Lattice QCD for charm at STCF

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Outline

- Introduction
- Charmed meson radiative decay
- Charmed meson semileptonic decay
- Charmed meson rare decay
- Summary

- "November Revolution"— The discovery of the J/ψ particle in 1974 greatly facilitated the establishment of the Standard Model.

- How to translate the "charm" in Chinese, "美" or "魅" ? 粲!

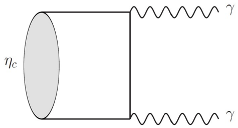
王竹溪

《诗经·唐风·绸缪》全文

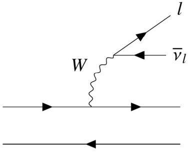
绸缪束薪，三星在天。
今夕何夕，见此良人。
子兮子兮，如此良人何！
绸缪束刍，三星在隅。
今夕何夕，见此邂逅。
子兮子兮，如此邂逅何！
绸缪束楚，三星在户。
今夕何夕，见此粲者。
子兮子兮，如此粲者何！

Charmed high-precision frontier

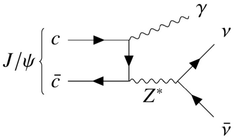
- Many puzzles to be resolved
  - Charmonium decay puzzle



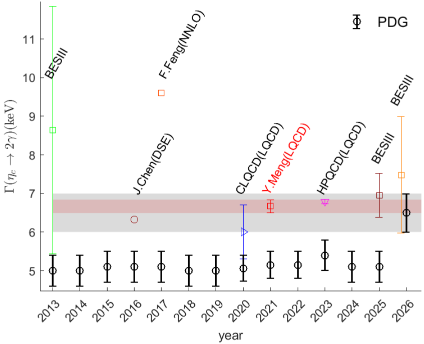
- Precise test for Standard Model
  — CKM matrix element $V_{cs(d)}$

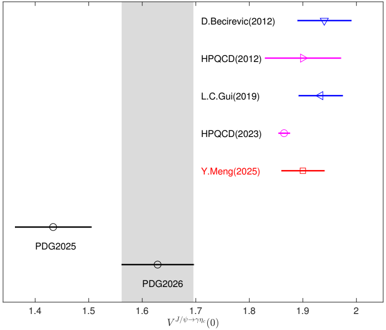


- Search for new physics
  — rare decay with $\mathrm{Br} < 10^{-10}$



Charmonium decay puzzle





- $\Gamma(\eta_c \to 2\gamma) = 6.67(16)(6)$ keV
  Y.M et al,Sci Bull68,1880(2023)

  $\Gamma(J/\psi \to \gamma\eta_c) = 2.30(10)$ keV
  Y.M et al,PRD111,014508(2025)
- A first step toward resolving the long-standing charmonium-decay puzzle
  - We find 2.9σ tension and PDG-fit is misleading, arXiv:2109.09381
  - H-P.Wang and C-Z.Yuan,New puzzle in charmonium decays, arXiv:2112.08584
  - HPQCD(2023) and BESIII(2025/2026)

PDG25 → PDG26

	x_1	x_6	x_9	x_{16}	x_{18}	x_{37}	x_{38}	x_{41}	x_{45}	x_{48}	x_{51}	x_{53}	x_{54}	x_{59}	$x^{h_c(1P)}_{25}$	$x^{\psi(2S)}_{182}$	$x^{J/\psi(1S)}_{237}$	$x^{B^\pm}_{254}$	$\Gamma_{\eta_c(1S)}$
x_1	100																		
x_6	14	100																	
x_9	11	13	100																
x_{16}	7	8	8	100															
x_{18}	9	11	11	7	100														
x_{37}	25	25	22	12	17	100													
x_{38}	13	13	11	6	9	51	100												
x_{41}	7	7	6	4	5	15	8	100											
x_{45}	5	5	5	2	3	12	6	4	100										
x_{48}	13	17	17	10	15	26	13	8	5	100									
x_{51}	19	20	20	11	16	39	20	11	11	24	100								
x_{53}	7	7	8	4	5	22	11	5	10	8	21	100							
x_{54}	5	7	7	4	6	12	6	3	4	10	13	9	100						
x_{59}	-38	-35	-27	-16	-22	-63	-32	-17	-12	-31	-47	-17	-11	100					
$x^{h_c(1P)}_{25}$	-8	-8	-7	-4	-6	-28	-18	-24	-9	-10	-16	-11	-4	20	100				
$x^{\psi(2S)}_{182}$	-1	-1	-1	-1	-1	-3	-2	-3	-1	-5	-3	-1	-1	3	2	100			
$x^{J/\psi(1S)}_{237}$	-17	-28	-32	-19	-30	-38	-19	-13	-7	-46	-40	-13	-20	39	14	3	100		
$x^{B^\pm}_{254}$	-14	-14	-17	-7	-11	-43	-22	-8	-19	-16	-43	-49	-18	34	15	2	26	100	
$\Gamma_{\eta_c(1S)}$	-1	-1	-1	0	-1	-2	-1	0	0	-1	1	0	0	-20	0	0	1	1	100

	Mode	Rate	Scale factor
Γ_1	$\eta_c(1S) \to \eta'(958)\pi\pi$	$(2.0 \pm 0.4) \times 10^{-2}$	1.4
Γ_6	$\eta_c(1S) \to K^*(892)\overline{K}^*(892)$	$(7.0 \pm 1.2) \times 10^{-3}$	
Γ_9	$\eta_c(1S) \to \phi\phi$	$(1.8 \pm 0.4) \times 10^{-3}$	2.3
Γ_{16}	$\eta_c(1S) \to \omega\omega$	$(2.7 \pm 0.9) \times 10^{-3}$	2.1
Γ_{18}	$\eta_c(1S) \to f_2(1270)f_2(1270)$	$(1.08 \pm 0.27) \times 10^{-2}$	
Γ_{37}	$\eta_c(1S) \to K\overline{K}\pi$	$(7.1 \pm 0.4) \times 10^{-2}$	1.1
Γ_{38}	$\eta_c(1S) \to K\overline{K}\eta$	$(1.32 \pm 0.15) \times 10^{-2}$	
Γ_{41}	$\eta_c(1S) \to K^+K^-\pi^+\pi^-$	$(8.3 \pm 1.8) \times 10^{-3}$	1.9
Γ_{45}	$\eta_c(1S) \to 2(K^+K^-)$	$(1.4 \pm 0.4) \times 10^{-3}$	1.4
Γ_{48}	$\eta_c(1S) \to 2(\pi^+\pi^-)$	$(9.6 \pm 1.5) \times 10^{-3}$	1.4
Γ_{51}	$\eta_c(1S) \to p\overline{p}$	$(1.33 \pm 0.11) \times 10^{-3}$	1.1
Γ_{53}	$\eta_c(1S) \to p\overline{p}\pi^+\pi^-$	$(3.7 \pm 0.5) \times 10^{-3}$	
Γ_{54}	$\eta_c(1S) \to \Lambda\overline{\Lambda}$	$(1.10 \pm 0.28) \times 10^{-3}$	1.5
Γ_{59}	$\eta_c(1S) \to \gamma\gamma$	$(1.66 \pm 0.13) \times 10^{-4}$	1.2

PDG25

	x_1	x_6	x_9	x_{16}	x_{18}	x_{37}	x_{38}	x_{41}	x_{45}	x_{48}	x_{52}	x_{54}	x_{55}	x_{60}	x^h
x_{60}	-41	-37	-29	-17	-22	-76	-45	-20	-16	-33	-51	-23	-20	100	
$x^{h_c(1P)}_{30}$	-12	-12	-10	-6	-8	-36	-25	-25	-12	-13	-21	-15	-9	29	
$x^{\psi(2S)}_{199}$	-1	-1	-1	-1	-1	-3	-2	-3	-1	-3	-3	-1	-1	3	
$x^{J/\psi(1S)}_{248}$	-20	-30	-34	-20	-30	-42	-25	-16	-10	-46	-43	-17	-32	43	
$x^{B^\pm}_{274}$	-18	-18	-19	-9	-12	-50	-30	-11	-23	-19	-39	-55	-30	41	
$\Gamma_{\eta_c(1S)}$	-2	-1	-1	-1	-1	-3	-2	-1	-1	-1	1	-1	-1	-15	

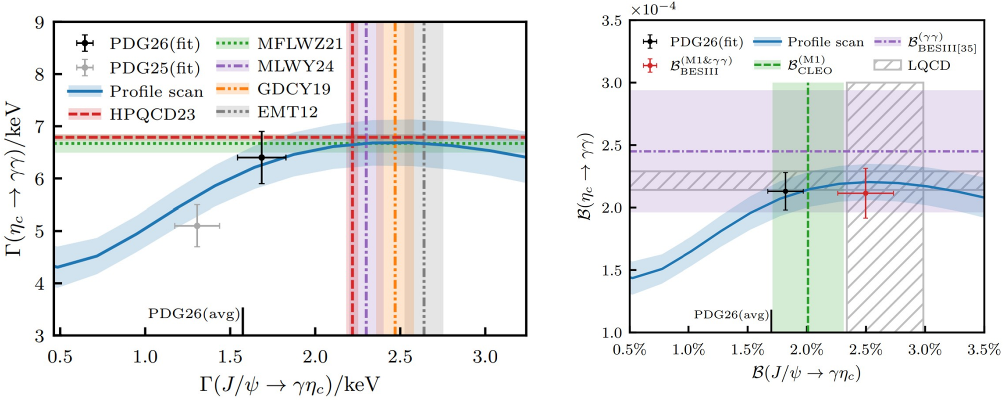
	Mode	Rate	Scale factor
Γ_1	$\eta_c(1S) \to \eta'(958)\pi\pi$	$(1.59 \pm 0.34) \times 10^{-2}$	1.7
Γ_6	$\eta_c(1S) \to K^*(892)\overline{K}^*(892)$	$(5.5 \pm 1.1) \times 10^{-3}$	1.2
Γ_9	$\eta_c(1S) \to \phi\phi$	$(1.4 \pm 0.4) \times 10^{-3}$	2.9
Γ_{16}	$\eta_c(1S) \to \omega\omega$	$(2.1 \pm 0.8) \times 10^{-3}$	2.4
Γ_{18}	$\eta_c(1S) \to f_2(1270)f_2(1270)$	$(8.4 \pm 2.4) \times 10^{-3}$	1.2
Γ_{37}	$\eta_c(1S) \to K\overline{K}\pi$	$(5.9 \pm 0.5) \times 10^{-2}$	1.8
Γ_{38}	$\eta_c(1S) \to K\overline{K}\eta$	$(1.11 \pm 0.15) \times 10^{-2}$	1.3
Γ_{41}	$\eta_c(1S) \to K^+K^-\pi^+\pi^-$	$(6.7 \pm 1.8) \times 10^{-3}$	2.4
Γ_{45}	$\eta_c(1S) \to 2(K^+K^-)$	$(1.2 \pm 0.4) \times 10^{-3}$	1.6
Γ_{48}	$\eta_c(1S) \to 2(\pi^+\pi^-)$	$(7.5 \pm 1.3) \times 10^{-3}$	1.6
Γ_{52}	$\eta_c(1S) \to p\overline{p}$	$(1.11 \pm 0.12) \times 10^{-3}$	1.4
Γ_{54}	$\eta_c(1S) \to p\overline{p}\pi^+\pi^-$	$(3.4 \pm 0.5) \times 10^{-3}$	1.2
Γ_{55}	$\eta_c(1S) \to \Lambda\overline{\Lambda}$	$(9.4 \pm 1.8) \times 10^{-4}$	1.2
Γ_{60}	$\eta_c(1S) \to \gamma\gamma$	$(2.13 \pm 0.15) \times 10^{-4}$	1.5
$\Gamma^{h_c(1P)}_{30}$	$h_c(1P) \to \gamma\eta_c(1S)$	$(64 \pm 5) \times 10^{-2}$	1.2
$\Gamma^{\psi(2S)}_{199}$	$\psi(2S) \to \gamma\eta_c(1S)$	$(3.6 \pm 0.5) \times 10^{-3}$	1.3
$\Gamma^{J/\psi(1S)}_{248}$	$J/\psi(1S) \to \gamma\eta_c(1S)$	$(1.82 \pm 0.15) \times 10^{-2}$	1.6
$\Gamma^{B^\pm}_{274}$	$B^+ \to \eta_c K^+$	$(1.20 \pm 0.08) \times 10^{-3}$	1.3
$\Gamma_{\eta_c(1S)}$	$\eta_c(1S)$ WIDTH	30.0 ± 0.5 (MeV)	1.2

PDG26

- (i) New constraints from experiments; (ii) Highly correlated with other decay channels

Towards resolving the puzzle

M. C. Schaaf and A. Vairo, arXiv:2606.19902



- PDG fit dependence on $\mathcal{B}(J/\psi \to \gamma\eta_c)$
- Model-independent photon line shape for $J/\psi \to \gamma\eta_c$
  — peak strength rather than integrating photon energy — no damping function

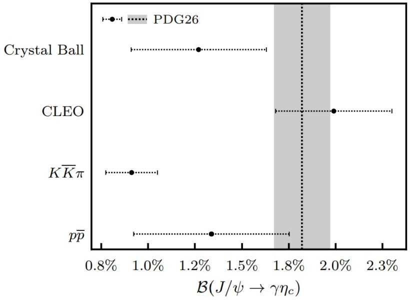
Role of STCF

Based on 10 billion J/ψ samples, BESIII have measured

- $\mathcal{B}(J/\psi \to \gamma\eta_c) \times \mathcal{B}(\eta_c \to 2\gamma) = 5.23(26)(30) \times 10^{-6}$ PRL134,181901(2025)
- $\mathcal{B}(J/\psi \to \gamma\eta_c) \times \mathcal{B}(\eta_c \to p\bar{p}) = 2.11(2)(7) \times 10^{-5}$ PRL136,051901(2026)
- Absolute branching fraction $\mathcal{B}(\eta_c \to 2\gamma) = 2.45(48)(09) \times 10^{-4}$. PRL accepted(2026)

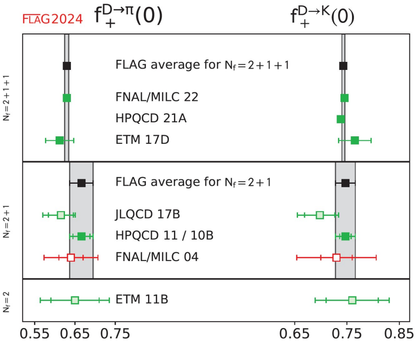
Based on 1000 billion J/ψ samples per year on STCF

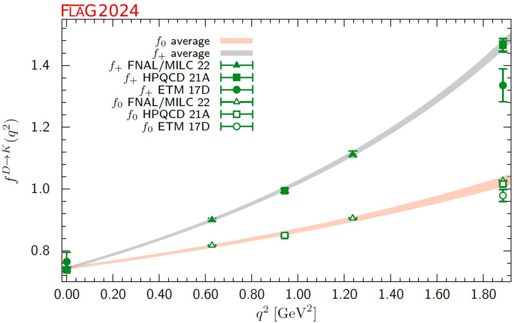
- A direct measurement $\mathcal{B}(J/\psi \to \gamma\eta_c)$
- More precise constraints on $\mathcal{B}(J/\psi \to \gamma\eta_c) \times \mathcal{B}(\eta_c \to X_i)$



Charmed meson semileptonic decay

FLAG24: no results away from $q^2 = 0$ provided due to discrepancy among ETM17 and others.





- More lattice QCD calculations on $D \to K$. in progress
- Independent determination on CKM matrix element from other channels, i.e. $P \to V$

Why $P \to V$ semileptonic decay?

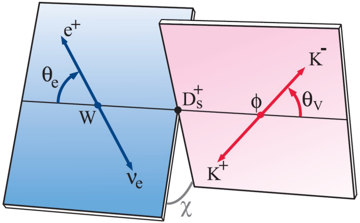
- For $P' \to P$:

$$\frac{d\Gamma(P' \to Pe\nu)}{dq^2} = \frac{G_F^2}{24\pi^3}|\vec{p}|^3|V_{xy}|^2|f_+(q^2)|^2$$

for example, $D \to \pi$ and $D \to K$

- For $P \to V$:

$$\frac{\mathrm{d}\Gamma(P \to Ve\nu)}{\mathrm{d}q^2\mathrm{d}\chi\mathrm{d}\cos\theta_\ell\mathrm{d}\cos\theta_V} = \frac{G_F^2|V_{xy}|^2|\vec{p}|q^2}{12(2\pi)^4 M^2} W\left(q^2, \theta_V, \theta_\ell, \chi\right)$$



  - Independent determination of the CKM matrix element
  - Stronger test with abundant polarization information

$P \to V$ semileptonic decay

- Experiments:

Channels	r_V	r_2	Refs
$D^0 \to K^*(892)$			
(μ)	$1.37 \pm 0.09 \pm 0.03$	$0.76 \pm 0.06 \pm 0.02$	PRL134,011803(2025)
(μ)	$1.46 \pm 0.11 \pm 0.04$	$0.71 \pm 0.08 \pm 0.03$	PRL135,111803(2025)
(e)	$1.48 \pm 0.05 \pm 0.02$	$0.70 \pm 0.04 \pm 0.02$	JHEP03,197(2025)
LQCD	1.48(12)&1.46(11)→1.44(28)	0.6(3)&1.0(2)→ 1.5(6)	NPB119,625(2003)
$D_s \to \phi$			
(μ)	$1.58 \pm 0.17 \pm 0.02$	$0.71 \pm 0.14 \pm 0.02$	JHEP12,072(2023)
(e)	$1.807 \pm 0.046 \pm 0.065$	$0.816 \pm 0.036 \pm 0.030$	PRD78,051101(2008)
LQCD	1.72(21)	0.74(12)	PRD90,074506(2014)
$D^0 \to \rho^- e^+ \nu_e$	$1.695 \pm 0.083 \pm 0.051$	$0.845 \pm 0.056 \pm 0.039$	PRL122,062001(2019)

- First-principle lattice calculation is urgent

Lattice setup

Configuration	Volume	a/fm	m_π/MeV	$N_{\text{conf}} \times N_t \times N_s$
C24P29	$24^3 \times 72$	0.10524(05)	292.3(1.0)	$450 \times 72 \times 2$
C32P29	$32^3 \times 64$	0.10524(05)	293.1(0.8)	$397 \times 64 \times 2$
C32P23	$32^3 \times 64$	0.10524(05)	227.9(1.2)	$333 \times 64 \times 3$
F32P30	$32^3 \times 96$	0.07753(03)	300.4(1.2)	$360 \times 96 \times 2$
F48P21	$48^3 \times 96$	0.07753(03)	207.5(1.1)	$241 \times 48 \times 4$
G36P29	$36^3 \times 108$	0.06887(12)	297.2(0.9)	$300 \times 54 \times 2$
F48P32	$48^3 \times 144$	0.05199(08)	316.6(1.0)	$300 \times 72 \times 2$

- Four lattice spacings $[0.052 - 0.105]$ fm $\rightarrow$ continuum limit
- $m_\pi = 208 - 317$ MeV $\rightarrow$ physical pion msss
- Same lattice spacings and pion mass: C24P29 vs C32P29 $\rightarrow$ finite-volume effect

$D_s \to \phi$: mass and decay constant

- CLQCD,PRD111,054504(2025)

$$m_{D_s} = 1966.7(1.5)\ \text{MeV}$$
$$f_{D_s} = 247.0(2.1)\ \text{MeV}$$

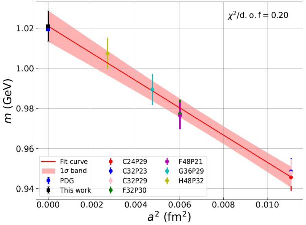
- This work 2510.14478

$$m_\phi = 1021.1(7.6)\ \text{MeV}$$
$$f_\phi = 246.2(4.1)\ \text{MeV}$$

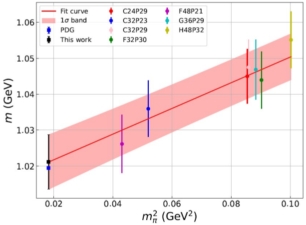
much improved than

$$m_\phi = 1032(16)\ \text{MeV}$$
$$f_\phi = 241(18)\ \text{MeV}$$

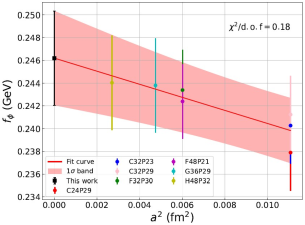
HPQCD,PRD90,074506(2014)



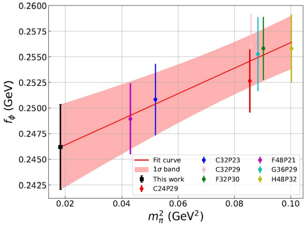
(i) ϕ mass at $m_\pi = 0.135$ GeV.



(ii) ϕ mass at $a = 0.0$ fm.



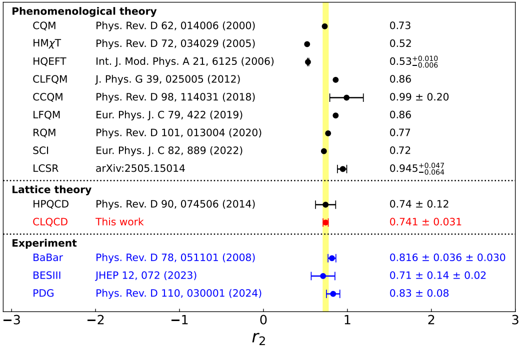
(iii) ϕ decay constant at $m_\pi = 0.135$ GeV.

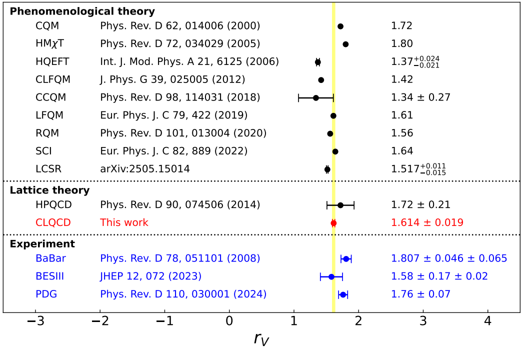


(iv) ϕ decay constant at $a = 0.0$ fm.

$D_s \to \phi$ form factors

$$r_2 \equiv A_2(0)/A_1(0), \quad r_V \equiv V(0)/A_1(0)$$





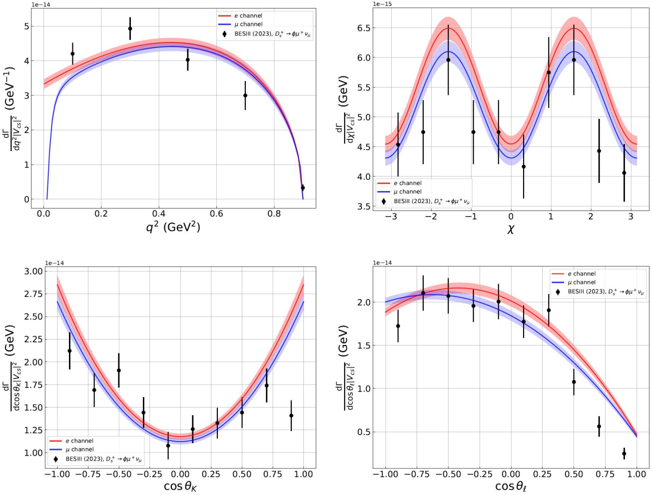
- Precision improved by up to an order of magnitude

G.-F.Fan,**Y.M**,C.Liu,Z.-F Liu,T.-X Wang,T.-H Shen,K.-L Zhang, and L.Zhang, arXiv:2510.14478

$D_s \to \phi$ differential decay width

$\mathcal{B}(D_s \to \phi \ell \nu_\ell) \times 10^{-2}$	μ channel	e channel	$\mathcal{R}_{\mu/e}$
This work (z-expansion)	2.378(80)	2.521(87)	0.9430(17)
This work (single pole)	2.378(55)	2.521(61)	0.9433(13)
This work (modified pole)	2.378(53)	2.520(58)	0.9437(12)
BaBar (2008) [1]	–	2.61(17)	–
CLEO (2015) [2]	–	2.14(19)	–
BESIII (2018) [3]	1.94(54)	2.26(46)	0.86(29)
BESIII (2023) [4]	2.25(11)	–	–
PDG [27]	2.24(11)	2.34(12)	0.957(68)

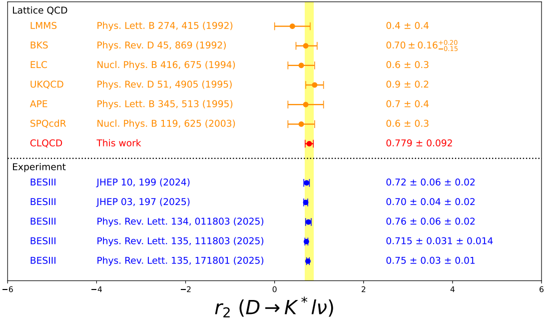
- $D_s \to \phi e \nu_e$

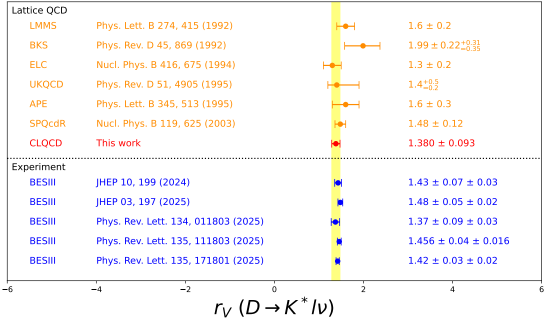


- Discrepancy in q^2-dsitribution compared to BESIII measurement on $D_s \to \phi \mu \nu_\mu$

$D \to K^*$ form factors(preliminary)

- Adding C48P14 and F48P30, 9 ensembles give: $m_{K^*} = 886(11)$ MeV





- First full lattice QCD calculation!
- Systematic study with a^2, m_π^2, and $e^{-m_\pi L}$ extrapolation

Charmonium weak decay

- Experiments

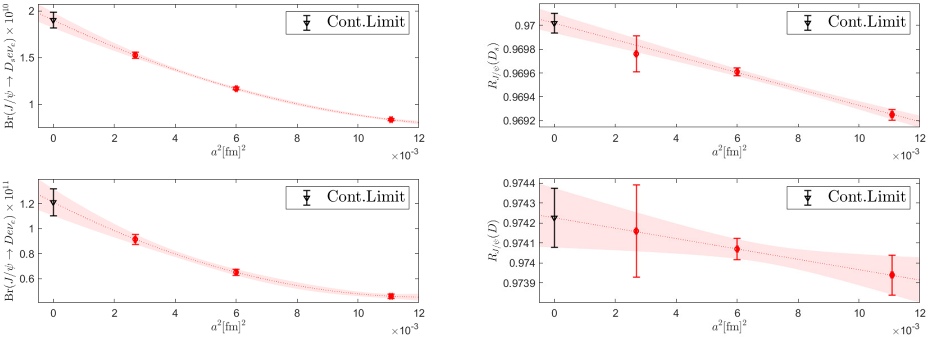
channels	Upper limit	J/ψ number	Refs
$J/\psi \to D_s e \nu_e$	4.9×10^{-5}	5.8×10^{7}	PLB639,418(2006)
$J/\psi \to D_s e \nu_e$	1.3×10^{-6}	2.3×10^{8}	PRD90,112014(2014)
$J/\psi \to D e \nu_e$	7.1×10^{-8}	1.01×10^{10}	JHEP06,157(2021)
$J/\psi \to D \mu \nu_\mu$	5.6×10^{-7}	1.01×10^{10}	JHEP01,126(2024)
$J/\psi \to D_s e \nu_e$	9.9×10^{-8}	1.01×10^{10}	2510.25100

BES & BESIII collaboration

- Future measurements ?

channels	Upper limit	J/ψ number	Refs
$J/\psi \to D_s \mu \nu_\mu$	—	1.01×10^{10}	BESIII
$J/\psi \to D_{(s)} e \nu_e$	—	$\sim 10^{12}$	STCF
$J/\psi \to D_{(s)} \mu \nu_\mu$	—	$\sim 10^{12}$	STCF

$J/\psi \to D_{(s)}$ branching fraction



- The branching fraction Y.M et al, PRD110,074510(2024).

$$\mathrm{Br}(J/\psi \to D_s e\nu_e) = 1.90(6)_{\mathrm{stat}}(5)_{V_{cs}} \times 10^{-10}$$
$$\mathrm{Br}(J/\psi \to D e\nu_e) = 1.21(6)_{\mathrm{stat}}(9)_{V_{cd}} \times 10^{-11}$$

Cuttent status of the dark matter search on the collider

- Remarkable candidates
  - Dark photon A': coupling to the photon through the kinetic mixing $\frac{\epsilon}{2}F^{\mu\nu}F'_{\mu\nu}$, free parameters are $m_{A'}$ and ϵ
  - CP-odd Higgs Boson A^0: m_{A^0} and $g_{al\bar{l}}$
  - Axion-like particle a: m_a and $g_{a\gamma\gamma}$
- Dark matter can be produced on the collider
  - Direct production: $e^+e^- \to \gamma X$

    BABAR,PRL119,131804(2017) BESIII,PLB839,137785(2023)
  - Resonant production: $e^+e^- \to J/\psi/\Upsilon(1S) \to \gamma + X$

    CLEO,PRD81,091101(2010) BABAR,PRL107,021804(2011)
    Belle,PRL122,011801(2019) Belle,PRL128,081804 (2022)
    BESIII,PRD101,112005(2020) BESIII,PRD105,012008(2022)
    BESIII,PLB838,137698(2023)

Light dark matter search at the collider

CLEO 2010

PHYSICAL REVIEW D 81, 091101(R) (2010)

Search for the decay J/ψ→γ + invisible

(CLEO Collaboration)

$J/\psi \to \gamma + X$

invisible to detector

light dark matter ?

The upper limit corresponding to $m_X = 0$ is 4.3×10^{-6} at the 90% confidence level

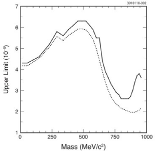


FIG. 4. The 90% confidence-level upper limits for $J/\psi \to \gamma X$, where X is invisible to the detector. The dashed line shows the results for statistical uncertainties alone, and the solid line includes systematic and statistical uncertainties.

BABAR
Belle

$\Upsilon(1S) \to \gamma + X$

PRL 107, 021804 (2011)

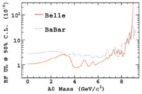
Search for Production of Invisible Final States in Single-Photon Decays of $\Upsilon(1S)$

(BABAR Collaboration)

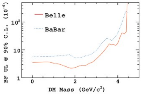
PRL 122, 011801 (2019)

Search for a Light CP-odd Higgs Boson and Low-Mass Dark Matter at the Belle Exper

(Belle Collaboration)



$\mathcal{B}(\Upsilon \to \gamma A^0) \times \mathcal{B}(A^0 \to \text{invisible})$



$\mathcal{B}(\Upsilon(1S) \to \gamma\chi\bar{\chi})$

BESIII

PHYSICAL REVIEW D 101, 112005 (2020)

Search for the decay J/ψ→γ + invisible

Upper bound $\mathcal{B} \sim 7.0 \times 10^{-7}$

PHYSICAL REVIEW D 105, 012008 (2022)

Search for a CP-odd light Higgs boson in $J/\psi \to \gamma A^0$

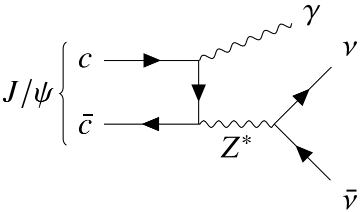
Upper bound $\mathcal{B}(J/\psi \to \gamma A^0) \times \mathcal{B}(A^0 \to \mu^+\mu^-) \sim (1.2 - 778.0) \times 10^{-9}$

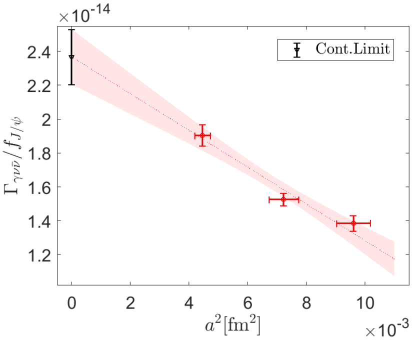
PHYSICS LETTERS B 838, 137698 (2023)

Search for an axion-like particle in radiative J/ψ decays

We search for an axion-like particle (ALP) a through the process $\psi(3686) \to \pi^+\pi^- J/\psi$, $J/\psi \to \gamma a$, $a \to \gamma\gamma$ in a data sample of $(2.71 \pm 0.01) \times 10^9$ $\psi(3686)$ events collected by the BESIII detector. No significant ALP signal is observed over the expected background, and the upper limits on the branching fraction of the decay $J/\psi \to \gamma a$ and the ALP-photon coupling constant $g_{a\gamma\gamma}$ are set at 95% confidence level in the mass range of $0.165 \le m_a \le 2.84$ GeV$/c^2$. The limits on $\mathscr{B}(J/\psi \to \gamma a)$ range from 8.3×10^{-8} to 1.8×10^{-6} over the search region, and the constraints on the ALP-photon coupling are the most stringent to date for $0.165 \le m_a \le 1.468$ GeV$/c^2$.

Standard model background in J/ψ invisible decay





- The first lattice QCD calculation **Y.M** et al, 2601.18209

$$\mathrm{Br}[J/\psi \to \gamma\nu\bar{\nu}] = 1.04(7)(8) \times 10^{-10}$$

- Phenomenological estimation $\sim 0.7 \times 10^{-10}$
  Dao-Neng Gao,PRD **90**,077501(2014)

Summary

Outlook and role of the STCF

- Charmonium decay puzzle
  - $J/\psi \to \gamma\eta_c$ and $\eta_c \to 2\gamma$
  - Model-independent determination of $\mathcal{B}(J/\psi \to \gamma\eta_c)$
- Semileptonic decay
  - $D_s \to \phi$ and $D \to K^*$
  - Real photon radiation: $D \to K\gamma l\nu_l$
- Rare decay
  - $J/\psi \to D_{(s)}$ and $J/\psi \to \gamma\nu\bar{\nu}$
  - Axion-like particle search: $D \to l\nu_l a$

Summary

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Thanks to the STCF!