非微扰哈密顿框架(NPHF)研究强子态

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PRL 128 (2022) 112001，2510.01564

西安

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Outline

- Background： Bare state + Coupled channel
- NPHF
- From $D_s(2317)$ to $T_{c\bar{s}}(2327)$
- Summary and Outlook

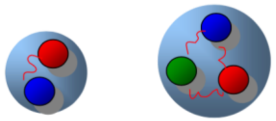
Background: Beyond Quark Model

Hadron state:
meson, baryon//Exotic

Quark model?

Quark/Gluon

conventional hadron

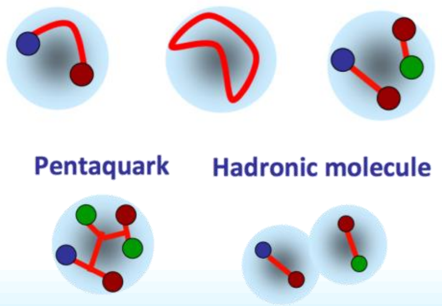


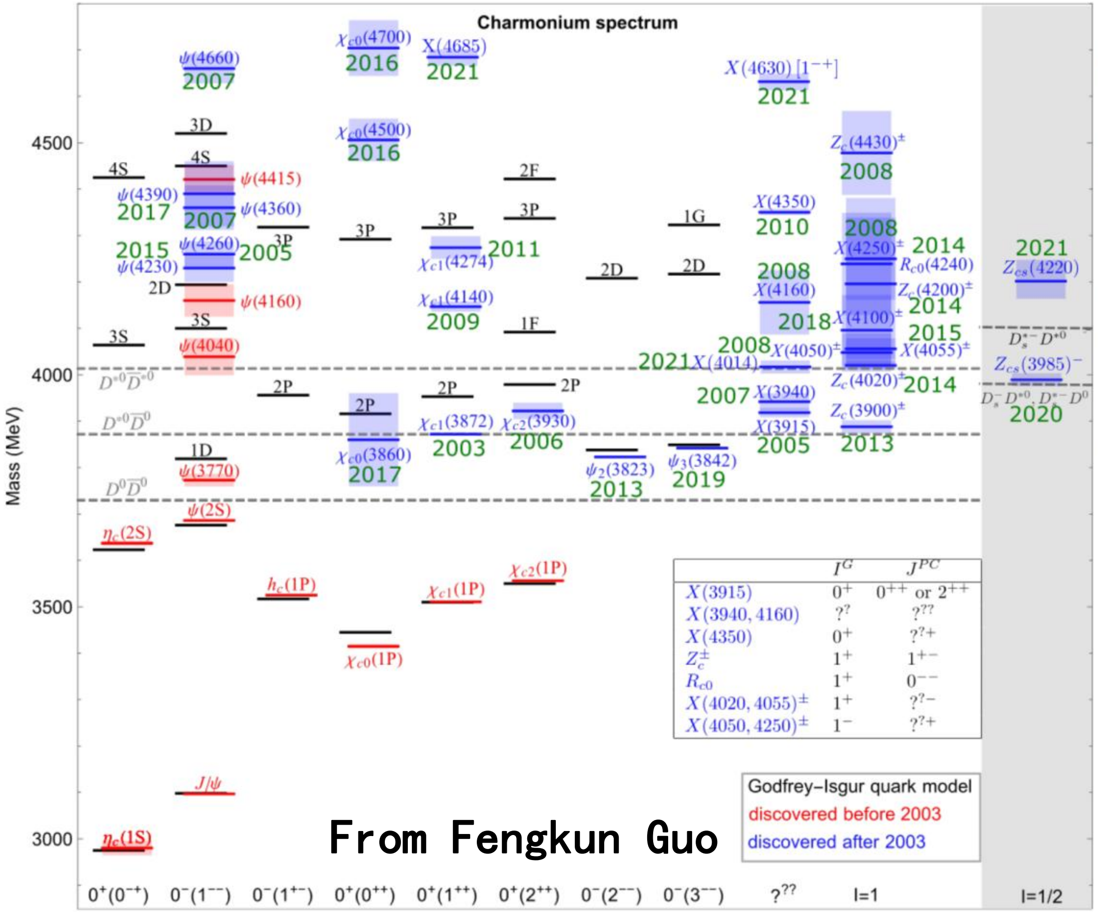
(q $\bar{q}$) (qqq)

Exotic

Hybrid Glueball Tetraquark

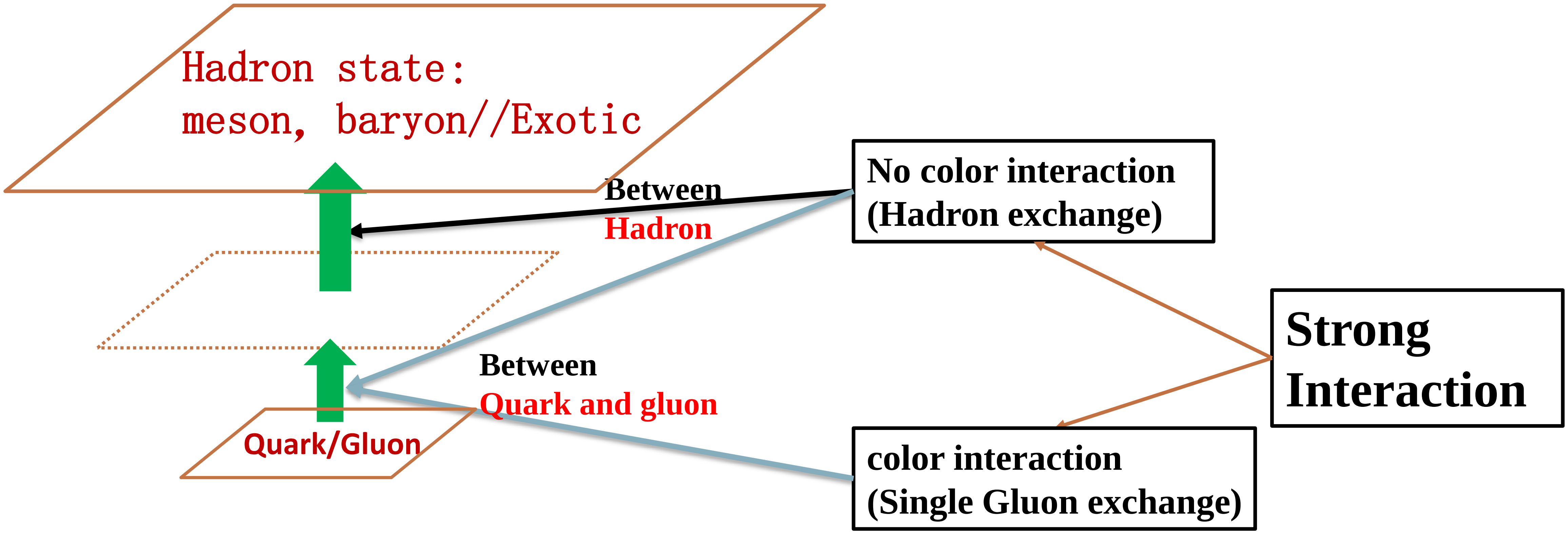
Pentaquark Hadronic molecule



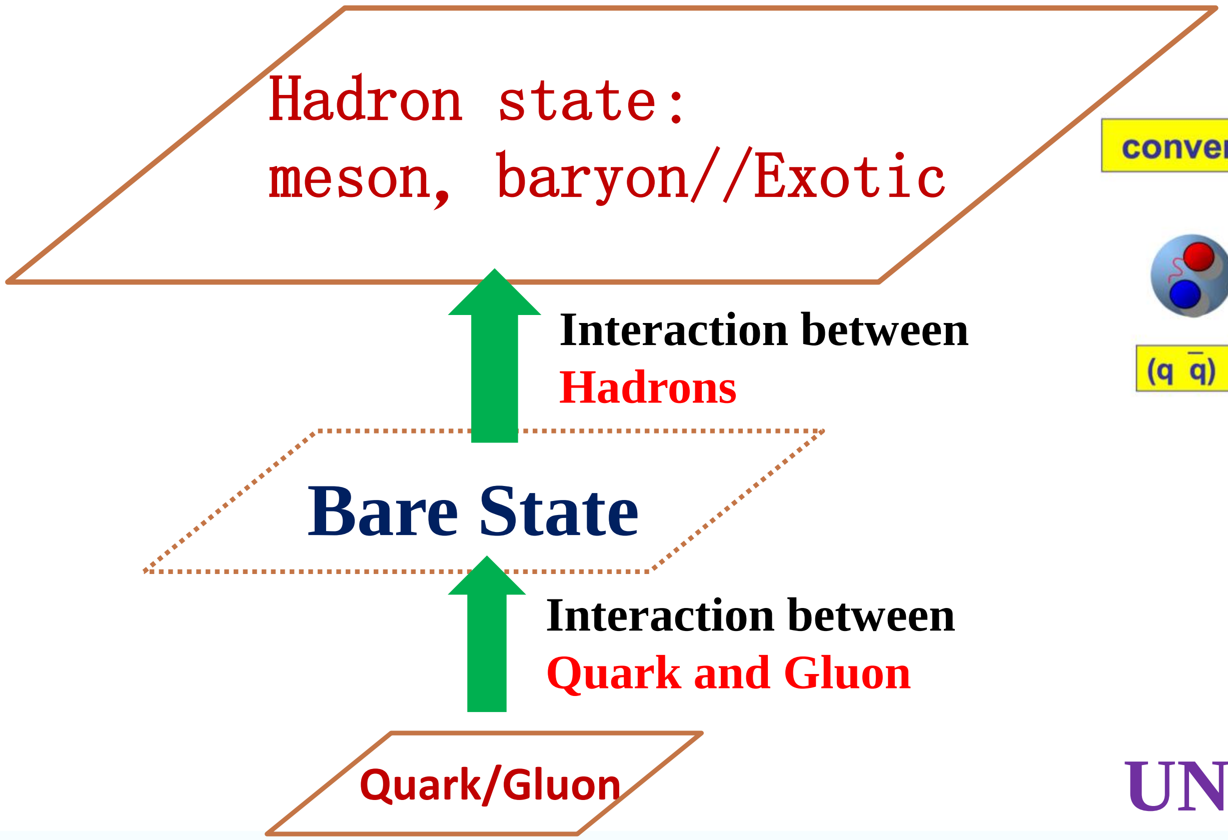


From Fengkun Guo

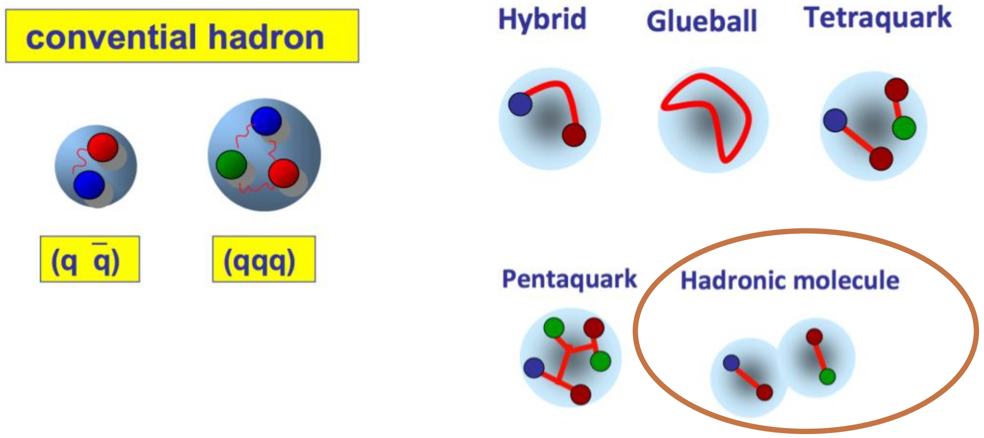
Background：Strong Interaction



Background： Bare state + Coupled channel

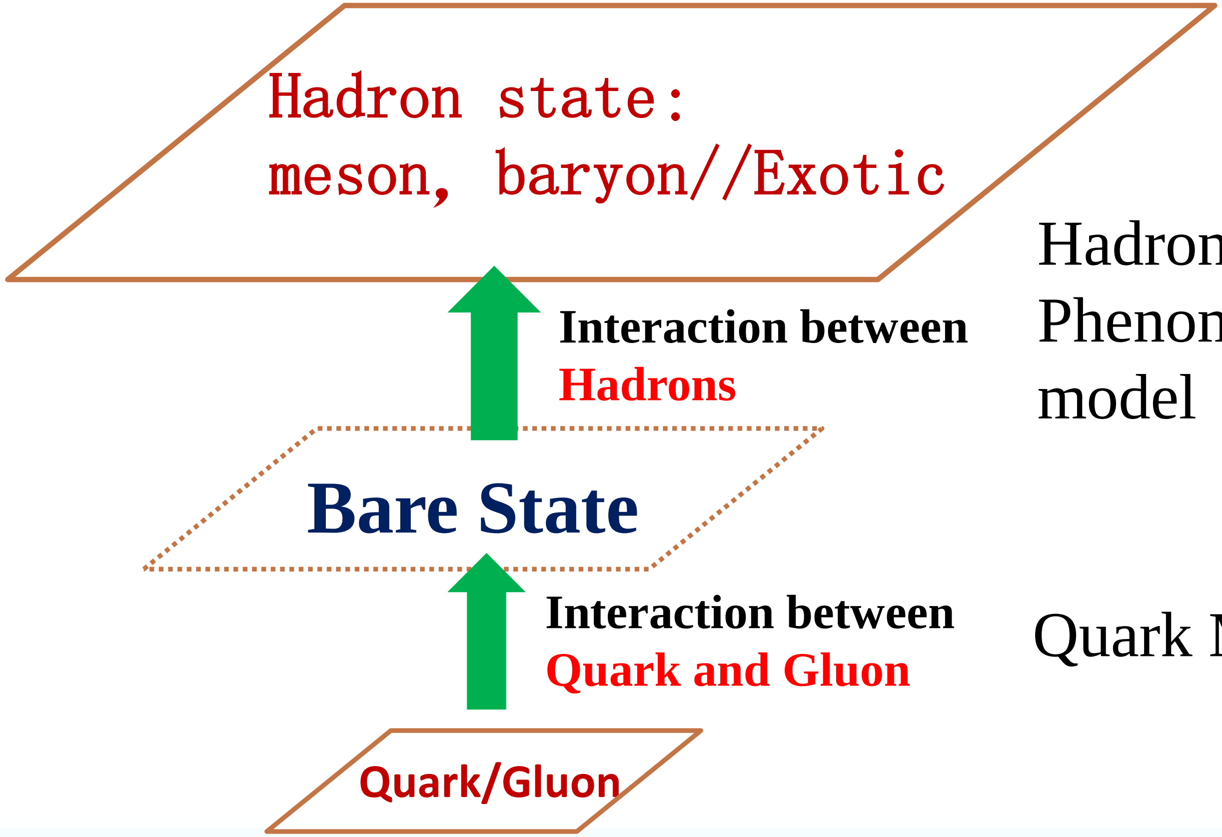


Exotic



UNQUENCHED quark model

Background： Bare state + Coupled channel



Hadron level Phenomenological model

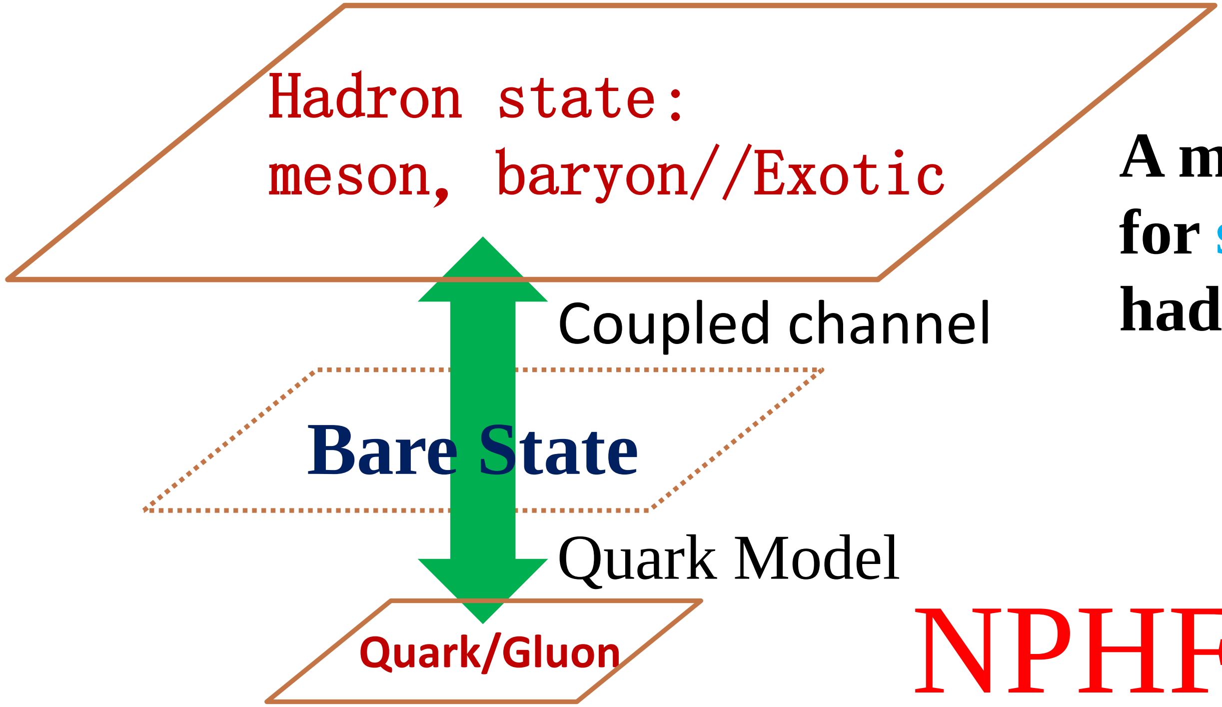
Quark Model

Coupled channel

Bare state(Single)
+multi-hadron

The number of hadrons is **not conserved**!

Background： Strong-Spectrum-Reaction



quark level &&
hadron level.

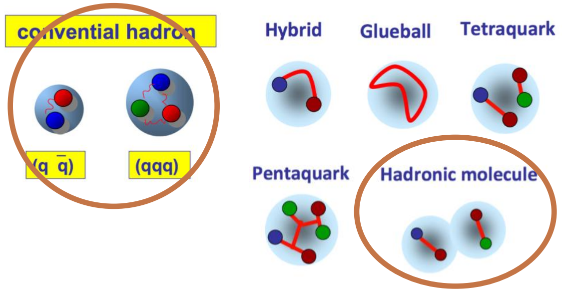
A more comprehensive framework for systematically describing hadrons

Not only one, at least a set of hadron.

NPHF

NonPerturbative Hamiltonian Framework

NPHF： bare state + hadron channel



$$H = H_0 + H_I$$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha} |\alpha(k_\alpha)\rangle \left[\sqrt{m_{\alpha 1}^2 + k_\alpha^2} + \sqrt{m_{\alpha 2}^2 + k_\alpha^2}\right] \langle \alpha(k_\alpha)|$$

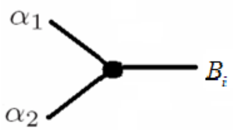
$|B_i>$ bare state， bare mass m_i ⟷ Quark level

$|\alpha(k_\alpha)>$ non-interaction channels ⟷ Hadron level

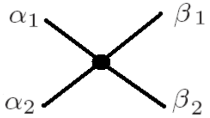
$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[|\alpha(k_\alpha)\rangle g_{i,\alpha}^{+} \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_\alpha)| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_\alpha)\rangle v_{\alpha,\beta} \langle \beta(k_\beta)|$$



From 3P0 model, and the wavefunction of quark model



One boson exchange (OBE)

Resonance
(Mass, width, pole position, coupling)

Hamiltonian

T matrix
(phase shift, inelasticity)

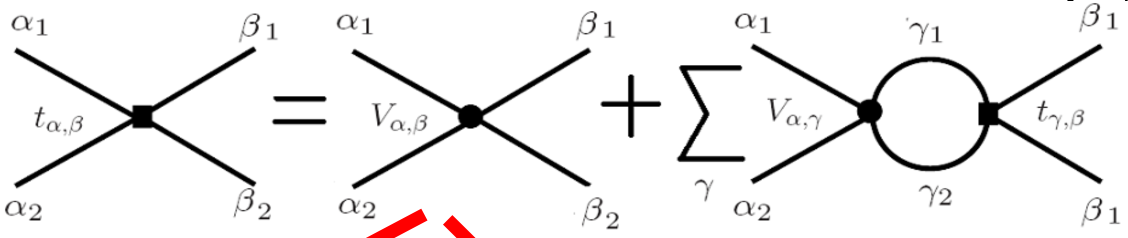
Lattice spectrum

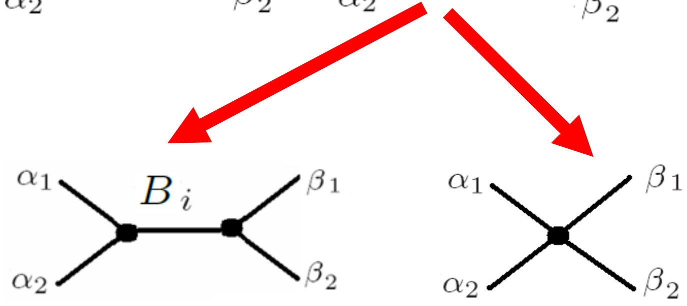
Introduction of HEFT

Argonne-Osaka Model

- **T Matrix:**

$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_{\gamma}\int k_\gamma{}^2 dk_\gamma \frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma 1}^2+k_\gamma^2}-\sqrt{m_{\gamma 2}^2+k_\gamma^2}+i\varepsilon}$$





$$g^*_{i,\alpha}\frac{1}{E-m_i}g_{i,\beta}$$

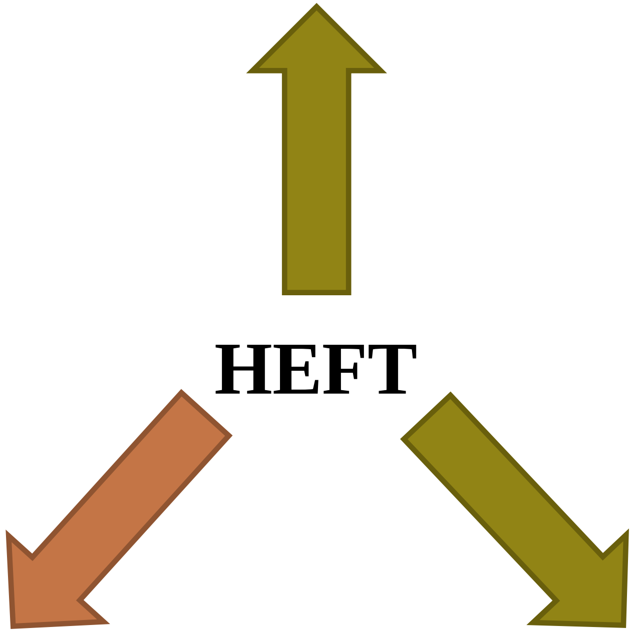
$$v_{\alpha,\beta}$$

$$S_{\alpha,\beta}=1-i2\sqrt{\rho_\alpha}\,t_{\alpha,\beta}(k_{0\alpha},k_{0\beta},E)\sqrt{\rho_\beta}$$

$$\rho_\alpha=\frac{\pi k_{0\alpha}\sqrt{m_{\alpha 1}^2+k_{0\alpha}^2}\sqrt{m_{\alpha 1}^2+k_{0\alpha}^2}}{E}$$

$$\eta e^{2i\delta_\alpha}=S_{\alpha,\alpha}$$

Resonance
(Mass , Width, Pole position, Coupling)



HEFT

T matrix
(Phase Shifts, inelasticity)

Lattice Spectrum

Introduction of HEFT

J. M. M. Hall etc. PRD 87(2013), 094510
J.-j. Wu etc. PRC90 (2014), 055206

- **Hamiltonian with discrete momentum**

Continuous → Discrete

$$\int d\vec{k} \quad \text{and} \quad |\alpha(\vec{k}_\alpha)\rangle \quad \text{and} \quad \langle\beta(\vec{k}_\beta)|\alpha(\vec{k}_\alpha)\rangle = \delta_{\alpha\beta}\delta(\vec{k}_\alpha - \vec{k}_\beta)$$

$$\downarrow$$

$$\sum_{\mathrm{i}}\left(\frac{2\pi}{\mathrm{L}}\right)^3 \quad \text{and} \quad \left(\frac{2\pi}{\mathrm{L}}\right)^{-3/2}|\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\rangle_\alpha \quad \text{and} \quad {}_\beta\langle\vec{k}_{\mathrm{j}},-\vec{k}_{\mathrm{j}}|\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\rangle_\alpha = \delta_{\alpha\beta}\delta_{\mathrm{ij}}$$

$$H_0 = \sum_{i=1,n}|B_i\rangle m_i\langle B_i| + \sum_{\alpha,\mathrm{i}}|\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\rangle_\alpha\left[\sqrt{m_{\alpha_B}^2+k_\alpha^2}+\sqrt{m_{\alpha_M}^2+k_\alpha^2}\right]{}_\alpha\langle\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}|$$

$$H_I = \sum_{\mathrm{j}}\left(\frac{2\pi}{\mathrm{L}}\right)^{3/2}\sum_\alpha\sum_{i=1,n}\left[|\vec{k}_{\mathrm{j}},-\vec{k}_{\mathrm{j}}\rangle_\alpha\, g_{i,\alpha}^{+}\langle B_i| + |B_i\rangle g_{i,\alpha}\,{}_\alpha\langle\vec{k}_{\mathrm{j}},-\vec{k}_{\mathrm{j}}|\right] + \sum\left(\frac{2\pi}{\mathrm{L}}\right)^3\sum|\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\rangle_\alpha\, v_{\alpha,\beta}\,{}_\beta\langle\vec{k}_{\mathrm{j}},-\vec{k}_{\mathrm{j}}|$$

$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g_1^V(k_0) & g_2^V(k_0) & \cdots & g_{n_c}^V(k_0) & g_1^V(k_1) & \cdots \\ g_1^V(k_0) & v_{1,1}^V(k_0,k_0) & v_{1,2}^V(k_0,k_0) & \cdots & v_{1,n_c}^V(k_0,k_0) & v_{1,1}^V(k_0,k_1) & \cdots \\ g_2^V(k_0) & v_{2,1}^V(k_0,k_0) & v_{2,2}^V(k_0,k_0) & \cdots & v_{2,n_c}^V(k_0,k_0) & v_{2,1}^V(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ g_{n_c}^V(k_0) & v_{n_c,1}^V(k_0,k_0) & v_{n_c,2}^V(k_0,k_0) & \cdots & v_{n_c,n_c}^V(k_0,k_0) & v_{n_c,1}^V(k_0,k_1) & \cdots \\ g_1^V(k_1) & v_{1,1}^V(k_1,k_0) & v_{1,2}^V(k_1,k_0) & \cdots & v_{1,n_c}^V(k_1,k_0) & v_{1,1}^V(k_1,k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$(H_0+H_I)\,|\Psi> = E\,|\Psi>$$

Eigen-Value ⟷ Lattice Spectrum

Eigen-Vector

Resonance
(Mass , Width, Pole position, Coupling)

HEFT

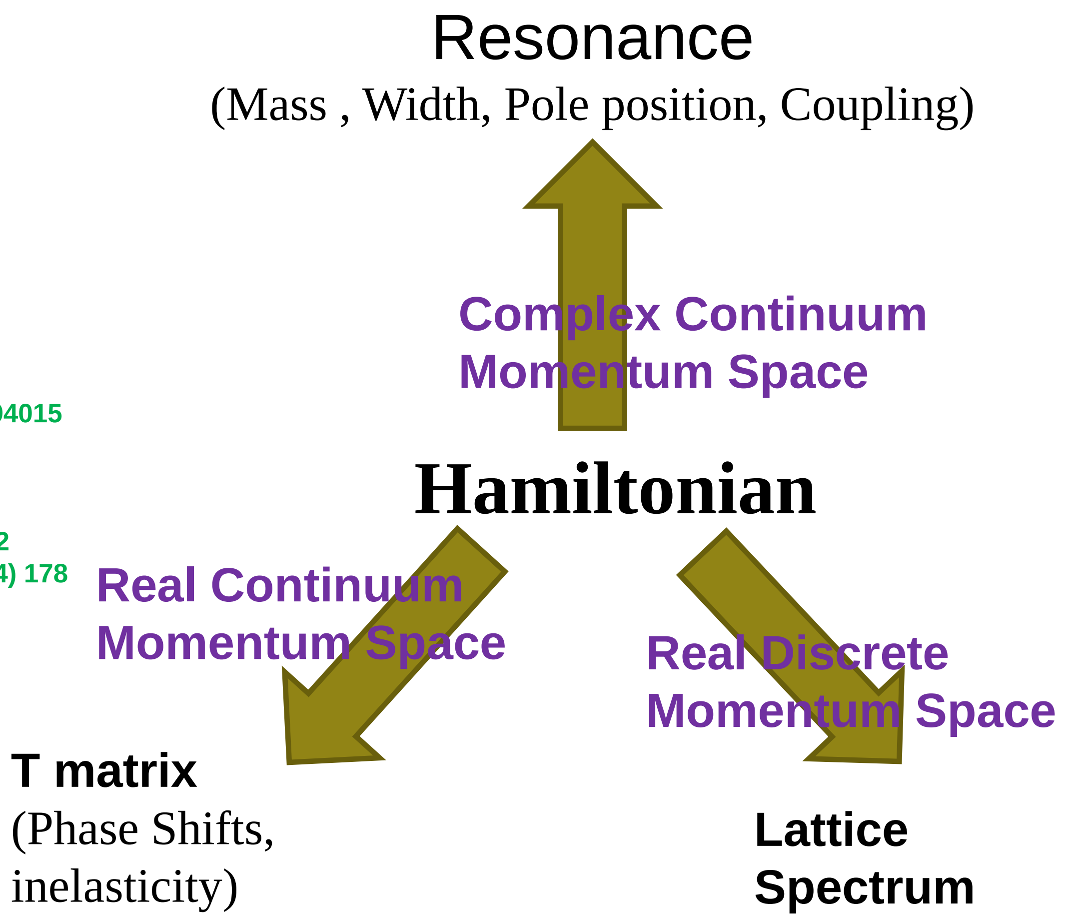
T matrix
(Phase Shifts, inelasticity)

Lattice Spectrum

1. Finite-volume matrix Hamiltonian model for a Δ→Nπ syste PRD 87 (2013) no.9, 094510
2. Finite-volume Hamiltonian method for coupled-channels interactions in lattice QCD PRC 90 (2014) no.5, 055206
3. Lattice QCD Evidence that the Λ(1405) Resonance is an Antikaon-Nucleon Molecule PRL 114 (2015) no.13, 132002
4. Hamiltonian effective field theory study of the N∗(1535) resonance in lattice QCD PRL 116 (2016) no.8, 082004
5. Hamiltonian effective field theory study of the N∗(1440) resonance in lattice QCD PRD 95 (2017) no.3, 034034
6. Structure of the Λ(1405) from Hamiltonian effective field theory PRD 95 (2017) no.1, 014506
7. Nucleon resonance structure in the finite volume of lattice QCD PRD 95 (2017) no.11, 114507
8. Structure of the Roper Resonance from Lattice QCD Constraints PRD 97(2018) no.9, 094509
9. Kaonic Hydrogen and Deuterium in Hamiltonian Effective Field Theory PLB 808(2020),135652
10. Partial Wave Mixing in Hamiltonian Effective Field Theory PRD 101(2020) no.11,114501
11. Hamiltonian effective field theory in elongated or moving finite volume PRD 103(2021) no.9, 094518
12. Regularization in Nonperturbative Extensions of Effective Field Theory PRD 106 (2022) no.3, 034506
13. Novel Coupled Channel Framework Connecting the Quark Model and Lattice QCD for the Near-threshold Ds States PRL128(2022),112001
14. The investigations of the P-wave Bs states combining quark model and lattice QCD in the coupled channel framework JHEP01 (2023) 058
15. Effects of multiple single-particle basis states in scattering systems Annals Phys. 459(2023) 169531
16. Low-lying odd-parity nucleon resonances as quark-model like states PRD 108 (2023) 9, 094519
17. Structure of the Λ(1670) resonance PRD 109 (2024) 5, 054025
18. Investigation on the bottom analogs Tbb of Tcc+ PRD 110 (2024) 7, 074007
19. Pion photoproduction of nucleon excited states with Hamiltonian effective field theory PRD 110 (2024) 9, 094015
20. Study of the pion-mass dependence of ρ-meson properties in lattice QCD PRD 109 (2024) 3, 034505
21. Searching for the first radial excitation of the Δ(1232) in lattice QCD JPG 51 (2024) 6, 065106
22. Three-coupled-channel analysis of Zc(3900) involving $D\bar{D}^*$, πJ/ψ, and ρηc PRD 110 (2024) 11, 114029
23. New insight into the exotic states strongly coupled with the $D\bar{D}^*$ from the Tcc+ Sci.Bull. 69 (2024) 3036-3042
24. Generalized boost transformations in finite volumes and application to Hamiltonian methods JHEP 08 (2024) 178
25. Spectral parameters of the ρ resonance from lattice QCD JHEP 08 (2025) 064
26. Finite volume Hamiltonian method for two-particle systems containing long-range potential on the lattice JHEP 04 (2025) 108
27. Structure of the Ω-(2012) with Hamiltonian effective field theory PRD 112 (2025) 5, L051503
28. Nucleon resonance structure up to 2 GeV and the nature of the Roper resonance PRD 111 (2025) 11, 116002
29. Exploring the Ω− spectrum in lattice QCD JPG 52 (2025) 6, 065103
30. The odd-parity strange baryons Σ (1/2−) below 1.8 GeV with Hamiltonian effective field theory 2509.17510

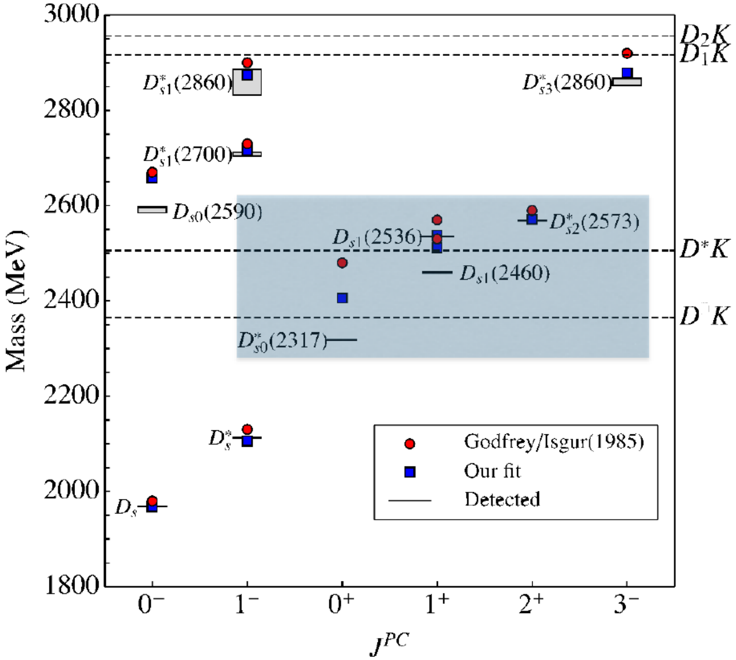
$N^*(1440), N^*(1535), N^*(1650), N^*(1710), \Delta(1232), \Delta(1600), \Lambda(1405), \Lambda(1670),$
$D_{s0}(2317), D_{s1}(2460), D_{s1}(2536), D_{s2}(2573), X(3872), T_{cc}, Z_c(3900), \ldots\ldots$

NPHF



$D_{s0}(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, $D_{s2}(2573)$

Z. Yang, G.-J. Wang, J.-j. Wu, S.-l. Zhu, M. Oka **Phys.Rev.Lett.128(2022),112001**



	$\bar{c}s$ cores		channel	
	$B(\vert^{2S+1}L_J\rangle)$	$B(\text{mass})$	α	L
$D_{s0}^*(2317)$	$\vert^3P_0\rangle$	2405.9	DK	S
$D_{s1}^*(2460)$	$0.68\vert^1P_1\rangle - 0.74\vert^3P_1\rangle$ $= -0.99\phi_s + 0.13\phi_d$	2511.5	D^*K	S, D
$D_{s1}^*(2536)$	$-0.74\vert^1P_1\rangle - 0.68\vert^3P_1\rangle$ $= -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D_{s2}^*(2573)$	$\vert^3P_2\rangle$	2571.2	DK, D^*K	D

1. Fix the bare mass and wave function from GI model;

2. The interaction of bare-channel and channel-channel;

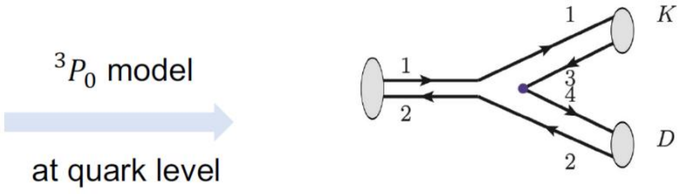
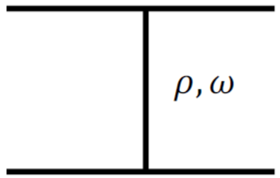
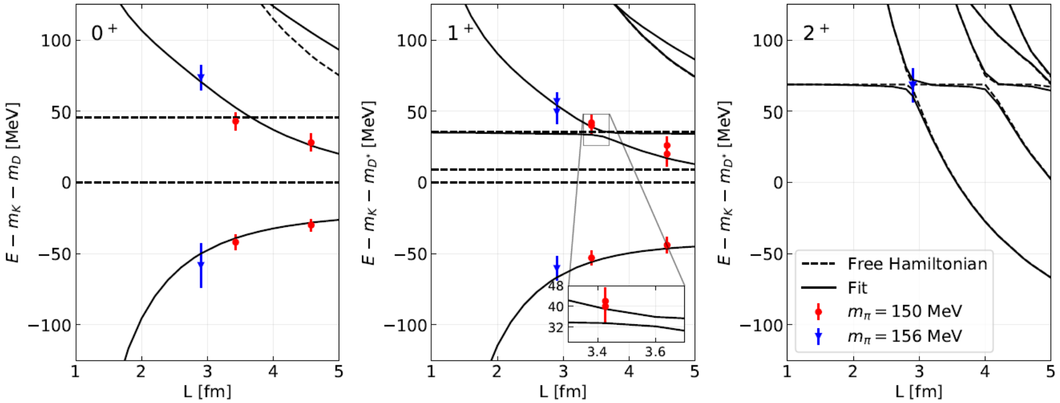


Figure 2: The diagram contribute to the process $D_s^*(2317) \to DK$.



	$P(c\bar{s})$
$D_{s0}^*(2317)$	$\approx 32.0\%$
$D_{s1}^*(2460)$	$\approx 52.4\%$
$D_{s1}^*(2536)$	$\approx 98.2\%$
$D_{s2}^*(2573)$	$\approx 95.9\%$

3. Fit Lattice data;



Conclusion:

$D^*_{s0}(2317)$-DK

$D_{s1}(2460)$-D^*K **S-wave**

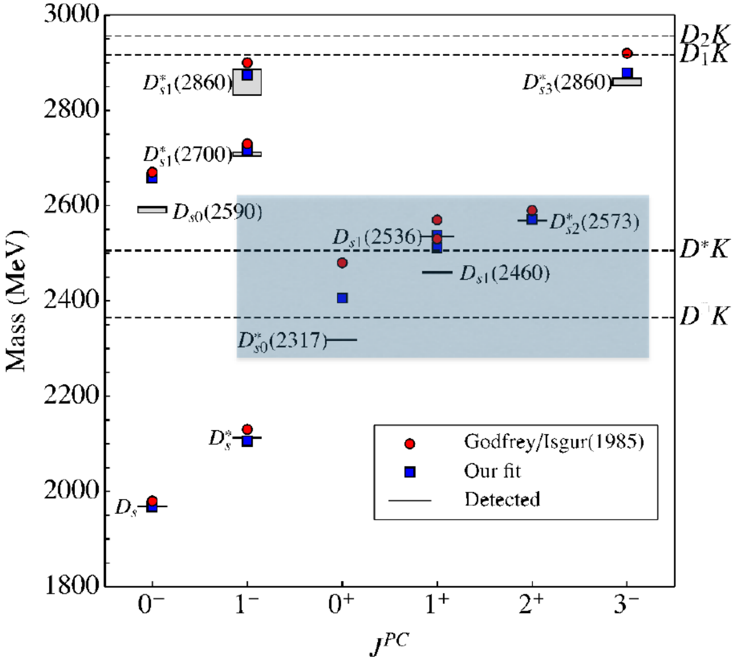
Mass moving vs GI Model

$D_{s1}(2536)$-D^*K

$D_{s2}(2573)$-$D^{(*)}K$ **D-wave**

Mass stable vs GI model

$D_{s0}(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, $D_{s2}(2573)$



	$\bar{c}s$ cores		channel	
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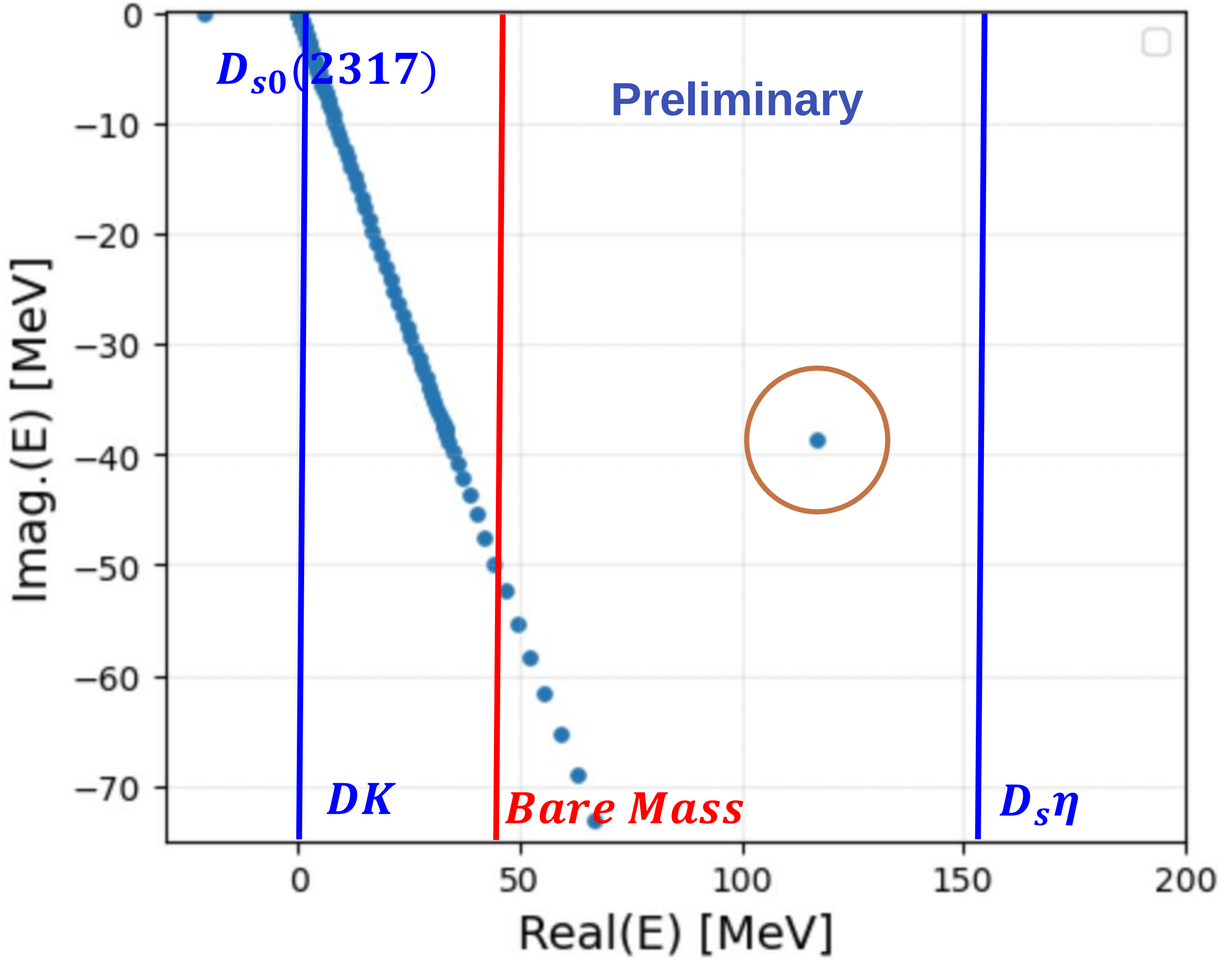
$D_{s1}^*(2536)$	$-0.74\vert^1P_1\rangle - 0.68\vert^3P_1\rangle = -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D_{s2}^*(2573)$	$\vert^3P_2\rangle$	2571.2	DK, D^*K	D

1

2

3



128(2022),112001

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–

on：

)-DK

)-D*K S-wave

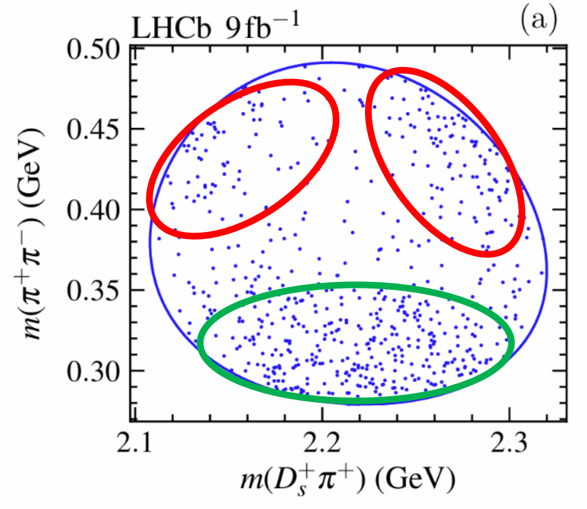
ving vs GI Model

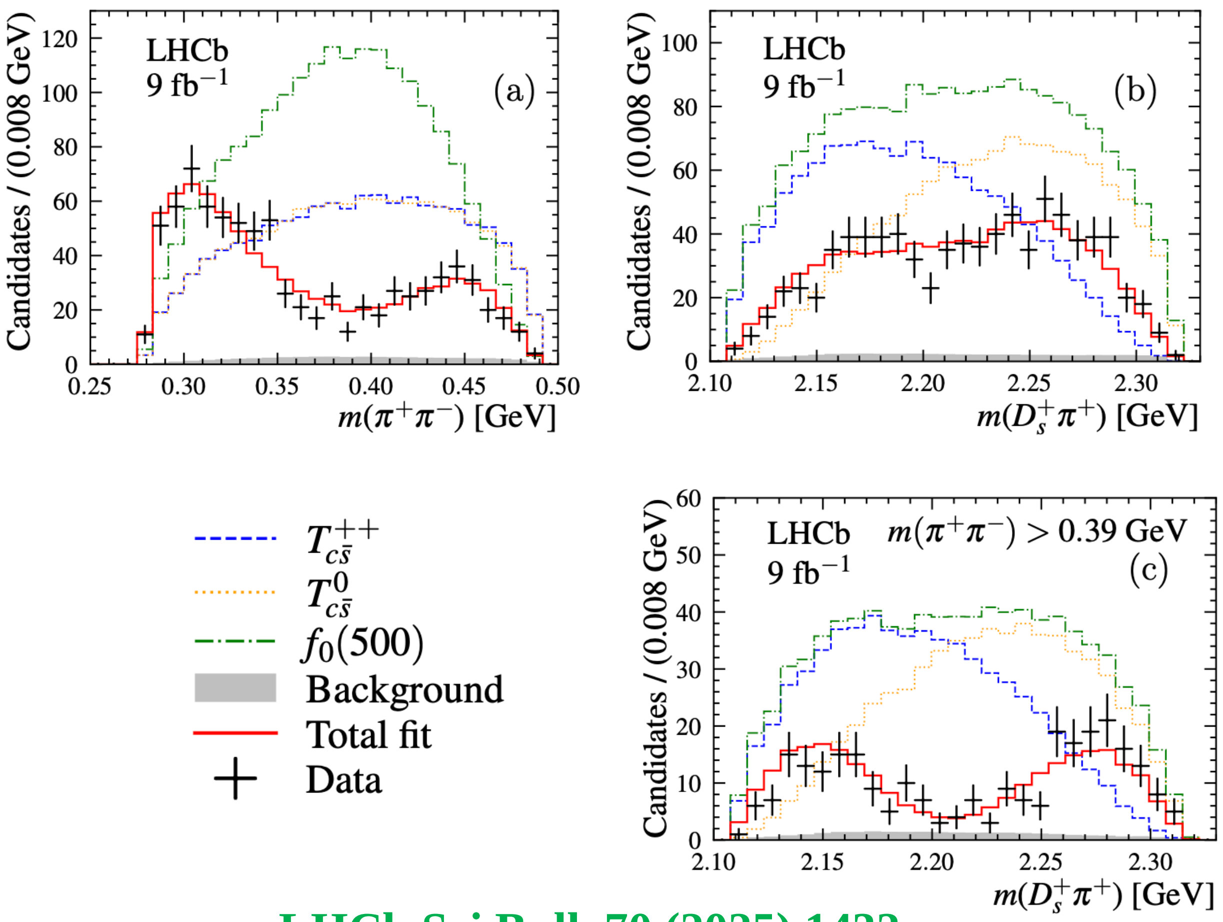
)-D*K

)-D(*)K D-wave

ole vs GI model

$B \to D^{(*)} D_{s1}(2460) \to D^{(*)} D_s^+ \pi^+ \pi^-$ decays





$D_s \to D_s\pi$ Isospin breaking

$D_s^+\pi^+$, $D_s^+\pi^0$, $D_s^+\pi^-$ are isospin triplet

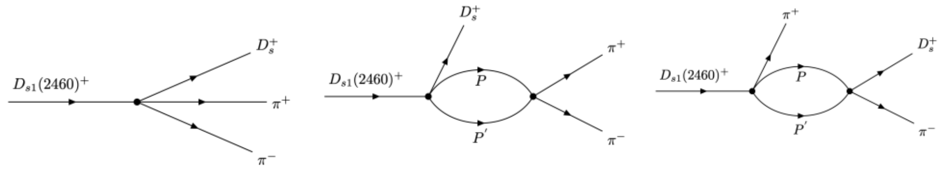
– Four quark state

Model	Resonance	Mass (MeV)	Width (MeV)
$f_0(500) + \text{RBW } T_{c\bar{s}}(0^+)$	$f_0(500)$	$464 \pm 23 \pm 14$	$214 \pm 28 \pm 8$
	$T_{c\bar{s}}^{++}/T_{c\bar{s}}^{0}$	$2312 \pm 27 \pm 11$	$264 \pm 46 \pm 21$
$f_0(500) + \text{K-matrix } T_{c\bar{s}}(0^+)$	$f_0(500)$	$472 \pm 32 \pm 19$	$226 \pm 24 \pm 18$
	$T_{c\bar{s}}^{++}/T_{c\bar{s}}^{0}$	$2328 \pm 12 \pm 12$	$96 \pm 16 \pm 23$

LHCb Sci.Bull. 70 (2025) 1432

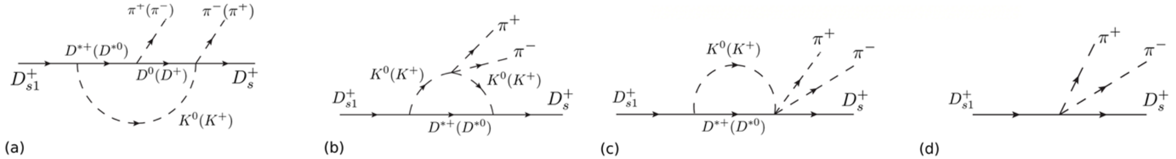
$B \to D^{(*)} D_{s1}(2460) \to D^{(*)} D_s^+ \pi^+ \pi^-$ decays

ZY Wang, YS Li and SQ Luo, PRD 111 (2025) 7, 076009

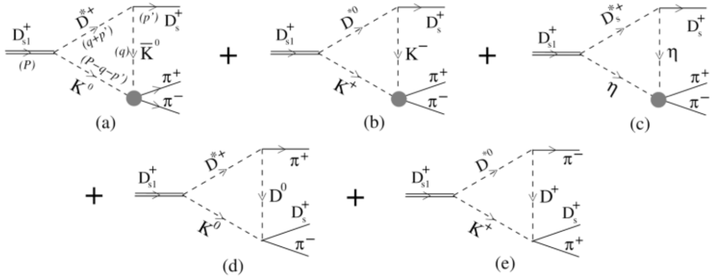


Missing the triangle loop, losing the nature of $D_{s1}(2460)$

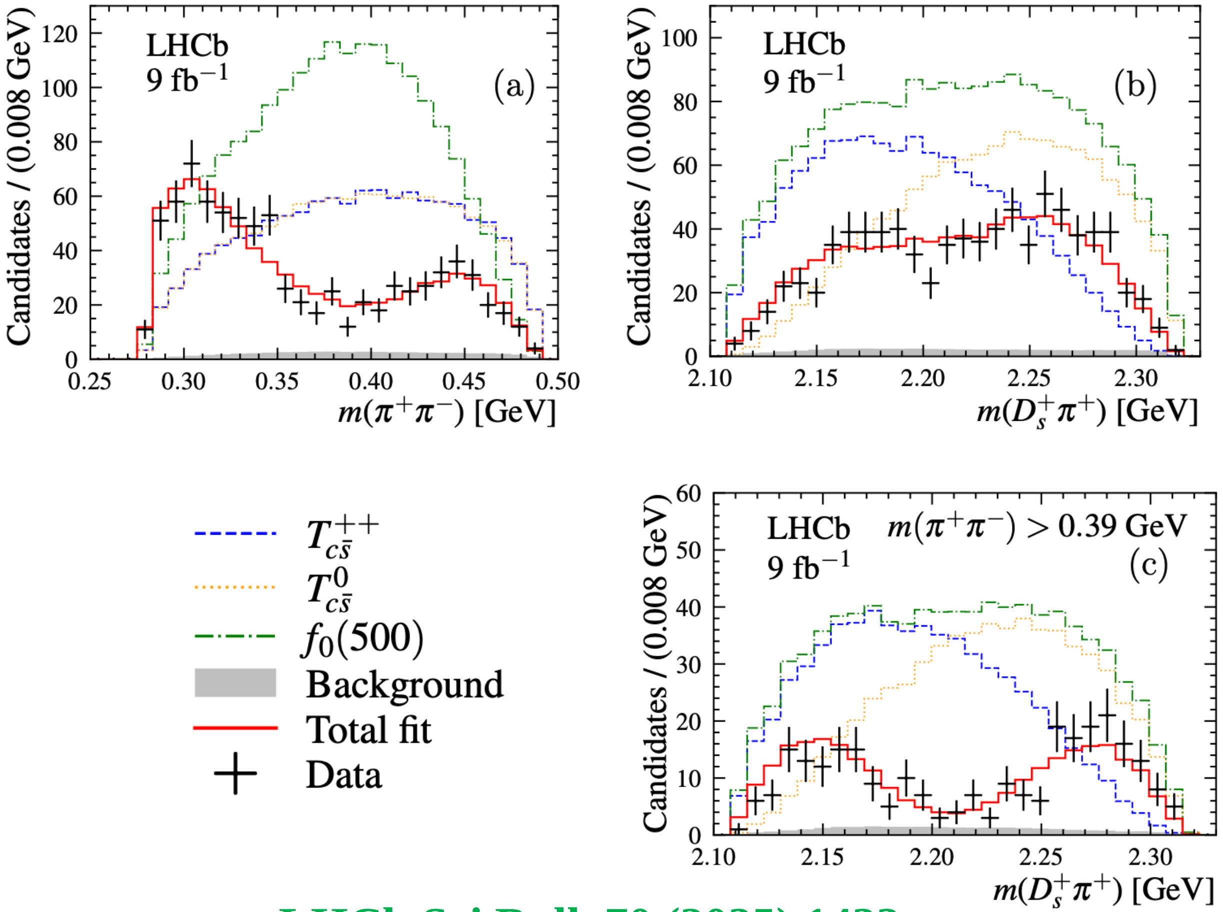
Tang, Lin, Guo, Hanhart and Meissner, CTP 75 (2023)5 055203



Roca, Dias and Oset, Eur.Phys.J.C 85 (2025) 7, 808



Missing the coupled channel effect of $D_s\pi - DK$

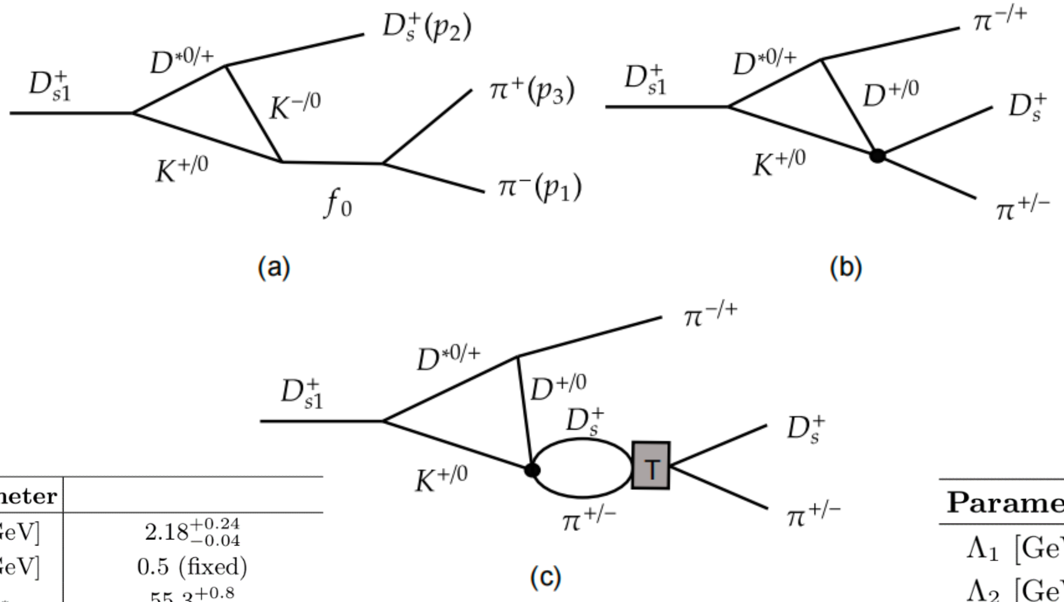


LHCb Sci.Bull. 70 (2025) 1432

$D_{s1}(2460) \to D_s^+ \pi^+ \pi^-$

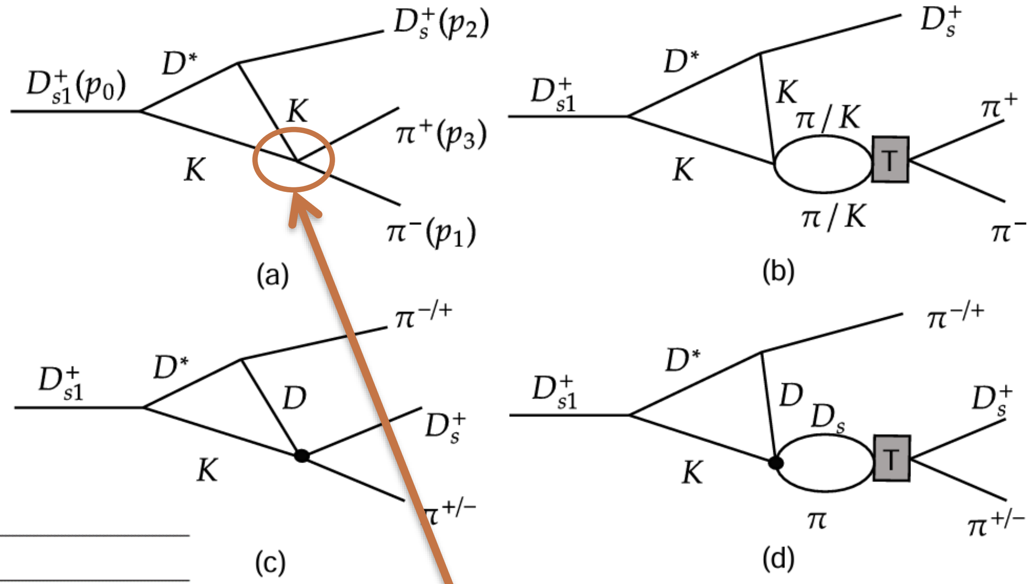
Z. Yang, G.-J. Wang, J.-J. Wu, M. Oka, 2510.01564

v1



Parameter	
Λ_1 [GeV]	$2.18^{+0.24}_{-0.04}$
Λ_2 [GeV]	0.5 (fixed)
g_{K^*}	$55.3^{+0.8}_{-2.5}$
ϕ [Rad]	$3.78^{+0.38}_{-0.26}$
r_1	215^{+51}_{-86}
r_2	$-9.0^{+4.3}_{-0.8}$
m_{f_0} [MeV]	519^{+31}_{-89}
Γ_{f_0} [MeV]	242^{+90}_{-88}
χ^2/d.o.f.	1.43
E_p [MeV]	$2288.4^{+11.7}_{-13.4} - 89.6^{+6.5}_{-6.4}i$

v2



Parameter	
Λ_1 [GeV]	1.60 ± 0.07
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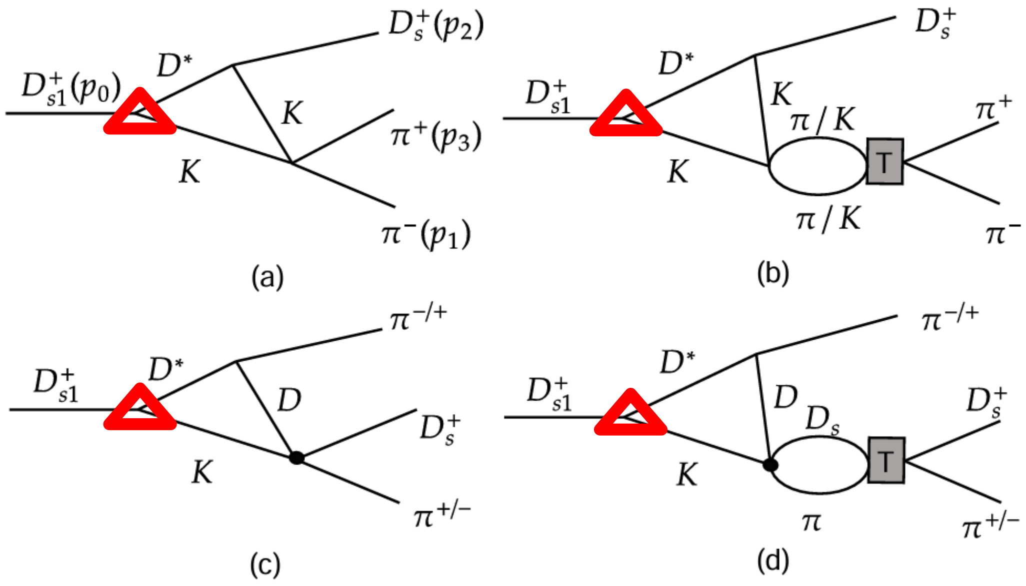
g_{K^*}	61.4 ± 2.4
r_1	7.1 ± 0.8
r_2	0.121 ± 0.014
χ^2/d.o.f.	1.43
E_p [MeV]	$2211.0^{+31.4}_{-22.6} - 154.5^{+20.7}_{-17.1}i$

J. A. Oller and E. Oset,
NPA 620, 438 (1997)

$D_{s1} \to D_s^+ \pi^+ \pi^-$



$$
\begin{aligned}
L_\mu &= P_{\mu\nu}(p_2+q, m_{D^*})(q-p_2)^\nu, \\
N_\mu &= P_{\mu\nu}(p_1+q, m_{D^*})(p_1-q)^\nu(p_2-q+2p_3)^\alpha \\
&\quad \times P_{\alpha\beta}(p_2-q, m_{K^*})(p_2+q)^\beta,
\end{aligned}
$$

$$
\begin{aligned}
i\mathcal{M}_a &= r_1 \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu}{[q^2-m_K^2][(q+p_2)^2-m_{D^*}^2][(p_0-p_2-q)^2-m_K^2]}, \\
i\mathcal{M}_b &= r_1 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu G_{\pi\pi/KK}(k, m_{23}) T_{\pi\pi/KK\to\pi\pi}(m_{13})}{[q^2-m_K^2][(q+p_2)^2-m_{D^*}^2][(p_0-p_2-q)^2-m_K^2]}, \\
i\mathcal{M}_c &= r_2 \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu}{[q^2-m_D^2][(q+p_1)^2-m_{D^*}^2][(p_0-p_1-q)^2-m_K^2]} + (p_1 \leftrightarrow p_3), \\
i\mathcal{M}_d &= r_2 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu G_{D_s\pi}(k, m_{23}) T_{D_s\pi\to D_s\pi}(\vec{k}, \vec{p}_{\rm on}, m_{23})}{[q^2-m_D^2][(q+p_1)^2-m_{D^*}^2][(p_0-p_1-q)^2-m_K^2]} + (p_1 \leftrightarrow p_3),
\end{aligned}
$$

Conclusion：

$D^*_{s0}(2317)$-DK
$D_{s1}(2460)$-D^*K — S-wave

$$\mathcal{M}_S = g_S \epsilon_i^\mu \epsilon_{j,\mu}^\dagger$$

$$\mathcal{M}_D = \frac{g_D}{M^2} \epsilon_i^\mu \epsilon_j^{\dagger\nu} \left(q_\mu q_\nu - \frac{g_{\mu\nu} q^2}{4}\right)$$

Mass moving vs GI Model

$D_{s1}(2460)$: $g_S \gg g_D$

$D_{s1}(2536)$-D^*K
$D_{s2}(2573)$-$D^{(*)}K$ — D-wave

$D_{s1}(2536)$: $\frac{g_S}{g_D} = 0.08\, e^{2.67i}$

LHCb，JHEP 10 (2023) 106 $\frac{g_S}{g_D} = 0.1\, e^{-0.7i}$

Mass stable vs GI model

Parameter	
Λ_1 [GeV]	1.60 ± 0.07
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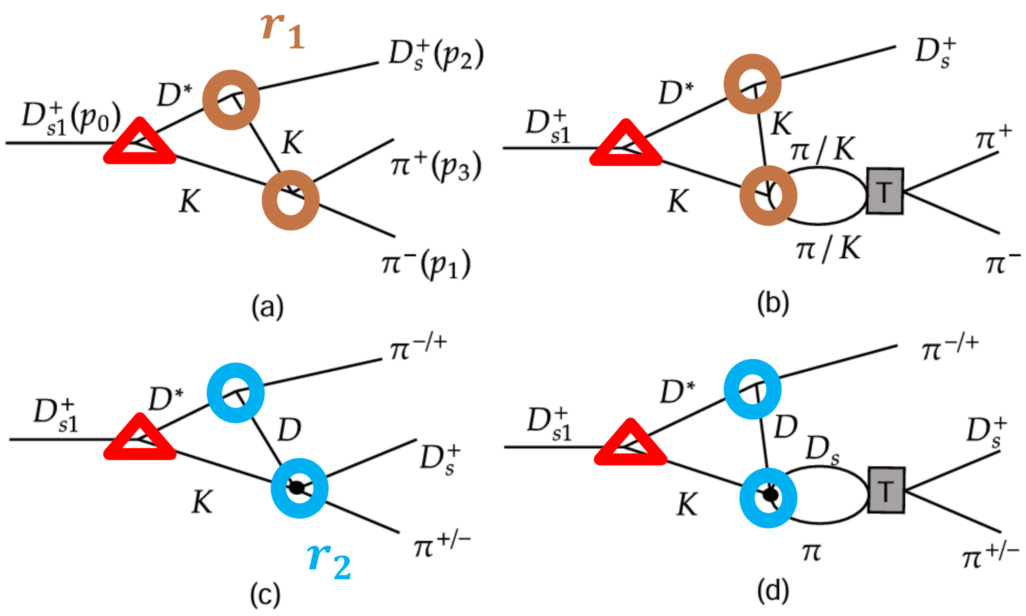
Elegant model.

1. **We successfully combine two D_{s1} states together.**
2. Various free parameters are combined together.
3. We seriously consider the T matrix of $D_s\pi - DK$ scattering.

$D_{s1} \to D_s^+ \pi^+ \pi^-$



$$\mathcal{M} = \mathcal{M}_a + e^{i\phi}(\mathcal{M}_b + \mathcal{M}_c)$$

$$\begin{aligned} L_\mu &= P_{\mu\nu}(p_2+q, m_{D^*})(q-p_2)^\nu, \\ N_\mu &= P_{\mu\nu}(p_1+q, m_{D^*})(p_1-q)^\nu (p_2-q+2p_3)^\alpha \\ &\quad \times P_{\alpha\beta}(p_2-q, m_{K^*})(p_2+q)^\beta, \end{aligned}$$

$$\begin{aligned} i\mathcal{M}_a &= \frac{r_1}{m_{13}^2 - m_{f_0}^2 + i m_{f_0}\Gamma_{f_0}} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu}{[q^2-m_K^2][(q+p_2)^2-m_{D^*}^2][(p_0-p_2-q)^2-m_K^2]}, \\ i\mathcal{M}_b &= r_2 \int \sqrt{\pi}\sin\theta d\theta \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu}{[q^2-m_D^2][(q+p_1)^2-m_{D^*}^2][(p_0-p_1-q)^2-m_K^2]} + (p_1 \leftrightarrow p_3), \\ i\mathcal{M}_c &= r_2 \int \frac{\sin\theta d\theta k^2 dk}{2\pi^{3/2}} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu G_{D_s\pi}(k, m_{23}) T_{D_s\pi\to D_s\pi}(k, p_{\rm on}, m_{23})}{[q^2-m_D^2][(q+p_1)^2-m_{D^*}^2][(p_0-p_1-q)^2-m_K^2]} + (p_1 \leftrightarrow p_3), \end{aligned}$$

Conclusion：

$D^*_{s0}(2317)$-DK

$D_{s1}(2460)$-D^*K

S-wave

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LHCb，JHEP 10 (2023) 106 $\quad \frac{g_S}{g_D} = 0.1\ e^{-0.7i}$

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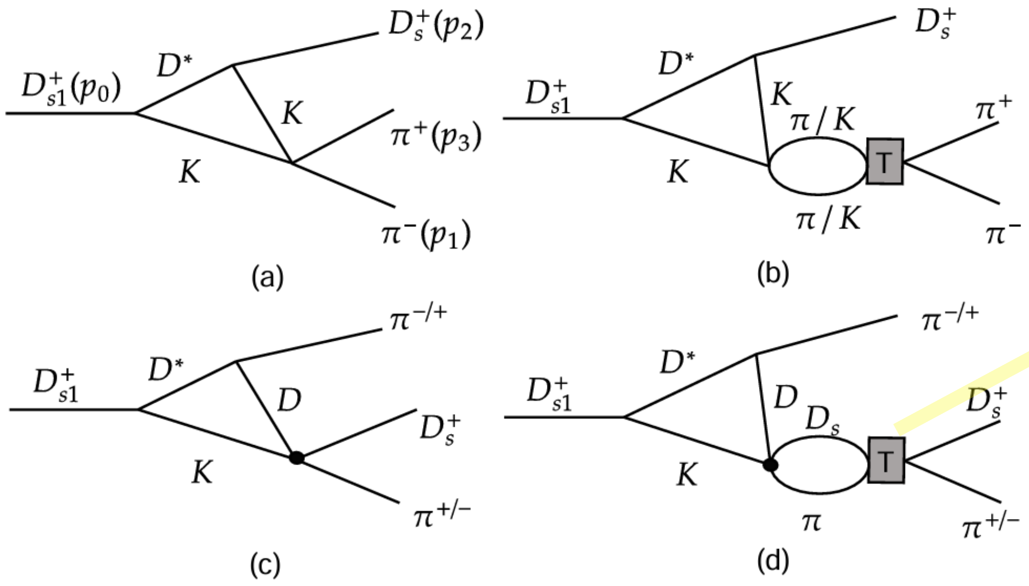
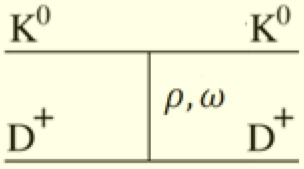
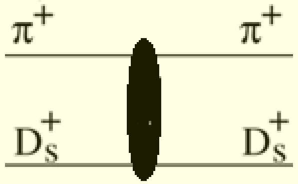
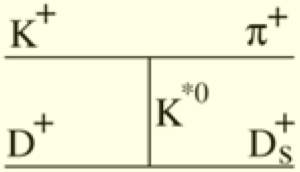




$I = 0, V_\rho + V_\omega < 0$
$I = 1, V_\rho - V_\omega \sim 0$



$V \sim 0$



$V \neq 0$

$T = (1 - VG)^{-1} V$
$\to \det T^{-1} \propto (1 - V^2 G_{DK} G_{D_s\pi})$

For DK, effective potential is $V^2 G_{D_s\pi}$
For $D_s\pi$, effective potential is $V^2 G_{DK}$

Control the value of cut off will adjust the effective potential.

$\Lambda_1, \Lambda_2, g_{K^*}$

$$T_{\alpha\beta}(\vec{p},\vec{p}';E) = \mathcal{V}_{\alpha\beta}(\vec{p},\vec{p}';E) + \sum_\gamma \int d\vec{q}\, \frac{\mathcal{V}_{\alpha\gamma}(\vec{p},\vec{q};E)T_{\gamma\beta}(\vec{q},\vec{p}';E)}{E - \sqrt{m_{\gamma_1}^2+q^2} - \sqrt{m_{\gamma_2}^2+q^2} + i\epsilon}$$

$$\mathcal{V} = \frac{g_{K^*}\,(p_\pi + p_K)\cdot(p_{D_s}+p_D)}{(p_\pi - p_K)^2 - m_{K^*}^2}\left(\frac{\Lambda_1^2}{\Lambda_1^2+p_1^2}\frac{\Lambda_2^2}{\Lambda_2^2+p_2^2}\right)^2$$

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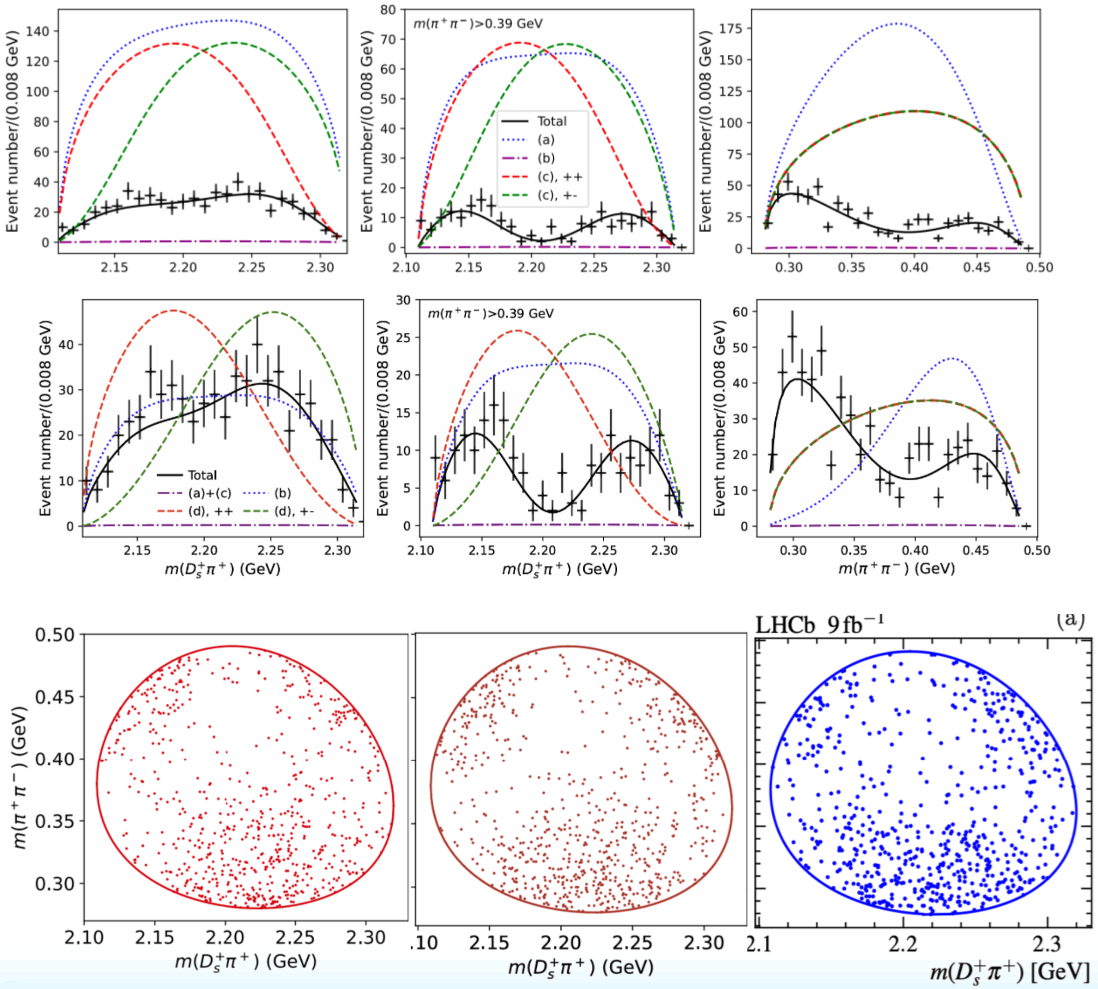
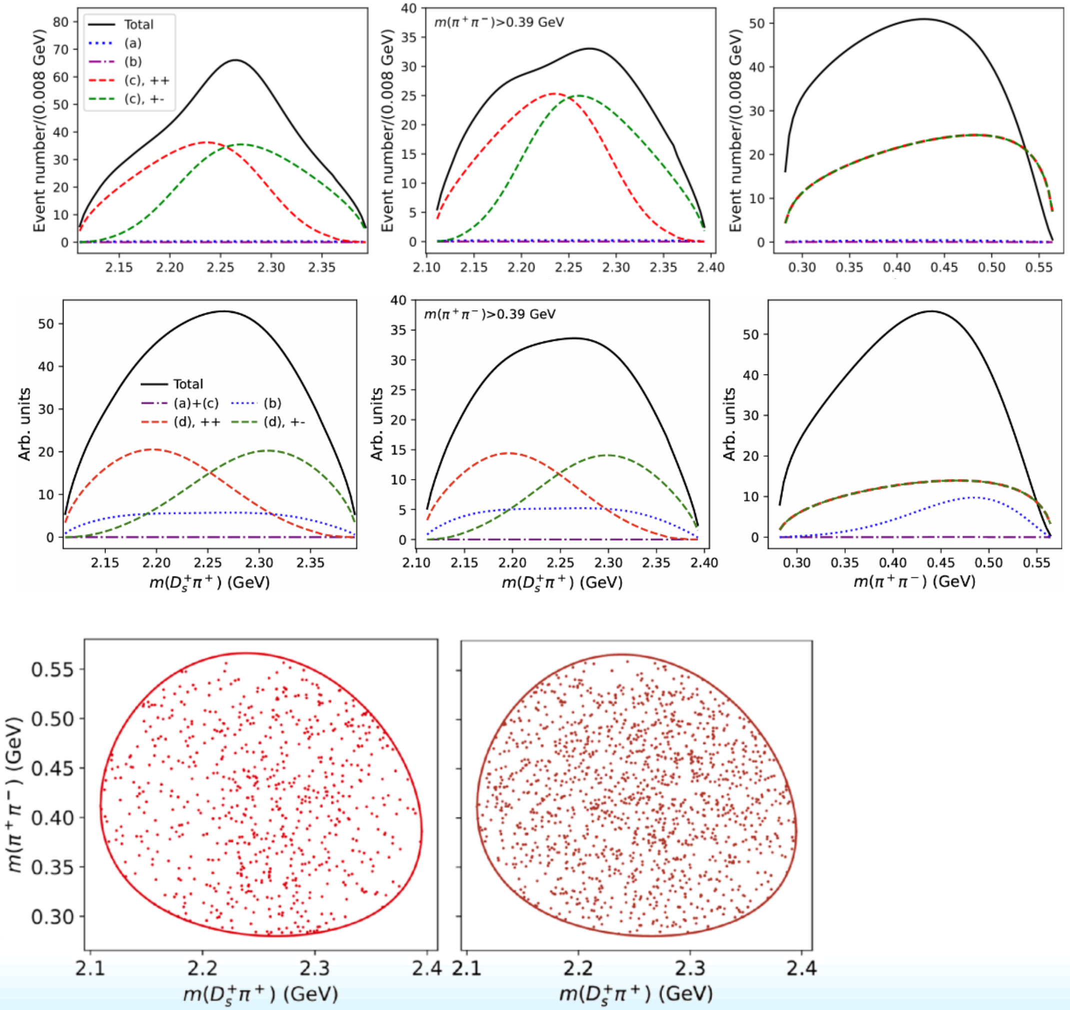
$D_{s1}(2460)$ χ^2/dof =1.43/



$D_{s1}(2536)$



Summary

- The strong interaction involves physics at two distinct levels and is extremely complex.
- We introduce the NonPerturbative Hamiltonian Framework (NPHF) approach, which connects the quark-gluon level to the hadronic level via a bare state, establishing a coupled-channel model with varying numbers of hadrons to achieve a comprehensive understanding of hadron properties.
- Using lattice QCD data, we explain why the masses of $D_{s0}(2317)$ and $D_{s1}(2460)$ are lower than predictions from conventional quark models, while those of $D_{s1}(2536)$ and $D_{s2}(2573)$ agree well.
- It further elucidates the $D_{s1}(2460) \to D_s\pi\pi$, reveals the existence of the $T_{c\bar{s}}$ state, and predicts the $D_{s1}(2536) \to D_s\pi\pi$.