Polarization transfer in $\psi' \to \psi\pi\pi$ alternative J/ψ samples at BESIII and STCF [1]

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[1] Based on http://arxiv.org/abs/2603.09858

Outline

Introduction

Theoretical Framework

Experimental Applications

Broader Implications

Conclusions

Motivation: why $\psi' \to \psi\pi\pi$?

- The question is how the polarization of ψ' transfers to J/ψ.
- Given that the small phase space of $\pi\pi$ is not sufficient to produce too much D-wave.
- But no clear and quantitative answer yet

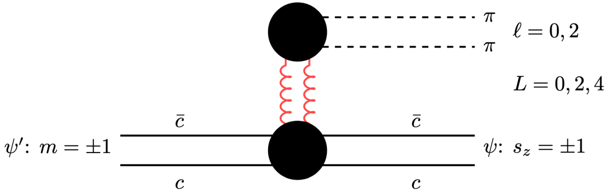

图 1: Schematic diagram for the transition $\psi' \to \psi\pi\pi$.

Motivation: why $\psi' \to \psi\pi\pi$?

- With high-statistics data (BESIII, Belle II, LHC, STCF), **systematics** dominate.
- $\psi' \to \psi\pi\pi$ provides a uniquely **clean laboratory**:
  - Large branching fraction ($> 50\%$) $\Rightarrow$ huge J/ψ yields from existing and future ψ' samples.
  - No continuum background under ψ peak.
  - Parent ψ' produced with **known polarization** in e^+e^- ($m = \pm 1$).
- **Key question**: How is polarization transferred through a non-perturbative transition?
- This work: a complete **Spin Density Matrix (SDM)** formalism to answer it.

Scientific value and impact

Theoretical:

- Derive general relation $\rho_\psi = T\rho_{\psi'}T^\dagger$.
- Prove that $\rho_\psi = \rho_{\psi'}$ in S-wave limit (polarization inheritance).
- Quantify D-wave corrections $\Rightarrow$ probe of sub-leading QCD dynamics (multipole expansion).

Experimental:

- “Create” a **continuum-free, polarized** ψ sample.
- Enable precision branching fraction measurements and amplitude analyses without continuum backgrounds.
- Universal formalism applicable to $\Upsilon(nS) \to \Upsilon(mS)\pi\pi$, $e^+e^- \to ZH$, etc.

Spin Density Matrix (SDM) formalism

- SDM $\rho_{mm'}$ describes polarization states (m, m' helicities).
- For decay $A \to B + X$, the daughter SDM transforms linearly:

$$\rho^{(B)}_{bb'} = \sum_{aa'} \mathcal{M}_{ba}\, \rho^{(A)}_{aa'}\, \mathcal{M}^{*}_{b'a'} \quad \Rightarrow \quad \boxed{\rho = T\rho' T^{\dagger}}$$

- Transition matrix T encodes the decay dynamics; ρ' encodes production.
- **Independence of production**: T depends on the decay dynamics, ρ' depends on the production environment.

Kinematics and partial wave decomposition

- Angles: Ω_ψ (direction of ψ in ψ' rest frame), Ω_π (π^+ direction in $\pi\pi$ rest frame), Ω_ℓ (ℓ^+ direction in ψ rest frame).
- Quantum numbers: ℓ ($\pi\pi$ orbital), L (relative motion of ψ and $\pi\pi$), $S = s_\psi + \ell$.
- Parity and C require ℓ even, $L + \ell$ even $\Rightarrow$ dominant waves: M_{001} ($\ell = L = 0$, $S = 1$), M_{201}, M_{021} .

General transformation law

Amplitude in helicity formalism (Cahn 1975):

$$T_{s_z,m}(\Omega_\psi,\Omega_\pi) = \sum_{\ell,L,S,\ell_z,L_z,S_z} M_{\ell LS}\, Y_L^{L_z}(\Omega_\psi)\, Y_\ell^{\ell_z}(\Omega_\pi)\, C^{S;\ell s}_{S_z;\ell_z s_z}\, C^{J;LS}_{m;L_z S_z}$$

After integrating over unobserved angles, the daughter SDM:

$$\rho_{s_z,s_z'} = \sum_{m,m'} T_{s_z,m}\, \rho'_{m,m'}\, T^\dagger_{s_z',m'} \quad \text{or} \quad \boxed{\rho = T\rho' T^\dagger}$$

- **Key**: This relation is independent of how ψ' was produced.

The S-wave identity: perfect polarization inheritance

For pure S-wave ($\ell = L = 0$): $T_{s_z,m} = M_{001}\delta_{s_z,m}$

$$\rho_{s_z,s_z'} = |M_{001}|^2 \rho'_{s_z,s_z'} \quad \Rightarrow \quad \boxed{\rho_\psi = \rho_{\psi'}}$$ [2]

- **Physical origin**: amplitude $\propto \boldsymbol{\epsilon}_{\psi'} \cdot \boldsymbol{\epsilon}^*_\psi$ (scalar product) $\Rightarrow$ no helicity mixing.
- **QCD interpretation**: Leading $E1E1$ transition in multipole expansion preserves polarization.
- **Experimental significance**: ψ inherits the full polarization of ψ'; any deviation quantifies non-S-wave contributions.

[2] Since the transition matrix T is not unitary in general, the daughter SDM needs to be renormalized after integration over phase space to ensure $\mathrm{Tr}\,\rho = 1$.

Corrections from D-wave contributions

- Include M_{201} and M_{021}:

$$\rho = T^{001}\rho' T^{001\dagger} + T^{201}\rho' T^{201\dagger} + T^{021}\rho' T^{021\dagger}$$

- For diagonal parent $\rho' = \mathrm{diag}(p_+, 0, p_-)$:

$$\begin{aligned}
\rho_{++} &= |M_{001}|^2 p_+ \quad + \tfrac{1}{10}\left[|M_{201}|^2 + |M_{021}|^2\right](p_+ + 6p_-) \\
\rho_{00} &= \tfrac{3}{10}\left[|M_{201}|^2 + |M_{021}|^2\right](p_+ + p_-) \\
\rho_{--} &= |M_{001}|^2 p_- \quad + \tfrac{1}{10}\left[|M_{201}|^2 + |M_{021}|^2\right](6p_+ + p_-)
\end{aligned}$$

- D-wave fraction $f_D \sim 3\%$ (from BES) yields $\rho_{00} \approx 0.01$ instead of 0.

Extracting SDM from dilepton angular distributions

- Angular distribution for $\psi^{(\prime)} \to \mu^+\mu^-$:

$$W(\theta, \phi) \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\phi \sin^2\theta \cos 2\phi + \lambda_{\theta\phi} \sin 2\theta \cos\phi$$

- Relation to SDM elements:

$$\lambda_\theta = \frac{\rho_{++} + \rho_{--} - 2\rho_{00}}{\rho_{++} + \rho_{--} + 2\rho_{00}}$$

$$\lambda_\phi = \frac{2\,\mathrm{Re}\,\rho_{+-}}{\rho_{++} + \rho_{--} + 2\rho_{00}}$$

$$\lambda_{\theta\phi} = \frac{\sqrt{2}\,\mathrm{Re}(\rho_{+0} - \rho_{0-})}{\rho_{++} + \rho_{--} + 2\rho_{00}}$$

Statistical sensitivity (Fisher information)

- **BESIII** ψ' sample: 2.7×10^9.
- **STCF** projected ψ' sample: $\sim 10^{11}$ (a year) (factor ~ 40 larger than BESIII).
- Projected uncertainties with Fisher information method:

$$\text{BESIII:} \quad \delta\lambda_\theta \approx 0.0003, \quad \delta\lambda_\phi \approx 0.0002$$
$$\text{STCF:} \quad \delta\lambda_\theta \approx 0.00005, \quad \delta\lambda_\phi \approx 0.00003$$

- Sensitivity to *D*-wave fractions down to 0.1% level.

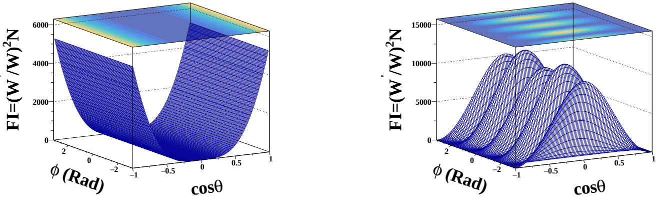

图 2: Observed Fisher information for (left) λ_θ and (right) λ_ϕ on the $(\cos\theta, \phi)$ plane. Higher Fisher information (bright regions) indicates phase space regions that provide more statistical sensitivity.

Three-path self-consistency test

Three independent measurements:

Path A: Measure $\rho'_{(\text{expt})}$ from $\psi' \to \mu^+\mu^-$.

Path B: Extract T from PWA of $\psi' \to \psi\pi\pi$.

Path C: Measure $\rho_{(\text{expt})}$ from $\psi \to \mu^+\mu^-$ in the decay chain.

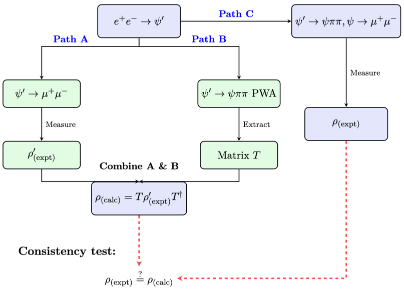

图 3: Three-path self-consistent test of polarization transfer.

Consistency check:

$$\rho_{(\text{calc})} = T\rho'_{(\text{expt})}T^\dagger \stackrel{?}{=} \rho_{(\text{expt})}$$

Agreement validates the whole framework and constrains partial waves.

Extension to $\psi' \to h_c \pi^0$

- Another hadronic transition with h_c ($J^{PC} = 1^{+-}$) and π^0 (0^{-+}).
- Only $L = 0, 2$ allowed between h_c and π^0.
- Amplitude: $T_{s_z,m}(\Omega_{h_c}) = M_0 \delta_{ms_z} + M_2 \sum_{L_z} Y_2^{L_z}(\Omega_{h_c}) C^{1;2,1}_{m;L_z s_z}$.
- Again, S-wave ($L = 0$) yields $\rho_{h_c} = \rho_{\psi'}$.
- Complementary probe: S-wave to P-wave transition.

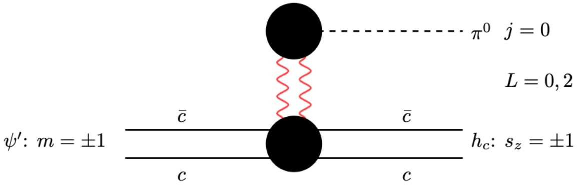

图 4: Schematic for $\psi' \to h_c \pi^0$. Only $L = 0, 2$ orbital angular momenta are allowed.

Universality of the formalism

- The transformation $\rho = T\rho' T^\dagger$ follows from angular momentum conservation alone.
- Applies to any $V_1 \to V_2 + X(0^+)$ process.
- **Bottomonium**: $\Upsilon(nS) \to \Upsilon(mS)\pi\pi$ –same $E1E1$ dominance expected.
- **Electroweak**: $e^+e^- \to Z^* \to ZH$ –S-wave preserves Z polarization, D-wave mixes [9].
- Provides a unified probe from charmonium to Higgs physics.

Summary and conclusions

- Developed complete SDM formalism for $\psi' \to \psi\pi\pi$.
- Proved *S*-wave dominance leads to **perfect polarization inheritance** $\rho_\psi = \rho_{\psi'}$.
- Quantified *D*-wave corrections; they are small but measurable.
- Proposed **three-path self-consistency test** to validate framework and constrain amplitudes.
- The formalism is universal and applicable to other quarkonia and electroweak processes.
- With BESIII's 2.7×10^9 ψ' we already obtain a **clean, polarized** ψ **sample** for precision studies.
- The future STCF with $\sim 10^{11}$ ψ' will reduce statistical uncertainties by an order of magnitude and open new windows into rare J/ψ decays and precision studies.

Thank you! Questions?

Key references

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