2026年超级陶粲装置研讨会

Dispersion relation and Hyperon Electromagnetic Structure

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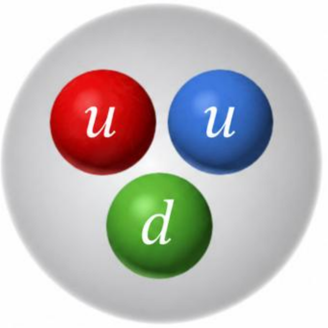
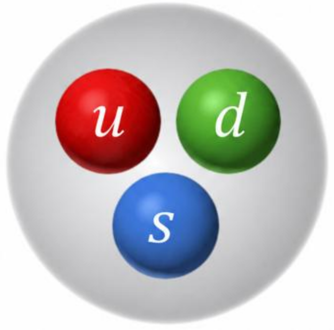
参考文献： EPJA 59 (2023), 54; EPJC 82 (2022), 1091; PLB 856 (2024) 138887

2026年7月，西安

❖ Nucleon vs Hyperons

	Nucleon: $p(\mathrm{uud})$, $n(\mathrm{udd})$	**Hyperon**: $\Lambda(\mathrm{uds})$, $\Sigma(\mathrm{uus})$, $\Xi(\mathrm{ssd})$, $\Omega(\mathrm{sss})$
		
Lifetime:	$p(\infty)$, $n(14.6\ \mathrm{min})$	10^{-10} s
Exp. Probe:	$e\,p \to e\,p$, $e^+e^- \to N\bar{N}$, eH(μH)-spectroscopy	$e^+e^- \to Y\bar{Y}$, $Y_1 \to Y_2 + \gamma^*(X)$
Status:	$r_p^E = 0.8407(6)\mathrm{fm}$, **0.07%** $r_p^M = 0.851(26)\mathrm{fm}$, **3.1%** $< r_n^2 >^E = -0.1155(17)\mathrm{fm}$, **1.5%** $r_n^M = 0.864(9)\mathrm{fm}$, **1.0%**	$r_{\Sigma^-}^E = 0.78(10)\mathrm{fm}$, **13%**

❖ Hyperon radius in theory

☐ Electromagnetic form factor of Hyperons

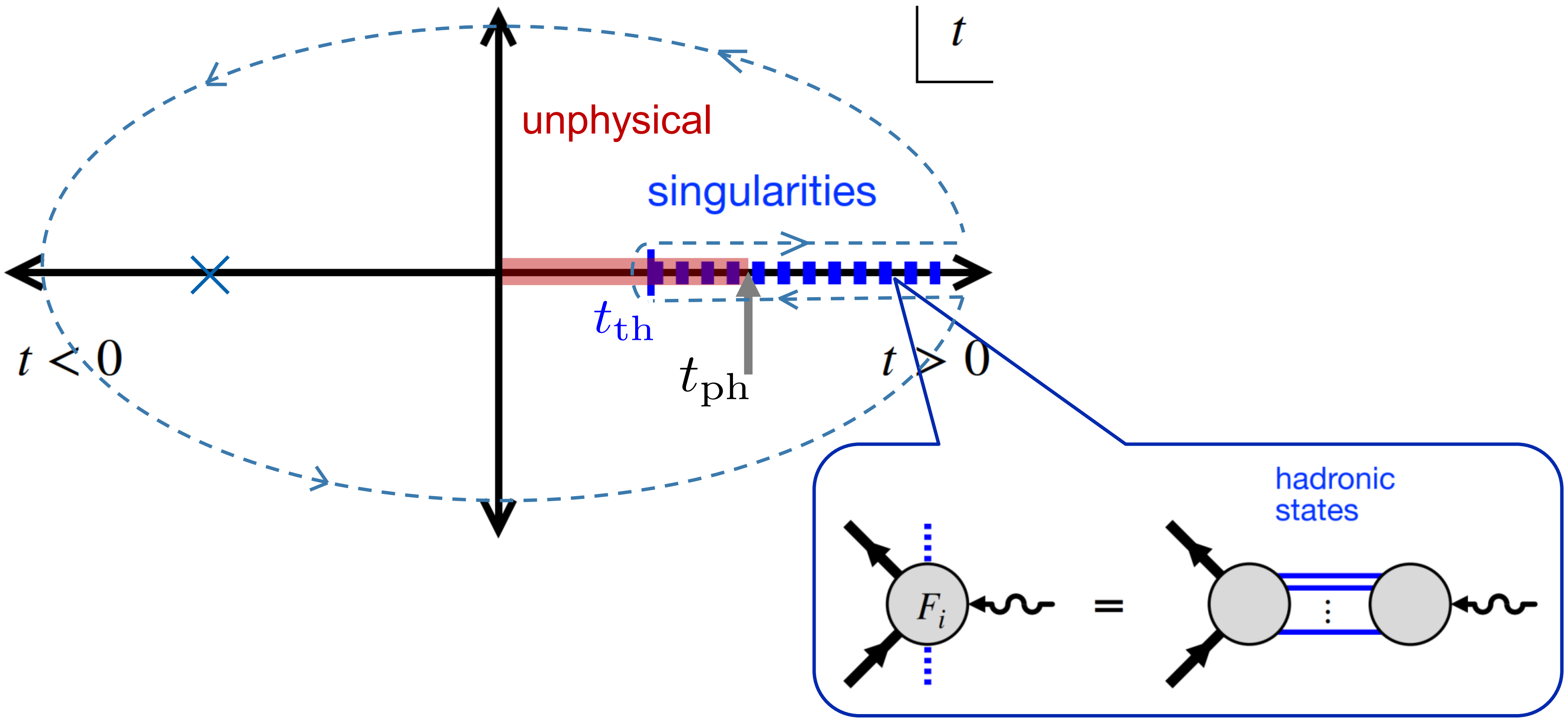
$$\langle Y_2(p')\,|j^{\mu}_{em}|\,Y_1(p)\rangle = \bar{u}(p')\left[\left(\gamma^{\mu} + \frac{m_{Y_1} - m_{Y_2}}{t} q^{\mu}\right) F_1(t) + \frac{i\sigma^{\mu\nu} q_{\nu}}{m_{Y_1} + m_{Y_2}} F_2(t)\right] u(p)$$

- Crossing symmetry: $\langle Y_2(p')\,|j^{\mu}_{em}|\,Y_1(p)\rangle \Longleftrightarrow \langle Y_2(p')Y_1(p)\,|j^{\mu}_{em}|\,0\rangle$
- $Y_1 = Y_2$ for elastic FFs, $Y_1 \neq Y_2$ for transition FFs
- Four-momentum transfer: $t \equiv q^2 = (p' - p)^2 = -Q^2$
- Sachs form factors: $G_E = F_1 + t/(m_{Y_1} + m_{Y_2})^2 F_2,\ G_M = F_1 + F_2$
- Normalization: $G_E(0)$, $F_1(0)$ gives the charge, $G_M(0)$ gives the magnetic moment
- Hyperon radius: $F(t) = F(0)\left(1 + t\langle r^2\rangle/6 + \cdots\right)$
- Model-independent approach: Dispersion relation

❖ Hyperon radius in theory

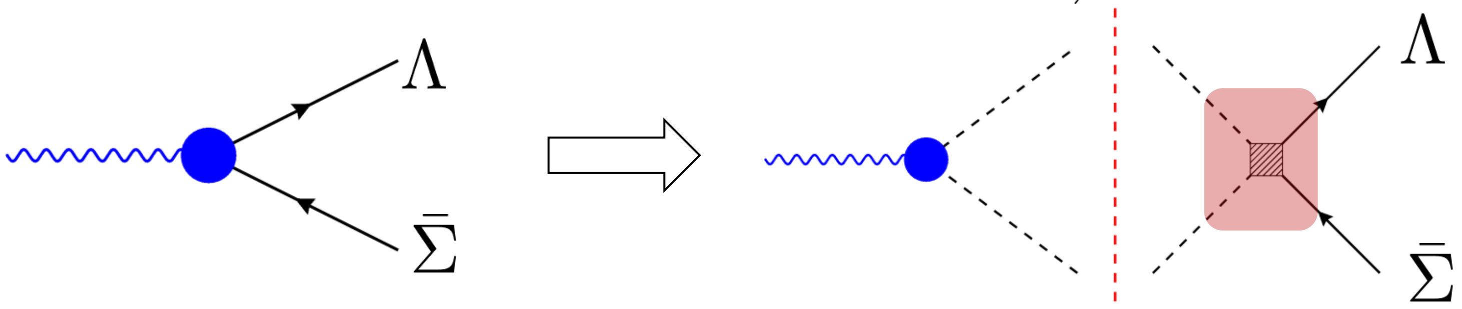
- Dispersion relation for EMFFs of Hyperons

$$F_i(t) = \int_{t_{\rm th}}^{\infty} \frac{dt'}{\pi} \frac{{\rm Im}F_i(t')}{t'-t-i0^+}$$



❖ Example1: Σ → Λ transition form factor

- Dispersion relation for Σ2Λ TFFs

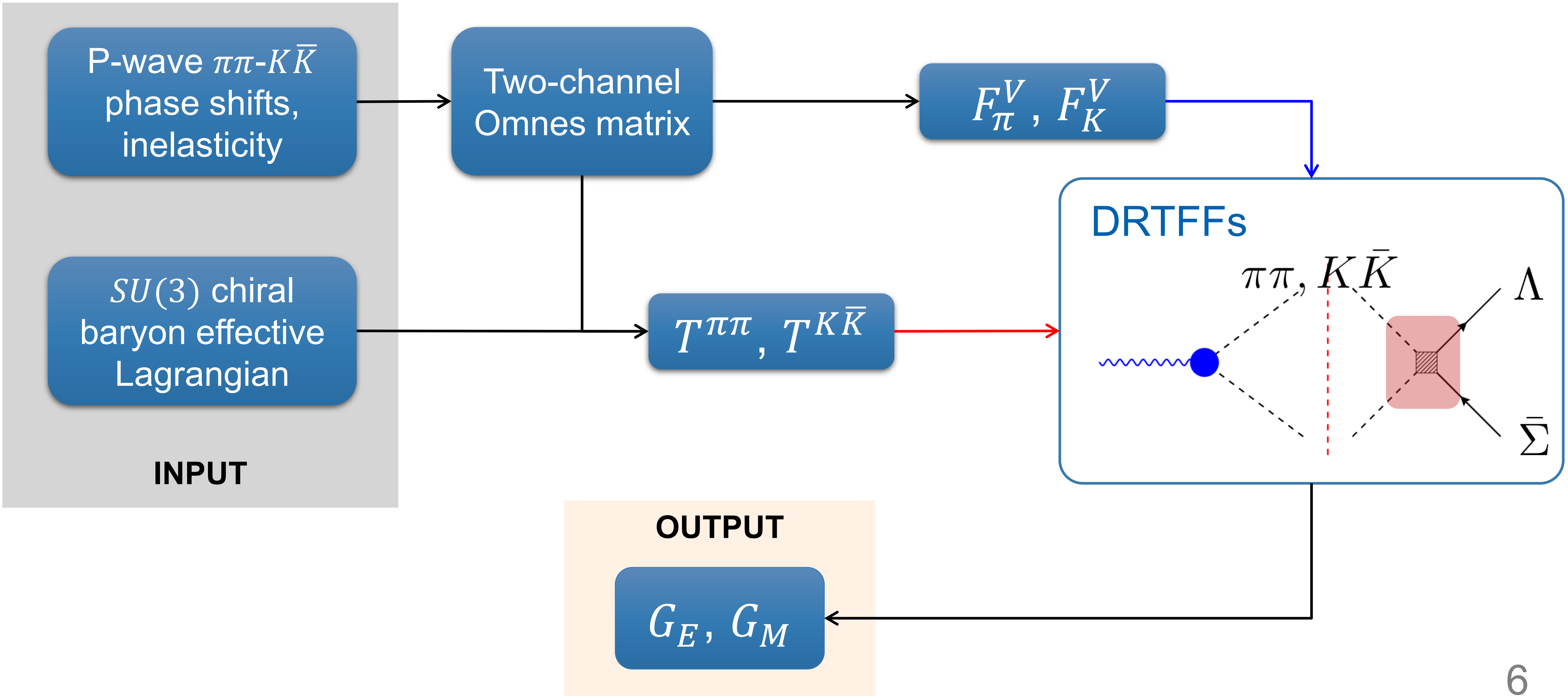


$$\frac{1}{2i}\mathrm{disc}_{\mathrm{uni.}} G_{E/M}(t) = \frac{1}{24\pi}\begin{pmatrix} T^{\pi\pi}_{E/M} & \sqrt{2}T^{K\bar{K}}_{E/M} \end{pmatrix}\begin{pmatrix} \Sigma_{\pi\pi} & 0 \\ 0 & \Sigma_{K\bar{K}} \end{pmatrix}\begin{pmatrix} F^V_\pi \\ \sqrt{2}F^V_K \end{pmatrix}^*$$

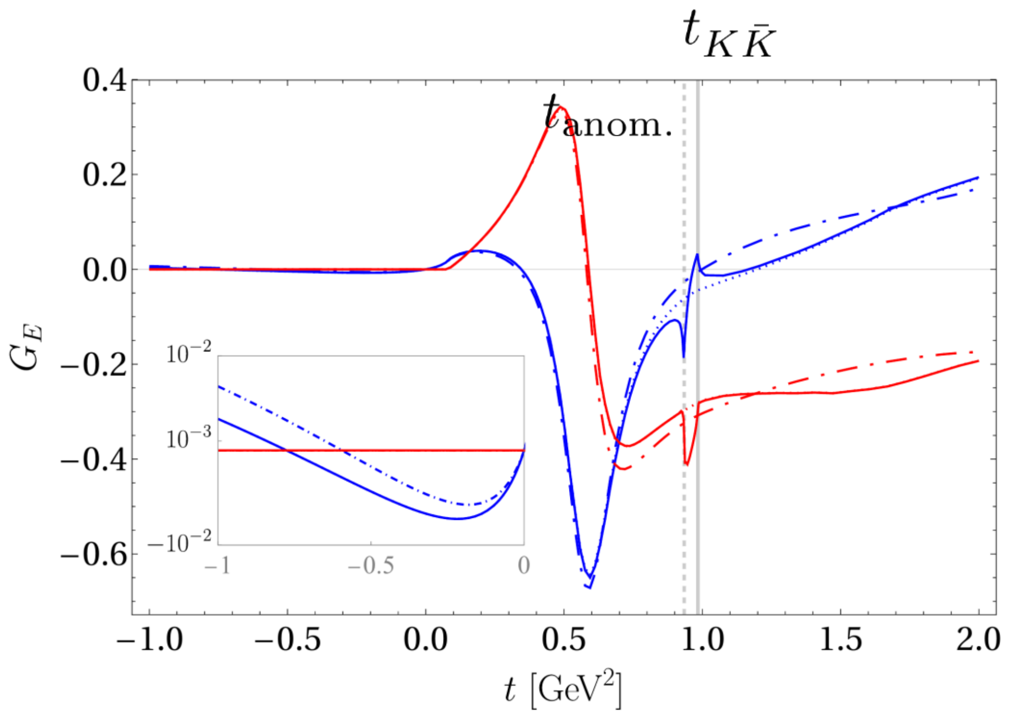
$$G_{E/M}(t) = G_{E/M}(0) + \frac{t}{2\pi i}\int_{4M_\pi^2}^{\infty} dt' \frac{\mathrm{disc}_{\mathrm{uni.}} G_{E/M}(t')}{t'(t'-t-i0^+)} + G^{\mathrm{anom.}}_{E/M}(t)$$

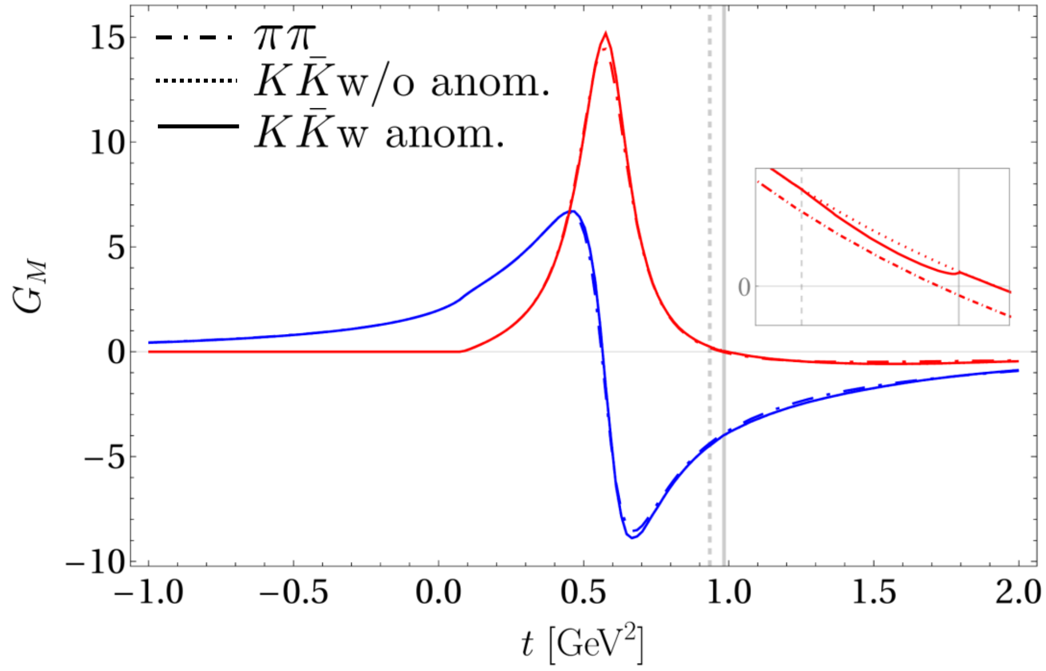
❖ Example1: Σ → Λ transition form factor

- Dispersion relation for Σ2Λ TFFs



❖ Example1: Σ → Λ transition form factor





☐ Insights from the DR-theoretic Σ2Λ TFFs

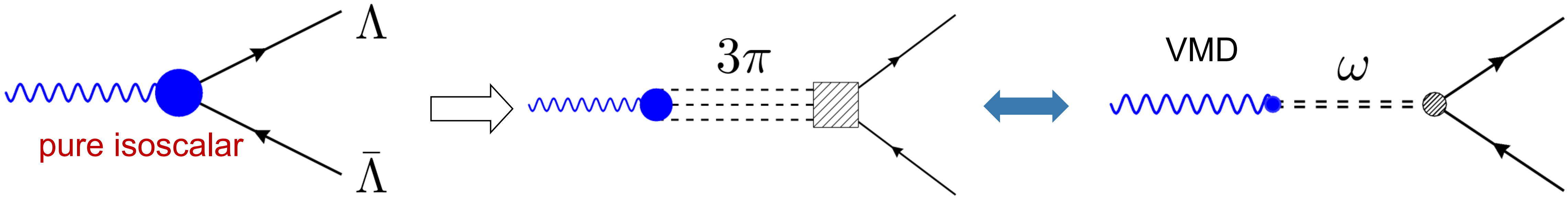
- At low energy region, $\pi\pi$ serves as the dominant contribution
- Poor knowledge of h_A, b_{10} causes the main theoretical errors

$$\mathcal{L}_{8+10}^{(1)} = \frac{D}{2}\langle \bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\rangle + \frac{F}{2}\langle \bar{B}\gamma^\mu\gamma_5[u_\mu, B]\rangle + \frac{1}{2\sqrt{2}} h_A \epsilon_{ade} g_{\mu\nu} (\bar{T}^\mu_{abc} u^\nu_{bd} B_{ce} + \bar{B}_{ec} u^\nu_{db} T^\mu_{abc}),$$

$$\mathcal{L}_8^{(2)} = \frac{i}{2} b_{10} \langle \bar{B}\{[u^\mu, u^\nu], \sigma_{\mu\nu} B\}\rangle$$

❖ Example2: Extraction of Λ magnetic radius

☐ Dispersion relation for Λ EMFFs

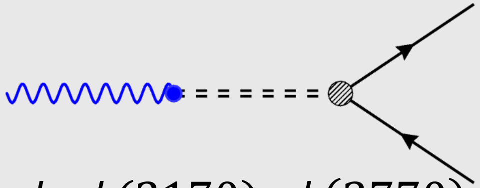


☐ Strategy for extracting Λ EMFFs from exp.

$e^+ + e^- \to \Lambda + \bar{\Lambda}$

⋯

VMD simplied DR-EMFFs



$\omega,\ \phi,\ \phi(2170),\ \psi(3770),\ \cdots$

$\int_{s_0}^{\infty} \mathrm{Im}\ F_i^s(s) s^n ds = 0$

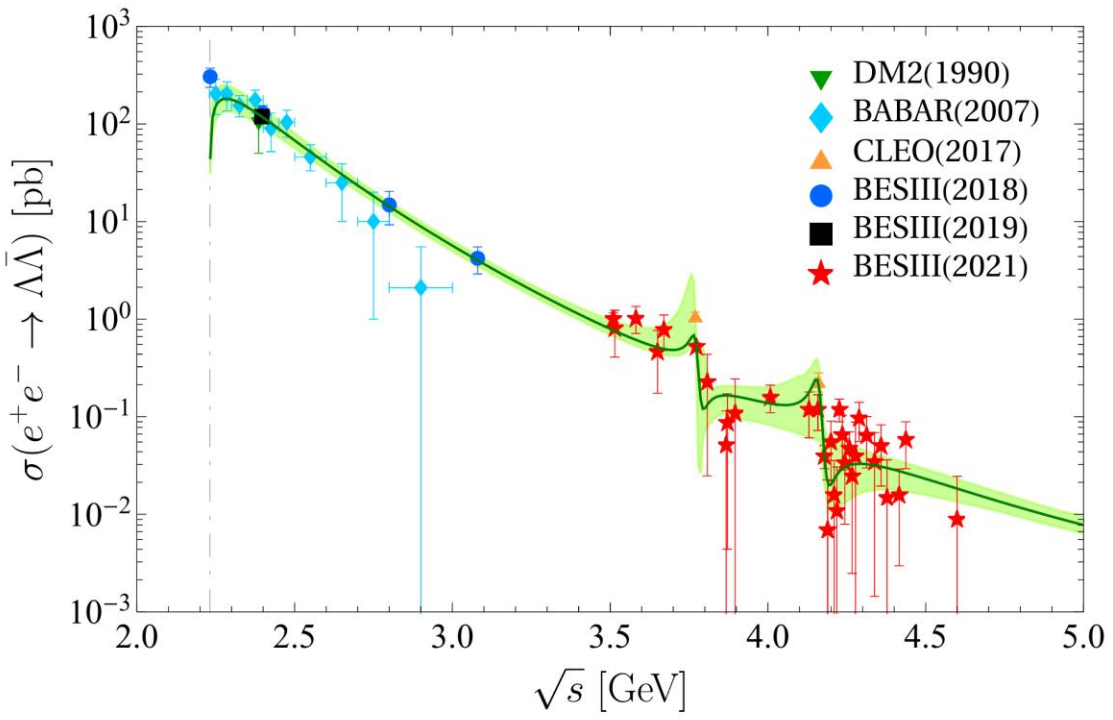
$< r_\Lambda^2 >_E$ fixed by CHPT

$F_1,\ F_2$

$r_\Lambda^M,\ g_{V\Lambda\Lambda}$

B. Kubis et al. EPJC (2001)

❖ Example2: Extraction of Λ magnetic radius



(g_1, g_2)	ω	ϕ
N	$(18.6 \pm 4.3, 8.4 \pm 6.6)$	-
Λ	$(18.04 \pm 0.13, -34.45 \pm 0.24)$	$(-16.81 \pm 0.24, 18.99 \pm 0.25)$

- CHPT constrained magnetic radius

$$r_\Lambda^M = 0.681(2)\,\mathrm{fm}$$

- Test the $SU(3)$ flavor symmetry

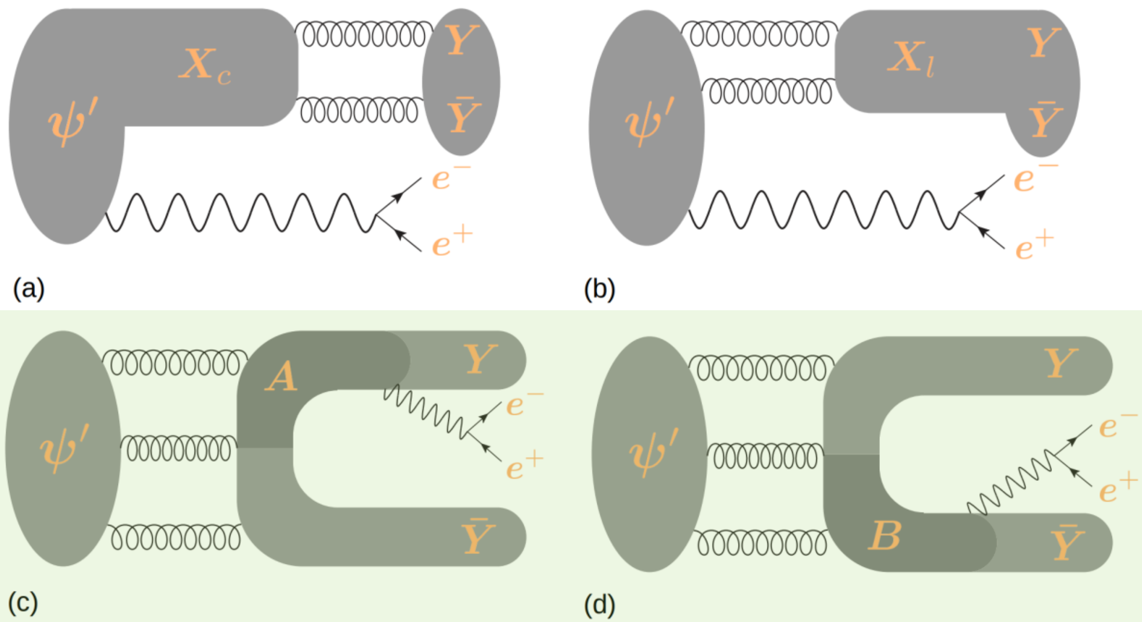
$$\frac{g_1^{\omega\Lambda\Lambda}}{g_1^{\omega NN}} = \frac{2}{3}\frac{5\alpha_{BBV}-2}{4\alpha_{BBV}-1} \longrightarrow \alpha_{BBV} \geq 0.82$$

$$\frac{g_1^{\phi\Lambda\Lambda}}{g_1^{\omega NN}} = \frac{-\sqrt{2}}{3}\frac{2\alpha_{BBV}+1}{4\alpha_{BBV}-1} \longrightarrow \alpha_{BBV} \leq 0.8$$

❖ Hyperon radius at STCF

- Radiative Dalitz decay of charmonium

$$\psi \to Y\bar{Y}e^+e^-$$



$$\psi \to Y\bar{Y}\gamma_{\mathrm{FSR}}$$

Advantages to access radius:

- No hyperon beam is required
- Extremely low momentum transfer $4m_e^2 = 1.05 \times 10^{-6}$ GeV^2
- Rich ψ' events at STCF
- Isospin symmetry:

$$\mathcal{M}_{(a)+(b)}(\psi \to Y^c\bar{Y}^c e^+e^-) = \mathcal{M}_{(a)+(b)}(\psi \to Y^0\bar{Y}^0 e^+e^-)$$

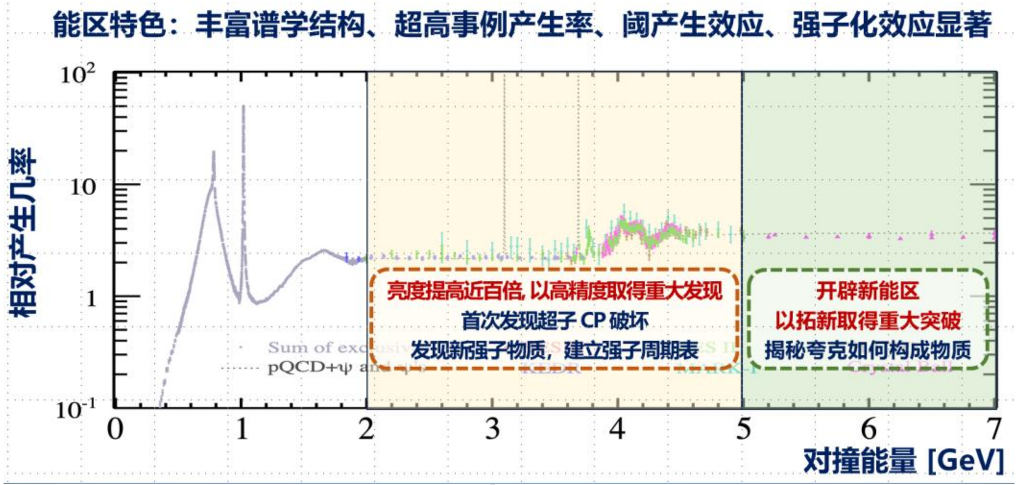


Image: H.-P. Peng’s talk

❖ Hyperon radius at STCF

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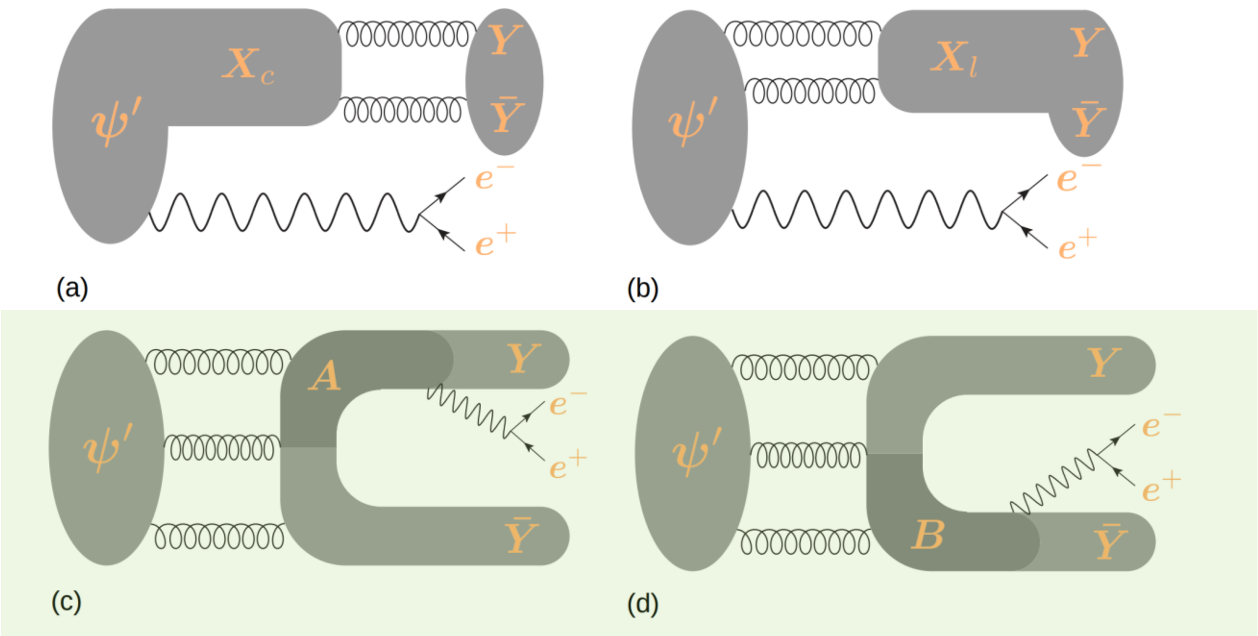
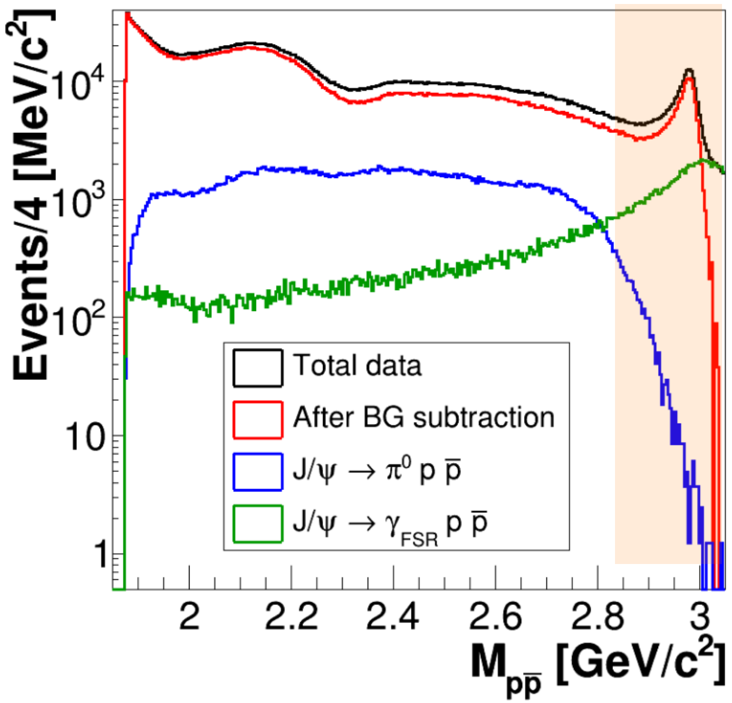


$$\psi \to Y\bar{Y}\gamma_{FSR}$$

Disadvantages: signal is small portion of the whole radiative decay

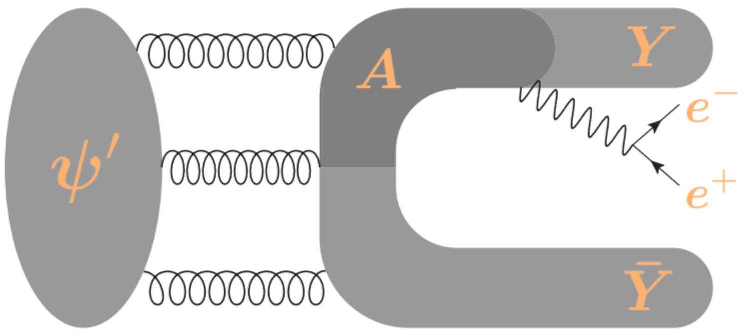
$$J/\psi \to p\bar{p}\gamma$$



R. Kappert, PhD thesis, Groningen U. (2022)

❖ Hyperon radius at STCF

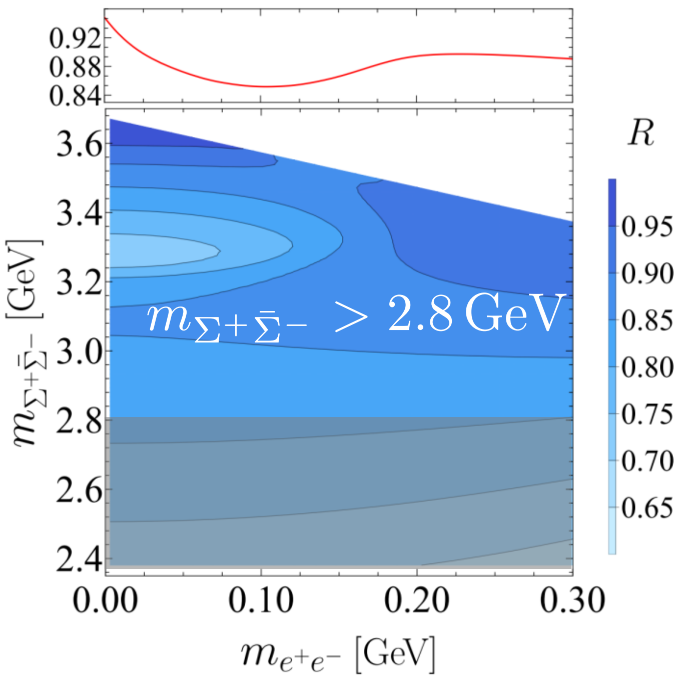
☐ Selection of kinematic region

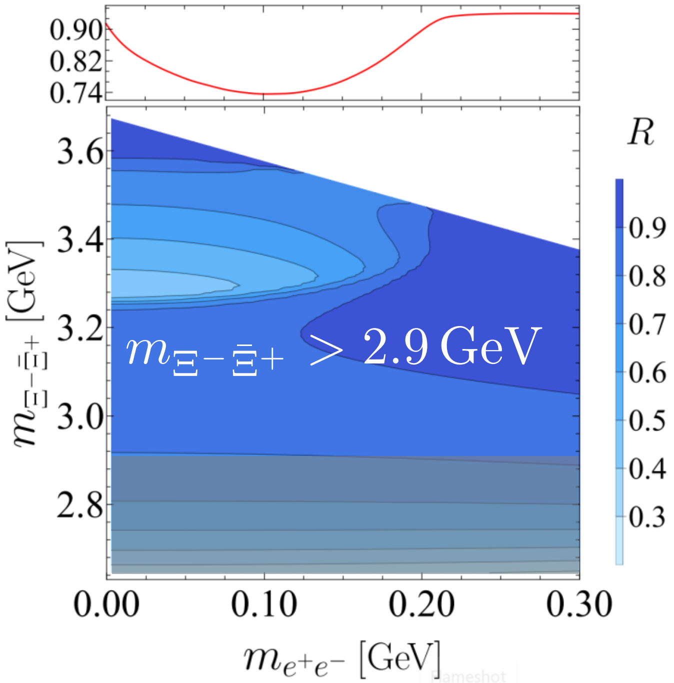


Identify a signal region dominated by Y pole:

$$\psi' \to \bar{Y} + A(A \to Y\gamma^*)$$

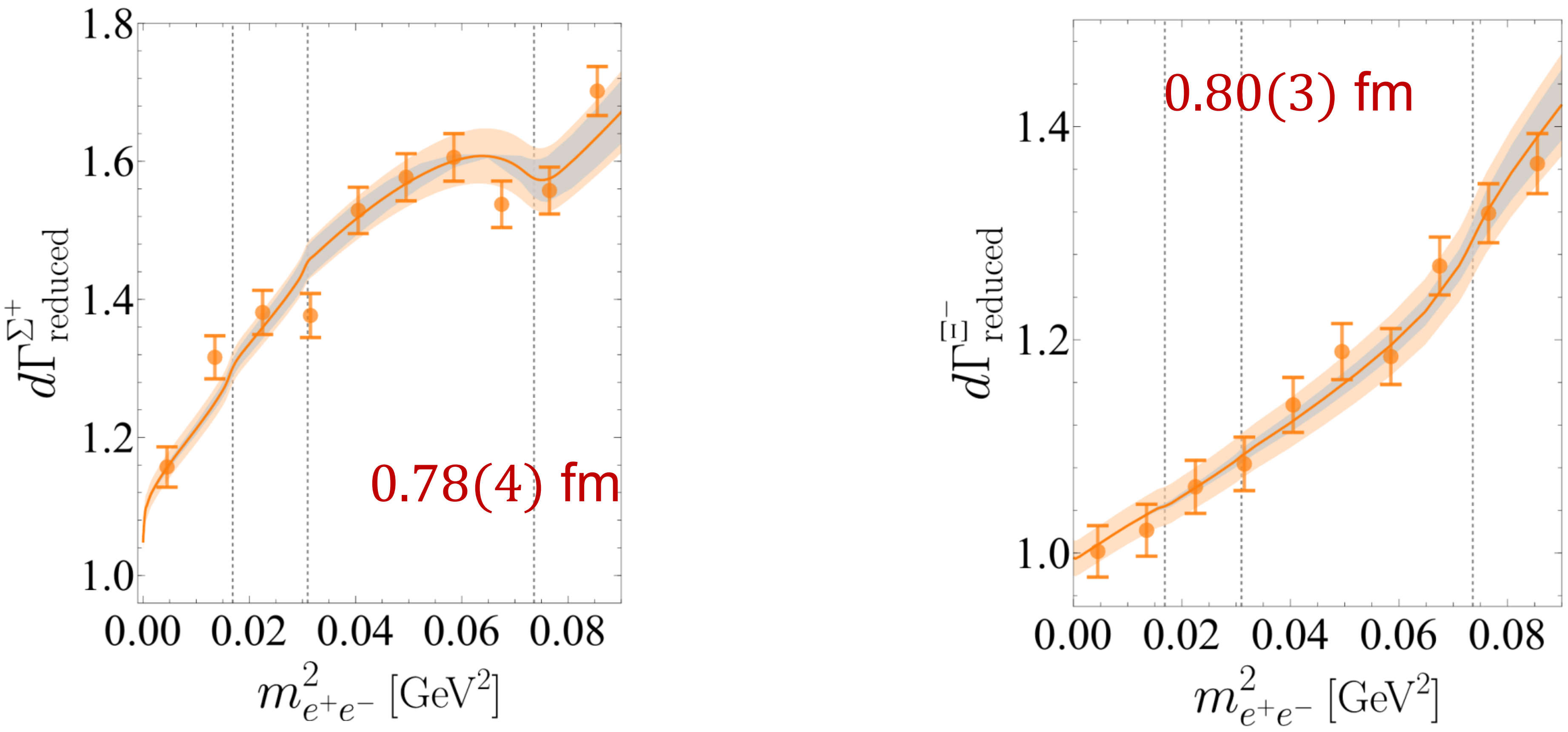
$$\frac{dR}{dm_{e^+e^-}\,dm_{Y\bar{Y}}} = \frac{\int d\cos\theta^*\, d\cos\theta'\, d\phi\, d\Gamma^{Y}_{A+B}}{\int d\cos\theta^*\, d\cos\theta'\, d\phi\, d\Gamma^{Y+Y^*}_{A+B}}$$





❖ Hyperon radius at STCF

- Sensitivity analysis by fitting to the Monte Carlo events (2×10^4 about $10^9\ \psi'$)



Summary

- Dispersion relation is a powerful tool for EMFFs analysis
  - Connecting unknown LECs to measurable observables
  - A systematic construction of nucleon and hyperon EMFFs becomes possible
- Radiative Dalitz decays of charmonium offer a feasible exp. approach to measuring hyperon charge radii

Thank you for your attention!