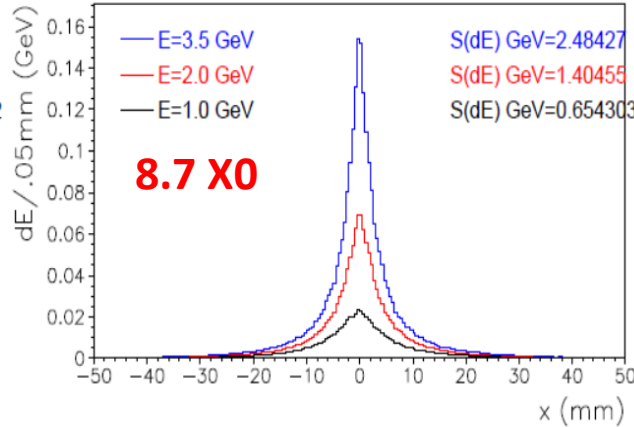
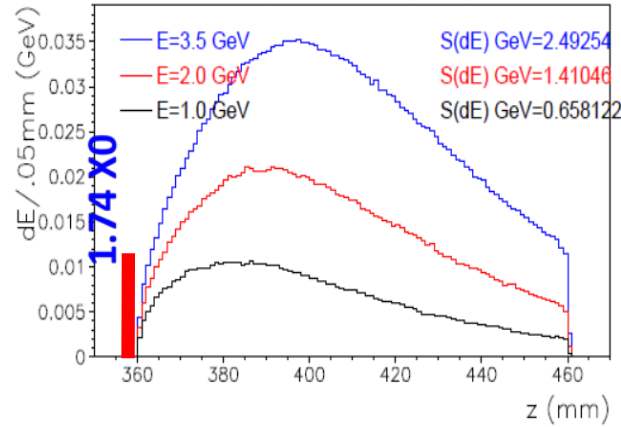


Cu 2mm pipe 1.74 X⁰ @80mrad. Sum dE/dx per event

electron at 80 mrad, LYSO 100 mm (8.7X⁰)

electron at 80 mrad, LYSO 100 mm (8.7X⁰)

electron at 80 mrad, LYSO 23 mm (2X⁰)

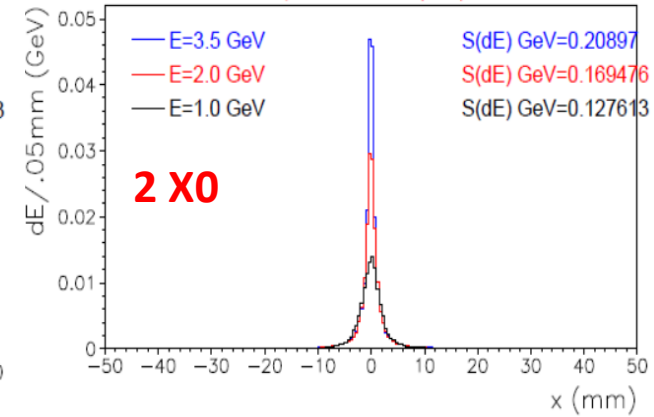
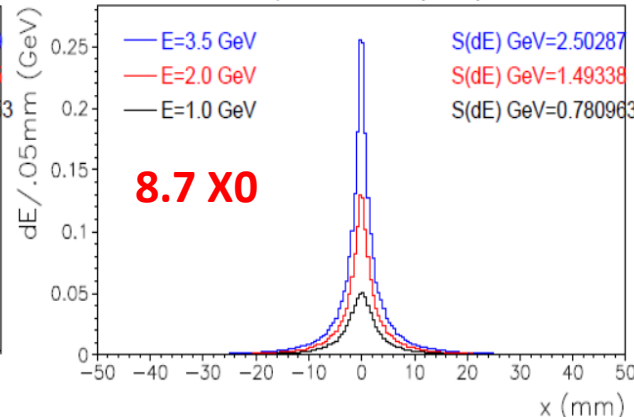
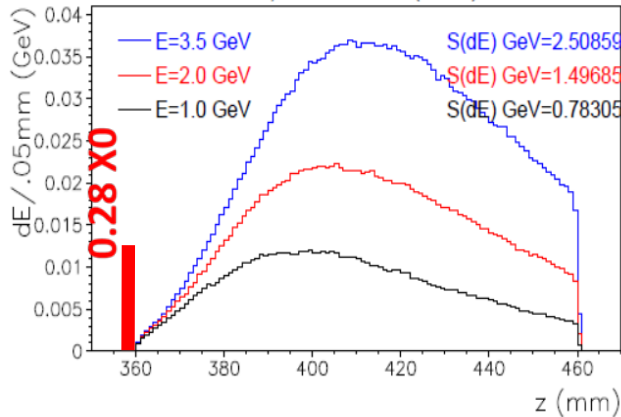


Al 2mm pipe 0.28 X⁰ @80mrad

electron at 80 mrad, LYSO 100 mm (8.7X⁰)

electron at 80 mrad, LYSO 100 mm (8.7X⁰)

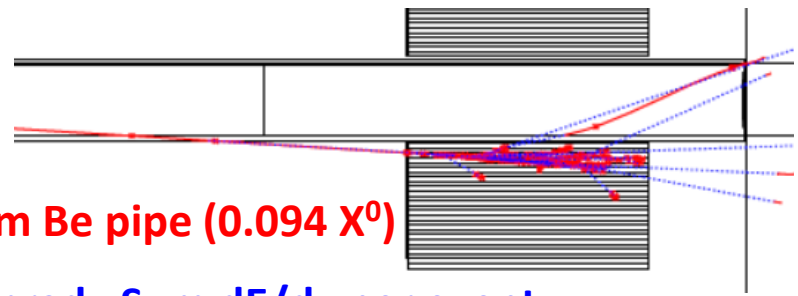
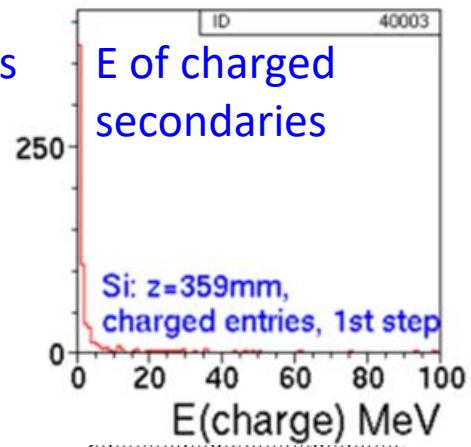
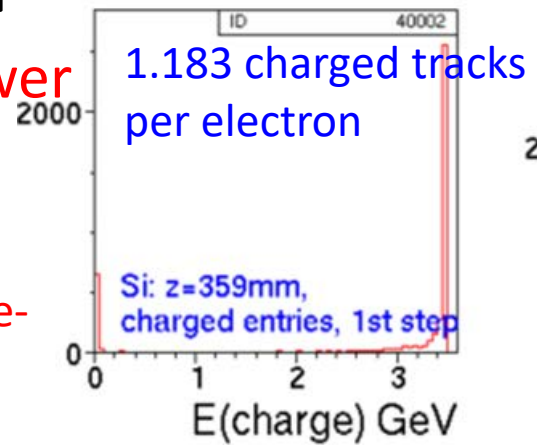
electron at 80 mrad, LYSO 23 mm (2X⁰)



2 mm Be pipe, mini pipe materials

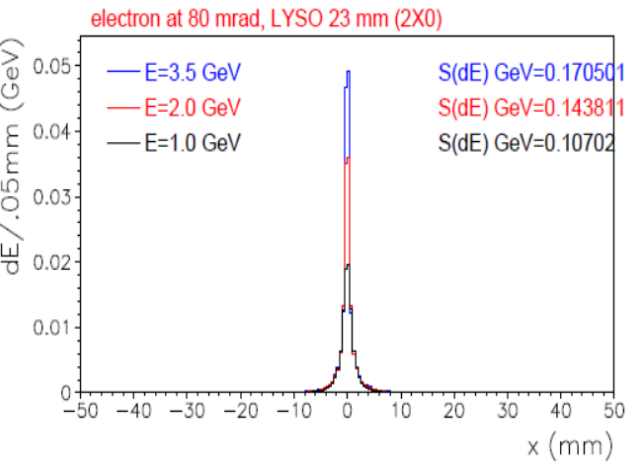
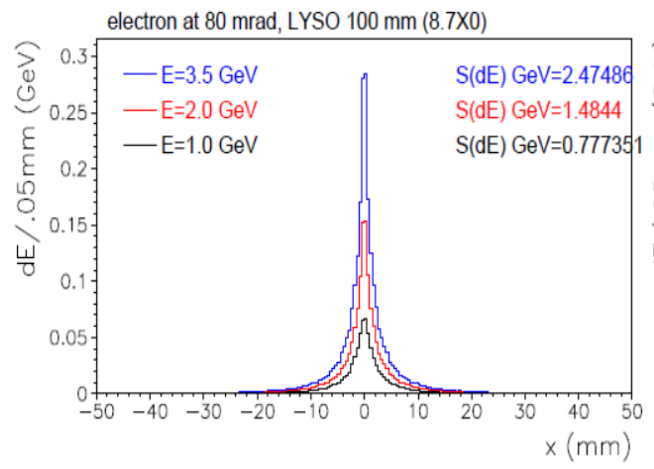
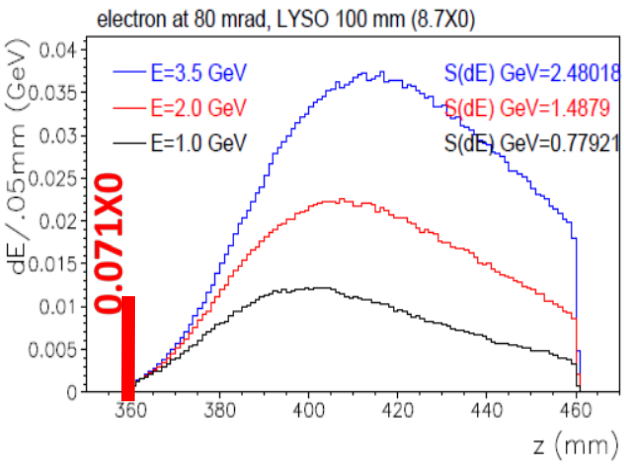
Be pipe to minimize electron shower in forward/tracking volume

- Si before LYSO,
- No. of incident charge tracks in Si : 1.18/e-
- Most secondaries <10 MeV
- >100 MeV cut → 1% contamination



3.5 GeV e⁻ 60mrad, 2 mm Be pipe (0.094 X⁰)

Be 2mm pipe 0.071 X⁰ @80mrad, Sum dE/dx per event



Discussion

- **Proposal for detecting QED radiative photons**
Forward Bhabha high statistics
Better than 10⁻³ prevision
- **Forward detector within ITKM cone**
Si + LYSO for e/ γ measurements
- **Be beam pipe**
Minimized preshower
minimum upstream background to tracking volume