上海交通大學
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Lattice QCD of baryon for STCF

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2026 Super Tau-Charm Facilities Workshop

Outline

Unitarity of CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

PDG 2024

First row:

$$|V_{ud}| = 0.97367 \pm 0.00032$$
$$|V_{us}| = 0.22431 \pm 0.00085$$
$$|V_{ub}| = 0.00382 \pm 0.00020$$

10^{-4} accuracy $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0007$

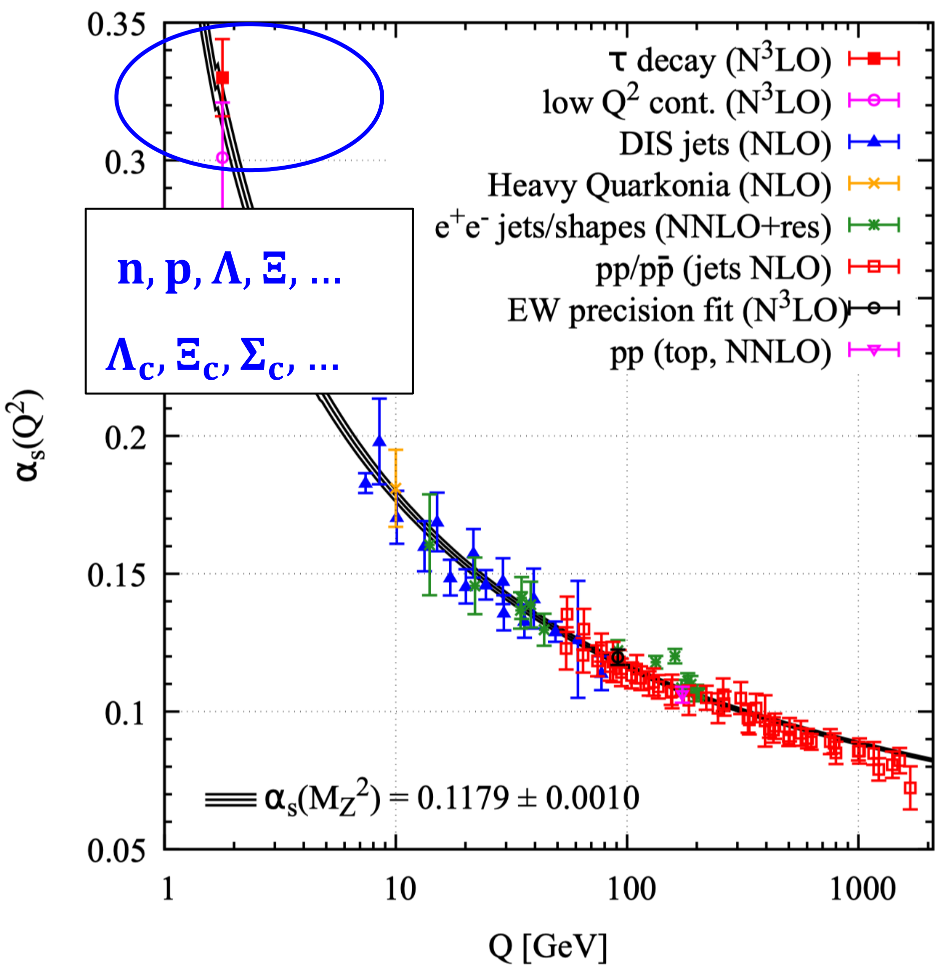
Second row:

$$|V_{cd}| = 0.221 \pm 0.004 \qquad 1.81\%$$
$$|V_{cs}| = 0.975 \pm 0.006 \qquad 0.62\%$$
$$|V_{cb}| = 0.0411 \pm 0.0012 \qquad 2.92\%$$

10^{-2} accuracy $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.001 \pm 0.012$

High precision determination of the second row provide strong tests of CKM unitarity, but this has to answered together with theoretical understanding!

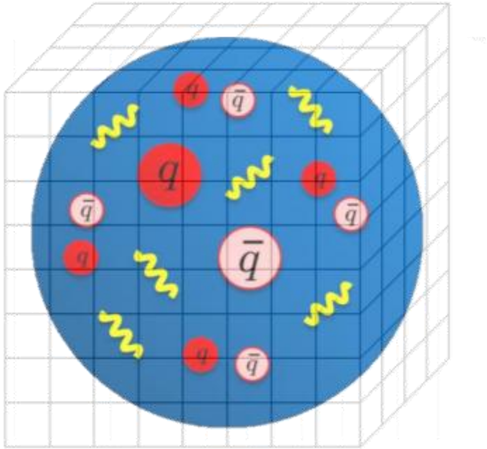
Tau-charm region： perturbative vs non-perturbative QCD



- Around the proton radius scale (~ 1 GeV), QCD cannot be solved analytically, and the theoretical calculation at this scale is one of the seven Millennium Prize Problems.
- The baryon of the Tau-charm region： is just in the region where the transition from perturbative to nonperturbative happens.

Lattice QCD

$$S_E^{latt} = \underbrace{\sum \frac{6}{g^2} ReTr(1 - U_P)}_{\text{wilson gauge action}} + \underbrace{\sum_q \bar{q}(D_\mu^{lat}\gamma_\mu + am_q)q}_{\text{lattice fermion action}}$$



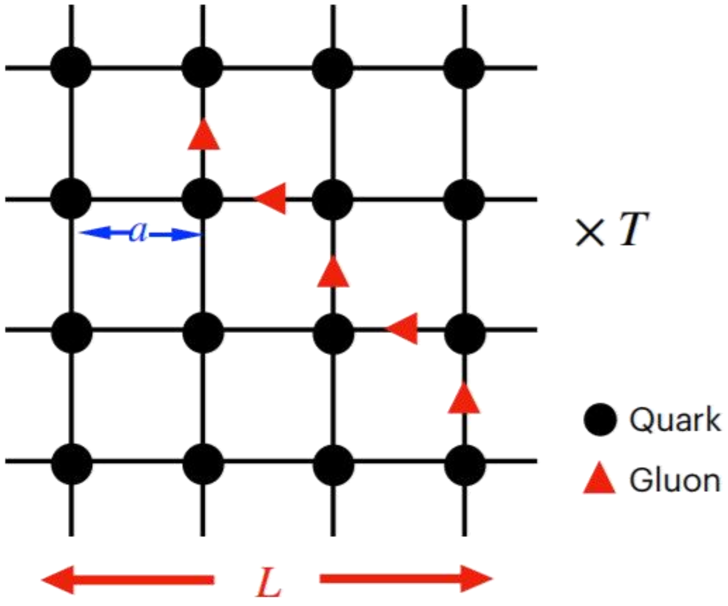
- **Charm quark on discrete lattice:**

consider both IR and UV effects:

$$m_\pi L \gtrsim 4, \quad \text{and} \quad a^{-1} \gg \text{mass scale}$$

for $m_\pi = m_\pi^{\text{phy}} \sim 140\text{MeV}$, **and** $m_c \simeq 1.3\text{GeV}$,

$$L \gtrsim 5.6\text{fm}$$
$$a^{-1} \gg 1.3\text{GeV} \simeq (0.15\text{fm})^{-1}$$

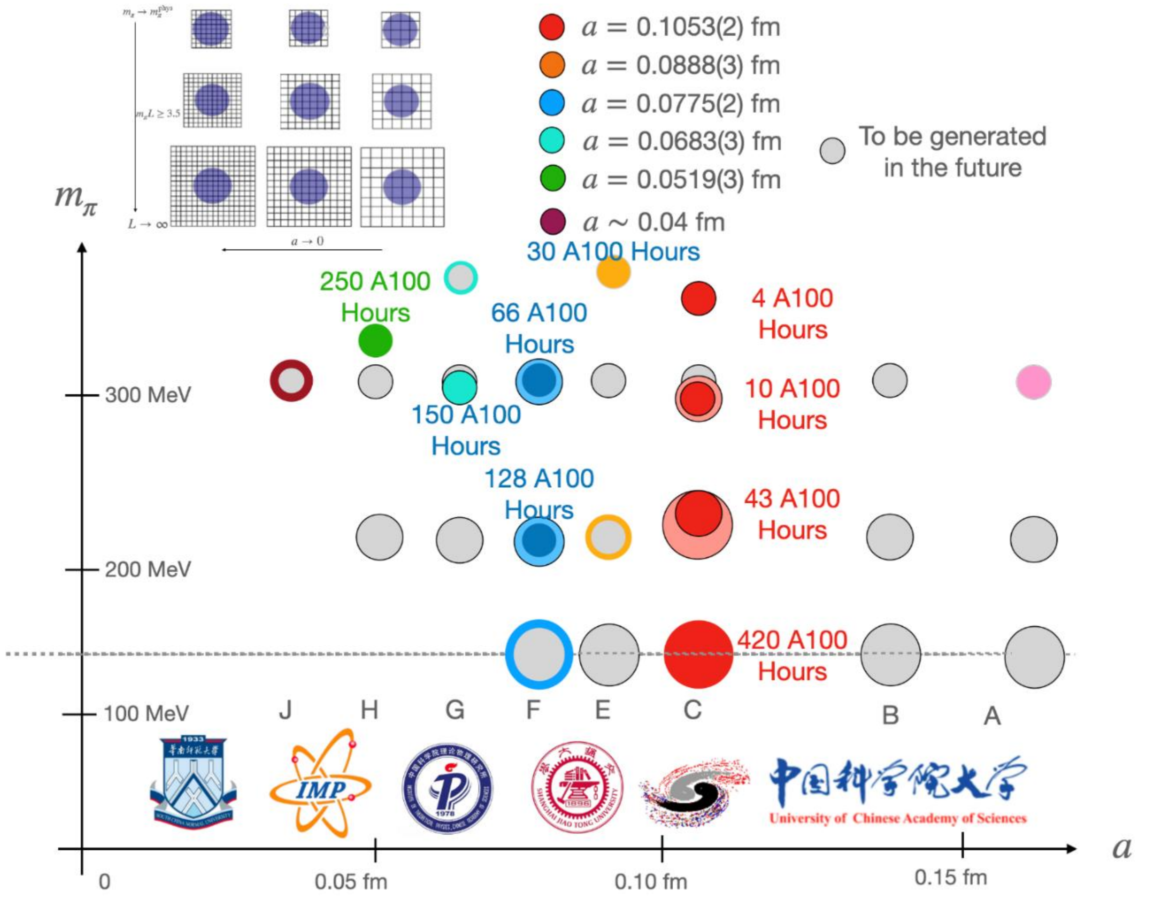


Chinese Lattice QCD configurations

Hu, et. al., CLQCD, PRD109, 054507 (2024)

✓ **2+1 flavor full-QCD ensembles with stout smeared clover fermion action and Symanzik gauge actions**

- **14 ensembles with more than 5000 configurations in total;**
- **5 lattice spacings from ~ 0.05 fm to 0.11 fm;**
- **7 pion masses from ~ 130 MeV to 350 MeV;**
- **4 Volumes from ~ 2.5 fm to 5.0 fm.**



☐ **Ensembles with another setup are preparing for better control on systematic uncertainties.**

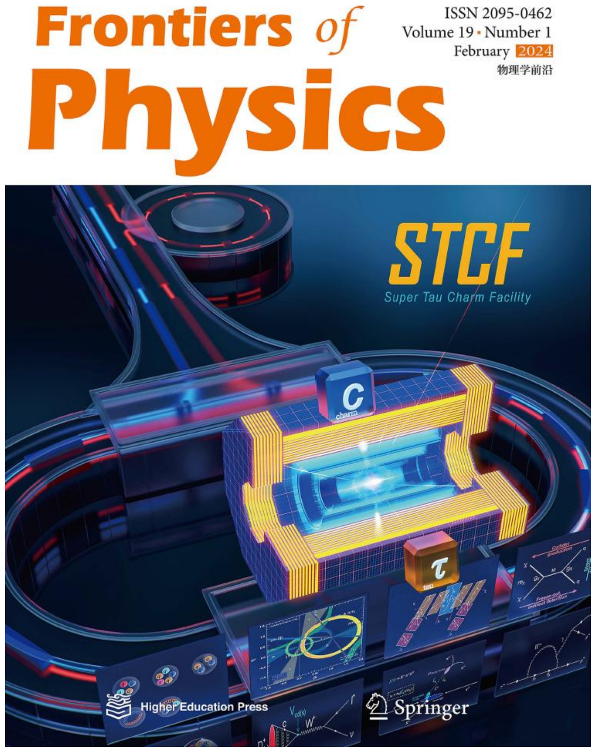
Baryon events produced by STCF

Branching fractions for some J/ψ; $\psi(3686)\to B\bar{B}$ decay form STCF

Decay mode	$\mathcal{B}$ (in units of 10^{-4})	Angular distribution parameter α_ψ	Detection efficiency	No. of events expected at the STCF
$J/\psi\to\Lambda\bar{\Lambda}$	$19.43\pm0.03\pm0.33$	0.469 ± 0.026	40%	1100×10^6
$\psi(3686)\to\Lambda\bar{\Lambda}$	$3.97\pm0.02\pm0.12$	0.824 ± 0.074	40%	130×10^6
$J/\psi\to\Xi^0\bar{\Xi}^0$	11.65 ± 0.04	0.66 ± 0.03	14%	230×10^6
$\psi(3686)\to\Xi^0\bar{\Xi}^0$	2.73 ± 0.03	0.65 ± 0.09	14%	32×10^6
$J/\psi\to\Xi^-\bar{\Xi}^+$	10.40 ± 0.06	0.58 ± 0.04	19%	270×10^6
$\psi(3686)\to\Xi^-\bar{\Xi}^+$	2.78 ± 0.05	0.91 ± 0.13	19%	42×10^6

The expected numbers of events per year at different STCF at 4.630 GeV.

$\psi(3686)\pi^+\pi^-$	0.033	3.3×10^7	
$\Lambda_c\bar{\Lambda}_c$	0.56	5.6×10^8	
$\Lambda_c\bar{\Lambda}_c$		6.4×10^7	Single tag
$\tau^+\tau^-$	3.4	3.4×10^9	



STCF, Front. Phys. 19, 14701 (2024)

High precision measurement in STCF

High precision calculation in LQCD

Outline

$\Lambda \to p\ell\nu$ semi-leptonic decay

Experimental measurement：

	BESIII	LHCb	STCF
$Br(\Lambda \to p\mu^-\bar{\nu}_\mu)$	$(1.48\pm0.21)\times10^{-4}$	$(1.46\pm0.10)\times10^{-4}$	-
$Br(\Lambda \to pe^-\bar{\nu}_e)$	$(8.16\pm0.27)\times10^{-4}$	-	$(8.32\pm0.07)\times10^{-4}$
$\lvert V_{us}\rvert_\mu$	-	0.235 ± 0.016	-
$\lvert V_{us}\rvert_e$	0.2332 ± 0.0042	-	0.9%

BESIII : PhysRevLett.127.121802 (2021)

BESIII : arXiv: 2509.09266

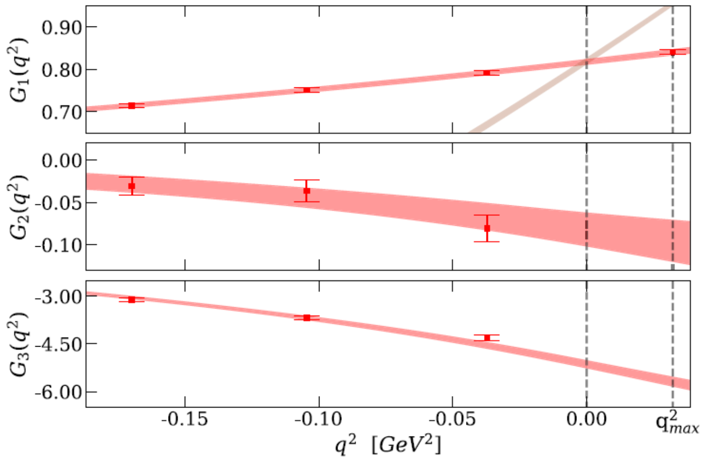
STCF : Eur. Phys. J. C 85, 1408 (2025)

LHCb : arXiv: 2511.15681

Lattice QCD calculation：

S. Bacchio, et al. Phys. Rev. Lett. 135, 231901 (2026)

Ensemble	Abrv.	V/a^4	β	a [fm]	m_π [MeV]
cB211.072.64	B64	$64^3\times128$	1.778	0.07957(13)	140.2(2)



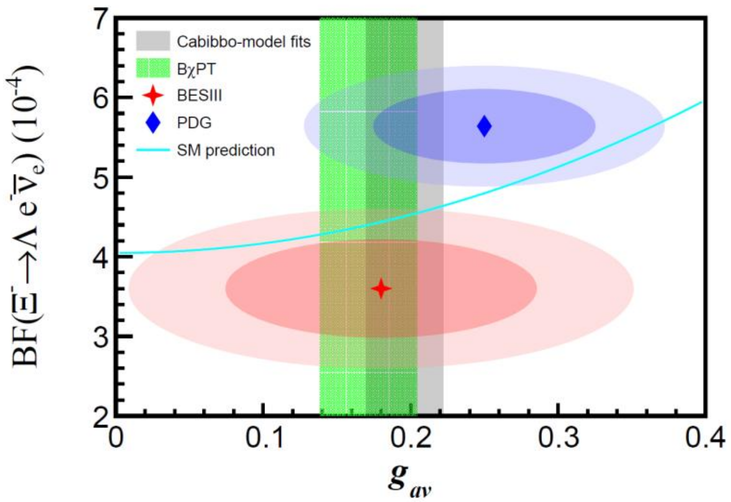
$\Xi \to \Lambda \ell \nu$ semi-leptonic decay

Branching fraction $\Xi \to \Lambda e \bar{\nu}_e$:

BESIII：$\left(3.60 \pm 0.40_{stat} \pm 0.10_{syst}\right) \times 10^{-4}$

3.9σ

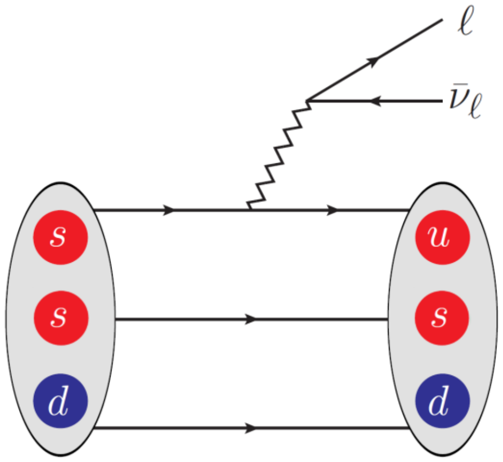
PDG2024：$\left(5.63 \pm 0.40_{stat} \pm 0.31_{syst}\right) \times 10^{-4}$



BESIII, arXiv: 2512.15273

Lattice QCD calculation：

$$
\begin{aligned}
&\langle \Lambda(p_f, s_f) | \bar{u} \gamma_\mu (1-\gamma_5) s | \Xi(p_i, s_i) \rangle \\
=&\bar{u}_\Lambda(p_f)\left[\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu} q^\nu \frac{F_2(q^2)}{m_\Xi} + q_\mu \frac{F_3(q^2)}{m_\Xi}\right] u_\Xi(p_i) \\
-&\bar{u}_\Lambda(p_f)\left[\gamma_\mu G_1(q^2) + i\sigma_{\mu\nu} q^\nu \frac{G_2(q^2)}{m_\Xi} + q_\mu \frac{G_3(q^2)}{m_\Xi}\right] \gamma_5 u_\Xi(p_i)
\end{aligned}
$$



In collaboration with：Z.F. Liu, Y. Meng, G.Y. Wang, W. Wang, Q.A. Zhang, F.W. Zhang

Hyperon: Lattice set-up

The three point correlation functions of hyperon：

$$C^{3\mathrm{pt}}_{\Xi\to\Lambda}(t,t_{seq},\vec{p},\vec{q})T=\sum_{\vec{x},\vec{y}}e^{-i\vec{p}\cdot\vec{x}}e^{-i\vec{q}\cdot\vec{y}}\langle O_{\Lambda,C}(x),J(y),\bar{O}_{\Xi,J}(0)\rangle T_{JC}$$

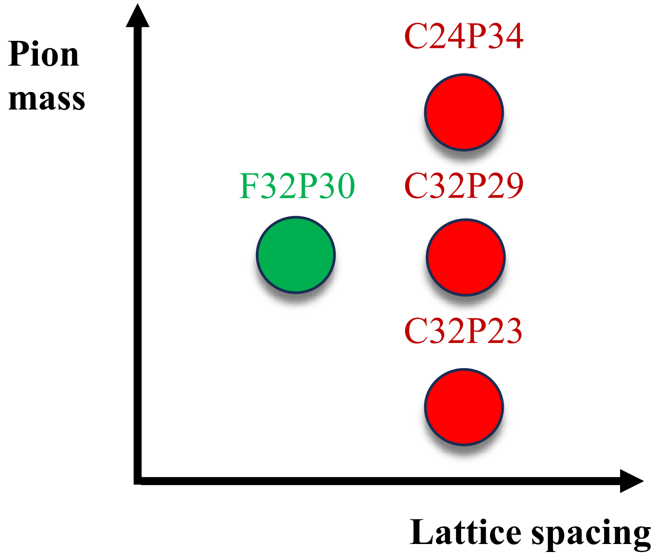
where

$$O_{\Xi^-}=\epsilon_{ijk}(d_i^T C\gamma_5 s_j)P_+ s_k,$$

$$O_{\Lambda}=\epsilon_{ijk}(u_i^T C\gamma_5 d_j)s_k,$$

$$J=\bar{u}\gamma_\mu(1-\gamma_5)s.$$

Ensemble	$L^3\times T$	a (fm)	smear size	Pion (MeV)	Nconf	Nsrc
C24P34	$24^3\times 64$	0.105	4 30	340	300	10
C32P29	$32^3\times 64$	0.105	4 30	290	984	2
C32P23	$32^3\times 64$	0.105	4 30	230	954	2
F32P30	$32^3\times 96$	0.077	5 50	300	772	2



Reduction formula

momentum： $P_1 = (m_1, 0, 0, 0), P_2 = (E_2, 0, 0, p)$

$$C^{3pt}_{\Xi\to\Lambda}(t, t_{seq}, \vec{p}, \vec{q})T = \sum_{\vec{y}} e^{-i\vec{q}\cdot\vec{y}} \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}$$

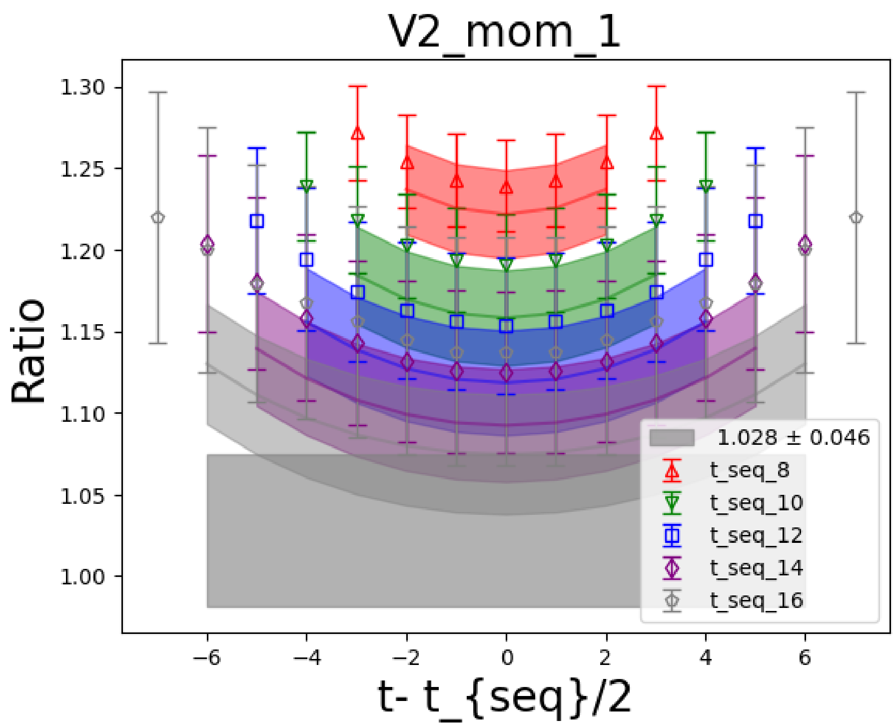
$$\times \langle O_\Lambda(x), \overline{u}(y)\underline{\Gamma} s(y), \overline{O}_\Xi(0)\rangle \underline{T}$$

current projector

mom$=(0,0,p)$	$\Gamma=\gamma_x$	$\Gamma=\gamma_y$	$\Gamma=\gamma_z$	$\Gamma=\gamma_t$
$\mathrm{T}=1$	0	0	V_1	V_2
$\mathrm{T}=\gamma^x$	V_3	0	0	0
$\mathrm{T}=\gamma^y$	0	V_3	0	0
$\mathrm{T}=\gamma^x\gamma^y$	0	0	0	0
$\mathrm{T}=\gamma^z$	0	0	V_4	V_5
$\mathrm{T}=\gamma^x\gamma^z$	V_6	0	0	0
$\mathrm{T}=\gamma^y\gamma^z$	0	V_6	0	0
$\mathrm{T}=\gamma^5\gamma^t$	0	0	0	0

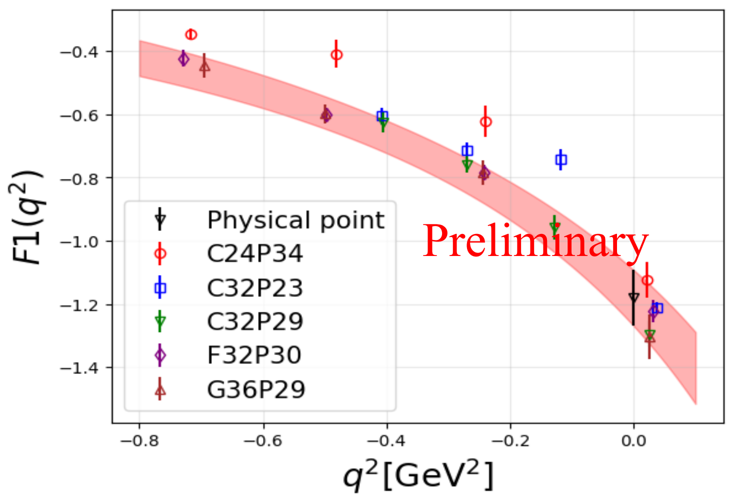
Linear combination of form factors：

$$V_i = AF_1(q^2) + BF_2(q^2) + CF_3(q^2), \qquad i = 1\sim 6$$

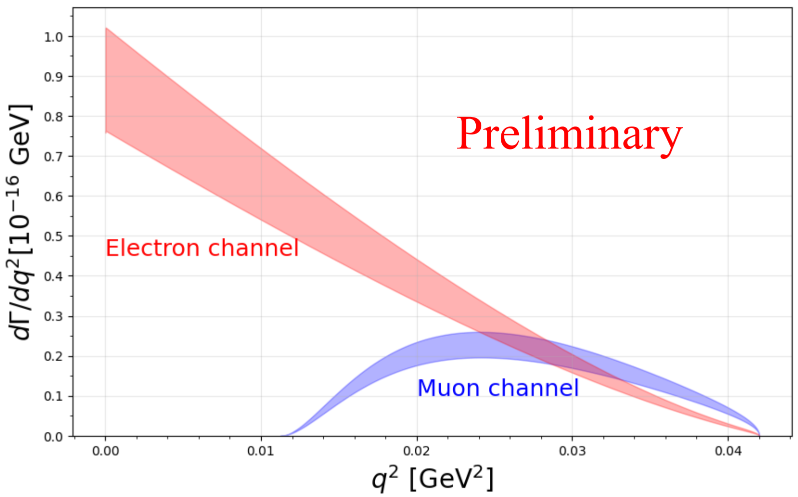


The ratio of 3pt/2pt

Preliminary result of $\Xi \to \Lambda \ell \nu$



vector form factor F1

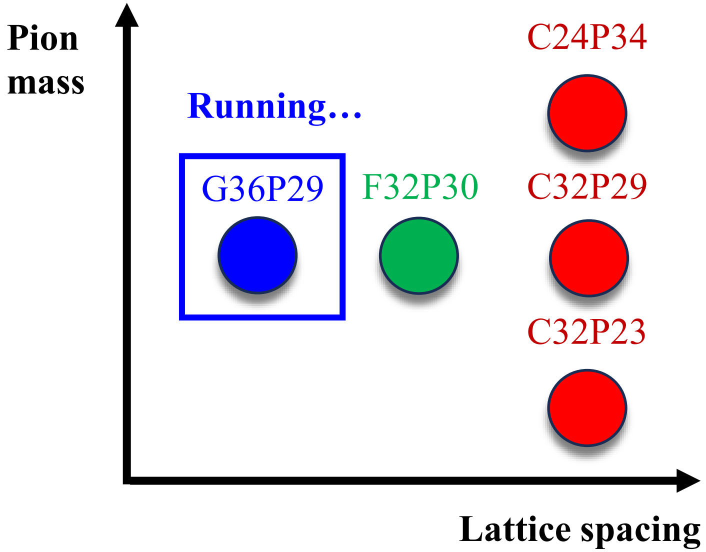


Differential decay width

The next :

1、New ensemble G36P29

2、Chiral and continuum limit extrapolations

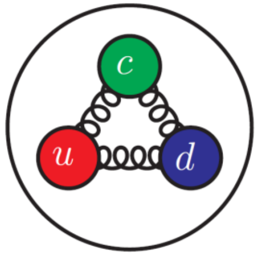


Outline

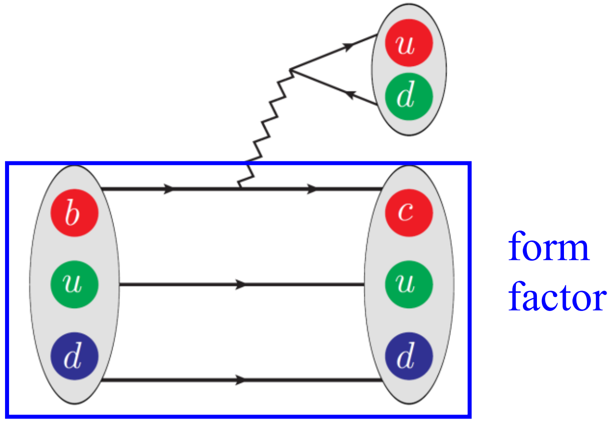
- ✓ **Charmed meson:**
  - ✓ **Decay constant:** H.Y. Du, el.al, PRD 111,054504 (2025)
  - ✓ $\boldsymbol{D_s \to \phi\ell\nu}$**:** G.F. Fan, et.al, 2510.14478 (2025)
  - ✓ $\boldsymbol{D_s^* \to D_s\gamma}$**:** Y. Meng, et.al, PRD 109, 074511 (2024)

- ✓ **Charmed baryon:**
  - ✓ **Decay constants:** L.Y. Li, et.al, Phys.Rev.D 113 (2026), 054509
  - ✓ $\boldsymbol{\Xi_c \to \Xi\ell\nu}$ **:** Q.A. Zhang, et.al, CPC 46, 011002 (2022),
  - ✓ $\boldsymbol{\Xi_c - \Xi_c'}$ **mixing:** H. Liu, et.al, PLB 841,137941 (2023), PRD109, 036037 (2024)

Charmed baryon: Decay Constants



Charmed baryon Λ_c



$\Lambda_b \to \Lambda_c \pi$ two-body decay

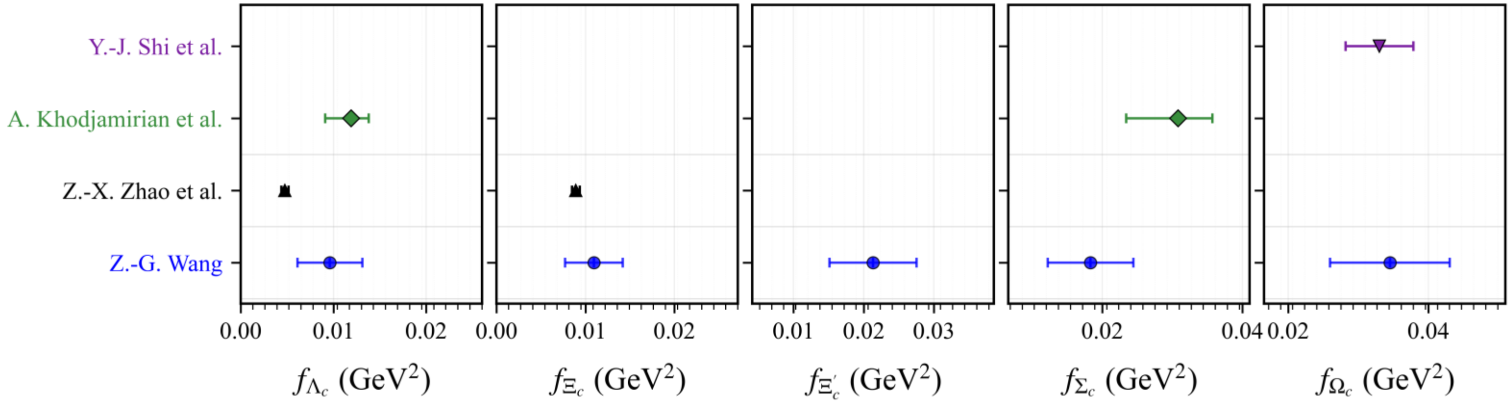
$\Lambda_b \to \Lambda_c$ form factor：

$$\langle \Lambda_c | iA | \Lambda_b \rangle \propto \langle \Lambda_c | O_{\Lambda_c} | 0 \rangle \propto f_{\Lambda_c}$$

The definition of decay constants：

$$\langle 0 | \epsilon_{ijk} (u_i^T C \gamma_5 d_j) c_k | \Lambda_c \rangle = m_{\Lambda_c} f_{\Lambda_c} u_{\Lambda_c}$$

Only QCD sum rule calculation：



The decay constants of charmed baryons have yet to be determined from first principles.

Interpolating operator

SU(3) symmetry of di-quark：

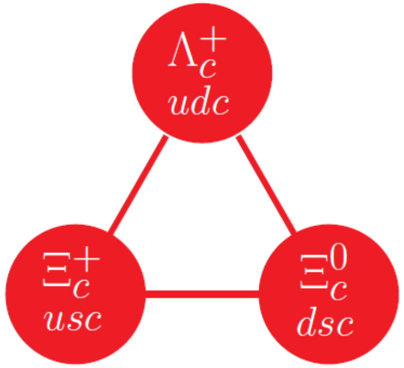
$3 \otimes 3 = \bar{3} \oplus 6$

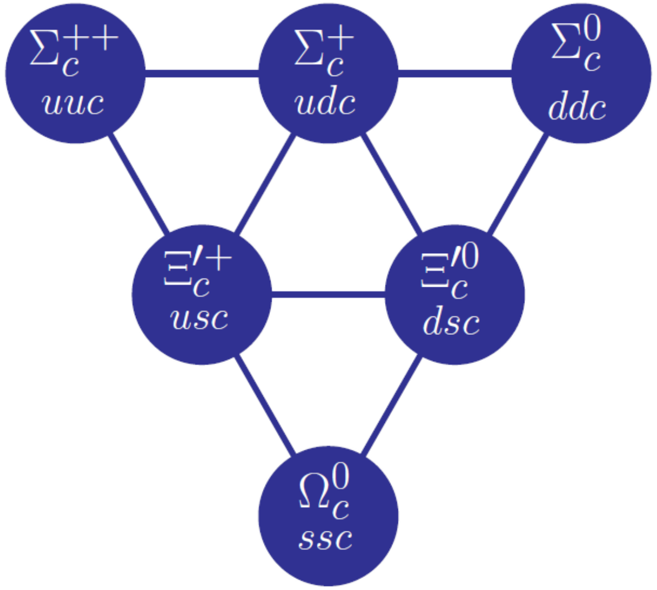
$$\bar{3}: \begin{cases} O_{\Lambda_c^+} = \epsilon_{ijk}(u_i^T C\gamma_5 d_j)c_k, \\ O_{\Xi_c^+} = \epsilon_{ijk}(u_i^T C\gamma_5 s_j)c_k, \\ O_{\Xi_c^0} = \epsilon_{ijk}(d_i^T C\gamma_5 s_j)c_k, \end{cases}$$

Pseudoscalar current

$$6: \begin{cases} O_{\Xi_c'^+} = \epsilon_{ijk}(u_i^T C\gamma_\mu s_j)\gamma^\mu\gamma_5 c_k, \\ O_{\Xi_c'^0} = \epsilon_{ijk}(d_i^T C\gamma_\mu s_j)\gamma^\mu\gamma_5 c_k, \\ O_{\Sigma_c^+} = \epsilon_{ijk}(u_i^T C\gamma_\mu d_j)\gamma^\mu\gamma_5 c_k, \\ O_{\Sigma_c^0} = \frac{1}{\sqrt{2}}\epsilon_{ijk}(d_i^T C\gamma_\mu d_j)\gamma^\mu\gamma_5 c_k, \\ O_{\Sigma_c^{++}} = \frac{1}{\sqrt{2}}\epsilon_{ijk}(u_i^T C\gamma_\mu u_j)\gamma^\mu\gamma_5 c_k, \\ O_{\Omega_c^0} = \frac{1}{\sqrt{2}}\epsilon_{ijk}(s_i^T C\gamma_\mu s_j)\gamma^\mu\gamma_5 c_k. \end{cases}$$

Vector current



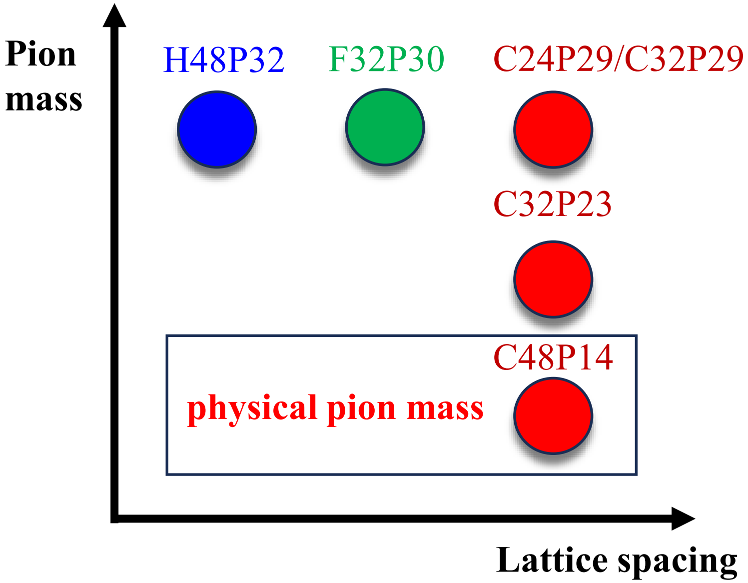


A. Khodjamirian, et al., JHEP 09, 106 (2011).

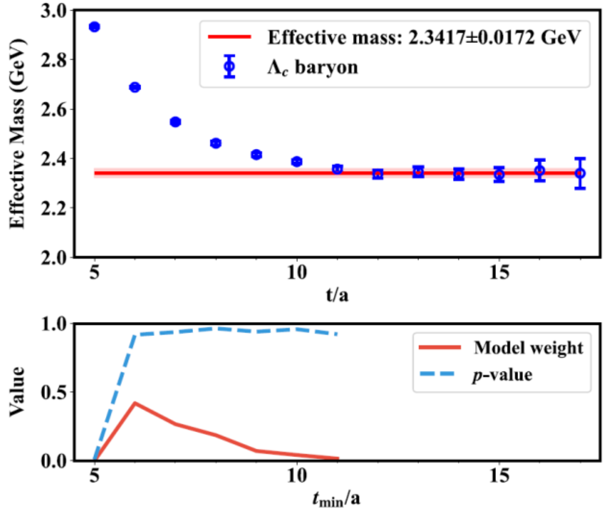
H. Liu, et al., Phys. Lett. B 841, 137941 (2023)

Charmed baryon: Lattice set-up

- Four lattice spacings [0.052 - 0.105] fm → continuum limit
- $m_\pi = 135 - 320$ MeV → physical pion mass

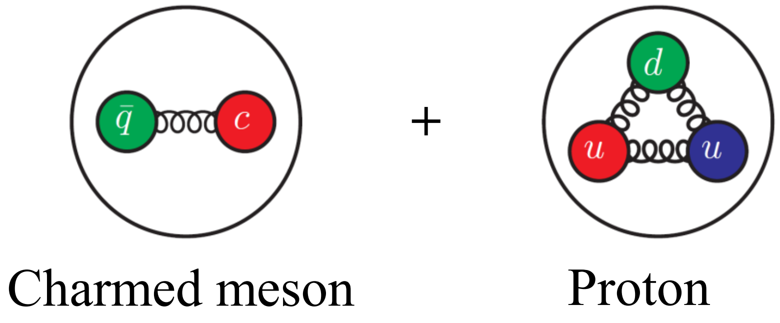


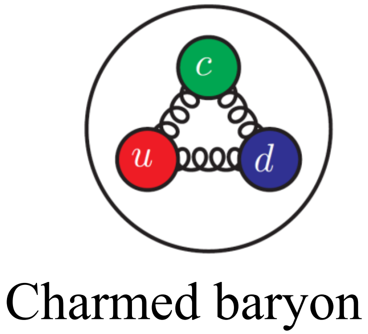
Ensemble	$L^3 \times T$	a (fm)	Pion (MeV)	Nconf	Nsrc
C24P29	$24^3 \times 72$	0.105	290	864	2×10
C32P29	$32^3 \times 64$	0.105	290	984	2×10
C48P14	$48^3 \times 96$	0.105	135	187	2×20
C32P23	$32^3 \times 64$	0.105	230	451	2×10
F32P30	$32^3 \times 96$	0.077	300	777	2×10
H48P32	$48^3 \times 144$	0.052	320	550	2×6

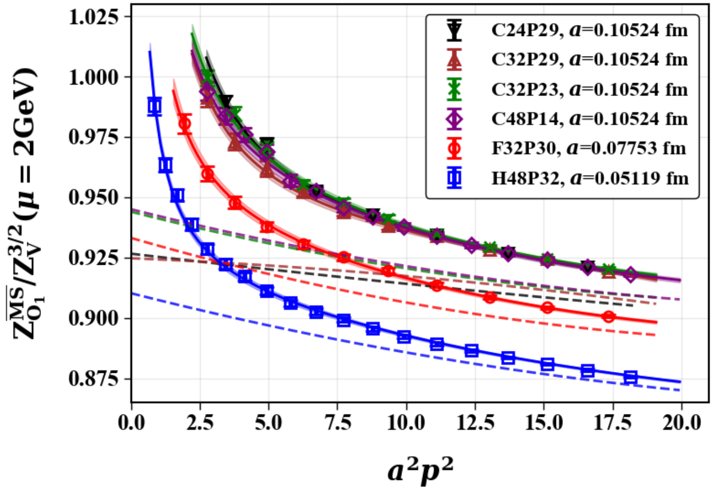


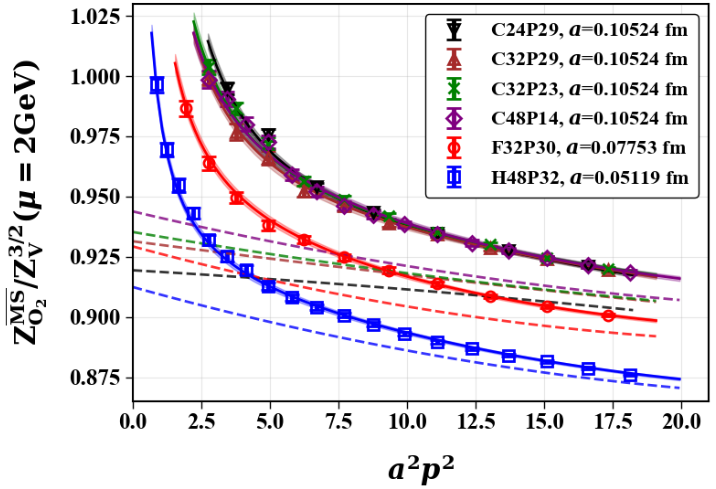
Non-perturbation renormalization

Renormalization on the lattice QCD:









Renormalization condition：

$$Z_{O_i} Z_{q1}^{-1/2} Z_{q2}^{-1/2} Z_{q3}^{-1/2} \Gamma_{O_i}|_{\mathrm{SMOM}_{\gamma_\mu}/\mathrm{SYM3q}} = 1$$

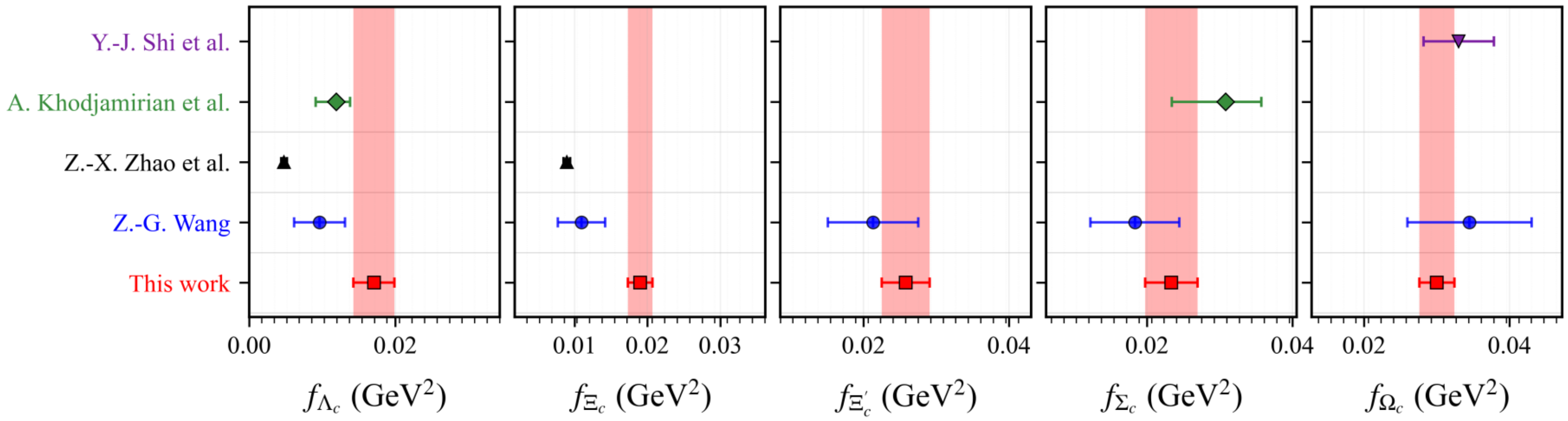
L.Y. Li, et.al, Phys.Rev.D 113 , 054509 (2026)

Numerical result of decay constants

Charmed baryon renormalization constants:

Ensemble	$Z^{\overline{\mathrm{MS}}}_{\Lambda_c}$	$Z^{\overline{\mathrm{MS}}}_{\Xi_c}$	$Z^{\overline{\mathrm{MS}}}_{\Xi'_c}$	$Z^{\overline{\mathrm{MS}}}_{\Sigma_c}$	$Z^{\overline{\mathrm{MS}}}_{\Omega_c}$
C24P29	0.9277(82)	0.9584(84)	0.9510(85)	0.9205(82)	0.9824(88)
C32P29	0.9252(91)	0.9557(95)	0.963(10)	0.9319(98)	0.994(10)
C32P23	0.9477(85)	0.9791(88)	0.9701(84)	0.9389(81)	1.0023(86)
C48P14	0.9481(86)	0.9796(89)	0.9783(80)	0.9469(77)	1.0108(83)
F32P30	0.8908(56)	0.9085(57)	0.9048(51)	0.8872(50)	0.9228(52)
H48P32	0.8400(27)	0.8492(28)	0.8513(26)	0.8420(26)	0.8607(27)

Charmed baryon decay constants:



L.Y. Li, et.al., Phys.Rev.D 113 , 054509 (2026)

Charmed Baryon Ξ_c Decays

$\Xi_c^0\,(dsc) \to \Xi^-(dss)\ell^+\nu_\ell$: Recent progresses

- **Experimental measurement: (In 2021)**

$$\frac{\mathcal{B}_{\rm Belle}(\Xi_c^0\to\Xi^-e^+\nu_e)}{\mathcal{B}_{\rm Belle}(\Xi_c^0\to\Xi^-\pi^+)} = (0.730\pm0.021\pm0.033)\%$$

$$\frac{\mathcal{B}_{\rm ALICE}(\Xi_c^0\to\Xi^-e^+\nu_e)}{\mathcal{B}_{\rm ALICE}(\Xi_c^0\to\Xi^-\pi^+)} = (1.38\pm0.14\pm0.22)\%$$

combining with a normalization mode: (in 2019)

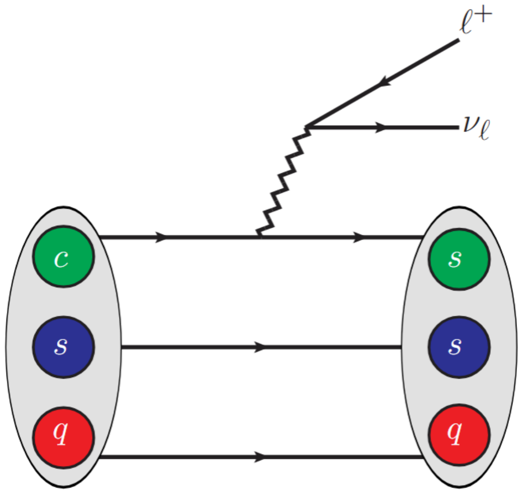
$$\mathcal{B}_{\rm Belle}(\Xi_c^0\to\Xi^-\pi^+) = (1.80\pm0.50\pm0.14)\%$$

giving the absolute branching fractions:

$$\mathcal{B}_{\rm Belle}(\Xi_c^0\to\Xi^-e^+\nu_e) = (1.31\pm0.04\pm0.07\pm0.38)\%$$
$$\mathcal{B}_{\rm ALICE}(\Xi_c^0\to\Xi^-e^+\nu_e) = (2.48\pm0.25\pm0.40\pm0.72)\%$$

- **PDG average: (In 2024)**

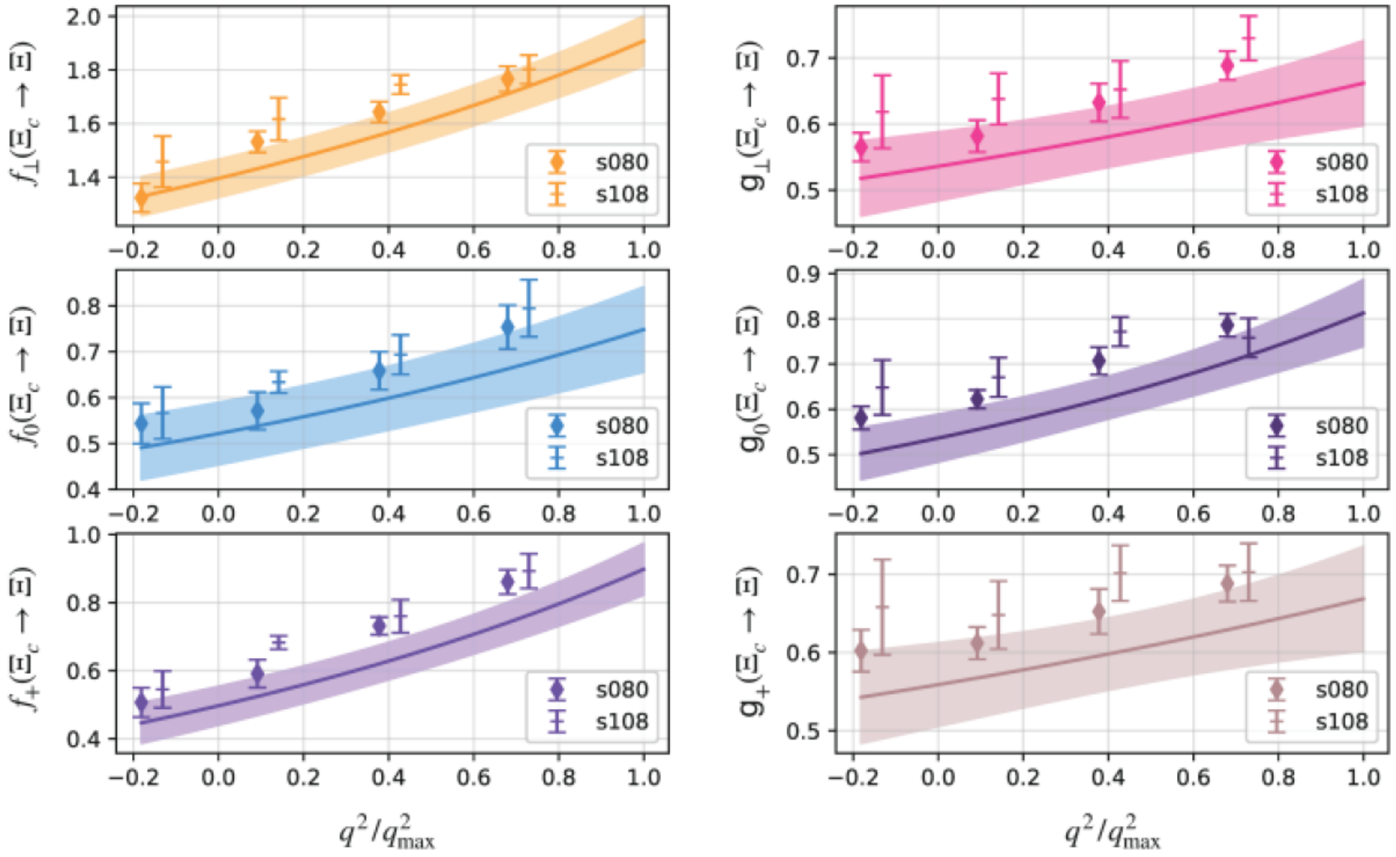
$$\mathcal{B}_{\rm PDG}(\Xi_c^0\to\Xi^-e^+\nu_e) = (1.05\pm0.20)\%$$



Belle Collaboration, PRL127, 121803 (2021)
ALICE Collaboration, PRL127, 272001 (2021)
Belle Collaboration, PRL122, 082001 (2019)
PDG Collaboration, PRD110, 030001 (2024)

Lattice QCD results in 2022:

Q.A. Zhang, et.al, CPC46, 011002 (2022)



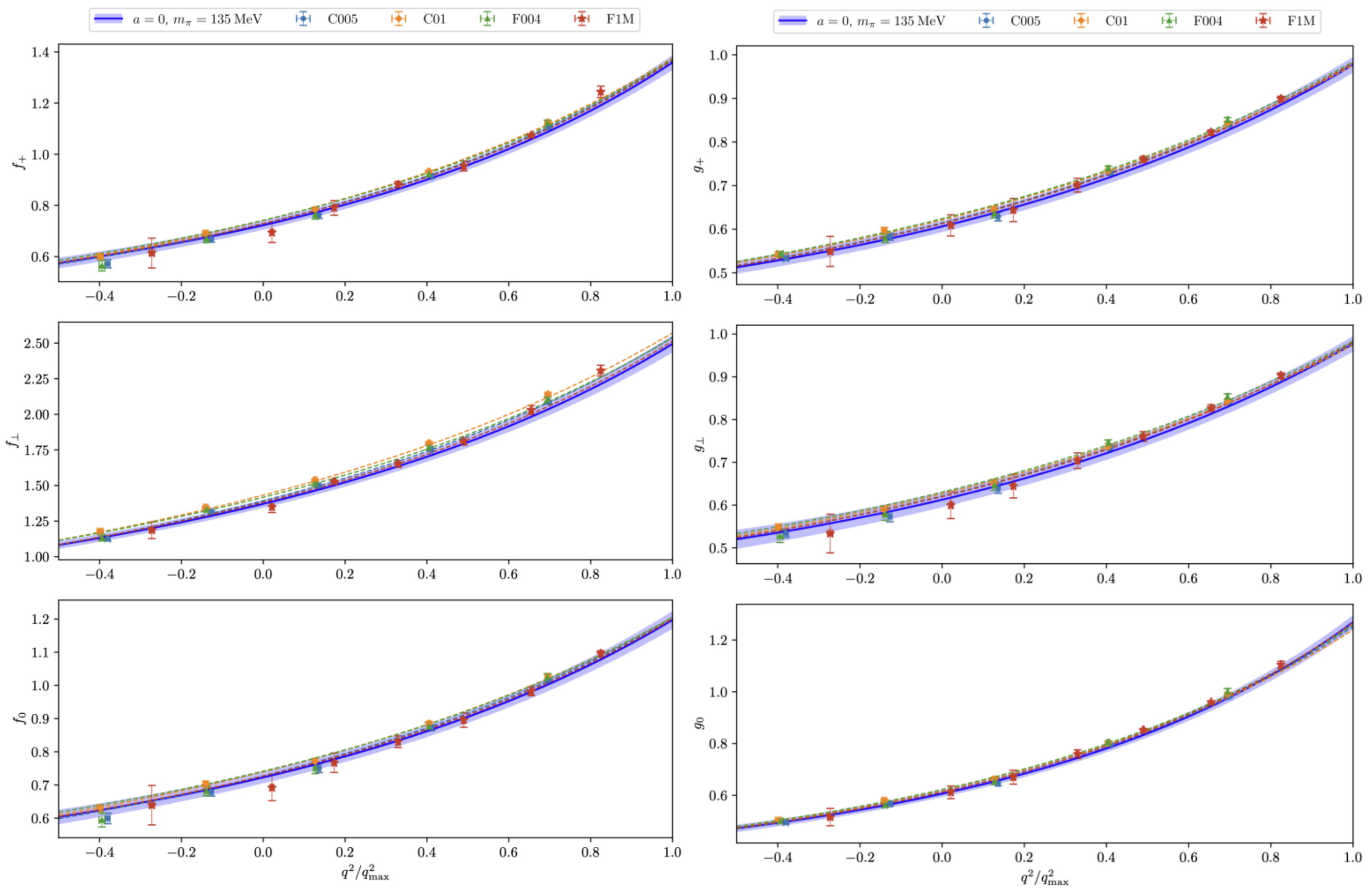
Ensembles by CLQCD collaboration:

	$\beta=10/g^2$	$L^3\times T$	a	$c_{\rm sw}$	κ_l	m_π	κ_s	m_{η_s}
s108	6.20	$24^3\times72$	0.108	1.161	−0.2770	285	0.1330	640
s080	6.41	$32^3\times96$	0.080	1.141	−0.2295	295	0.1318	650

- 2+1 flavor stout smeared clover fermion + Symanzik gauge actions
- 2 lattice spacings @ $a = 0.105$fm and 0.077fm;
- Valence quark mass @ m_π~300MeV m_{η_s}~650MeV;
- Charm quark tuned by physical J/ψ mass;
- Measurements: 18,000 ~ 31,000.

Lattice QCD results in 2025:

C. Farrell et al., Phys. Rev. D 111, 114521 (2025)



Ensembles by RBC&UKQCD collaborations:

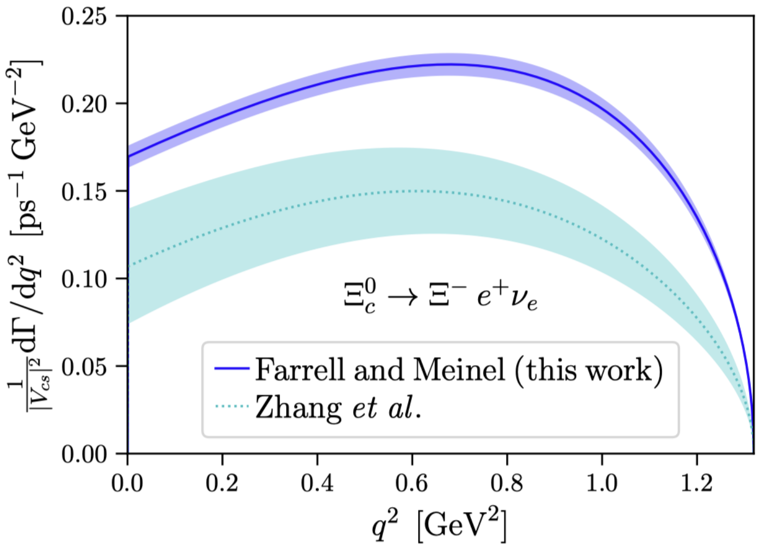
Label	$N_s^3 \times N_t \times N_5$	a [fm]	$am_{u,d}$	$am_s^{(\mathrm{sea})}$	$am_s^{(\mathrm{val})}$	m_π [MeV]	N_{meas}
C01	$24^3 \times 64 \times 16$	0.111	0.01	0.04	0.0323	420	2264 sl, 283 ex
C005	$24^3 \times 64 \times 16$	0.111	0.005	0.04	0.0323	340	2488 sl, 311 ex
F004	$32^3 \times 64 \times 16$	0.083	0.004	0.03	0.0248	300	2008 sl, 251 ex
F1M	$48^3 \times 96 \times 12$	0.073	0.002144	0.02144	0.02217	230	1808 sl, 113 ex

- 2+1 flavor domain-wall fermion + Iwasaki gauge actions;
- 3 lattice spacings @ $a \sim 0.07 - 1.11$ fm;
- Valence quark mass @ $m_\pi \sim$ 230-420 MeV;
- Charm quark use anisotropic clover action, m_c is tuned to match the mass of D_s and hyperfine splitting.
- All-mode-averaging is used on each ensemble.

Charmed baryon: Ξ_c Decay

Discrepancies in decay rates of $\Xi_c^0 \to \Xi^- e^+ \nu_e$

Belle	$(1.31 \pm 0.39)\%$
ALICE	$(2.48 \pm 0.86)\%$
PDG average	$(1.05 \pm 0.20)\%$
Zhang *et al.*	$(2.38 \pm 0.44)\%$
Farrel and Meinel	$(3.58 \pm 0.12)\%$



C. Farrell and S. Meinel, arXiv:2504.07302 [hep-lat]

	$\beta = 10/g^2$	$L^3 \times T$	a	c_{sw}	κ_l	m_π	κ_s	m_{η_s}
s108	6.20	$24^3 \times 72$	0.108	1.161	−0.2770	285	0.1330	640
s080	6.41	$32^3 \times 96$	0.080	1.141	−0.2295	295	0.1318	650

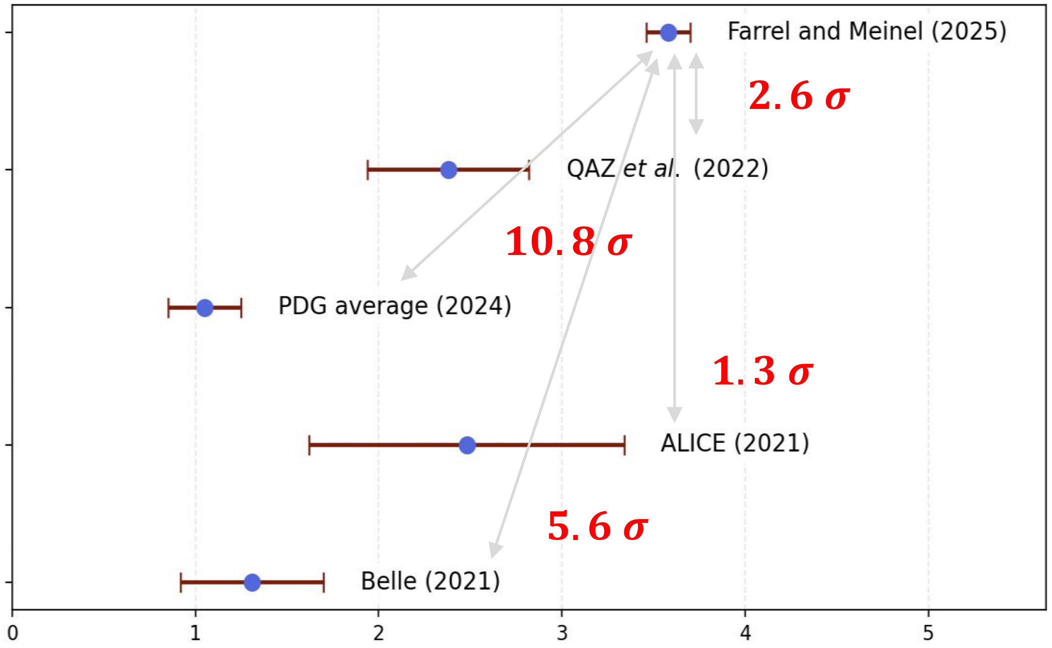
CLQCD

Label	$N_s^3 \times N_t \times N_5$	β	a [fm]	$2\pi/L$ [GeV]	$am_{u,d}$	m_π [GeV]	$am_s^{(\mathrm{sea})}$	$am_s^{(\mathrm{val})}$	$am_Q^{(c)}$	$\nu^{(c)}$	$c_{E,B}^{(c)}$	N_{ex}	N_{sl}
C01	$24^3 \times 64 \times 16$	2.13	0.1106(3)	0.4673(13)	0.01	0.4312(13)	0.04	0.0323	0.1541	1.2004	1.8407	283	2264
C005	$24^3 \times 64 \times 16$	2.13	0.1106(3)	0.4673(13)	0.005	0.3400(11)	0.04	0.0323	0.1541	1.2004	1.8407	311	2488
F004	$32^3 \times 64 \times 16$	2.25	0.0828(3)	0.4680(17)	0.004	0.3030(12)	0.03	0.0248	−0.05167	1.1021	1.4483	251	2008
F1M	$48^3 \times 96 \times 12$	2.31	0.0728(3)	0.3545(13)	0.002144	0.2320(10)	0.02144	0.02144	−0.05874	1.0941	1.5345	113	1808

RBC

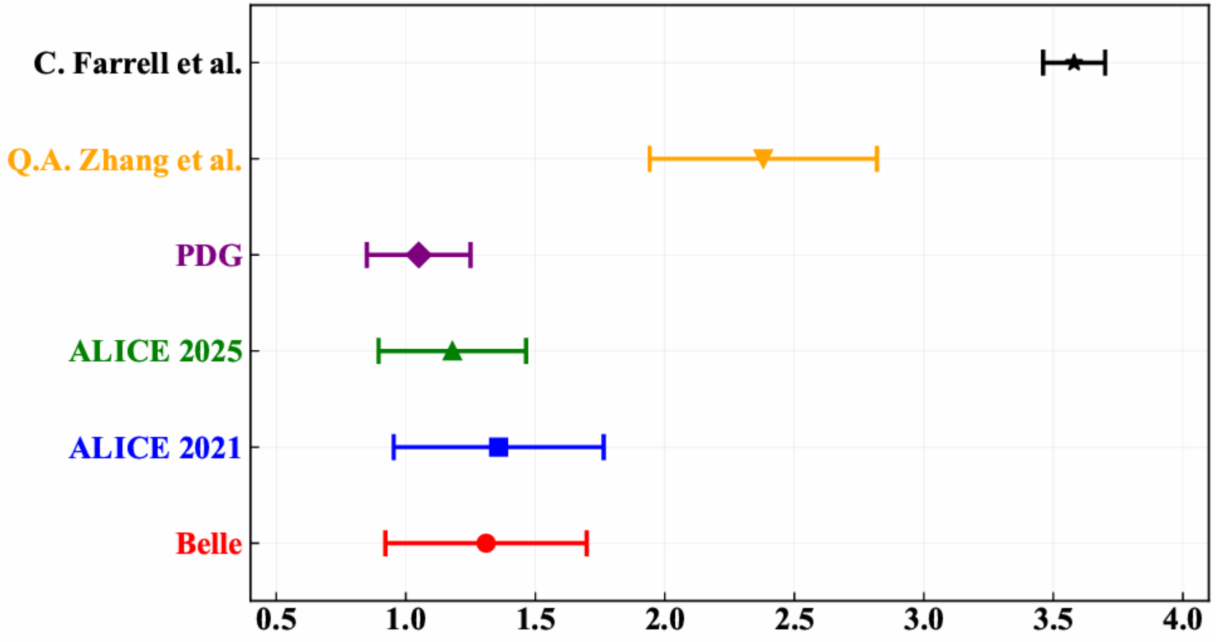
Puzzle of $\Xi_c^0 \to \Xi^- e^+ \nu_e$ decay

ALICE's result is updated



by Q.A. Zhang PPT

https://doi.org/10.1142/S0217751X2645003X



L.Y. Li and W. Wang, Int. J. Mod. Phys. A (2026)

$\Xi_c \to \Xi$ form factors from lattice QCD with domain-wall quarks: A new piece in the $\boxed{\text{puzzle}}$ of Ξ_c^0 decay rates

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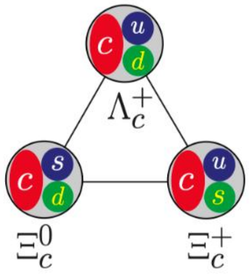
(Received 22 April 2025; accepted 10 June 2025; published 27 June 2025)

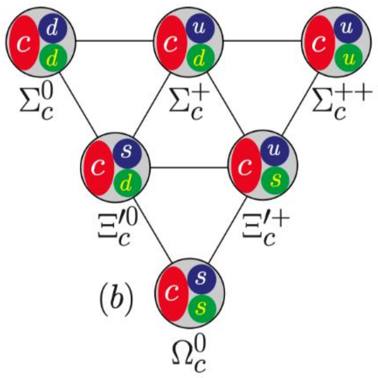
Charmed baryon: $\Xi_c - \Xi_c'$ mixing

- Baryonic currents of $SU(3)_F$ eigenstates: **$3 \otimes 3 = \bar{3} \oplus 6$**



Anti-symmetric triplet

$$O^{\bar{3}} = \epsilon^{abc}\underbrace{\left(\ell^{Ta} C\gamma_5 s^b\right)}_{J=0} P_+ \tilde{c}^c,$$



Symmetric sextet

$$O^6 = \epsilon^{abc}\underbrace{\left(\ell^{Ta} C\vec{\gamma} s^b\right)}_{J=1} \cdot \vec{\gamma}\gamma_5 P_+ \tilde{c}^c$$

- Mixing comes from the breaking of $SU(3)_F$ symmetry:

Mass eigenstates

$$\begin{pmatrix} |\Xi_c\rangle \\ |\Xi_c'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\Xi_c^{\bar{3}}\rangle \\ |\Xi_c^{6}\rangle \end{pmatrix}$$

Flavor eigenstates

the mass and flavor eigenstates satisfying:

$$H_0|\Xi_c^{\bar{3}/6}(\vec{p}=0)\rangle = m_{\Xi_c^{\bar{3}/6}}|\Xi_c^{\bar{3}/6}(\vec{p}=0)\rangle,$$

$$H|\Xi_c\rangle = m_{\Xi_c}|\Xi_c\rangle, \qquad H|\Xi_c'\rangle = m_{\Xi_c'}|\Xi_c'\rangle,$$

Charmed baryon: $\Xi_c - \Xi_c'$ mixing

- **Several works from various methods are controversial……**

Sum rule	**5.5°±1.8°**	**T.M. Aliev, et al., Phys. Rev. D 83, 016008 (2011)**
HQET	**8.12°±0.80°**	**Y. Matsui, Nucl. Phys. A 1008, 122139 (2021)**
Quark model	**16.27°±2.30°**	**H.W. Ke, et al., Phys. Rev. D 105, 096011 (2022)**
	24.66°±0.90°	**C.Q. Geng, et al., Phys.Lett.B 838, 137736 (2023)** **C.Q. Geng, et al., Phys.Lett.B 839, 137831 (2023)**
Lattice QCD	**Negligibly small**	**Brown, *et al.*, Phys. Rev. D 90, 094507 (2014)**

Charmed baryon: $\Xi_c - \Xi_c'$ mixing

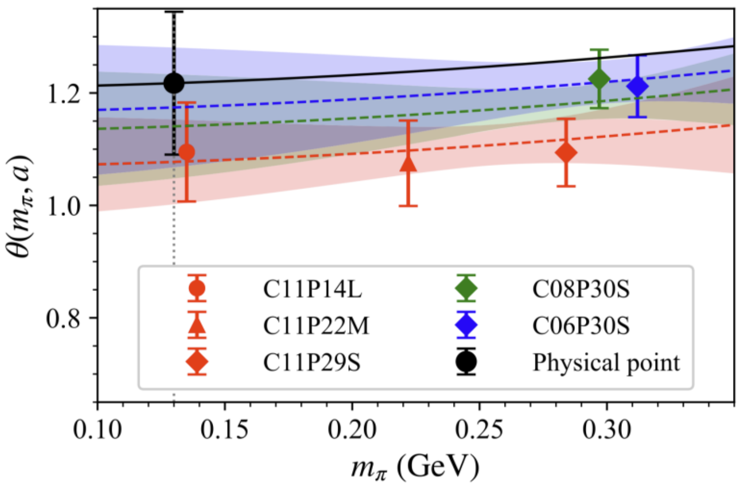
Strategy I: From matrix of correlation function

H. Liu, *et al.*, PLB841, 137941 (2023)

Lattice Calculation

$$C(t) = \sum_{\vec{x}} \begin{pmatrix} \langle O^{\bar{3}}(\vec{x},t)\bar{O}^{\bar{3}}(\vec{0},0)\rangle & \langle O^{\bar{3}}(\vec{x},t)\bar{O}^{6}(\vec{0},0)\rangle \\ \langle O^{6}(\vec{x},t)\bar{O}^{\bar{3}}(\vec{0},0)\rangle & \langle O^{6}(\vec{x},t)\bar{O}^{6}(\vec{0},0)\rangle \end{pmatrix}$$

Theoretical Parametrization $\rightarrow C(m_{\Xi_c}, m_{\Xi_c'}, \theta)$



Strategy II: From the matrix of Hamiltonian

H. Liu, *et al.*, PRD109, 036037 (2024)

Lattice Calculation

$$M_F(\vec{p}) = \int \frac{d^3\vec{p'}}{(2\pi)^3} \begin{pmatrix} \langle \Xi_c^{\bar{3}}(\vec{p})|(H_0+\Delta H)|\Xi_c^{\bar{3}}(\vec{p'})\rangle & \langle \Xi_c^{\bar{3}}(\vec{p})|\Delta H|\Xi_c^{6}(\vec{p'})\rangle \\ \langle \Xi_c^{6}(\vec{p})|\Delta H|\Xi_c^{\bar{3}}(\vec{p'})\rangle & \langle \Xi_c^{6}(\vec{p})|(H_0+\Delta H)|\Xi_c^{6}(\vec{p'})\rangle \end{pmatrix}$$

Theoretical Parametrization

$$M_F(\vec{p}=0) = \begin{pmatrix} 2m^2_{\Xi_c^{\bar{3}}} & 0 \\ 0 & 2m^2_{\Xi_c^{6}} \end{pmatrix} + m_8 \begin{pmatrix} \langle \Xi_c^{\bar{3}}|O_8|\Xi_c^{\bar{3}}\rangle & \langle \Xi_c^{\bar{3}}|O_8|\Xi_c^{6}\rangle \\ \langle \Xi_c^{6}|O_8|\Xi_c^{\bar{3}}\rangle & \langle \Xi_c^{6}|O_8|\Xi_c^{6}\rangle \end{pmatrix}$$

$$\rightarrow M_F(m_{\Xi_c}, m_{\Xi_c'}, \theta)$$

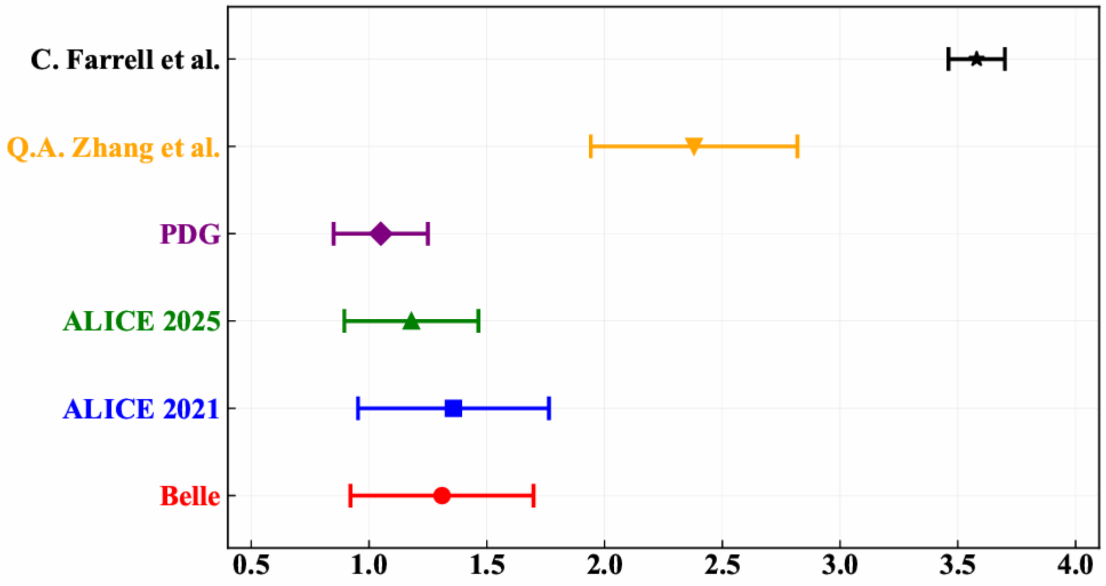
- Results from LQCD (C11P29S) agree with each other:

Strategy I:	$\theta = (1.094 \pm 0.060)^\circ$
Strategy II:	$\theta = (0.810 \pm 0.206)^\circ$

Challenges and Prospect

A small mixing angle is not a sufficient to explain Ξ_c decays!

$$\text{Strategy } I: \quad \theta = (1.094 \pm 0.060)^\circ$$
$$\text{Strategy II}: \quad \theta = (0.810 \pm 0.206)^\circ$$



- ✓ A new LQCD calculation using CLQCD configurations is under progress
- ✓ An independent experimental measurement of $\Xi_c^0 \to \Xi^- \pi^+$ is needed.

Summary

- ✓ Lattice QCD has provided a nonperturbative approach to explore baryon physics for STCFs
- ✓ Some progresses have been made in the hyperon
- ✓ Many progresses have been made in the charmed baryon:
  - ✓ Decay constants,
  - ✓ $\Xi_c \to \Xi \ell \nu$ form factor
  - ✓ $\Xi_c - \Xi_c{}'$ mixing
- ✓ There are still many challenges:
  - ✓ $\Xi_c \to \Xi \ell \nu$ puzzle

Thank you very much for your attention !