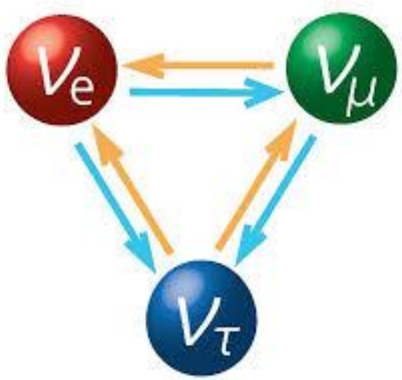
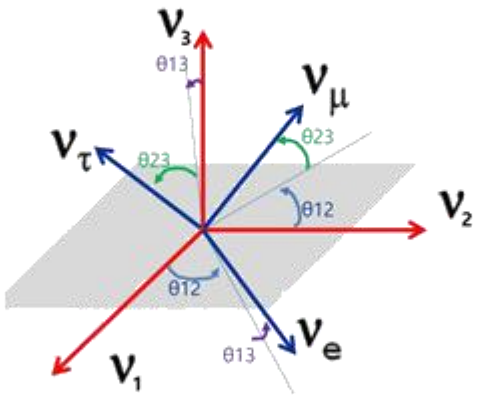
Neutrino oscillation and origin of neutrino mass

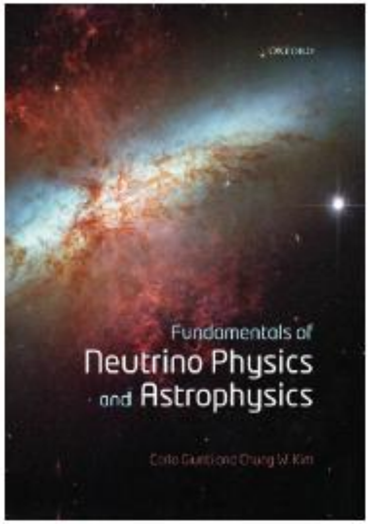
Gui-Jun Ding

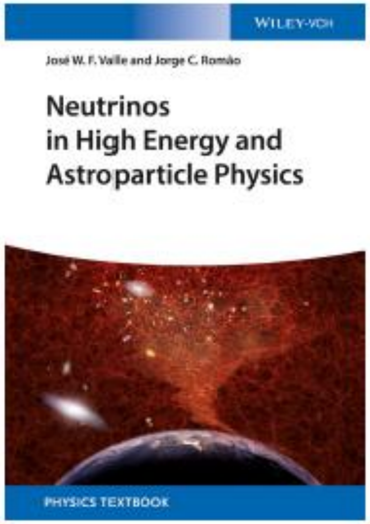
University of Science and Technology of China

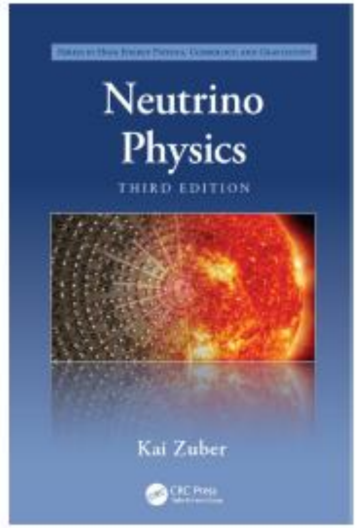


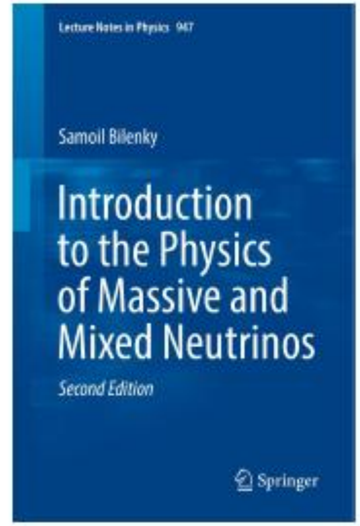


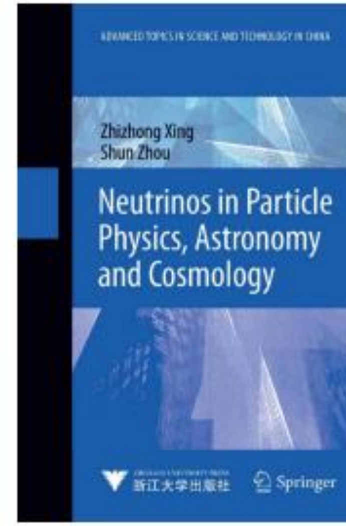
References: many excellent books…

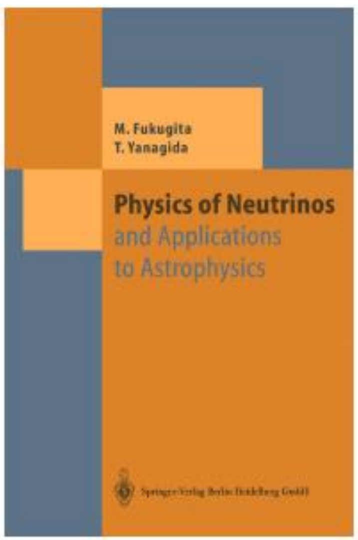


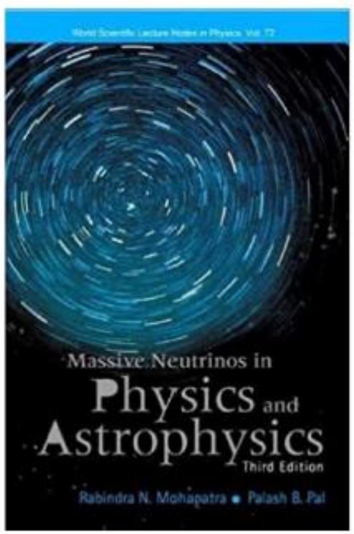


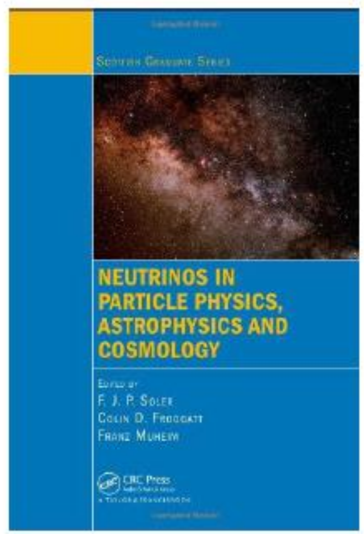


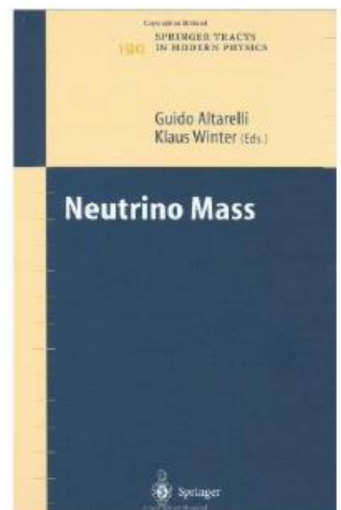


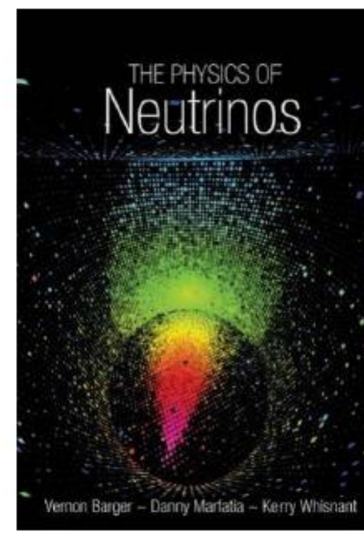












Many excellent reviews/lecture notes…

- **“Neutrino Physics”,** P. Hernandez, arXiv:1708.01046
- **“2004 TASI Lectures on Neutrino Physics”,** Andre de Gouvea, hep-ph/0411274
- **“TASI 2002 lectures on neutrinos”,** Yuval Grossman, hep-ph/0305245
- **“Neutrino masses and mixings and…”,** Alessandro Strumia, Francesco Vissani, hep-ph/0606054
- **“Neutrino Physics (CERN-2014-001)”,** Zhi-Zhong Xing, arXiv:1406.7739
- **“Neutrino Mass and New Physics”,** R. N. Mohapatra, A. Y. Smirnov, Ann.Rev.Nucl.Part.Sci. 56 (2006) 569-628, hep-ph/0603118
- **“Leptonic CP Violation”,** G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim, Rev.Mod.Phys. 84 (2012) 515-565, arXiv:1111.5332
- **“From the trees to the forest: a review of radiative neutrino mass models”,** Yi Cai, Juan Herrero-García, Michael A. Schmidt, Avelino Vicente, Raymond R. Volkas, arXiv:1706.08524
- **“Lepton flavor symmetries”,** Ferruccio Feruglio, Andrea Romanino, Rev.Mod.Phys. 93 (2021) 1, 015007, arXiv:1912.06028
- **“The symmetry approach to quark and lepton masses and mixing”,** Gui-Jun Ding, Jose W.F. Valle, Phys.Rept. 1109 (2025) 1-105, arXiv:2402.16963

Outline

Lecture 1: introduction to neutrino physics

Lecture 2: neutrino mixing and neutrino oscillation

Lecture 3: quark masses and mixing in SM

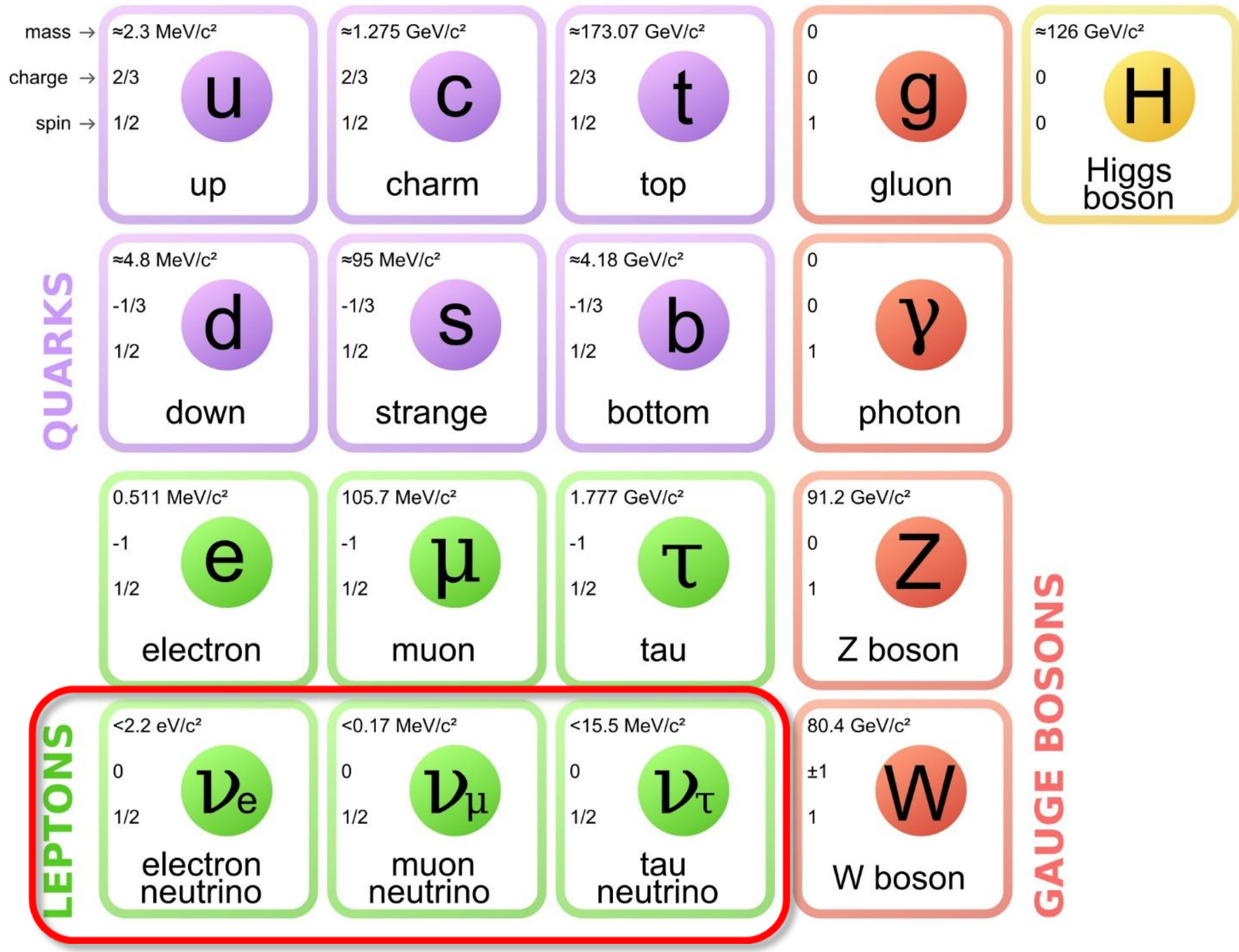
Lecture 4:neutrino mass generation mechanism

Lecture 5:neutrino Probe absolute neutrino mass and neutrinoless double beta decay

Lecture 1

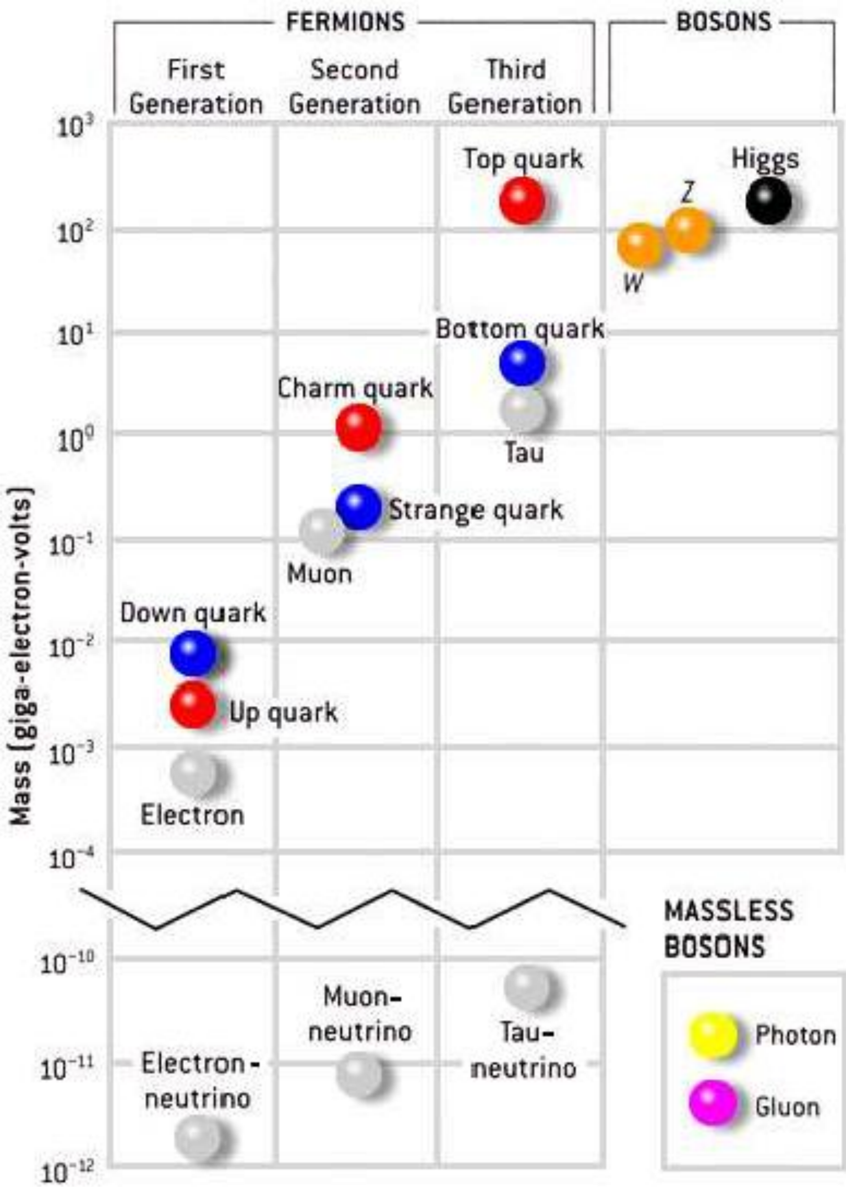
- Neutrino: concept and discovery
- Helicity and Handedness
- Neutrinos in standard model

The standard model (SM)



Neutrino is a portal to new physics!

SM mass spectrum



- **Neutrino Properties:**
  charge = 0
  spin = 1/2
  mass <1eV
  speed≃c

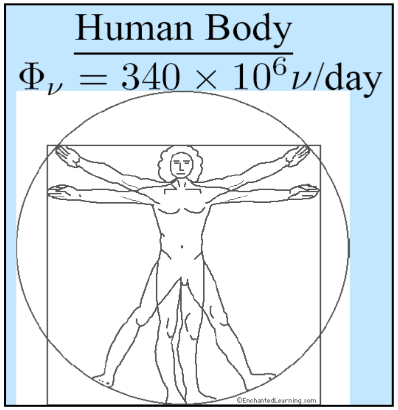
- Neutrinos are lighter than all other fermions by at least a factor 10^6

- Units

$$1\text{GeV} = 1.78 \times 10^{-27}\text{kg}$$

$$1\text{GeV} = 10^3\text{MeV} = 10^9\text{eV}$$

Where are neutrinos produced?



Reactors

Sun

Accelerators

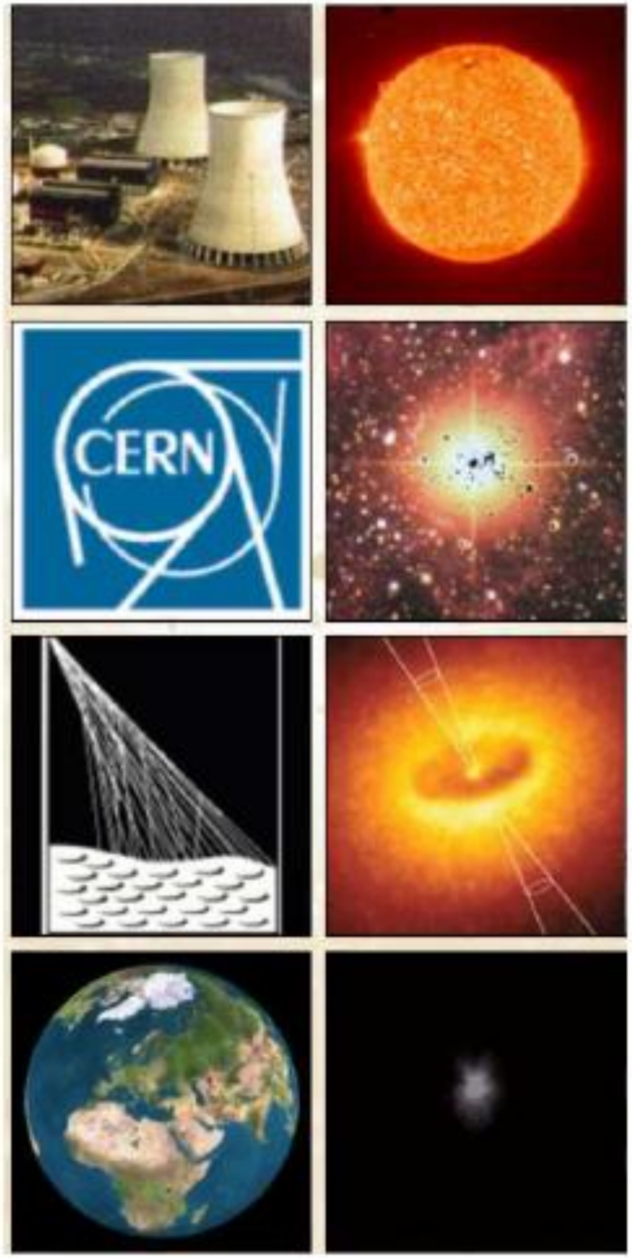
Supernovae

Earth atmosphere

Active galaxies

Earth crust radioactivity

Big Bang



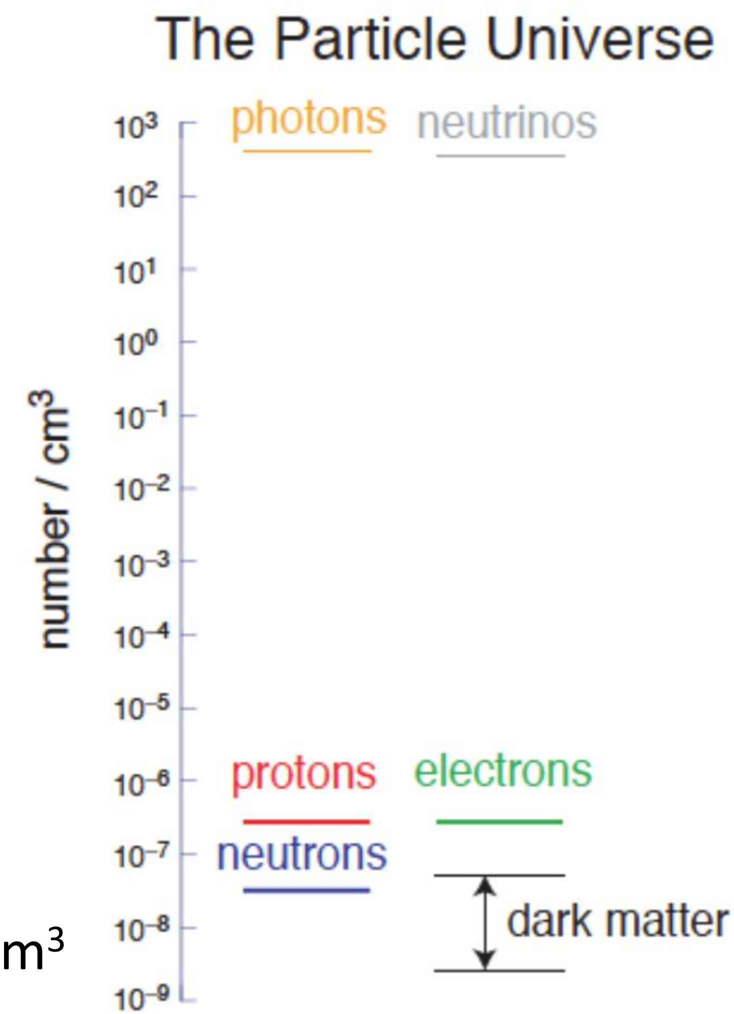
Neutrinos are everywhere

Every second we are traversed by:

- 400×10^{12} neutrinos from the Sun
- 50×10^{9} neutrinos from natural radioactivity
- 10×10^{9} neutrinos from nuclear power plants

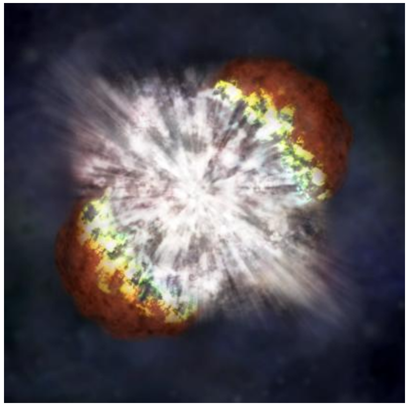
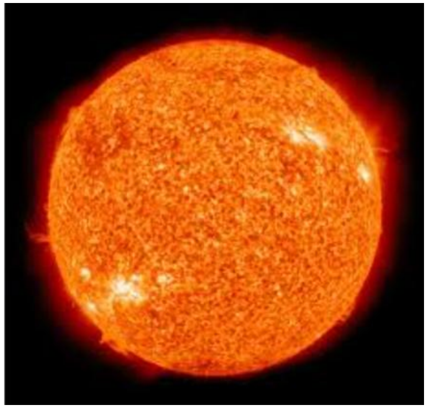
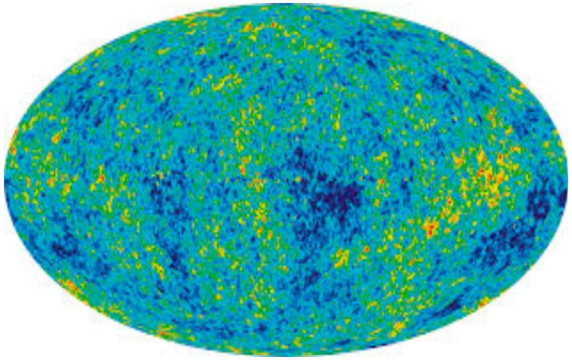
Moreover:

- our body emits 400 neutrinos/s (^{40}K decay)
- the Universe contains ~ 330 neutrinos/cm^3



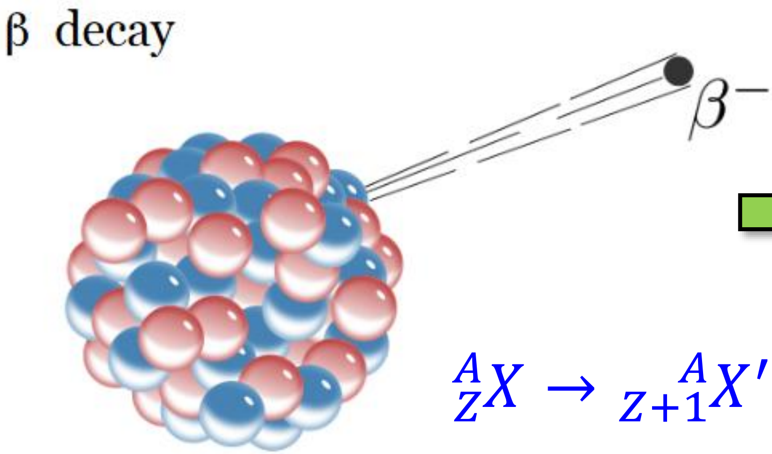
Why neutrinos are so important?

- they can probe environments that other techniques cannot: SN explosions, core of the Sun,...
- their role is crucial for the evolution of the universe (Big Bang Nucleosynthesis, structure formation)
- they could help explaining the matter-antimatter asymmetry of the Universe (leptogenesis mechanism)
- they could be a component of the dark matter of the universe.


Beta decay and enegy crisis?

1900 Radioactivity: Becquerel, M & P Curie, Rutherford…

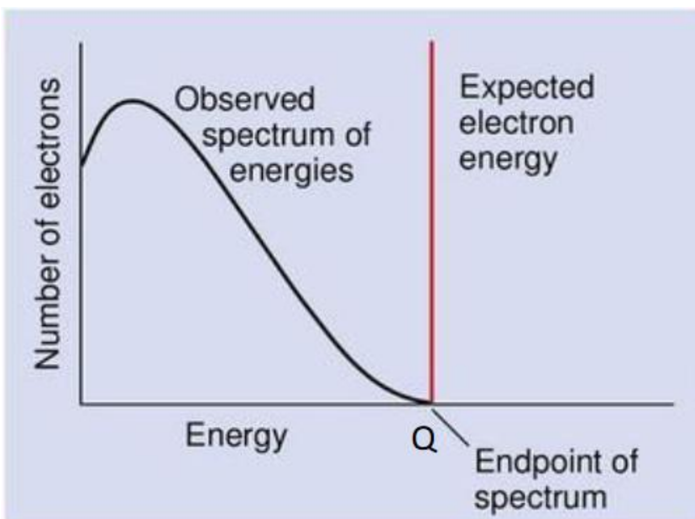


Energy conservation:

$$E_{\text{electron}} \simeq (M_N - M_{N'})c^2 = Q$$

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}X' + e^-$$

- Electron spectrum of β decay: why continuous not single peak?



ν hypothesis and first kinematical properties at 1930

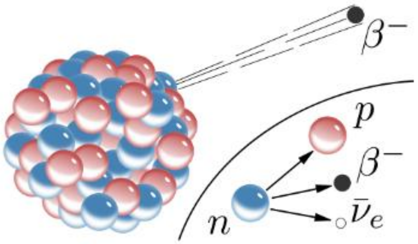
- Pauli introduced the neutrino **in a famous letter** to explain continuous electron spectrum in nuclear beta decay.

Dear Radioactive Ladies and Gentlemen,
As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle, and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...
Unfortunately, I cannot personally appear in Tübingen since I am indispensable here in Zürich because of a ball on the night from December 6 to 7….

zero electric charge, spin 1/2, tiny mass

1930: $m_\nu < 0.01$ GeV

Today: $m_\nu < 0.1 \sim 1$ eV



$${}^{A}_{Z}X \rightarrow {}_{Z+1}^{\;\;A}X' + e^- + \bar{\nu}$$

Three years later: ν name and first dynamical properties

- At 1933, Fermi postulated the first theory of nuclear beta decay, the theory of weak interactions


ANNO IV - VOL. II - N. 12 QUINDICINALE 31 DICEMBRE 1933 - XII

LA RICERCA SCIENTIFICA

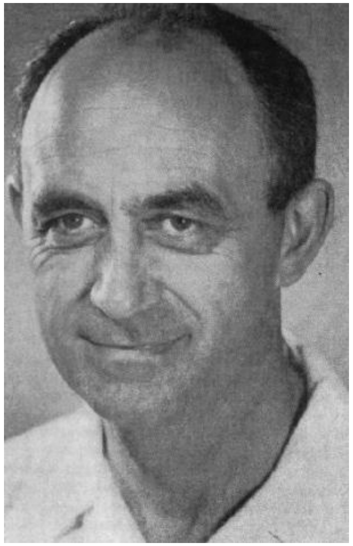
ED IL PROGRESSO TECNICO NELL'ECONOMIA NAZIONALE

Tentativo di una teoria dell'emissione dei raggi "beta"

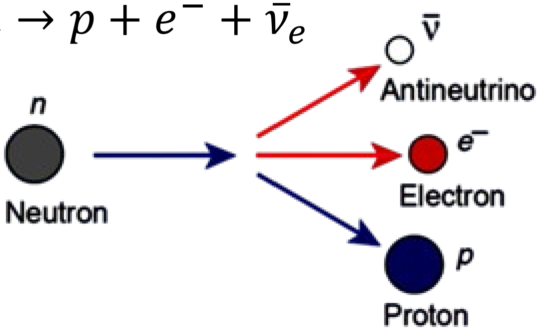
Nota del prof. ENRICO FERMI

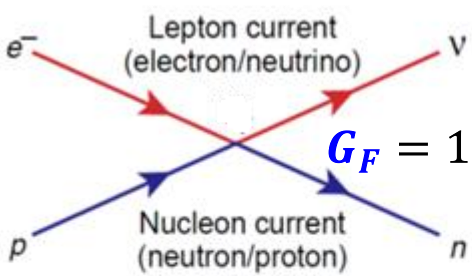
Riassunto: Teoria della emissione dei raggi β delle sostanze radioattive, fondata sull'ipotesi che gli elettroni emessi dai nuclei non esistano prima della disintegrazione ma vengano formati, insieme ad un neutrino, in modo analogo alla formazione di un quanto di luce che accompagna un salto quantico di un atomo. Confronto della teoria con l'esperienza.

Nature did not publish this article: "contained speculations too remote from reality to be of interest to the reader…"



$n \to p + e^- + \bar{\nu}_e$





$G_F = 1.166 \times 10^{-5}\text{GeV}^{-2}$

set the energy scale of $\sqrt{1/G_F} \sim O(100)\text{GeV}$ of weak interactions

→new name for particle: neutrino

How to detect neutrinos

- At 1934, Bethe and Peierls calculated the cross section σ for the processes:

$$\nu + n \rightarrow p + e^-$$
$$\bar{\nu} + p \rightarrow n + e^+$$

Fermi theory prediction:

$$\sigma \approx 10^{-44}\text{cm}^2 \left(\frac{E_\nu}{m_e c^2}\right)^2$$

For $E_\nu \sim 2$ MeV , $\sigma \sim 10^{-43}\text{cm}^2$, to be compared with $\sigma_{\gamma p} \sim 10^{-25}\text{cm}^2$

"It is therefore absolutely impossible to observe the processes of this kind with the neutrinos created in nuclear transformations."

- mean free path of neutrinos: $\lambda \approx \frac{1}{n\sigma}$

neutrino mean free path in water: $\lambda_{\text{water}} \approx 1.7 \times 10^{17}\text{m} \sim 15$ Light years

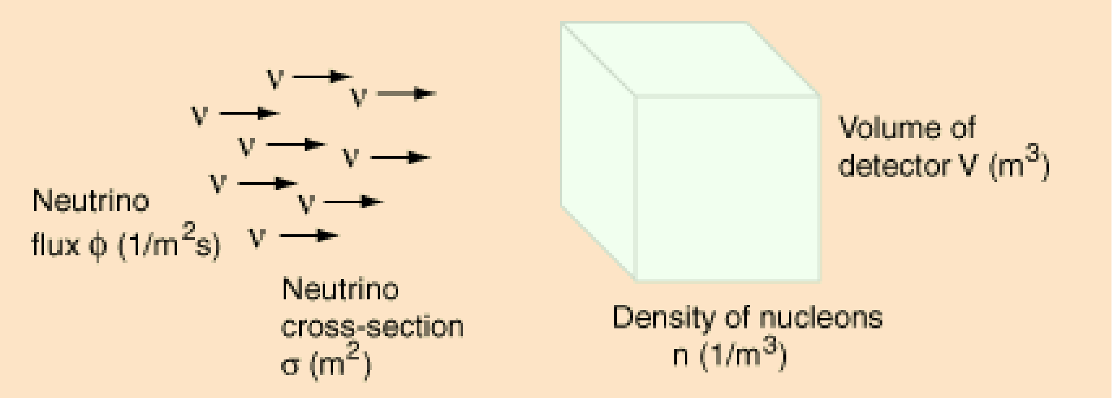
neutrino mean free path in lead: $\lambda_{\text{lead}} \approx 1.5 \times 10^{16}\text{m} \sim 1.5$ Light years

Neutrino: impossible to detect?

"I have done something very bad today by proposing a particle that cannot be detected. It is something that no theorist should ever do."—Pauli, 1930

Pauli's worst insult to a theory: "Not even wrong"

- Event number in a neutrino experiment:



$$N = \Phi \sigma N_{targ} \Delta t$$

with a 1000 kg detector and a flux of 10^{10} ν/cm²/s: few ν events/day

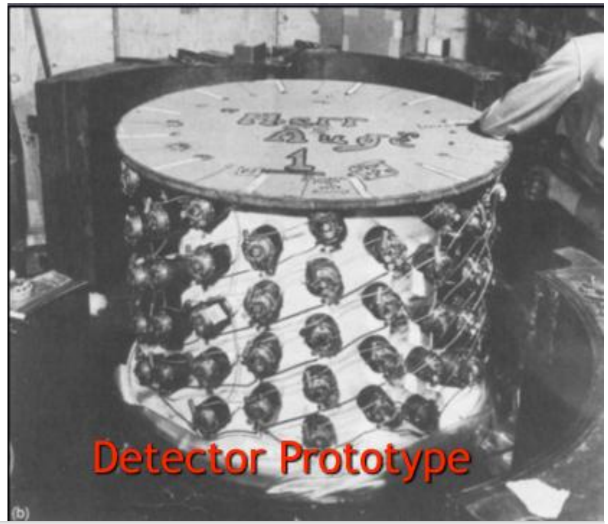
—solar neutrino flux $\sim 7 \times 10^{10}$ ν/cm²/s

—reactor neutrino flux $\sim 10^{20}$ ν/cm²/s

Difficult but not impossible!

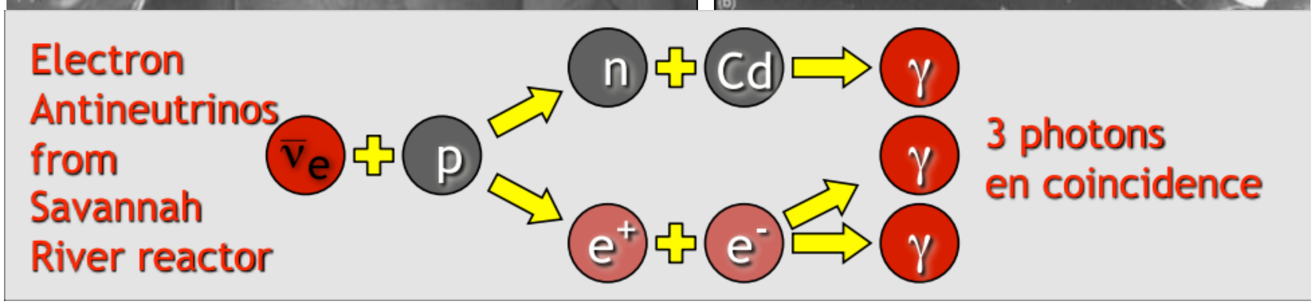
Discovery of the neutrino

- At 1956, first observation of reactor ν_e by Reines and Cowan.



Scintillator
$H_2O + CdCl_2$

1995 Nobel Prize in Physics to Reines



Telegram to Pauli on 12/06/1956

"We are happy to inform you that we have definitely detected neutrinos from fission fragments by observing inverse beta decay of protons. Observed cross section agrees well with expected six times ten to minus forty-four square centimeters"

Modern versions of Reines&Cowan experiment: Chooz, Dchooz, Daya Bay, RENO… still making discoveries today !

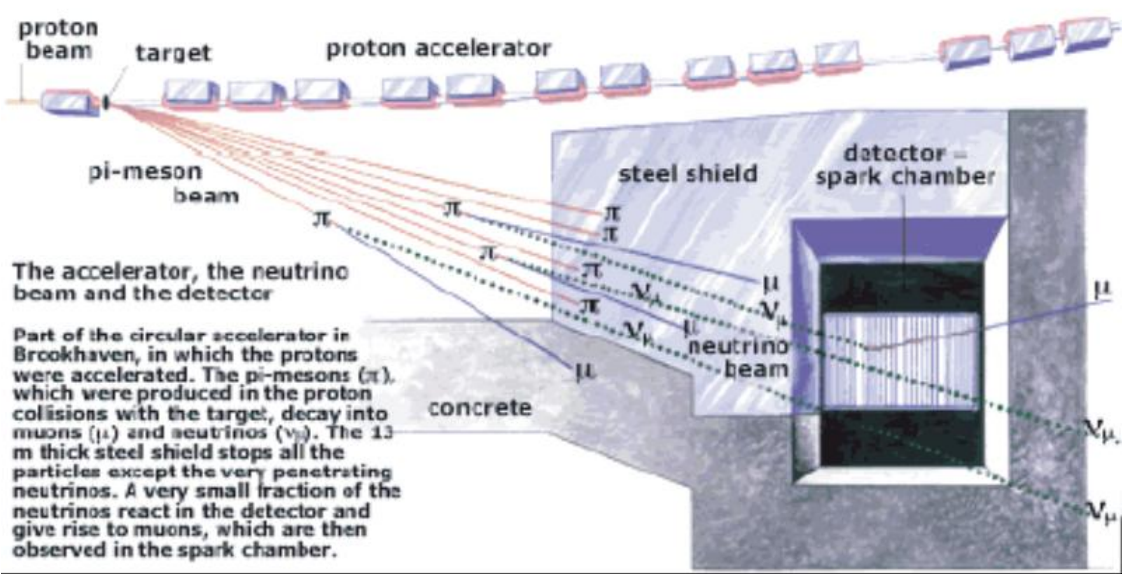
More than one neutrino flavours?

- At 1959, Pontecorvo suggested the existence of a different neutrino, associated to muon decay and proposed an experiment to check it

$$\nu_{\text{acc}} + n \rightarrow p + (e^- \ or \ \mu^-?\,)$$

$$\binom{\nu_e}{e} \quad \binom{\nu_\mu}{\mu}$$

- At 1962: Discovery of ν_μ by Lederman, Schwartz and Steinberger



$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\nu_\mu + n \rightarrow p + \mu^-$$

not e^-

Leon M. Lederman | Melvin Schwartz | Jack Steinberger

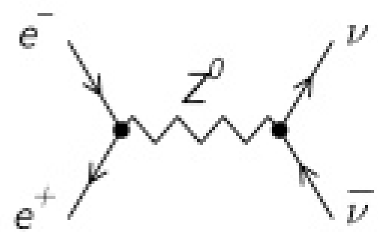
1988 Nobel Prize in Physics

Modern versions of Lederman, Schwartz, Steinberger experiment are accelerator neutrino experiments: MINOS, OPERA, T2K, NoVA,…

More than two neutrino flavours?

- At 1978: Discovery of τ at SLAC → imbalance of energy in τ decay suggests existence of a third neutrino.

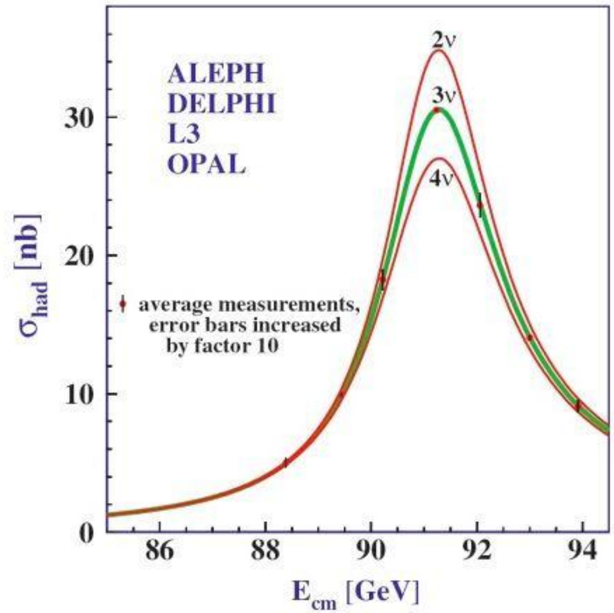
- At 1989: LEP measurements of the invisible decay width of Z boson



$$\Gamma_{\text{inv}} \equiv \Gamma_Z - \Gamma_{\text{had}} - 3\Gamma_{\text{lep}}$$

$$\text{N}_\nu = \Gamma_{\text{inv}}/\Gamma_{\text{SM}}(Z \to \nu_i \overline{\nu_i})$$

$$\to \text{N}_\nu = 2.984 \pm 0.008$$



- At 2000: Discovery of ν_τ by the DONUT Collaboration.

800 GeV p → D_s meson ($\equiv c\bar{s}$) → $\nu_\tau$beam → τ detected

Parity violation in weak interactions

- $\tau - \theta$ puzzle

$$\theta^+ \rightarrow \pi^+ + \pi^0 \qquad P = (-1) \times (-1) = +1$$

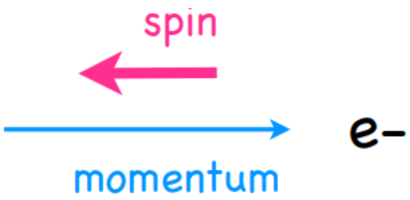
$$\tau^+ \rightarrow \pi^+ + \pi^+ + \pi^- \qquad P = (-1) \times (-1) \times (-1) = -1$$

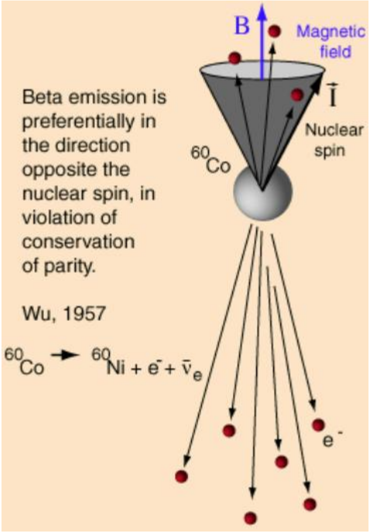
$$\tau^+ = \theta^+ = K^+$$

- At 1956: Lee and Yang proposed parity violation in weak interactions

1957 Nobel Prize in Physics

- At 1957: using a radioactive source of ^{60}Co, Wu et. al. determined that weak interaction violates parity conservation maximally:





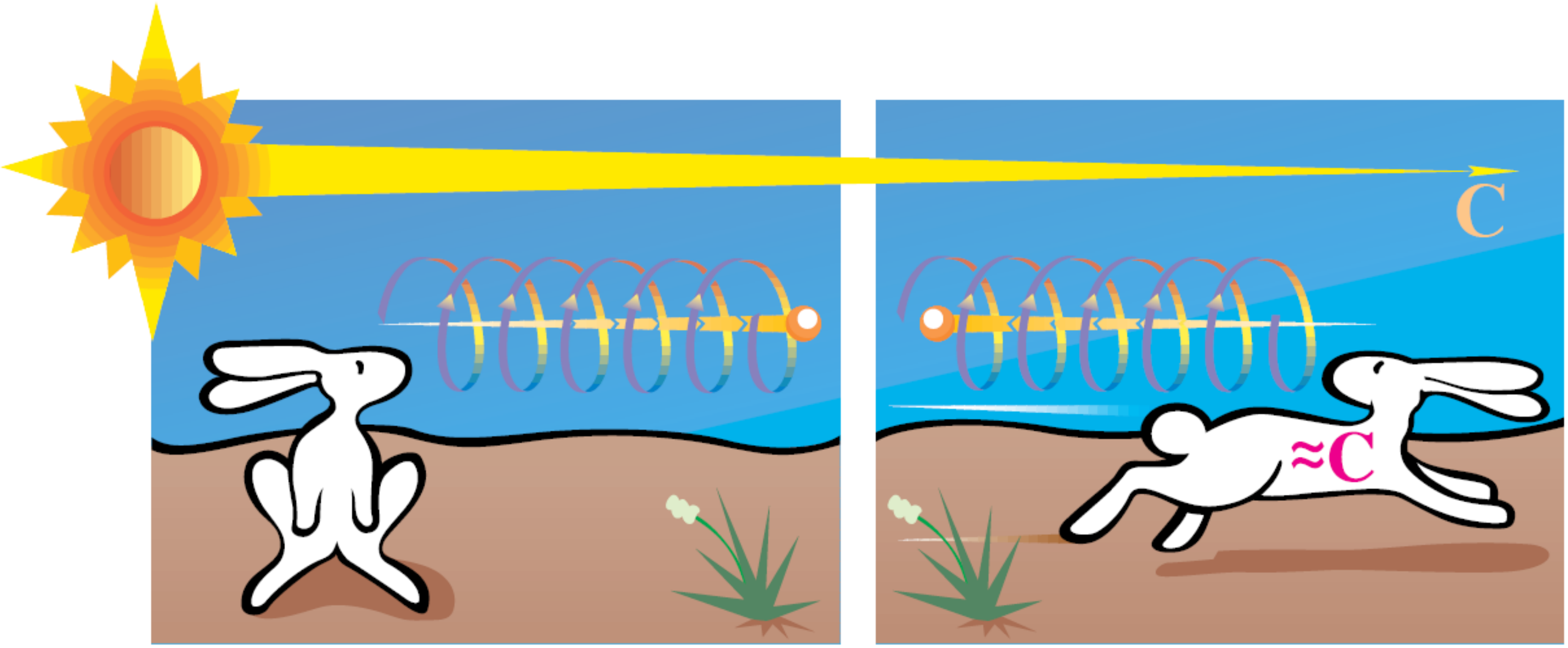
- At 1958: Goldhaber et. al. measured the helicity of the neutrino → only left-handed neutrinos (spin antiparallel to neutrino direction) participate in the weak interaction (right handed antineutrinos)

Helicity

The Helicity of a fermion relates its spin to its direction of motion

$$\lambda = \boldsymbol{s} \cdot \boldsymbol{p} / |\boldsymbol{p}| = \pm 1/2$$

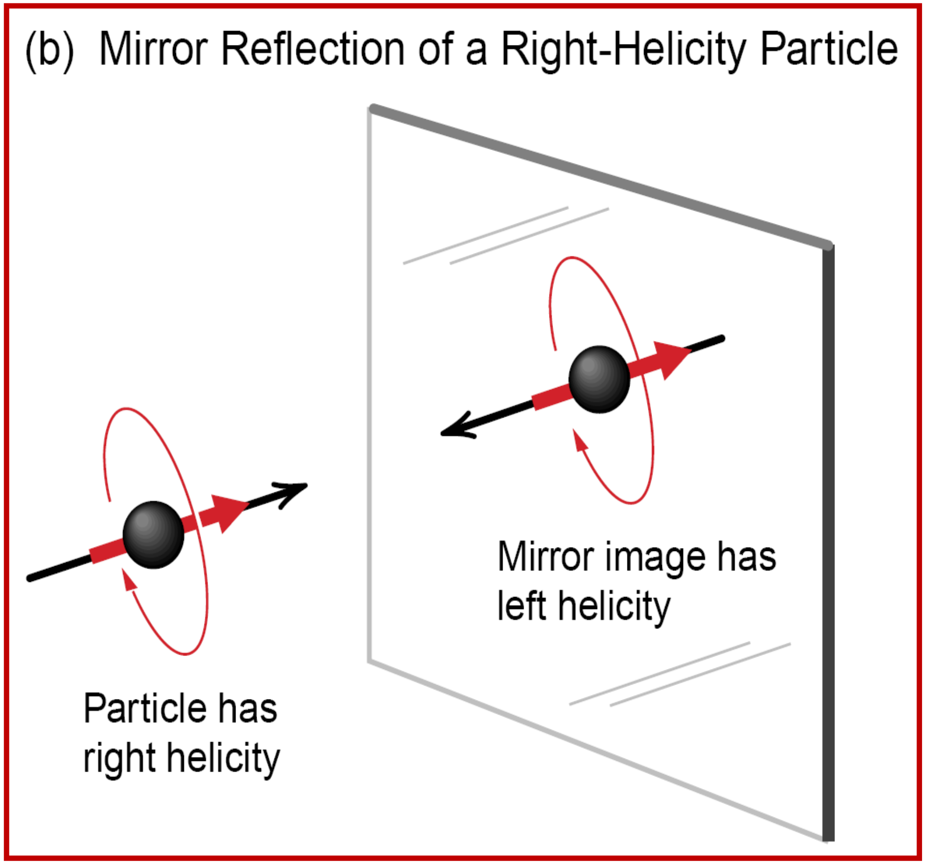
A massive particle's helicity isn't relativistically invariant.

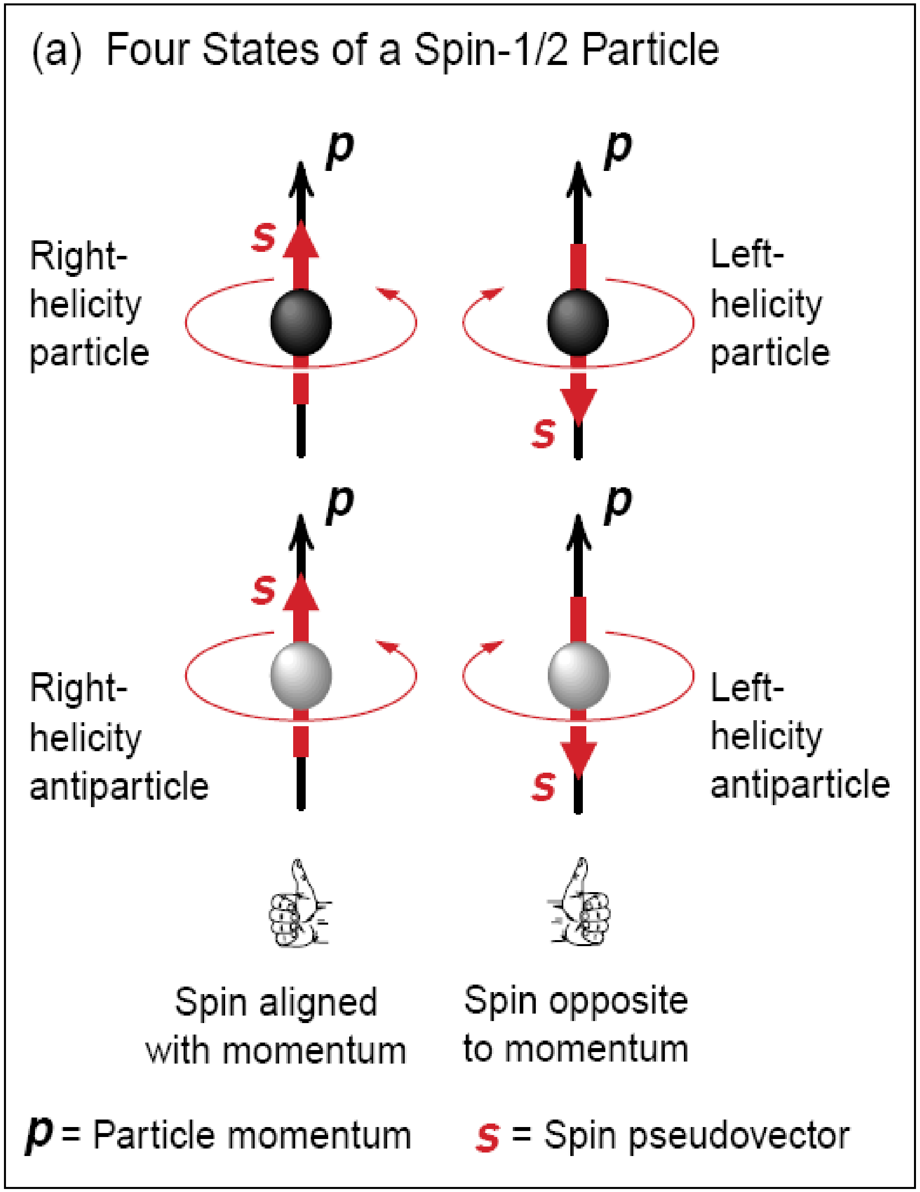


Looks like a left-handed corkscrew. No—like a right-handed corkscrew!

Parity and helicity

Parity and Helicity: if parity were conserved, a spin-1/2 particle would exist in both left- & right-helicity states.





Handedness

- The Handedness of a spin-1/2 particle is a relativistically invariant quantity to describe this particle's spin states.

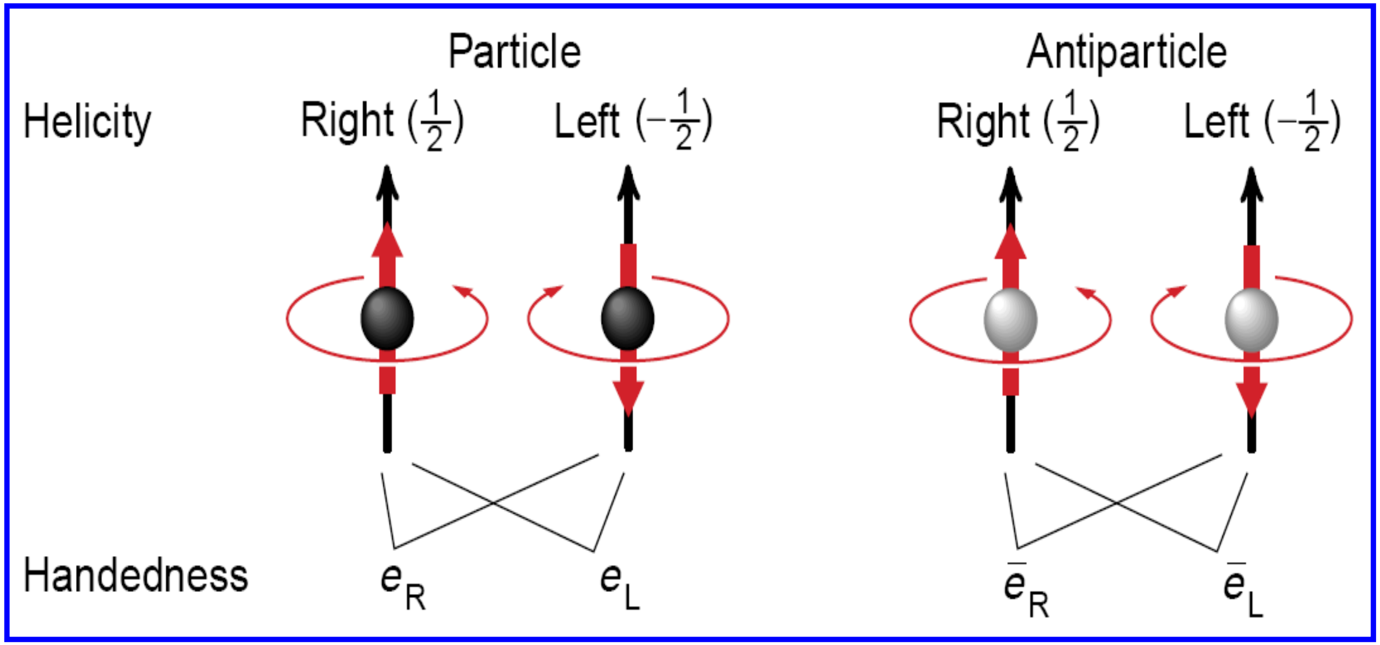
- 2 independent handedness or chirality states: left & right.

- A massless or massive particle can always be decomposed into two independent components: left- and right-handed.

$$\psi = \psi_L + \psi_R$$

$$\psi_L = \frac{1+\gamma_5}{2}\psi$$

$$\psi_R = \frac{1-\gamma_5}{2}\psi$$



Neutrino interactions in SM

- SM Gauge group: $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$
- Fermion fields and transformation under SM $(SU(3), SU(2)_L)_{U(1)_Y}$

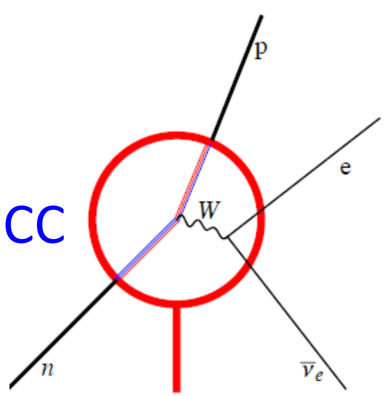
$(1,2)_{-\frac{1}{2}}$	$(3,2)_{-\frac{1}{6}}$	$(1,1)_{-1}$	$(3,1)_{-\frac{2}{3}}$	$(3,1)_{-\frac{1}{3}}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

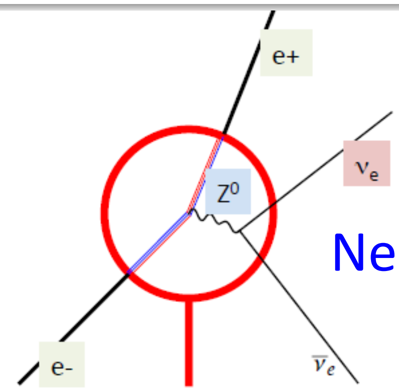
No right-handed neutrinos, thus neutrino mass is zero in renormalizable SM

- Neutrino interactions: charged currents and neutral current interactions

$$\mathcal{L}_{SM} \supset -\frac{g}{\sqrt{2}} \sum_f \bar{\nu}_{Lf} \gamma_\mu l_{Lf} W^+_\mu - \frac{g}{2\cos\theta_W} \sum_f \bar{\nu}_{Lf} \gamma_\mu \nu_{Lf} Z_\mu + h.c.$$

Charged currents: CC

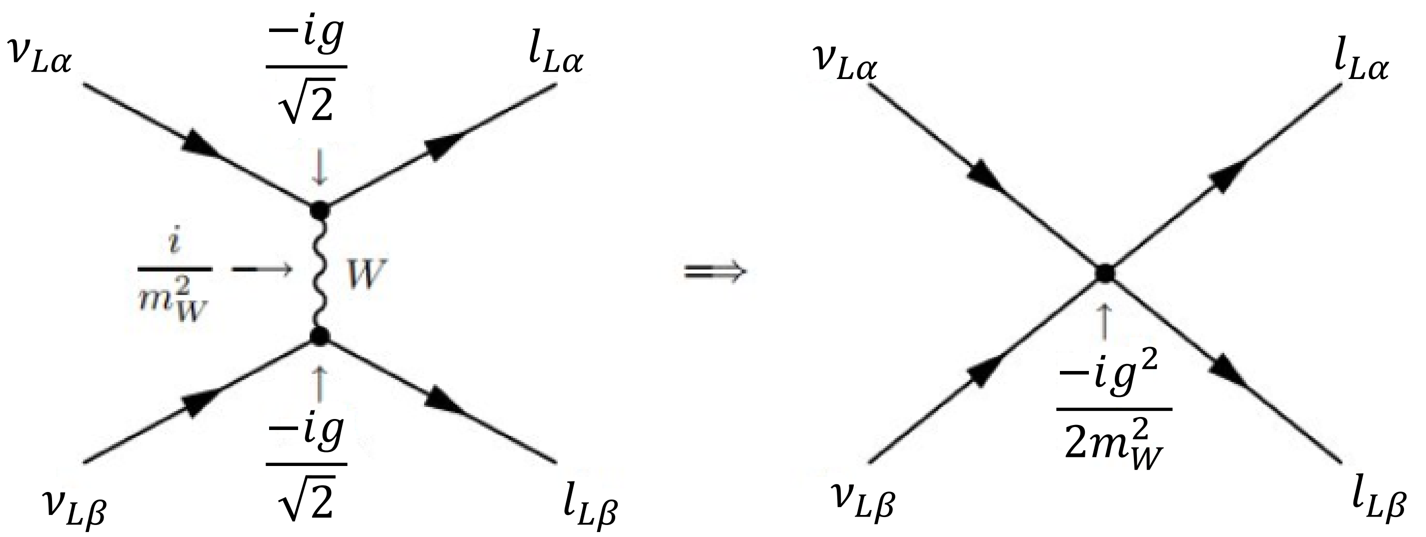




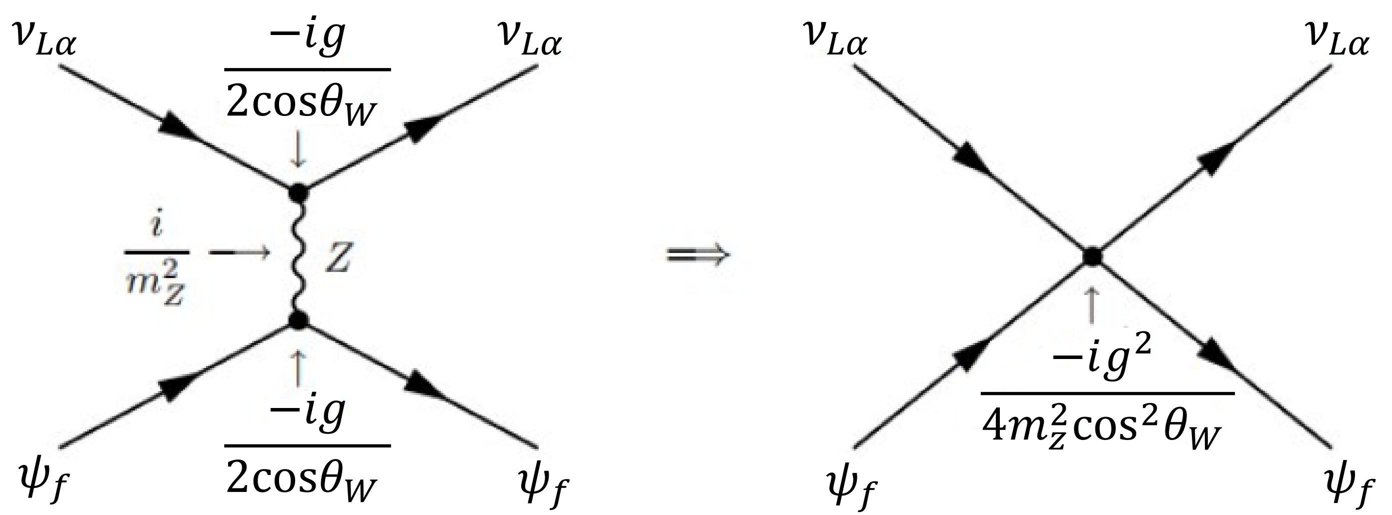
Neutral currents: NC

➢ Four-fermion interactions at low energy

CC



NC

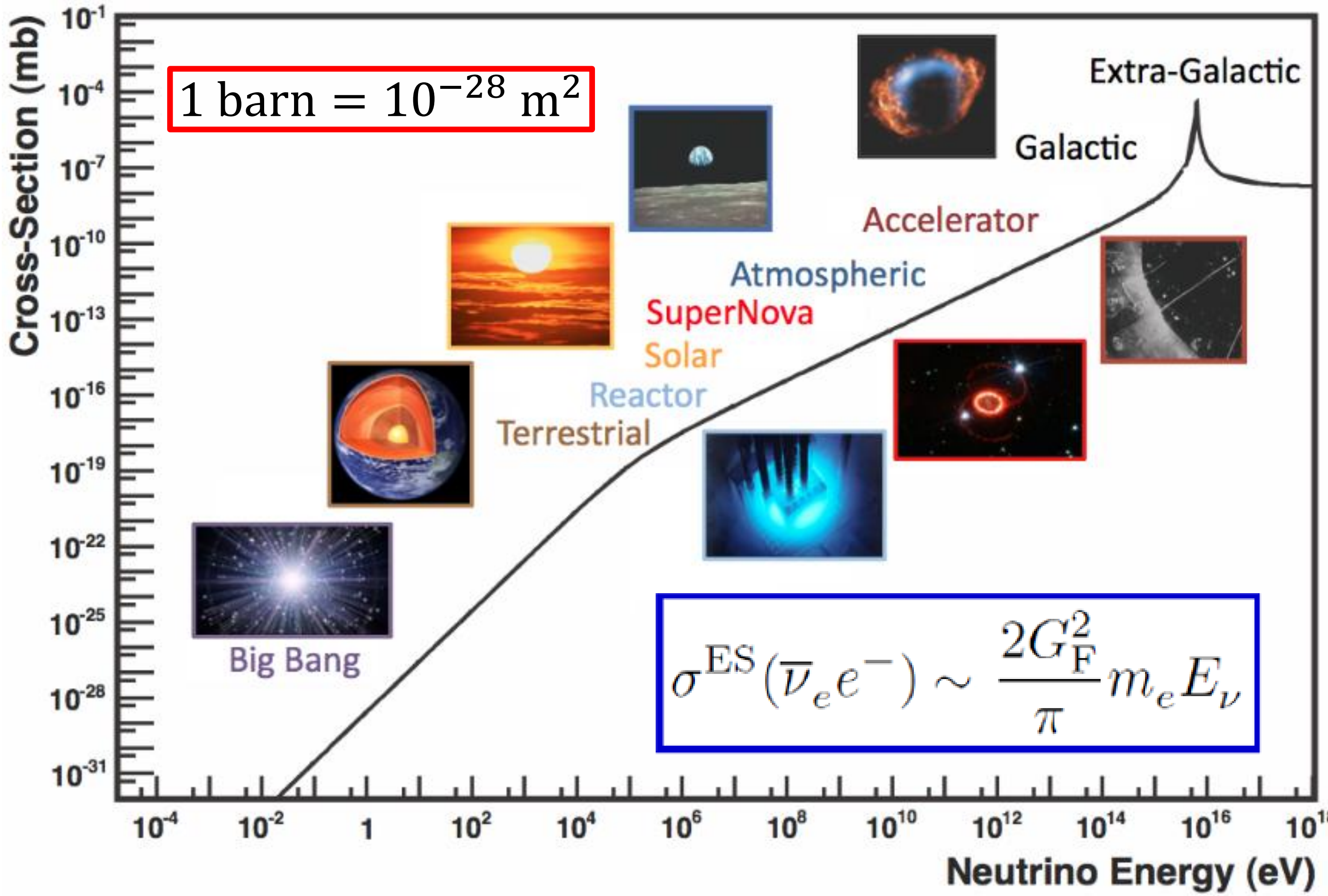


ψ_f can be quark and lepton fields of SM

Fermi constant:

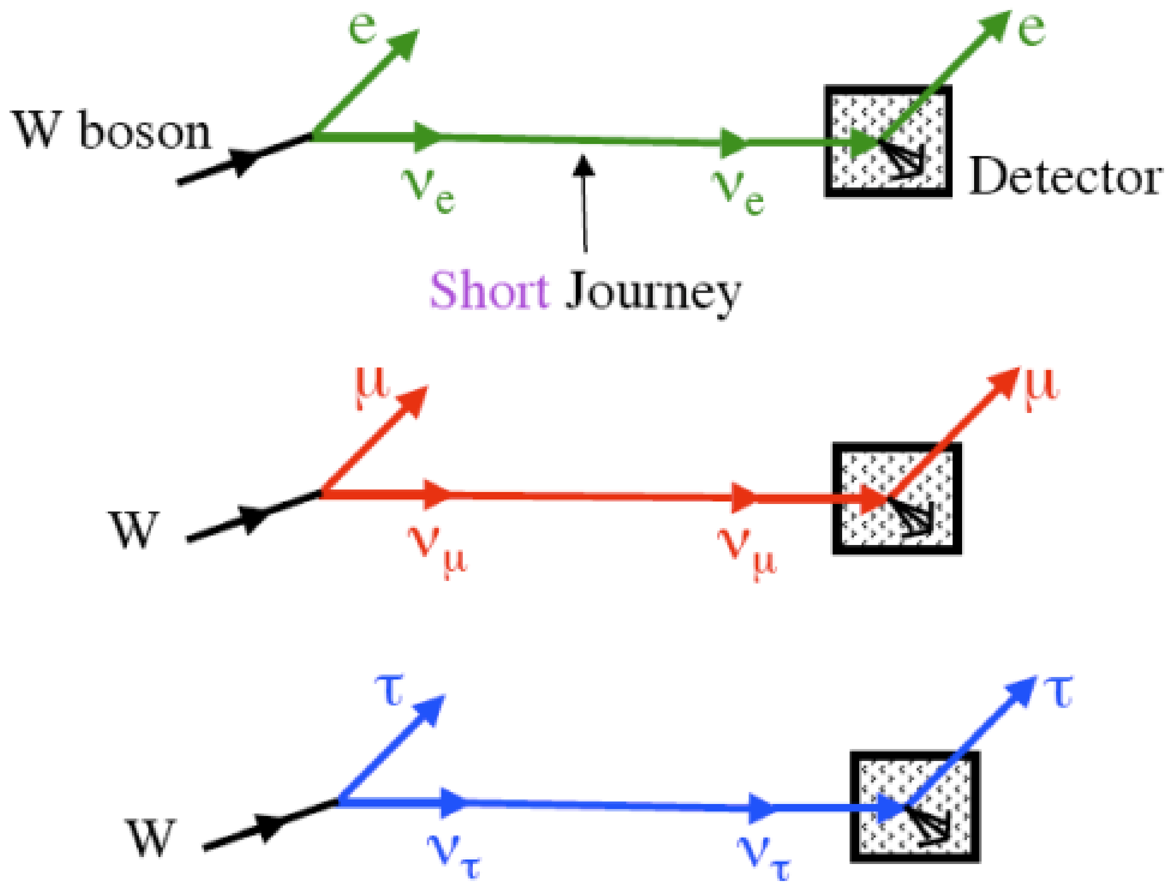
$$\frac{G_F}{\sqrt{2}} \equiv \frac{g^2}{8m_W^2} = 1.166 \times 10^{-5}\text{GeV}^{-2}$$

- The neutrino interactions are very weak and very difficult to be observed



[Formaggio,Zeller, Rev.Mod.Phys. 84 (2012) 1307, arXiv: 1305.7513]

- Picture of neutrino interactions before discovery of neutrino oscillation



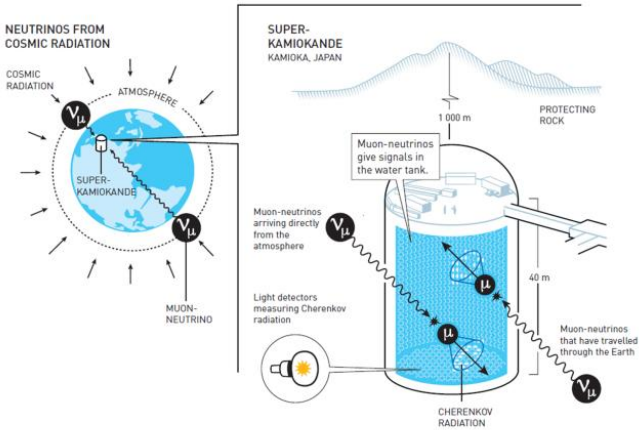
Lecture 2

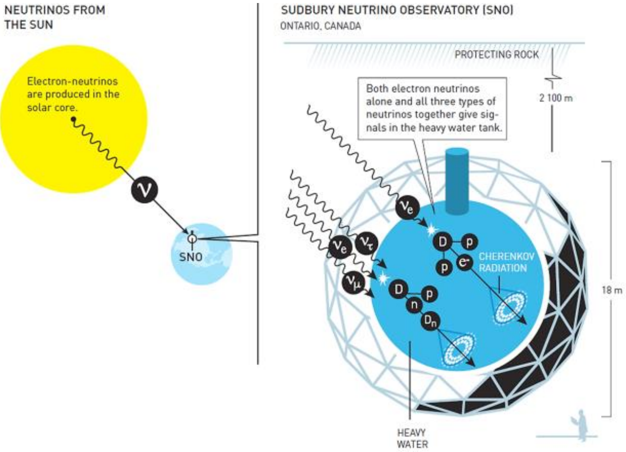
- **The 3x3 neutrino mixing matrix**
- **Neutrino oscillations in vacuum**
- **Neutrino oscillations in matter**

2015 Nobel Prize in Physics

Takaaki Kajita

Arthur B. McDonald





"for the discovery of neutrino oscillations, which shows that neutrinos have mass"

Neutrino mixing

- Neutrino interactions: charged currents and neutral current interactions

$$\mathcal{L}_{SM} \supset -\frac{g}{\sqrt{2}}\sum_f \bar{\nu}_{Lf}\gamma_\mu l_{Lf} W_\mu^+ - \frac{g}{2\cos\theta_W}\sum_f \bar{\nu}_{Lf}\gamma_\mu \nu_{Lf} Z_\mu + h.c.$$

- If the flavor states are transformed into the mass states, the source of flavor mixing and CP violation will show up in the *CC* interactions:

Quarks: $\mathcal{L}_{CC}^{q} = -\frac{g}{\sqrt{2}}\overline{(u\ c\ t)_L}\gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + h.c.$

Leptons: $\mathcal{L}_{CC}^{l} = -\frac{g}{\sqrt{2}}\overline{(e\ \mu\ \tau)_L}\gamma^\mu U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W_\mu^- + h.c.$

- V: quark mixing matrix, it is named as CKM (Cabibbo-Kobayashi-Maskawa) matrix which is a 3x3 unitary matrix.
- U: neutrino mixing matrix, it is also called PMNS(Pontecorvo–Maki–Nakagawa–Sakata) matrix, it is a 3x3 unitary matrix too at leading order.

Parameterization of neutrino mixing matrix

The 3×3 unitary matrix U can always be parametrized as a product of 3 unitary rotation matrices in the complex planes:

$$O_1(\theta_1, \alpha_1, \beta_1, \gamma_1) = \begin{pmatrix} c_1 e^{i\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix}$$

$$O_2(\theta_2, \alpha_2, \beta_2, \gamma_2) = \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{i\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix}$$

$$O_3(\theta_3, \alpha_3, \beta_3, \gamma_3) = \begin{pmatrix} c_3 e^{i\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix}$$

$s_i \equiv \sin\theta_i, c_i \equiv \cos\theta_i$
$(i = 1,2,3)$

- **Category A: 3 possibilities**

$$U = O_i O_j O_i \ (i \neq j)$$

- **Category B: 6 possibilities**

$$U = O_i O_j O_k \ (i \neq j \neq k)$$

Phases in neutrino mixing matrix

For example, the standard parametrization of neutrino mixing matrix is given by:

U

$$= \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix} \begin{pmatrix} c_3 e^{\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix} \begin{pmatrix} c_1 e^{\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix}$$

$$= \begin{pmatrix} c_1 c_3 e^{i(\alpha_1+\gamma_2+\alpha_3)} & s_1 c_3 e^{i(-\beta_1+\gamma_2+\alpha_3)} & s_3 e^{i(\gamma_1+\gamma_2-\beta_3)} \\ -s_1 c_2 e^{i(\beta_1+\alpha_2+\gamma_3)} - c_1 s_2 s_3 e^{i(\alpha_1-\beta_2+\beta_3)} & c_1 c_2 e^{i(-\alpha_1+\alpha_2+\gamma_3)} - s_1 s_2 s_3 e^{i(-\beta_1-\beta_2+\beta_3)} & s_2 c_3 e^{i(\gamma_1-\beta_2-\alpha_3)} \\ s_1 s_2 e^{i(\beta_1+\beta_2+\gamma_3)} - c_1 c_2 s_3 e^{i(\alpha_1-\alpha_2+\beta_3)} & -c_1 s_2 e^{i(-\alpha_1+\beta_2+\gamma_3)} - s_1 c_2 s_3 e^{i(-\beta_1-\alpha_2+\beta_3)} & c_2 c_3 e^{i(\gamma_1-\alpha_2-\alpha_3)} \end{pmatrix}$$

$$= \begin{pmatrix} e^{ia} & 0 & 0 \\ 0 & e^{ib} & 0 \\ 0 & 0 & e^{ic} \end{pmatrix} \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix}$$

$$a = (\alpha_1 - \beta_1) - (\alpha_2 + \beta_2 - \gamma_2) - \gamma_3 \,, \quad b = -\beta_2 - \alpha_3 \,, \quad c = -\alpha_2 - \alpha_3 \,;$$
$$x = \beta_1 + (\alpha_2 + \beta_2) + (\alpha_3 + \gamma_3) \,, \quad y = -\alpha_1 + (\alpha_2 + \beta_2) + (\alpha_3 + \gamma_3) \,, \quad z = \gamma_1 \,.$$

The phases a, b, c can be absorbed by redefinition of charged lepton fields.

Physical Phases

If neutrinos are Dirac particles $\nu \neq \bar{\nu}$, the phases x, y and z can be removed. Then the neutrino mixing matrix is

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

for Dirac neutrinos

If neutrinos are Majorana particles $\nu = \bar{\nu}$, left- and right-handed fields are correlated. Hence only a common phase of three left-handed fields can be redefined (e.g., x= 0). Then

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

for Majorana neutrinos.

- Three mixing angles: solar neutrino mixing angle θ_{12}, atmospheric neutrino mixing angle θ_{23}, reactor neutrino mixing angle θ_{13}
- Three CP violation angles: Dirac CP violation phase δ, Majorana CP violation phase α_{21}, α_{31} only for Majorana neutrinos

- In the limit of two neutrino only, the neutrino mixing depends on 1 angle only (+ 1 Majorana phase)

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$ for Dirac neutrinos

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha_{21}/2} \end{pmatrix}$$ for Majorana neutrinos

- Three flavor neutrino mixing can be written as the product of two flavor mixing, and it depends on 3 angles and 1 Dirac (+2 Majorana) CP violating phases.

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

Atmospheric mixing | Reactor mixing & Dirac CP phase | Solar mixing | Majorana CP phases

Ultra-relativistic Approximation

Only neutrinos with energy $\gtrsim 0.1 MeV$ are detectable!

Charged-Current Processes: Threshold

$$\nu + A \to B + C$$
$$\Downarrow$$
$$s = 2Em_A + m_A^2 \geq (m_B + m_C)^2$$
$$\Downarrow$$
$$E_{\text{th}} = \frac{(m_B + m_C)^2}{2m_A} - \frac{m_A}{2}$$

$$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^- \qquad E_{\text{th}} = 0.233\,\text{MeV}$$
$$\nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^- \qquad E_{\text{th}} = 0.81\,\text{MeV}$$
$$\bar{\nu}_e + p \to n + e^+ \qquad E_{\text{th}} = 1.8\,\text{MeV}$$
$$\nu_\mu + n \to p + \mu^- \qquad E_{\text{th}} = 110\,\text{MeV}$$
$$\nu_\mu + e^- \to \nu_e + \mu^- \qquad E_{\text{th}} \simeq \frac{m_\mu^2}{2m_e} = 10.9\,\text{GeV}$$

Elastic Scattering Processes: Cross Section $\propto$ Energy

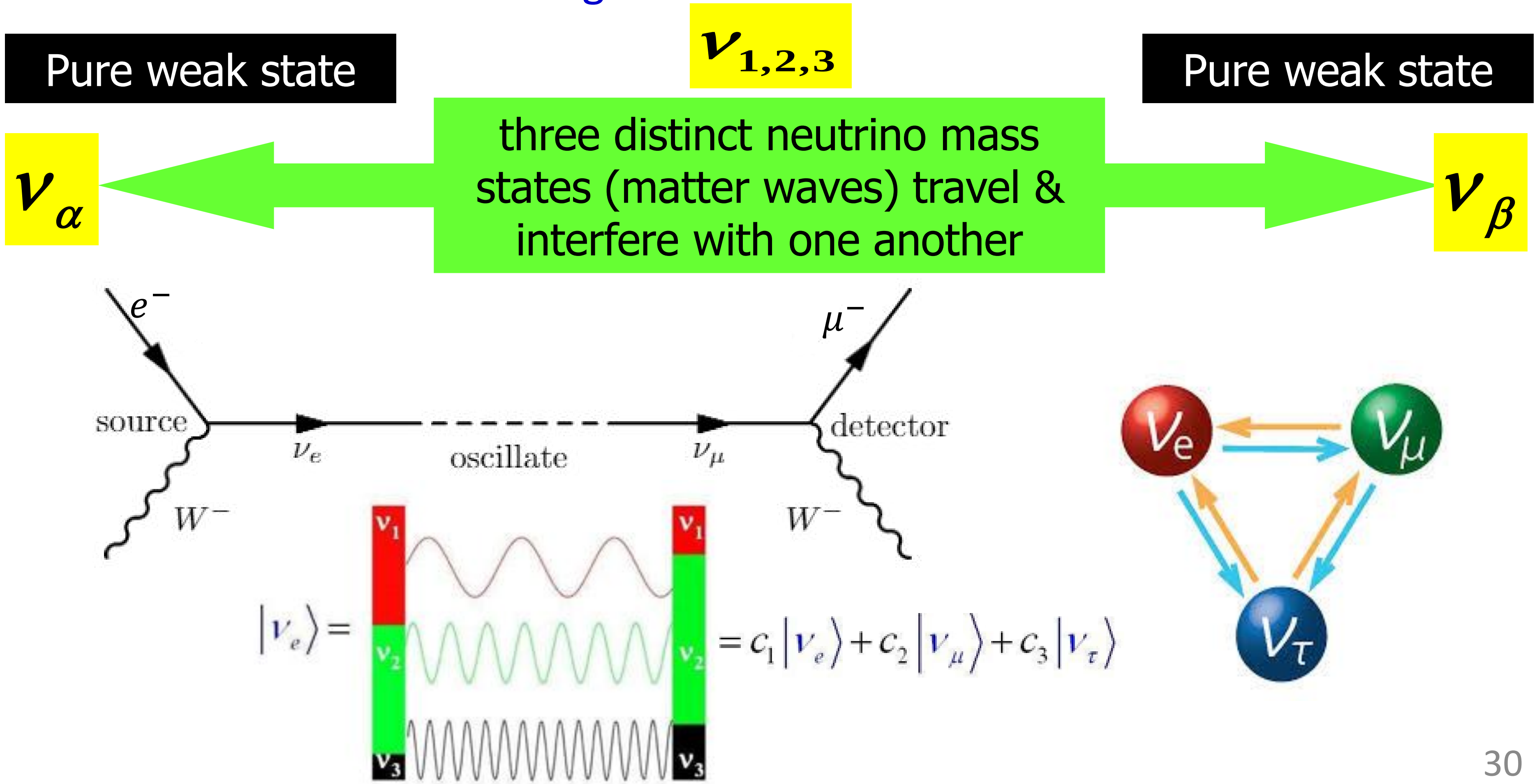
$$\nu + e^- \to \nu + e^- \qquad \sigma(E) \sim \sigma_0\, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\text{cm}^2$$

Background $\Longrightarrow$ $E_{\text{th}} \simeq 5\,\text{MeV}$ (SK, SNO), $0.25\,\text{MeV}$ (Borexino)

Laboratory and Astrophysical Limits $\Longrightarrow$ $m_\nu \lesssim 1\,\text{eV}$

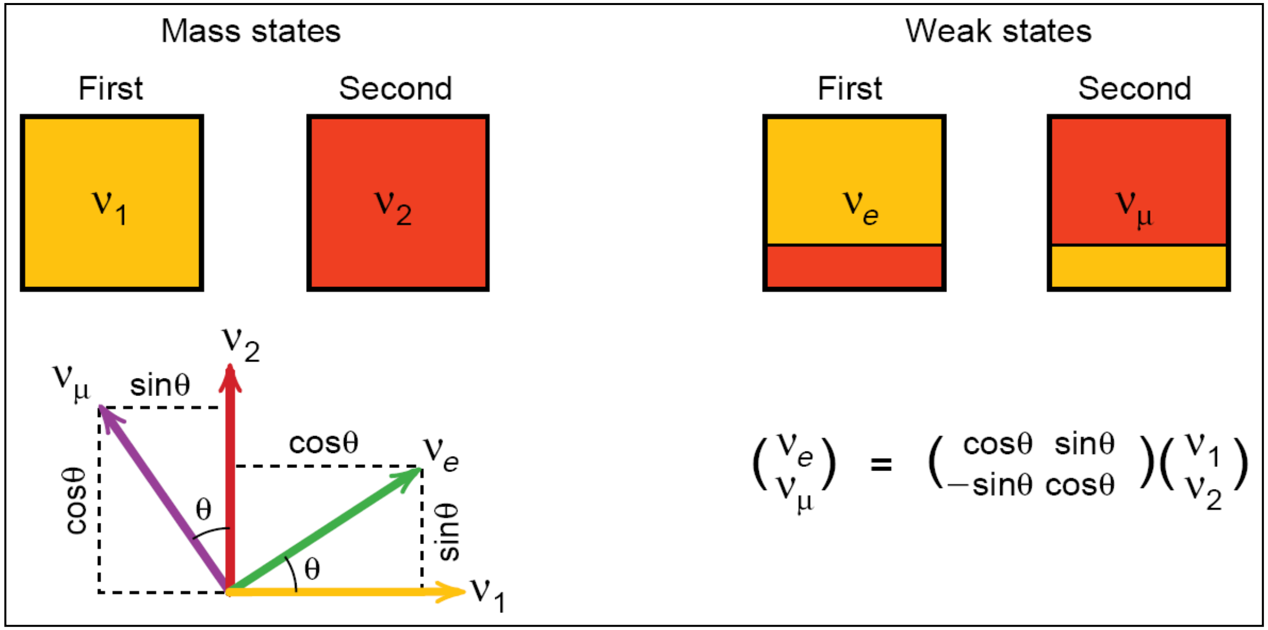
Why neutrino oscillation?

- Oscillation — a spontaneous periodic change from one neutrino flavor state to another, is a spectacular quantum phenomenon. It can occur as a natural consequence of neutrino mixing.
- In a neutrino oscillation experiment, the neutrino beam is produced and detected via the weak charged-current interactions.



2-flavor neutrino oscillation?

For simplicity, we consider two-flavor neutrino mixing and oscillation:



Approximation: a plane wave with a common momentum for each mass state

$$
\begin{aligned}
|\nu_\mu(0)\rangle &= |\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle \\
|\nu_\mu(t)\rangle &= -\sin\theta e^{-iE_1 t}|\nu_1\rangle + \cos\theta e^{-iE_2 t}|\nu_2\rangle \\
&= e^{-iE_1 t}\left(-\sin\theta|\nu_1\rangle + \cos\theta e^{-i\Delta E t}|\nu_2\rangle\right)
\end{aligned}
$$

$$
\begin{aligned}
\Delta E &\equiv E_2 - E_1 = \sqrt{p^2+m_2^2} - \sqrt{p^2+m_1^2} \\
&\approx \left(p + \frac{m_2^2}{2p}\right) - \left(p + \frac{m_1^2}{2p}\right) \approx \frac{\Delta m^2}{2E}
\end{aligned}
$$

$\Delta m^2 \equiv m_2^2 - m_1^2$, $E \approx p \gg m_{1,2}$ (relativistic neutrino beam) , $\hbar = c = 1$ (natural units)

- The oscillation probability for appearance ν experiments:

$$\begin{aligned} P\left(\nu_\mu \to \nu_e\right) &= \left|\langle \nu_e | \nu_\mu(t) \rangle\right|^2 = \left|(\cos\theta\langle\nu_1| + \sin\theta\langle\nu_2|)\left(-\sin\theta|\nu_1\rangle + \cos\theta e^{-i\Delta E t}|\nu_2\rangle\right)\right|^2 \\ &= \left|\sin\theta\cos\theta\left(1 - e^{-i\Delta E t}\right)\right|^2 = 2\left(\sin\theta\cos\theta\right)^2\left(1 - \cos\frac{\Delta m^2 t}{2E}\right) \\ &= \sin^2 2\theta \sin^2\frac{\Delta m^2 L}{4E} \end{aligned}$$

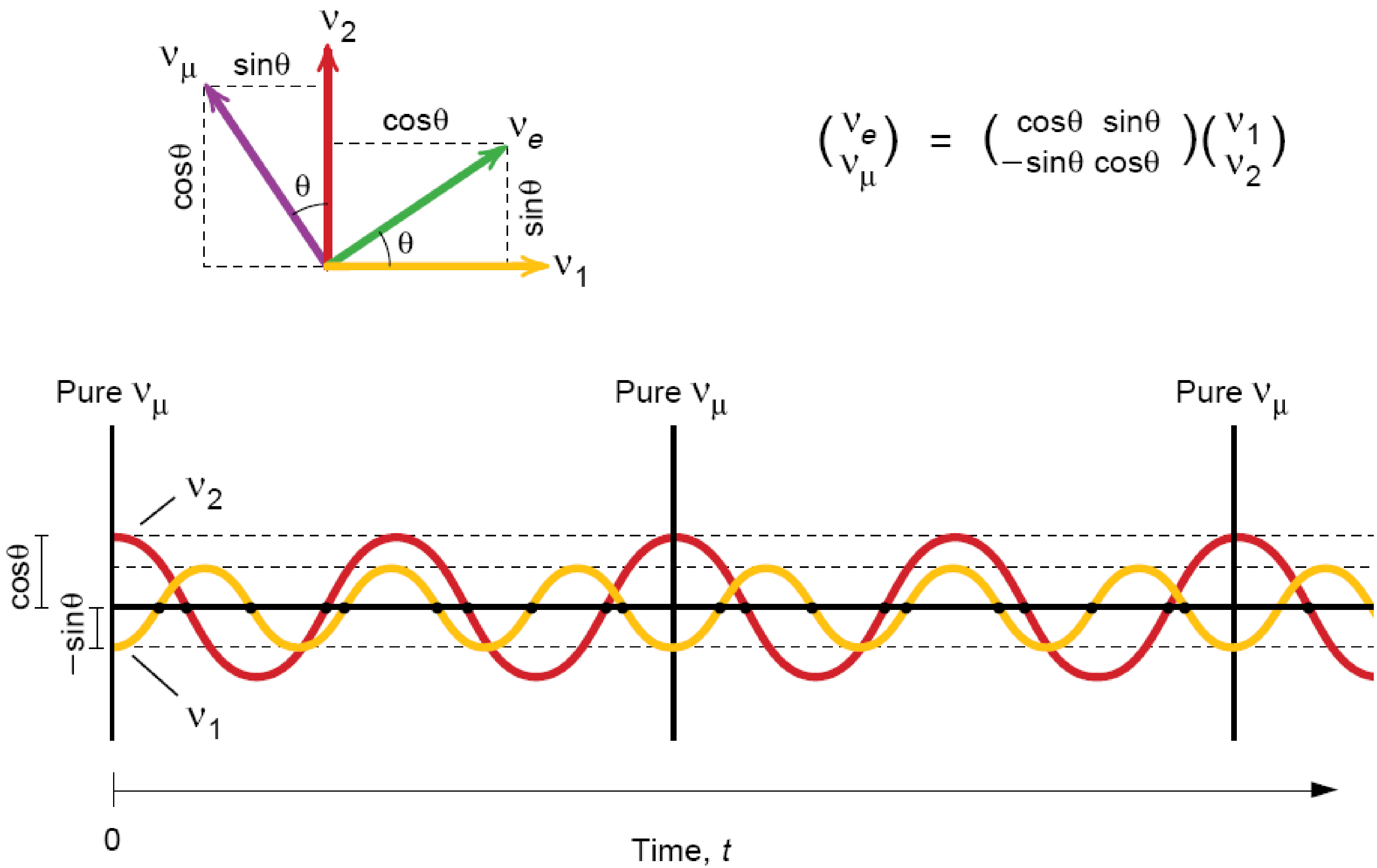
The conversion and survival probabilities in realistic units:

$$\begin{aligned} P\left(\nu_\mu \to \nu_e\right) &= \sin^2 2\theta \sin^2\frac{1.27\Delta m^2 L}{E} \\ P\left(\nu_\mu \to \nu_\mu\right) &= 1 - \sin^2 2\theta \sin^2\frac{1.27\Delta m^2 L}{E} \end{aligned}$$

Due to the smallness of (1,3) mixing, both solar & atmospheric neutrino oscillations are roughly the 2-flavor oscillation.

Δm^2 in unit of eV^2 , L in unit of km , E in unit of GeV

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2 x}{E_\nu}\right)$$



$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Why 1.27 ?

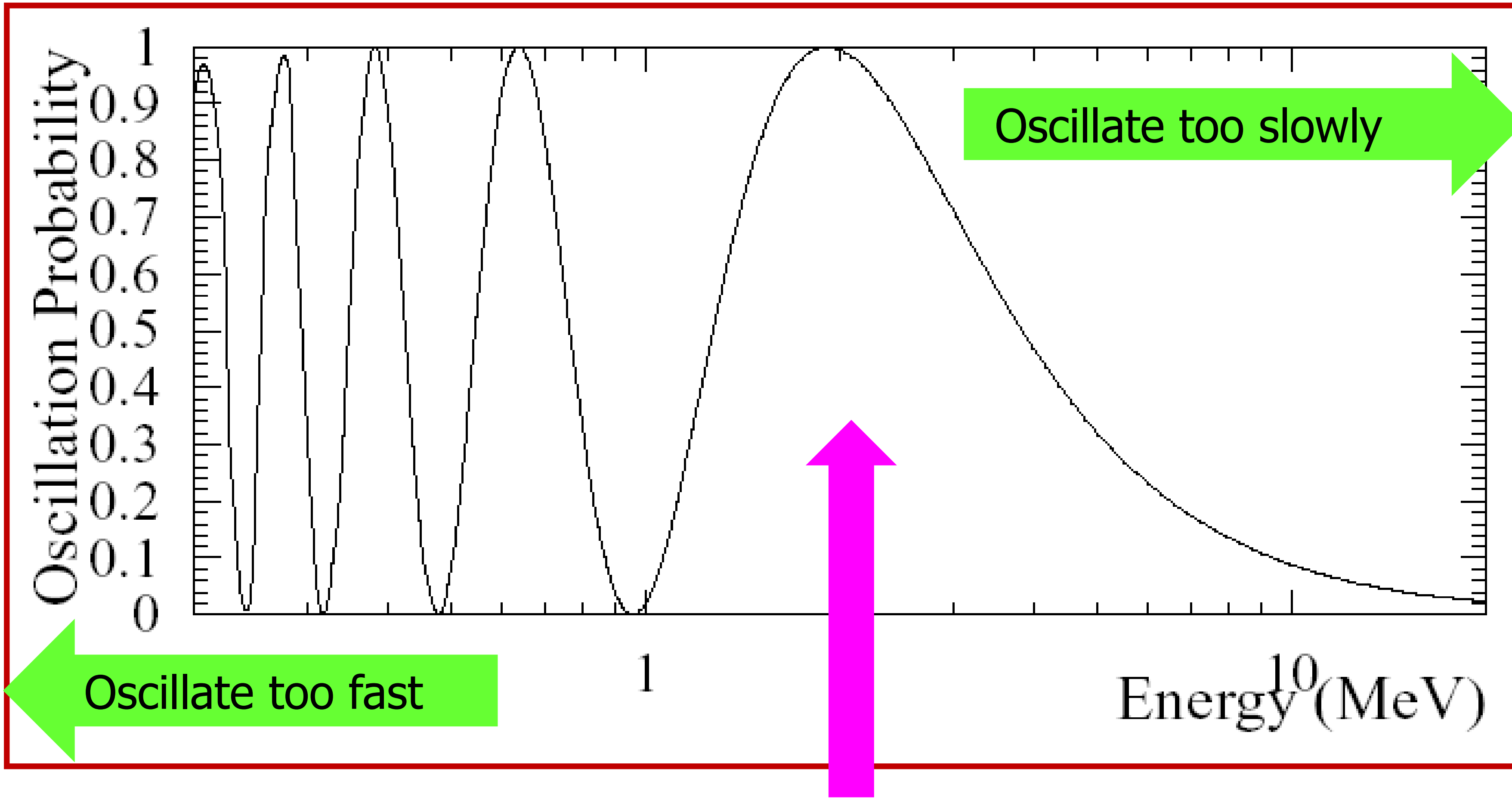
	Natural units	Realistic units
Phase factors	$\exp\left(-iE_{1,2}t\right)$	$\exp\left(-i\frac{E_{1,2}}{\hbar}t\right)$
Energies and momentum	$E_{1,2}=\sqrt{p^2+m_{1,2}^2}$	$E_{1,2}=\sqrt{p^2c^2+m_{1,2}^2c^4}$
Energy difference	$\Delta E=\frac{\Delta m^2}{2E}$	$\Delta E=\frac{\Delta m^2c^3}{2p}=\frac{\Delta m^2c^4}{2E}$
Time and distance	$t=L$	$t=\frac{L}{c}$
Oscillation argument	$\frac{1}{2}\Delta Et=\frac{\Delta m^2L}{4E}$	$\frac{1}{2}\frac{\Delta E}{\hbar}t=\frac{c^3}{\hbar}\cdot\frac{\Delta m^2L}{4E}$

$$c=2.998\times10^5\ \text{km s}^{-1}$$
$$\hbar=6.582\times10^{-25}\ \text{GeV}\cdot\text{s}$$

$$\frac{c^3}{4\hbar}\Rightarrow\frac{1}{4\times0.1973}=1.267\approx1.27$$

$$c=1\quad\Rightarrow\quad\hbar=6.582\times10^{-25}\ \text{GeV}\times2.998\times10^5\ \text{km}$$
$$=1.973\times10^{-19}\ \text{GeV km}=0.1973\text{eV}^2\ \text{GeV}^{-1}\ \text{km}$$

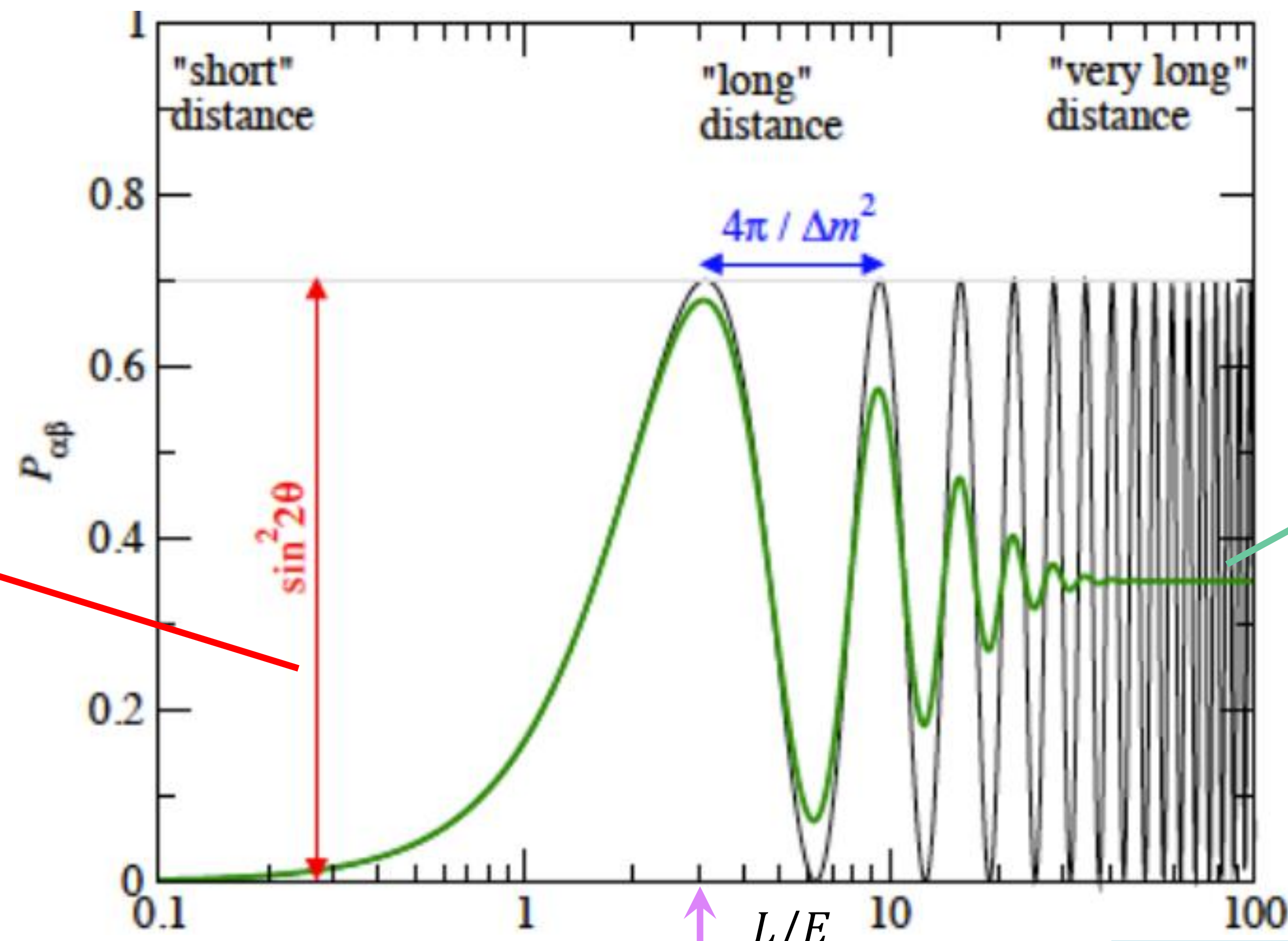
- 2-flavor neutrino oscillation with neutrino energy:



$$P(\nu_e \rightarrow \nu_\mu) = |\langle \nu_\mu | \nu(t) \rangle|^2 = \sin^2 2\theta \sin^2 \left(\frac{1.27 \Delta m^2 L}{E} \right)$$

- 2-neutrino oscillation probability with L/E:

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \quad \text{in natural unit}$$



oscillation amplitude

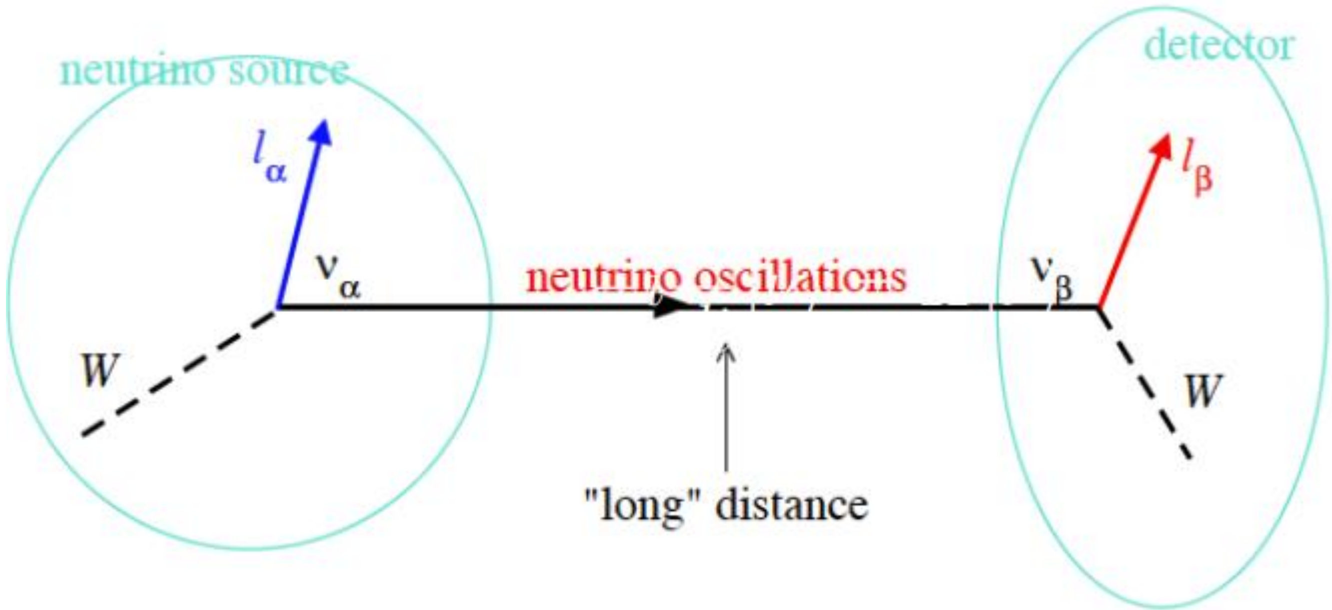
averaged oscillation

first oscillation maximum

oscillation length:

$$L_{ocs} \equiv \frac{4\pi E}{\Delta m^2}$$

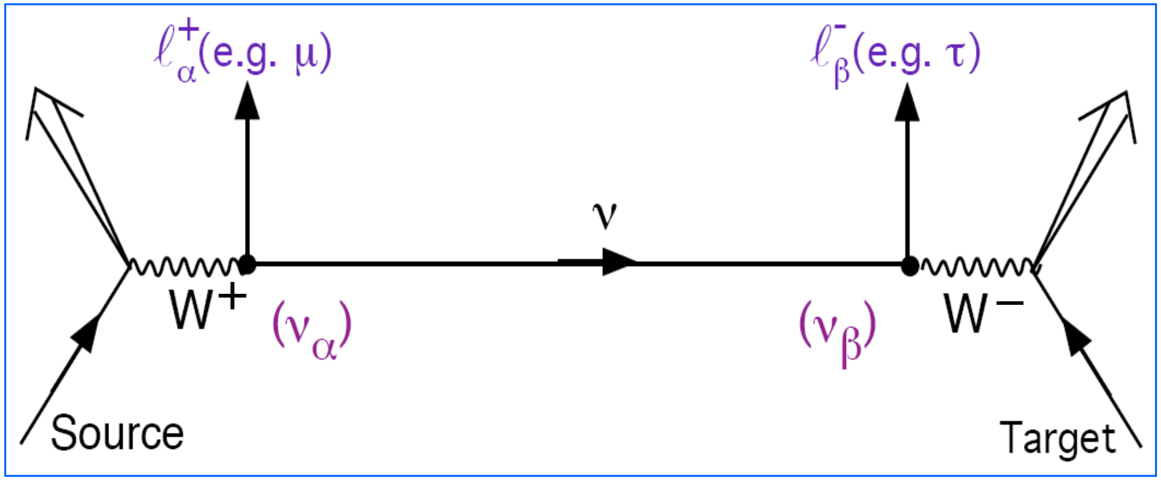
Neutrino oscillation picture



Production	Propagation	Detection
$\|\nu_\alpha\rangle = \sum_j U^*_{\alpha j}\|\nu_j\rangle$	$\nu_j : e^{-iE_j t}$	$\langle\nu_\beta\| = \sum_j \langle\nu_j\|U_{\beta j}$
coherent superposition of massive states	different propagation phases change ν_j composition	projection over flavor eigenstates

3-flavor neutrino oscillation

Production and detection of a neutrino beam by CC weak interactions:



$$|\nu_\alpha(0)\rangle = |\nu_\alpha\rangle = \sum_{i=1}^{3} V_{\alpha i}^* |\nu_i\rangle$$

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^{3} V_{\alpha i}^* e^{-iE_i t} |\nu_i\rangle$$

$$\boldsymbol{\alpha,\beta,\gamma = e,\mu,\tau}$$
$$\boldsymbol{i,j,k = 1,2,3}$$

$$A\left(\nu_\alpha \to \nu_\beta\right) = \langle \nu_\beta | \nu_\alpha(t)\rangle = \left(\sum_{j=1}^{3} V_{\beta j}\langle \nu_j|\right)\left(\sum_{i=1}^{3} V_{\alpha i}^* e^{-iE_i t}|\nu_i\rangle\right) = \sum_{i=1}^{3} V_{\alpha i}^* V_{\beta i} e^{-iE_i t}$$

$$P\left(\nu_\alpha \to \nu_\beta\right) = \left|\langle \nu_\beta | \nu_\alpha(t)\rangle\right|^2 = \left|\sum_{i=1}^{3} V_{\alpha i}^* V_{\beta i} e^{-iE_i t}\right|^2$$

$$= \sum_{i=1}^{3} \left|V_{\alpha i}^* V_{\beta i}\right|^2 + 2\sum_{i<j}^{3} \mathrm{Re}\left[V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^* e^{i\left(E_j - E_i\right)t}\right]$$

- The neutrino mixing matrix is denoted as V here, sorry for the notation change.

The formula of three-flavor oscillation probability with CP/T violation:

$$
\begin{aligned}
P\left(\nu_\alpha \to \nu_\beta\right) &= \sum_{i=1}^{3} \left|V_{\alpha i}^* V_{\beta i}\right|^2 + 2\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*\right) \cos\frac{\Delta m_{ji}^2 L}{2E} \\
&\quad - 2\sum_{i<j}^{3} \mathrm{Im}\left(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*\right) \sin\frac{\Delta m_{ji}^2 L}{2E} \\
&= \sum_{i=1}^{3} \left|V_{\alpha i}^* V_{\beta i}\right|^2 + 2\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*\right) \\
&\quad -4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*\right) \sin^2\frac{\Delta m_{ji}^2 L}{4E} - 2\sum_{i<j}^{3} \mathrm{Im}\left(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*\right) \sin\frac{\Delta m_{ji}^2 L}{2E} \\
&= \left|\sum_{i=1}^{3} V_{\alpha i}^* V_{\beta i}\right|^2 - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*\right) \sin^2\frac{\Delta m_{ji}^2 L}{4E} \\
&\quad + 2\sum_{i<j}^{3} \mathrm{Im}\left(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*\right) \sin\frac{\Delta m_{ji}^2 L}{2E}
\end{aligned}
$$

Jarlskog

$$\left|\sum_{i=1}^{3} V_{\alpha i}^* V_{\beta i}\right|^2 = \delta_{\alpha\beta}$$

$$\mathrm{Im}\left(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*\right) = \mathcal{J} \sum_{\gamma,k} \left(\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\right)$$

The final formula of 3-flavor oscillation probabilities with CP violation:

$$P\left(\nu_\alpha \to \nu_\beta\right) = \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right)\sin^2\frac{\Delta m^2_{ji}L}{4E} + 8\mathcal{J}\sum_\gamma \epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m^2_{21}L}{4E}\sin\frac{\Delta m^2_{31}L}{4E}\sin\frac{\Delta m^2_{32}L}{4E}$$

- How the last term is derived?

$$\begin{aligned}
& 2\sum_{i<j}^{3} \mathrm{Im}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right)\sin\frac{\Delta m^2_{ji}L}{2E} \\
= & +2\mathcal{J}\sum_\gamma \epsilon_{\alpha\beta\gamma}\left(\sin\frac{\Delta m^2_{21}L}{2E} - \sin\frac{\Delta m^2_{31}L}{2E} + \sin\frac{\Delta m^2_{32}L}{2E}\right) \\
= & -2\mathcal{J}\sum_\gamma \epsilon_{\alpha\beta\gamma}\left(\sin\frac{\Delta m^2_{12}L}{2E} + \sin\frac{\Delta m^2_{23}L}{2E} + \sin\frac{\Delta m^2_{31}L}{2E}\right) \\
= & +8\mathcal{J}\sum_\gamma \epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m^2_{12}L}{4E}\sin\frac{\Delta m^2_{23}L}{4E}\sin\frac{\Delta m^2_{31}L}{4E}
\end{aligned}$$

General properties of neutrino oscillations

- Conservation of probability: $\sum_\beta P(\nu_\alpha \to \nu_\beta) = 1$
- For antineutrino: $V \to V^*$
- Neutrino oscillations violate flavour lepton number conservation(expected from mixing) but conserve total lepton number
- Complex phases in the mixing matrix induce CP violation:

$$P(\nu_\alpha \to \nu_\beta) \neq P(\overline{\nu_\alpha} \to \overline{\nu_\beta})$$

- Neutrino oscillations do not depend on the absolute neutrino mass scale and Majorana phases.
- Neutrino oscillations are sensitive only to mass squared differences:

$$\Delta m_{ji}^2 \equiv m_j^2 - m_i^2$$

Discrete Symmetries

Basic expression

$$P\left(\nu_\alpha \to \nu_\beta\right) = \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right)\sin^2\frac{\Delta m^2_{ji}L}{4E} + 8\mathcal{J}\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m^2_{21}L}{4E}\sin\frac{\Delta m^2_{31}L}{4E}\sin\frac{\Delta m^2_{32}L}{4E}$$

CP transformation

$V \to V^*$

$J \to -J$

$$P\left(\overline{\nu}_\alpha \to \overline{\nu}_\beta\right) = \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right)\sin^2\frac{\Delta m^2_{ji}L}{4E} - 8\mathcal{J}\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m^2_{21}L}{4E}\sin\frac{\Delta m^2_{31}L}{4E}\sin\frac{\Delta m^2_{32}L}{4E}$$

T transformation

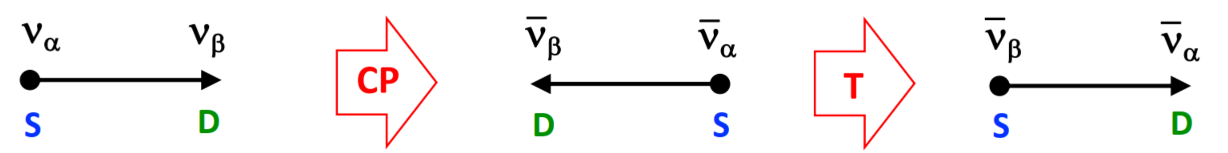
$\alpha \leftrightarrow \beta$

$$P\left(\nu_\beta \to \nu_\alpha\right) = \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right)\sin^2\frac{\Delta m^2_{ji}L}{4E} - 8\mathcal{J}\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m^2_{21}L}{4E}\sin\frac{\Delta m^2_{31}L}{4E}\sin\frac{\Delta m^2_{32}L}{4E}$$

CPT invariance

$$P\left(\overline{\nu}_\beta \to \overline{\nu}_\alpha\right) = P\left(\nu_\alpha \to \nu_\beta\right)$$

Action of **CP** and **T** on $\nu_\alpha \to \nu_\beta$ oscillations from source **S** to detector **D**:



C = charge conjugation (particle-antiparticle exchange)

P = parity (space reversal)

T = time reversal

CP & T Violation

Under CPT invariance, CP- and T-violating asymmetries are identical:

$$P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$$
$$= 16\mathcal{J} \sum_\gamma \epsilon_{\alpha\beta\gamma} \sin\frac{\Delta m^2_{21} L}{4E} \sin\frac{\Delta m^2_{31} L}{4E} \sin\frac{\Delta m^2_{32} L}{4E}$$

Comments:

- CP / T violation cannot show up in the disappearance neutrino oscillation experiments ($\alpha = \beta$);
- CP / T violation is a small three-family flavor effect;
- CP / T violation in normal lepton-number-conserving neutrino oscillations depends only upon the Dirac phase of V; hence such oscillation experiments cannot tell us whether neutrinos are Dirac or Majorana particles.

$$J = \sin\theta_{12}\cos\theta_{12}\sin\theta_{23}\cos\theta_{23}\sin\theta_{13}\cos^2\theta_{13}\sin\delta \le 1/6\sqrt{3} \approx 9.6\%$$

Disappearance experiments

Most neutrino oscillation experiments are of the disappearance type:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4|V_{\alpha 1}|^2 |V_{\alpha 2}|^2 \sin^2 \frac{\Delta m^2_{21} L}{4E}$$
$$-4|V_{\alpha 1}|^2 |V_{\alpha 3}|^2 \sin^2 \frac{\Delta m^2_{31} L}{4E}$$
$$-4|V_{\alpha 2}|^2 |V_{\alpha 3}|^2 \sin^2 \frac{\Delta m^2_{32} L}{4E}$$

$$|\Delta m^2_{21}| = \Delta m^2_{\rm sun} \ll \Delta m^2_{\rm atm} = |\Delta m^2_{32}| \approx |\Delta m^2_{31}|$$

$\sim 7.6\times 10^{-5}\ \mathrm{eV}^2$ $\sim 2.4\times 10^{-3}\ \mathrm{eV}^2$

This hierarchy & the small (1,3) mixing lead to the 2-flavor oscillation approximation for many experiments. A few upcoming experiments (long-baseline experiments) will probe the complete 3-flavor effects.

2-flavor Approximation

Solar, reactor, atmospheric and accelerator ν oscillation experiments:

Experiment	Survival probability	Oscillation factor
Solar $\nu_e \to \nu_e$	$1 - \sin^2 2\theta_{12} \sin^2\left(1.27\frac{\Delta m^2_{21} L}{E}\right)$	$\sin^2 2\theta_{12} = 4\|V_{e1}\|^2\|V_{e2}\|^2$
KamLAND $\overline{\nu}_e \to \overline{\nu}_e$	$1 - \sin^2 2\theta_{12} \sin^2\left(1.27\frac{\Delta m^2_{21} L}{E}\right)$	$\sin^2 2\theta_{12} = 4\|V_{e1}\|^2\|V_{e2}\|^2$
Atmospheric $\nu_\mu \to \nu_\mu$	$1 - \sin^2 2\theta_{23} \sin^2\left(1.27\frac{\Delta m^2_{32} L}{E}\right)$	$\sin^2 2\theta_{23} = 4\|V_{\mu 3}\|^2\left(1 - \|V_{\mu 3}\|^2\right)$
K2K $\nu_\mu \to \nu_\mu$	$1 - \sin^2 2\theta_{23} \sin^2\left(1.27\frac{\Delta m^2_{32} L}{E}\right)$	$\sin^2 2\theta_{23} = 4\|V_{\mu 3}\|^2\left(1 - \|V_{\mu 3}\|^2\right)$
MINOS $\nu_\mu \to \nu_\mu$	$1 - \sin^2 2\theta_{23} \sin^2\left(1.27\frac{\Delta m^2_{32} L}{E}\right)$	$\sin^2 2\theta_{23} = 4\|V_{\mu 3}\|^2\left(1 - \|V_{\mu 3}\|^2\right)$
CHOOZ $\overline{\nu}_e \to \overline{\nu}_e$	$1 - \sin^2 2\theta_{13} \sin^2\left(1.27\frac{\Delta m^2_{32} L}{E}\right)$	$\sin^2 2\theta_{13} = 4\|V_{e3}\|^2\left(1 - \|V_{e3}\|^2\right)$

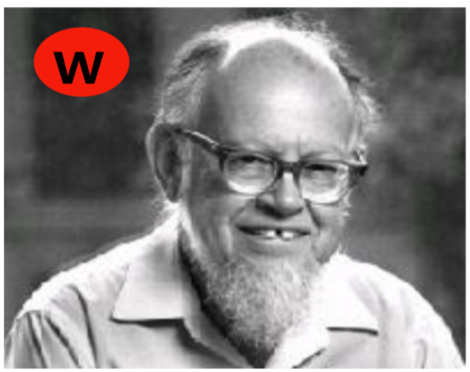
Matter Effect in neutrino oscillation

PHYSICAL REVIEW D VOLUME 17, NUMBER 9 1 MAY 1978

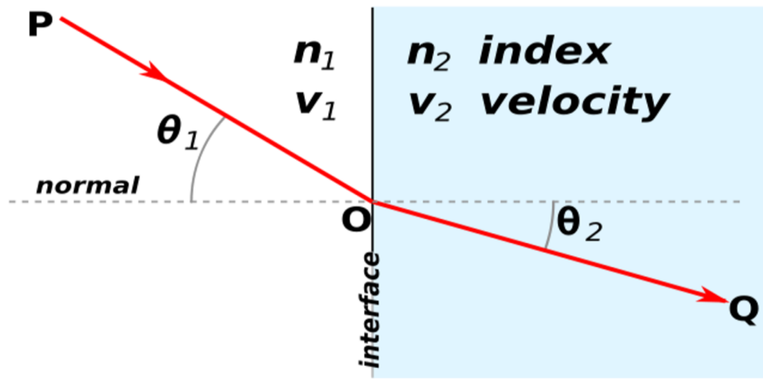
Neutrino oscillations in matter

L. Wolfenstein
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.



Lincoln Wolfenstein
(1923-2015)



Refraction of light in media

$$\nu_e : \exp[ipx(n_{nc} + n_{cc} - 1)]$$
$$\nu_\mu : \exp[ipx(n_{nc} - 1)]$$
$$\nu_\tau : \exp[ipx(n_{nc} - 1)]$$

Refraction of neutrinos in media, where both CC and NC interactions contribute to refractive indices (not far from 1)

When neutrinos are traveling in matter, the effect of coherent forward scattering with background particles leads to a modification of their energies. Such a modification can be described by an potential energy. The difference between the potentials of distinct neutrino flavors is relevant for neutrino oscillations.

formalism of neutrino oscillation in matter

① Flavor neutrino ν_α with momentum p: $|\nu_\alpha(p)\rangle = \sum_k U_{\alpha k}^* |\nu_k(p)\rangle$

②Evolution of neutrino states is determined by Hamiltonian

③Hamiltonian in vacuum: $H = H_0$

$$H_0|\nu_k(p)\rangle = E_k|\nu_k(p)\rangle, \quad E_k = \sqrt{p^2 + m_k^2}$$

④Hamiltonian in matter: $H = H_0 + H_I$, $H_I|\nu_\alpha(p)\rangle = V_{\beta\alpha} |\nu_\beta(p)\rangle$

⑤Schrödinger evolution equation: $i\frac{d}{dt}|\nu(p,t)\rangle = H|\nu(p,t)\rangle$

⑥Initial condition: $|\nu(p,0)\rangle = |\nu_\alpha(p)\rangle$

⑦At $t > 0$, the neutrino state is a superposition of all flavors:

$$|\nu(p,t)\rangle = \sum_\beta \varphi_\beta(p,t)|\nu_\beta(p)\rangle \Rightarrow \varphi_\beta(p,t) = \langle\nu_\beta(p)|\nu(p,t)\rangle, \quad \varphi_\beta(p,0) = \delta_{\alpha\beta}$$

⑧Transition probability:

$$P(\nu_\alpha \to \nu_\beta) = |\varphi_\beta(p,t)|^2$$

$\boldsymbol{\varphi_\beta(p,t)}$: **flavor transition amplitude**

Evolution equation of flavor transition amplitude

Schrödinger equation: $i\frac{d}{dt}|\nu(p,t)\rangle = H|\nu(p,t)\rangle$

$$|\nu(p,t)\rangle = \sum_{\beta} \varphi_\beta(p,t)\big|\nu_\beta(p)\big\rangle$$

$$i\frac{d}{dt}\varphi_\beta(p,t) = \langle\nu_\beta(p)|H|\nu(p,t)\rangle = \langle\nu_\beta(p)|H_0|\nu(p,t)\rangle + \langle\nu_\beta(p)|H_I|\nu(p,t)\rangle$$

- $$\langle\nu_\beta(p)|H_0|\nu(p,t)\rangle = \sum_\rho\sum_{k,j}\underbrace{\langle\nu_\beta(p)\big|\nu_k(p)\rangle}_{U_{\beta k}}\underbrace{\langle\nu_k(p)|H_0|\nu_j(p)\rangle}_{\delta_{kj}E_k}\underbrace{\langle\nu_j(p)\big|\nu_\rho(p)\rangle}_{U^*_{\rho j}}\underbrace{\langle\nu_\rho(p)\big|\nu(p,t)\rangle}_{\varphi_\rho(p,t)}$$

$$= \sum_\rho\sum_{k,j} U_{\beta k}E_k U^*_{\rho k}\,\varphi_\rho(p,t)$$

- $$\langle\nu_\beta(p)|H_I|\nu(p,t)\rangle = \sum_\rho\underbrace{\langle\nu_\beta(p)|H_I|\nu_\rho(p)\rangle}_{V_{\beta\rho}}\underbrace{\langle\nu_\rho(p)\big|\nu(p,t)\rangle}_{\varphi_\rho(p,t)} = \sum_\rho V_{\beta\rho}\,\varphi_\rho(p,t)$$

$$\Rightarrow \quad i\frac{d}{dt}\varphi_\beta = \sum_\rho\left(\sum_k U_{\beta k}E_k U^*_{\rho k} + V_{\beta\rho}\right)\varphi_\rho$$

Ultrarelativistic neutrinos: $E_k = p + \frac{m_k^2}{2E}, \quad E \simeq p, \quad t \simeq x$

$$\Rightarrow \quad i \frac{d}{dx} \varphi_\beta(p,x) = p\, \varphi_\beta + \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + V_{\beta\rho} \right) \varphi_\rho(p,x)$$

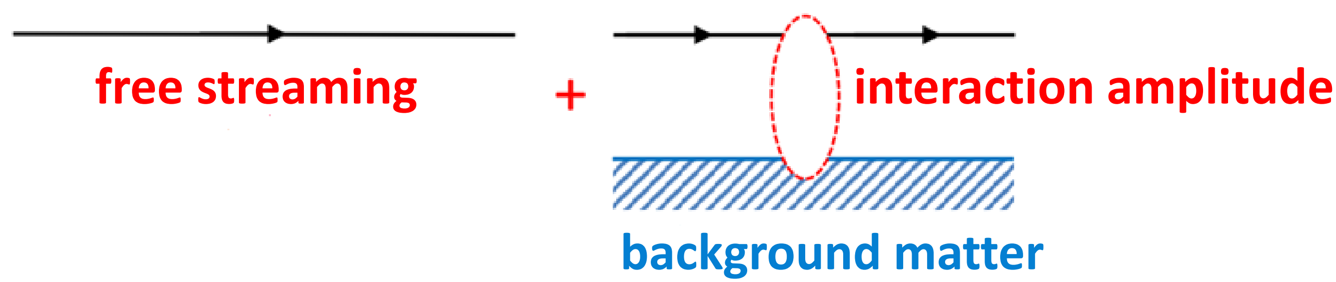
Redefine the flavor transition amplitude: $\varphi_\beta(p,x) \to e^{-ipx} \varphi_\beta(p,x)$

$$i \frac{d}{dx} \varphi_\beta(p,x) = \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + V_{\beta\rho} \right) \varphi_\rho(p,x)$$

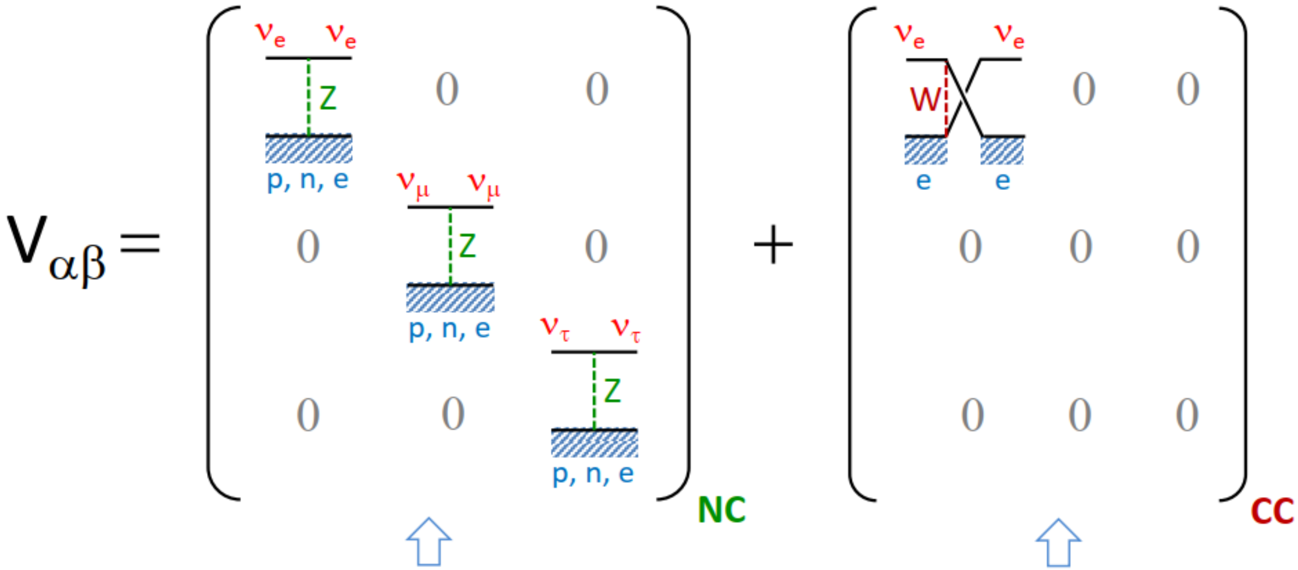
Evolution of flavor transition amplitudes in matrix form:

vacuum (kinematics) — matter (dynamics)

$$i \frac{d}{dx} \begin{pmatrix} \varphi_e \\ \varphi_\mu \\ \varphi_\tau \end{pmatrix} = \Bigg[\underbrace{\frac{1}{2E} U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger}_{\text{vacuum (kinematics)}} + \underbrace{\begin{pmatrix} V_{ee} & V_{e\mu} & V_{e\tau} \\ V_{\mu e} & V_{\mu\mu} & V_{\mu\tau} \\ V_{\tau e} & V_{\tau\mu} & V_{\tau\tau} \end{pmatrix}}_{\text{matter (dynamics)}} \Bigg] \begin{pmatrix} \varphi_e \\ \varphi_\mu \\ \varphi_\tau \end{pmatrix}$$



effective potential of neutrino interactions with matter



proportional to unit→**unobservable**   observable in ν_e related oscillation

Full calculation (omitted) gives

$$V = V_{NC}\,\mathrm{diag}(1,1,1) + V_{CC}\,\mathrm{diag}(1,0,0)$$

$$V_{NC} = -\frac{G_F N_n}{\sqrt{2}}, \qquad V_{CC} = \sqrt{2} G_F N_e$$

N_e: number density of electron
N_n: number density of neutron

For anti-neutrinos in matter, the sign of potential is inversed

$$V = -V_{NC}\,\mathrm{diag}(1,1,1) - V_{CC}\,\mathrm{diag}(1,0,0)$$

neutrino evolution in matter for three flavors

$$i\frac{d}{dx}\begin{pmatrix}\varphi_e\\ \varphi_\mu\\ \varphi_\tau\end{pmatrix} = \frac{1}{2E}\left[U\begin{pmatrix}m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix}U^\dagger + \begin{pmatrix}A(x) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]\begin{pmatrix}\varphi_e\\ \varphi_\mu\\ \varphi_\tau\end{pmatrix}$$

where $A = 2EV_{CC}$ is introduced to make the vacuum and matter terms more similar:

$$A = 2EV_{CC} = 2\sqrt{2}G_F N_e(x)E, \qquad [-A(x) \text{ for anti–neutrinos}]$$

Expect sizeable matter effects when the contributions of matter and vacuum are comparable

$$\frac{A}{\Delta m_{ij}^2} \sim O(1)$$

Exercise: Units for matter effects

$$\frac{A}{\Delta m_{ij}^2} = 1.526 \times 10^{-7} Y_e \left(\frac{\rho(x)}{\mathrm{g/cm^3}}\right)\left(\frac{E}{\mathrm{MeV}}\right)\left(\frac{\mathrm{eV}^2}{\Delta m_{ij}^2}\right)$$

$Y_e \equiv \frac{n_e}{n_N} \sim \frac{1}{2}$: the ratio of electron and nucleon number densities

matter effect for 2 flavors in constant matter density

Lepton mixing matrix for 2-flavor neutrinos ν_e, ν_μ:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\Delta m^2 \equiv m_2^2 - m_1^2$$

$$\Rightarrow U \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} U^\dagger = \begin{pmatrix} m_1^2 \cos^2\theta + m_2^2 \sin^2\theta & \Delta m^2 \sin\theta\cos\theta \\ \Delta m^2 \sin\theta\cos\theta & m_1^2 \sin^2\theta + m_2^2 \cos^2\theta \end{pmatrix}$$

$$= \frac{m_1^2 + m_2^2}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{\Delta m^2}{2} \begin{pmatrix} -\cos2\theta & \sin2\theta \\ \sin2\theta & \cos2\theta \end{pmatrix}$$

irrelevant common phase

- Evolution equation of flavor transition amplitudes (after removing irrelevant common phase):

$$i \frac{d}{dx} \begin{pmatrix} \varphi_e \\ \varphi_\mu \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos2\theta + A & \Delta m^2 \sin2\theta \\ \Delta m^2 \sin2\theta & \Delta m^2 \cos2\theta - A \end{pmatrix} \begin{pmatrix} \varphi_e \\ \varphi_\mu \end{pmatrix}$$

diagonalization of effective Hamiltonian:

$$\begin{pmatrix} \cos\theta_M & -\sin\theta_M \\ \sin\theta_M & \cos\theta_M \end{pmatrix} \begin{pmatrix} -\Delta m^2 \cos2\theta + A & \Delta m^2 \sin2\theta \\ \Delta m^2 \sin2\theta & \Delta m^2 \cos2\theta - A \end{pmatrix} \begin{pmatrix} \cos\theta_M & \sin\theta_M \\ -\sin\theta_M & \cos\theta_M \end{pmatrix} = \begin{pmatrix} -\Delta m_M^2 & 0 \\ 0 & \Delta m_M^2 \end{pmatrix}$$

θ_M: effective mixing angle in matter

Δm_M^2: effective squared-mass difference

$$\begin{cases} \tan 2\theta_M = \dfrac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A} \\ \Delta m_M^2 = \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2} \end{cases}$$

MSW (Wolfenstein-Mykheev-Smirnov) resonance condition: $\theta_M = \pi/4$

$$\Delta m^2 \cos 2\theta - A = 0 \Rightarrow N_e^R = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}\, E G_F}$$

- Mass eigenstates and transition probability in matter

$$\begin{pmatrix} \varphi_e \\ \varphi_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_M & \sin\theta_M \\ -\sin\theta_M & \cos\theta_M \end{pmatrix} \begin{pmatrix} \varphi_1^M \\ \varphi_2^M \end{pmatrix} \Rightarrow \begin{pmatrix} \varphi_1^M \\ \varphi_2^M \end{pmatrix} = \begin{pmatrix} \cos\theta_M & -\sin\theta_M \\ \sin\theta_M & \cos\theta_M \end{pmatrix} \begin{pmatrix} \varphi_e \\ \varphi_\mu \end{pmatrix}$$

Evolution equation:

$$i \frac{d}{dx} \begin{pmatrix} \varphi_1^M \\ \varphi_2^M \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m_M^2 & 0 \\ 0 & \Delta m_M^2 \end{pmatrix} \begin{pmatrix} \varphi_1^M \\ \varphi_2^M \end{pmatrix}$$

Solution:

$$\begin{cases} \varphi_1^M(x) = \varphi_1^M(0) \exp\left(i \dfrac{\Delta m_M^2 x}{4E} \right) \\ \varphi_2^M(x) = \varphi_2^M(0) \exp\left(-i \dfrac{\Delta m_M^2 x}{4E} \right) \end{cases}$$

Transition probability of $\nu_e \to \nu_\mu$:

$$\text{initial } \nu_e \Rightarrow \begin{pmatrix} \varphi_e(0) \\ \varphi_\mu(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \Longrightarrow \quad \begin{pmatrix} \varphi_1^M(0) \\ \varphi_2^M(0) \end{pmatrix} = \begin{pmatrix} \cos\theta_M \\ \sin\theta_M \end{pmatrix}$$

$$P(\nu_e \to \nu_\mu) = \left|\varphi_\mu(x)\right|^2 = \left|-\sin\theta_M\, \varphi_1^M(x) + \cos\theta_M\, \varphi_2^M(x)\right|^2$$

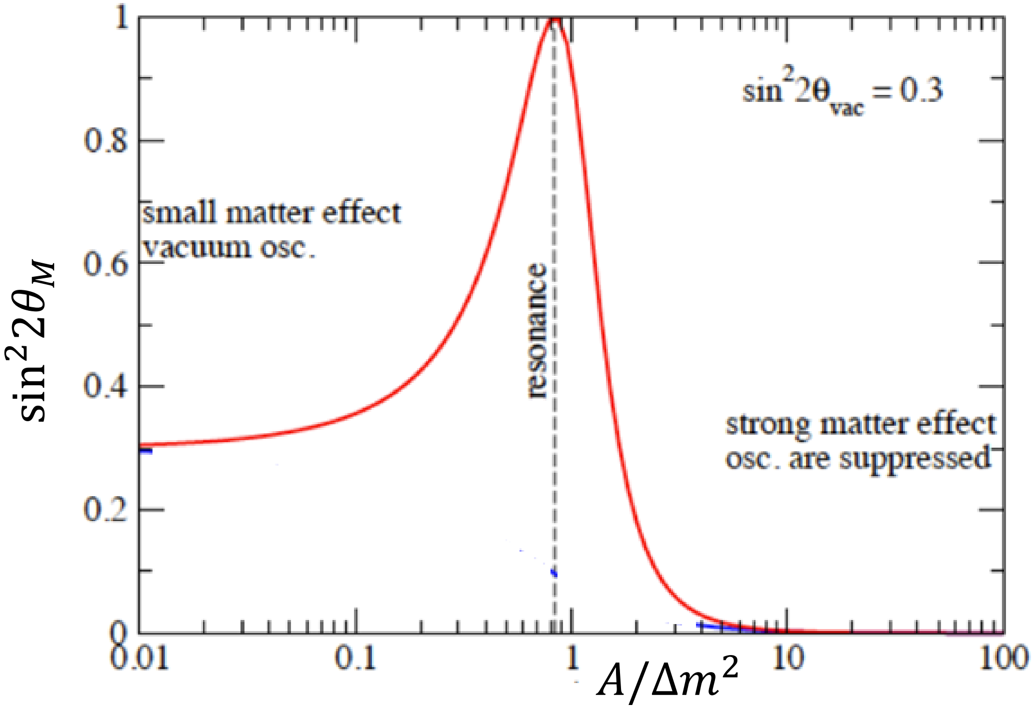
$$= \left|-\sin\theta_M \cos\theta_M \exp\left(i\frac{\Delta m_M^2 x}{4E}\right) + \cos\theta_M \sin\theta_M \exp\left(-i\frac{\Delta m_M^2 x}{4E}\right)\right|^2$$

$$= \sin^2 2\theta_M \sin^2\left(\frac{\Delta m_M^2 x}{4E}\right)$$

- we can use vacuum expression for oscillation probability by replacing "vacuum" parameters by "matter" parameters

- Maximal transition probability for $\theta_M = \pi/4$

MSW resonance



Effective mixing angle in matter

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + \left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2}$$

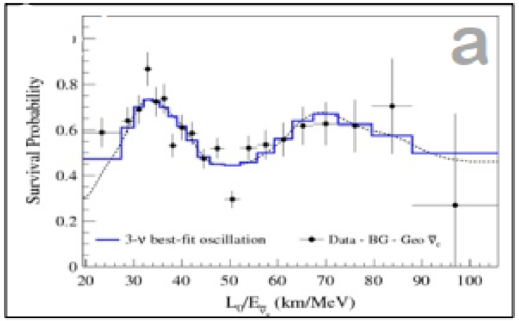
- $A \ll \Delta m^2 \cos 2\theta$, small matter effect→vacuum oscillation: $\theta_M \simeq \theta$
- $A \gg \Delta m^2 \cos 2\theta$, matter effect dominate→ oscillation is suppressed : $\theta_M \simeq \frac{\pi}{2}$
- $A = \Delta m^2 \cos 2\theta$, MSW resonance takes place→maximal mixing: $\theta_M = \frac{\pi}{4}$

- In the case of $\theta < \pi/4$, resonance condition is satisfied for neutrinos for $\Delta m^2 > 0$

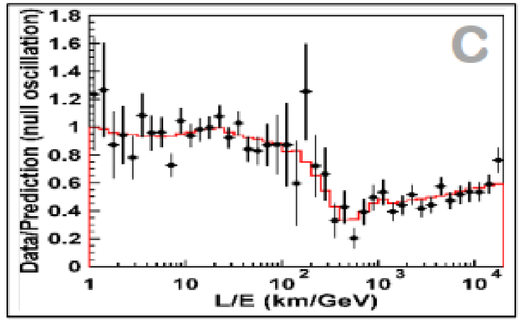
for anti-neutrinos for $\Delta m^2 < 0$

3ν can explain $\alpha \rightarrow \beta$ oscillations seen in vacuum and matter…

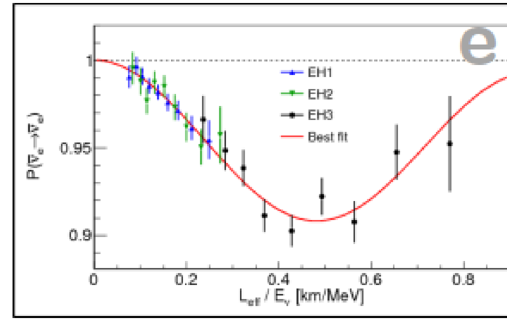
e→e (KamLAND, KL)



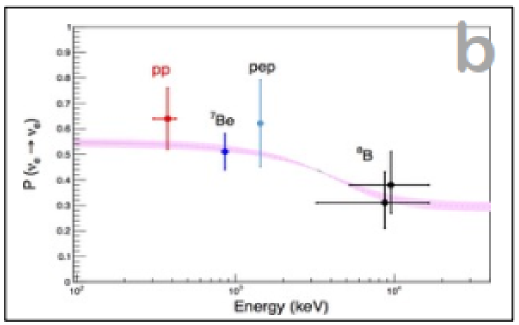
μ→μ (Atmospheric)



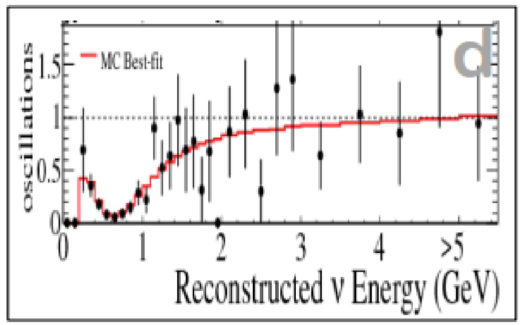
e→e (SBL Reac.)



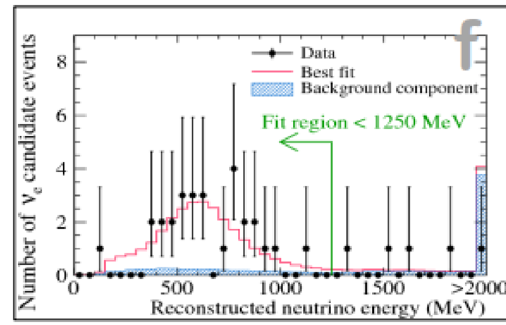
e→e (Solar)



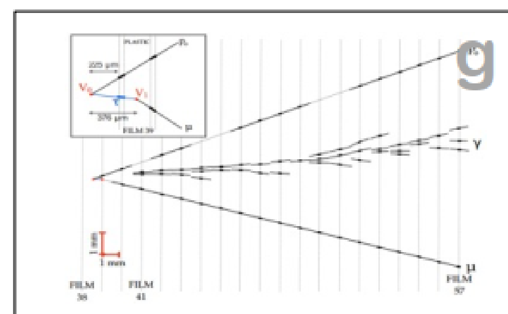
μ→μ (LBL Accel)



μ→e (LBL Accel)



μ→τ (OPERA, SK, DC)

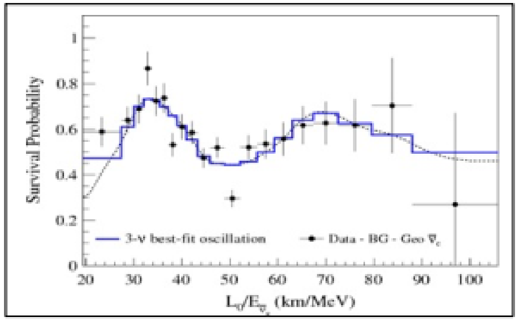


LBL = Long baseline (few x 100 km); SBL = short baseline (~1 km)

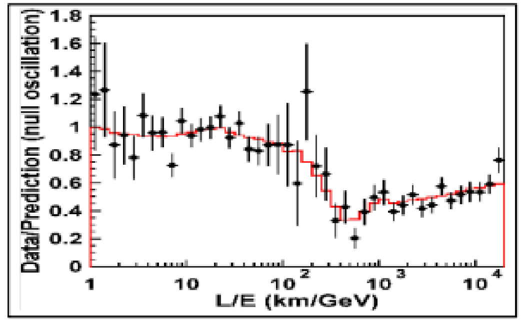
(a) KamLAND reactor [plot]; (b) Borexino [plot], Homestake, Super-K, SAGE, GALLEX/GNO, SNO; (c) Super-K atmosph. [plot], DeepCore, MACRO, MINOS etc.; (d) T2K (plot), NOvA, MINOS, K2K LBL accel.; (e) Daya Bay [plot], RENO, Double Chooz SBL reactor; (f) T2K [plot], MINOS, NOvA LBL accel.; (g) OPERA [plot] LBL accel., Super-K and IC-CD atmospheric.

… with dominant parameters

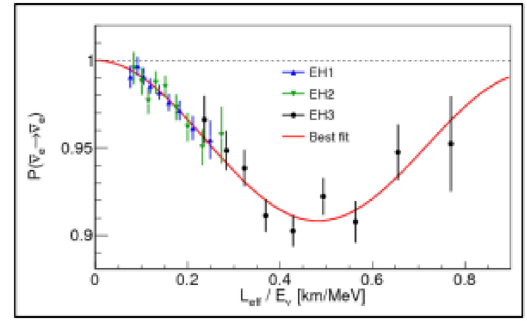
$e \rightarrow e$ $(\delta m^2, \theta_{12})$



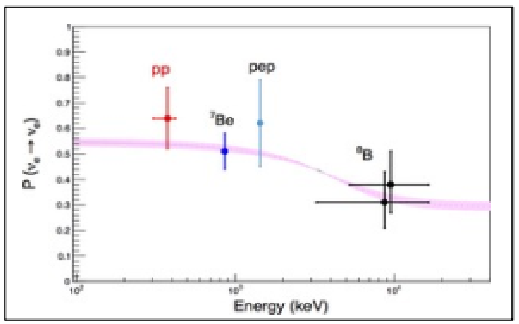
$\mu \rightarrow \mu$ $(\Delta m^2, \theta_{23})$



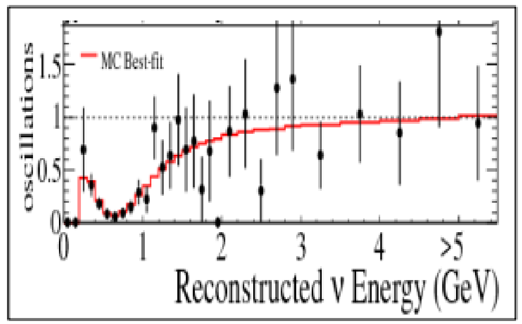
$e \rightarrow e$ $(\Delta m^2, \theta_{13})$



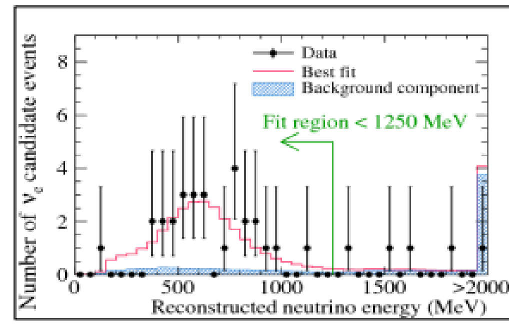
$e \rightarrow e$ $(\delta m^2, \theta_{12})$



$\mu \rightarrow \mu$ $(\Delta m^2, \theta_{23})$



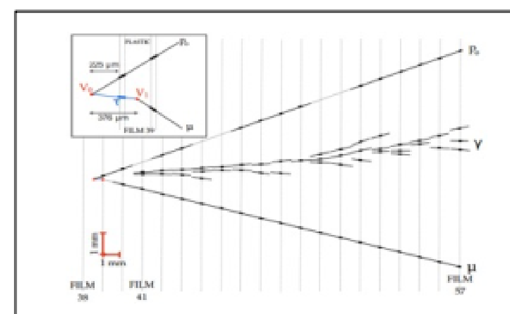
$\mu \rightarrow e$ $(\Delta m^2, \theta_{13}, \theta_{23})$



Established so far:

δm^2 $|\Delta m^2|$ θ_{12} θ_{23} θ_{13}

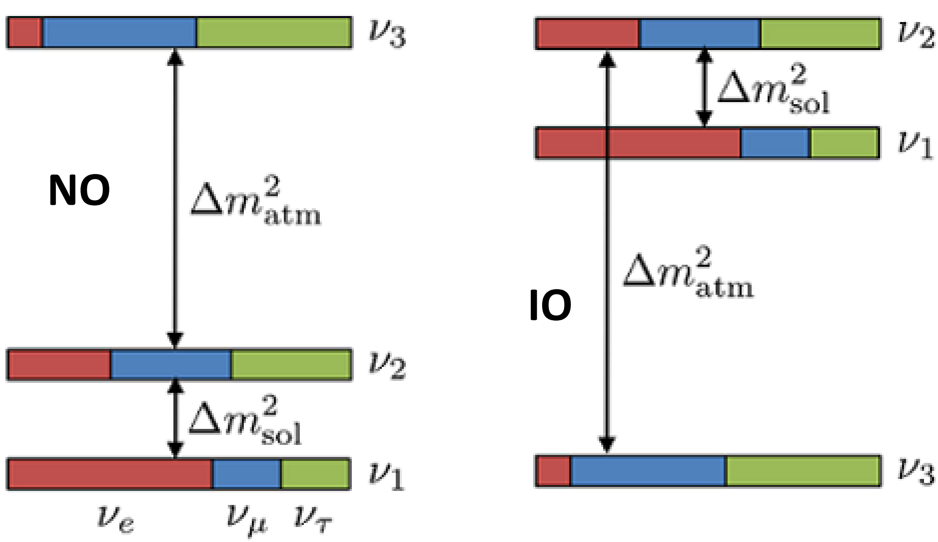
$\mu \rightarrow \tau$ $(\Delta m^2, \theta_{23})$



3-flavor Global Fit

Parameter	Ordering	Best fit	1σ range	2σ range	3σ range	"1σ" (%)
$\delta m^2/10^{-5}$ eV^2	NO, IO	7.37	7.21 – 7.52	7.06 – 7.71	6.93 – 7.93	2.3
$\sin^2\theta_{12}/10^{-1}$	NO, IO	3.03	2.91 – 3.17	2.77 – 3.31	2.64 – 3.45	4.5
$\|\Delta m^2\|/10^{-3}$ eV^2	NO	2.495	2.475 – 2.515	2.454 – 2.536	2.433 – 2.558	0.8
	IO	2.465	2.444 – 2.485	2.423 – 2.506	2.403 – 2.527	0.8
$\sin^2\theta_{13}/10^{-2}$	NO	2.23	2.17 – 2.27	2.11 – 2.33	2.06 – 2.38	2.4
	IO	2.23	2.19 – 2.30	2.14 – 2.35	2.08 – 2.41	2.4
$\sin^2\theta_{23}/10^{-1}$	NO	4.73	4.60 – 4.96	4.47 – 5.68	4.37 – 5.81	5.1
	IO	5.45	5.28 – 5.60	4.58 – 5.73	4.43 – 5.83	4.3
δ/π	NO	1.20	1.07 – 1.37	0.88 – 1.81	0.73 – 2.03	18
	IO	1.48	1.36 – 1.61	1.24 – 1.72	1.12 – 1.83	8
$\Delta\chi^2_{\mathrm{IO-NO}}$	IO−NO	+5.0				

[Capozzi, Giare, Lisi, et al, arXiv: 2503.07752]

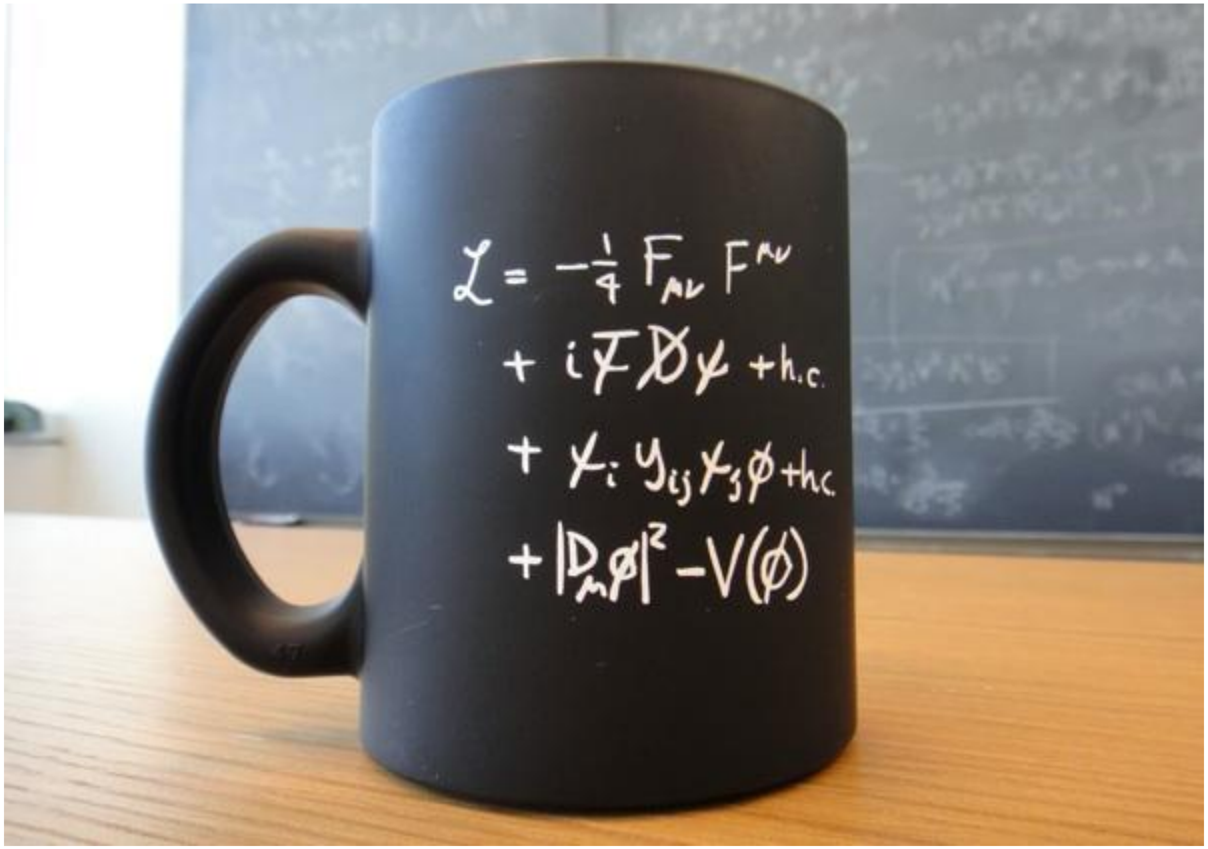


- What is the value of δ ?
- Mass hierarchy: NO or IO?
- Octant of θ_{23} : > or <45° ?
- Majorana or Dirac neutrinos?
- Why m_ν so small?

Lecture 3

- **Fermion masses in SM**
- **Quark flavor mixing**
- **Massive neutrino and lepton flavor mixing**

CERN mug with SM Lagrangian



←vector bosons and …

←fermion interactions …

←Higgs & fermion Yukawa …

←Higgs & vector bosons (+Higgs self-interaction potential V)

Flavor structure of SM

SM Gauge group: $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$

i=1,2,3→generation index	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$q_{iL}=\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$	**3**	**2**	**1/6**
$U_{iR}=u_R, c_R, t_R$	**3**	**1**	**2/3**
$D_{iR}=d_R, s_R, b_R$	**3**	**1**	**−1/3**
$\ell_{iL}=\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	**1**	**2**	**−1/2**
$E_{iR}=e_R, \mu_R, \tau_R$	**1**	**1**	**−1**
$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	**1**	**2**	**1/2**

- The fermion content is "chiral": left-handed and right-handed fermions transform differently under G_{SM}
- Explicit fermion mass term is forbidden
- No right-handed neutrinos→neutrinos are massless at renormalizable level

Quark and charged lepton masses

- The Yukawa interactions are the most general **gauge invariant** and **renormalizable** terms in the Lagrangian that involve fermions and the Higgs doublet

$$\mathcal{L}_{Yukawa} = -(Y_u)_{ij}\,\overline{q_{iL}}\,\tilde{\phi}U_{jR} - (Y_d)_{ij}\overline{q_{iL}}\phi D_{jR} - (Y_\ell)_{ij}\overline{\ell_{iL}}\phi E_{jR} + \mathrm{h.c.}$$

Up-type quark masses — down-type quark masses — charged lepton masses

Conjugated Higgs doublet $\tilde{\phi} \equiv i\sigma_2\phi^*$: $(1, 2, -1/2)$ under $(SU(3)_c, SU(2)_L, U(1)_Y)$

Yukawa matrices $Y_{u,d,\ell}$ are arbitrary 3×3 complex matrices in flavor space

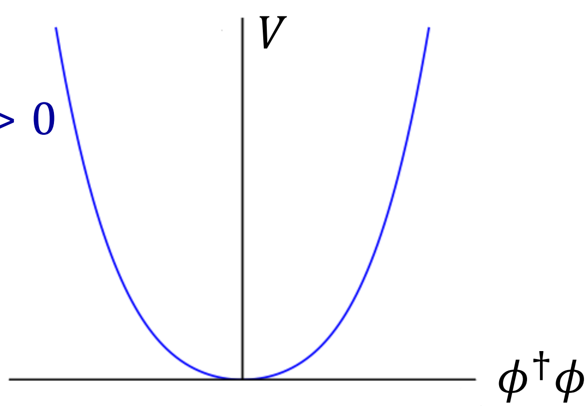
- **Higgs mechanism**

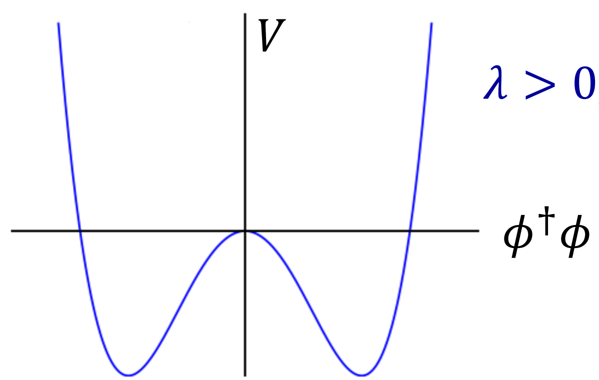
The most general scalar potential allowed by SM gauge group

$$V(\phi) = \mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$$

$$\phi \equiv \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$$

$\lambda > 0, \mu^2 > 0$



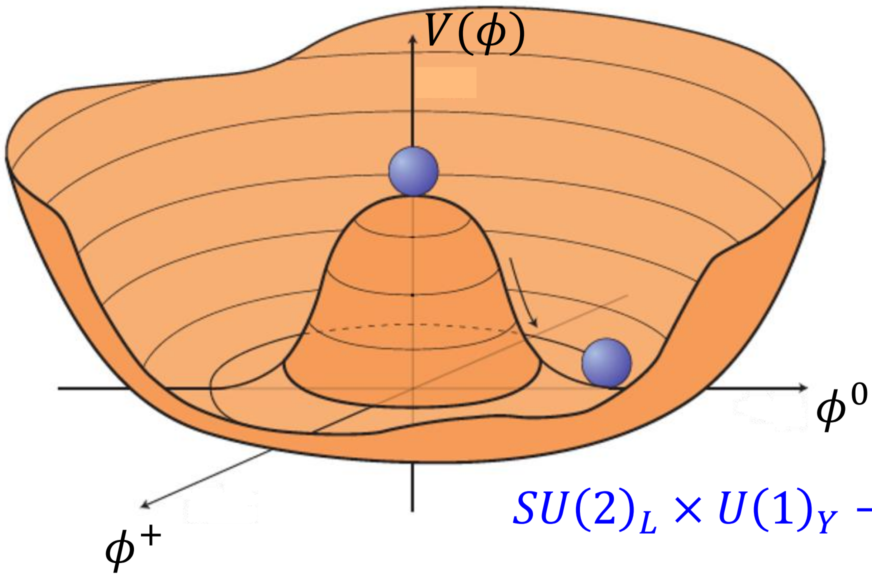


$\lambda > 0, \mu^2 < 0$

Minimizing the scalar potential, one finds the Higgs vacuum expectation value (VEV):

$$\langle\phi\rangle \equiv \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \Rightarrow \langle\tilde{\phi}\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$$

$$v = \sqrt{-\mu^2/\lambda} \simeq 246.22\text{GeV}$$



$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$

Unitary gauge: $$\phi \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

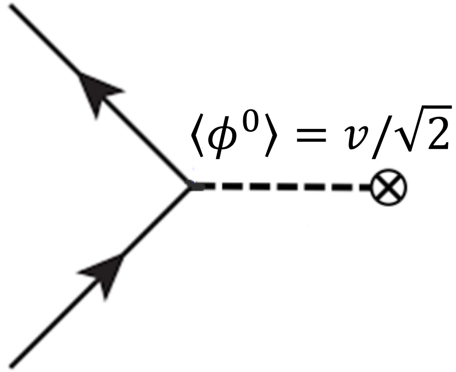
The neutral real component h is the only physical state

- After electroweak symmetry breaking, Yukawa interactions give rise to fermion mass matrices

$$(Y_u)_{ij}\,\overline{Q_{iL}}\,\tilde{\phi}U_{jR}\xrightarrow{\langle\phi\rangle}\overline{u_{iL}}(M_u)_{ij}U_{jR},\quad M_u=\frac{v}{\sqrt{2}}Y_u$$

$$(Y_d)_{ij}\overline{Q_{iL}}\phi D_{jR}\xrightarrow{\langle\phi\rangle}\overline{d_{iL}}(M_d)_{ij}D_{jR},\quad M_d=\frac{v}{\sqrt{2}}Y_d$$

$$(Y_\ell)_{ij}\overline{\ell_{iL}}\phi E_{jR}\xrightarrow{\langle\phi\rangle}\overline{e_{iL}}(M_\ell)_{ij}E_{jR},\quad M_\ell=\frac{v}{\sqrt{2}}Y_\ell$$



$$\mathcal{L}_{SM}\supset-(\overline{u_L},\overline{c_L},\overline{t_L})\underbrace{M_u}_{3\text{x}3}\begin{pmatrix}u_R\\c_R\\t_R\end{pmatrix}-(\overline{d_L},\overline{s_L},\overline{b_L})\underbrace{M_d}_{3\text{x}3}\begin{pmatrix}d_R\\s_R\\b_R\end{pmatrix}-(\overline{e_L},\overline{\mu_L},\overline{\tau_L})\underbrace{M_\ell}_{3\text{x}3}\begin{pmatrix}e_R\\\mu_R\\\tau_R\end{pmatrix}$$

The fermion mass matrices $M_{u,d,\ell}$ can be diagonalized through bi-unitary transformations

$$\left.\begin{aligned}V_{Lu}^\dagger M_u V_{Ru}&=\mathrm{diag}(m_u,m_c,m_t)\\V_{Ld}^\dagger M_d V_{Rd}&=\mathrm{diag}(m_d,m_s,m_b)\\V_{L\ell}^\dagger M_\ell V_{R\ell}&=\mathrm{diag}(m_e,m_\mu,m_\tau)\end{aligned}\right\}$$

Quark and charged lepton masses are proportional to the Higgs VEV v

➢ rotating from the interaction basis to the mass basis by field redefinition

LH fields:

$$\begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} = \underbrace{V_{Lu}}_{3\text{x}3} \begin{pmatrix} u'_L \\ c'_L \\ t'_L \end{pmatrix}, \quad \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} = \underbrace{V_{Ld}}_{3\text{x}3} \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}, \quad \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} = \underbrace{V_{L\ell}}_{3\text{x}3} \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix}$$

RH fields:

$$\begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} = \underbrace{V_{Ru}}_{3\text{x}3} \begin{pmatrix} u'_R \\ c'_R \\ t'_R \end{pmatrix}, \quad \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} = \underbrace{V_{Rd}}_{3\text{x}3} \begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix}, \quad \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} = \underbrace{V_{R\ell}}_{3\text{x}3} \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix}$$

The primed fields are mass eigenstates

- $V_{Lu},\ V_{Ru}, V_{Ld},\ V_{Rd}, V_{L\ell},\ V_{R\ell}$ are 3-dimensional unitary matrices

$$V_{Lu}^\dagger V_{Lu} = V_{Lu}V_{Lu}^\dagger = V_{Ru}^\dagger V_{Ru} = V_{Ru}V_{Ru}^\dagger = 1$$

$$V_{Ld}^\dagger V_{Ld} = V_{Ld}V_{Ld}^\dagger = V_{Rd}^\dagger V_{Rd} = V_{Rd}V_{Rd}^\dagger = 1$$

$$V_{L\ell}^\dagger V_{L\ell} = V_{L\ell}V_{L\ell}^\dagger = V_{R\ell}^\dagger V_{R\ell} = V_{R\ell}V_{R\ell}^\dagger = 1$$

- LH up-type quarks $(u_L, c_L, t_L)^T$ and down-type quarks $(d_L, s_L, b_L)^T$ are rotated separately

Quark mixing: Cabbibo-Kobayashi-Maskawa (CKM)

basis where CC and NC diagonal ≠ mass eigenbasis

The charged current (CC) interaction in weak basis:

$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}\left[\overline{u_{iL}}\gamma^{\mu}d_{iL} + \overline{\nu_{iL}}\gamma^{\mu}e_{iL}\right]W_{\mu}^{+} + \text{h.c.}$$

In the mass eigenstate basis

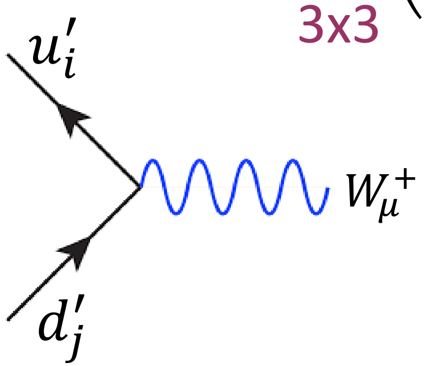
$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}\left[\overline{u'_{iL}}\gamma^{\mu}\underbrace{\left(V_{Lu}^{\dagger}V_{Ld}\right)_{ij}}_{\text{CKM}}d'_{jL} + \overline{\nu_{iL}}\gamma^{\mu}V_{L\ell}e'_{jL}\right]W_{\mu}^{+} + \text{h.c.}$$

$$V_{CKM} \equiv V_{Lu}^{\dagger}V_{Ld}$$

Neutrinos are massless in the SM, one can always redefine neutrino fields as

$$\begin{pmatrix}\nu_{eL}\\ \nu_{\mu L}\\ \nu_{3L}\end{pmatrix} = \underbrace{V_{L\ell}}_{3\text{x}3}\begin{pmatrix}\nu'_{eL}\\ \nu'_{\mu L}\\ \nu'_{\tau L}\end{pmatrix}$$

$$\frac{g}{\sqrt{2}}\overline{\nu_{iL}}\gamma^{\mu}V_{L\ell}e'_{iL}W_{\mu}^{+} = \frac{g}{\sqrt{2}}\overline{\nu'_{iL}}\gamma^{\mu}e'_{iL}W_{\mu}^{+}$$



$$i\frac{g}{\sqrt{2}}\gamma^{\mu}\frac{1-\gamma_5}{2}(V_{CKM})_{ij}$$

No lepton mixing and no neutrino oscillations in SM!

CKM Parametrization

Not all entries are independent: 3 Euler angles $\theta_{12}, \theta_{13}, \theta_{23}$ and 1 complex phase δ

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_{ij} \equiv \cos\theta_{ij}, s_{ij} \equiv \sin\theta_{ij}$

$$|V_{ud}| \sim |V_{cs}| \sim |V_{tb}| \sim 1$$

$$|V_{us}| \sim |V_{cd}| \sim 0.22$$

$$|V_{cb}| \sim |V_{ts}| \sim 0.04$$

$$|V_{ub}| \sim |V_{td}| \sim 0.005$$

hierarchical

$s_{13} << s_{23} << s_{12} << 1$

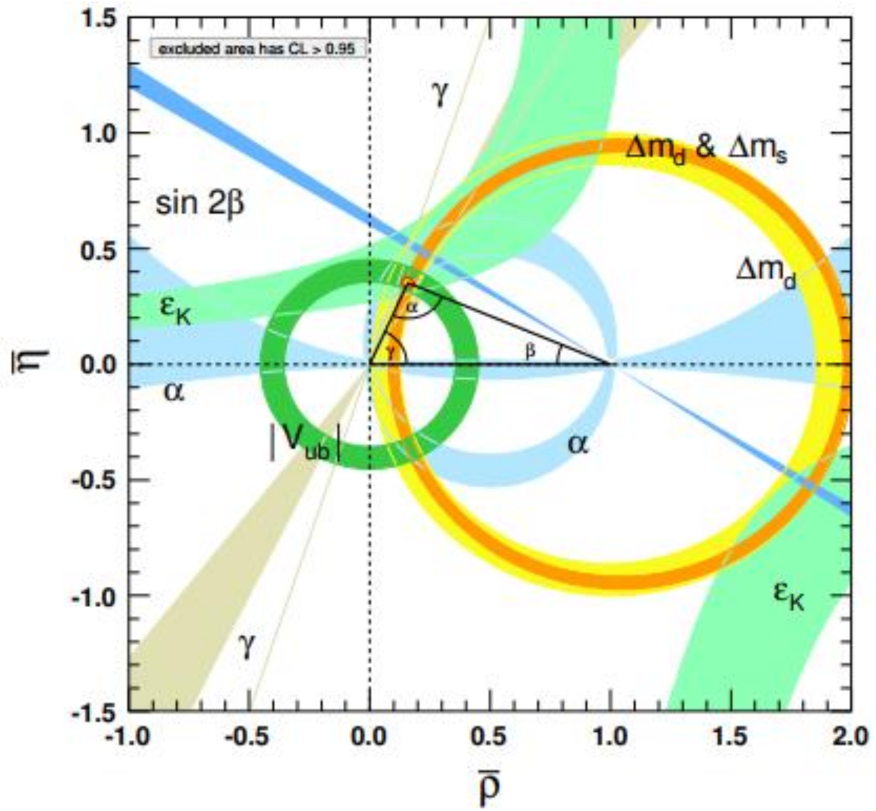
Wolfenstein parametrization

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda = s_{12}, \;\; A\lambda^2 = s_{23}, \;\; A\lambda^3(\rho - i\eta) = s_{13}e^{-i\delta}$$

➢ The fitting values of the CKM elements **[Particle Data Group 2024]**

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22501 \pm 0.00068 & 0.003732^{+0.000090}_{-0.000085} \\ 0.22487 \pm 0.00068 & 0.97349 \pm 0.00016 & 0.04183^{+0.00079}_{-0.00069} \\ 0.00858^{+0.00019}_{-0.00017} & 0.04111^{+0.00077}_{-0.00068} & 0.999118^{+0.000029}_{-0.000034} \end{pmatrix}$$



$$\sin\theta_{12} = 0.22501 \pm 0.00068$$

$$\sin\theta_{13} = 0.003732^{+0.000090}_{-0.000085}$$

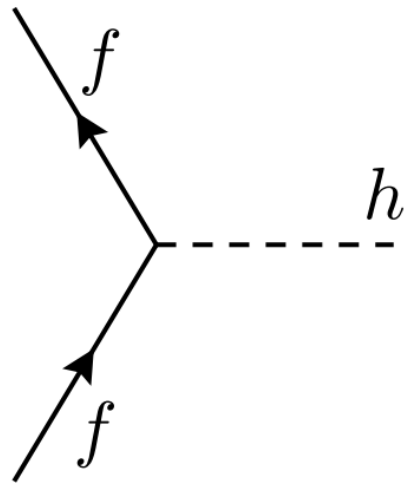
$$\sin\theta_{23} = 0.04183^{+0.00079}_{-0.00069}$$

$$\delta = 1.147 \pm 0.026$$

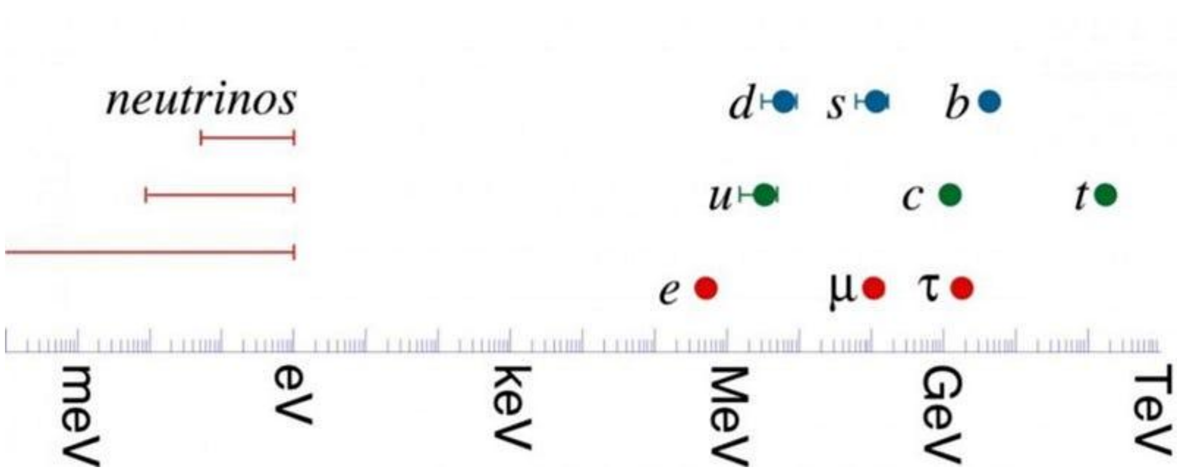
Higgs-fermion couplings

The Higgs-fermion coupling is proportional to the fermion mass

$$\left(1+\frac{h}{v}\right)\left(-m_{u_i}\overline{u_{iL}}\;u_{iR}-m_{d_i}\overline{d_{iL}}\;u_{iR}-m_{\ell_i}\overline{e_{iL}}\;e_{iR}\right)+\text{h.c.}$$

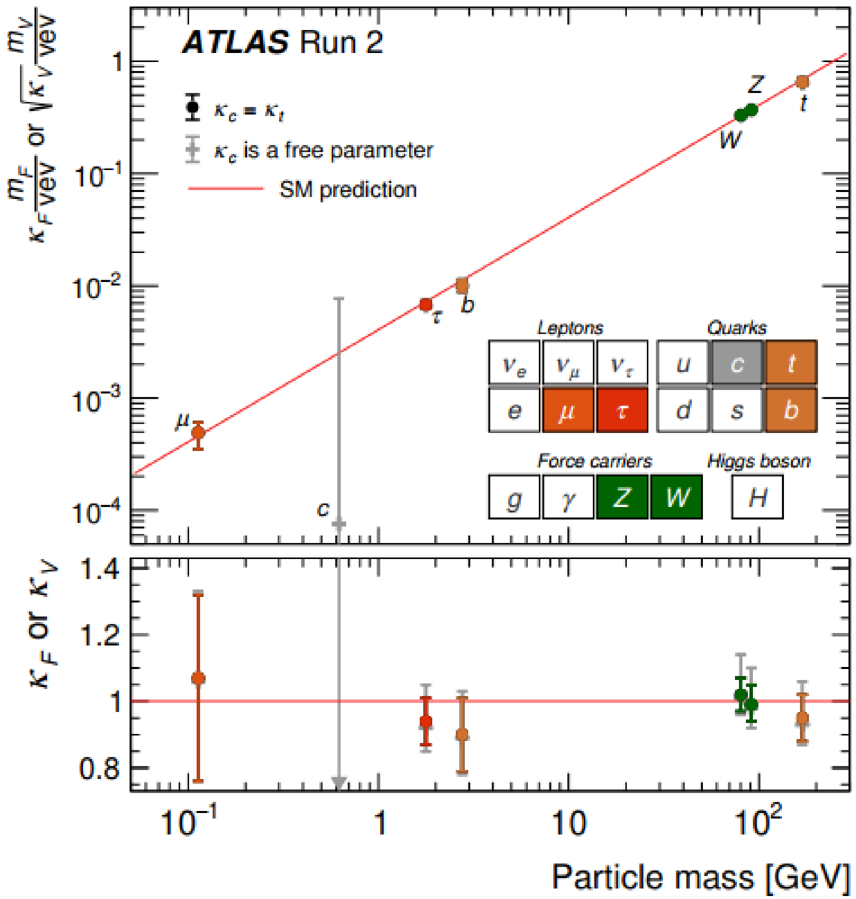


$$-i\frac{g}{2}\frac{m_f}{m_W}$$

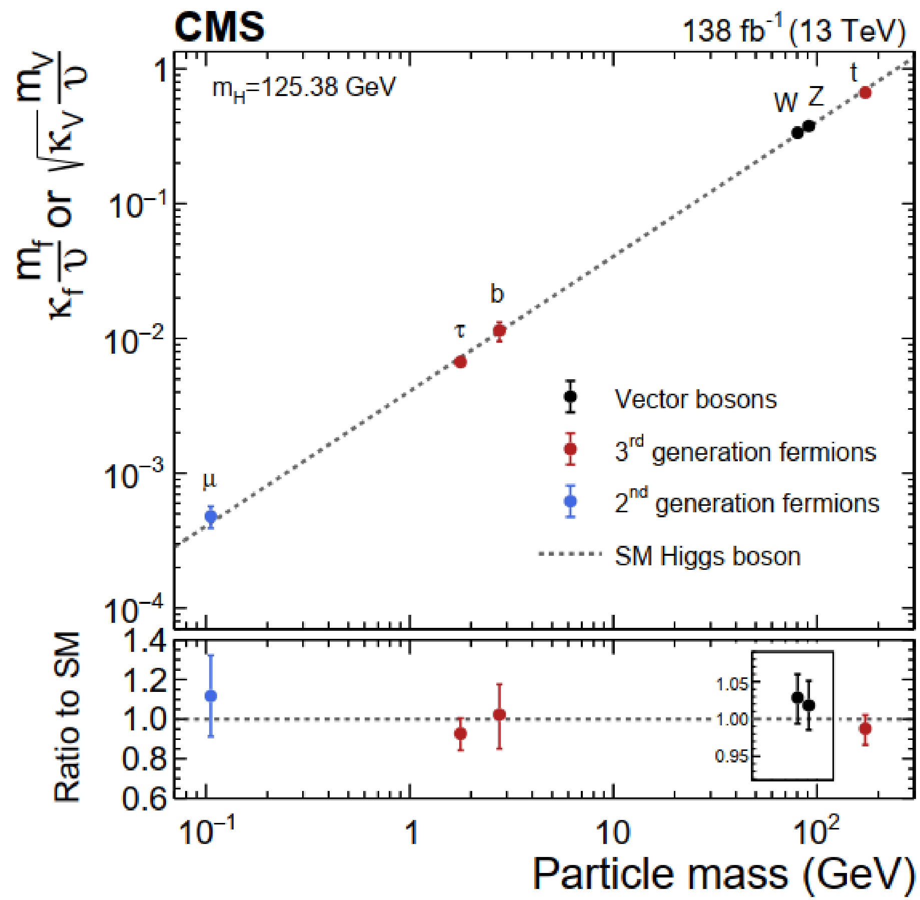


Huge hierarchies among the quark and lepton masses

Nature 607(2022) 52, arXiv:2207.00092



Nature 607(2022) 60, arXiv:2207.00043



Massive neutrinos: Dirac versus Majorana

Dirac neutrino of mass m:

$$\mathcal{L}^{m}_{Dirac} = -m\overline{\nu_L}\nu_R - m\overline{\nu_R}\nu_L$$
$$= -m\overline{\nu_D}\,\nu_D$$

$$\nu_D \equiv \nu_L + \nu_R \Rightarrow \nu_D^c \neq \nu_D$$

Dirac: $\nu_D = \nu_L + \nu_R \rightarrow$ LH and RH components are **independent**

- break SM gauge invariance

Majorana neutrino of mass m:

$$\mathcal{L}^{m}_{Majorana} = -\frac{1}{2}m\overline{\nu_L^c}\nu_L - \frac{1}{2}m\overline{\nu_L}\nu_L^c$$
$$= -\frac{1}{2}m\nu_L^T C\nu_L - \frac{1}{2}m\overline{\nu_L}C\overline{\nu_L}^T$$
$$= -\frac{1}{2}m\overline{\nu_M}\nu_M$$

$$\nu_M \equiv \nu_L + \nu_L^c \Rightarrow \nu_M = \nu_M^c$$

Majorana: $\nu_M = \nu_L + \nu_L^c \rightarrow$ LH and RH components are **NOT independent**

- break SM gauge invariance

$$P_L = \frac{1-\gamma_5}{2},\quad P_R = \frac{1+\gamma_5}{2},\quad \psi_L \equiv P_L\psi,\quad \psi_R \equiv P_R\psi,$$
$$\bar{\psi} \equiv \psi^\dagger\gamma^0,\quad \psi^c = C\bar{\psi}^T = C\gamma^0\psi^*,\quad \overline{\psi^c} \equiv -\psi^T C^{-1},$$
$$C = i\gamma^2\gamma^0,\quad C^\dagger = C^{-1},\quad C^T = -C$$

Massive Dirac neutrinos in SM

Introduce three right-handed neutrino ν_{iR} which are SM singlets,

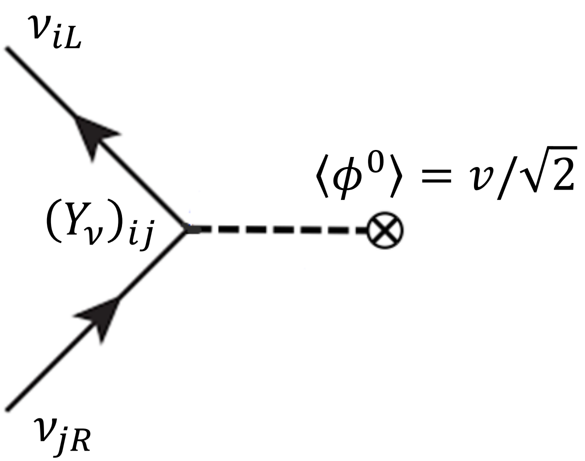
$$\nu_{iR} \sim (1,1,0) \text{ under } (SU(3)_c, SU(2)_L, U(1)_Y)$$

Massive Dirac neutrino via Yukawa coupling: SM+ν_R

$$\mathcal{L}^{\nu}_{Yukawa} = -(Y_\nu)_{ij}\, \overline{\ell_{iL}}\, \tilde{\phi}\nu_{jR} + \mathrm{h.c.}$$

EWSB: $\langle\tilde{\phi}\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$ $\qquad M_\nu = \frac{v}{\sqrt{2}} Y_\nu$

$$\mathcal{L}^{\nu}_{Dirac} = -\,\overline{\nu_{iL}}\,(M_\nu)_{ij}\nu_{jR} + \mathrm{h.c.}$$



Neutrino mass-eigenstate basis:

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \underbrace{V_{L\nu}}_{3\text{x}3} \begin{pmatrix} \nu'_{1L} \\ \nu'_{1L} \\ \nu'_{1L} \end{pmatrix}, \quad \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix} = \underbrace{V_{R\nu}}_{3\text{x}3} \begin{pmatrix} \nu'_{1R} \\ \nu'_{2R} \\ \nu'_{3R} \end{pmatrix}$$

Lepton number conservation:
$\ell_{iL} \to e^{i\varphi}\ell_{iL}$,
$\nu_{iR} \to e^{i\varphi}\nu_{iR}$

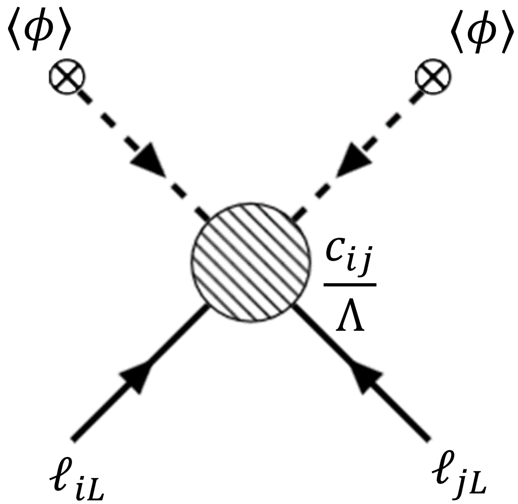
$$\Rightarrow \quad V^\dagger_{L\nu} M_\nu V_{R\nu} = \mathrm{diag}(m_1, m_2, m_3)$$

tiny Yukawa couplings $\quad (Y_\nu)_{ij} \sim \frac{\sqrt{2} m_\nu}{v} \sim \frac{0.2\ \mathrm{eV}}{200\ \mathrm{GeV}} \sim 10^{-12}$

Massive Majorana neutrinos in SM

Massive Majorana neutrino via dim-5 Weinberg operator

$$\mathcal{L}^{\nu}_{\text{Weinberg}} = -\frac{1}{2}\frac{c_{ij}}{\Lambda}\left(\overline{\ell^{c}_{iL}}\,\tilde{\phi}^{*}\right)\left(\tilde{\phi}^{\dagger}\ell_{iL}\right) + \text{h.c.}$$



EWSB: $\langle\tilde{\phi}\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$ $\qquad M_{\nu} = c_{ij}\frac{v^2}{2\Lambda}$

$$\mathcal{L}^{m}_{\text{Majorana}} = -\frac{1}{2}\overline{\nu^{c}_{iL}}\,(M_{\nu})_{ij}\nu_{jL} + \text{h.c.}$$

Neutrino mass-eigenstate basis:

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \underbrace{V_{L\nu}}_{3\text{x}3} \begin{pmatrix} \nu'_{1L} \\ \nu'_{2L} \\ \nu'_{3L} \end{pmatrix} \Longrightarrow V^{T}_{L\nu} M_{\nu}\, V_{L\nu} = \text{diag}(m_1, m_2, m_3)$$

Rough estimate: $\Lambda \sim \frac{v^2}{2m_{\nu}} \sim \frac{200\text{^}2}{2\times 0.1}\text{GeV} \sim 10^{14}\text{GeV}$

Lepton number **violation** under $\ell_{iL} \to e^{i\varphi}\ell_{iL}$:

$$\mathcal{L}^{\nu}_{\text{Weinberg}} \to e^{2i\varphi}\,\mathcal{L}^{\nu}_{\text{Weinberg}}$$

Lepton mixing: Pontecorvo–Maki–Nakagawa–Sakata (PMNS)

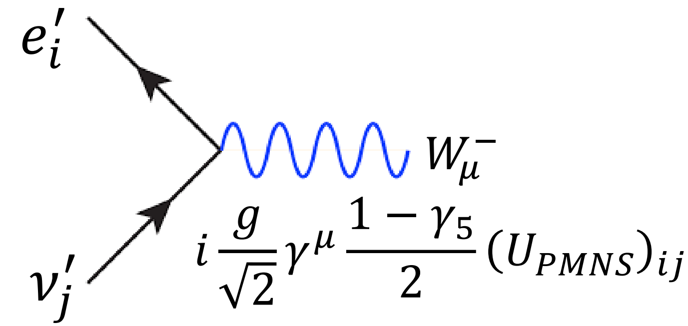
Lepton charged current (CC) interaction in weak basis:

$$-\mathcal{L}_{CC}^{\ell} = \frac{g}{\sqrt{2}}\overline{e_{iL}}\gamma^{\mu}\nu_{iL}W_{\mu}^{-} + \text{h.c.}$$

In the mass eigenstate basis $e_{iL} = (V_{Le})_{ij}e'_{jL}$, $\nu_{iL} = (V_{L\nu})_{ij}\nu'_{jL}$

$$-\mathcal{L}_{CC}^{\ell} = \frac{g}{\sqrt{2}}\overline{e'_{iL}}\gamma^{\mu}\underbrace{\left(V_{L\ell}^{\dagger}V_{L\nu}\right)_{ij}}_{\text{PMNS}}\nu'_{jL}W_{\mu}^{-} + \text{h.c.}$$

$$U_{PMNS} \equiv V_{L\ell}^{\dagger}V_{L\nu}$$



In standard parametrization

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

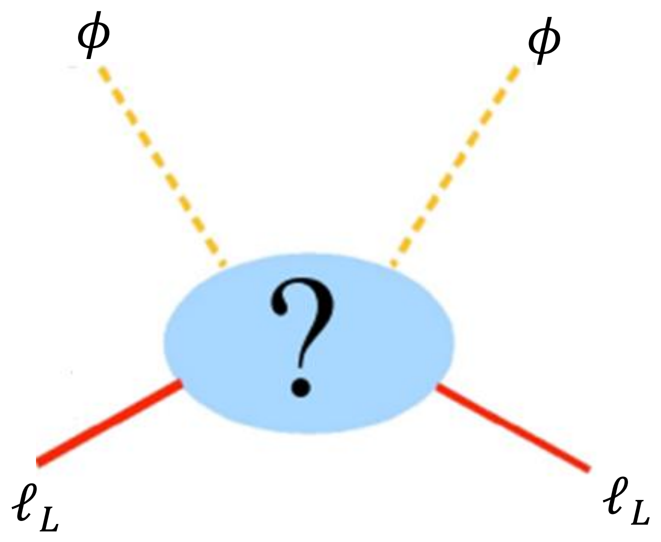
Atmospheric mixing	Reactor mixing & Dirac CP phase	Solar mixing	Majorana CP phases
$\theta_{23} \sim 43.3°$	$\theta_{13} \sim 8.56°,\ \delta_{CP} \sim 212°?$	$\theta_{12} \sim 33.68°$	$\alpha_{21}, \alpha_{31} \sim ?$

The Majorana CP violation phases α_{21}, α_{31} are unphysical for Dirac neutrinos

Lecture 4

- **Seesaw mechanisms**
- **Radiative generation of neutrino mass**

UV completion of Weinberg operator at tree-level



- Transformation properties:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (2, 1/2)$$

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (2, -1/2)$$

- $SU(2)_L$ contractions

$$\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} = (\mathbf{3} \oplus \mathbf{1}) \otimes (\mathbf{3} \oplus \mathbf{1})$$

similar to the composition of four ½-spin

$\mathcal{O}_1 = (\ell_{iL}\phi)_1(\ell_{jL}\phi)_1$ $\qquad$ $\mathcal{O}_2 = (\ell_{iL}\ell_{jL})_1(\phi\phi)_1$ ✗ vanishing because of antisymmetry

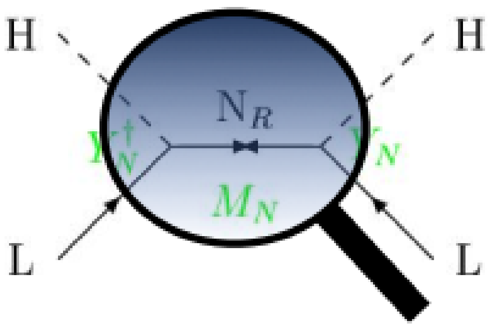
$\mathcal{O}_3 = (\ell_{iL}\phi)_3(\ell_{jL}\phi)_3$ $\qquad$ $\mathcal{O}_4 = (\ell_{iL}\ell_{jL})_3(\phi\phi)_3$

If neutrino are Majorana particles, a **Majorana mass** can arise as the **low energy realization of a higher energy theory (new mass scale!)**

Three types of seesaw mechanism

$\mathcal{O}_1 = (\ell_{iL}\phi)_1(\ell_{jL}\phi)_1$

Type I see-saw:
a heavy singlet fermion

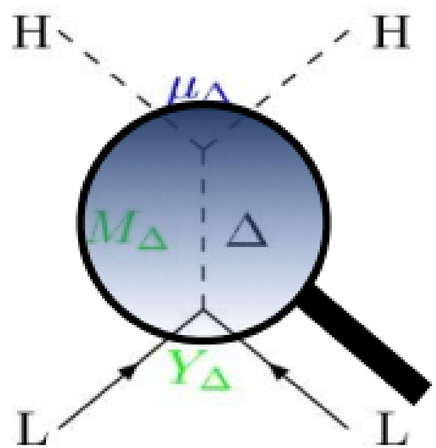


$$m_\nu = \frac{\mathcal{C} v^2}{\Lambda} \equiv Y_N^T \frac{v^2}{M_N} Y_N$$

Minkowski;
Yanagida; Glashow;
Gell-Mann, Ramond Slansky;
Mohapatra, Senjanovic...

$\mathcal{O}_4 = (\ell_{iL}\ell_{jL})_3(\phi\phi)_3$

Type II see-saw:
a heavy triplet scalar

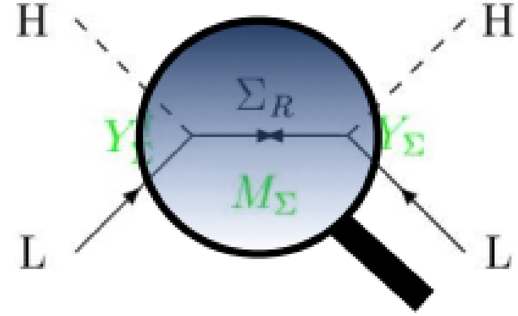


$$m_\nu = \frac{\mathcal{C} v^2}{\Lambda} \equiv Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Konetschny, Kummer;
Cheng, Li;
Lazarides, Shafi, Wetterich ...

$\mathcal{O}_3 = (\ell_{iL}\phi)_3(\ell_{jL}\phi)_3$

Type III see-saw:
a heavy triplet fermion

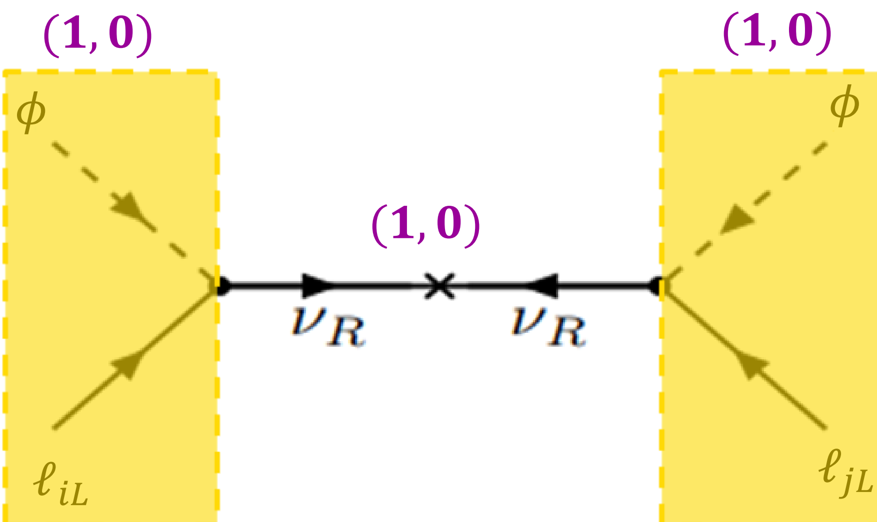


$$m_\nu = \frac{\mathcal{C} v^2}{\Lambda} \equiv Y_\Sigma^T \frac{v^2}{M_\Sigma} Y_\Sigma$$

Foot et al; Ma;
Bajc, Senjanovic...

type-I seesaw mechanism

Higgs-lepton coupling: $\overline{\ell_{iL}^c}\,\tilde{\phi}^*, \tilde{\phi}^\dagger \ell_{iL} \sim (2,-1/2)\otimes(2,1/2) = (3,0)\oplus\mathbf{(1,0)}$



$(1,0)\otimes(1,0) = (1,0)$

Field content: SM fields+ n_R RH neutrinos ν_R

$$\mathcal{L}_{\rm full} = \mathcal{L}_{\rm SM} + i\overline{\nu_{iR}}\partial\!\!\!/\,\nu_{iR} - \left[(Y_\nu)_{ij}\,\overline{\ell_{iL}}\,\tilde{\phi}\,\nu_{jR} + \frac{1}{2}(M_R)_{ij}\overline{\nu_{iR}^c}\,\nu_{jR} + \text{h.c.}\right]$$

$(M_R)_{ij}$ is not protected by any symmetry, hence it can be very large

EWSB: $\langle\phi\rangle = \binom{0}{v/\sqrt{2}}$ $\Rightarrow$ $$\mathcal{L}_{\rm mass}^{\nu} = -(M_D)_{ij}\overline{\nu_{iL}}\,\nu_{jR} - \frac{1}{2}(M_R)_{ij}\overline{\nu_{iR}^c}\,\nu_{jR} + \text{h.c.}$$

$$M_D \equiv Y_\nu \frac{v}{\sqrt{2}}$$

Define a vector of LH fields with $3 + n_R$ components

$$N_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}, \quad \nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \quad \nu_R^c = \begin{pmatrix} \nu_{1R}^c \\ \vdots \\ \nu_{n_R R}^c \end{pmatrix}$$

The neutrino Dirac-Majorana mass Lagrangian

$$\mathcal{L}_{\text{mass}}^{\nu} = -\frac{1}{2}(M_{D+M})_{ij}\overline{N_{iL}}N_{jL}^c + \text{h.c.}$$

$$M_{D+M} = \begin{pmatrix} \overbrace{0}^{3\text{x}3} & \overbrace{M_D}^{3\text{x}n_R} \\ \underbrace{M_D^T}_{n_R\text{x}3} & \underbrace{M_R}_{n_R\text{x}n_R} \end{pmatrix}$$

$(3 + n_R) \times (3 + n_R)$
complex symmetric mass matrix

In mass eigenstate basis

weak states ← $N_L = U_L N_L'$ → mass eigenstates

↓

unitary $(3 + n_R) \times (3 + n_R)$ complex matrix

$$N_L' = \begin{pmatrix} \nu_{1L}' \\ \vdots \\ \nu_{nL}' \end{pmatrix}$$

$n \equiv 3 + n_R$

$U_L^\dagger M_{D+M} U_L^* = \text{diag}(m_1, m_2, \ldots, m_n)$, $\quad m_1, m_2, \ldots, m_n$ **positive** neutrino masses massive Majorana neutrinos

$$\mathcal{L}^{\nu}_{\text{mass}} = -\frac{1}{2}(M_{D+M})_{ij}\overline{N_{iL}}N^{c}_{jL} + \text{h.c.}$$

$$N'_L = \begin{pmatrix} \nu'_{1L} \\ \vdots \\ \nu'_{nL} \end{pmatrix}, \quad n \equiv 3 + n_R$$

$$U_L^{\dagger} M_{D+M} U_L^{*} = \text{diag}(m_1, m_2, \dots, m_n)$$

$$N_L = U_L N'_L$$

$$\mathcal{L}^{\nu}_{\text{mass}} = -\frac{1}{2}\overline{N'_L}\, U_L^{\dagger} M_{D+M} U_L^{*}\, N'_L + \text{h.c.} = -\frac{1}{2}\sum_{i=1}^{n} m_i \overline{\nu'_{iL}}\, \nu'^{c}_{iL} + \text{h.c.}$$

Majorana fields $\nu'_i = \nu'_{iL} + \nu'^{c}_{iL}$

$$\mathcal{L}^{\nu}_{\text{mass}} = -\frac{1}{2}\sum_{i=1}^{n} m_i \bar{\nu}'_i\, \nu'_i$$

The general Dirac-Majorana mass term leads to $3 + n_R$ massive Majorana neutrinos.

➢ Seesaw formula

In the limit $M_R \gg M_D$, there will be definitely a light and a heavy neutrino sector

Block-diagonalize M_{D+M} to separate the heavy and light sectors

$$N_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}, \quad N_L \to W_L N_L$$

$$W_L = \begin{pmatrix} \sqrt{1-BB^\dagger} & B \\ -B^\dagger & \sqrt{1-B^\dagger B} \end{pmatrix} \approx \begin{pmatrix} 1-\frac{1}{2}BB^\dagger & B \\ -B^\dagger & 1-\frac{1}{2}B^\dagger B \end{pmatrix}, \quad B \approx M_D M_R^{-1}$$

$$\Rightarrow \quad W_L^\dagger M_{D+M} W_L^* = \begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix}$$

The diagonal blocks are:

$$M_\nu = -M_D M_R^{-1} M_D^T + \frac{1}{2} M_D M_R^{-1}\left(M_D^T M_D^* M_R^{-1*} + M_R^{-1*} M_D^\dagger M_D\right) M_R^{-1} M_D^T + \cdots$$

$$M_N = M_R + \frac{1}{2}\left(M_D^T M_D^* M_R^{-1*} + M_R^{-1*} M_D^\dagger M_D\right) + \cdots$$

at lowest order in the expansion, one has:

$$M_\nu = -M_D M_R^{-1} M_D^T, \quad M_N = M_R$$

The blocks M_ν, M_N are generally **non-diagonal**

$$U_\nu^\dagger M_\nu U_\nu^* = \mathrm{diag}(m_1, m_2, m_3), \quad U_N^\dagger M_N U_N^* = \mathrm{diag}\left(M_1, M_2, \ldots, M_{n_R}\right)$$

$$\begin{cases} U_\nu^\dagger M_\nu U_\nu^* = \mathrm{diag}(m_1, m_2, m_3) \\ U_N^\dagger M_N U_N^* = \mathrm{diag}\left(M_1, M_2, \ldots, M_{n_R}\right) \end{cases} \qquad M_i \gg m_j$$

Rotation to the **mass eigenstate** basis:

$$N_L = W_L \mathrm{diag}(U_\nu, U_N) N_L'$$

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_{1R}^c \\ \vdots \\ \nu_{n_R R}^c \end{pmatrix} = \begin{pmatrix} \overbrace{\sqrt{1 - BB^\dagger}\, U_\nu}^{3\text{x}3} & \overbrace{B U_N}^{3\text{x}n_R} \\ \underbrace{-B^\dagger U_\nu}_{n_R\text{x}3} & \underbrace{\sqrt{1 - B^\dagger B}\, U_N}_{n_R\text{x}n_R} \end{pmatrix} \begin{pmatrix} \nu_{1L}' \\ \nu_{2L}' \\ \nu_{3L}' \\ N_{1L}' \\ \vdots \\ N_{n_R L}' \end{pmatrix}$$

Heavy-light neutrino mixing is **small** in type-I seesaw

CC and NC interactions in type-I seesaw

- in weak basis

CC interactions: $\mathcal{L}_{CC}^{\ell} = -\frac{g}{\sqrt{2}} \overline{e_{iL}} \gamma^{\mu} \nu_{iL} W_{\mu}^{-} + \text{h.c.}$

NC interactions: $\mathcal{L}_{NC}^{\nu} = -\frac{g}{2c_W} \overline{\nu_{iL}} \gamma^{\mu} \nu_{iL} Z_{\mu}$

- in mass eigenstate basis of type-I seesaw

$$\mathcal{L}_{CC}^{\ell} = -\frac{g}{\sqrt{2}} \left[\overline{e'_{iL}} \gamma^{\mu} \left(V_{\ell L}^{\dagger} \sqrt{1 - BB^{\dagger}}\, U_{\nu} \right)_{ij} \nu'_{jL} + \overline{e'_{iL}} \gamma^{\mu} \left(V_{\ell L}^{\dagger} B U_{N} \right)_{ij} N'_{jL} \right] W_{\mu}^{-} + \text{h.c.}$$

$$\mathcal{L}_{NC}^{\nu,N} = -\frac{g}{2c_W} \left[\overline{\nu'_{iL}} \gamma^{\mu} \left(U_{\nu}^{\dagger} (1 - BB^{\dagger})\, U_{\nu} \right)_{ij} \nu'_{jL} + \overline{N'_{iL}} \gamma^{\mu} \left(U_{N}^{\dagger} B^{\dagger} B U_{N} \right)_{ij} N'_{jL} \right.$$
$$\left. + \overline{\nu'_{iL}} \gamma^{\mu} \left(U_{\nu}^{\dagger} \sqrt{1 - BB^{\dagger}}\, B U_{N} \right)_{ij} N'_{jL} + \overline{N'_{iL}} \gamma^{\mu} \left(U_{N}^{\dagger} B^{\dagger} \sqrt{1 - BB^{\dagger}}\, U_{\nu} \right)_{ij} \nu'_{jL} \right] Z_{\mu}$$

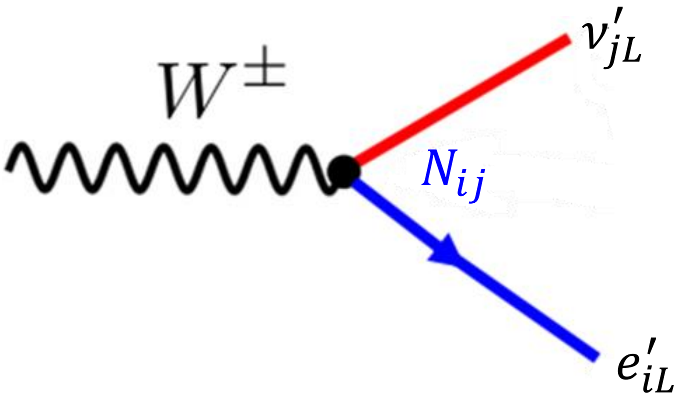
- Non-unitary (active neutrino) interactions

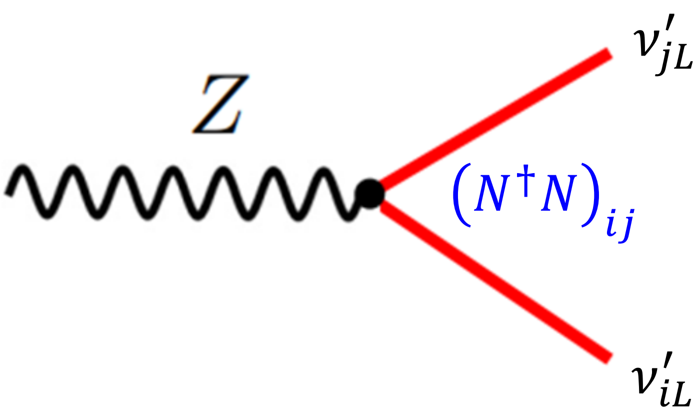
$$\mathcal{L}_{W,Z}^{\nu} = -\frac{g}{2c_W} \overline{\nu'_{iL}} \gamma^{\mu} \left(N^{\dagger} N \right)_{ij} \nu'_{jL} Z_{\mu} - \frac{g}{\sqrt{2}} \left[\overline{e'_{iL}} \gamma^{\mu} N_{ij} \nu'_{jL} W_{\mu}^{-} + \text{h.c.} \right]$$

$$N \equiv \left(1 - \frac{\varepsilon}{2}\right) U_{PMNS}, \quad U_{PMNS} = V_{\ell L}^{\dagger} U_{\nu}, \quad \varepsilon = V_{\ell L}^{\dagger} B B^{\dagger} V_{\ell L} \sim \frac{m_{\nu}}{M_R} \ll 1$$

Non-unitary: $NN^{\dagger} \approx 1 - \varepsilon \neq 1\,, \quad N^{\dagger} N \approx U_{PMNS}^{\dagger} (1 - \varepsilon)\, U_{PMNS} \neq 1$

➢ Experimental constraints on non-unitary interactions



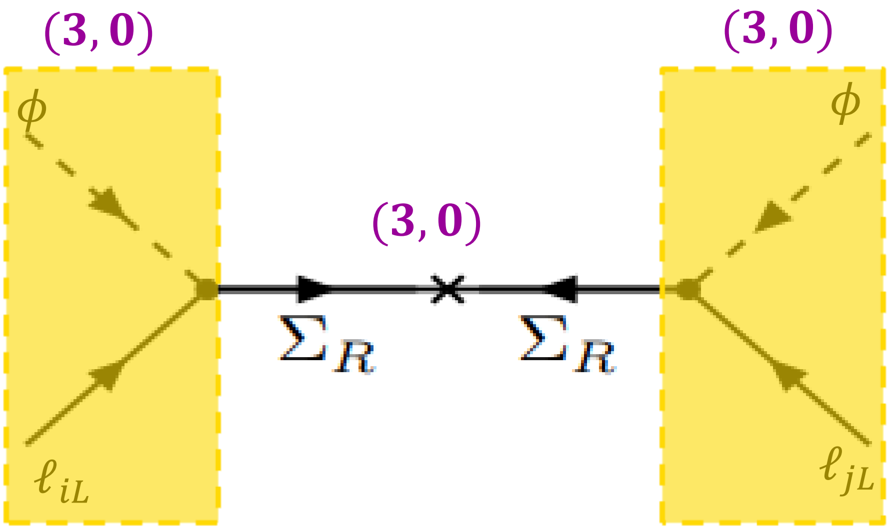


$\sqrt{2\eta_{ij}} \equiv \sqrt{\varepsilon_{ij}}$		G-SS	
		LFC	LFV
$\sqrt{2\eta_{ee}}$, $\lvert\theta_e\rvert$	1σ	$\mathbf{0.031^{+0.010}_{-0.020}}$	–
	2σ	$< \mathbf{0.050}$	–
$\sqrt{2\eta_{\mu\mu}}$, $\lvert\theta_\mu\rvert$	1σ	$< \mathbf{0.011}$	–
	2σ	$< \mathbf{0.021}$	–
$\sqrt{2\eta_{\tau\tau}}$, $\lvert\theta_\tau\rvert$	1σ	$\mathbf{0.044^{+0.019}_{-0.027}}$	–
	2σ	$< \mathbf{0.075}$	–
$\sqrt{2\eta_{e\mu}}$, $\sqrt{\lvert\theta_e\theta_\mu\rvert}$	1σ	< 0.018	$< \mathbf{4.1 \cdot 10^{-3}}$
	2σ	< 0.026	$< \mathbf{4.9 \cdot 10^{-3}}$
$\sqrt{2\eta_{e\tau}}$, $\sqrt{\lvert\theta_e\theta_\tau\rvert}$	1σ	$< \mathbf{0.045}$	< 0.107
	2σ	$< \mathbf{0.052}$	< 0.127
$\sqrt{2\eta_{\mu\tau}}$, $\sqrt{\lvert\theta_\mu\theta_\tau\rvert}$	1σ	$< \mathbf{0.024}$	< 0.115
	2σ	$< \mathbf{0.035}$	< 0.137

[Fernandez-Martinez, Hernandez-Garcia, Lopez-Pavon, arXiv:1605.08774]

type-III seesaw mechanism

Higgs-lepton coupling: $\overline{\ell^c_{iL}}\,\tilde{\phi}^*, \tilde{\phi}^\dagger \ell_{iL} \sim (2,+1/2) \otimes (2,-1/2) = (3,0) \oplus (\mathbf{1},\mathbf{0})$



$$(3,0) \otimes (3,0) = (1,0) \oplus (3,0) \oplus (5,0)$$

Field content: SM fields+ n_Σ **RH fermion triplets** $\vec{\Sigma}_R$

We can write each fermionic triplet as: $\vec{\Sigma}_{iR} = \left(\Sigma^{(1)}_{iR}, \Sigma^{(2)}_{iR}, \Sigma^{(3)}_{iR}\right)$

It is convenient to work with charge eigenstates and use a 2x2 representation:

$$\Sigma_{iR} = \begin{pmatrix} \frac{\Sigma^0_{iR}}{\sqrt{2}} & \Sigma^+_{iR} \\ \Sigma^-_{iR} & -\frac{\Sigma^0_{iR}}{\sqrt{2}} \end{pmatrix}, \quad \Sigma^\pm_{iR} = \frac{\Sigma^{(1)}_{iR} \mp \Sigma^{(2)}_{iR}}{\sqrt{2}}, \quad \Sigma^0_{iR} = \Sigma^{(3)}_{iR}$$

In terms of which the Lagrangian is

$$\mathcal{L}_{\rm full} = \mathcal{L}_{\rm SM} + {\rm Tr}\left[\overline{\Sigma_{iR}} i \not{D}\, \Sigma_{iR}\right] - \left[(Y_\Sigma)_{ij}\, \overline{\ell_{iL}}\, \Sigma_{jR}\, \widetilde{\phi} + \frac{1}{2}(M_\Sigma)_{ij} {\rm Tr}\left[i\sigma_2 \overline{\Sigma^c_{iR}}\, i\sigma_2\, \Sigma_{jR}\right] + {\rm h.c.}\right]$$

Similar to type-I seesaw mechanism, the neutrino mass terms read

$$\mathcal{L}^{\nu}_{\rm mass} = -\frac{1}{2}(\mathcal{M}_\nu)_{ij}\overline{N_{iL}}N^c_{jL} + {\rm h.c.}, \quad N_L = \left(\nu_{eL}\; \nu_{\mu L}\; \nu_{\tau L}\; \Sigma^{0c}_{1R} \ldots \Sigma^{0c}_{n_\Sigma R}\right)^T$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_\Sigma \end{pmatrix}, \quad M_D = \frac{v}{\sqrt{2}} Y_\Sigma$$

The effective light neutrino mass matrix is

$$M_\nu = -M_D M_\Sigma^{-1} M_D^T$$

There are corrections to the charged-lepton masses due to the presence of the heavy fields:

$$\mathcal{M}_\ell = \begin{pmatrix} m_\ell & M_D \\ 0 & M_\Sigma \end{pmatrix}$$

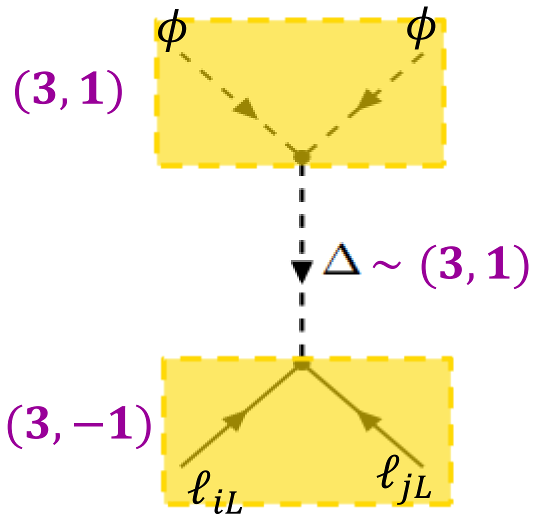
$$\mathcal{L}^{\pm}_{\rm mass} = -\frac{1}{2}(\mathcal{M}_\ell)_{ij}\overline{E_{iL}}E_{jR} + {\rm h.c.},$$

$$E_L = \left(e_L\; \mu_L\; \tau_L\; \Sigma^{+c}_{1R} \ldots \Sigma^{+c}_{n_\Sigma R}\right)^T, \quad E_R = \left(e_R\; \mu_R\; \tau_R\; \Sigma^{-}_{1R} \ldots \Sigma^{-}_{n_\Sigma R}\right)^T$$

- The charged and neutral current interactions are a bit more complicated but nothing drastically different with respect to the type I seesaw case....

type-II seesaw mechanism

Higgs-lepton coupling: $\overline{\ell_{iL}^c}\ell_{jL} \sim (2,-1/2)\otimes(2,-1/2) = (1,-1)\oplus(3,-1)$



The singlet combination $\overline{\nu_{iL}^c}\, e_{jL} - \overline{e_{iL}^c}\, \nu_{jL}$ doesn't lead to a neutrino mass term

Field content: SM fields+ **Y=1 scalar triplet** $\vec{\Delta} = \left(\Delta^{(1)}, \Delta^{(2)}, \Delta^{(3)}\right)$

It is convenient to work with charge eigenstates and use a 2x2 representation:

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}, \quad \Delta^{++} = \frac{\Delta^{(1)} - i\Delta^{(2)}}{\sqrt{2}}, \quad \Delta^0 = \frac{\Delta^{(1)} + i\Delta^{(2)}}{\sqrt{2}}, \quad \Delta^+ = \Delta^{(3)}$$

The Lagrangian is

$$\mathcal{L}_{\rm full} = \mathcal{L}_{\rm SM} + {\rm Tr}\left[(D^\mu\Delta)^\dagger\,(D_\mu\Delta)\right] + \left[(Y_\Delta)_{ij}\,\overline{\ell_{iL}}\,\Delta^\dagger\,i\sigma_2\ell^c_{jL} + {\rm h.c.}\right] - V(\phi,\Delta)$$

- The most general gauge-invariant scalar potential is

$$V(\phi,\Delta) = M_\Delta^2{\rm Tr}(\Delta^\dagger\Delta) + \mu\tilde{\phi}^T i\sigma_2\Delta\tilde{\phi} + \lambda\phi^\dagger\Delta^\dagger\Delta\phi + \lambda'\phi^\dagger\phi{\rm Tr}(\Delta^\dagger\Delta) + \tilde{\lambda}{\rm Tr}[(\Delta^\dagger\Delta)^2] + \hat{\lambda}\left({\rm Tr}[\Delta^\dagger\Delta]\right)^2 + {\rm h.c.}$$

In the limit $M_\Delta^2 \gg v^2$, minimize the potential

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2}\end{pmatrix}, \quad \langle\Delta\rangle = \begin{pmatrix} 0 & 0 \\ u/\sqrt{2} & 0\end{pmatrix} \Rightarrow u = \frac{-\mu v^2}{\sqrt{2}M_\Delta^2}$$

The VEV of Δ^0 is supressed by the large triplet mass

- Neutrino mass

$$\mathcal{L} \supset (Y_\Delta)_{ij}\overline{\ell_{iL}}\,\Delta^\dagger i\sigma_2\ell^c_{jL} + {\rm h.c.} = -(Y_\Delta)_{ij}\overline{\nu_{iL}}\,\Delta^0\nu^c_{jL} + \cdots + {\rm h.c.}$$

$\langle\Delta^0\rangle = u/\sqrt{2}$

$$\mathcal{L}^\nu_{\rm mass} = -\frac{u}{\sqrt{2}}(Y_\Delta)_{ij}\overline{\nu_{iL}}\,\nu^c_{jL} + {\rm h.c.}$$

$$\Rightarrow M_\nu = \sqrt{2}\,Y_\Delta u = -\mu\frac{v^2}{M_\Delta^2}Y_\Delta$$

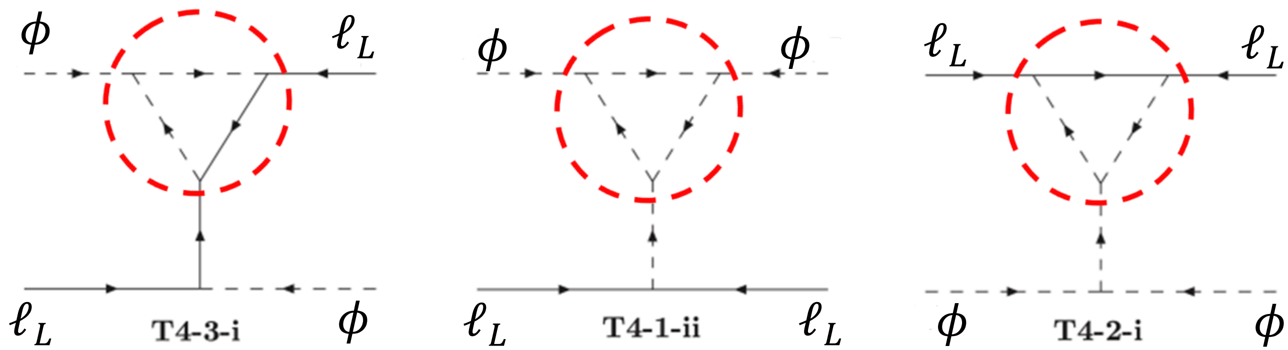
The μ term breaks lepton number and, thus, can be naturally small

Radiative origin of neutrino mass

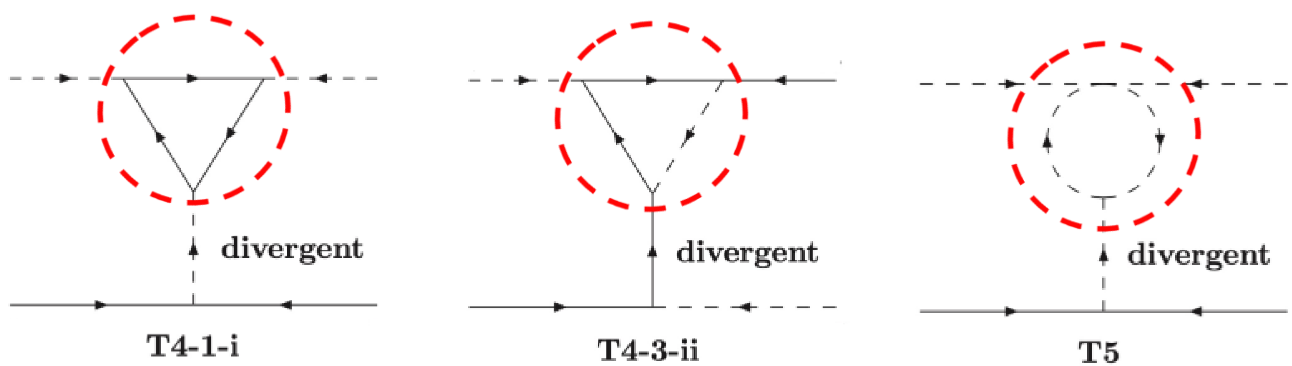
- neutrino mass vanishes at tree level, generated radiatively at n −loop order
- neutrino masses suppressed by loop factors: intermediate states can be light and probed at existing facilities such as colliders, charged lepton flavour violation. (NOTE: ν_R in seesaw at the GUT scale)

➢ **General classification of one-loop neutrino mass models**

- Topologies which are just finite corrections to tree-level seesaws

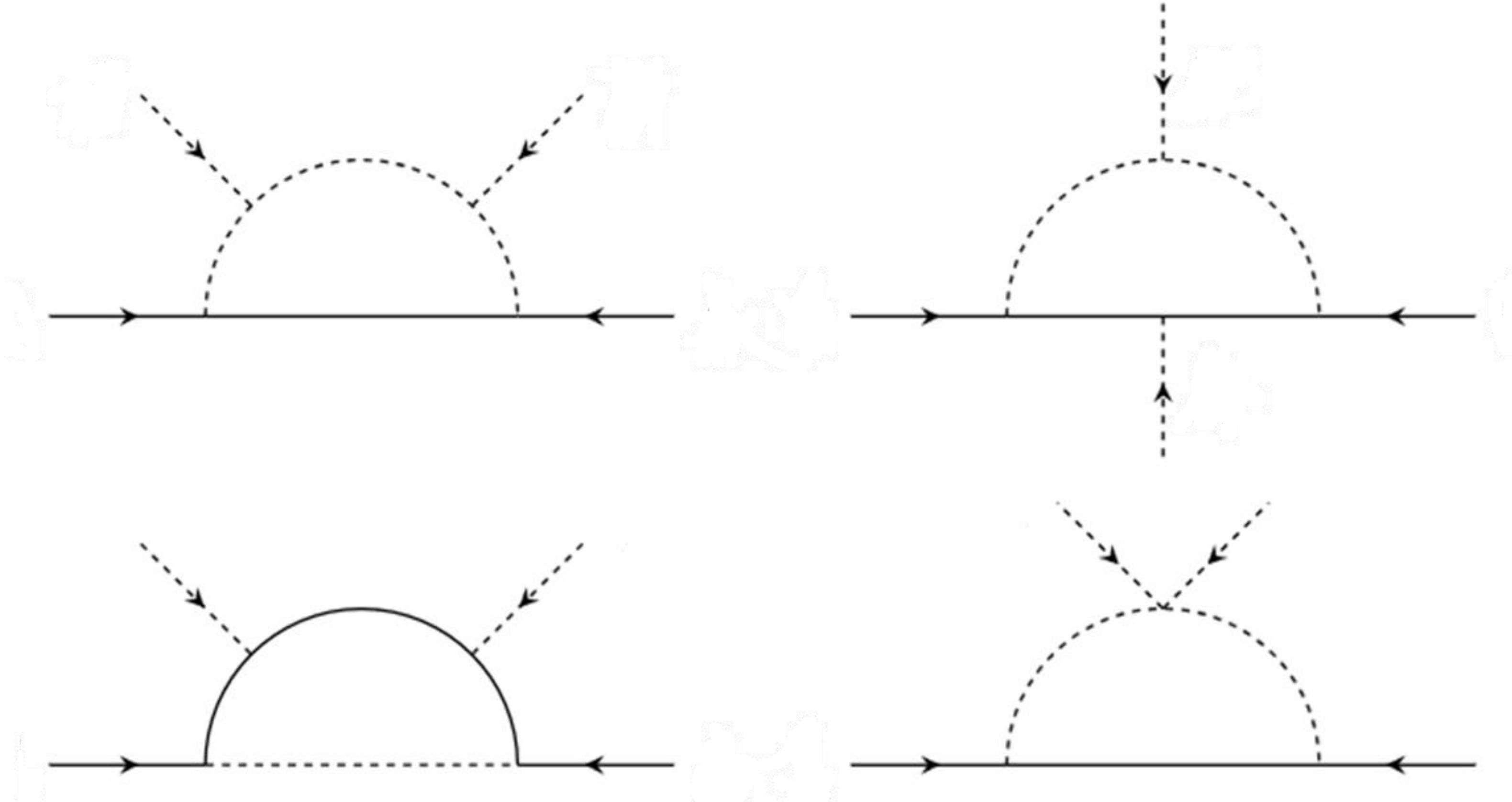


- Divergent diagrams:



- Genuine 1-loop topologies

Genuine n-loop order means that only diagrams starting from n-loop order contribute to neutrino mass. There are no lower-order terms contributing to neutrino masses.



Dashed lines denote scalars or gauge bosons; solid lines refer to fermions

- The quantum numbers of the messenger fields are fixed by SM gauge invariance
- uniqueness of tree-level seesaw lost, large (∞) number of models

[Bonnet, Hirsch, Ota, Winter, 1411.7038]

An example: Zee model

Zee model is an extension of SM with two Higgs doublets $\phi_1, \phi_2 \sim (1,2,1/2)$ and a singly-charged scalar singlet $h^+ \sim (1,1,1)$

- **Higgs basis:** only Higgs field takes a VEV

[Zee,Phys.Lett.B 93 (1980) 389; Cheng, Li, Phys. Rev. D22 (1980) 2860]

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \varphi_1^0 + iG^0) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\varphi_2^0 + iA) \end{pmatrix}, \quad h^+$$

- **scalar potential (16 independent terms)**

$$\begin{aligned} V = &\ \mu_1^2 H_1^\dagger H_1 + \mu_2^2 H_2^\dagger H_2 - \left(\mu_3^2 H_2^\dagger H_1 + \text{H.c.}\right) + \frac{1}{2}\lambda_1 \left(H_1^\dagger H_1\right)^2 \\ &+ \frac{1}{2}\lambda_2 \left(H_2^\dagger H_2\right)^2 + \lambda_3 \left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + \lambda_4 \left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right) \\ &+ \left\{\frac{1}{2}\lambda_5 \left(H_1^\dagger H_2\right)^2 + \left[\lambda_6 \left(H_1^\dagger H_1\right) + \lambda_7 \left(H_2^\dagger H_2\right)\right] H_1^\dagger H_2 + \text{H.c.}\right\} \\ &+ \mu_h^2 \left|h^+\right|^2 + \lambda_h \left|h^+\right|^4 + \lambda_8 \left|h^+\right|^2 H_1^\dagger H_1 + \lambda_9 \left|h^+\right|^2 H_2^\dagger H_2 \\ &+ \lambda_{10} \left|h^+\right|^2 \left(H_1^\dagger H_2 + \text{H.c.}\right) + \left(\mu \epsilon_{\alpha\beta} H_1^\alpha H_2^\beta h^- + \text{H.c.}\right) \end{aligned}$$

Scalar mass spectrum

Two charged Higgs: $$m^2_{h_1^+,h_2^+} \equiv \frac{1}{2}\left[M^2_{H^+} + M^2_{33} \mp \sqrt{\left(M^2_{H^+} - M^2_{33}\right)^2 + 2v^2\mu^2}\,\right]$$

$$M^2_{H^+} = \mu_2^2 + \frac{1}{2}v^2\lambda_3\,, \quad M^2_{33} = \mu_h^2 + v^2\lambda_8$$

one CP-odd Higg $$m_A^2 = M^2_{H^+} - \frac{1}{2}v^2\left(\lambda_5 - \lambda_4\right)$$

two CP-even Higgs

$$m^2_{H,h} \equiv \frac{1}{2}\left\{m_A^2 + v^2\left(\lambda_1 + \lambda_5\right) \pm \sqrt{\left[m_A^2 + v^2\left(\lambda_5 - \lambda_1\right)\right]^2 + 4v^4\lambda_6^2}\right\}$$

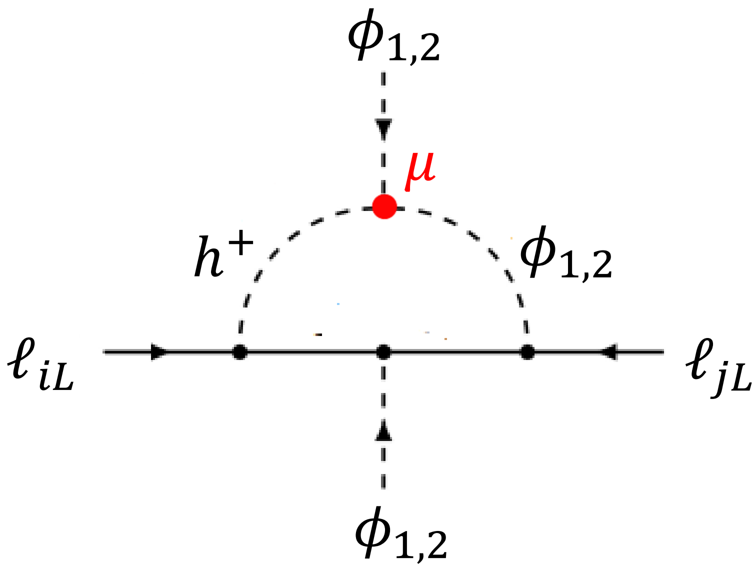
Very rich phenomenology for LHC, FCNC, LFV: stability of the potential, naturality and perturbativity, Higgs signals, Higgs lepton flavor violation…

[e.g., Herrero-Garcia,Ohlsson,Riad,Wiren,1701.05345]

➢ Lepton Yukawa interactions and neutrino mass

$f_{ij} = -f_{ji}$

$$-\mathcal{L}^{\ell}_{Yuk} = \overline{\ell_{iL}} \left[(Y_1^\dagger)_{ij}\phi_1 + (Y_2^\dagger)_{ij}\phi_2 \right] E_{jR} + f_{ij}\overline{\ell^c_{iL}} i\sigma_2 \ell_{jL} h^+ + \text{h.c.}$$



In the scalar mass eigenstate basis, calculating the one-loop diagram

$$M_\nu = \frac{s_{2\varphi}\, t_\beta}{8\sqrt{2}\pi^2\, v} \ln \frac{m^2_{h_2^+}}{m^2_{h_1^+}} \left[f\, m_E^2 + m_E^2 f^T - \frac{v}{\sqrt{2}\, s_\beta} (f\, m_E\, Y_2 + Y_2^T\, m_E\, f^T) \right],$$

$$m_E \equiv \frac{v}{\sqrt{2}} \left(c_\beta Y_1^\dagger + s_\beta Y_2^\dagger \right), \quad \tan\beta \equiv v_2/v_1\,, \quad s_{2\varphi} \equiv \frac{\sqrt{2} v \mu}{m^2_{h_2^+} - m^2_{h_1^+}}$$

If $\mu = 0$ lepton number conservation is restored and neutrino masses vanish

Scotogenic model:combining neutrino mass with Dark Matter

Scotogenic comes from the Greek word skotos (σκοτος) -darkness. Then, the meaning of scotogenic is "created from darkness". [E. Ma, hep-ph/0601225]

- version of inert Higgs doublet model
- SM + 2nd Higgs doublet η + RH neutrinos N
- η and N are odd under a discrete Z_2 symmetry ⇒ the lightest of them is a DM candidate
- neutrino masses generated at 1-loop (Tree-level forbidden by Z_2)

Field	$SU(2)_L \times U(1)_Y$	Z_2
ℓ_{iL}	$(2,-1/2)$	$+$
e_i	$(1,-1)$	$+$
ϕ	$(2,\ 1/2)$	$+$
N_i	$(1,0)$	$-$
η	$(2,\ 1/2)$	$-$

➢ Fermion Yukawa and mass terms

$$\mathcal{L}_N = \overline{N_i} i \partial\!\!\!/ N_i - \frac{M_{N_i}}{2} \overline{N_i^c} N_i - (Y_N)_{ij} \tilde{\eta}^\dagger \overline{N_i} \ell_{jL} + \text{h.c.}$$

The Z_2 symmetry does not allow couplings $\tilde{\phi}^\dagger \overline{N} \ell_L$

➢ Scalar potential and electroweak symmetry breaking

$$V(\phi,\eta) = -m_\phi^2\phi^\dagger\phi + m_\eta^2\eta^\dagger\eta + \frac{\lambda_1}{2}\left(\phi^\dagger\phi\right)^2 + \frac{\lambda_2}{2}\left(\eta^\dagger\eta\right)^2 + \lambda_3\left(\phi^\dagger\phi\right)\left(\eta^\dagger\eta\right)$$
$$+\lambda_4\left(\phi^\dagger\eta\right)\left(\eta^\dagger\phi\right) + \frac{\lambda_5}{2}\left[\left(\phi^\dagger\eta\right)^2 + \left(\eta^\dagger\phi\right)^2\right]$$

If $\lambda_5 = 0$ then we can assign $L(\eta) = -1, L(N = 0), L(\ell_L) = 1$, and lepton number conservation is restored. Small λ_5 is natural in the 't Hooft sense.

We want the Z_2 symmetry to be maintained after EWSB. Thus we must guarantee that:

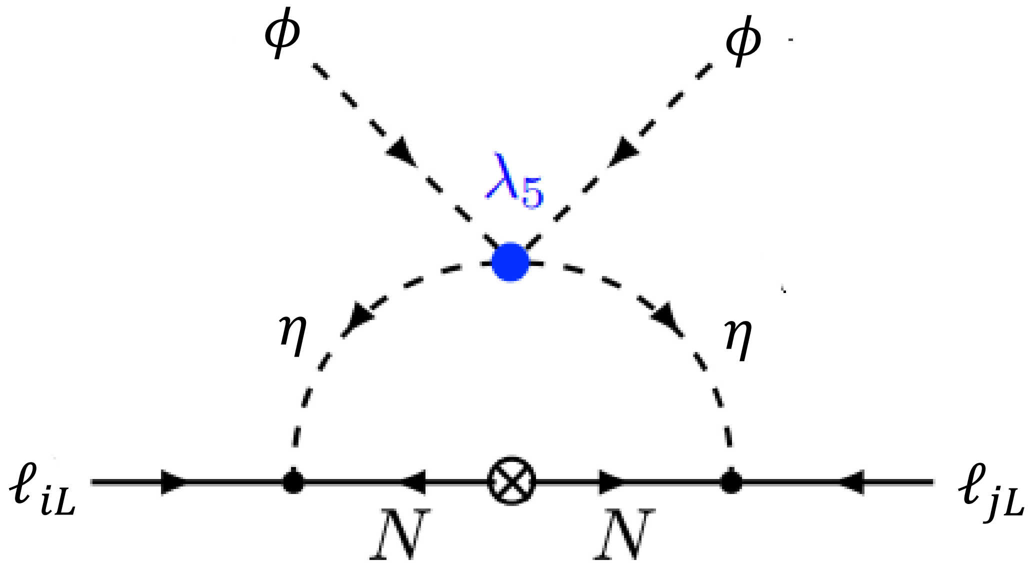
$$\langle\eta\rangle \equiv \begin{pmatrix}0\\0\end{pmatrix},\ \ \langle\phi\rangle \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}$$ ⟹ Inert scalar sector: $\eta^\pm,\ \ \eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$

Scalar mass spectrum:

$$\begin{cases} m_{\eta^+}^2 = m_\eta^2 + \lambda_3 v^2/2 \\ m_R^2 = m_\eta^2 + \left(\lambda_3+\lambda_4+\lambda_5\right)v^2/2 \\ m_I^2 = m_\eta^2 + \left(\lambda_3+\lambda_4-\lambda_5\right)v^2/2 \end{cases}$$ ⟹ $$\boxed{m_R^2 - m_I^2 = \lambda_5 v^2}$$

- Z_2 remains unbroken, the lightest particle η_R, η_I or N_i will be stable→dark matter candidate.

➢ Scotogenic neutrino mass matrix



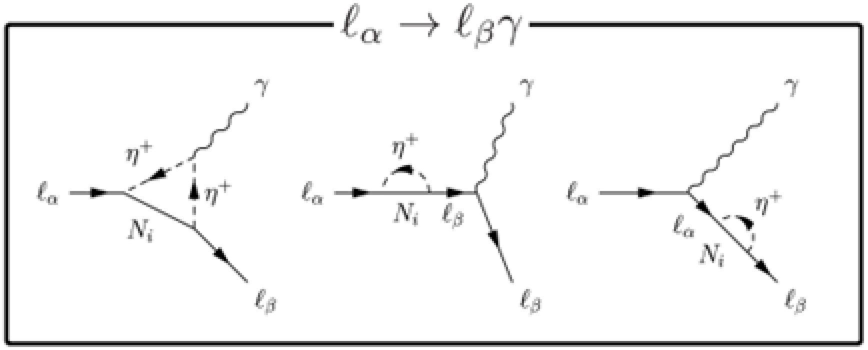
$$(M_\nu)_{\alpha\beta} = \sum_{k=1}^{3} \frac{(Y_N)_{ki}(Y_N)_{kj}}{32\pi^2} M_{N_k} \left[\frac{m_R^2}{m_R^2 - M_{N_k}^2} \log\left(\frac{m_R^2}{M_{N_k}^2}\right) - \frac{m_I^2}{m_I^2 - M_{N_k}^2} \log\left(\frac{m_I^2}{M_{N_k}^2}\right) \right]$$

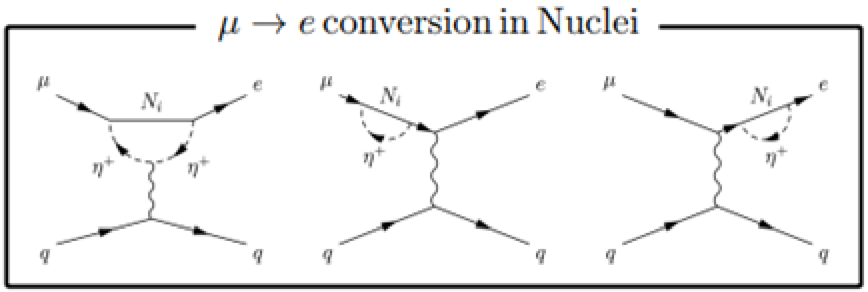
small scalar masses splitting: $m_R^2 - m_I^2 = \lambda_5 v^2 \ll m_0^2 \equiv (m_R^2 + m_I^2)/2$

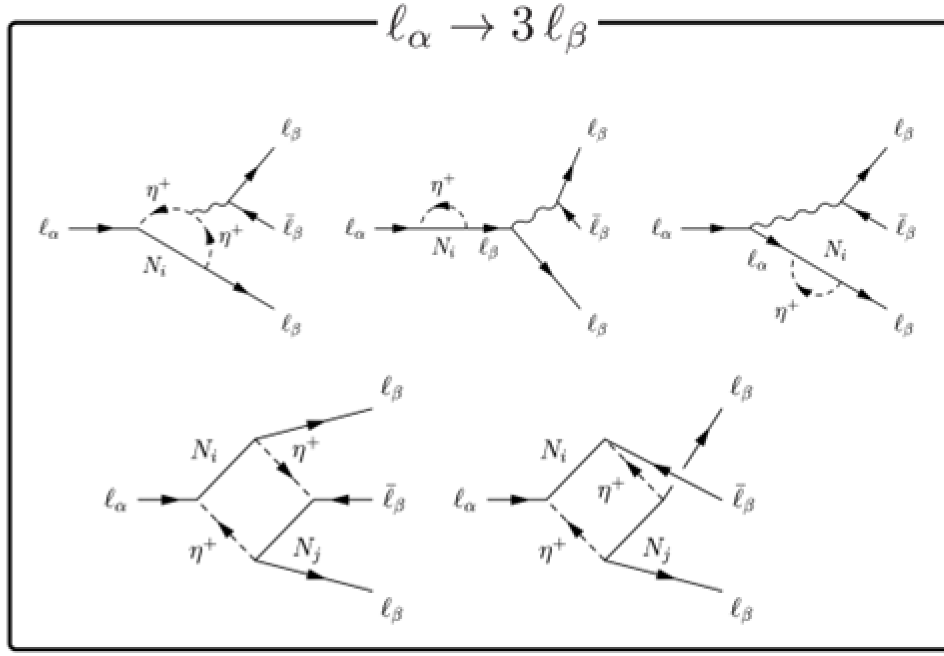
$$(M_\nu)_{\alpha\beta} \approx \frac{\lambda_5 v^2}{32\pi^2} \sum_{k=1}^{3} \frac{(Y_N)_{ki}(Y_N)_{kj} M_{N_k}}{m_0^2 - M_{N_k}^2} \left[1 + \frac{M_{N_k}^2}{m_0^2 - M_{N_k}^2} \log\left(\frac{M_{N_k}^2}{m_0^2}\right) \right]$$

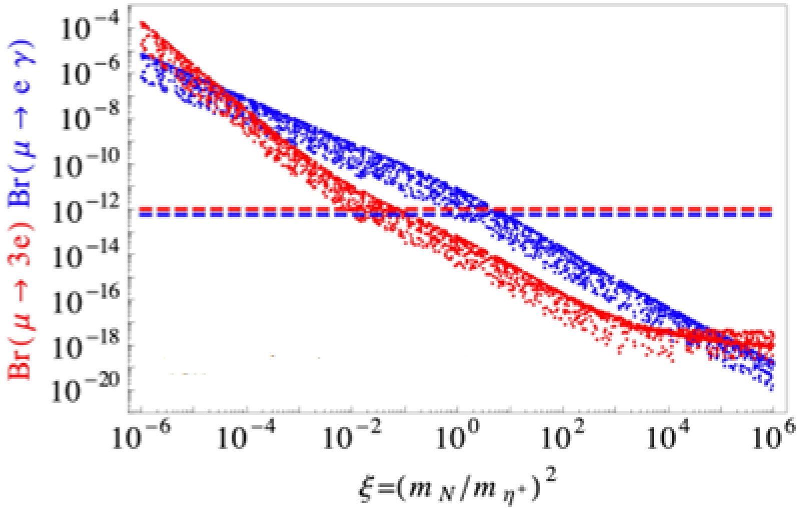
Many phenomenologically interesting aspects: LHC signals, dark matter relic abundance, LFV, etc

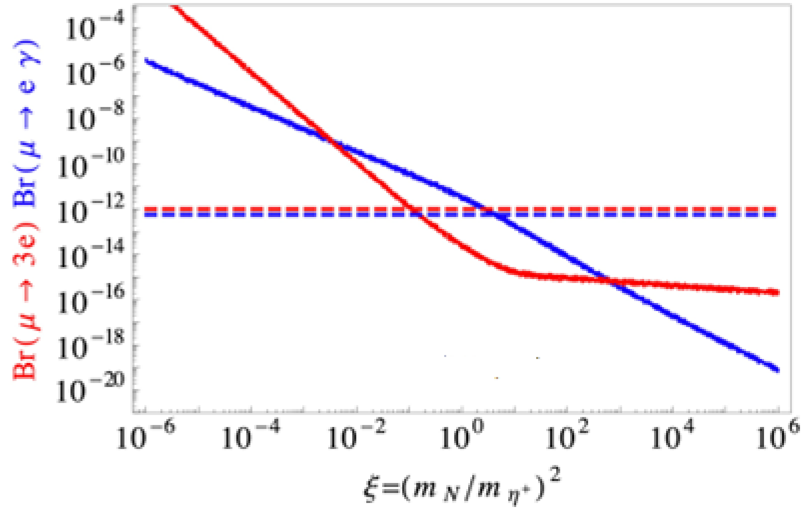
Lepton flavor violation in Scotogenic model











[Toma, Vicente, 1312.2840]

Lecture 5

- **Absolute neutrino mass in beta decay and cosmology**
- **Neutrinoless double beta decay**

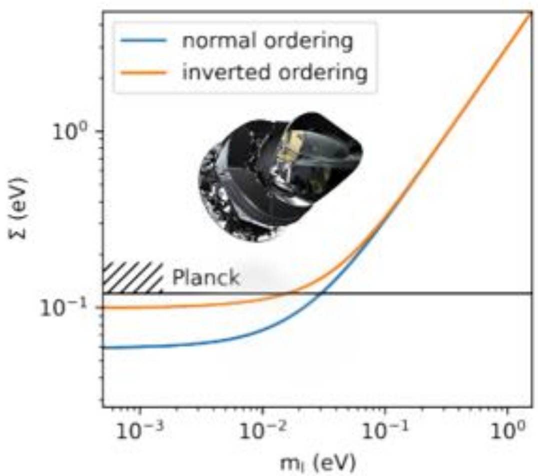
Absolute neutrino mass

neutrino oscillation determines Δm_{ij}^2, absolute mass scale is unconstrained

- Three different ways to measure absolute neutrino mass: sensitive to different quantities

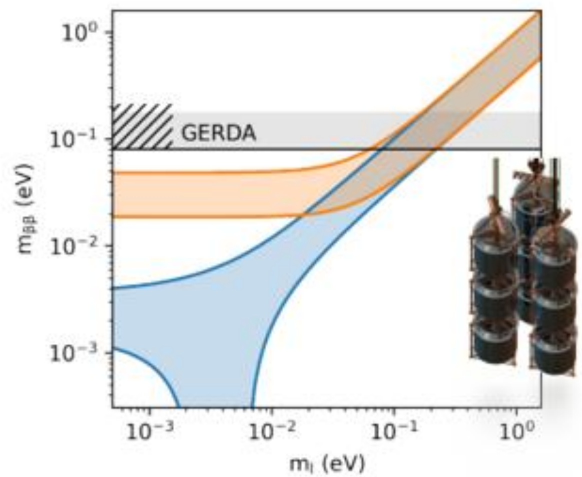
Cosmology

$$\Sigma = \sum_i m_i$$



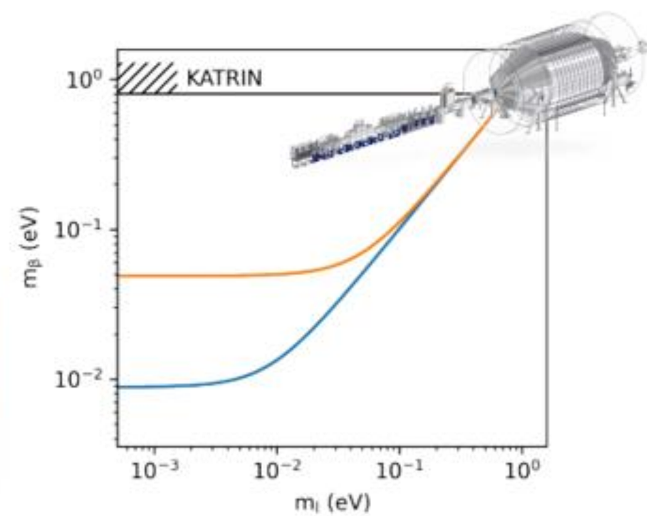
Neutrinoless ββ decay

$$m_{\beta\beta} = \left| \sum_i U_{ei}^2 m_i \right|$$



β-decay kinetics

$$m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

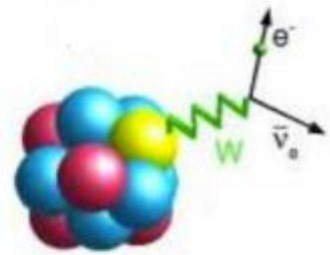


The β spectrum and neutrino mass

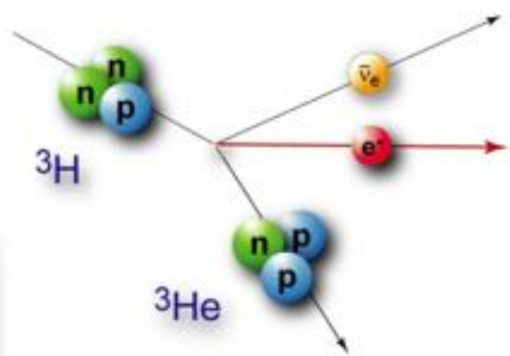
- β decay

$$n \rightarrow p + e^- + \overline{\nu}_e$$

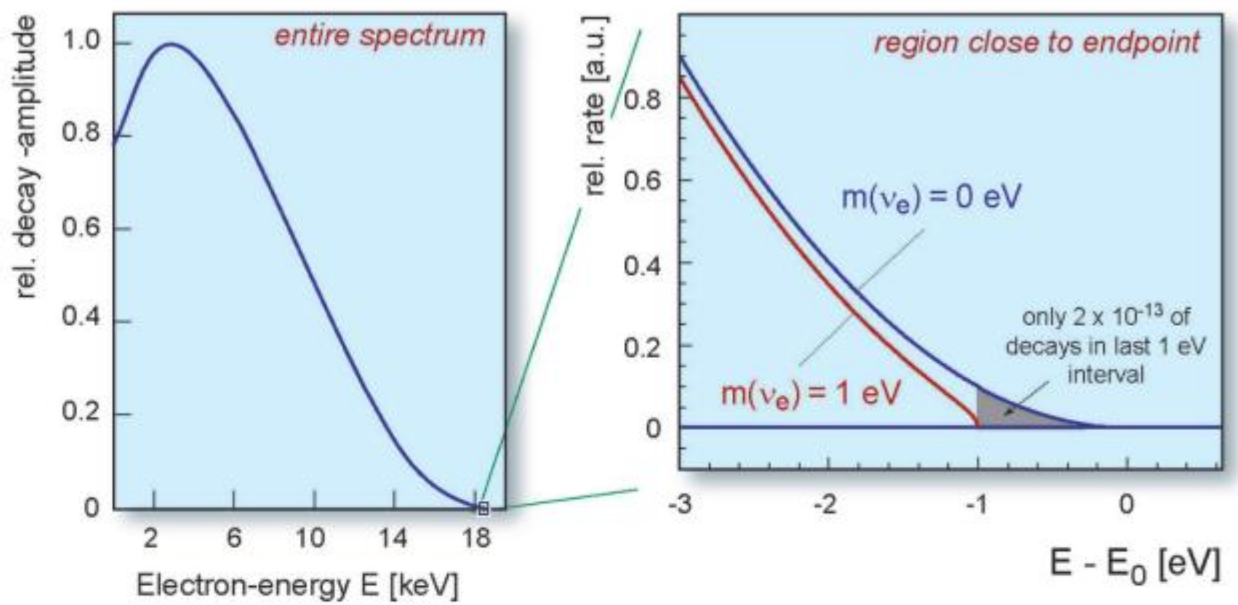
$$(A, Z) \rightarrow (A, Z+1) + e^- + \bar{\nu}_e$$



- General idea: distortion of the endpoint of the electron spectrum due to tiny $m_i \neq 0$



Tritium beta decay $\quad {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^- + \bar{\nu}_e$

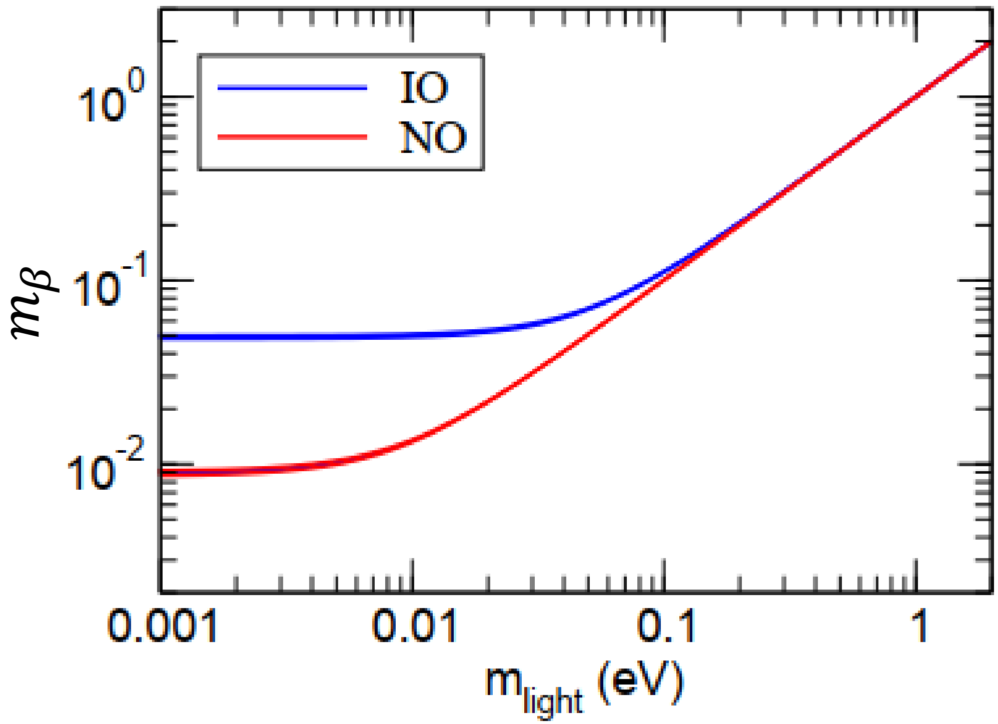


$$\frac{dN}{dE} \propto \sqrt{(E_0 - E)^2 - m_\beta^2}$$

$$m_\beta^2 = \sum_i m_i^2 |U_{ei}|^2$$

The effective mass

$$m_\beta = \sqrt{\sum_i m_i^2 |U_{ei}|^2} = \begin{cases} \sqrt{m_1^2 + \Delta m_{21}^2(1 - c_{13}^2 c_{12}^2) + \Delta m_{32}^2 s_{13}^2} & \text{for NO} \\ \sqrt{m_3^2 - \Delta m_{21}^2 c_{13}^2 c_{12}^2 - \Delta m_{32}^2 c_{13}^2} & \text{for IO} \end{cases}$$



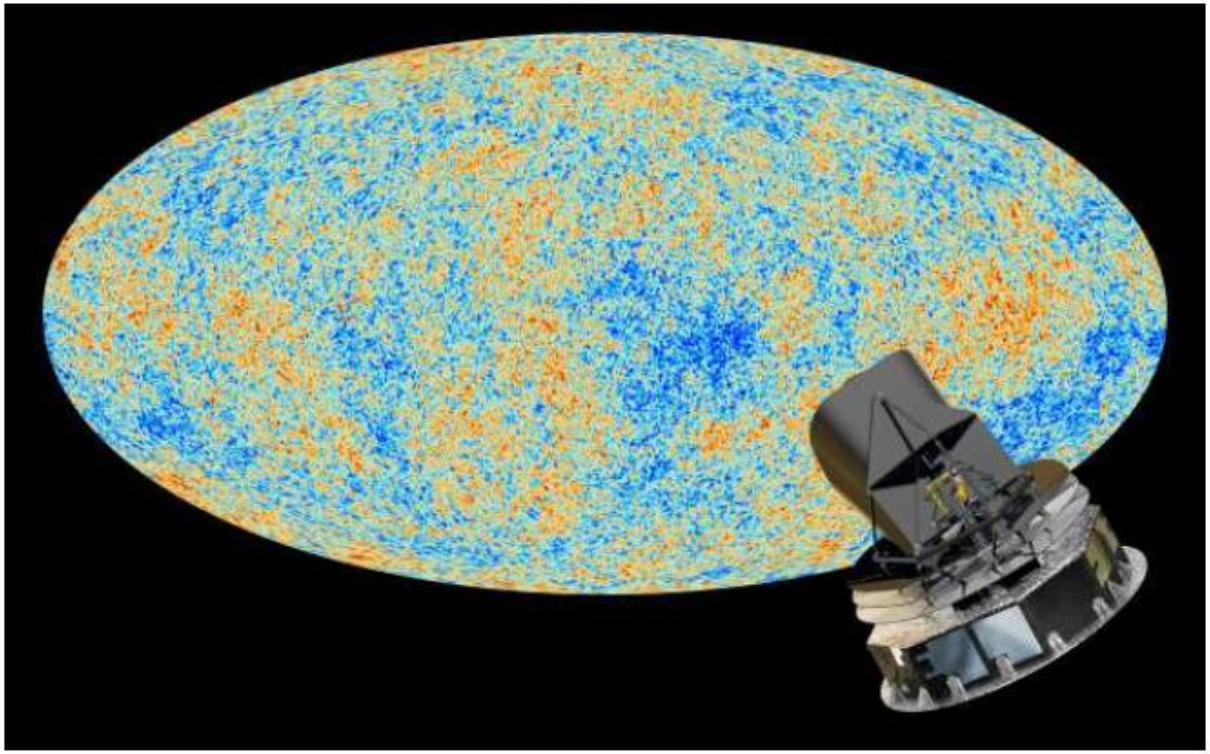
Minimum value for $m_{light} = 0$: $m_\beta^{min} = 8.5\ (48)$ meV for NO (IO)

For $m_{light} \gg |\Delta m_{31}^2|$: $m_\beta \approx m_{light}$

KATRIN = KArlsruhe TRitium Neutrino Experiment

Sensitivity of KATRIN aims to $m_\beta \leq 0.2\text{eV}$

Cosmology and neutrinos



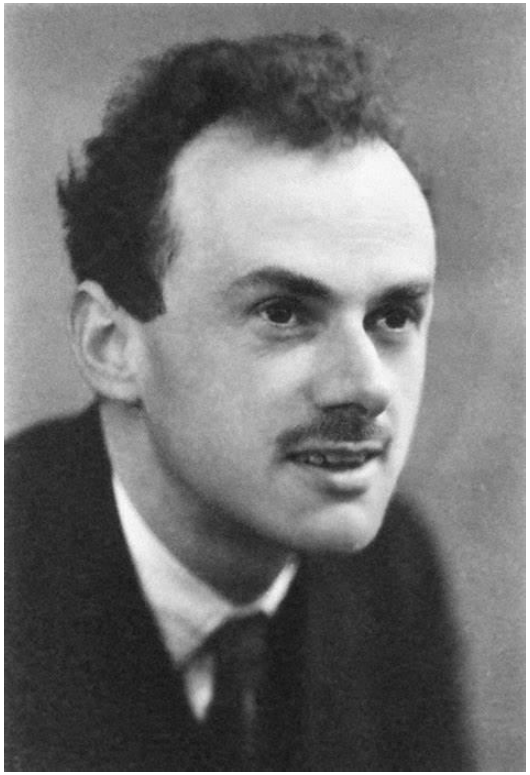
A bound on the sum over all neutrino masses is provided by the large-scale structure of the universe and the temperature fluctuations of the cosmic microwave background (CMB)

$$\Omega_\nu h^2 = \frac{\sum_i m_i}{93.5\ \mathrm{eV}},\quad \Omega_\nu \equiv \frac{\rho_\nu}{\rho_{cr}}, h = 0.7$$

The current constraint from Planck: $\sum_i m_i \leq 0.12\ \mathrm{eV}$

Massive neutrinos: Dirac or Majorana?

$\nu \neq \nu^c$



vs.

$\nu = \nu^c$

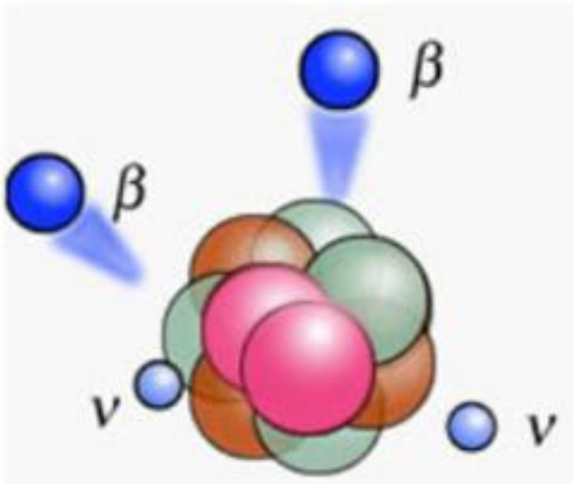
- **neutrinoless double beta decay**
- lepton number violation at collider
- cosmology

The 2β-decays

2νββ decays

$$(A,Z) \rightarrow (A,Z+2) + 2e^{-} + 2\overline{\nu}_e$$

$\Delta L = 0$

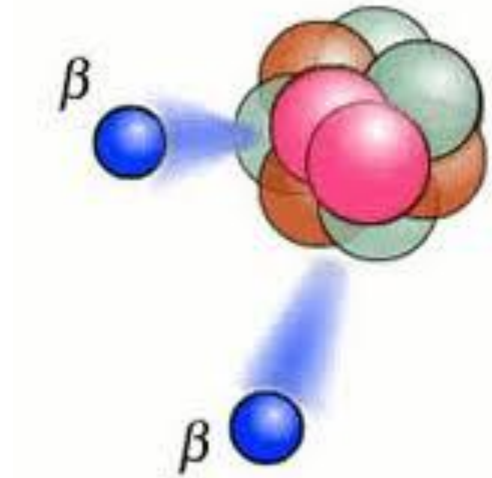


[Goeppert-Mayer, Phys. Rev.48,512(1935)]

- Allowed in SM
- second order in weak interaction
- Natural background for decay

0νββ decays

$$(A,Z) \rightarrow (A,Z+2) + 2e^{-}$$



$\Delta L = 2$

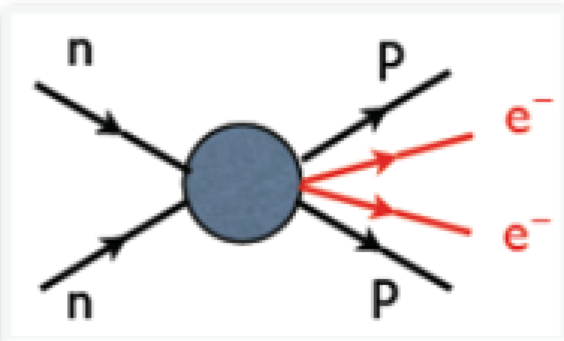
[Furry, Phys.Rev.56,1184(1939)]

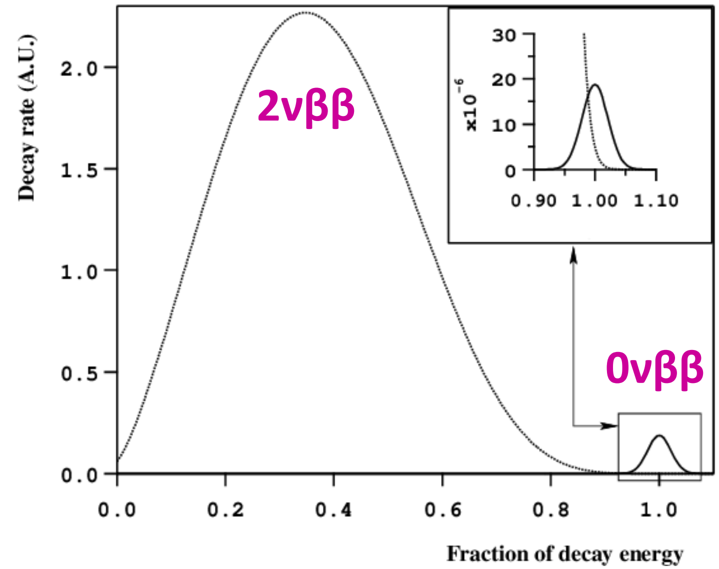
- **NOT** allowed in SM→rare
- Lepton number violation→ neutrinos are Majorana fermions

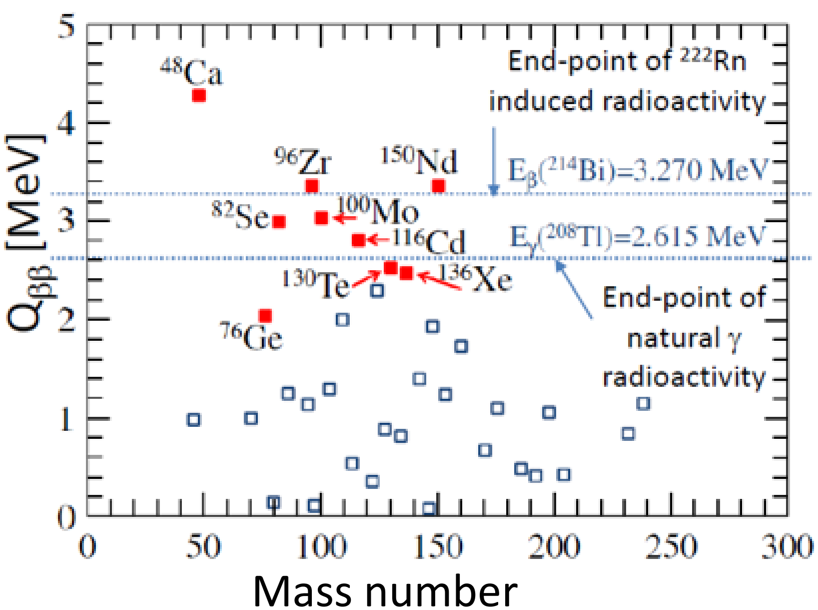
The 0νββ-decays: signature and candidate nuclei

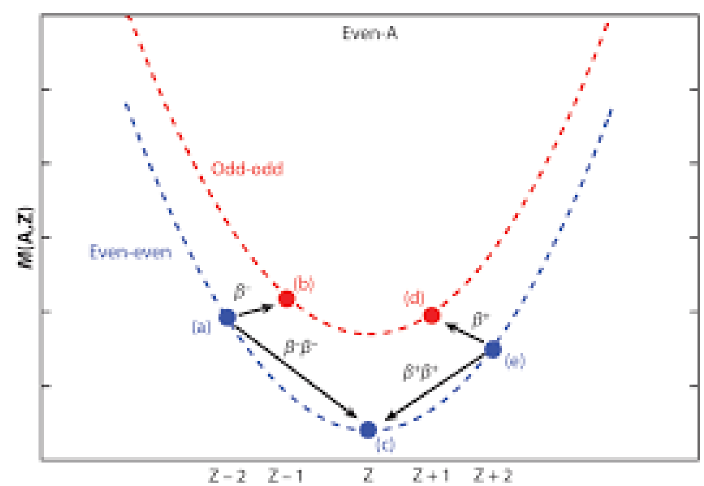
0νββ is potentially observable in certain even-even nuclei (9 isotopes including ^{48}Ca, ^{76}**Ge**, ^{100}Mo, ^{130}Te, ^{136}**Xe**) for which single beta decay is energetically forbidden. The decay rate is less than 1 event per ton and year.

$$T_{1/2} > 10^{25}\ \mathrm{yr}$$









The 9 experimentally most feasible isotopes:

$$Q_{\beta\beta} = M(A,Z) - M(A,Z+2)$$

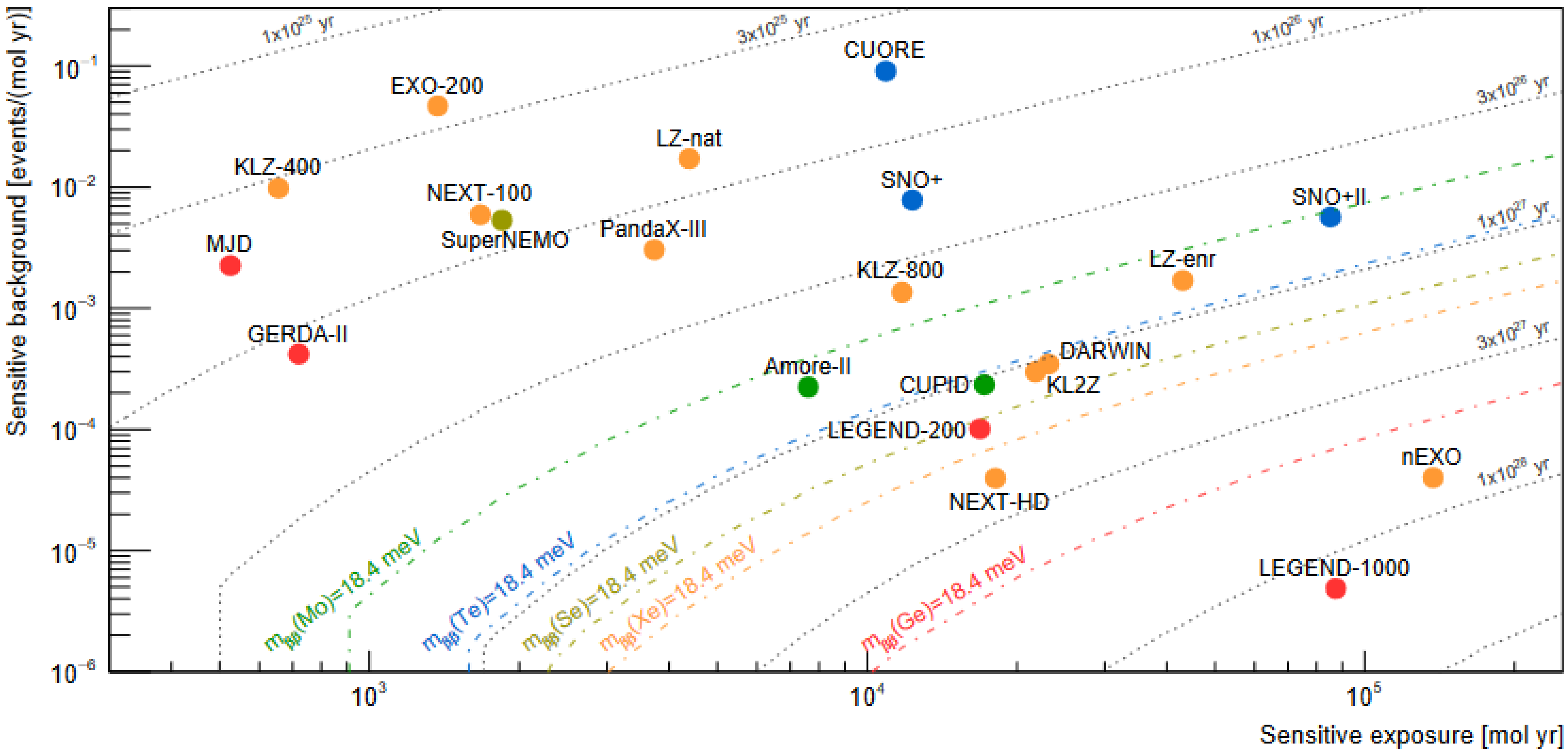
Isotope	Abundance (%)	$Q_{\beta\beta}$ (MeV)
^{48}Ca	0.187	4.263
^{76}Ge	7.8	2.039
^{82}Se	9.2	2.998
^{96}Zr	2.8	3.348
^{100}Mo	9.6	3.035
^{116}Cd	7.6	2.813
^{130}Te	34.08	2.527
^{136}Xe	8.9	2.459
^{150}Nd	5.6	3.371

Current and future experiments

Most stringent constraints on the half life:

- ^{136}Xe (KamLAND-Zen): $T_{1/2} > 2.3\text{x}10^{26}$ yrs [KamLAND-Zen Collaboration, 2203.02139]
- ^{76}Ge (GERDA): $T_{1/2} > 1.8\text{x}\ 10^{26}$ yrs [GERDA collaboration, 2009.06079]
- ^{130}Te (CUORE): $T_{1/2} > 2.2\text{x}\ 10^{25}$ yrs [CUORE collaboration, 2104.06906]

There are many 0νββ decay experiments in plan and construction



[Agostini, Benato, Detwiler, Menendez, Vissani, 2202.01787]

Half life of Neutrinoless Double Beta Decay

$$T_{1/2}^{-1} = |\, m_{\beta\beta} \,|^2 G^{0\nu} \,|\, M^{0\nu} \,|^2$$

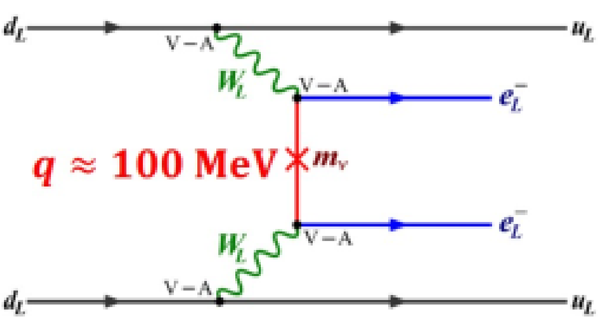
particle physics

phase-space factors

nuclear physics

$$A_{\mu\nu}^{\text{lep}} = \frac{1}{4}\sum_{i=1}^{3} U_{ei}^2 \gamma_\mu (1+\gamma_5) \frac{\not{q} + m_{\nu_i}}{q^2 - m_{\nu_i}^2} \gamma_\nu (1-\gamma_5)$$

$$\approx \frac{\gamma_\mu (1+\gamma_5)\gamma_\nu}{4q^2} \boxed{\sum_{i=1}^{3} U_{ei}^2 m_{\nu_i}} \quad m_{\beta\beta}$$



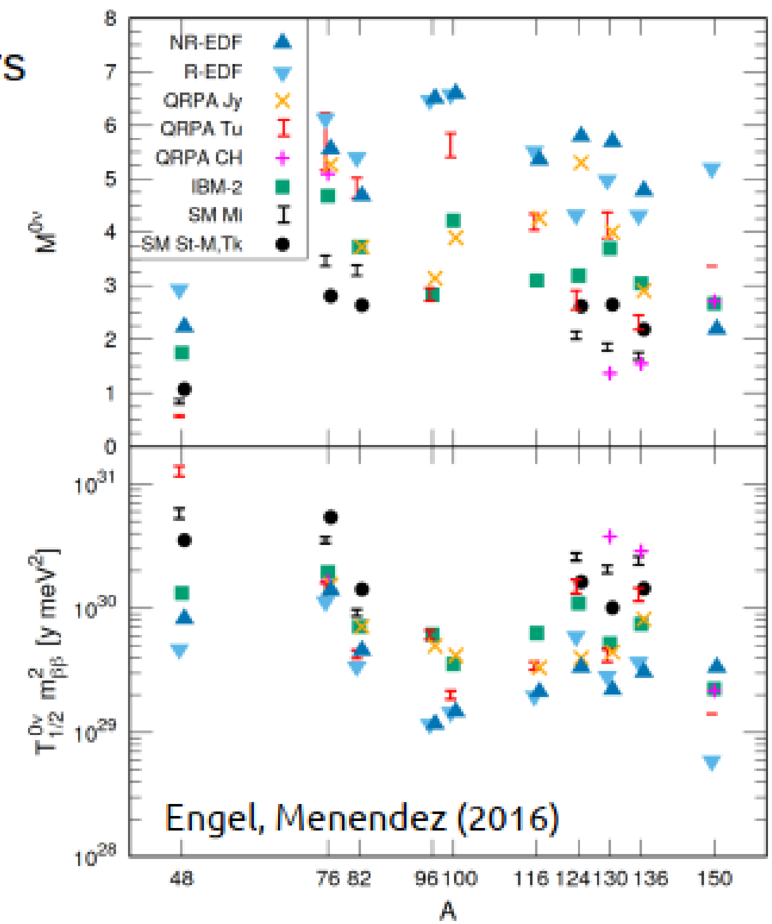
$$T_{1/2}^{0\nu} = \left(G\,|\mathcal{M}|^2 \langle m_{\beta\beta}\rangle^2\right)^{-1} \simeq 10^{27-28} \left(\frac{0.01\,\text{eV}}{\langle m_{\beta\beta}\rangle}\right)^2 \text{y}$$

$$T_{1/2}^{-1} = | m_{\beta\beta} |^2 \, G^{0\nu} | M^{0\nu} |^2$$

particle physics

phase-space factors

nuclear physics



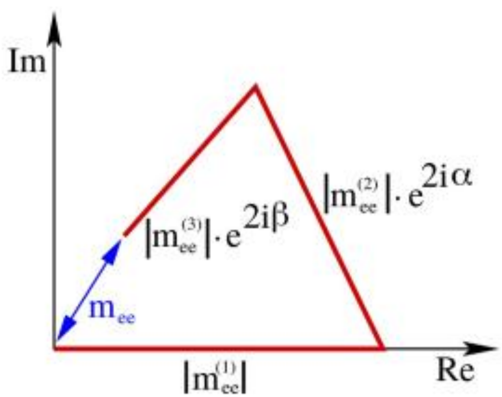
- **Dependence** on isotope and specific operator
- Differences between different **nuclear models**
- "**the g_A problem**" quenching of the axial-vector coupling

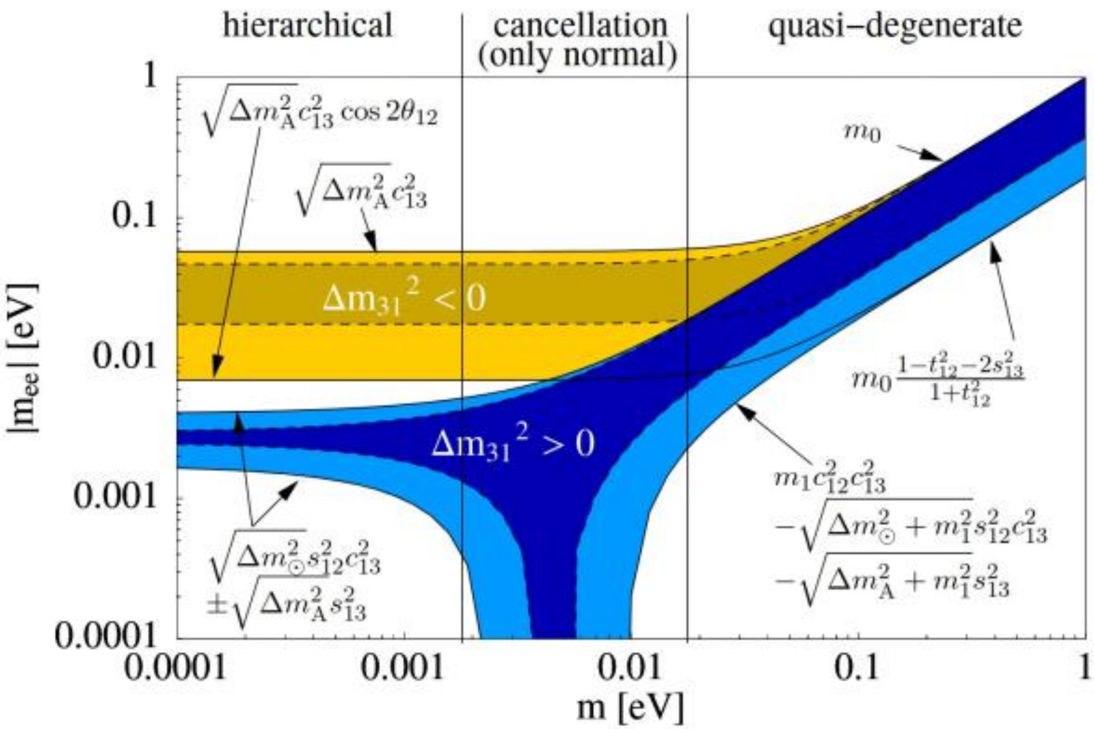
Effective mass $m_{\beta\beta}$ for 0νββ-decay

$$|m_{ee}| \equiv \left|\sum U_{ei}^2\, m_i\right| \quad \text{with } m_{ee} = |m_{ee}^{(1)}| + |m_{ee}^{(2)}|\, e^{2i\alpha} + |m_{ee}^{(3)}|\, e^{2i\beta}$$

$$\begin{aligned} |m_{ee}^{(1)}| &= m_1\, |U_{e1}|^2 = m_1\, c_{12}^2\, c_{13}^2 \\ |m_{ee}^{(2)}| &= m_2\, |U_{e2}|^2 = m_2\, s_{12}^2\, c_{13}^2 \\ |m_{ee}^{(3)}| &= m_3\, |U_{e3}|^2 = m_3\, s_{13}^2\ . \end{aligned}$$

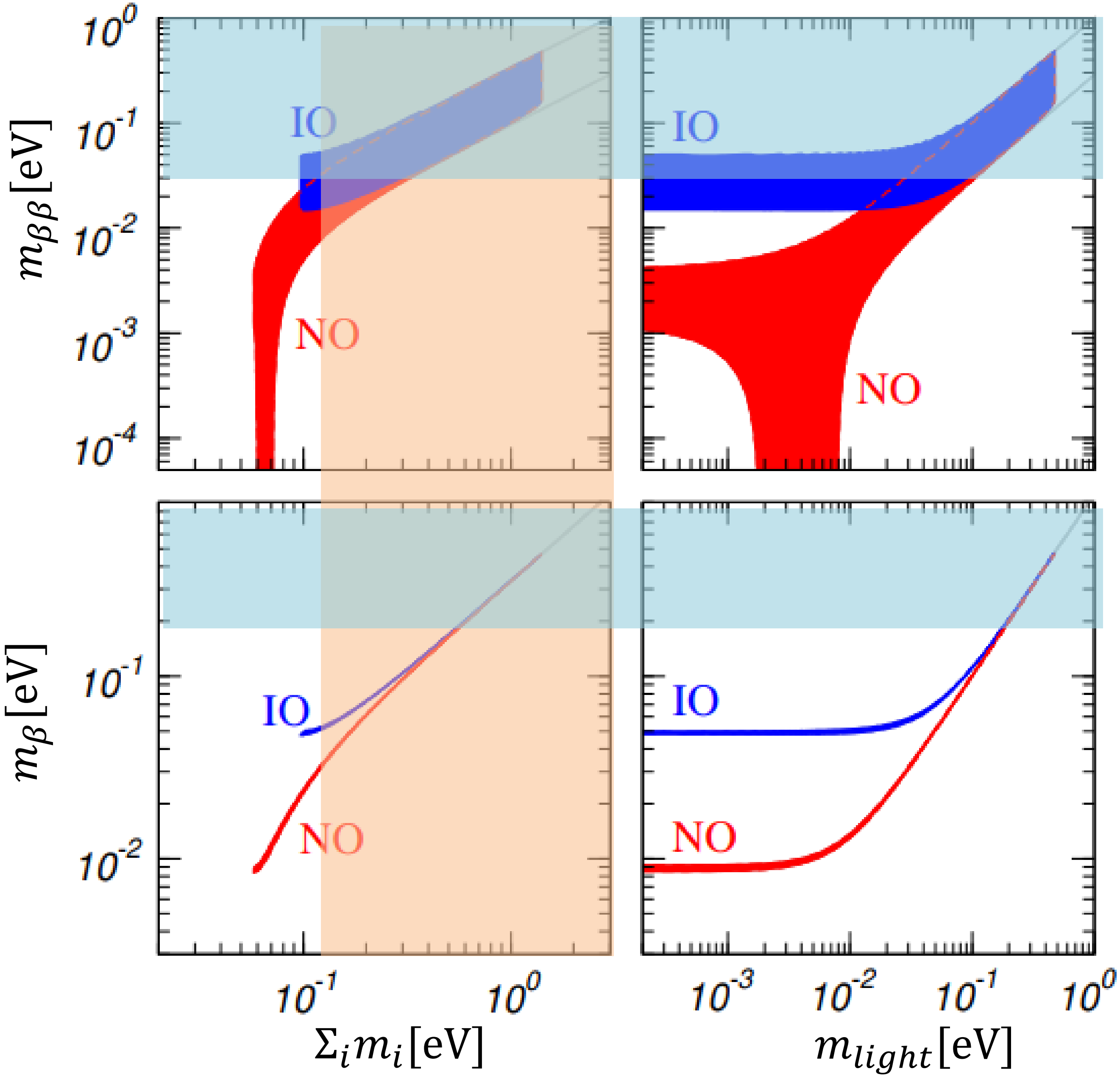
- **unknown Majorana phase and lightest neutrino**
- **Quasi-degenerate** region above 0.2 eV
- Accidental **cancellation** for NO





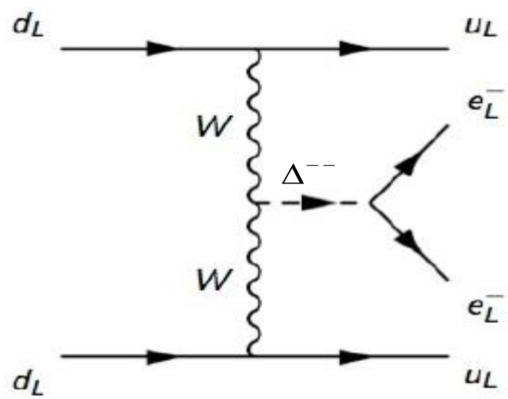
[Lindner, Merle, Rodejohann, hep-ph/0512143]

Absolute neutrino masses from synergies of neutrino facilities

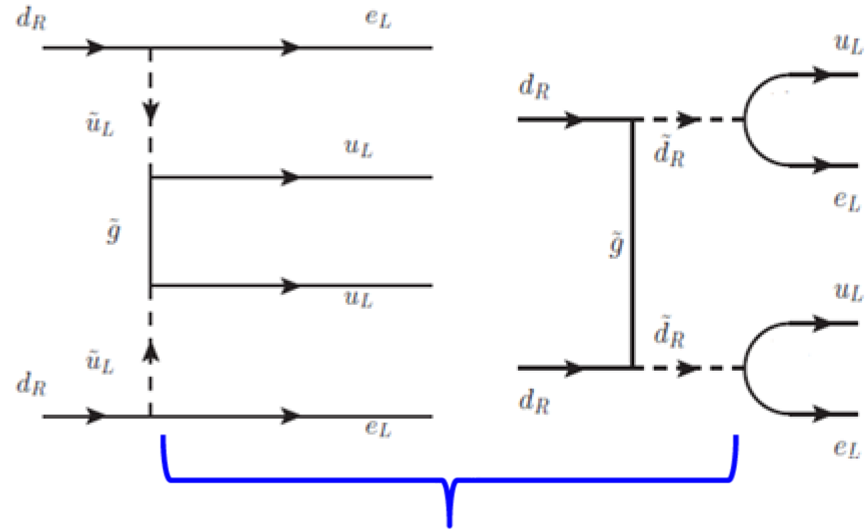


Possible BSM physics in 0νββ decay

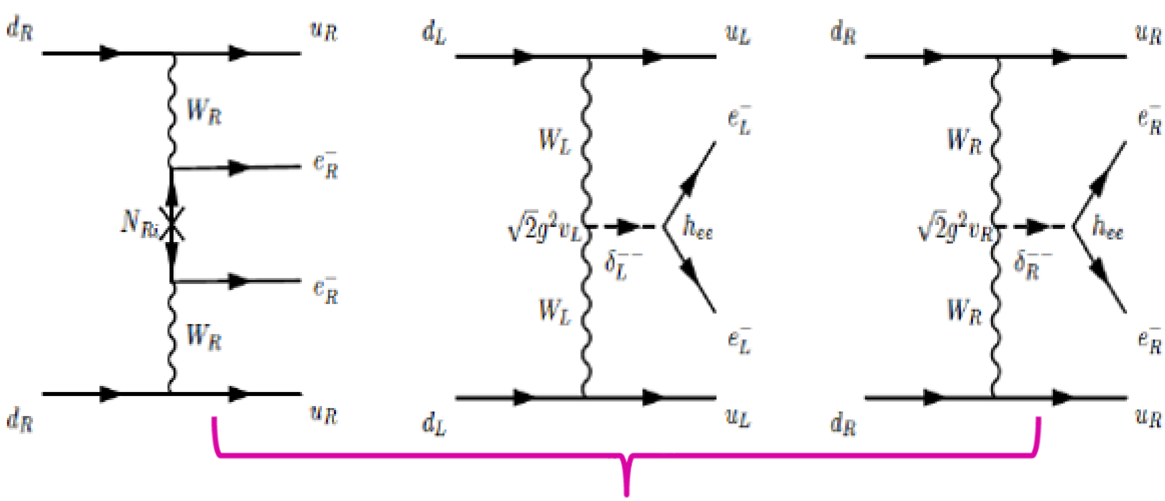
The 0νββ decay can also be induced by other ΔL=2 physics besides the Majorana neutrino mass. There are many possible scenarios:



Type II seesaw



R-parity violating SUSY



Left-right model

- 0νββ decay is connected to TeV scale physics and LFV.
- **A plenty of possible new physics scenario leading to $0\nu\beta\beta$**

Conclusions

- Neutrino oscillation shows that neutrinos are massive, extension of SM is required to accommodate tiny neutrino masses.
- Many fundamental questions remain to be answered: the nature of neutrinos (Majorana vs. Dirac) , absolute neutrino mass, CP violation in lepton sector, flavor structure of quarks and lepton, the possible connection with dark matter and baryon-asymmetry…
- A rich experimental neutrino program lies ahead, complementarity of different experimental approaches: neutrino oscillation, $0\nu\beta\beta$ decay, β decay, cosmology, colliders, lepton flavor violation etc. A bright future!

Thank you for your attention!