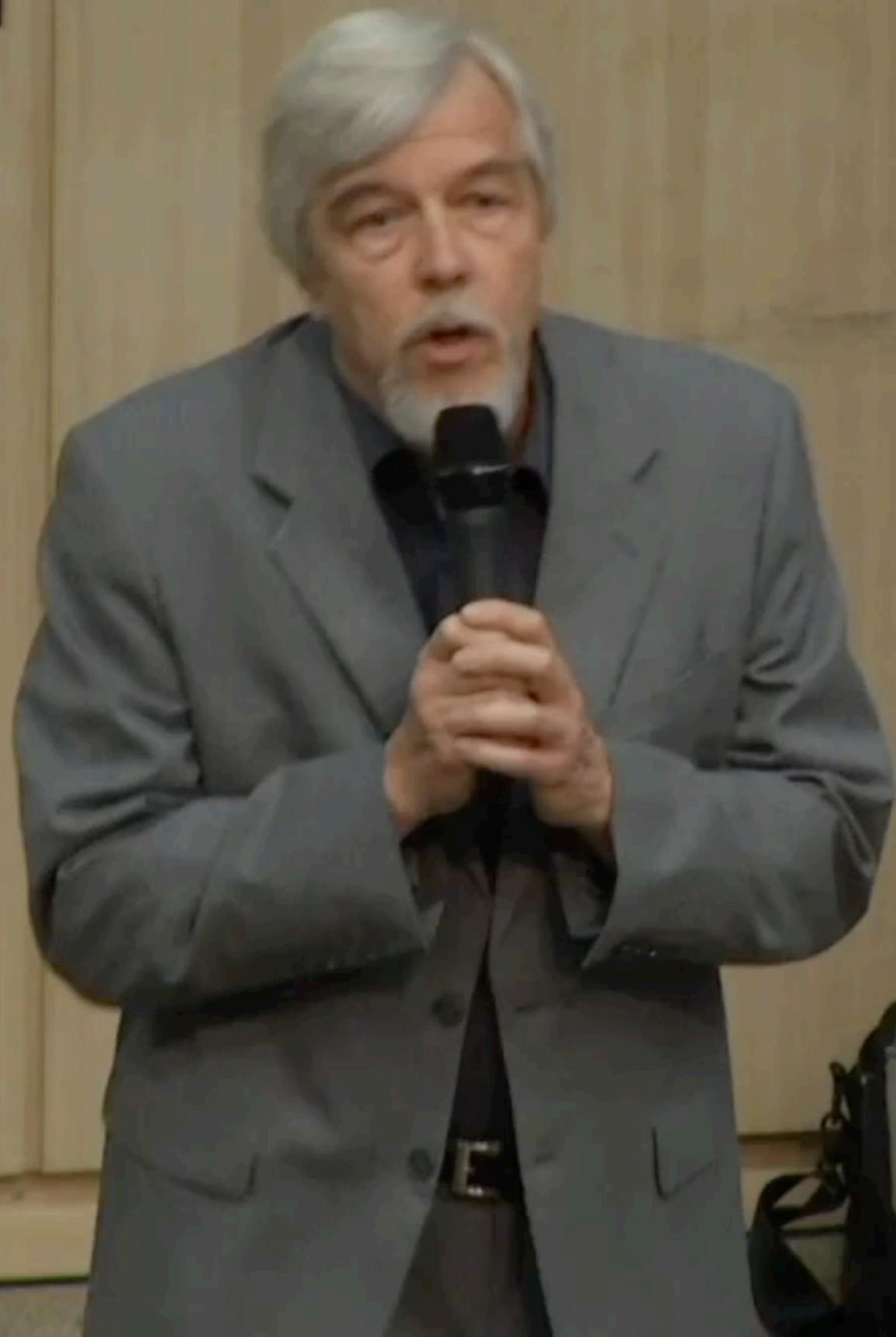


Supersymmetry

Beyond the Standard Model

Chiung Hwang

4 JULY 2012



- The Higgs boson (1964, 2012) was the last piece of the standard model, describing three fundamental forces in nature.
- Gravitational waves (1916, 2015) are ripples of spacetime, proving that spacetime is really dynamical and is the source of gravity.

These two events complete two major pillars of 20c physics:

Quantum Field Theory and General Relativity.

What's next?

What's next?

Supersymmetry

What Is Supersymmetry?

- A symmetry between boson (integer spin) and fermion (half-integer spin).



- Since spin is a representation of the spacetime symmetry, supersymmetry inevitably mixes with that of spacetime.

$$\text{Supersymmetry} \longleftarrow QQ^\dagger + Q^\dagger Q = 2H \longrightarrow \text{Time translation}$$

- For 4-dimensional Minkowski spacetime,

$$\left\{ Q_a, Q_{\dot{a}}^\dagger \right\} = -2 \sigma_{a\dot{a}}^\mu P_\mu$$

- How does it act on fields?
- E.g., for a scalar field A and a fermion field ψ_a ,

$$[A, Q_a] = -i\sqrt{2}\psi_a, \quad \left\{ \psi_a, Q_{\dot{a}}^\dagger \right\} = -\sqrt{2}\sigma_{a\dot{a}}^\mu \partial_\mu A$$

- Leads to a theory with symmetry between bosonic spectrum and fermionic spectrum.

- Why interesting?

- Why interesting?
- Haag–Łopuszański–Sohnius theorem (1975)

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Euclidean symmetry

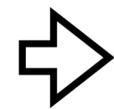
- Translations, rotations
- Euclidean space+time
- Newtonian mechanics



- Why interesting?
- Haag–Łopuszański–Sohnius theorem (1975)

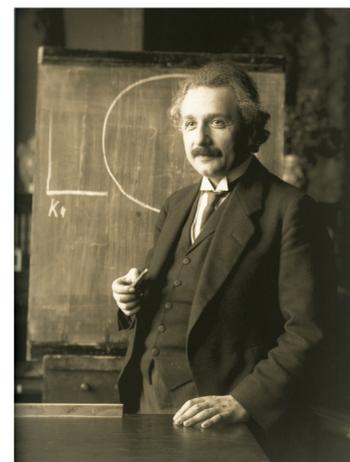
Euclidean symmetry

- Translations, rotations
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Poincare symmetry

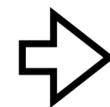
- Translations, rotations + boosts
- Minkowskian spacetime
- (Special) Relativity



- Why interesting?
- Haag–Łopuszański–Sohnius theorem (1975)

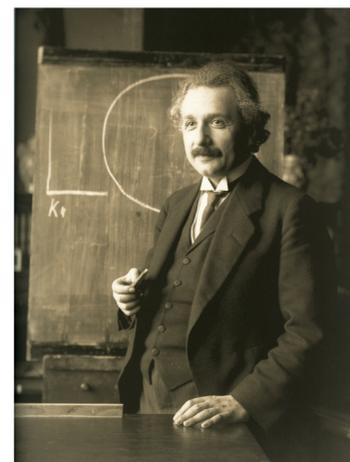
Euclidean symmetry

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Poincare symmetry

- Translations, rotations + boosts
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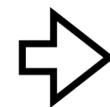
Super-Poincare symmetry

- Translations, rotations, boosts + supersymmetry
- Superspace
- Supersymmetric theories

- Why interesting?
- Haag–Łopuszański–Sohnius theorem (1975)

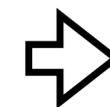
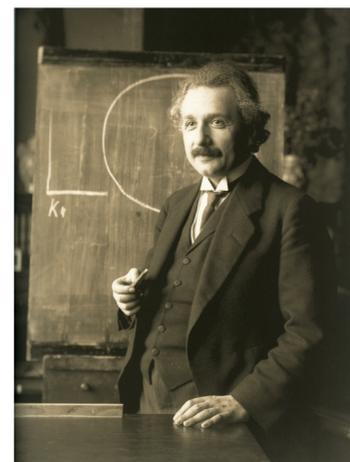
Euclidean symmetry

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Poincare symmetry

- Translations, rotations + boosts
- Minkowskian spacetime
- (Special) Relativity



Super-Poincare symmetry

- Translations, rotations, boosts + supersymmetry
- Superspace
- Supersymmetric theories

Supersymmetry is the only possible extension of the Poincare symmetry!

Early History of SUSY

- 1971 Ramond, Neveu-Schwarz develop **string theory** with fermions and bosons.
- 1971 Gervais-Sakita show that this theory obeys **supersymmetry algebra in two dimensions**.
- 1971 Golfand-Likhtman extend the Poincare algebra into a superalgebra and discover **supersymmetry in four spacetime dimensions**.
- 1974 Wess-Zumino rediscover **supersymmetry in four spacetime dimensions**.
- 1975 Motivated by WZ, **Haag-Lopuszański-Sohnius** prove that the super-Poincare symmetry is the unique generalization of the Poincare symmetry.

Why do we need it?

Outline

- Part I: Why Supersymmetry?
- Part II: Application: From Higgs Mass to Black Hole Entropy

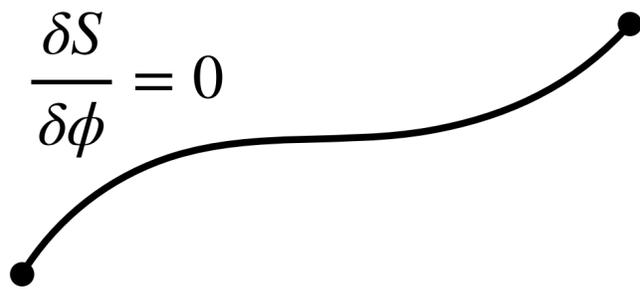
Part I

Why Supersymmetry?

Preliminary: Classical vs Quantum

Classical vs Quantum

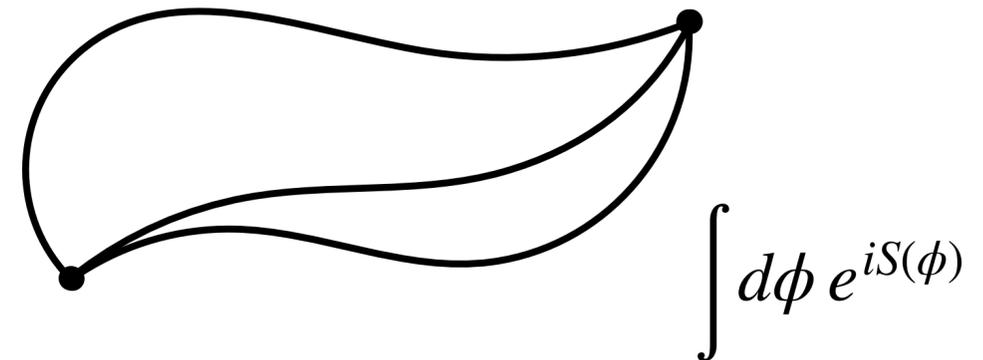
$$S(\phi) = \int dt L(\phi) = \int d^4x \mathcal{L}(\phi)$$



$$G(p) = \frac{1}{p^2 - m_0^2}$$

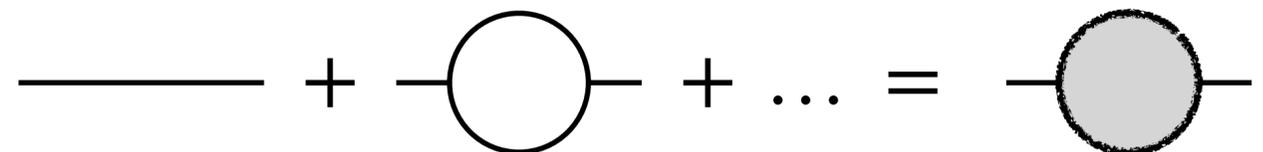


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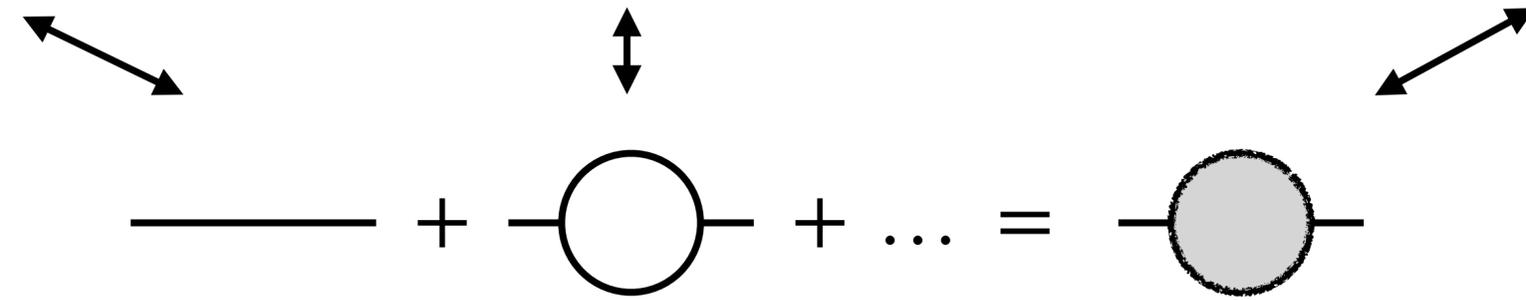
$$G(p) = \frac{1}{p^2 - m_0^2} + \int d^4l \frac{g^2}{(p^2 - m_0^2)^2 (l^2 - m_0^2) ((p+l)^2 - m_0^2)} + \dots$$

$$= \frac{1}{p^2 - m_{phy}^2}$$



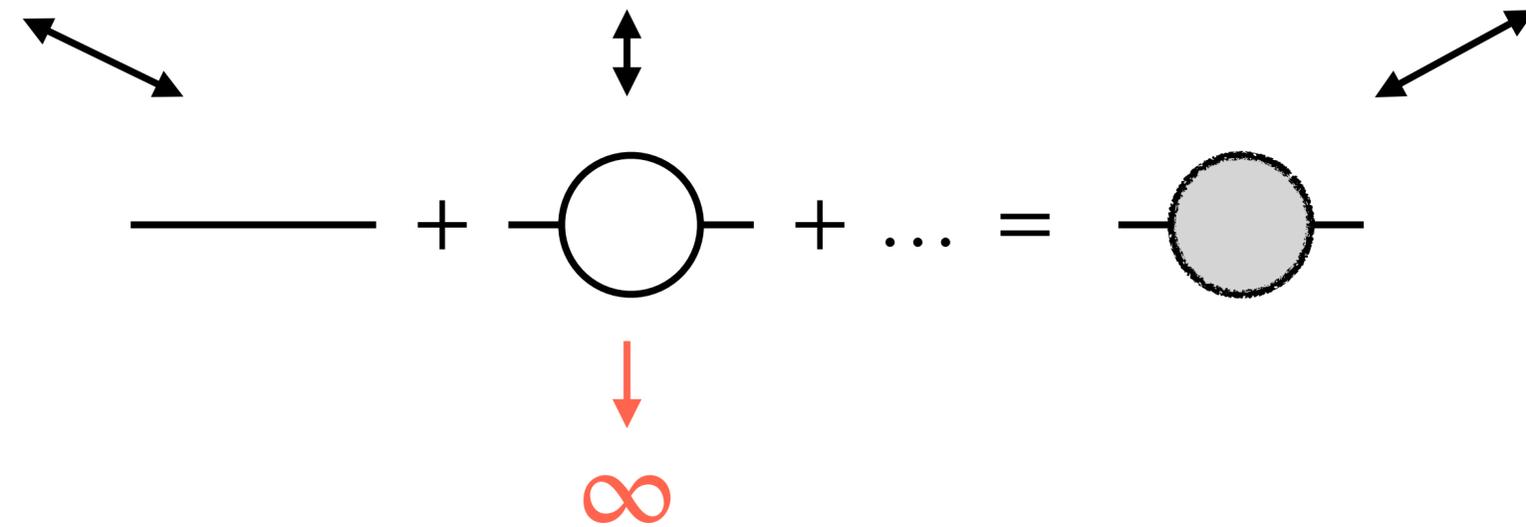
- Propagator of the ϕ^3 theory $\rightarrow \mathcal{L} = \frac{1}{2}(\partial\phi_0)^2 - \frac{1}{2}m_0^2\phi_0^2 - \frac{1}{3!}g_0\phi_0^3$

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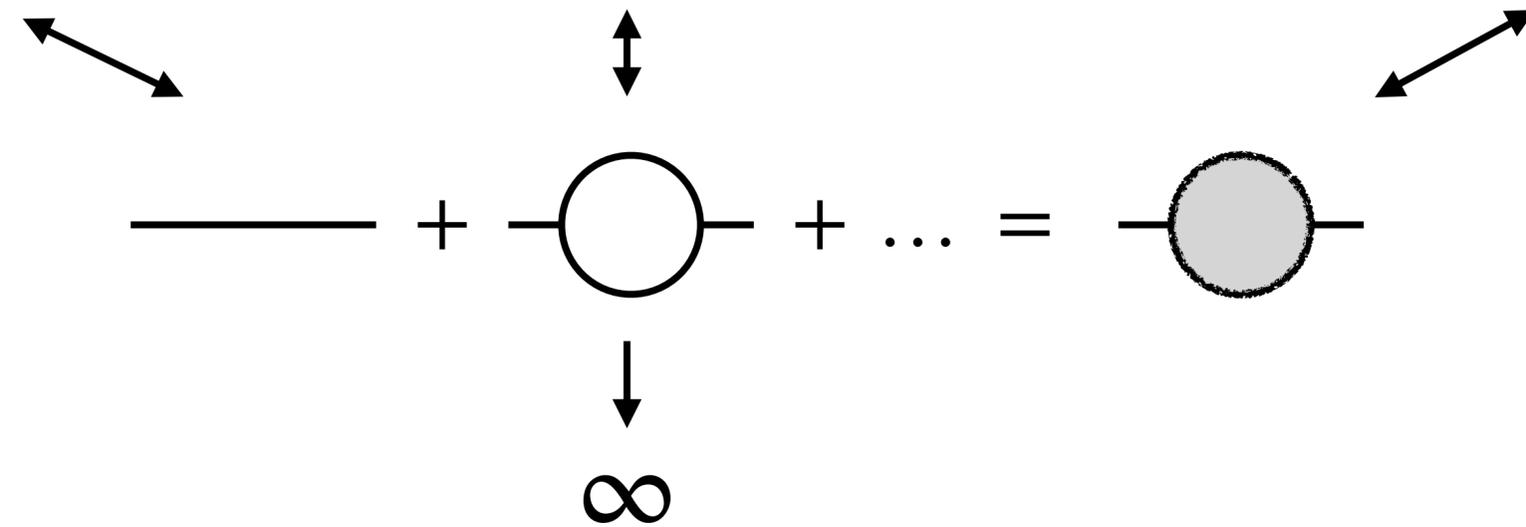
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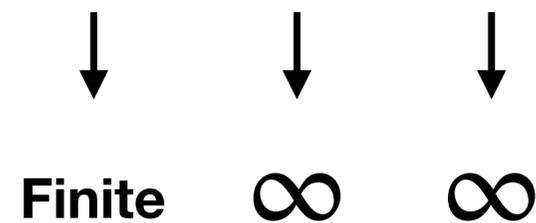


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$$\Rightarrow m_{phy}^2 = m_0^2 + \dots$$



$$m_{phy}^2 = m_0^2 + \dots$$



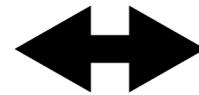
Finite



∞



∞



Taylor series

$$\frac{1}{x} = \frac{1}{0} - \frac{x}{0^2} + \dots$$

$$m_{phy}^2 = m_0^2 + \dots$$



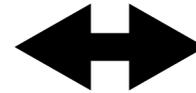
Finite



∞



∞



Taylor series

~~$$\frac{1}{x} = \frac{1}{0} - \frac{x}{0^2} + \dots$$~~

$$\frac{1}{a + (x - a)} = \frac{1}{a} - \frac{x - a}{a^2} + \dots$$

- **Renormalization**

$$\mathcal{L} = \frac{1}{2}(\partial\phi_0)^2 - \frac{1}{2}m_0^2\phi_0^2 - \frac{1}{3!}g_0\phi_0^3$$

- **Renormalization**

$$\mathcal{L} = \frac{1}{2}(\partial\phi_0)^2 - \frac{1}{2}m_0^2\phi_0^2 - \frac{1}{3!}g_0\phi_0^3$$



$$\phi_0 = \alpha\phi + \beta, \quad m_0 = \dots$$

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{3!}g\phi^3 \\ & + \frac{1}{2}A(\partial\phi)^2 + B\phi + \frac{1}{2}C\phi^2 + \frac{1}{3!}D\phi^3 \end{aligned}$$

- **Renormalization**

$$\mathcal{L} = \frac{1}{2}(\partial\phi_0)^2 - \frac{1}{2}m_0^2\phi_0^2 - \frac{1}{3!}g_0\phi_0^3$$



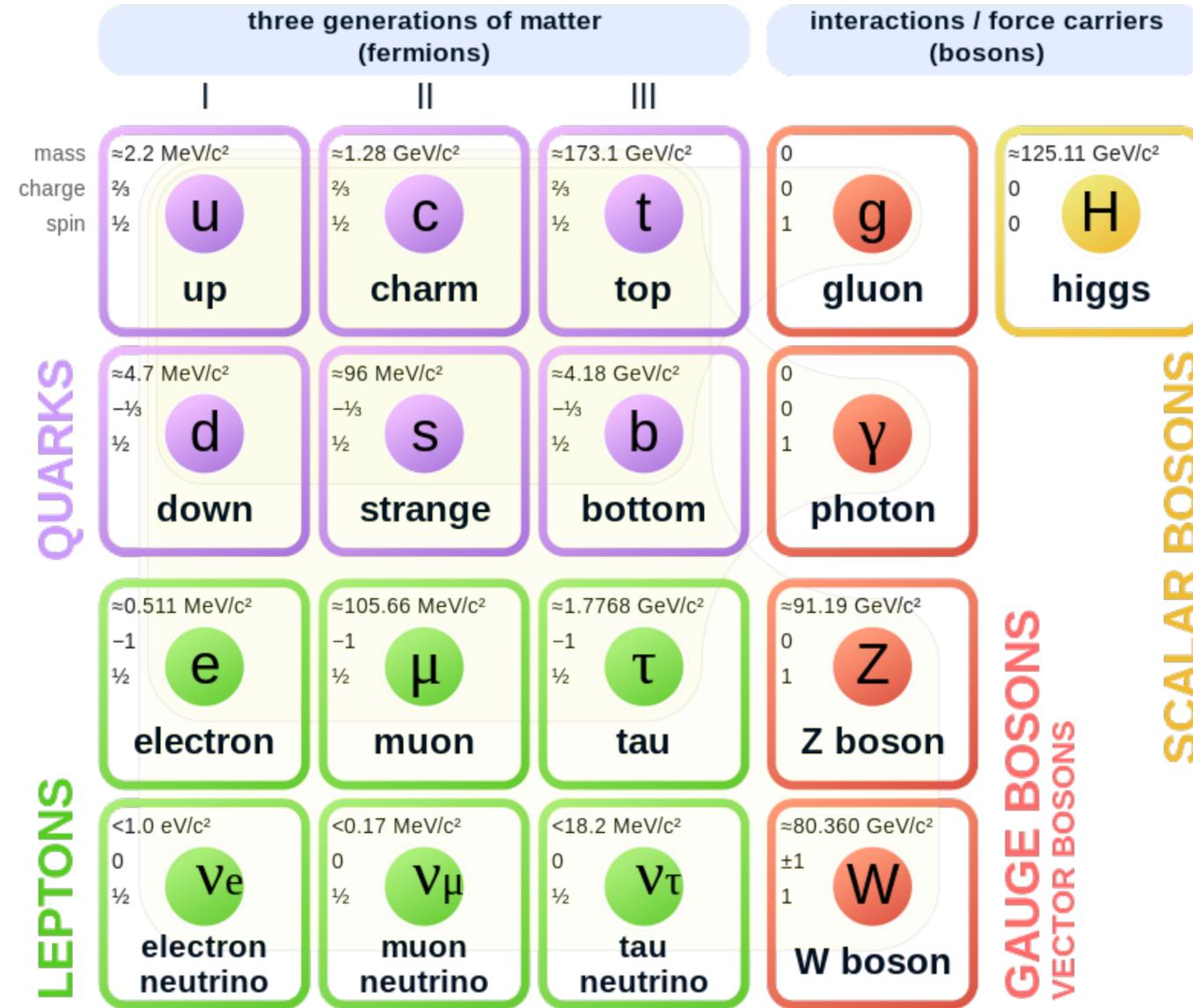
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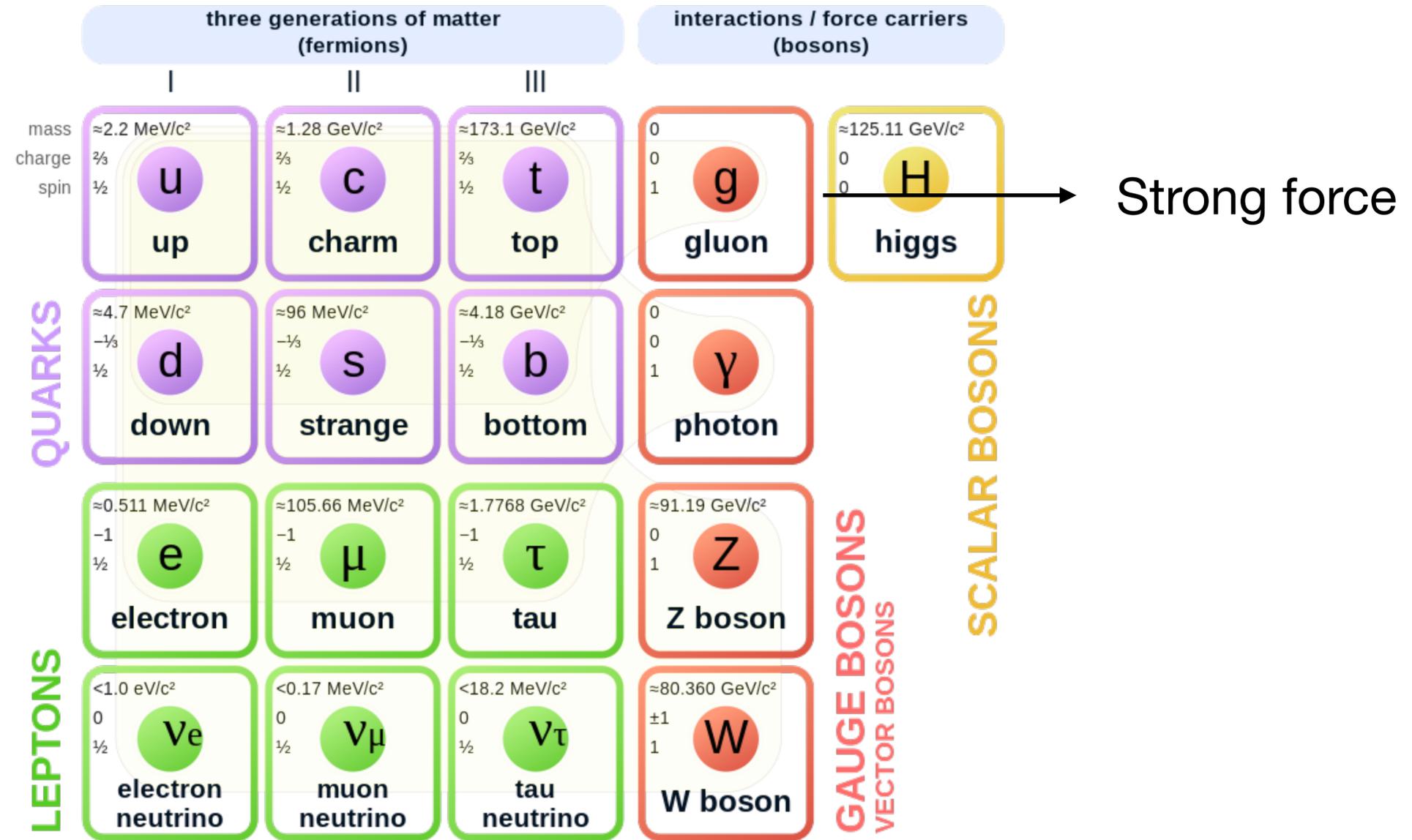
→ Counter-terms, tuned to cancel the infinities from the first line

Called *renormalizable* if a finite number of counter-terms are required

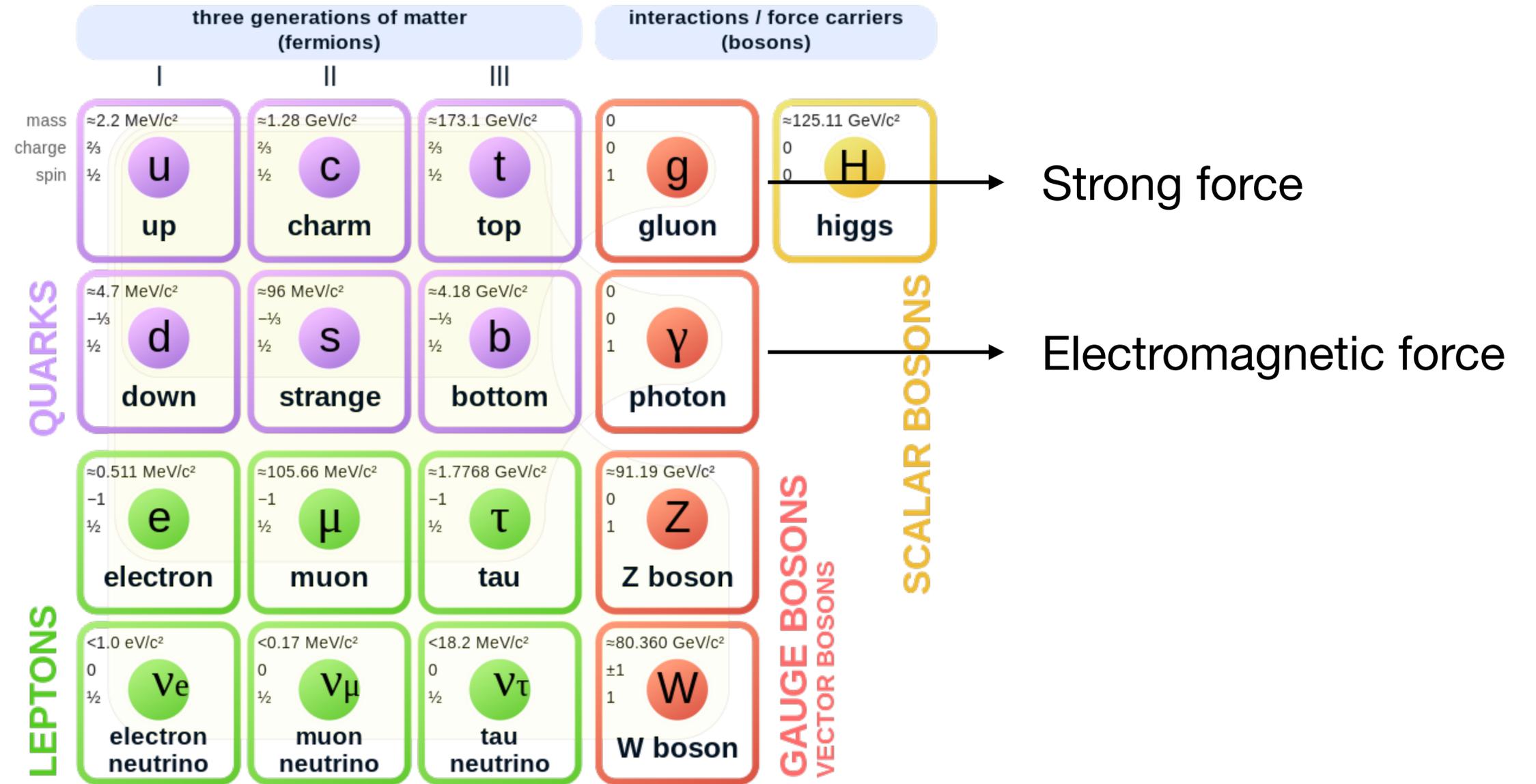
Standard Model of Elementary Particles



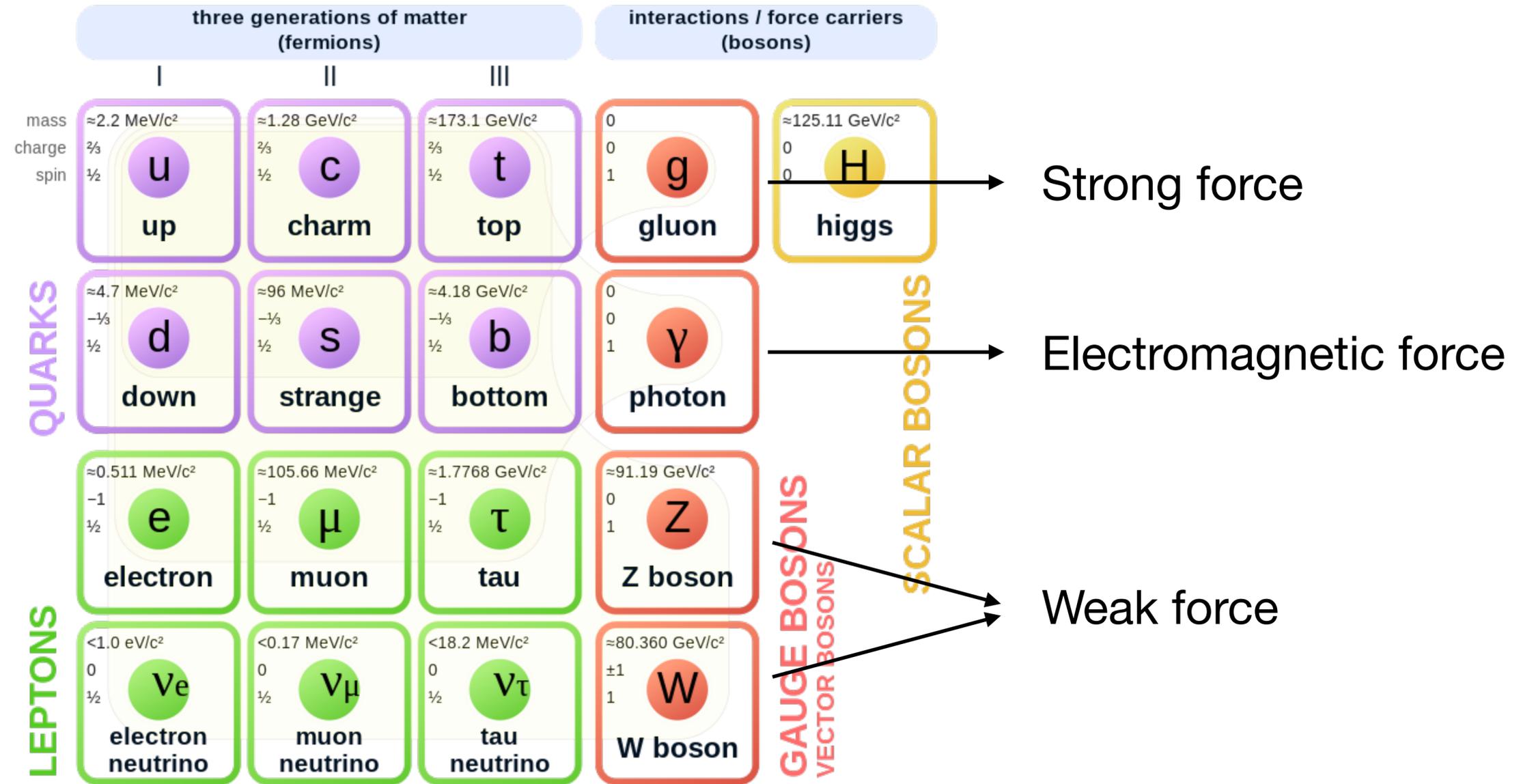
Standard Model of Elementary Particles



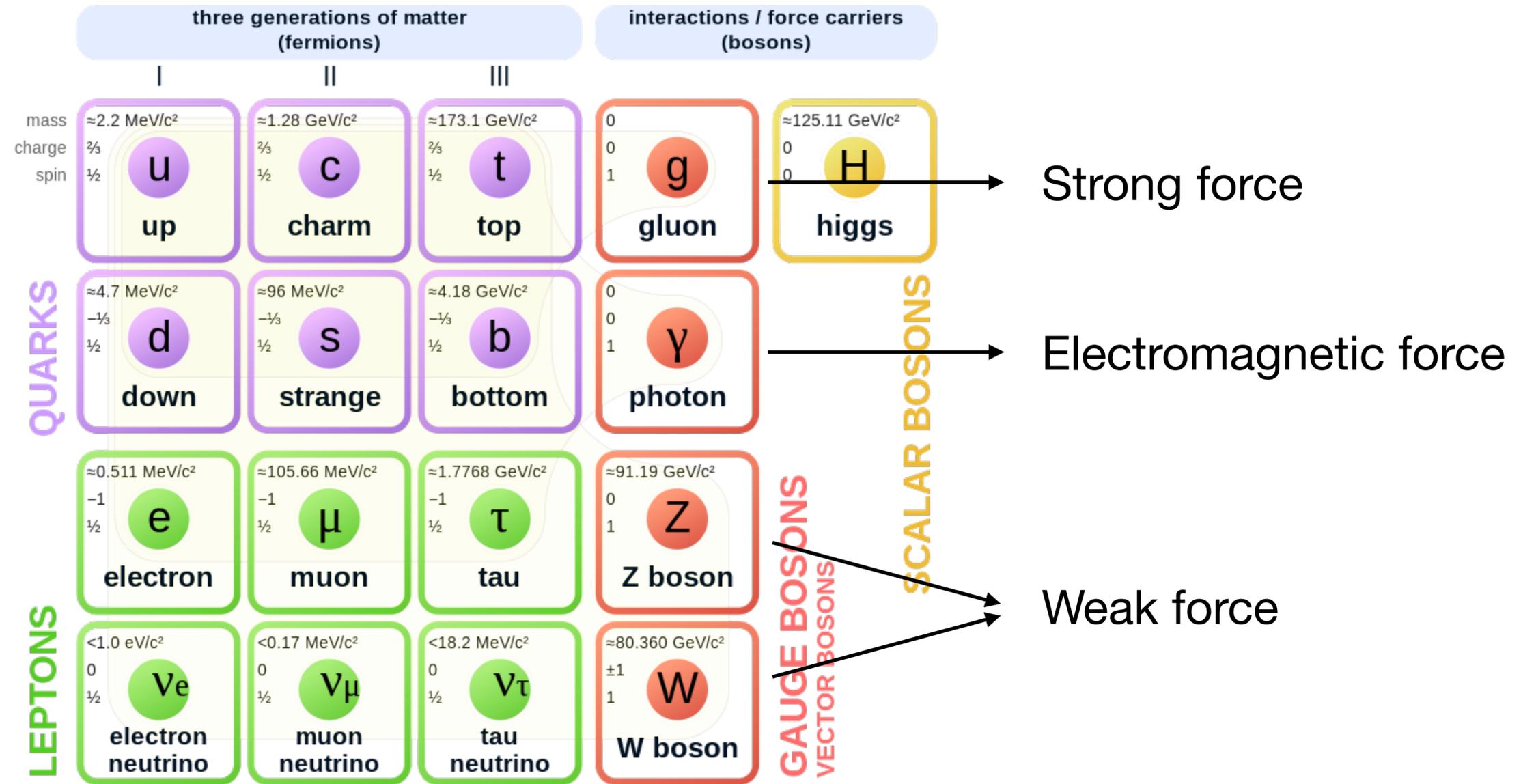
Standard Model of Elementary Particles



Standard Model of Elementary Particles



Standard Model of Elementary Particles



QFT is the most successful framework describing fundamental forces in nature.

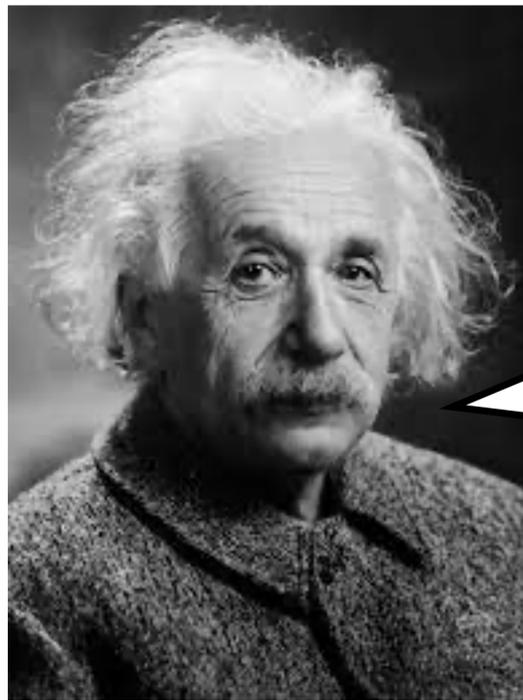
Even though the standard model and the QFT framework are so successful...

- *What about gravity?*
- *Other than gravity, is the standard model enough?*

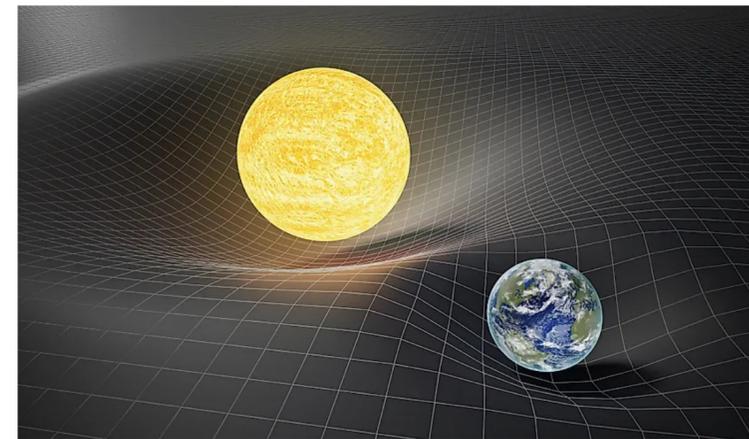
Q1. What about gravity?

Gravity = Curved Spacetime

- Gravity is explained by Einstein's general relativity (1915), which teaches us the gravitational force originates from dynamical and curved spacetime.



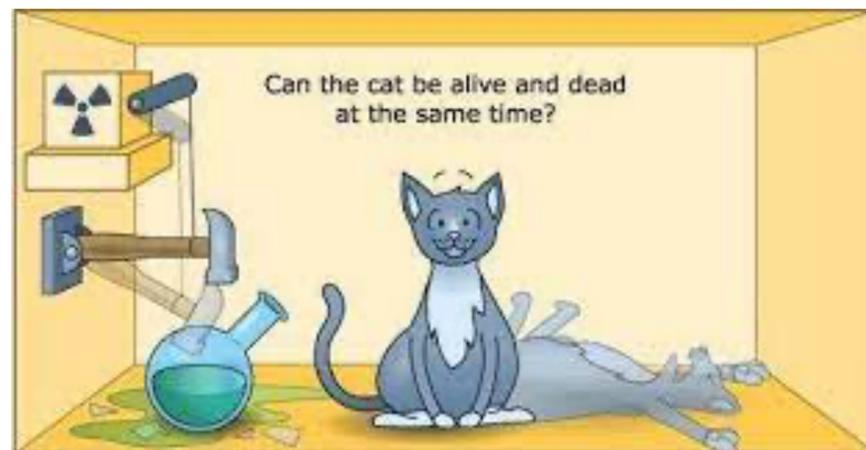
=



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Gravity as QFT

- General relativity is a classical theory.
- On the other hand, spacetime interacts with matters, which are quantum —so is the spacetime?
- What is quantum spacetime?

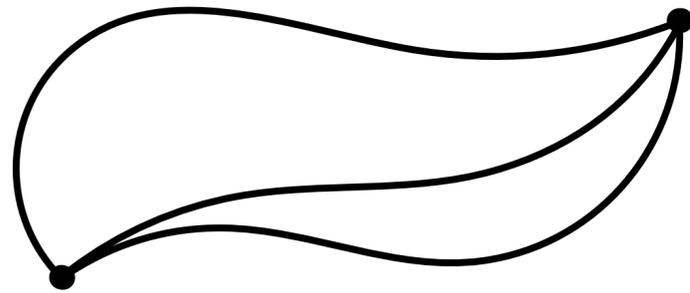


Interacting



Schrödinger's spacetime?

Gravity Is Non-renormalizable

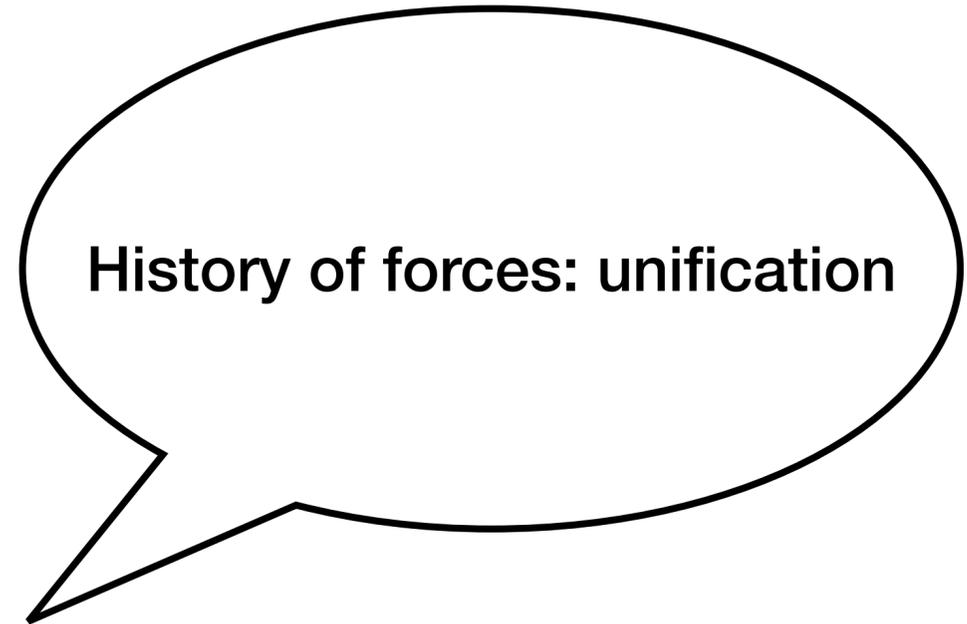


$$\text{---} + \text{---} \bigcirc \text{---} + \dots = \text{---} \bigcirc \text{---}$$

- However, GR is non-renormalizable—there are infinitely many counter-terms to be determined.
- No predictability in the high energy regime.
- Big bang? Black holes?

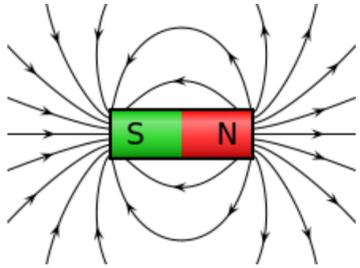
What do we learn from history?

Gravitational force = curved spacetime



Gravitational force = curved spacetime

Electric Force

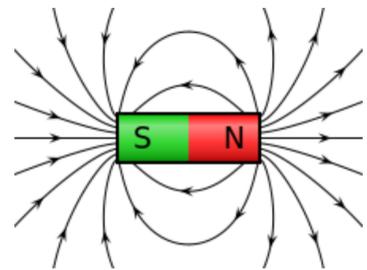


Magnetic Force

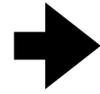
Electric Force



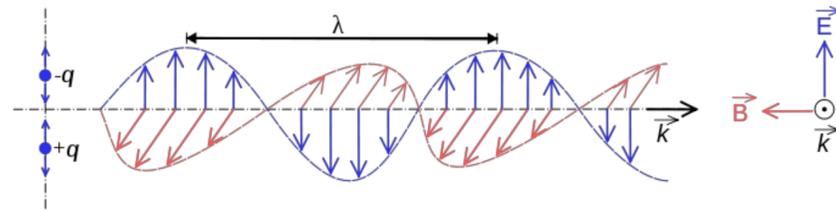
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Magnetic Force



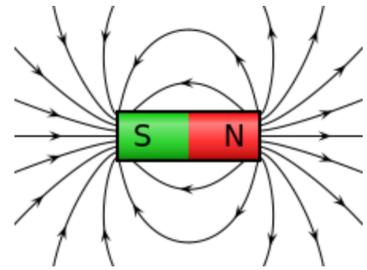
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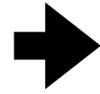
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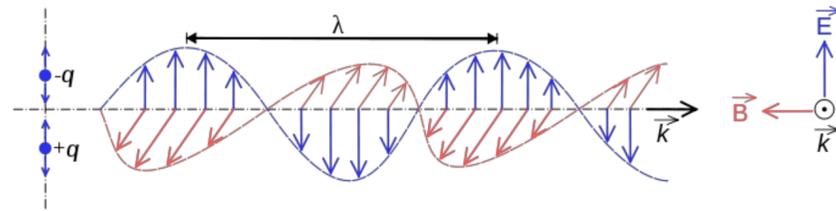
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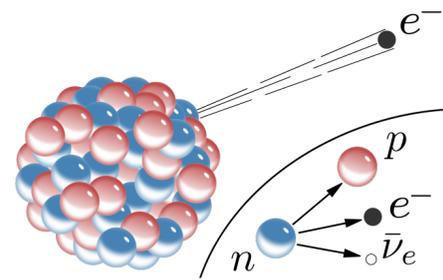
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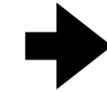
Electromagnetic Force



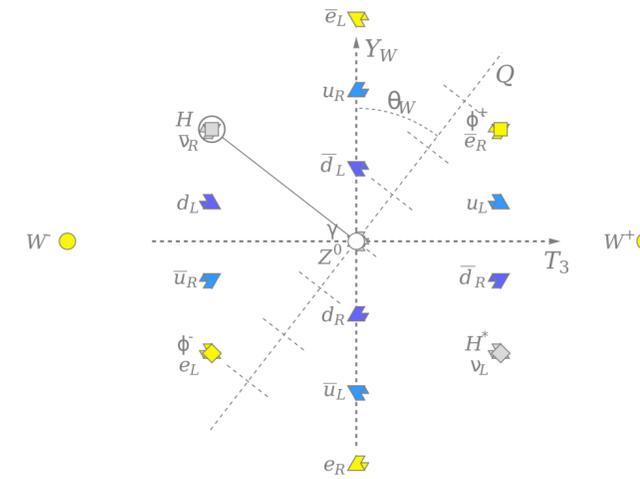
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Weak Force



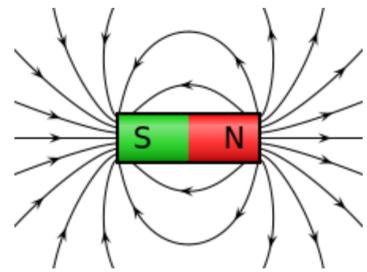
Electroweak Force



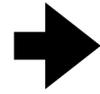
Electric Force



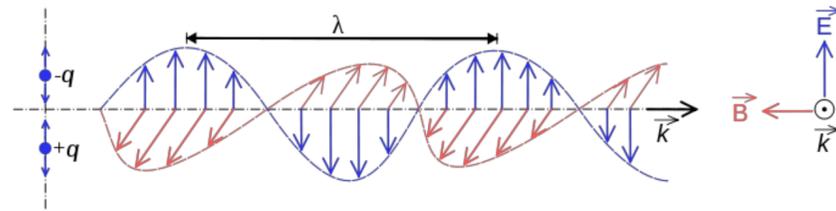
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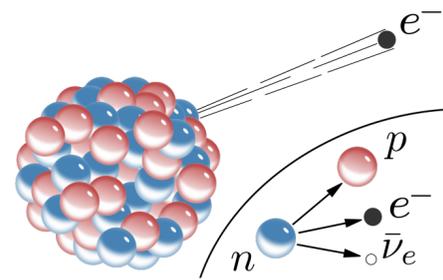
Magnetic Force



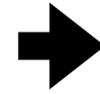
Electromagnetic Force



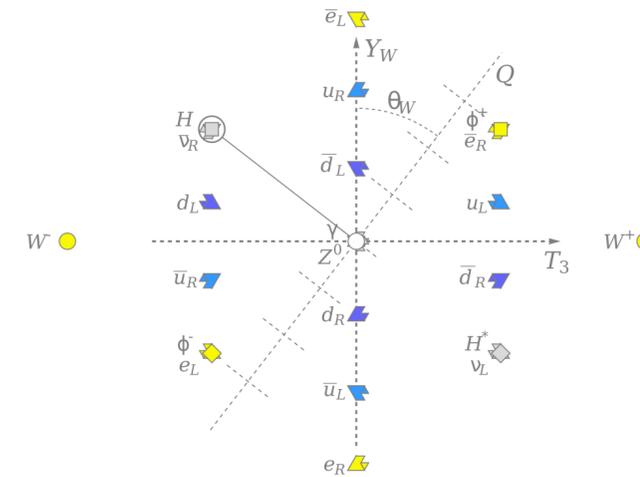
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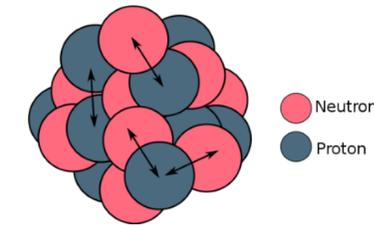
Weak Force



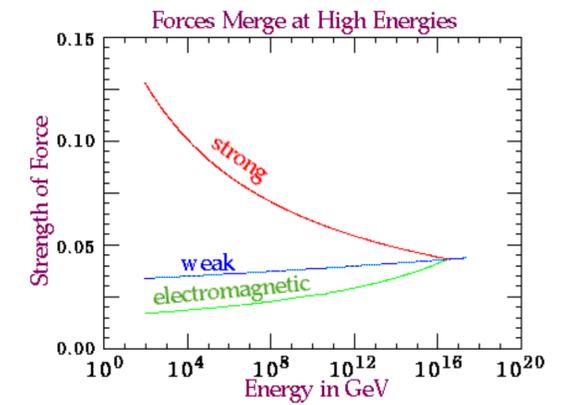
Electroweak Force



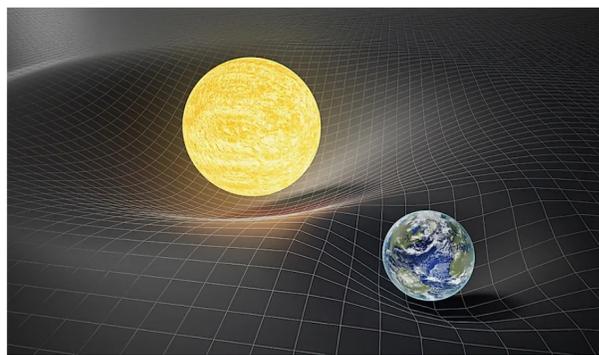
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Strong Force

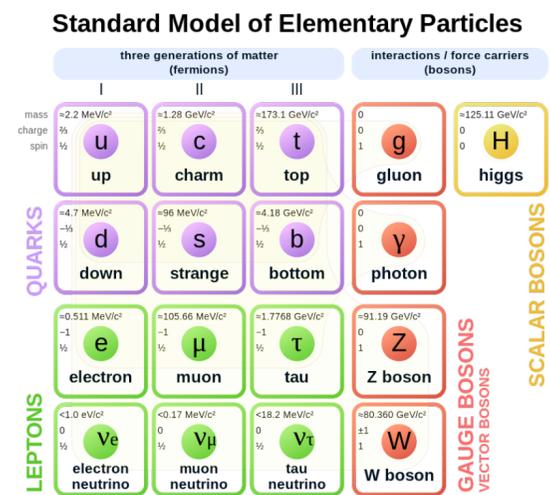


GUT?

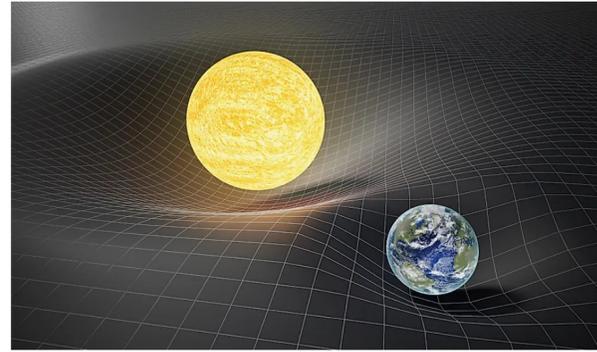


Gravity = curved spacetime

+



The other forces?



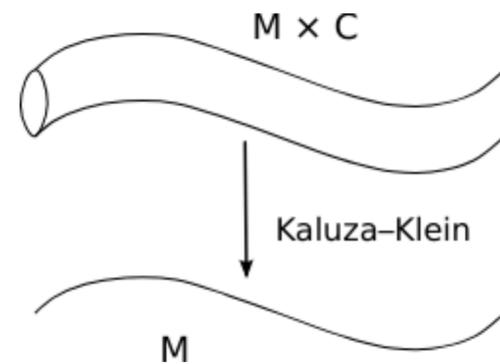
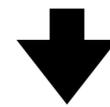
Gravity = curved spacetime

+

Standard Model of Elementary Particles

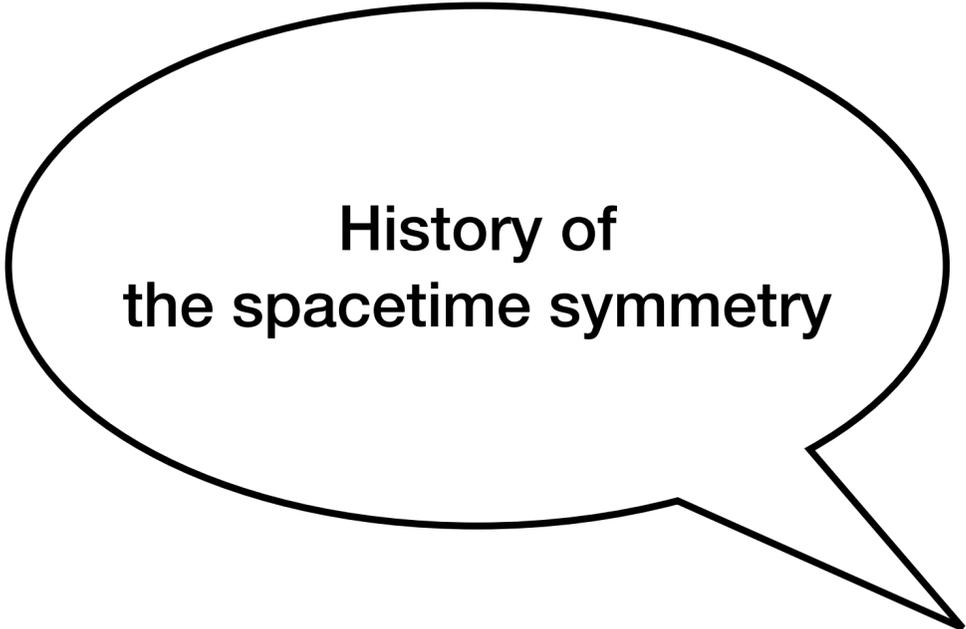
three generations of matter (fermions)			interactions / force carriers (bosons)	
	I	II	III	
mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	$\approx 125.11 \text{ GeV}/c^2$
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
QUARKS	u up	c charm	t top	g gluon
	d down	s strange	b bottom	γ photon
	e electron	μ muon	τ tau	Z Z boson
LEPTONS	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson
				H higgs
				SCALAR BOSONS
				GAUGE BOSONS VECTOR BOSONS

The other forces?



Kaluza-Klein theory—the other forces originate from gravity in the **extra dimensions**

Gravitational force = curved spacetime



History of
the spacetime symmetry

Gravitational force = curved spacetime

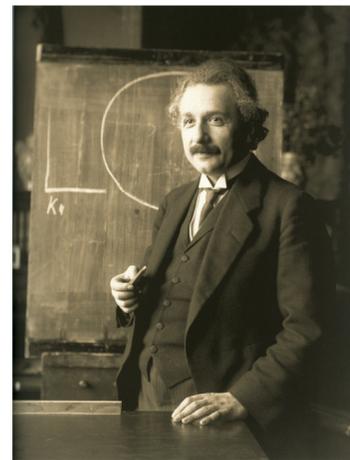
Euclidean symmetry

- Translations, rotations
- Euclidean space+time
- Newtonian mechanics



Poincare symmetry

- Translations, rotations + boosts
- Minkowskian spacetime
- (Special) Relativity



Super-Poincare symmetry

- Translations, rotations, boosts + supersymmetry
- Superspace
- Supersymmetric theories

Kaluza-Klein Theory

+

Supersymmetry

Kaluza-Klein Theory

+

Supersymmetry



11D Supergravity

- Large enough dimension to include the standard model
- Small enough dimension to exclude higher spin (> 2) particles
- However, still *non-renormalizable*
- Solution?

Kaluza-Klein Theory

+

Supersymmetry



11D Supergravity

- Large enough dimension to include the standard model
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UV completion

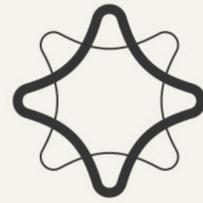
M-Theory



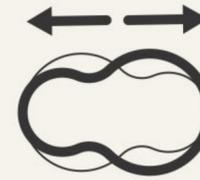
Type I
string theory



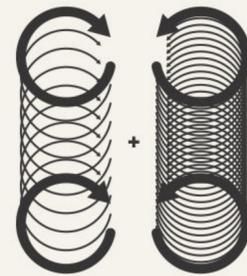
Type IIA
string theory



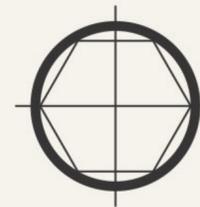
Type IIB
string theory



SO(32)
Heterotic
string theory



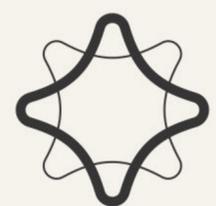
$E_8 \times E_8$
Heterotic
string theory



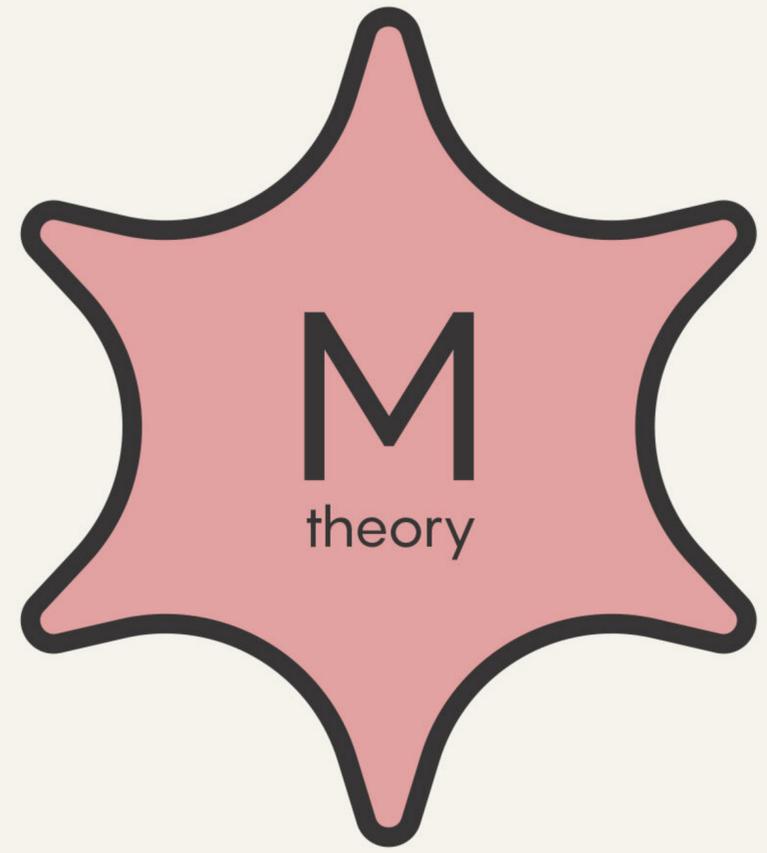
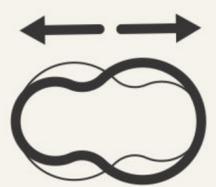
Type I
string theory



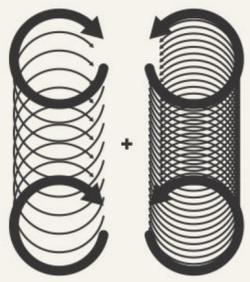
Type IIA
string theory



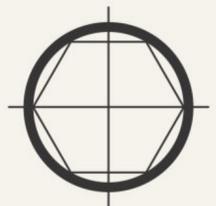
Type IIB
string theory



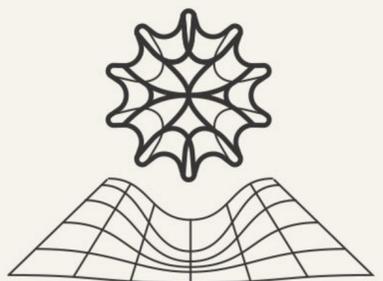
SO(32)
Heterotic
string theory



E₈ × E₈
Heterotic
string theory



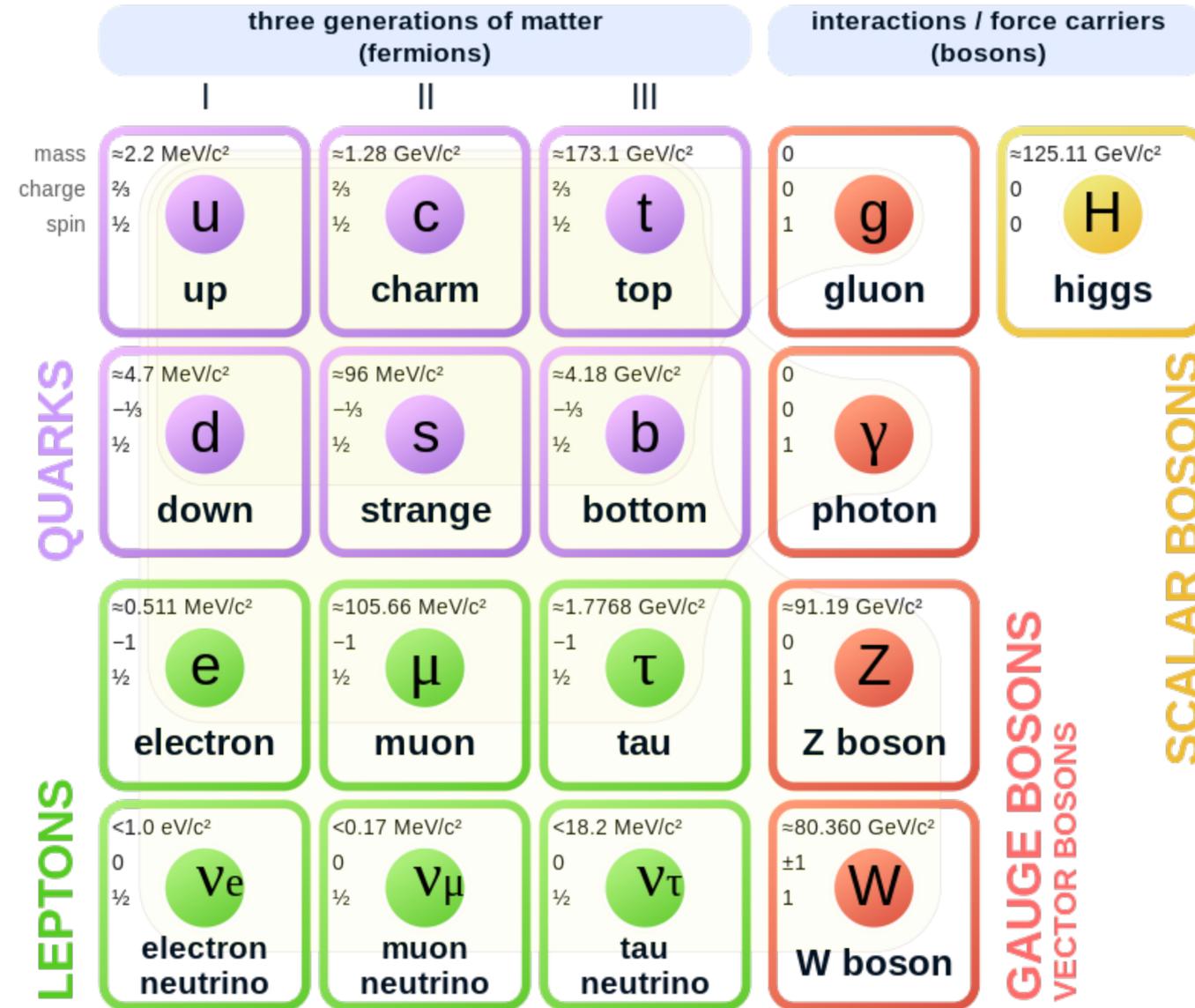
11-Dimensional
Supergravity
particle theory



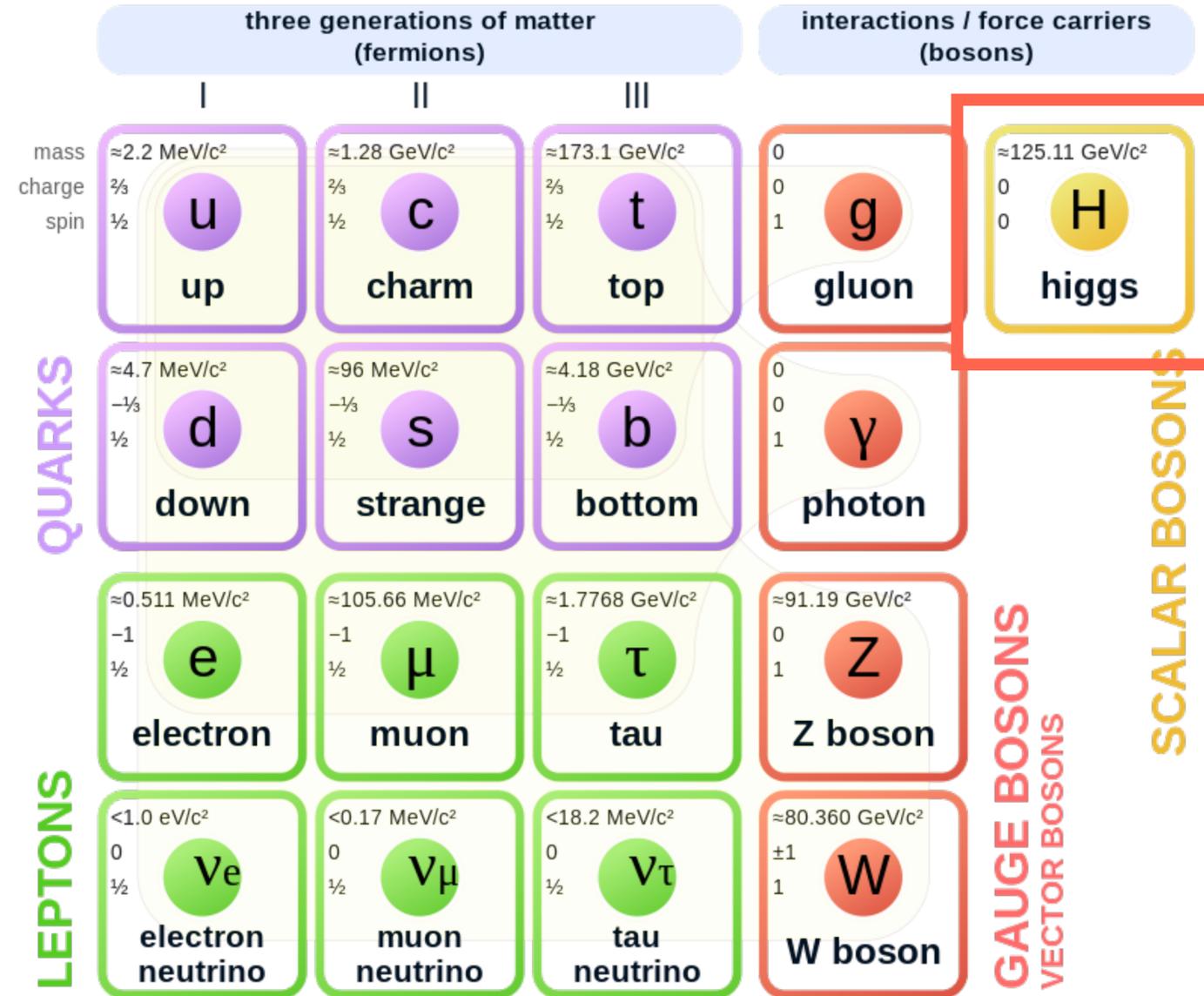
- Supersymmetry and Kaluza-Klein's idea lead to M/string theory, which provides a consistent framework for quantum gravity.
- Among other things, M/string theory provides the most concrete understanding of black hole entropy and its microstates. (Part II)

**Q2. Other than gravity,
is the standard model enough?**

Standard Model of Elementary Particles



Standard Model of Elementary Particles

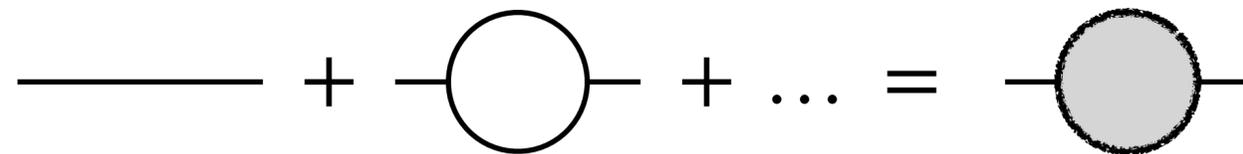


GAUGE BOSONS
VECTOR BOSONS

SCALAR BOSONS

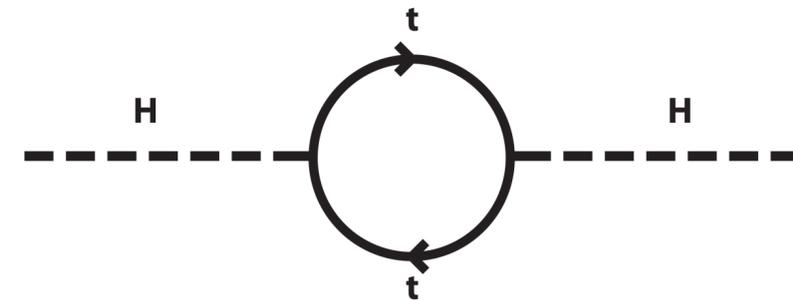
I. Hierarchy Problem

- Recall that QFT adds quantum corrections to classical (bare) quantities



- For Higgs mass,

$$\Delta m_{Higgs}^2 = -\frac{|\lambda_f|^2}{8\pi^2} \left[\Lambda_{UV}^2 + m_f^2 \times \dots \right]$$



- Λ_{UV} : the energy scale at which new physics kicks in

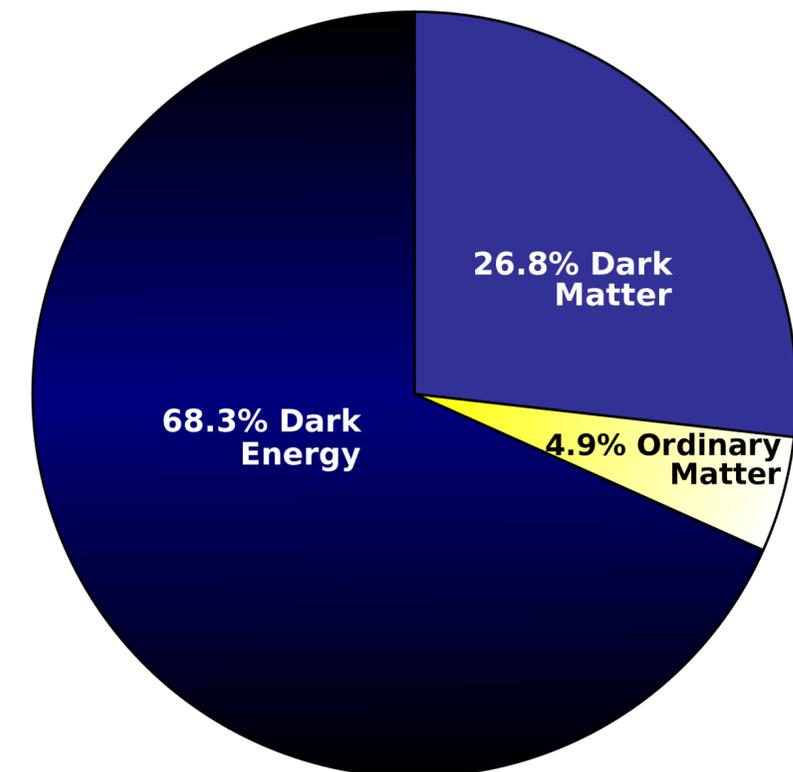
- For example, the Planck scale ($\sim 10^{19}$ GeV) where the quantum gravity effect becomes important.
- The GUT scale ($\sim 10^{16}$ GeV) where all the forces except gravity are unified.
- Both are much higher than the observed Higgs mass ($\sim 10^2$ GeV).
- A unnaturally precise cancelation for Higgs mass

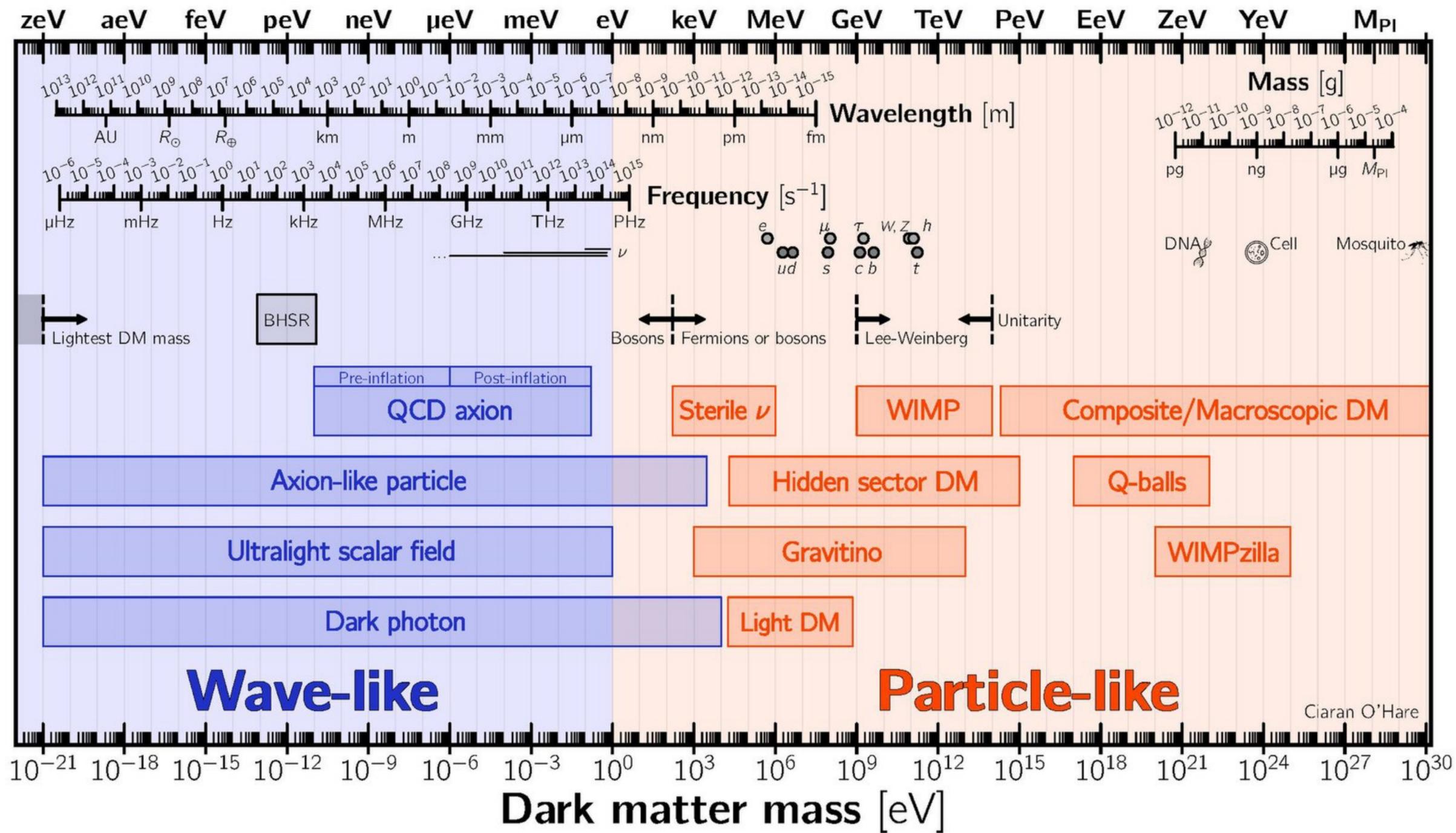
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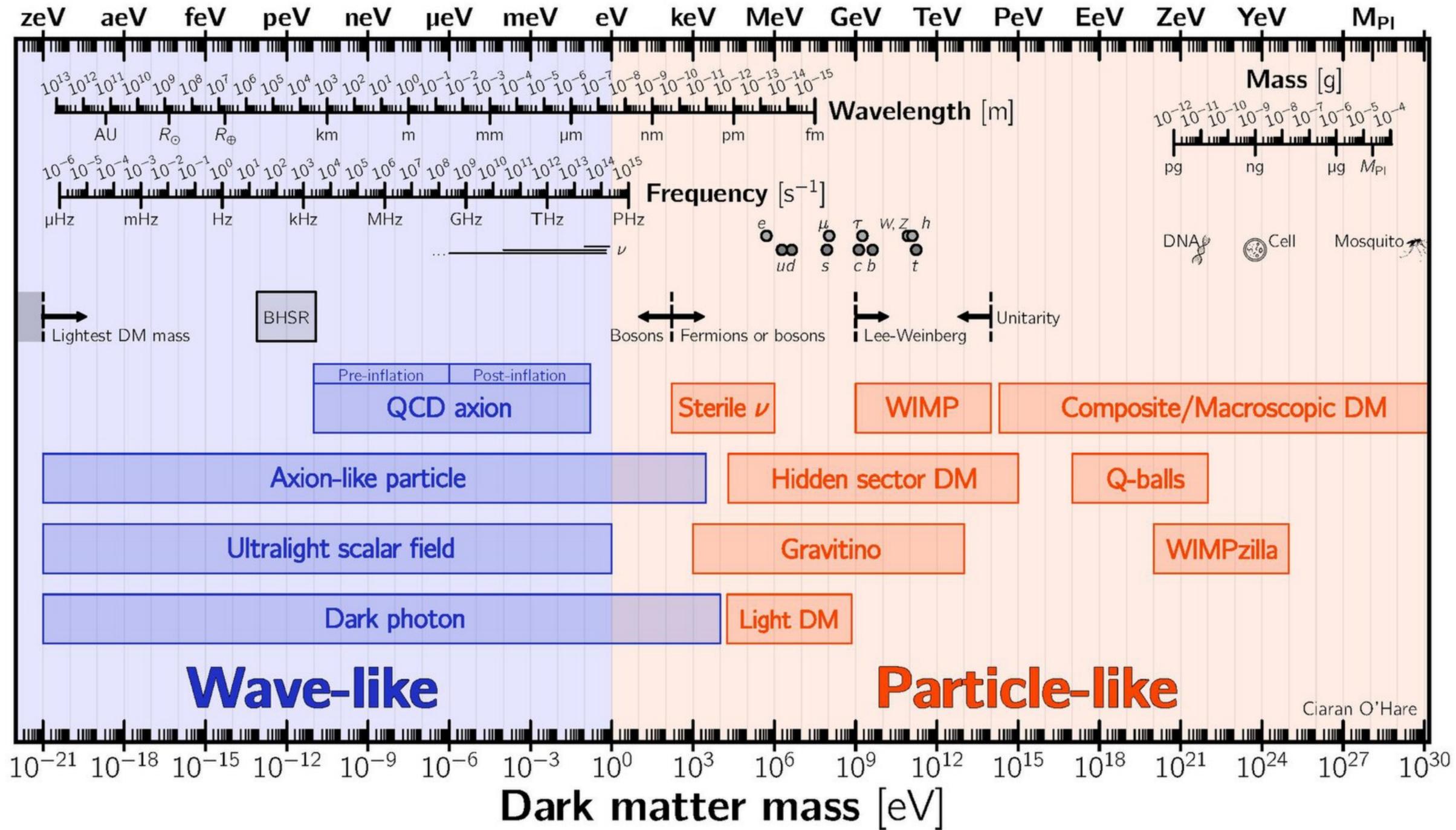
Why is Higgs mass so fine-tuned to cancel such a large quantum correction?

II. Dark Matter

- Our universe consists of ordinary matter, dark matter, and dark energy.
- Dark matter, 85% of the total amount of matter, doesn't interact with light and is only observed by gravitational effects.
- Many scenarios.







Supersymmetry requires new particles, some of which are DM candidates.

III. Grand Unified Theory

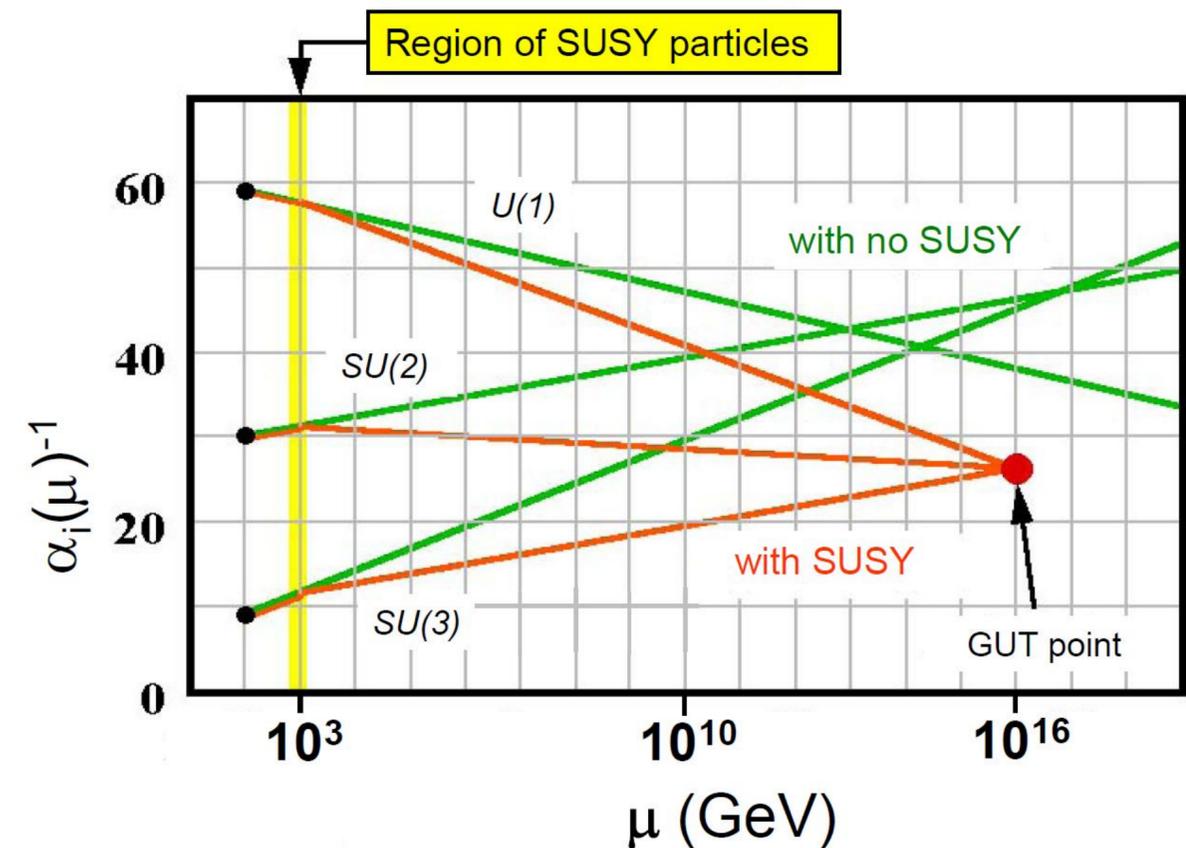
- Why do the electric charges of the electron and the proton exactly cancel each other?
- $U(1)$ symmetry alone doesn't necessarily require quantized charges.
- On the other hand, if $U(1)$ is part of a larger simple Lie group, charges must be quantized.

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➔ Grand Unified Theory

- There are several models based on $SU(5)$, $SO(10)$, E_6 , ... , which should be broken to the standard model gauge group $SU(3) \times SU(2) \times U(1)$.
- However, the couplings of $SU(3) \times SU(2) \times U(1)$ do not seem to agree at any energy scale.
- **Supersymmetry can cure this problem.**



Supersymmetry

provides (at least partial) solutions to these problems.

Part II

Application: From Higgs Mass to Black Hole Entropy

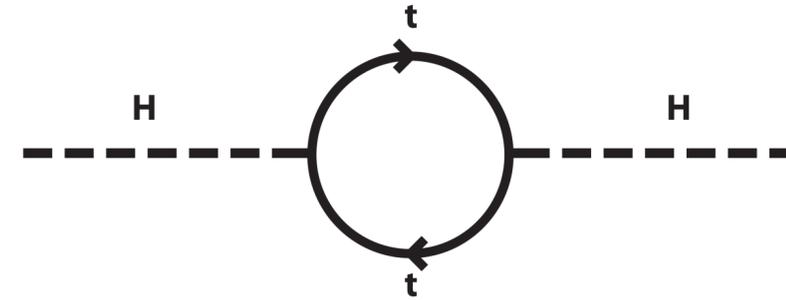
- Example I: Higgs Mass and the Hierarchy Problem
- Example II: Holographic Duality and Black Hole Entropy

Example I: Higgs Mass and the Hierarchy Problem

Higgs Mass

- The quantum correction to Higgs mass

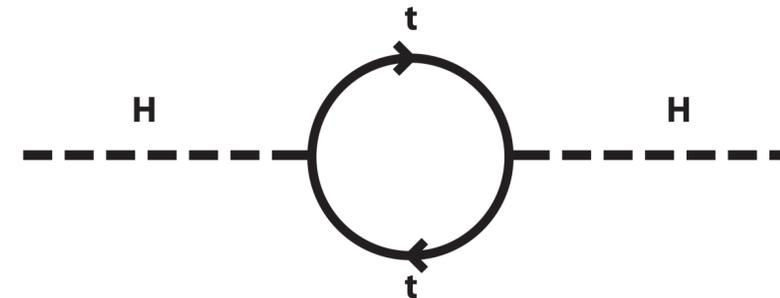
$$\Delta m_{Higgs}^2 = -\frac{|\lambda_f|^2}{8\pi^2} \left[\Lambda_{UV}^2 + m_f^2 \times \dots \right]$$



Higgs Mass

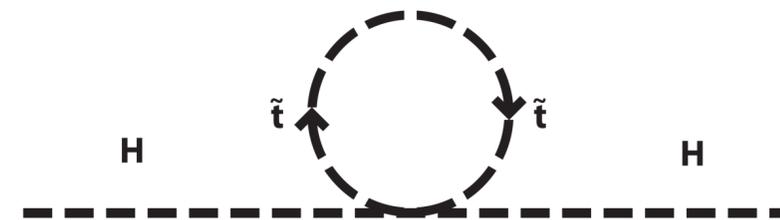
- The quantum correction to Higgs mass

$$\Delta m_{Higgs}^2 = -\frac{|\lambda_f|^2}{8\pi^2} \left[\Lambda_{UV}^2 + m_f^2 \times \dots \right]$$



- If there are two complex scalar fields with $\lambda_S = |\lambda_f|^2$ for each of the quarks and the leptons, the Λ_{UV}^2 contributions completely cancel.

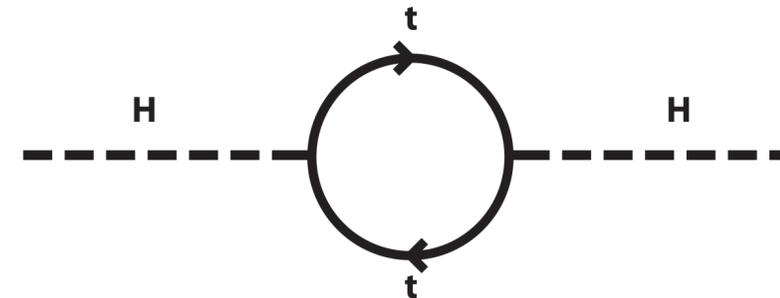
$$\Delta m_{Higgs}^2 \leftarrow + 2 \times \frac{\lambda_S}{16\pi^2} \left[\Lambda_{UV}^2 + m_S^2 \times \dots \right]$$



Higgs Mass

- The quantum correction to Higgs mass

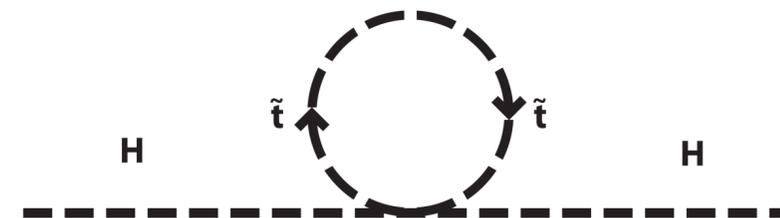
$$\Delta m_{Higgs}^2 = -\frac{|\lambda_f|^2}{8\pi^2} \left[\Lambda_{UV}^2 + m_f^2 \times \dots \right]$$



Very large

- If there are two complex scalar fields with $\lambda_S = |\lambda_f|^2$ for each of the quarks and the leptons, the Λ_{UV}^2 contributions completely cancel.

$$\Delta m_{Higgs}^2 \leftarrow + 2 \times \frac{\lambda_S}{16\pi^2} \left[\Lambda_{UV}^2 + m_S^2 \times \dots \right]$$



- Such cancelations persist to higher orders if the theory is **supersymmetric!**

Universal Properties of SUSY Theories

- The supersymmetric ground state, satisfying $Q_\alpha |vac\rangle = Q_{\dot{\alpha}}^\dagger |vac\rangle = 0$, must have the vanishing energy.

$$0 = \langle vac | \{Q_\alpha, Q_{\dot{\alpha}}^\dagger\} | vac \rangle = -2 \sigma_{\alpha\dot{\alpha}}^\mu \langle vac | P_\mu | vac \rangle$$

- For a supersymmetric multiplet with given four-momentum p^μ , the number of the bosonic states and that of the fermionic states must be the same unless $p^\mu = 0$.

$$0 = \text{tr} [(-1)^F \{Q_\alpha, Q_{\dot{\alpha}}^\dagger\}] \sim \text{tr} [(-1)^F P^\mu] = p^\mu \text{tr} (-1)^F = p_\mu (n_B - n_F)$$

The Simplest Model: A Free Chiral Supermultiplet

- Consider a complex scalar field ϕ and a Weyl fermion ψ with action

$$S = \int d^4x \left(\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} \right)$$

$$\mathcal{L}_{\text{scalar}} = -\partial^\mu \phi^* \partial_\mu \phi, \quad \mathcal{L}_{\text{fermion}} = i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

- S is invariant under the following transformation

$$\begin{aligned} \delta\phi &= \epsilon\psi, & \delta\phi^* &= \epsilon^\dagger\psi^\dagger \\ \delta\psi_\alpha &= -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi, & \delta\psi^\dagger_{\dot{\alpha}} &= i(\epsilon\sigma^\mu)_{\dot{\alpha}}\partial_\mu\phi^* \end{aligned}$$

Useful identities

$$[\sigma^\mu\bar{\sigma}^\nu + \sigma^\nu\bar{\sigma}^\mu]_\alpha{}^\beta = -2\eta^{\mu\nu}\delta_\alpha^\beta,$$

$$[\bar{\sigma}^\mu\sigma^\nu + \bar{\sigma}^\nu\sigma^\mu]^{\dot{\beta}}{}_{\dot{\alpha}} = -2\eta^{\mu\nu}\delta_{\dot{\alpha}}^{\dot{\beta}}$$

- Satisfy the SUSY algebra?

$$\left\{ Q_\alpha, Q_{\dot{\alpha}}^\dagger \right\} = -2 \sigma_{\alpha\dot{\alpha}}^\mu P_\mu$$

$$\left(\delta_{\epsilon_2} \delta_{\epsilon_1} - \delta_{\epsilon_1} \delta_{\epsilon_2} \right) \phi = i \left(-\epsilon_1 \sigma^\mu \epsilon_2^\dagger + \epsilon_2 \sigma^\mu \epsilon_1^\dagger \right) \partial_\mu \phi$$

$$\left(\delta_{\epsilon_2} \delta_{\epsilon_1} - \delta_{\epsilon_1} \delta_{\epsilon_2} \right) \psi_\alpha = i \left(-\epsilon_1 \sigma^\mu \epsilon_2^\dagger + \epsilon_2 \sigma^\mu \epsilon_1^\dagger \right) \partial_\mu \psi_\alpha + \underbrace{i\epsilon_{1\alpha} \epsilon_2^\dagger \bar{\sigma}^\mu \partial_\mu \psi - i\epsilon_{2\alpha} \epsilon_1^\dagger \bar{\sigma}^\mu \partial_\mu \psi}$$

= 0 only when EOM is satisfied

- To make it closed regardless of EOM, i.e., off-shell, introduce an auxiliary complex scalar field F with

$$\mathcal{L}_{\text{auxiliary}} = F^* F$$

- The off-shell SUSY action

$$S = \int d^4x \left(\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{auxiliary}} \right)$$

$$\mathcal{L}_{\text{scalar}} = -\partial^\mu \phi^* \partial_\mu \phi, \quad \mathcal{L}_{\text{fermion}} = i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi, \quad \mathcal{L}_{\text{auxiliary}} = F^* F$$

- The SUSY transformation

$$\delta\phi = \epsilon\psi, \quad \delta\phi^* = \epsilon^\dagger\psi^\dagger$$

$$\delta\psi_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi + \epsilon_\alpha F, \quad \delta\psi_{\dot{\alpha}}^\dagger = i(\epsilon\sigma^\mu)_{\dot{\alpha}} \partial_\mu \phi^* + \epsilon_{\dot{\alpha}}^\dagger F^*$$

$$\delta F = -i\epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi, \quad \delta F^* = i\partial_\mu \psi^\dagger \bar{\sigma}^\mu \epsilon$$

- The SUSY algebra is closed regardless of EOM for arbitrary field X :

$$\left(\delta_{\epsilon_2} \delta_{\epsilon_1} - \delta_{\epsilon_1} \delta_{\epsilon_2} \right) X = i \left(-\epsilon_1 \sigma^\mu \epsilon_2^\dagger + \epsilon_2 \sigma^\mu \epsilon_1^\dagger \right) \partial_\mu X$$

- $\Phi = (\phi, \psi_\alpha, F)$ forms an irreducible representation of the SUSY algebra, called a **chiral multiplet**.
- It also preserves the $U(1)$ R-symmetry acting on Φ as follows:

$$e^{ir\theta} \phi, \quad e^{i(r-1)\theta} \psi, \quad e^{i(r-2)\theta} F$$

Gauge Theory: A Vector Multiplet

- A **vector multiplet** consists of a gauge field A_μ , a Weyl fermion field λ , and an auxiliary real scalar field D .
- A renormalizable Lagrangian of a vector multiplet is completely fixed by SUSY.

$$\mathcal{L}_{vector} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + i\lambda^{\dagger a}\bar{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2}D^a D^a$$

$$\delta A_\mu^a = -\frac{1}{\sqrt{2}} \left(\epsilon^\dagger \bar{\sigma}_\mu \lambda^a + \lambda^{\dagger a} \bar{\sigma}_\mu \epsilon \right),$$

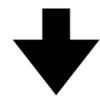
$$\delta \lambda_\alpha^a = \frac{i}{2\sqrt{2}} (\sigma^\mu \bar{\sigma}^\nu \epsilon)_\alpha F_{\mu\nu}^a + \frac{1}{\sqrt{2}} \epsilon_\alpha D^a,$$

$$\delta D^a = \frac{i}{\sqrt{2}} \left(-\epsilon^\dagger \bar{\sigma}^\mu \nabla_\mu \lambda^a + \nabla_\mu \lambda^{\dagger a} \bar{\sigma}^\mu \epsilon \right)$$

Gauged Chiral Multiplet

- The gauge invariant Lagrangian

$$\mathcal{L}_{chiral} = -\partial^\mu \phi^* \partial_\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^* F$$



$$\mathcal{L}_{chiral} = -\nabla^\mu \phi^* \nabla_\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \nabla_\mu \psi + F^* F$$

$$-\sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a \phi) + g(\phi^* T^a \phi)D^a \longrightarrow$$

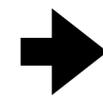
Gaugino
Yukawa couplings

- The SUSY transformation

$$\delta\phi = \epsilon\psi,$$

$$\delta\psi_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi + \epsilon_\alpha F,$$

$$\delta F = -i\epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$



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$$\delta\psi_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha \nabla_\mu \phi + \epsilon_\alpha F,$$

$$\delta F = -i\epsilon^\dagger \bar{\sigma}^\mu \nabla_\mu \psi + \sqrt{2}g(T^a \phi)\epsilon^\dagger \lambda^{\dagger a}$$

Supersymmetric Gauge Theory

$$\mathcal{L}_{vector} + \mathcal{L}_{chiral} + \mathcal{L}_{superpotential}$$

$$\mathcal{L}_{vector} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + i\lambda^{\dagger a} \bar{\sigma}^{\mu} \nabla_{\mu} \lambda^a + \frac{1}{2}D^a D^a$$

$$\begin{aligned} \mathcal{L}_{chiral} = & -\nabla^{\mu} \phi^* \nabla_{\mu} \phi + i\psi^{\dagger} \bar{\sigma}^{\mu} \nabla_{\mu} \psi + F^* F \\ & -\sqrt{2}g(\phi^* T^a \psi) \lambda^a - \sqrt{2}g\lambda^{\dagger a} (\psi^{\dagger} T^a \phi) + g(\phi^* T^a \phi) D^a \end{aligned} \quad \rightarrow \quad V = \left| \frac{\partial W}{\partial \Phi_i} \right|^2 + \frac{1}{2} \sum_a g_a^2 (\phi^* T^a \phi)^2$$

$$\mathcal{L}_{superpotential} = \frac{\partial W}{\partial \Phi_i} F_i - \frac{1}{2} \frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j} \psi_i \psi_j + h.c.$$

Supersymmetric Gauge Theory

Gauge group & representations W

$$\mathcal{L}_{vector} + \mathcal{L}_{chiral} + \mathcal{L}_{superpotential}$$

$$\mathcal{L}_{vector} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + i\lambda^{\dagger a} \bar{\sigma}^{\mu} \nabla_{\mu} \lambda^a + \frac{1}{2}D^a D^a$$

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Minimal Supersymmetric Standard Model

- One can construct a supersymmetric version of the standard model by introducing extra superpartners.
- Vector multiplets

Names	spin 1/2	spin 1	$SU(3)_C, SU(2)_L, U(1)_Y$
gluino, gluon	\tilde{g}	g	(8 , 1 , 0)
winos, W bosons	$\tilde{W}^\pm \quad \tilde{W}^0$	$W^\pm \quad W^0$	(1 , 3 , 0)
bino, B boson	\tilde{B}^0	B^0	(1 , 1 , 0)

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Minimal Supersymmetric Standard Model

- Chiral multiplets

Names		spin 0	spin 1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks, quarks ($\times 3$ families)	Q	$(\tilde{u}_L \ \tilde{d}_L)$	$(u_L \ d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
	\bar{u}	\tilde{u}_R^*	u_R^\dagger	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	\bar{d}	\tilde{d}_R^*	d_R^\dagger	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons, leptons ($\times 3$ families)	L	$(\tilde{\nu} \ \tilde{e}_L)$	$(\nu \ e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
	\bar{e}	\tilde{e}_R^*	e_R^\dagger	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs, higgsinos	H_u	$(H_u^+ \ H_u^0)$	$(\tilde{H}_u^+ \ \tilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$
	H_d	$(H_d^0 \ H_d^-)$	$(\tilde{H}_d^0 \ \tilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

- Superpotential

$$W_{MSSM} = \bar{u}y_u QH_u - \bar{d}y_d QH_d - \bar{e}y_e LH_d + \mu H_u H_d$$

Minimal Supersymmetric Standard Model

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sleptons, leptons ($\times 3$ families)	L	$(\tilde{\nu} \ \tilde{e}_L)$	$(\nu \ e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
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- Superpotential

$$W_{MSSM} = \bar{u}y_u QH_u - \bar{d}y_d QH_d - \bar{e}y_e LH_d + \mu H_u H_d$$

Holomorphic

Minimal Supersymmetric Standard Model

- Chiral multiplets

Names		spin 0	spin 1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks, quarks ($\times 3$ families)	Q	$(\tilde{u}_L \ \tilde{d}_L)$	$(u_L \ d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
	\bar{u}	\tilde{u}_R^*	u_R^\dagger	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	\bar{d}	\tilde{d}_R^*	d_R^\dagger	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons, leptons ($\times 3$ families)	L	$(\tilde{\nu} \ \tilde{e}_L)$	$(\nu \ e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
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	H_d	$(H_d^0 \ H_d^-)$	$(\tilde{H}_d^0 \ \tilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

- Superpotential

Two Higgs'

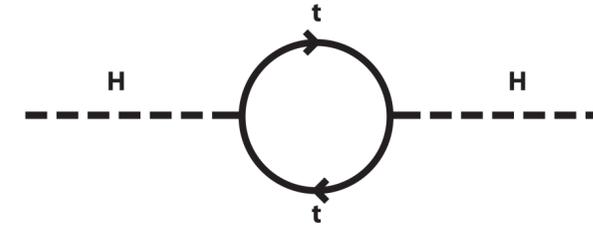
$$W_{MSSM} = \bar{u}_i y_u Q_i H_u - \bar{d}_i y_d Q_i H_d - \bar{e}_i y_e L_i H_d + \mu H_u H_d$$

Holomorphic

- However, we haven't observed any of superpartners
- *Supersymmetry should be spontaneously broken.*

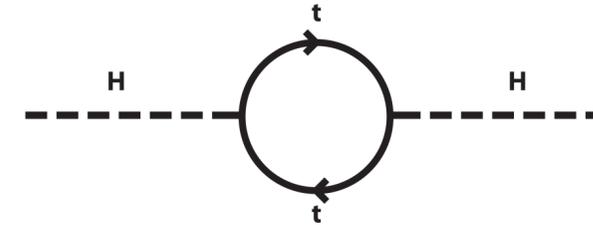
Soft SUSY Breaking

$$\Delta m_{Higgs}^2 \leftarrow -\frac{|\lambda_f|^2}{8\pi^2} \left[\Lambda_{UV}^2 + m_f^2 \times \dots \right]$$

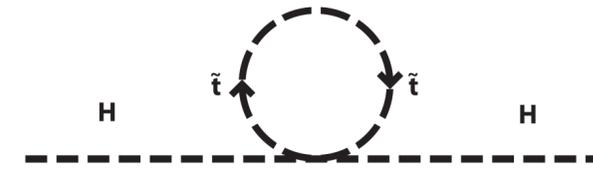


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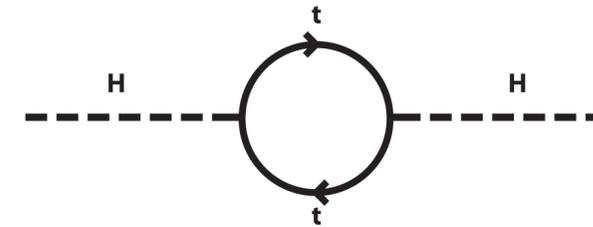


$$\Delta m_{Higgs}^2 \leftarrow + 2 \times \frac{\lambda_S}{16\pi^2} \left[\Lambda_{UV}^2 + m_S^2 \times \dots \right]$$

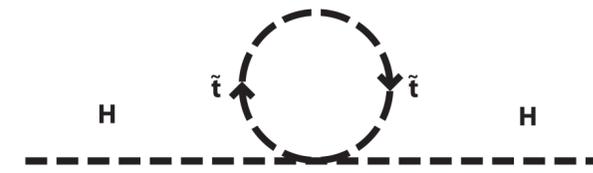


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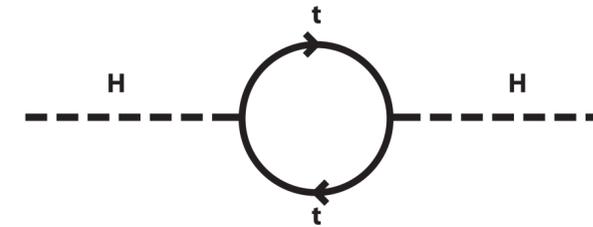


- If SUSY is broken severely such that $\lambda_S \neq |\lambda_f|^2$ anymore, the quantum correction to Higgs mass would be

$$\Delta m_{Higgs}^2 = \frac{1}{8\pi^2} \left(\lambda_S - |\lambda_f|^2 \right) \Lambda_{UV}^2 + \dots$$

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- SUSY must be broken maintaining $\lambda_S = |\lambda_f|^2$.

- Soft supersymmetry breaking:

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}$$

- \mathcal{L}_{soft} contains **SUSY breaking terms** maintaining $\lambda_S = |\lambda_f|^2$ with an **additional scale** m_{soft} , supposed to originate from the spontaneous SUSY breaking of the microscopic theory.

$$m_{soft} \rightarrow 0 \quad \Rightarrow \quad \mathcal{L}_{soft} \rightarrow 0$$

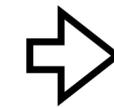
$$\Delta m_{Higgs}^2 = m_{soft}^2 \left[\frac{\lambda}{16\pi^2} \ln \left(\Lambda_{UV}/m_{soft} \right) + \dots \right]$$

- Soft SUSY breaking terms in the MSSM

$$\begin{aligned}
\mathcal{L}_{soft}^{MSSM} = & -\frac{1}{2} (M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + h.c.) \\
& - \left(\tilde{u} a_u \tilde{Q} H_u - \tilde{d} a_d \tilde{Q} H_d - \tilde{e} a_e \tilde{L} H_d + h.c. \right) \\
& - \tilde{Q}^\dagger m_Q^2 \tilde{Q} - \tilde{L}^\dagger m_L^2 \tilde{L} - \tilde{u} m_{\tilde{u}}^2 \tilde{u}^\dagger - \tilde{d} m_{\tilde{d}}^2 \tilde{d}^\dagger - \tilde{e} m_{\tilde{e}}^2 \tilde{e}^\dagger \\
& - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u H_d + h.c.)
\end{aligned}$$

$$M_1, M_2, M_3, a_u, a_d, a_e \sim m_{soft}$$

$$m_Q^2, m_L^2, m_{\tilde{u}}^2, m_{\tilde{d}}^2, m_{\tilde{e}}^2, m_{H_u}^2, m_{H_d}^2, b \sim m_{soft}^2$$



105 parameters

Phenomenology of MSSM

$$\Delta m_{Higgs}^2 = m_{soft}^2 \left[\frac{\lambda}{16\pi^2} \ln \left(\Lambda_{UV}/m_{soft} \right) + \dots \right]$$

- Assuming $\Lambda_{UV} \sim M_p$ and $\lambda \sim 1$, the SUSY breaking scale m_{soft} shouldn't be much greater than the 10^3 GeV scale to avoid further miraculous cancelations.
- The lightest superpartner at this scale?
- Seems no, but not conclusive.

At SUSY 2023...

Status and Future of Supersymmetry

Stephen P. Martin
Northern Illinois University
spmartin@niu.edu

SUSY 2023
July 17-21, 2023
University of Southampton

The LHC vs. Supersymmetric Models



However, constraints on SUSY are sometimes colloquially overstated, perhaps due to temptation to make grand statements.

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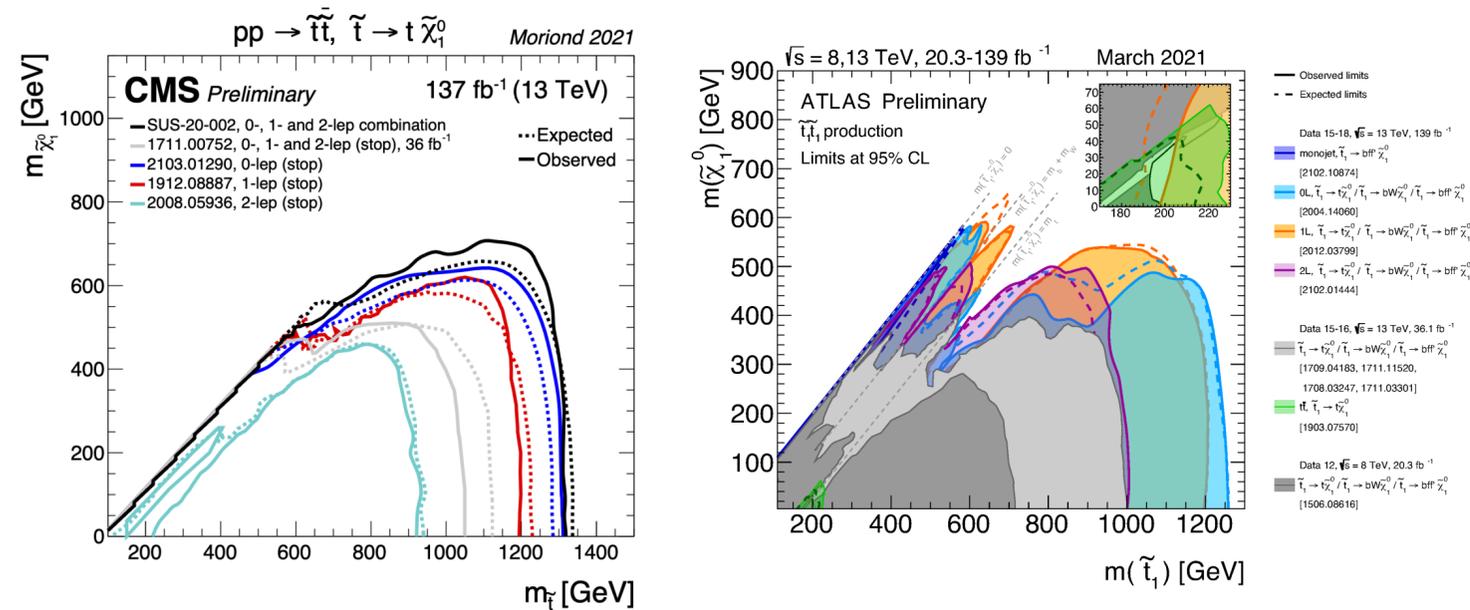


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At SUSY 2023...

Limits on top squarks

See Vellidis (CMS) and Maurer (ATLAS) talks for details.

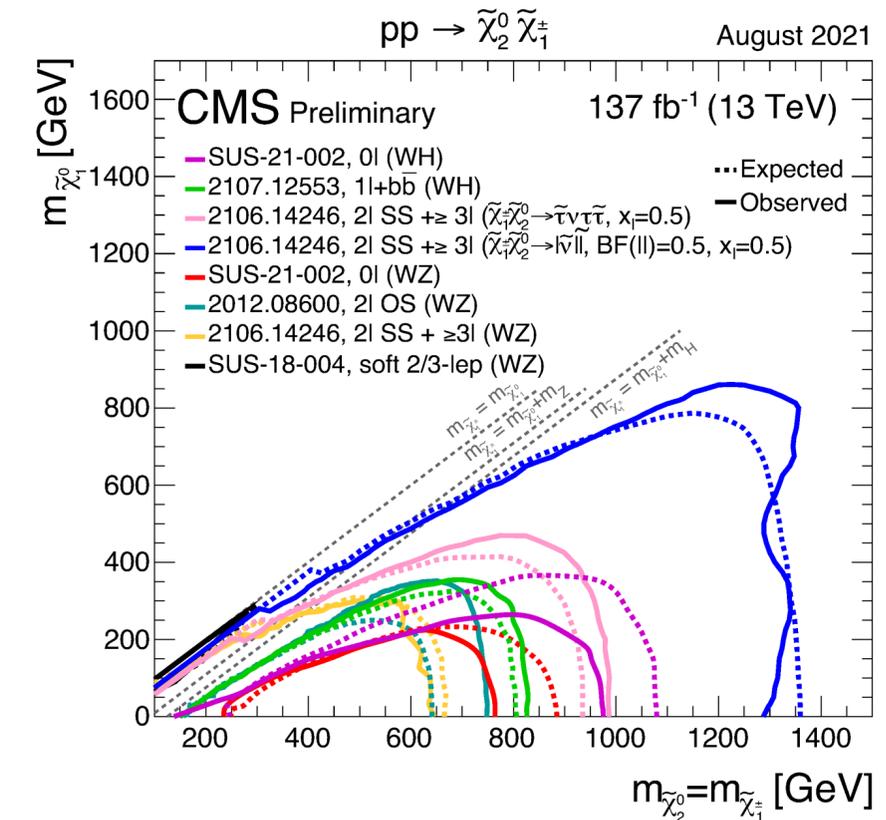


Pessimist: Exclusion of top-squark masses now up to 1300 GeV!

Optimist: No constraints at all on direct pair production of top squarks, if LSP mass exceeds 700 GeV.

More generally, “Compressed SUSY” models with small mass differences are more difficult because visible energy in each event is smaller.

Constraints on wino-like charginos and neutralinos that decay through sleptons: $pp \rightarrow \tilde{C}_1 \tilde{N}_2 \rightarrow \text{leptons} + \cancel{E}_T$



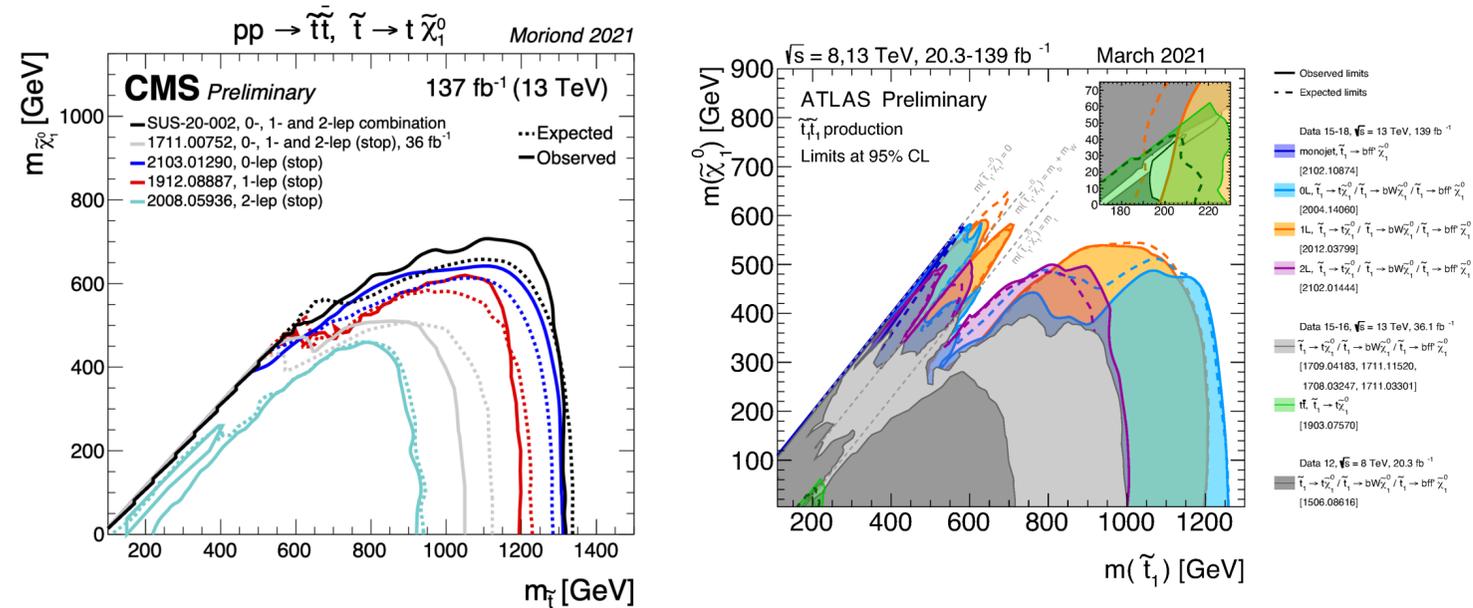
Pessimist: Exclusion of electroweakinos above 1300 GeV!

Optimist: Constraints on decays through staus are much weaker, with no exclusion for $m_{\tilde{C}_1} > 1000 \text{ GeV}$ or LSP mass $> 450 \text{ GeV}$. Furthermore...

At SUSY 2023...

Limits on top squarks

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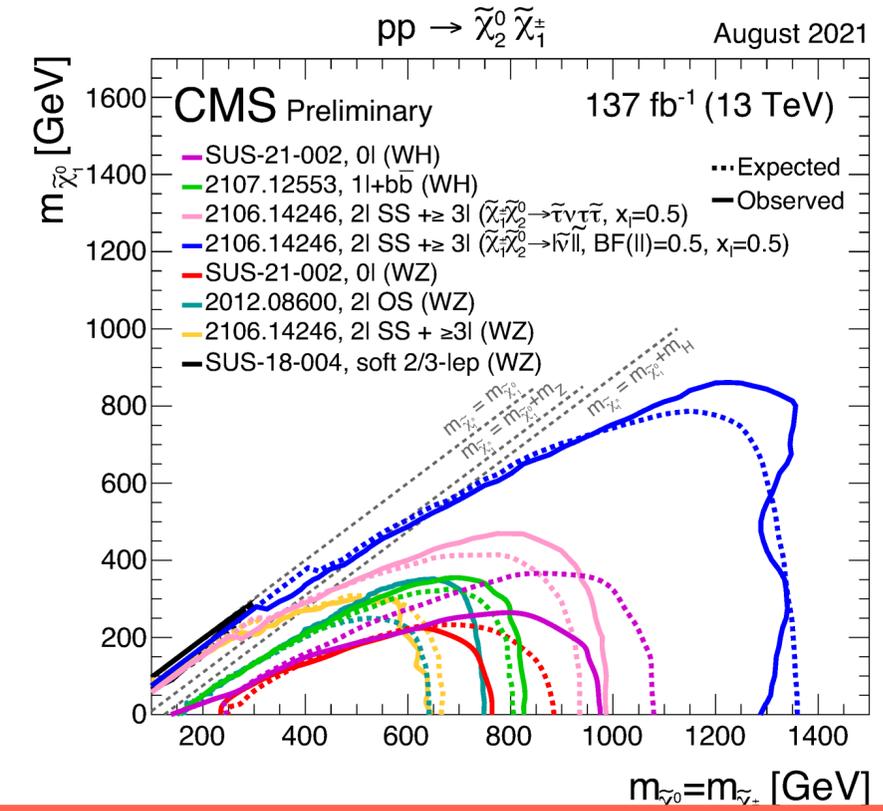


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Furthermore...

The current analysis relies on some simplifying assumptions. It is hard to make a conclusion in full generality due to the large number of parameters.

- Our nature seems not explained by the MSSM in the simplest way but doesn't rule it out.
- More models: Next-to-Minimal Supersymmetric Standard Model, Split supersymmetry, ...

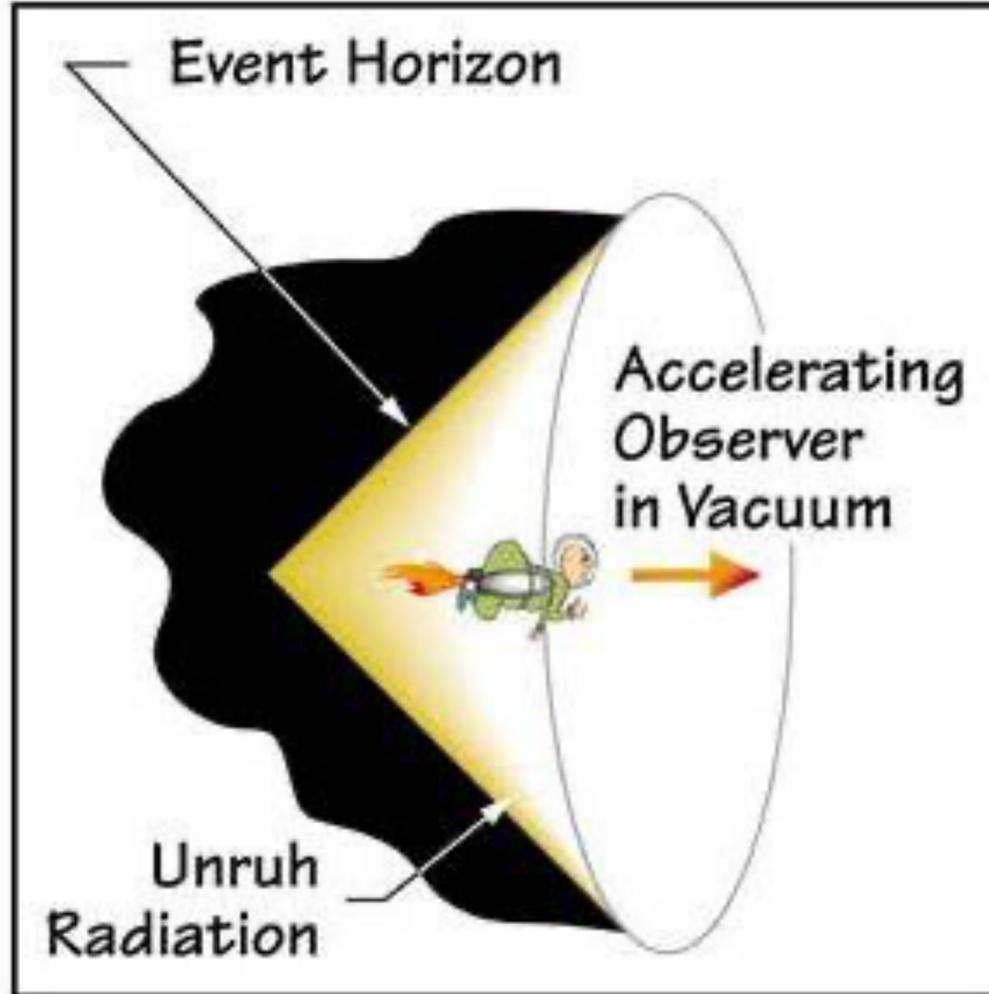
Example II: Holographic Duality and Black Hole Entropy

Black Hole Entropy: The Heart of Quantum Gravity

Unruh Effect

- Although we don't know the complete quantum theory of gravity, one can consider *quantum field theory of matters* in curved (still *classical*) spacetime.
- Surprisingly, the vacuum in QFT is an observer dependent notion.
- E.g., the Unruh effect

$$|vac\rangle_{inertial} = |thermal\rangle_{accelerating}$$

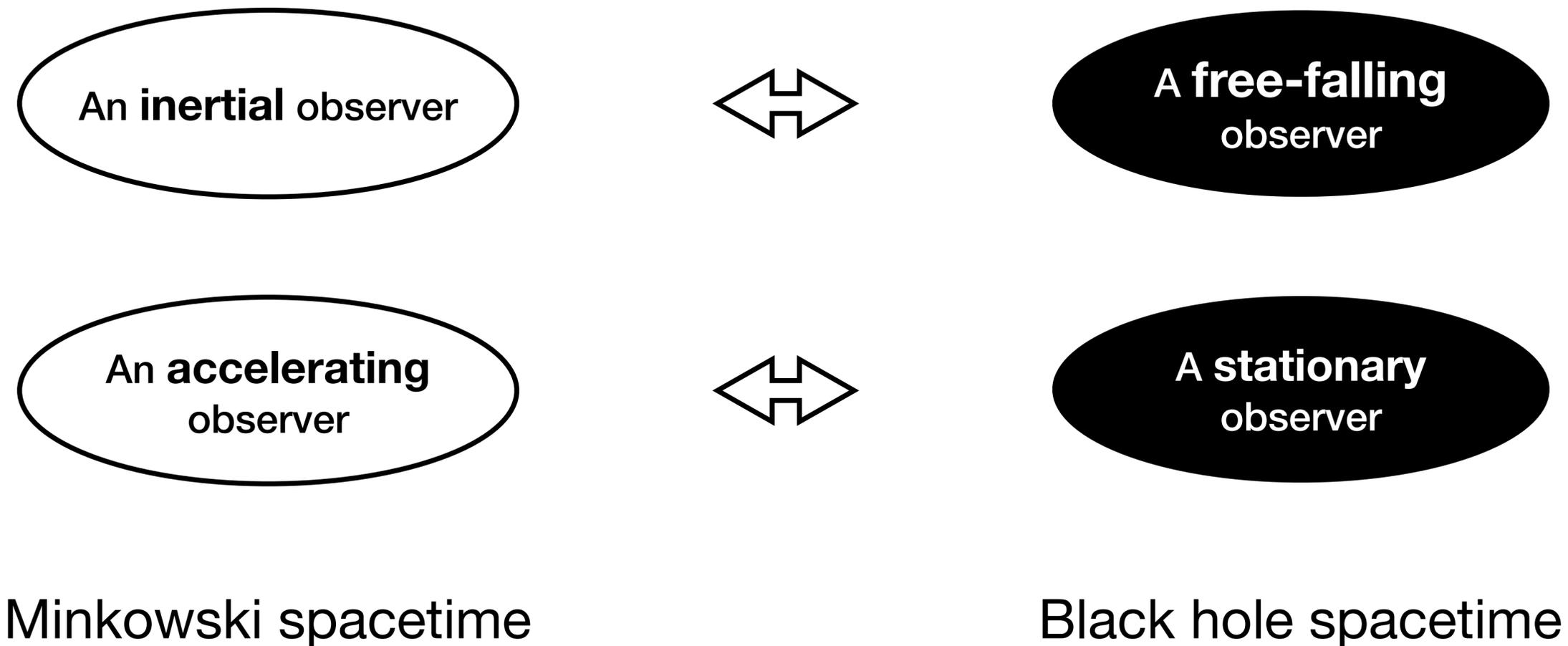


An accelerating observer in vacuum would see a similar Hawking-like radiation called Unruh radiation.

$$T_{Unruh} = \frac{\hbar a}{2\pi c k_B}$$

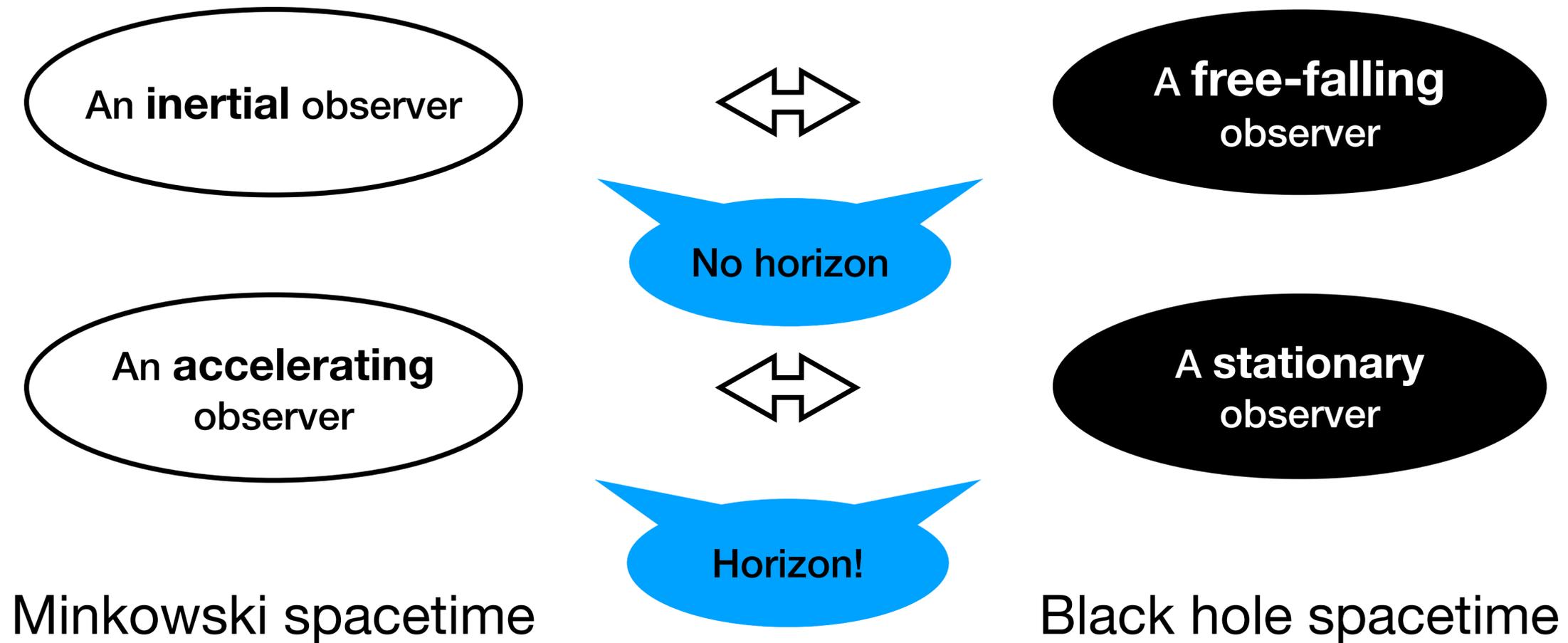
Hawking Radiation and Black Hole Entropy

- Exactly the same phenomenon occurs in a black hole spacetime.

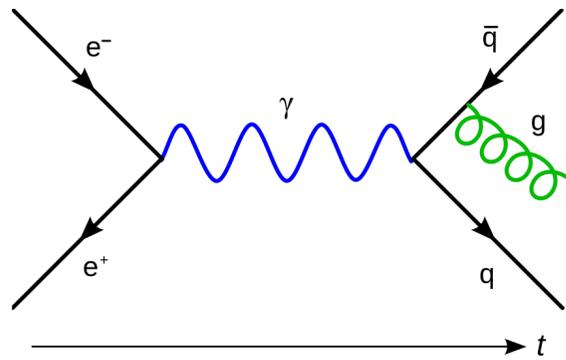


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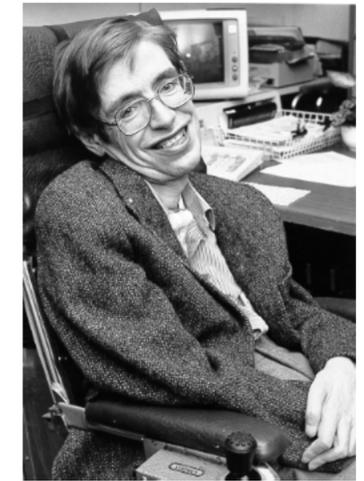
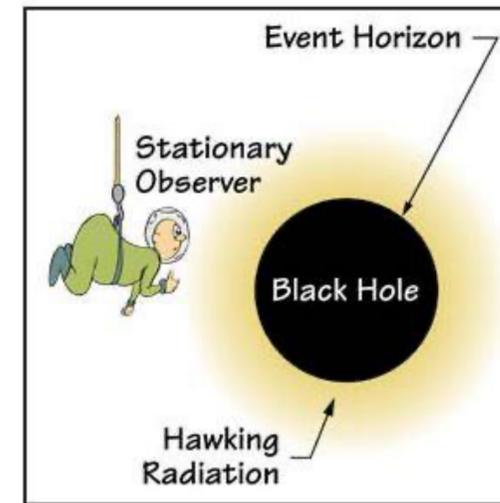
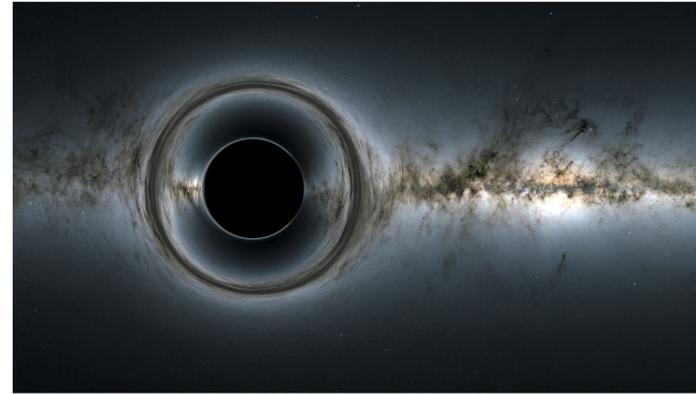
- Exactly the same phenomenon occurs in a black hole spacetime.



- **Quantum** field theory + the **curved** spacetime -> the Hawking radiation (1974)



+



A stationary observer outside the black hole would see the thermal Hawking radiation.

$$T_H = \frac{\hbar c^3}{8\pi GM k_B},$$

$$S_{BH} = \frac{k_B A c^3}{4G\hbar}$$

A black hole is not black!

- A black hole is a thermal object.
- The microstates of a black hole? $e^{S/k_B} \sim e^{10^{44}}$ for $10 M_\odot$.
- However, everything is squeezed into a single point in a (Schwarzschild) black hole.
- Where does this vast number of states come from?

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- Where does this vast number of states come from?

Quantum nature of the spacetime itself!

- A black hole entropy is a key property that any quantum theory of gravity must be able to explain.
- Our current understanding?

I'm a charged
metal cylinder



I'm a charged
metal cylinder



I'm a spherical
chicken



I'm a charged metal cylinder



What is the electric field I'm producing?

I'm a spherical chicken



I'm a charged metal cylinder



What is the electric field I'm producing?

I'm a spherical chicken

I don't know



Let me pretend to be a ball, like you. Then?



Let me pretend to be a ball, like you. Then?



Easy!

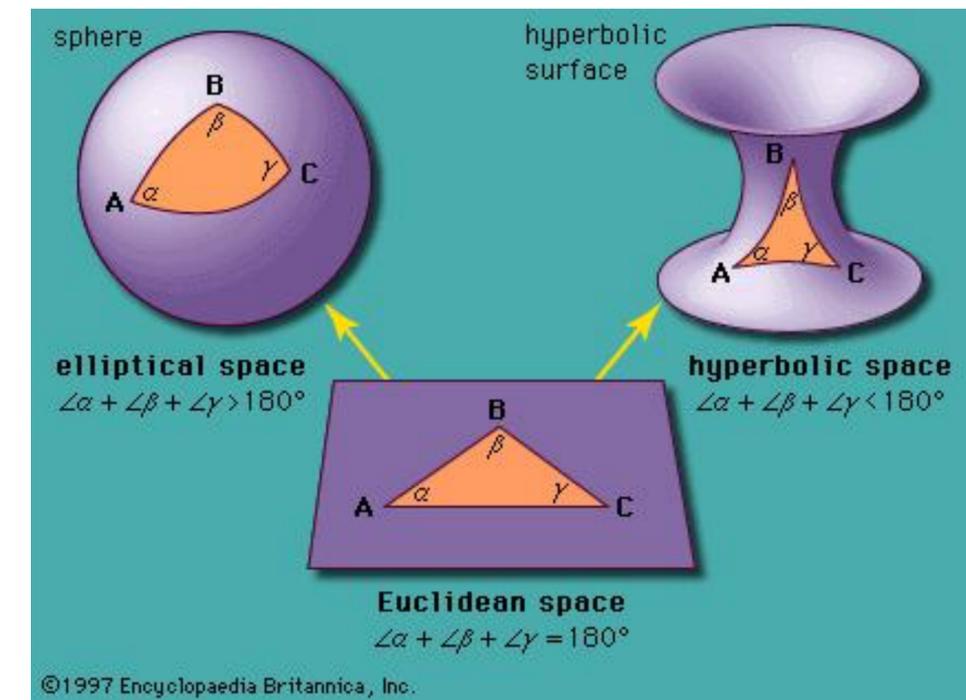


- Let's consider the most symmetric (but exotic) black hole.
- A **supersymmetric** black hole in the 5-dimensional **Anti-de Sitter** spacetime.

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Three types of spacetime with the constant curvature

- De Sitter (positive curvature)
- Minkowski (vanishing curvature)
- Anti-de Sitter (negative curvature)



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Spacetime version

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Why AdS?

- The Hawking temperature of a black hole in the flat spacetime:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$

- Negative specific heat.
- On the other hand, the AdS spacetime is a gravitational potential trap.
- An AdS black hole has positive specific heat and is thermodynamically stable.

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AdS is a nice playground to study black hole thermodynamics!

Why Supersymmetry?

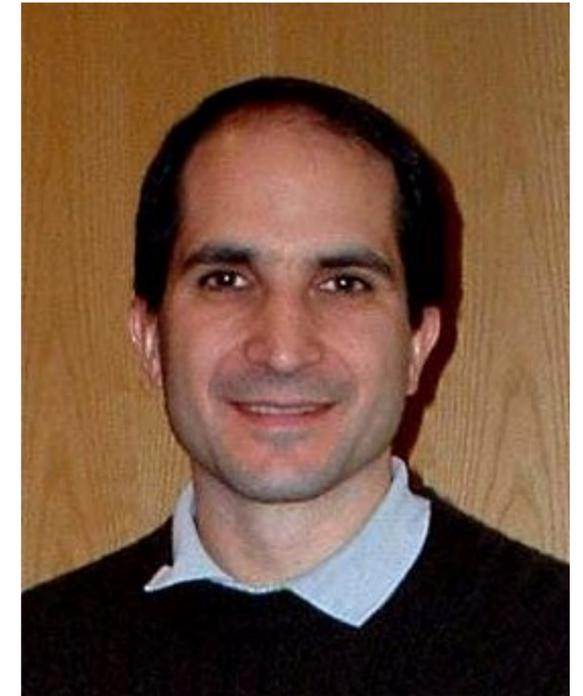
As mentioned, the largest spacetime symmetry. Furthermore...

Why Supersymmetry?

As mentioned, the largest spacetime symmetry. Furthermore...

The AdS/CFT correspondence

- A ground breaking duality between d -dimensional QFT and $d+1$ -dimensional gravity (Maldacena 1997)
- The most cited paper in High-Energy Physics (>20,000 citations, HEP database INSPIRE)



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Observation of Gravitational Waves from a Binary Black Hole Merger #9

LIGO Scientific and Virgo Collaborations • B.P. Abbott (Caltech) et al. (Feb 11, 2016)

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Planck 2015 results. XIII. Cosmological parameters #11

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PYTHIA 6.4 Physics and Manual #12

Torbjorn Sjostrand (Lund U., Dept. Theor. Phys.), Stephen Mrenna (Fermilab), Peter Z. Skands (Fermilab) (Mar, 2006)

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The Large N limit of superconformal field theories and supergravity #1
Juan Martin Maldacena (Harvard U.) (Nov, 1997)
Published in: *Int.J.Theor.Phys.* 38 (1999) 1113-1133 (reprint), *Adv.Theor.Math.Phys.* 2 (1998) 231-252 • e-Print: [hep-th/9711200](#) [hep-th]
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GEANT4 Collaboration • S. Agostinelli (Genoa U.) et al. (Aug, 2002)
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Planck Collaboration • N. Aghanim (Orsay, IAS) et al. (Jul 17, 2018)
Published in: *Astron.Astrophys.* 641 (2020) A6, *Astron.Astrophys.* 652 (2021) C4 (erratum) • e-Print: [1807.06209](#) [astro-ph.CO]
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String theory picture

String length
of D3-branes
String coupling

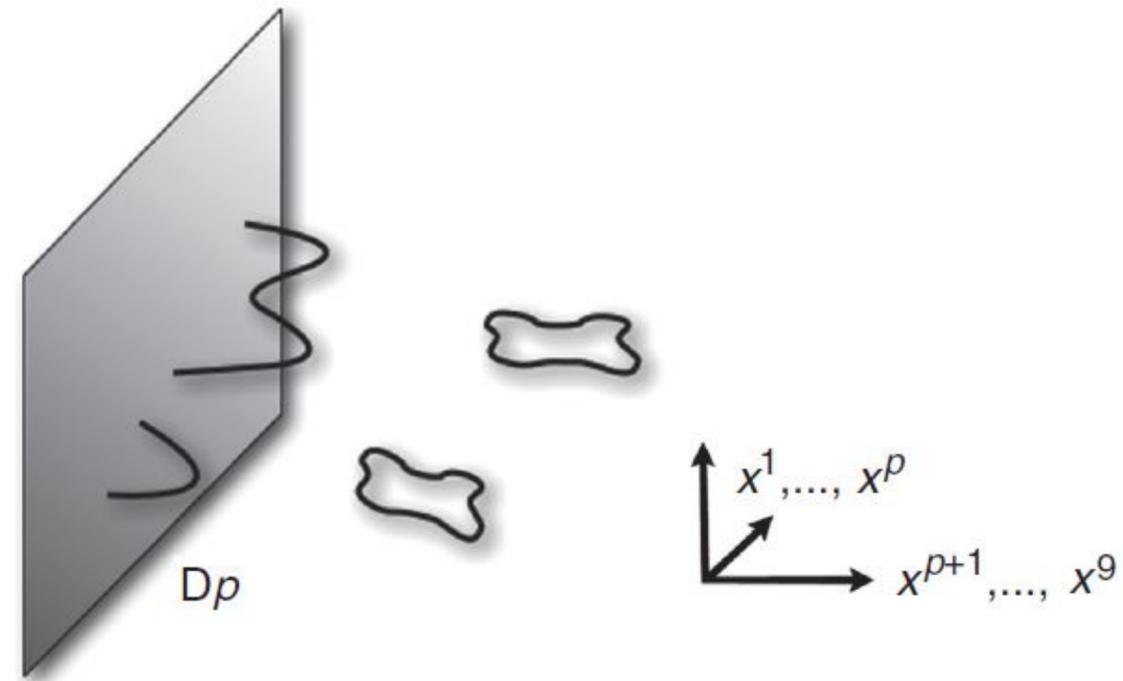


Figure 6.1 String theory in the presence of a Dp -brane, along the directions x^0, \dots, x^p . The closed string sector describes the fluctuations of the theory around the vacuum, while the sector of open strings characterizes the fluctuations of the non-perturbative object.

String theory picture

String length $\rightarrow 0$

of D3-branes $\rightarrow \infty$

String coupling $\rightarrow 0$

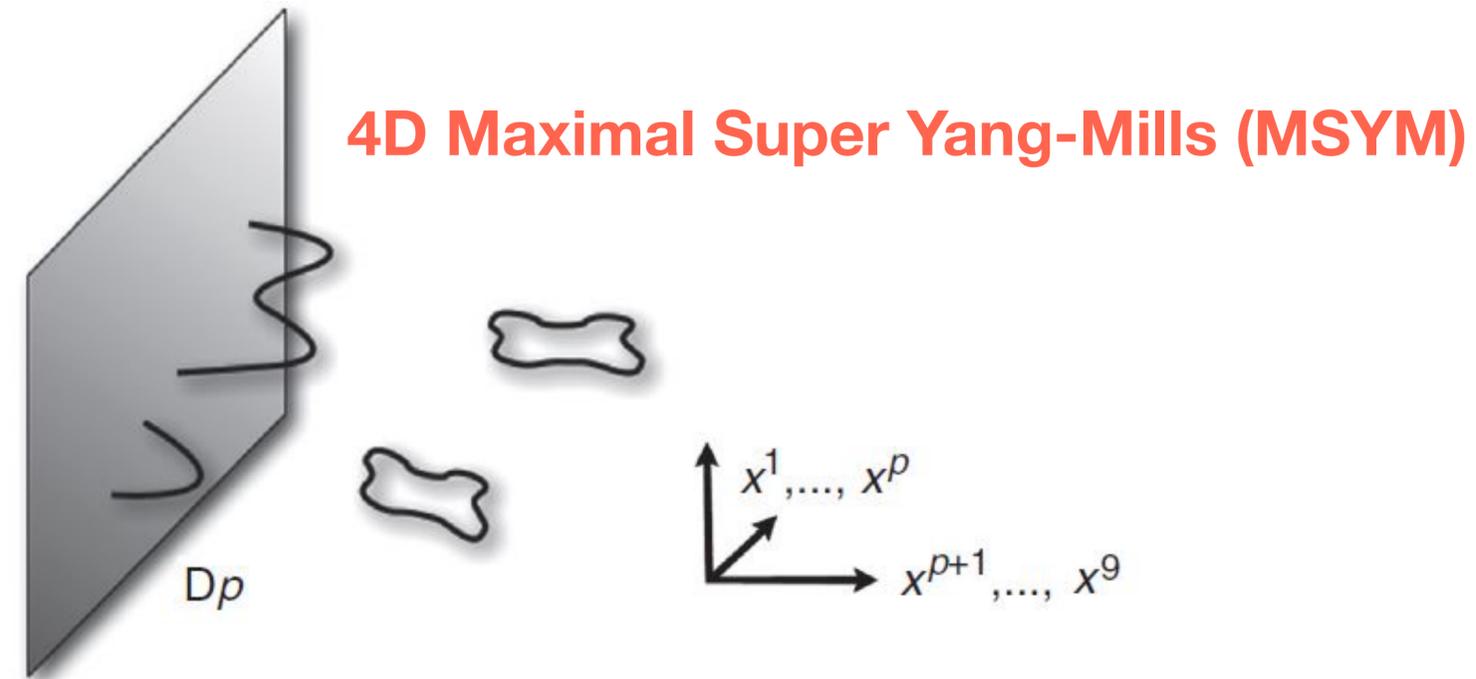


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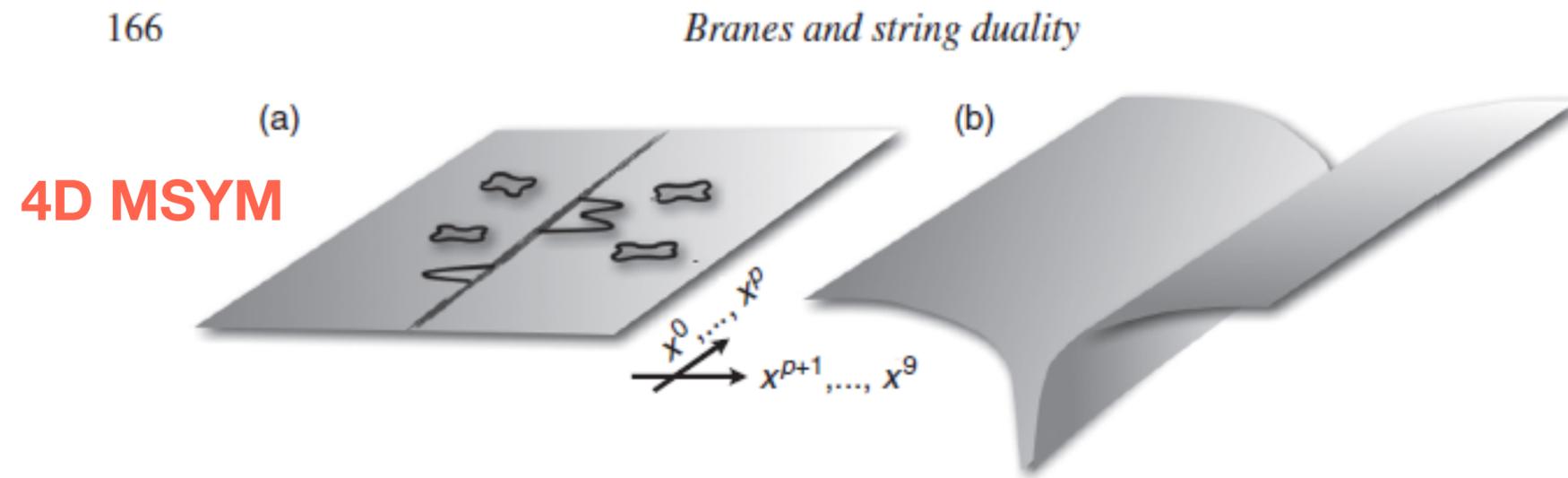


Figure 6.4 A Dp -brane interacts with closed strings via open strings (a), creating an effective supergravity background (b) which describes the backreaction of the D-brane tension and charge on the configuration.

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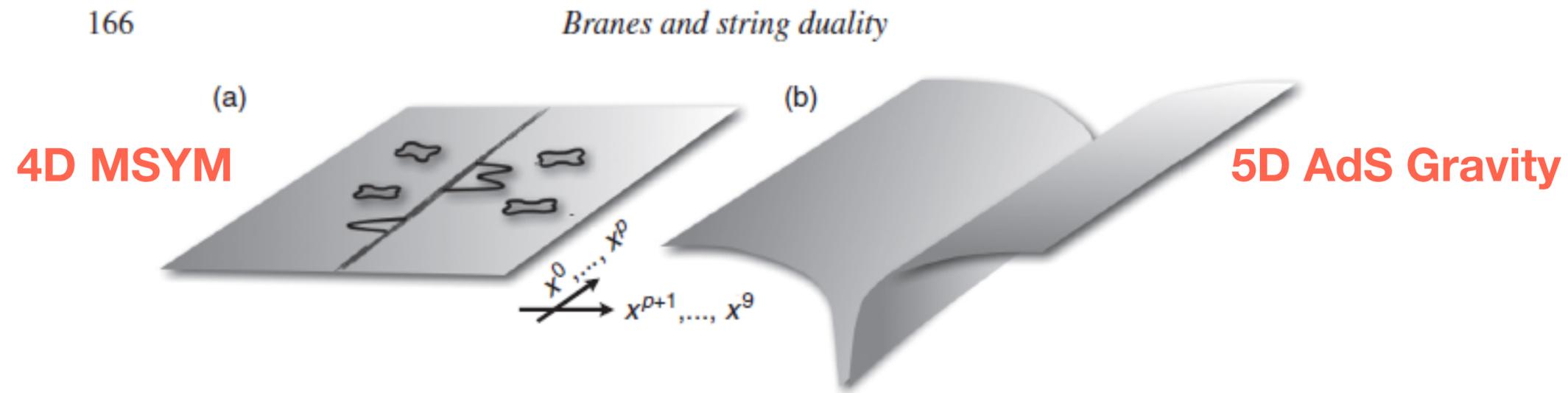


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String coupling

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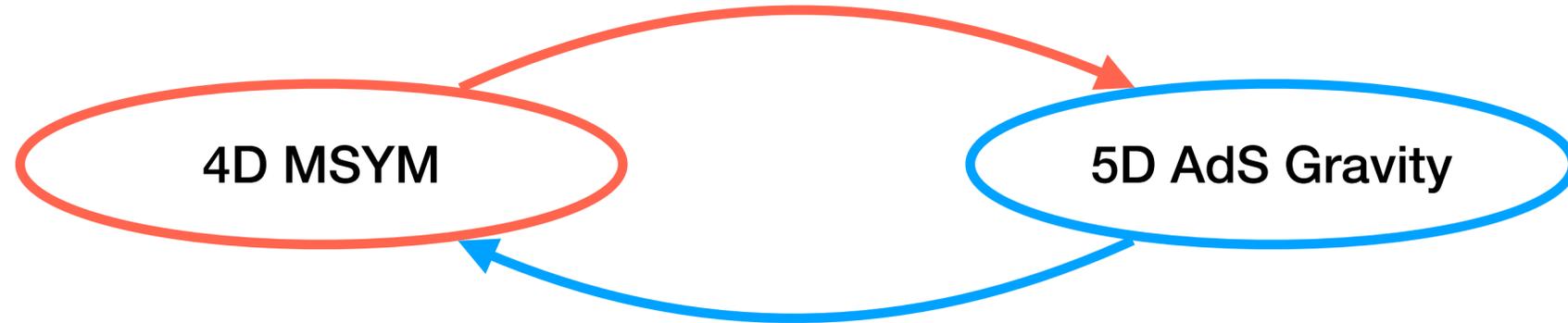
4D MSYM

5D AdS Gravity

String coupling

0 <-----> ∞

Strong coupling limit



4D MSYM

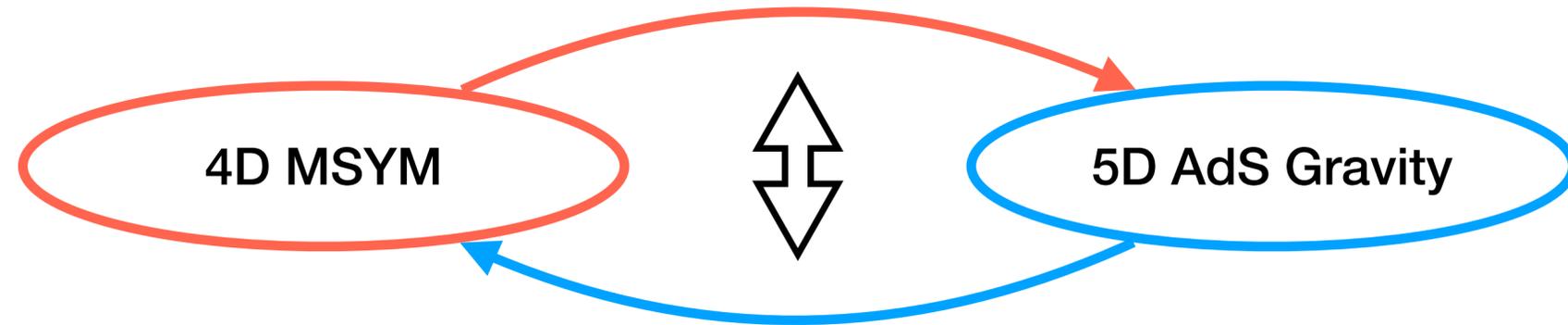
5D AdS Gravity

Strong coupling limit

String coupling

0 \leftarrow ----- \rightarrow ∞

Strong coupling limit



Strong coupling limit

AdS/CFT correspondence

AdS/CFT Correspondence

	4d MSYM	5d AdS Gravity
More (Less) D3-Branes	More (Less) Colors	Small (Large) Quantum Correction
Strong (Weak) String Coupling	Strong (Weak) (’t Hooft) Coupling	Small (Large) Higher Curvature Correction

- Weakly coupled 5d AdS gravity is equivalent to strongly coupled 4d MSYM, and vice-versa.
- Quantum corrections to the gravity correspond to $1/N$ corrections to the MSYM.

Microstates of an AdS Black Hole

- Black hole entropy is obtained from the horizon area of the (classical) black hole solution.
- A consistent quantum gravity should provide a microscopic explanation of this entropy.
- Via the AdS/CFT correspondence

$$Z_{(\text{strongly coupled})\ 4\text{d MSYM}} = Z_{5\text{d AdS (Einstein) gravity}}$$

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↓
Hard

Partition Function vs Index

$$Z_g = \text{Tr}_g e^{-\beta H - \mu Q} \longrightarrow \text{Hard}$$

Partition Function vs Index

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$$\hat{Z}_g = \text{Tr}_g (-1)^F e^{-\beta H - \mu Q}$$

Partition Function vs Index

$$Z_g = \text{Tr}_g e^{-\beta H - \mu Q} \longrightarrow \text{Hard}$$

(Note: In the original image, the Tr_g is circled in red, and a red arrow points from the circle to the expression $n_B + n_F$ above it.)

$$\hat{Z}_g = \text{Tr}_g (-1)^F e^{-\beta H - \mu Q}$$

(Note: In the original image, the Tr_g and $(-1)^F$ are circled in red, and a red arrow points from the circle to the expression $n_B - n_F$ above it.)

Partition Function vs Index

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$$\hat{Z}_g = \text{Tr}_g (-1)^F e^{-\beta H - \mu Q}$$
$$= \text{Tr}_{g \rightarrow 0} (-1)^F e^{-\beta H - \mu Q} \longrightarrow \text{Easy}$$

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(Again, magic of supersymmetry)

Partition Function vs Index

$$Z_g = \text{Tr}_g e^{-\beta H - \mu Q} \longrightarrow \text{Hard}$$

(Note: A red circle highlights the Tr_g operator, and a red arrow points from it to the expression $n_B + n_F$ above.)

$$\hat{Z}_g = \text{Tr}_g (-1)^F e^{-\beta H - \mu Q}$$
$$= \text{Tr}_{g \rightarrow 0} (-1)^F e^{-\beta H - \mu Q} \longrightarrow \text{Easy}$$

(Note: A red circle highlights the Tr_g operator and the $(-1)^F$ term, and a red arrow points from the circle to the expression $n_B - n_F$ above.)

(Again, magic of supersymmetry)

If the cancelation is not too large, the index would also capture enough microstates to reproduce the black hole entropy.

Entropy of an AdS5 Black Hole

- Let us first consider $N \gg 1$, i.e., classical gravity regime.
- With some computational techniques

$$\log \hat{Z} = \frac{N^2}{2} \frac{\Delta_1 \Delta_2 \Delta_3}{\omega^2}$$

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Legendre transformation \downarrow $S = \log \hat{Z} + \sum_i \Delta_i Q_i + \omega J$

$$S = 2\pi \sqrt{Q_1 Q_2 + Q_2 Q_3 + Q_3 Q_1 - N^2 J}$$

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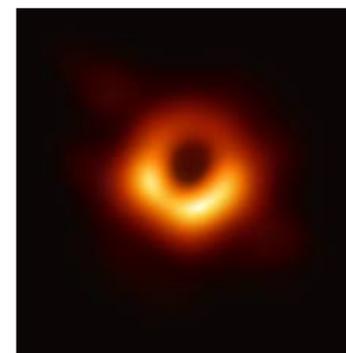
Exactly match the geometric entropy of the black hole solution!

QFT

GR

Strong QFT

A non-perturbative description of QFT

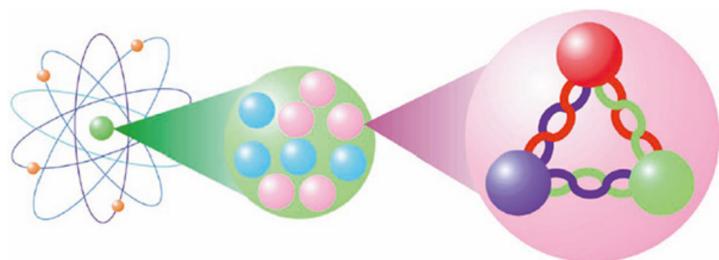


Weak gravity

AdS/CFT correspondence



Weak QFT



A non-geometric description of gravity

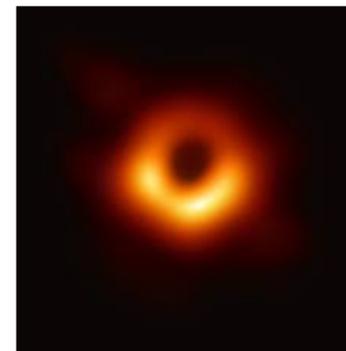
Strong gravity

QFT

GR

Strong QFT

A non-perturbative description of QFT



Weak gravity

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A black hole

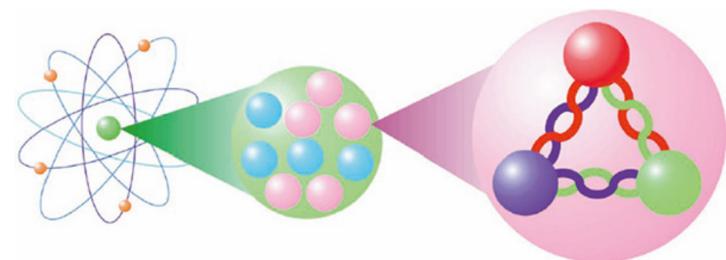


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A thermal ensemble of perturbative quantum states

Weak QFT



A non-geometric description of gravity

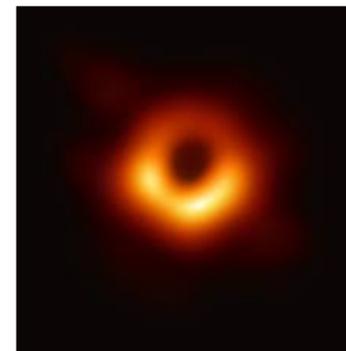
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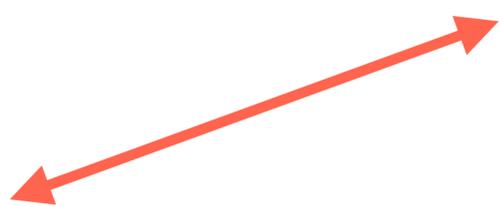
A black hole



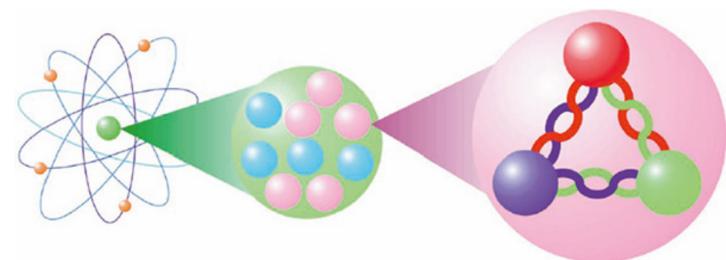
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Weak QFT



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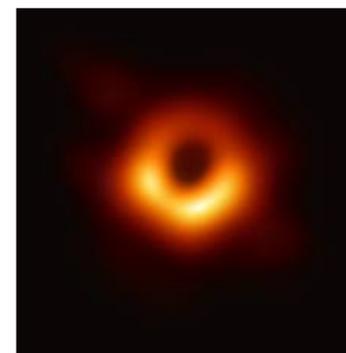
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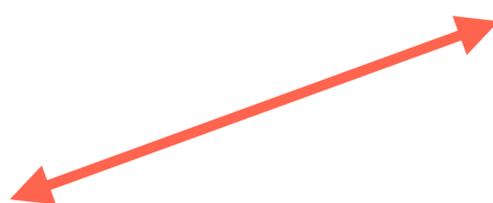


Weak gravity

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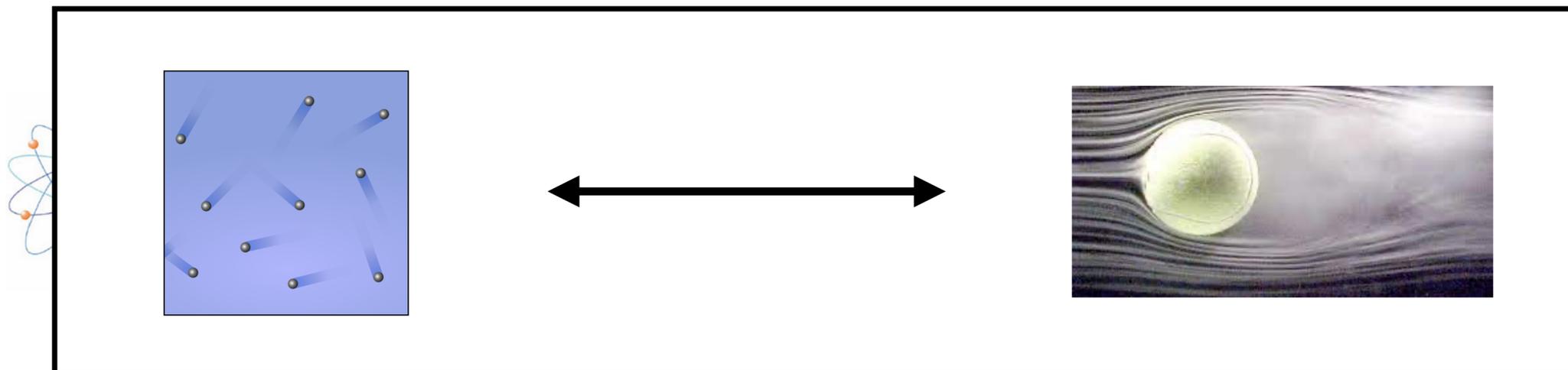


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Weak QFT

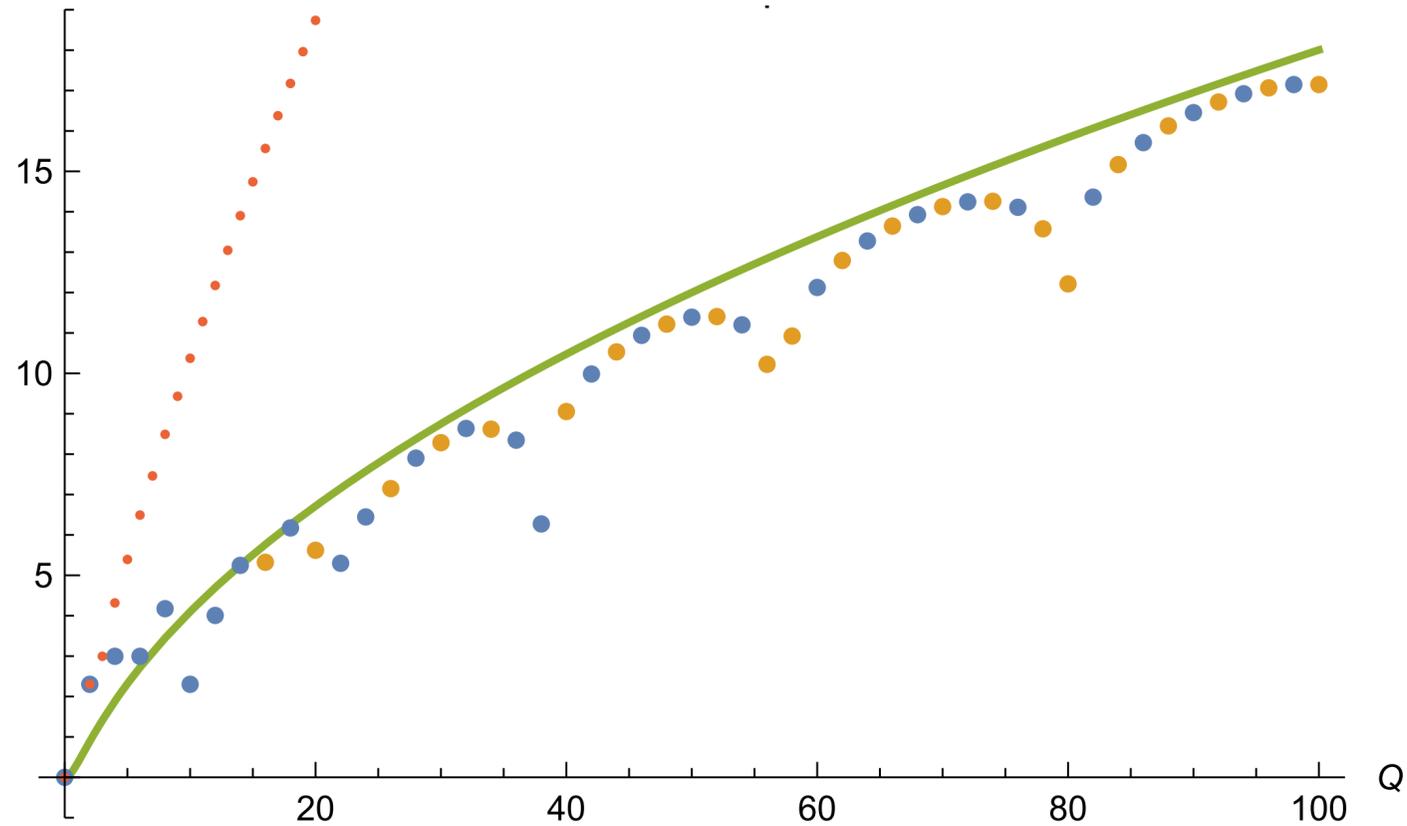


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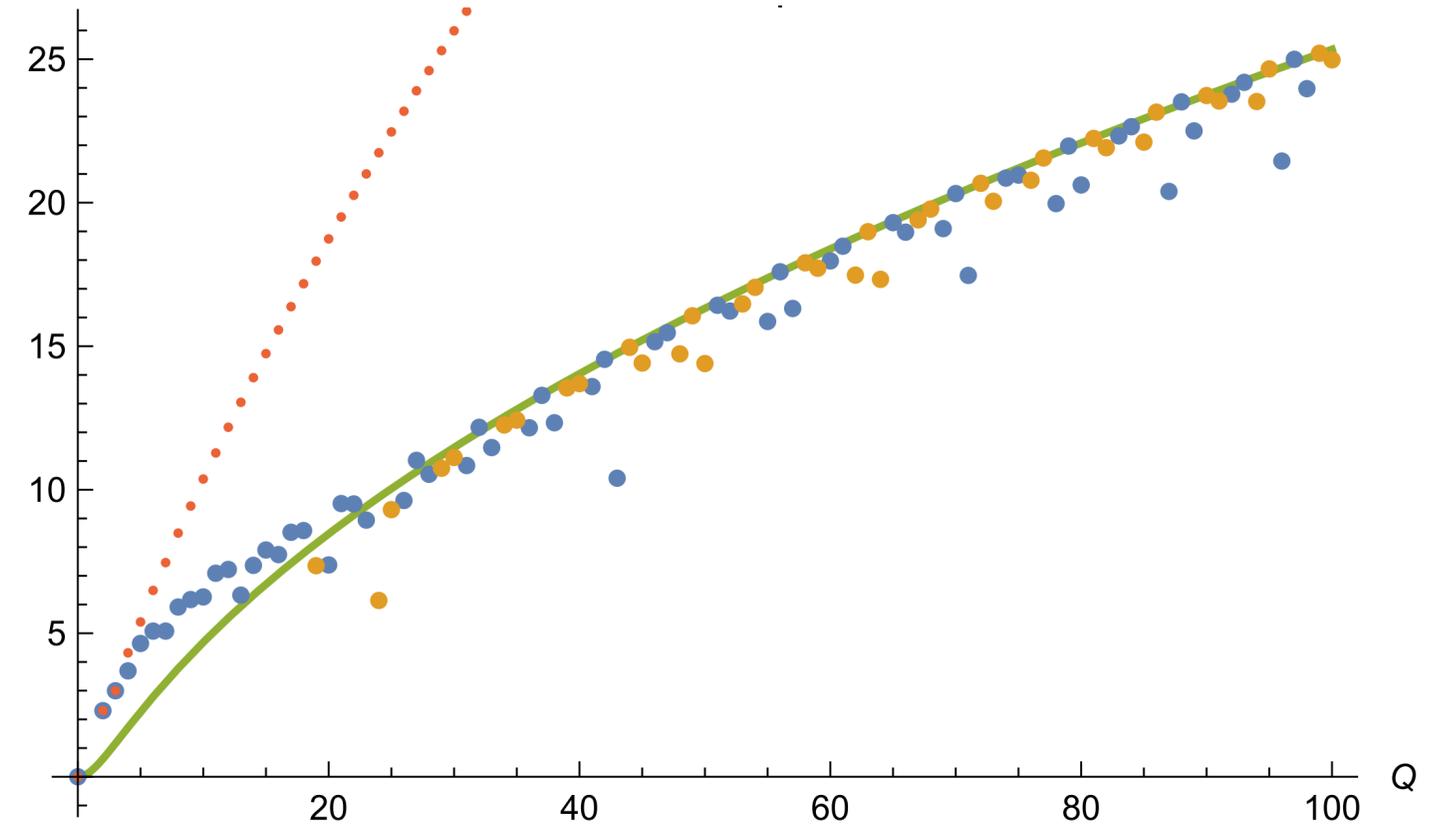
Quantum Gravity?

- Subleading terms in the $1/N$ expansion correspond to quantum corrections to classical gravity.
- Furthermore, QFT with small N ($= 2, 3, \dots$) can be viewed as a description of gravity in *deeply quantum* regimes.
- Black hole entropy \rightarrow ensemble of quantum states
- Can they be explicitly constructed?

Green line: classical black hole entropy
Red dots: classical graviton spectrum
Blue and yellow dots: quantum spectrum



$N = 2$



$N = 3$

Construction (and distinction) of graviton and black hole states -> black hole cohomology

- Considering a supersymmetric black hole, we derive the black hole entropy microscopically, which is a key feature of quantum gravity.
- More applications to strongly coupled theories, exhibiting confinement, duality, and other non-perturbative phenomena.
- New mathematics: mirror symmetry, the Seiberg-Witten theory, ...

Conclusion

- Phenomenologically, supersymmetry remains one of the most intriguing directions to pursue beyond the Standard Model, although minimal supersymmetry appears not to be favored by nature.
- Theoretically, supersymmetry provides a powerful framework for addressing unresolved problems in modern physics.

Much more to explore in the SUSY world!