Quantum entanglement in τ-pairs with $\tau \to \pi\nu$ decay at STCF

The workshop on Super Tau Charm Facility
Xi'an, Shanxi, China

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Overview

Previously reported — 7th Workshop on Future Tau-Charm Facilities, Nov 2025

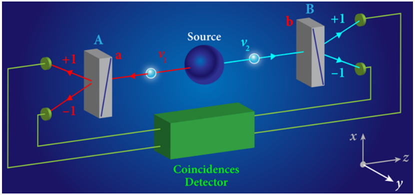
- Framework for $\tau\tau$ quantum entanglement
- τ reconstruction with $\tau \to \pi\nu$
- Validation of the full PHSP sample against theory
- Publication as a proceeding [1]

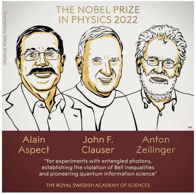
Updates in this talk

- Considering ISR effect on the theoretical predictions
- $d_{\rm min}$ and z-scan: explore two methods to resolve the two-fold ambiguity, cross-checked
- Understanding the limitations
  - two-fold ambiguity vs. detector resolution
  - significant bias on the entanglement observables

[1] X. Li et al., "Quantum entanglement of τ-pair and τ reconstruction at STCF", Int. J. Mod. Phys. A, 10.1142/S0217751X26450028 (2026)

Entanglement & Quantum Tomography





Entanglement

- Quantum Entanglement (QE) — inseparability of past-interacted physical systems
- Violation of Bell Inequality — nonlocality beyond entanglement.
- QE can be observed in correlations of spin or flavor space.

Quantum Tomography

- Helicity and polarization amplitudes are very sensitive probes into the details of the underlying physics.
- Aim: reveal the full quantum state of the final state in a scattering process, which means to reconstruct its density matrix.

Density Matrix

Density Matrix ρ

- Density matrix describes the statistical mixture of states $\{p_i, |\psi_i\rangle\}$

$$\rho = \sum_i p_i \, |\psi_i\rangle\langle\psi_i| \,, \quad p_i \geq 0, \quad \sum_i p_i = 1.$$

- For a biparticle qubits system,

$$\rho = \frac{1}{4}\left[\mathbb{1}\otimes\mathbb{1} + \sum_i B_i^{+}(\sigma_i \otimes \mathbb{1}) + \sum_j B_j^{-}(\mathbb{1}\otimes\sigma_j) + \sum_{i,j} C_{ij}(\sigma_i\otimes\sigma_j)\right].$$

σ_i are Pauli matrices, $B_i^{\pm}$ represent the polarization and C_{ij} represent the correlation of spins.

Entanglement & Bell Inequality [2] biparticle qubits system

- Entanglement can be described by concurrence $\mathcal{C}$

$$\mathcal{C}[\rho] = \max\{0, r_1 - r_2 - r_3 - r_4\} \; (? > 0), \quad R \equiv \rho(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y).$$

$r_1 \geq r_2 \geq r_3 \geq r_4$ are square roots of the eigenvalues of R.

- Bell Inequality is simplified as the Horodecki condition

$$\mathfrak{m}_{12}[\mathbf{C}] = m_1 + m_2 \; (? > 1), \quad M = \mathbf{C}\mathbf{C}^{\mathrm{T}}$$

$m_1 \geq m_2 \geq m_3$ are eigenvalues of M.

[2] A. J. Barr et al., "Quantum entanglement and Bell inequality violation at colliders", Prog. Part. Nucl. Phys. **139**, 104134 (2024)

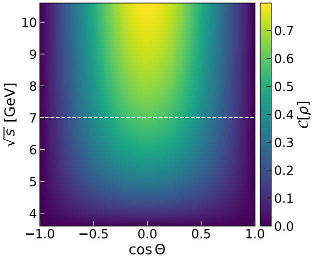
Theoretical Prediction

Analytical expression of $\mathcal{C}$ and $\mathfrak{m}_{12}$ [3]

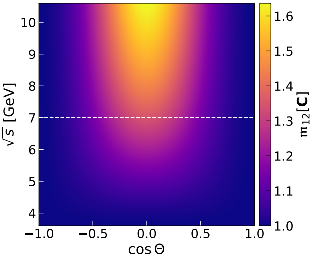
- Specifically, for tree-level QED process $e^+e^- \to \tau^+\tau^-$,

$$\mathcal{C} = \frac{\left(s - 4m_\tau^2\right)\sin^2\Theta}{4m_\tau^2\sin^2\Theta + s(\cos^2\Theta + 1)}, \quad \mathfrak{m}_{12} = 1 + \left[\frac{\left(s - 4m_\tau^2\right)\sin^2\Theta}{4m_\tau^2\sin^2\Theta + s(\cos^2\Theta + 1)}\right]^2.$$

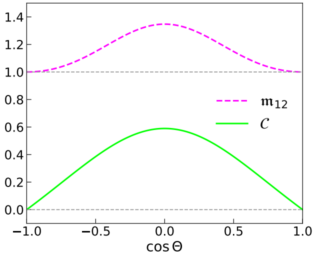
- Represents only the entanglement at production of the τ-pair.



- $\mathcal{C}(\sqrt{s}, \cos\Theta)$



- $\mathfrak{m}_{12}(\sqrt{s}, \cos\Theta)$



- $\mathcal{C}$ and $\mathfrak{m}_{12}$ at $\sqrt{s} = 7$ GeV

[3] K. Ehatäht et al., "Probing entanglement and testing Bell inequality violation with e+e-→τ+τ- at Belle II", Phys. Rev. D **109**, 032005 (2024)

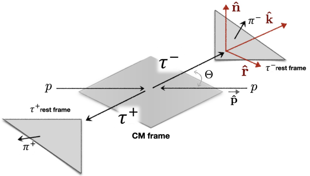
Reconstruction of Density Matrix

Helicity basis $\{\hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}}\}$ defined in the c.m.s frame of τ-pair as

$$\hat{\mathbf{p}} \cdot \hat{\mathbf{k}} = \cos\Theta$$

$$\hat{\mathbf{n}} = (\hat{\mathbf{p}} \times \hat{\mathbf{k}}) / \sin\Theta$$

$$\hat{\mathbf{r}} = (\hat{\mathbf{p}} - \hat{\mathbf{k}}\cos\Theta) / \sin\Theta.$$



Reconstruction from $\mathbf{B}^{\pm} = \{B_i^{\pm}\}, \mathbf{C} = \{C_{ij}\}$ [3, 4]

- For a biparticle qubits system $\quad (\hat{\mathbf{e}}_i = \hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}}, \quad h_i^{\pm} = \mathbf{h}^{\pm} \cdot \hat{\mathbf{e}}_i)$

$$B_i^{\pm} = \frac{3}{\kappa_{\pm}} \cdot \frac{1}{\sigma} \int \mathrm{d}\Omega^{\pm} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^{\pm}} (\mathbf{h}^{\pm} \cdot \hat{\mathbf{e}}_i) = \frac{3}{\kappa_{\pm}} \langle h_i^{\pm} \rangle \,,$$

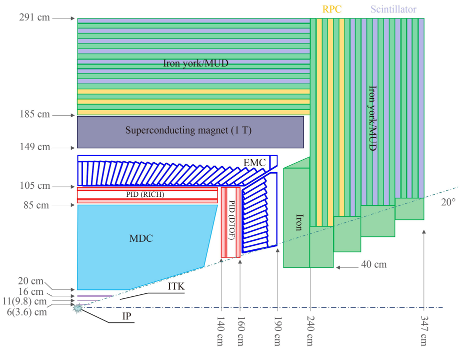
$$C_{ij} = \frac{9}{\kappa_- \kappa_+} \cdot \frac{1}{\sigma} \iint \mathrm{d}\Omega^- \mathrm{d}\Omega^+ \frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega^- \mathrm{d}\Omega^+} (\mathbf{h}^- \cdot \hat{\mathbf{e}}_i)(\mathbf{h}^+ \cdot \hat{\mathbf{e}}_j) = \frac{9}{\kappa_- \kappa_+} \langle h_i^- h_j^+ \rangle \,.$$

- $\mathbf{h}^{\pm}$ are polarimeter vectors of $\tau^{\pm}$ in its c.m.s frame, $\kappa_{\pm}$ is analyzing power.
- Both vary with decay channels and depend on kinematic information of decay products.
- For $\tau \to \pi\nu$, $\kappa_{\pm} = \pm 1$ and $\mathbf{h}^{\pm} = -\hat{\mathbf{n}}_{\pi^{\pm}}$. $\hat{\mathbf{n}}_{\pi^{\pm}}$ is direction vector in $\tau^{\pm}$ rest frame.

[3] K. Ehatäht et al., "Probing entanglement and testing Bell inequality violation with e+e-→τ+τ- at Belle II", Phys. Rev. D **109**, 032005 (2024)

[4] M. Fabbrichesi and L. Marzola, "Quantum tomography with τ leptons at the FCC-ee: Entanglement, Bell inequality violation, sinθW, and anomalous couplings", Phys. Rev. D **110**, 076004 (2024)

Detector Geometry [5]



Subdetector	from inside out

- ITK: Inner Tracker
  - ITKW: cylindrical MPGD
  - ITKM: CMOS M-MAPS
- MDC: Main Drift Chamber
- PIDE: PID in the Endcap
  - DTOF: DIRC-like TOF
- PIDB: PID in the Barrel
  - RICH: CsI+MPGD
  - BTOF: Barrel-TOF
- ECAL (EMC): Electromagnetic Calorimeter (CsI+APD)
- MUD: Muon Detector RPC/scintillator+iron york

[5] M. Achasov et al., "STCF conceptual design report (Volume 1): Physics & detector", Front. Phys. (Beijing) **19**, 14701 (2024)

Data Set

- Main decay channels of τ^-

Channel	Abs. BR	Norm. BR
$\mu^-\bar{\nu}_\mu\nu_\tau$	17.39%	19.30%
$e^-\bar{\nu}_e\nu_\tau$	17.82%	19.78%
$\pi^-\nu_\tau$	10.82%	12.01%
$\pi^-\pi^0\nu_\tau$	25.49%	28.29%
$\pi^-\pi^0\pi^0\nu_\tau$	9.26%	10.28%
$\pi^-\pi^+\pi^-\nu_\tau$	9.31%	10.33%
Sum	90.09%	100.00%

Signal Monte Carlo

- OSCAR 2.6.2 fullsim + digi + reco
- $\sqrt{s} = 7\ \mathrm{GeV}$
- 30 million τ-pair events including main decay channels of τ with ISR
- Selected via MC truth (1.44%, 4.33×10^5 events)

$$e^-e^+ \to \begin{cases} \tau^- \to \pi^-\nu_\tau \\ \tau^+ \to \pi^+\bar{\nu}_\tau \end{cases}$$

- Generator aMC@NLO MadGraph

Initial Selection

Charged Track Selection

- $D_0 < 0.1$ cm, $Z_0 < 0.5$ cm
- $|\cos\theta| < 0.94$
- $N_{\text{charge}} = 2$, $Q_{\text{total}} = 0$

Shower Selection

- Barrel $|\cos\theta| \in (0.00, 0.83)$
  $E_\gamma > 25$ MeV
- End cap $|\cos\theta| \in (0.83, 0.94)$
  $E_\gamma > 50$ MeV
- $t \in (-10, 10)$ ns
- $\theta_{\text{iso}} > 10^\circ$

Particle Identification

- GlobalPIDSvc based on data-driven machine learning methods, includes pre-trained BDT (based on XGBoost).
- For π
  - $\mathcal{P}_\pi(\pi) > \mathcal{P}_\pi(e), \mathcal{P}_\pi(K)$
- $N_\pi = 2$

$$e^- e^+ \to \begin{cases} \tau^- \to \pi^- \nu_\tau \\ \tau^+ \to \pi^+ \bar{\nu}_\tau \end{cases}$$

τ Reconstruction

Two-fold ambiguity [6] $\mathbf{k} = \mathbf{p}_{\tau^+}$

- In c.m.s frame of $\tau\tau$, assume no ISR.
- Invariant mass constraints

$$(p_\pi + p_\nu)^2 = m_\tau^2, \quad p_\nu^2 = 0.$$

- Initial four-momentum constraints

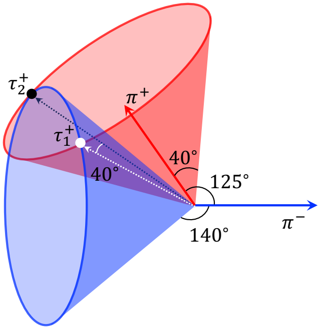
$$\sum_i p_i = p_{\rm ini} \equiv (E_{\rm cms}, 0, 0, 0).$$

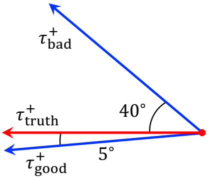
- Two-fold ambiguity of $\hat{\mathbf{k}}$
  - $p_{\rm ini}$ fixes $|\mathbf{k}|$

$$\cos\theta_i = \frac{2E_iE_\tau - m_i^2 - m_\tau^2}{2|\mathbf{k}||\mathbf{p}_i|} \ (i = \pi^-, \pi^+).$$

Selection

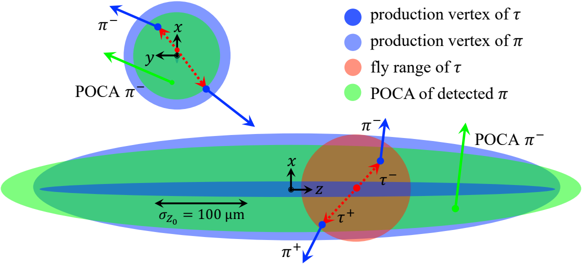
- $N_{\rm sol} > 0$.

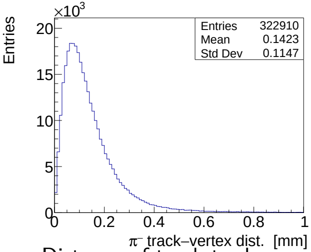




[6] K. Inami et al. (Belle), "An improved search for the electric dipole moment of the τ lepton", JHEP **04**, 110 (2022)

τ Reconstruction — Vertex Distribution





- Distance of track to decay vertex (mean=142 μm)

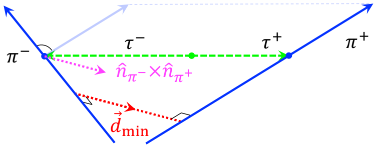
Size

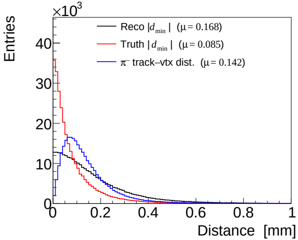
- Beam spot size at IP $\sigma_x \sim 10\ \mu m$, $\sigma_y \sim 0.1\ \mu m$
- Bunch length (too large?) $\sigma_z \sim 10$ mm
- Decay length of τ $L_\tau = 140\ \mu m$

Resolution

- ITKM+MDC (not good enough?) $\sigma_{D_0} \sim 20\ \mu m$, $\sigma_{Z_0} \sim 100\ \mu m$

τ Reconstruction — Method 1: $d_{\min}$ (66%)



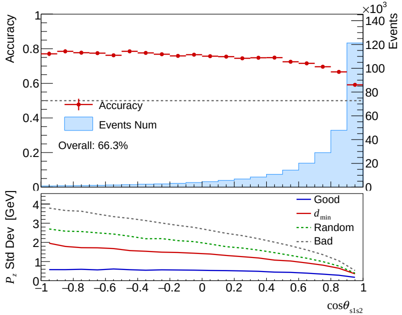


Methodology

- Compare $d_{\min}$ & $\hat{n}_{\pi^-} \times \hat{n}_{\pi^+}$
- Their relative orientation distinguishes two solutions.

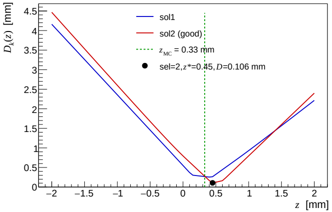
Influence factor

- τ decay length $148\ \mu m (\times 2)$
- $\pi\pi$ colinearity $\rightarrow d_{\min}^{\text{truth}} = 85\ \mu m$
- POCA resolution $\sigma_{\text{POCA}} = 142\ \mu m$
- $d_{\min}^{\text{reco}}$ $168\ \mu m \approx \sqrt{85^2 + 142^2}$

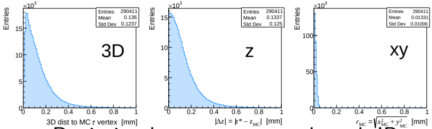


- Accuracy and P_z^τ Std Dev as to $\cos\theta_{s_1 s_2}$

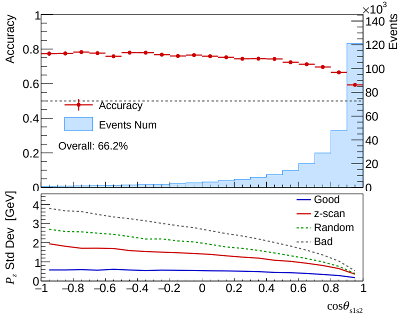
τ Reconstruction — Method 2: z-Scan (66%)



- Scan z for minimum distance (single event).



- Deviation between reco and truth IP



- Accuracy and P_z^{τ} Std Dev as to $\cos\theta_{s_1 s_2}$

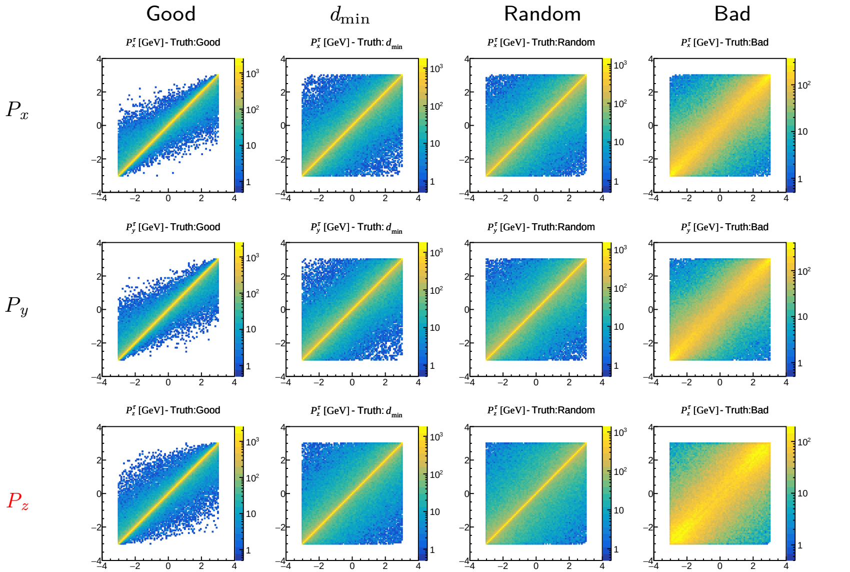
Methodology

- Assumption: IP @ $(0, 0, z)$, straight track
- Scan z to minimize distance between τ & 2π tracks for the two solutions
- Select solution with minimum distance

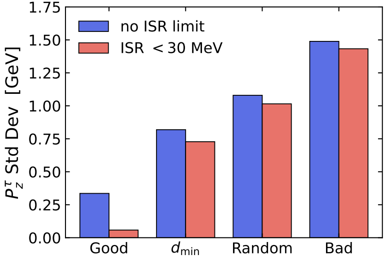
Precision

- z/3D direction $134/136\ \mu\text{m}$
- POCA resolution $\sigma_{\text{POCA}} = 142\ \mu\text{m}$

τ Reconstruction — Deviation



τ Reconstruction — ISR Effect

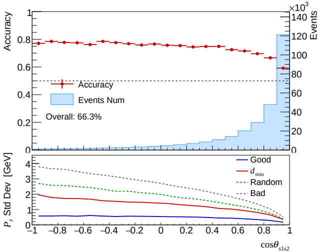


- P_z^τ Std Dev in different cases and ISR.

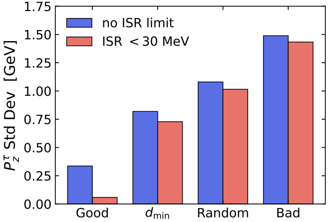
ISR effect on τ momentum resolution

- ISR < 30 MeV loses relatively 32% of events.
- Significant for good solution.
- No obvious effect for other conditions.
- Momentum resolution is dominated by the small fraction of bad solutions.

τ Reconstruction — Conclusion



- Accuracy & P_z^τ Std Dev vs $\cos\theta_{s_1 s_2}$



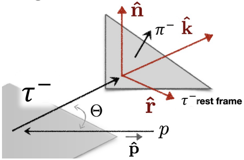
- P_z^τ Std Dev in different cases.

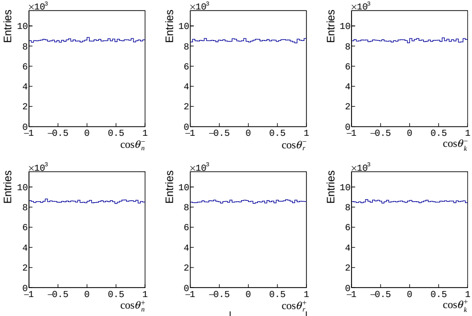
Method Crosscheck

- Momentum resolution is dominated by the small fraction of bad solutions.
- $d_{\min}$ and z-scan agree: same accuracy (66%) and same $\cos\theta_{s_1 s_2}$ dependence.
  - Close (60%): hard to distinguish, but also not necessary.
  - Different (80%): easy to distinguish, and these dominate the resolution.
- POCA-based two-fold resolution reaches its limit — set by the limited detector resolution, not the bunch length.

Full PHSP — $\mathbf{B}^{\pm}, \mathbf{C}$

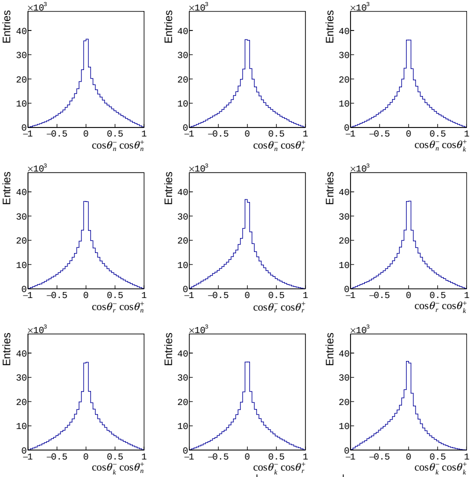
- Full PHSP: truth info, no detector effect.
- To check algorithm.





- $\cos\theta_i^{\pm} = -h_i^{\pm}$

- Experimentally, $h_i^{\pm} = \mathbf{h}^{\pm} \cdot \hat{\mathbf{e}}_i = -\cos\theta_i^{\pm}$
  $B_i^{\pm} = \pm 3 \langle h_i^{\pm} \rangle , \quad C_{ij} = -9 \langle h_i^- h_j^+ \rangle .$



- $\cos\theta_i^- \cos\theta_j^+ = h_i^- h_j^+$

Full PHSP — $\mathcal{C}, \mathfrak{m}_{12}$

Theoretical Result [3]

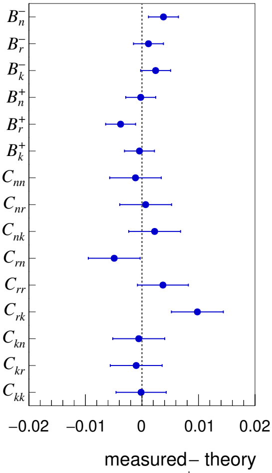
For tree-level QED calculation, considering angle average and ISR effect [7]

$$\mathbf{B} = \mathbf{0}, \quad \mathbf{C} = \begin{pmatrix} -0.314 & 0 & 0 \\ 0 & 0.562 & 0 \\ 0 & 0 & 0.751 \end{pmatrix}.$$

$$\mathcal{C} = 0.314, \quad \mathfrak{m}_{12} = 0.881.$$

Measured Result

- $\mathcal{C} = 0.315 \pm 0.004$
- $\mathfrak{m}_{12} = 0.885 \pm 0.008$
- Only statistical errors from 1000 times bootstrapping



- Deviation of $B_i^\pm$, C_{ij}

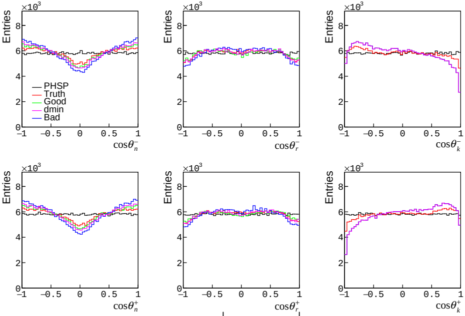
[3] K. Ehatäht et al., "Probing entanglement and testing Bell inequality violation with e+e-→$\tau+\tau$- at Belle II", Phys. Rev. D **109**, 032005 (2024)

[7] E. A. Kuraev and V. S. Fadin, "On Radiative Corrections to e+ e- Single Photon Annihilation at High Energy", Sov. J. Nucl. Phys. **41**, 466 (1985)

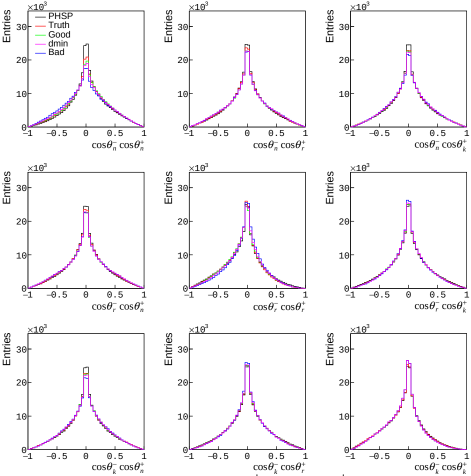
Truth & Reco — $\mathbf{B}^{\pm}, \mathbf{C}$

- Truth: corresponding to Good solution
- Good: detected info
- $d_{\min}$: 66% of Good
- Bad: kinematic equivalent to Good
- Normalize to Reco

- Experimentally, $h_i^{\pm} = \mathbf{h}^{\pm} \cdot \hat{\mathbf{e}}_i = -\cos\theta_i^{\pm}$
  $B_i^{\pm} = \pm 3 \langle h_i^{\pm} \rangle, \quad C_{ij} = -9 \langle h_i^- h_j^+ \rangle .$



- $\cos\theta_i^{\pm} = -h_i^{\pm}$



- $\cos\theta_i^- \cos\theta_j^+ = h_i^- h_j^+$

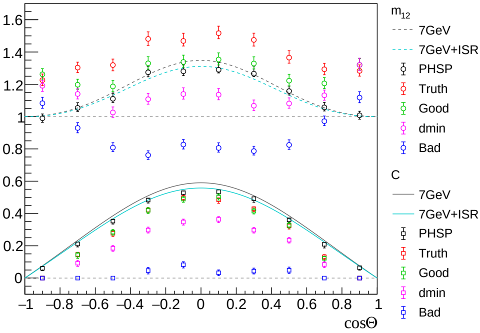
Truth & Reco — $\mathcal{C}, \mathfrak{m}_{12}$

Stepwise Consideration

- Theory (+ISR) — base line
- Full PHSP — statistical fluctuation
- Truth — detector acceptance
- Good — detector resolution
- $d_{\min}$ — two-fold ambiguity

Conclusion

- $\mathfrak{m}_{12}$
  - Sensitive to acceptance
  - Sensitive to wrong solutions
  - Bias must be studied
- $\mathcal{C}$
  - Robust against wrong solutions
  - Reduced by ISR



Summary

What we have done

- τ momentum and vertex reconstruction with $\tau \to \pi\nu$ signal MC
- Exploration of d_{min} and z-scan for the two-fold ambiguity
  both aaccuracy at $\sim$66% and in agreement.
- Stepwise decomposition to attribute the bias
  Theory (+ISR) $\to$ Full PHSP $\to$ Truth $\to$ Good $\to$ d_{min}

Where we are limited

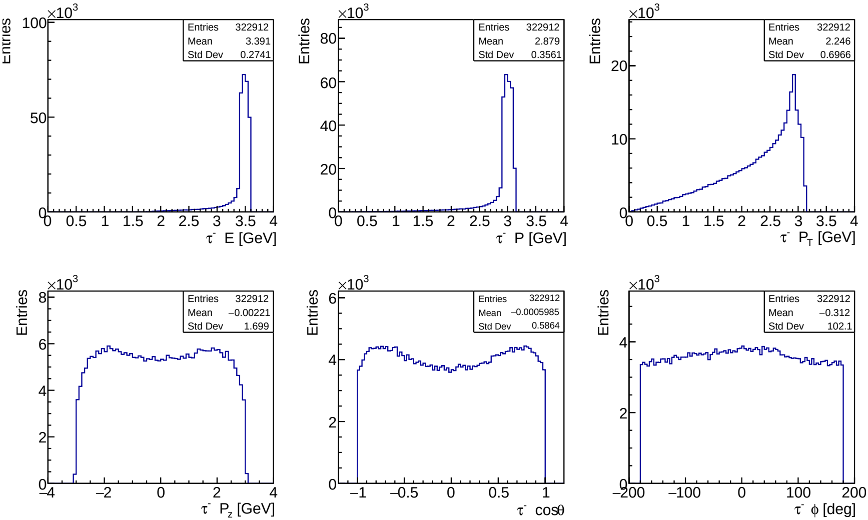
- Two-fold ambiguity is limited by detector spatial resolution, not bunch length.
- Entanglement observables are biased by detector acceptance and wrong solutions.

Next steps

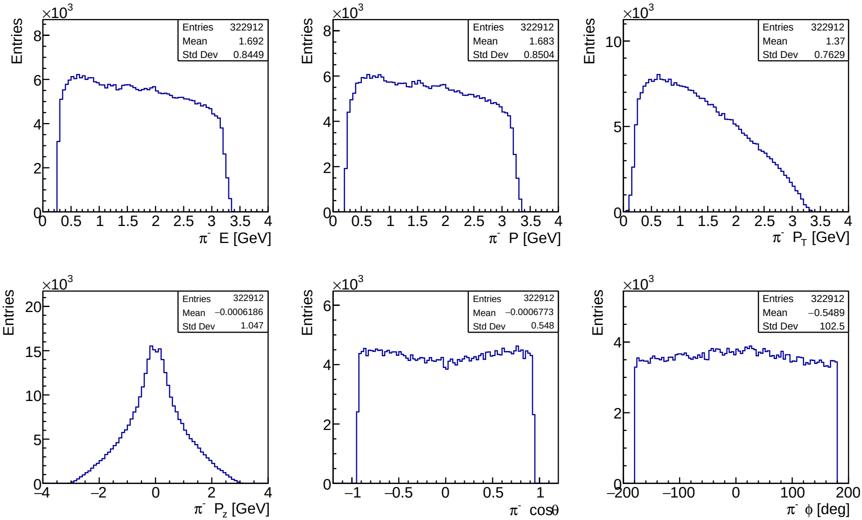
- Quantify and correct the bias from wrong solutions and acceptance.
- Background study

Thanks for your attention !

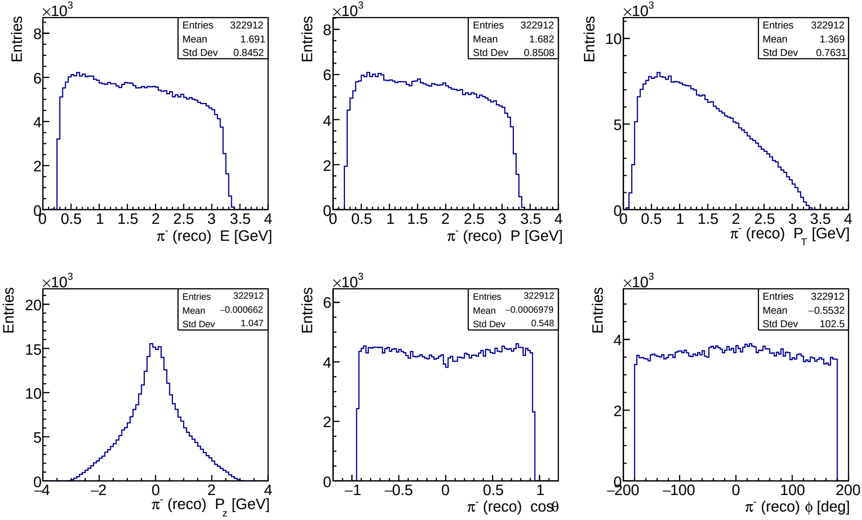
Truth — four momentum of τ^-



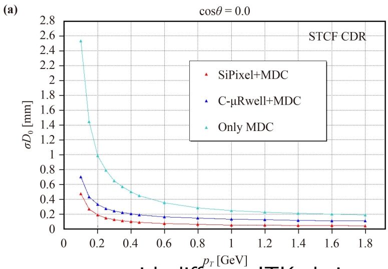
Truth — four momentum of π^-



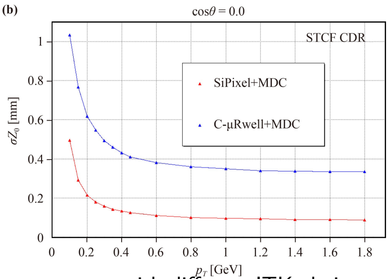
Reco — four momentum of π^-



Difficulty of vertex resolution — limitation of spatial resolution [6]



- σ_{D_0}-p_{T} with different ITK choice



- σ_{Z_0}-p_{T} with different ITK choice

τ Momentum Reconstruction

- Invariant mass constraints

$$p_{\tau^-}^2 = p_{\tau^+}^2 = m_\tau^2, \quad p_{\pi^-}^2 = p_{\pi^+}^2 = m_\pi^2, \quad p_\nu^2 = p_{\bar{\nu}}^2 = 0. \tag{1}$$

- Conservation of four momentum (in center-of-mass frame)

$$p_{\tau^-} = p_{\pi^-} + p_\nu, \quad p_{\tau^+} = p_{\pi^+} + p_{\bar{\nu}}, \tag{2}$$

$$p_{\tau^-} + p_{\tau^+} = p_0 \equiv (E_{\text{cms}}, 0, 0, 0). \tag{3}$$

- 2-fold ambiguity: $\mathbf{k}$ is the momentum of τ^+

$$\widehat{\mathbf{k}}_\pm = u\widehat{\mathbf{p}}_{\pi^+} + v\widehat{\mathbf{p}}_{\pi^-} \pm w\widehat{\mathbf{p}}_\perp, \quad \mathbf{p}_\perp = \mathbf{p}_{\pi^+} \times \mathbf{p}_{\pi^-}. \tag{4}$$

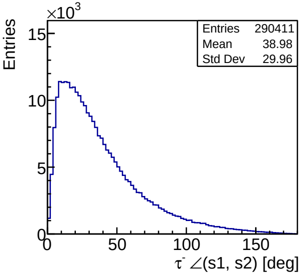
$$u = +\frac{\cos\theta_{\pi^+} + p_{\text{dot}}\cos\theta_{\pi^-}}{1 - p_{\text{dot}}^2} \tag{5}$$

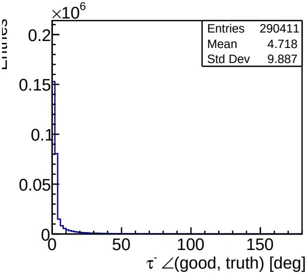
$$v = -\frac{\cos\theta_{\pi^-} + p_{\text{dot}}\cos\theta_{\pi^+}}{1 - p_{\text{dot}}^2} \tag{6}$$

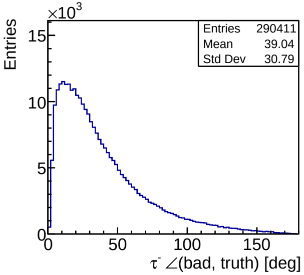
$$w = \sqrt{1 - u^2 - v^2 - 2uvp_{\text{dot}}}. \tag{7}$$

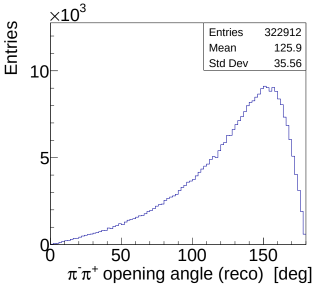
$$\cos\theta_i = \frac{2E_iE_\tau - m_i^2 - m_\tau^2}{2|\mathbf{k}||\mathbf{p}_i|} \ (i = \pi^-, \pi^+), \quad p_{\text{dot}} = \widehat{\mathbf{p}}_{\pi^+} \cdot \widehat{\mathbf{p}}_{\pi^-}. \tag{8}$$

τ Reconstruction — Two-fold Ambiguity

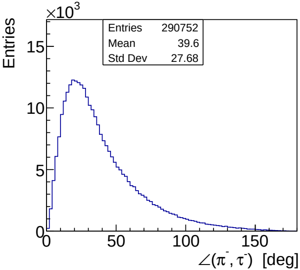


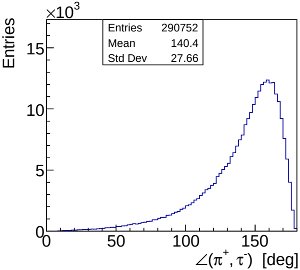


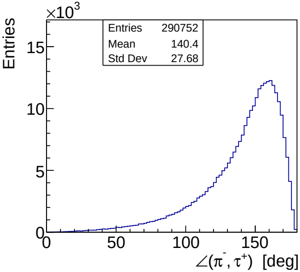


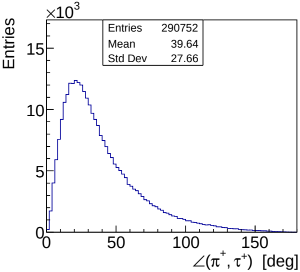


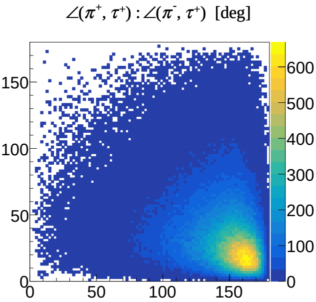
τ Reconstruction — Two-fold Ambiguity



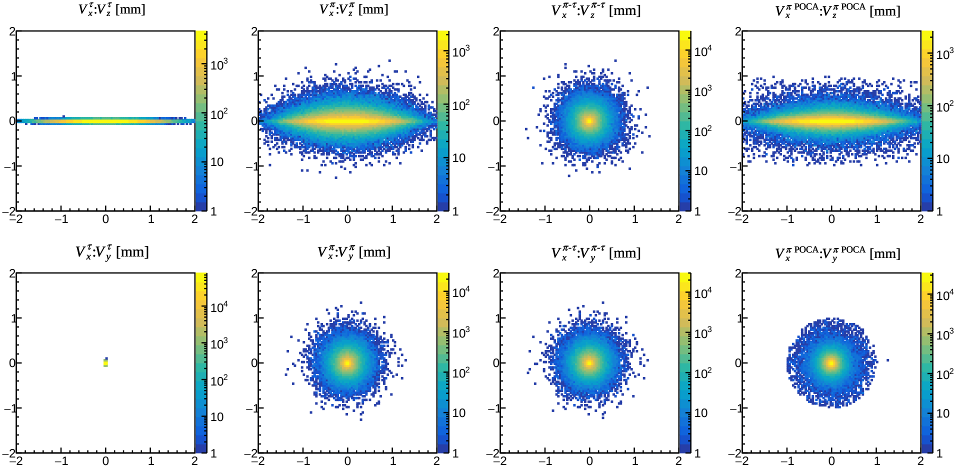






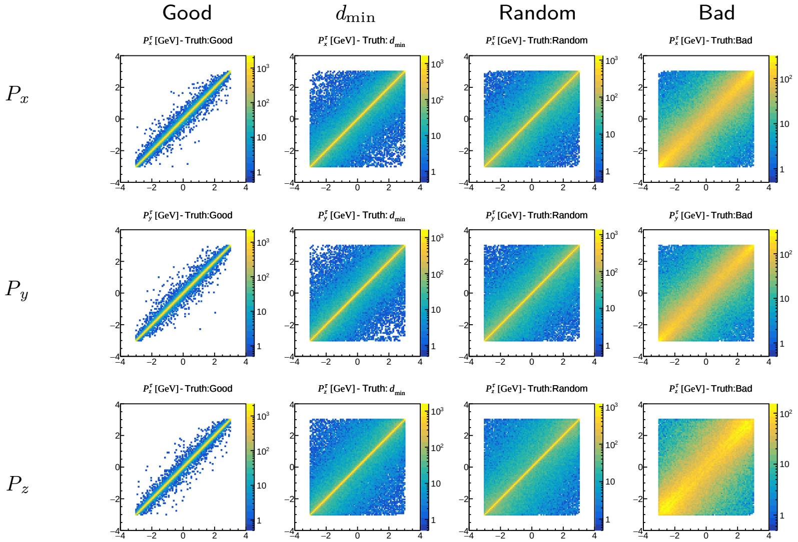


τ Reconstruction — Vertex Distribution



Production vertex of τ | Production vertex of π | Fly vector of τ | POCA of π

τ Reconstruction — Deviation (ISR < 30 MeV)



τ Reconstruction — ISR

- P_z^τ/GeV Std Dev in different solutions and ISR.

Solution	no ISR limit	ISR<30 MeV	Change
Good	0.336	0.058	-83%
$d_{\min}$	0.819	0.728	-11%
Random	1.080	1.015	-6%
Bad	1.489	1.433	-4%

Effect of ISR on τ momentum resolution

- ISR < 30 MeV loses relatively 32% of events.
- Significant for good solution.
- No obvious effect for other conditions.
- Momentum resolution is dominated by the small fraction of bad solutions.

Full PHSP — Measured $B_i^{\pm}$, C_{ij}

- Measured $B_i^{\pm}$, C_{ij}

	n	r	k
B_i^-	0.004 ± 0.003	0.001 ± 0.003	0.002 ± 0.003
B_i^+	0.000 ± 0.003	-0.004 ± 0.003	0.000 ± 0.003
C_{ni}	-0.315 ± 0.005	0.001 ± 0.005	0.002 ± 0.005
C_{ri}	-0.005 ± 0.005	0.566 ± 0.004	0.010 ± 0.005
C_{ki}	-0.001 ± 0.005	-0.001 ± 0.004	0.751 ± 0.005

Theoretical Calculation — ISR Convolution

ISR photons reduce the effective collision energy:

$$s' = s(1-x), \qquad 0 \le x \le 1 - \frac{4m_\tau^2}{s},$$

where x is the energy fraction carried by ISR photons.
The ISR-corrected differential cross section is

$$\frac{d\sigma_{\mathrm{ISR}}}{d\Phi}(s,\Phi) = \int dx\, F(x,s)\, \frac{d\sigma_0}{d\Phi}\big(s(1-x),\Phi\big),$$

where $F(x,s)$ is the ISR radiator function [7].

Physical meaning

The observed sample contains events produced at different effective energies s', weighted by the ISR probability and production cross section.

[7] E. A. Kuraev and V. S. Fadin, "On Radiative Corrections to e+ e- Single Photon Annihilation at High Energy", Sov. J. Nucl. Phys. **41**, 466 (1985)

Theoretical Calculation — Angular and Bin-wise Averaging

For $X = B_i^{\pm}$ or C_{ij}, the prediction in bin b is

$$\overline{X}_b = \frac{\displaystyle\int dx\, F(x,s) \int d\Phi\, \mathbf{1}_b(\Phi)\, \frac{d\sigma_0}{d\Phi}(s',\Phi)\, X(s',\Phi)}{\displaystyle\int dx\, F(x,s) \int d\Phi\, \mathbf{1}_b(\Phi)\, \frac{d\sigma_0}{d\Phi}(s',\Phi)}.$$

- $\Phi = \cos\theta$ for an angular distribution.
- $\mathbf{1}_b(\Phi)$ selects events in bin b.
- For Full PHSP, integrate over the full angular range.

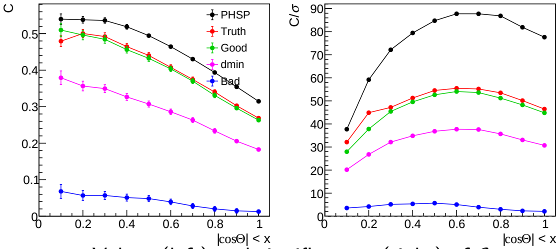
Calculation order

First average $B_i^{\pm}$ and C_{ij}, then calculate

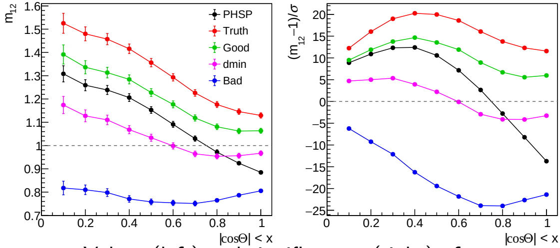
$$\mathcal{C} = \mathcal{C}(\overline{\rho}), \qquad m_{12} = \lambda_1 + \lambda_2,$$

where $\lambda_{1,2}$ are the two largest eigenvalues of $\overline{C}^T\overline{C}$.

Significance of $\mathcal{C}, \mathfrak{m}_{12}$ as to $|\cos\Theta|$ cut



- Values (left) and significances (right) of $\mathcal{C}$



- Values (left) and significances (right) of $\mathfrak{m}_{12}$

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[1] X. Li et al., "Quantum entanglement of τ-pair and τ reconstruction at STCF", Int. J. Mod. Phys. A, 10.1142/S0217751X26450028 (2026).

[2] A. J. Barr et al., "Quantum entanglement and Bell inequality violation at colliders", Prog. Part. Nucl. Phys. **139**, 104134 (2024).

[3] K. Ehatäht et al., "Probing entanglement and testing Bell inequality violation with e+e-$\rightarrow\tau+\tau$- at Belle II", Phys. Rev. D **109**, 032005 (2024).

[4] M. Fabbrichesi and L. Marzola, "Quantum tomography with τ leptons at the FCC-ee: Entanglement, Bell inequality violation, sinθW, and anomalous couplings", Phys. Rev. D **110**, 076004 (2024).

[5] M. Achasov et al., "STCF conceptual design report (Volume 1): Physics & detector", Front. Phys. (Beijing) **19**, 14701 (2024).

[6] K. Inami et al. (Belle), "An improved search for the electric dipole moment of the τ lepton", JHEP **04**, 110 (2022).

[7] E. A. Kuraev and V. S. Fadin, "On Radiative Corrections to e+ e- Single Photon Annihilation at High Energy", Sov. J. Nucl. Phys. **41**, 466 (1985).