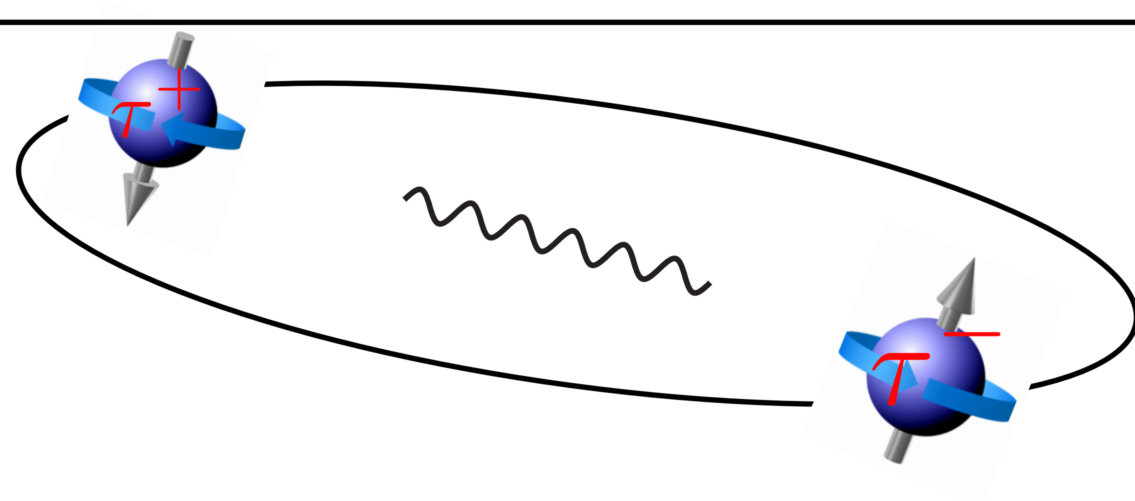
True Tauonium



Hua-Sheng Shao

w/ David d'Enterria (arXiv:2202.02316, arXiv:2302.07365)

+ Redamy Perez-Ramos (arXiv:2204.07269)

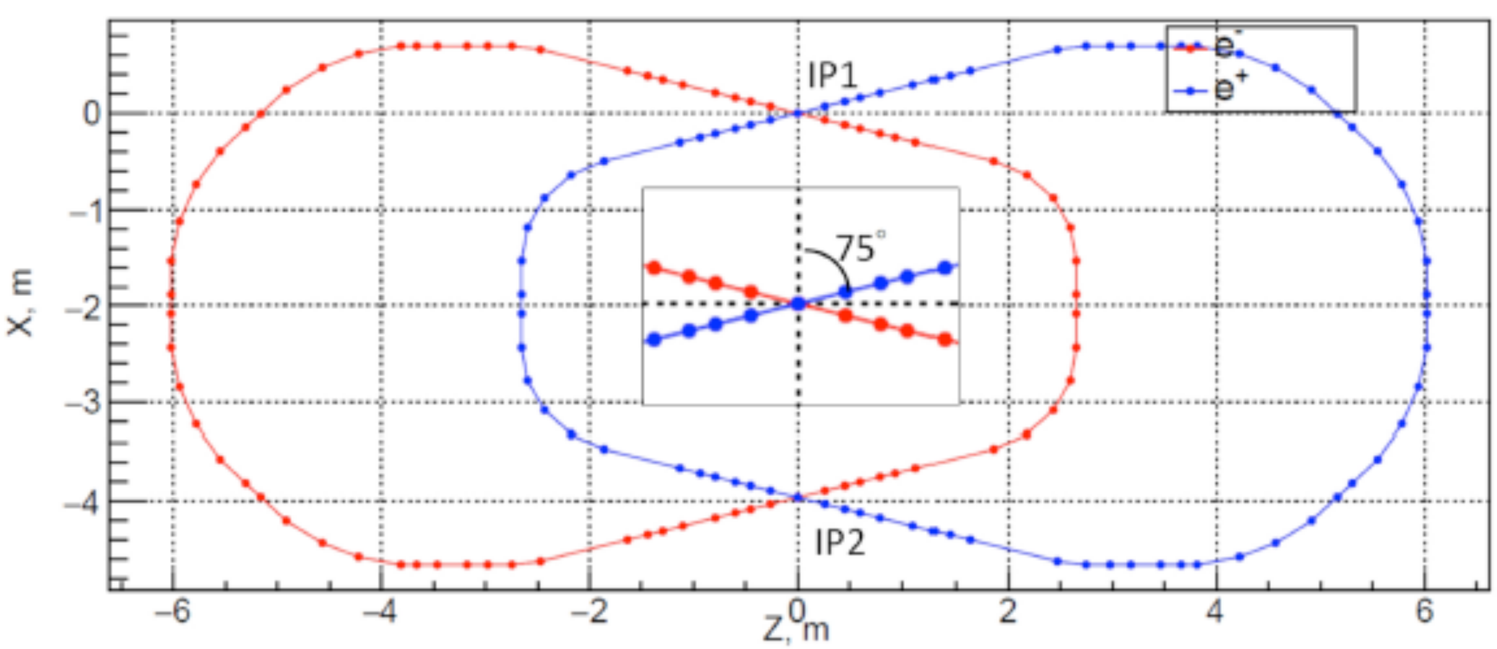
Workshop on Tau Physics at Belle II and STCF

01 September 2026

Leptonium: QED atoms

- **Opposite-charge leptons can form onium bound states**
  - Analogous to the hydrogen atom (a proton and an electron)
  - Pure QED interaction (unlike hydrogen atom)

    ⇒ *simplest atoms allow high-precision theory control*
  - Out of 6 possible combinations, only positronium (e^+e^-) and muonium ($\mu^\pm e^\mp$) have been observed experimentally
  - True muonium ($\mu^+\mu^-$) has been proposed to look for in the dedicated e+e- machines with the intersecting storage rings (Crab Waist)



Novosibirsk (1708.05819), Fermilab DIMUS (2203.07144)

... + other proposals:

η meson decay at LHCb

Cid Vidal et al. (PRD'19)

Fixed-target experiments

Lamm and Ji. (1712.03429)

e+e- *with standard crossing angle*

Gargiulo et al. (J. Phys. G'24)

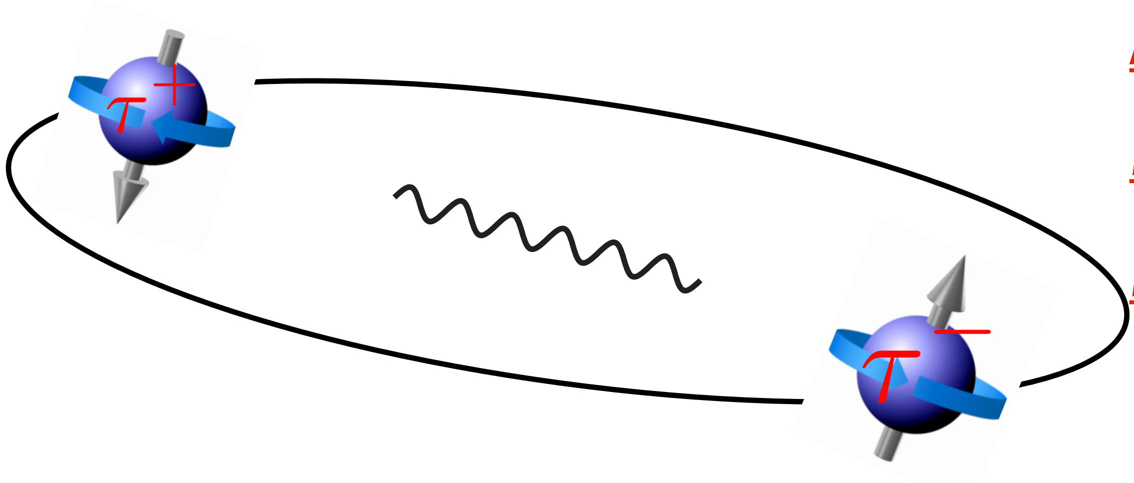
Main background: pions/electrons/photons/muon pair

Leptonium: QED atoms

- **Opposite-charge leptons can form onium bound states**
  - Unlike electron and muon, tau lepton is heaviest and also has a shortest lifetime

Type	Name	Symbol	Mass (MeV)	Mean lifetime
Lepton	Electron / Positron	e^- / e^+	0.511	$> 4.6 \times 10^{26}$ years
	Muon / Antimuon	μ^- / μ^+	105.7	2.2×10^{-6} seconds
	Tau lepton / Antitau	τ^- / τ^+	1777	2.9×10^{-13} seconds

  - Both $(\tau^{\pm}e^{\mp})$ and $(\tau^{\pm}\mu^{\mp})$ cannot form real bound states before tau decays, though some bound-state effects may exist
  - True tauonium $(\tau^+\tau^-)$ is the heaviest and most compact leptonium, which can undergo annihilation decays



Mass: 3553.696 MeV

Bohr radius : 30.4 fm (x3500 smaller than positronium)

Rydberg const (γ ionisation) : 3.76 keV

Why studying true tauonium ?

- **The properties of the tau lepton**
  - Precise tau mass determination *(will show you later, promising)*
  - Precise tau width determination *(14-20% via tau decays, not competitive)*
  - Anomalous magnetic dipole moment *(from hfs, challenging, or xs, not competitive)*
- **Precision tests of the Standard Model of particle physics**
  - Precise tests of QED — Karshenboim (Phys. Rept. 05)
  - Basic symmetry test (like CPT as done in positronium but at GeV scale)

    Bernreuther et al. (Z. Phys. C 88), Yamazaki et al. (PRL'10)
- **Direct or indirect sensitives to the BSM effects**
  - BSM effect is enhanced via $\mathcal{O}(m_\ell/\Lambda_{\rm BSM})$
  - Lepton flavour universality violation *(positronium vs dimuonium vs ditauonium)*
  - Tau-philic BSM interactions and particles without missing energy
  - …

A quotation from Steven Weinberg

- **The first (and probably the most) important step is to discovery it at experiments**

STEVEN WEINBERG (1977)

The Search for Unity: Notes for a History of Quantum Field Theory

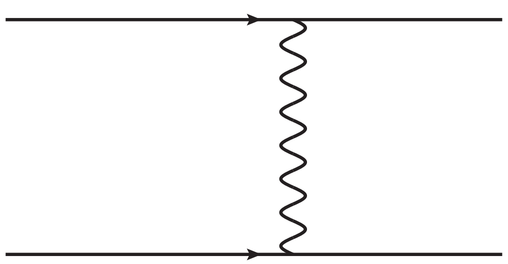
Why was quantum field theory not taken more seriously? One reason is the tremendous prestige of the Dirac theory[24] of 1928, which had worked so well in accounting for the fine structure of the hydrogen spectrum without including self-energy effects. Even more important, the appearance of infinities discredited quantum field theory altogether in many physicists' minds. But I think that the deepest reason is a psychological difficulty, that may not have been sufficiently appreciated by historians of science.[57] There is a huge apparent distance between the equations that theorists play with at their desks, and the practical reality of atomic spectra and collision processes. It takes a certain courage to bridge this gap, and to realize that the products of thought and mathematics may actually have something to do with the real world. Of course, when a branch of science is well under way, there is continual give and take between theory and experiment, and one gets used to the idea that the theory is about something real. Without the pressure of experimental data, the realization comes harder. The great thing accomplished by the discovery of the Lamb shift was not so much that it forced us to change our physical theories, as that it forced us to take them seriously.

Energy levels of true tauonium

- **The masses of the states** d'Enterria, Perez-Ramos, HSS (EPJC'22)
  - Determined within non-relativistic quantum mechanics

$n = 3\ (E = -2.6\ \mathrm{keV})$

$n = 2\ (E = -5.9\ \mathrm{keV})$



$$V(r) = -\frac{\alpha}{r}$$

$n = 1\ (E = -23.6\ \mathrm{keV})$

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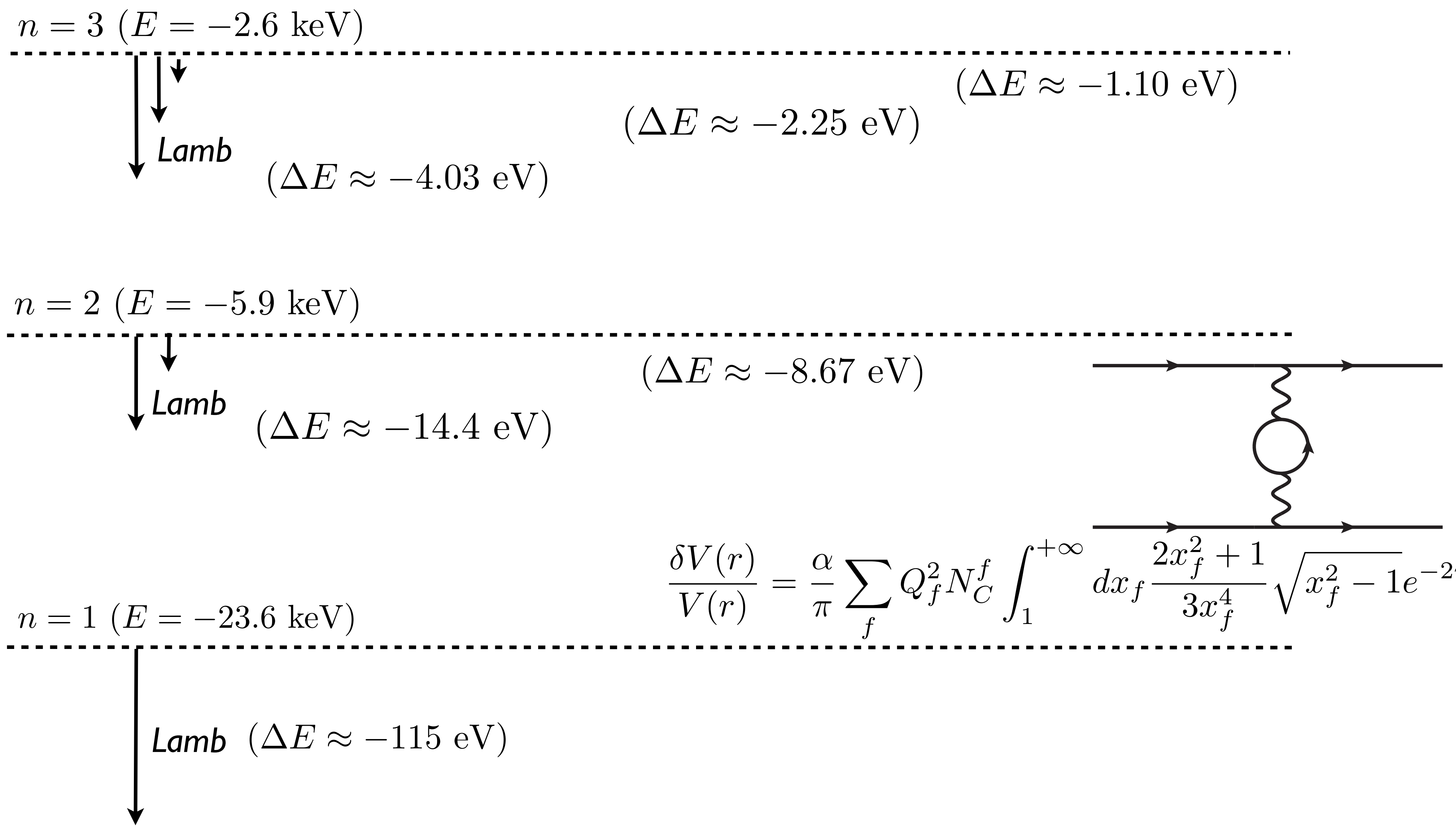

$$\frac{\delta V(r)}{V(r)} = \frac{\alpha}{\pi}\sum_f Q_f^2 N_C^f \int_1^{+\infty} dx_f \frac{2x_f^2+1}{3x_f^4}\sqrt{x_f^2-1}e^{-2m_f x_f r}$$

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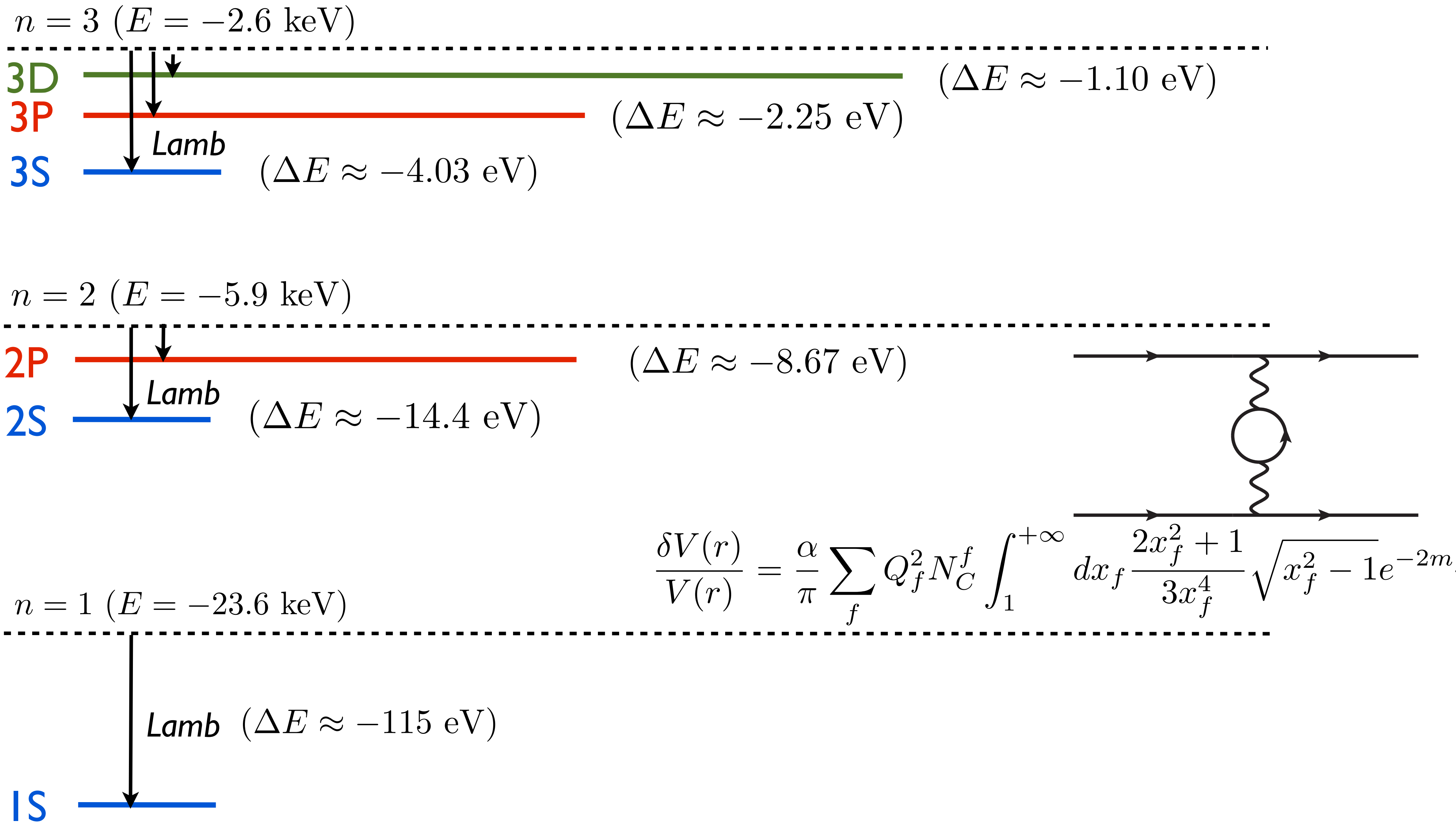

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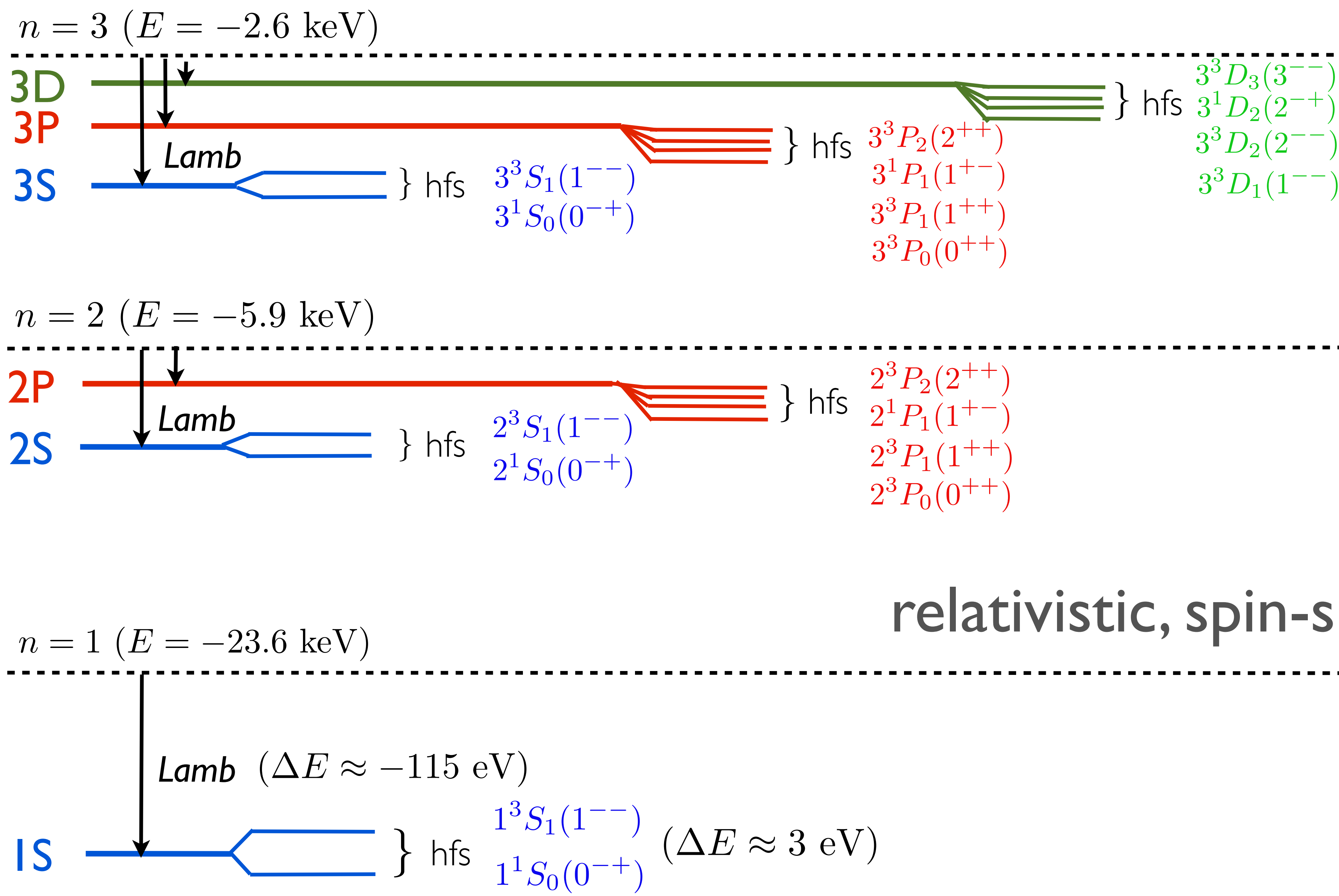


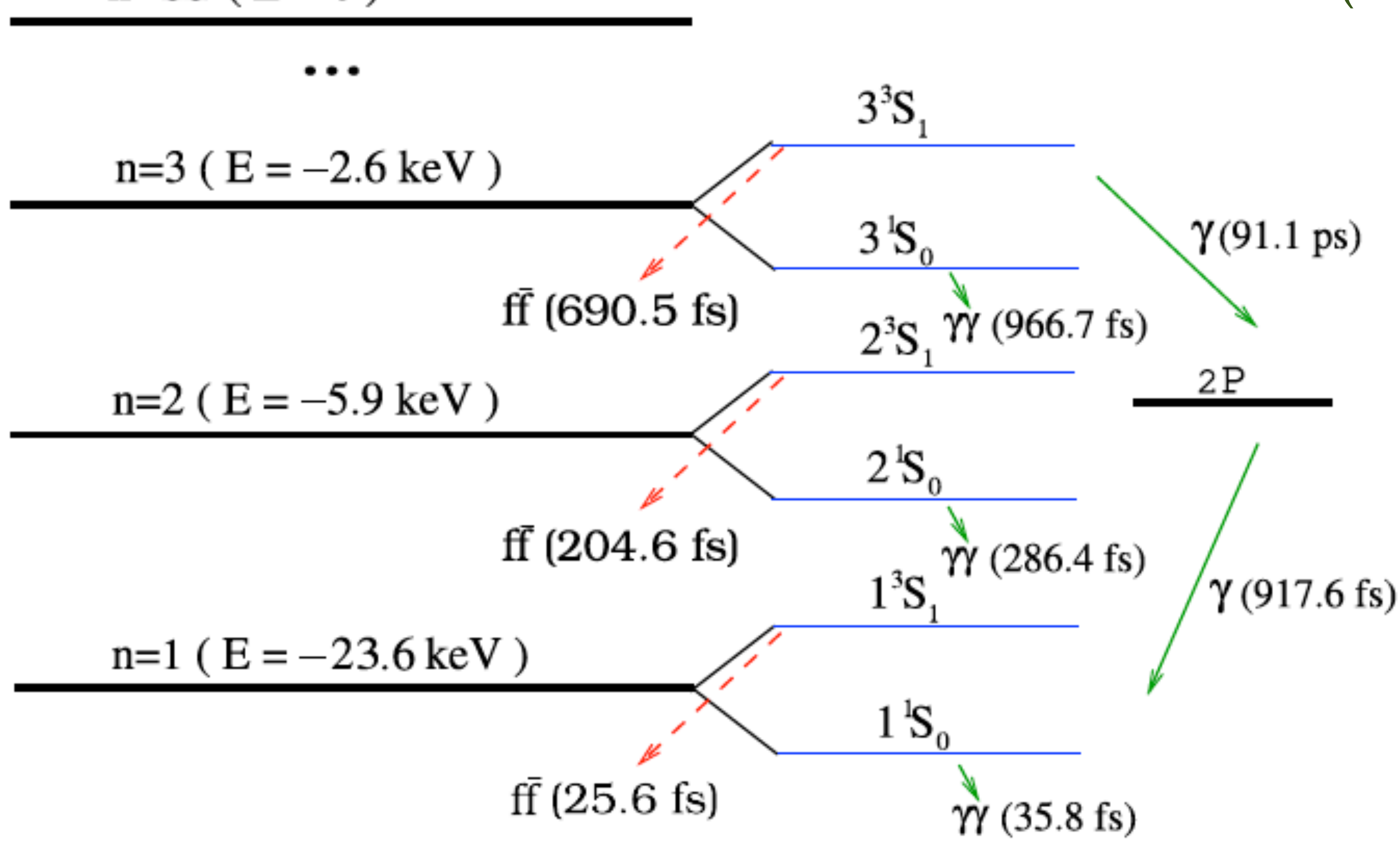
relativistic, spin-spin, LS, ...

Decay channels of true tauonium

- **Which are real bound states ?** d'Enterria, Perez-Ramos, HSS (EPJC'22)
  - Only the annihilation decays of the ground states (n=1) are faster than the weak decays of each constitute tau (290 fs)
    ⇨ *We will only focus on n=1 states*
  - Excited states will impact the di-tau cross section at threshold

Fu et al. (Sci.Bull.'24)

Leading order



Decay channels of true tauonium

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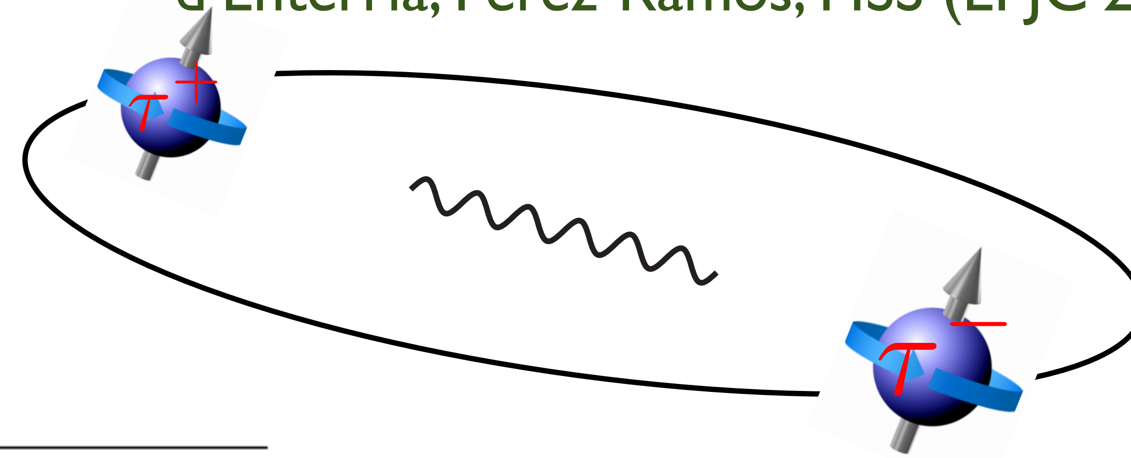
- **The spin-triplet state (ortho-true tauonium $\mathcal{T}_1$)**

d'Enterria, Perez-Ramos, HSS (EPJC'22)

$$J^{PC} = 1^{--}$$

- Mainly to di-fermions
- O(10^{-4}) four-fermion decay
- 14% tau weak decay



$$|\uparrow\uparrow\rangle$$

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

$$|\downarrow\downarrow\rangle$$

$\mathcal{T}$ state	$m_{\mathcal{X}}$ (MeV)	J^{PC}	$\Gamma_{\rm tot}$ (eV)	Lifetime (fs)	Decay mode	Γ_X (eV)	$\mathcal{B}_X$
1^3S_1	3553.696 ± 0.240	1^{--}	0.03159	20.83	$e^+e^-(\gamma)$	0.006436	20.37%
					∘ e^+e^-	$2.95 \cdot 10^{-3}$	9.33%
					∘ $e^+e^-\gamma$	$3.49 \cdot 10^{-3}$	11.04%
					$\mu^+\mu^-(\gamma)$	0.006436	20.37%
					∘ $\mu^+\mu^-$	$6.10 \cdot 10^{-3}$	19.30%
					∘ $\mu^+\mu^-\gamma$	$3.38 \cdot 10^{-4}$	1.07%
					$q\bar{q}(\gamma)$	0.01416	44.82%
					$\gamma\gamma\gamma$	$1.62 \cdot 10^{-5}$	0.051%
					$e^+e^-e^+e^-$	$5.55 \cdot 10^{-6}$	0.0176%
					$e^+e^-\mu^+\mu^-$	$4.21 \cdot 10^{-6}$	0.0133%
					$e^+e^-q\bar{q}$	$1.85 \cdot 10^{-6}$	0.0058%
					$\mu^+\mu^-\mu^+\mu^-$	$1.23 \cdot 10^{-7}$	$\mathcal{O}(10^{-6})$
					$\mu^+\mu^-q\bar{q}$	$7.36 \cdot 10^{-8}$	$\mathcal{O}(10^{-6})$
					$q\bar{q}q'\bar{q}'$	$9.73 \cdot 10^{-9}$	$\mathcal{O}(10^{-7})$
					$\nu_\tau\bar{\nu}_\tau$	$1.32 \cdot 10^{-8}$	$\mathcal{O}(10^{-7})$
					$\nu_e\bar{\nu}_e$	$4.30 \cdot 10^{-11}$	$\mathcal{O}(10^{-9})$
					$\nu_\mu\bar{\nu}_\mu$	$4.30 \cdot 10^{-11}$	$\mathcal{O}(10^{-9})$
					$(2)\tau \to X$	0.004535	14.35%

Decay channels of true tauonium

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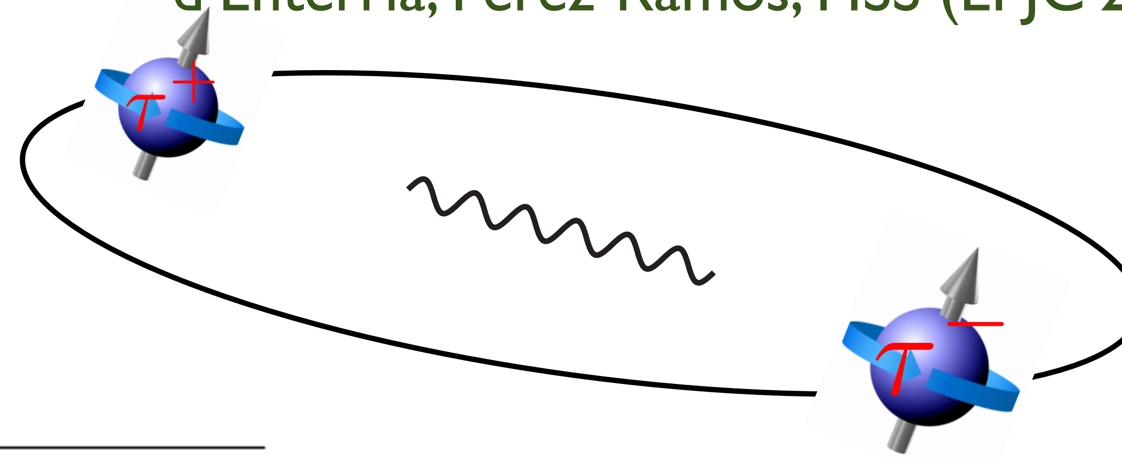

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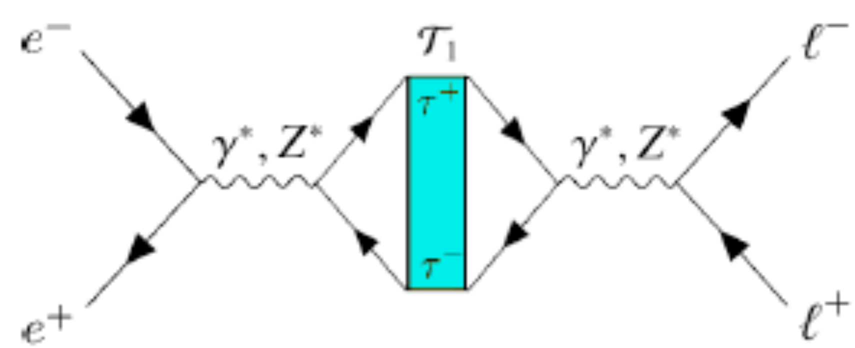
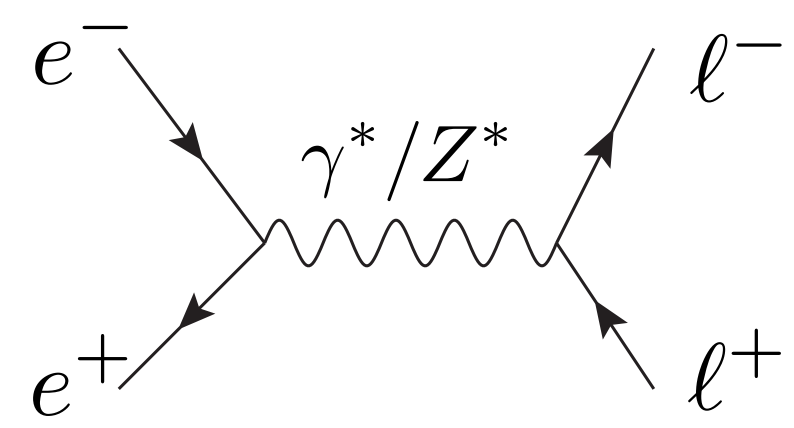
- **The spin-triplet state (ortho-true tauonium $\mathcal{T}_1$)**
  - Threshold scan at e+e-

d'Enterria, HSS (PLB'23)

Signal



Background



$$\sigma^{\text{ideal}}(e^+e^- \to \mathcal{T}_1) = \frac{12\pi\Gamma_{\text{tot}}(\mathcal{T}_1)\,\Gamma_{e^+e^-}(\mathcal{T}_1)}{(s-m_{\mathcal{T}}^2)^2+\Gamma_{\text{tot}}^2(\mathcal{T}_1)\,m_{\mathcal{T}}^2} \overset{\sqrt{s}=m_{\mathcal{T}}}{=} 236.6\ \mu\text{b}$$

⇩

Huge cross section ! Why not observed yet ?

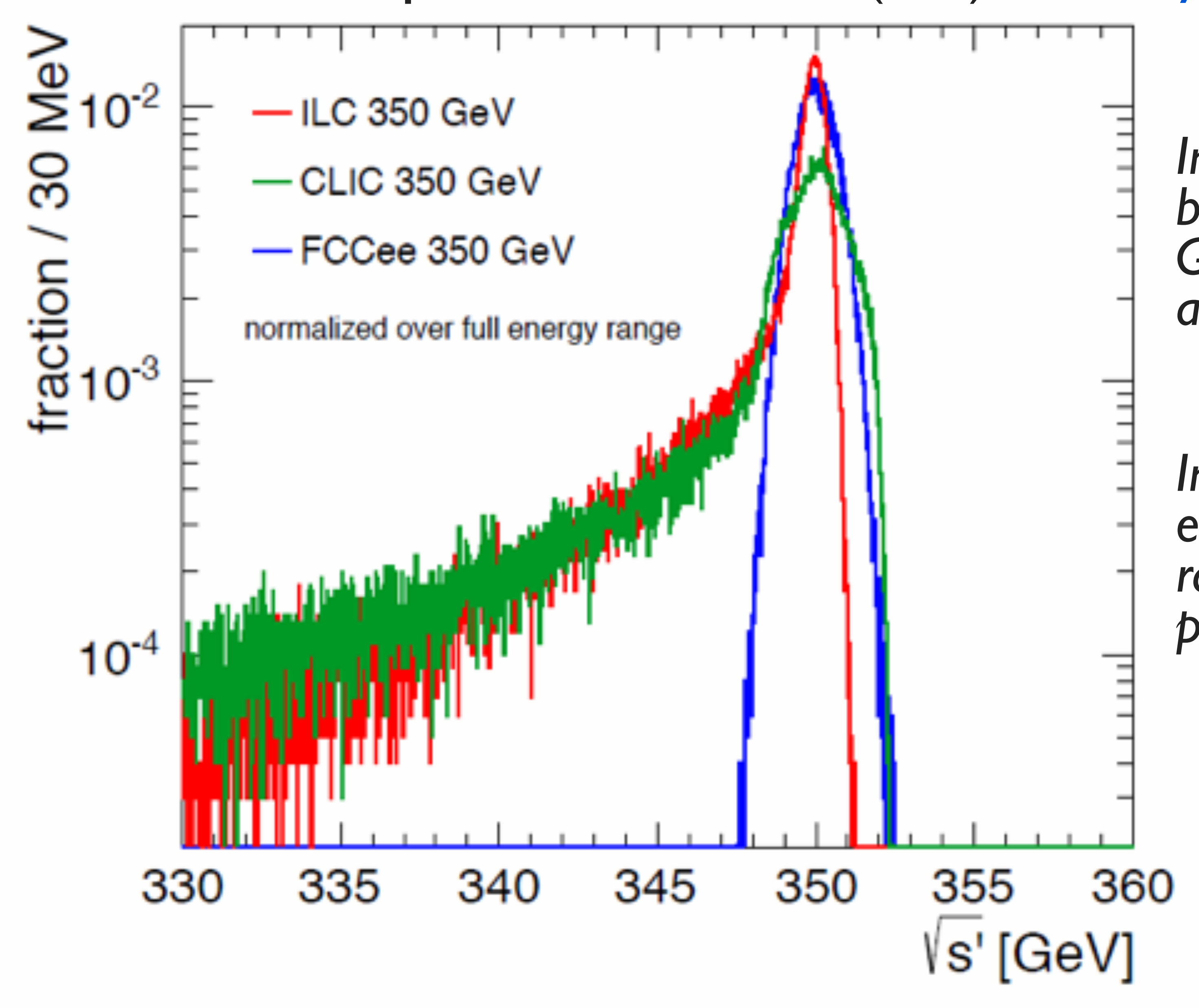
How to observe true tauonium ?

- **The spin-triplet state (ortho-true tauonium $\mathcal{T}_1$)**
  - Two important effects are missing ...

d'Enterria, HSS (PLB'23)

Monochromatization of beam energies

Initial-state photon radiation (ISR) *-> always lower center-of-mass energy*



In the high-energy circular collider, beamstrahlung causes an additonal Gaussian energy spread because particles are trapped on synchortron oscillation

In the low-energy circular collider, relative energy spread is mainly due to synchrotron radiation emitted when an ultra-relativistic particle passes through a bending magnet

How to observe true tauonium ?

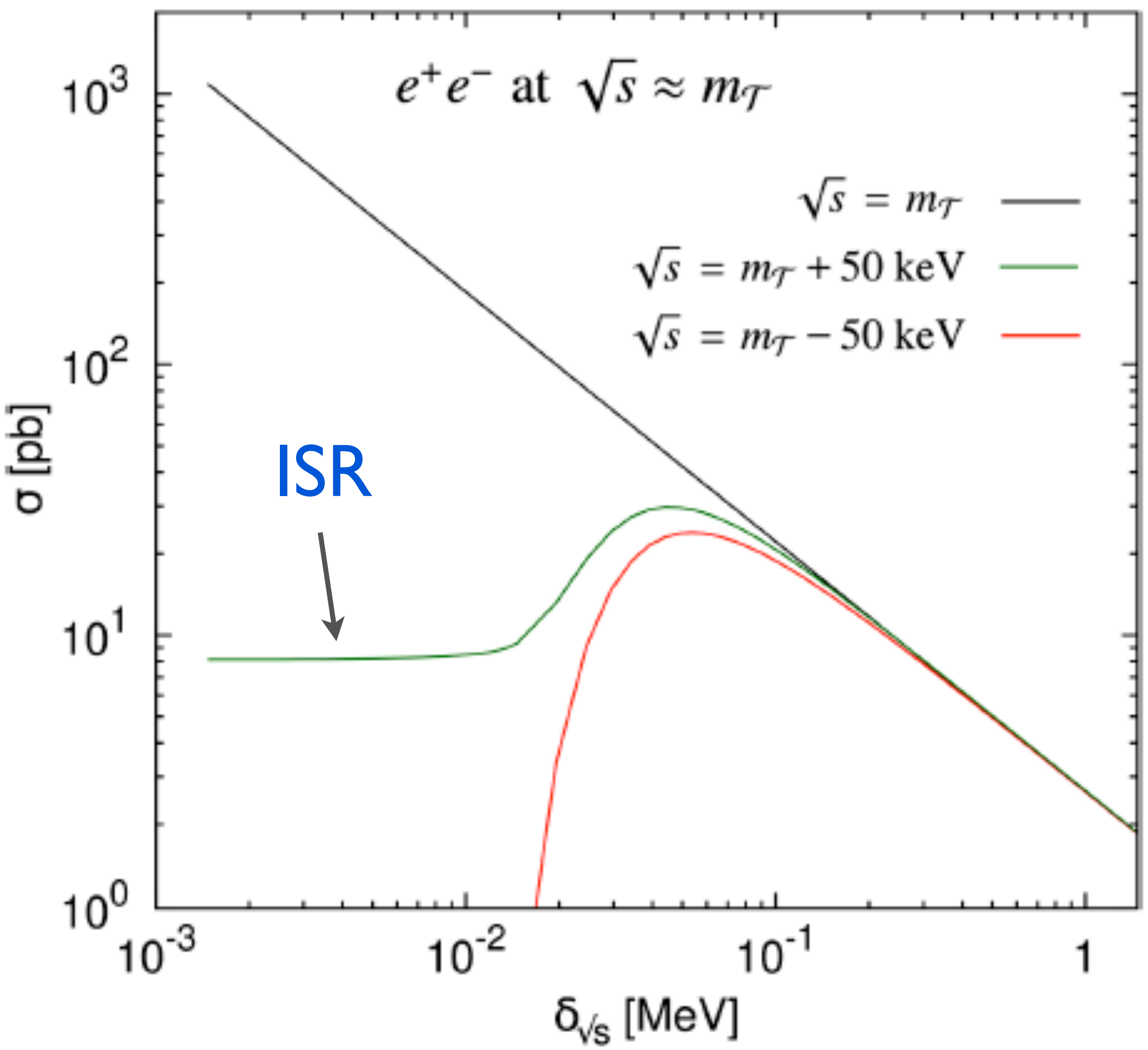
- **The spin-triplet state (ortho-true tauonium $\mathcal{T}_1$)**
  - Threshold scan at e+e-

d'Enterria, HSS (PLB'23)

Voigtian function

$$\sigma^{\rm actual}(e^+e^- \to \mathcal{T}_1) = \frac{12\pi^2\Gamma_{e^+e^-}(\mathcal{T}_1)}{m_{\mathcal{T}}}\int_0^1 dx_1\int_0^1 dx_2 f_{e^-/e^-}(x_1,s)f_{e^+/e^+}(x_2,s)V_2\left(\sqrt{x_1x_2s};m_{\mathcal{T}},\Gamma_{\rm tot}(\mathcal{T}_1),\sqrt{x_1x_2}\delta_{\sqrt{s}}\right)$$

ISR   energy spread



BEPC II / BESIII

How to observe true tauonium ?

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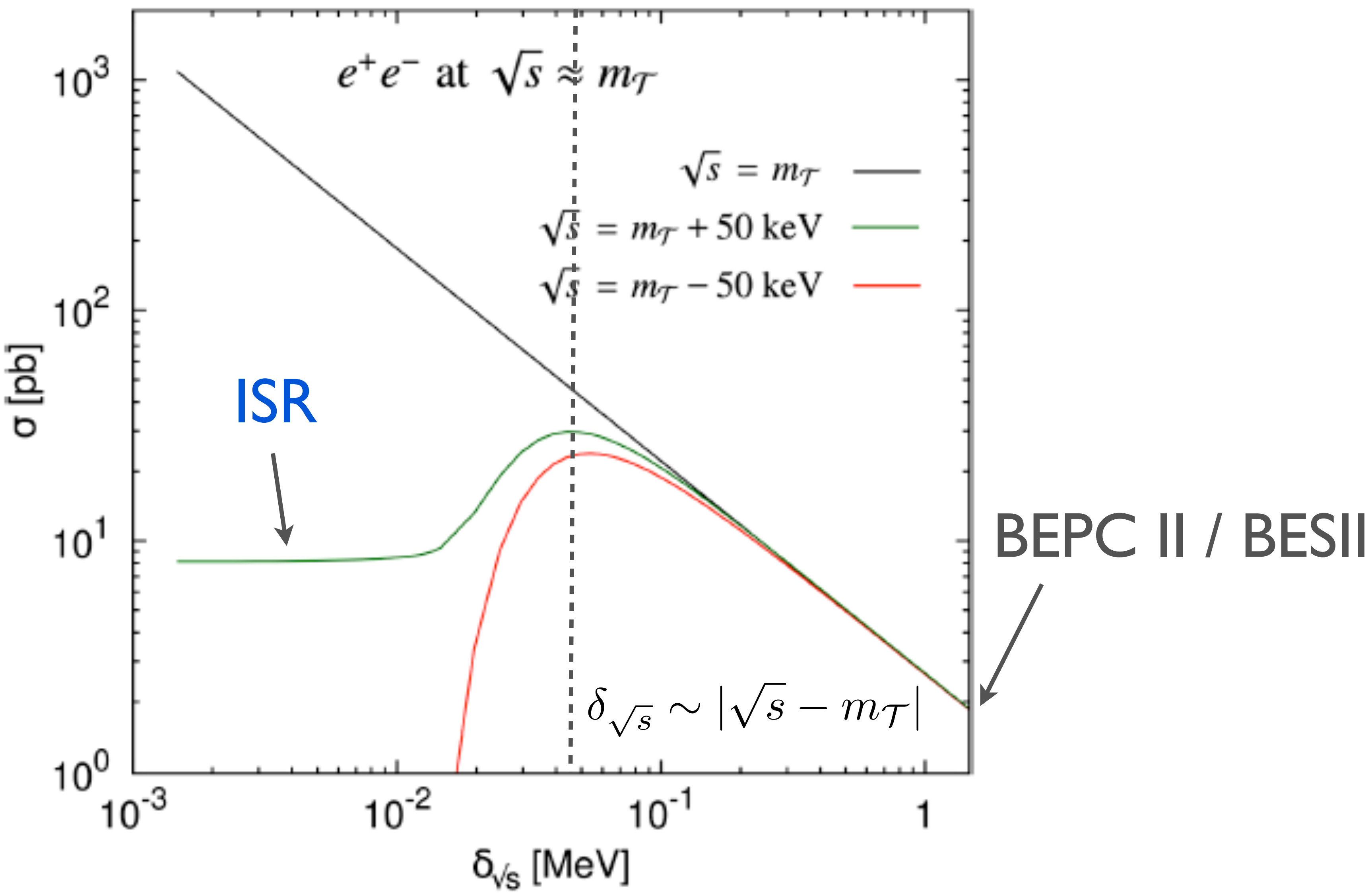
ISR — energy spread



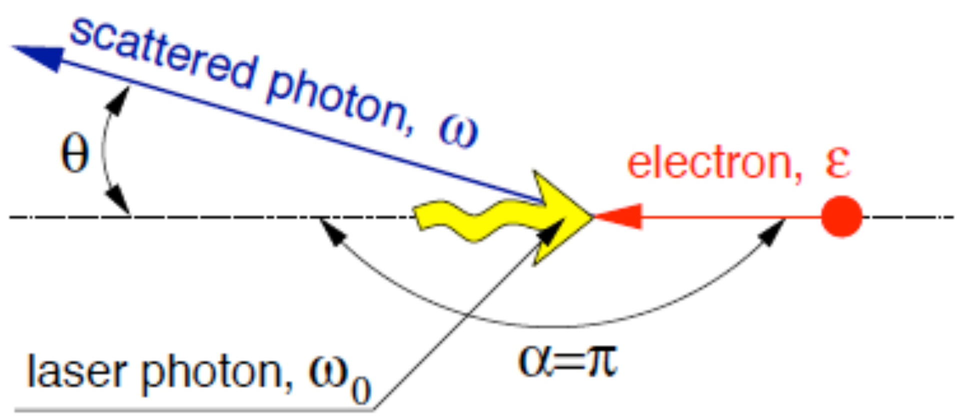
How to observe true tauonium ?

- **The spin-triplet state (ortho-true tauonium $\mathcal{T}_1$)**
  - Larger signal yields requires:

d'Enterria, HSS (PLB'23)

The good accuracy of $\sqrt{s}$ (as close as possible to $m_{\mathcal{T}}$)

Beam energy measurement system at BEPC II allows 2×10^{-5} accuracy



Abakumova et al. (1109.5771)

The applications of the same technique at three different lepton colliders can reach accuracies of the actual c.m. energy not worse than 50 keV. Achasov et al. (JINST'20)

E.g. BEPC II+VEPP-2000

The good precision of $\sqrt{s}$ (as small as possible of $\delta_{\sqrt{s}}$)

Monochromatization has been studied in the literature since Renieri in 1975 for ADONE.

Active studies in the FCC-ee community (see a recent talk by A. Faus-Golfe)

Possible to reach $\delta_{\sqrt{s}} = 50\ \mathrm{keV}$, but the price to pay is the reduction of the integrated luminosity

How to observe true tauonium ?

- **The spin-triplet state (ortho-true tauonium $\mathcal{T}_1$)**
  - Threshold scan at e+e-

d'Enterria, HSS (PLB'23)

Colliding system, $\sqrt{s}$ ($\delta_{\sqrt{s}}$ spread), $\mathcal{L}_{\rm int}$, experiment	σ			N			$S/\sqrt{B}$
	$\mathcal{T}_1$	$\tau^+\tau^-$	$\mu^+\mu^-$	$\mathcal{T}_1$	$\mathcal{T}_1 \to \mu^+\mu^-$	$\mu^+\mu^-$	
e^+e^- at 3.5538 GeV (1.47 MeV), 5.57 pb^{-1}, BES III	1.9 pb	117 pb	6.88 nb	10.4	2.1	38 300	0.01σ
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- Ortho-true tauonium can be observed at STCF without monochromatization

How to observe true tauonium ?

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- Ortho-true tauonium can be observed at STCF without monochromatization
- With monochromatization to $\delta_{\sqrt{s}} = 100\ \mathrm{keV}$ and 10 times smaller luminosity, we can use true tauonium to precisely determine the mass of the tau lepton

How to observe true tauonium ?

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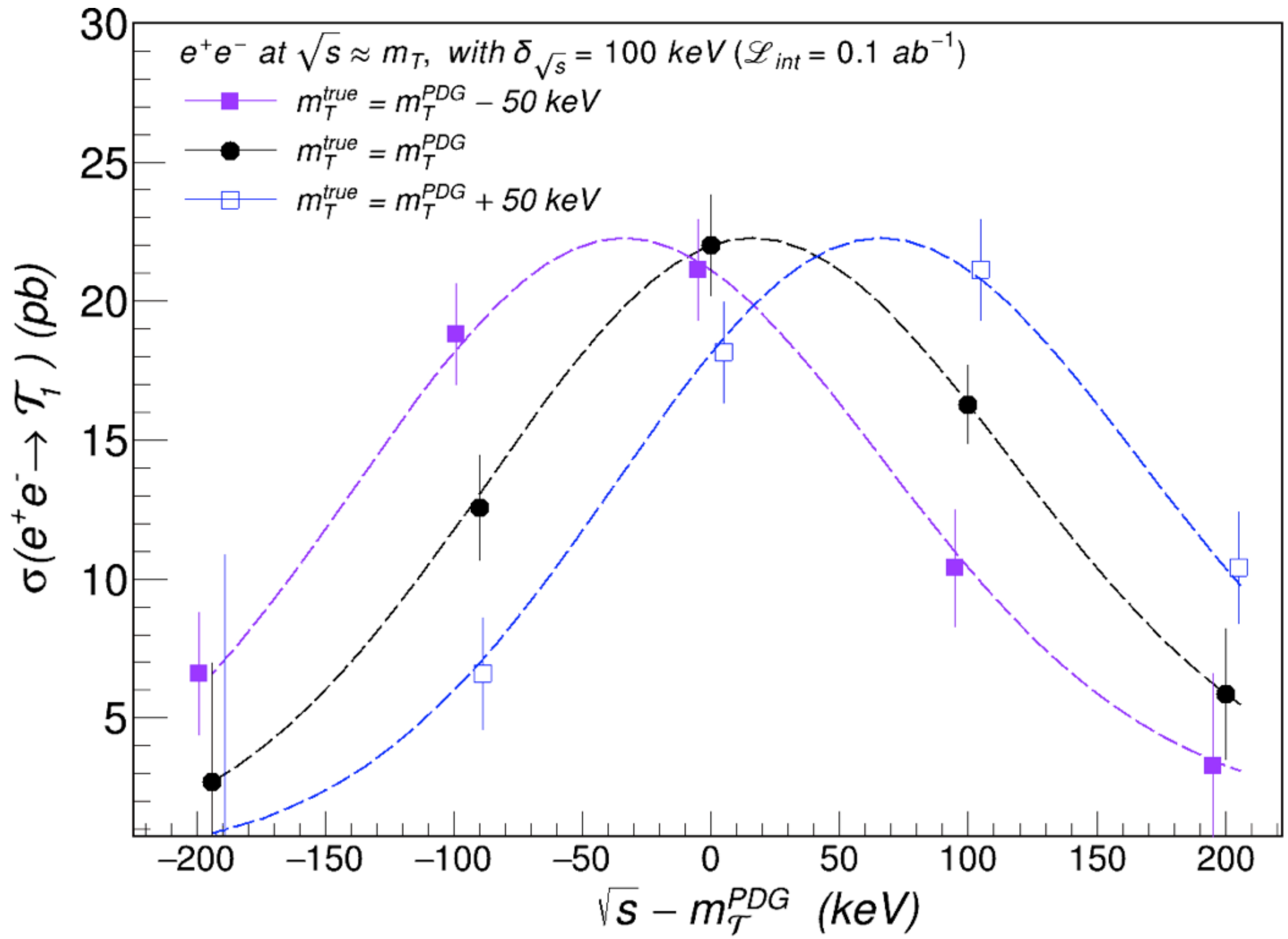
- With monochromatization to $\delta_{\sqrt{s}} = 100$ keV and 10 times smaller luminosity, we can use true tauonium to precisely determine the mass of the tau lepton

- True tauonium contributes 2% (50%) of the di-tau total cross section w/o (w/) beam energy monochromatization.

How to observe true tauonium ?

- **The spin-triplet state (ortho-true tauonium $\mathcal{T}_1$)**
  - Threshold scan at e+e- — d'Enterria, HSS (PLB'23)
    - As a bonus of the true tauonium observation, we can also precisely determine the mass of the tau lepton with monochromatization

$$m_\tau = (m_{\mathcal{T}} - E_{\rm bind})/2$$



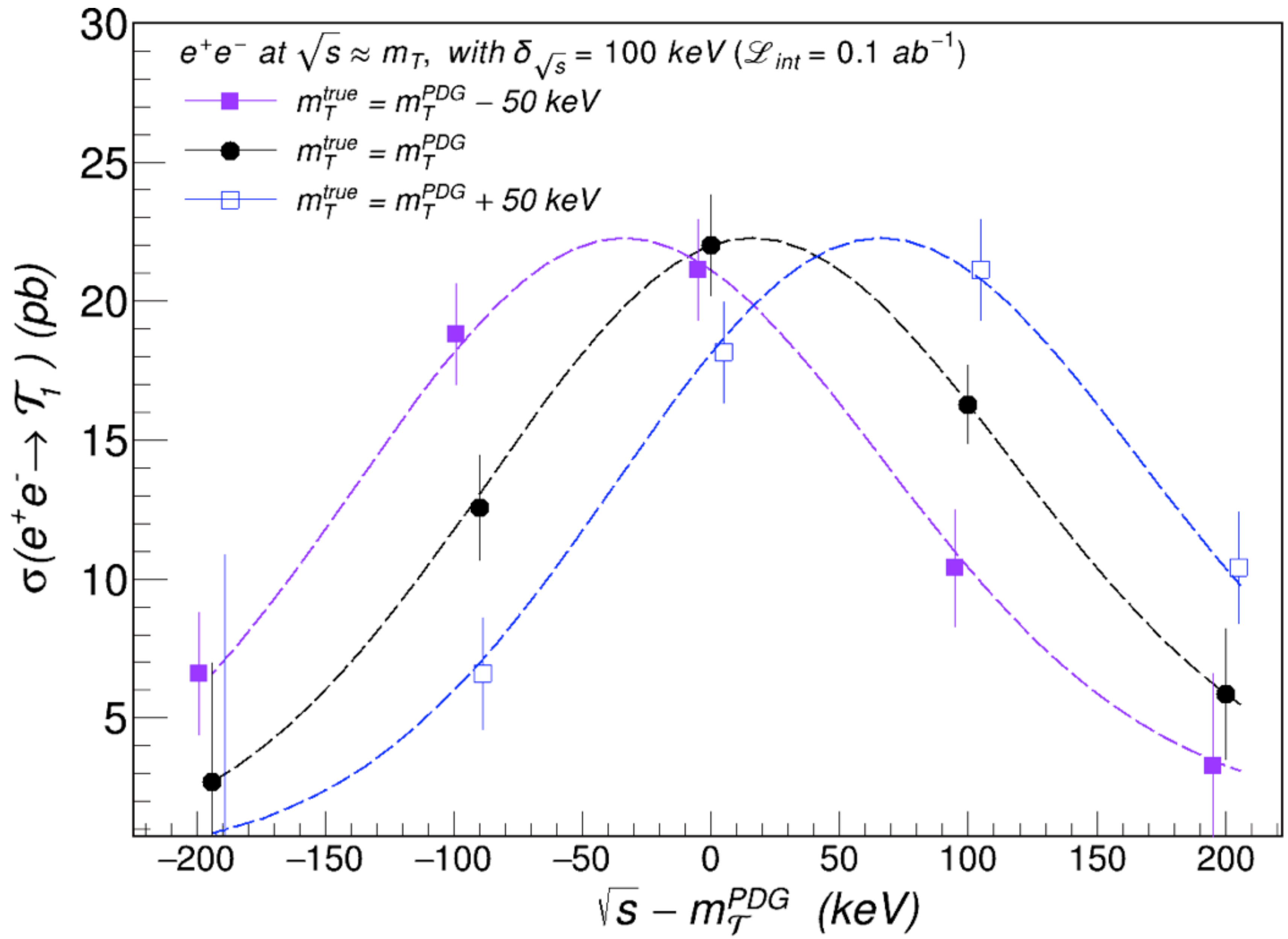
How to observe true tauonium ?

- **The spin-triplet state (ortho-true tauonium $\mathcal{T}_1$)**
  - Threshold scan at e+e-

d'Enterria, HSS (PLB'23)

    - As a bonus of the true tauonium observation, we can also precisely determine the mass of the tau lepton with monochromatization

$$m_\tau = (m_\mathcal{T} - E_{\text{bind}})/2 \longrightarrow \delta m_\tau = 25 \text{ keV}$$



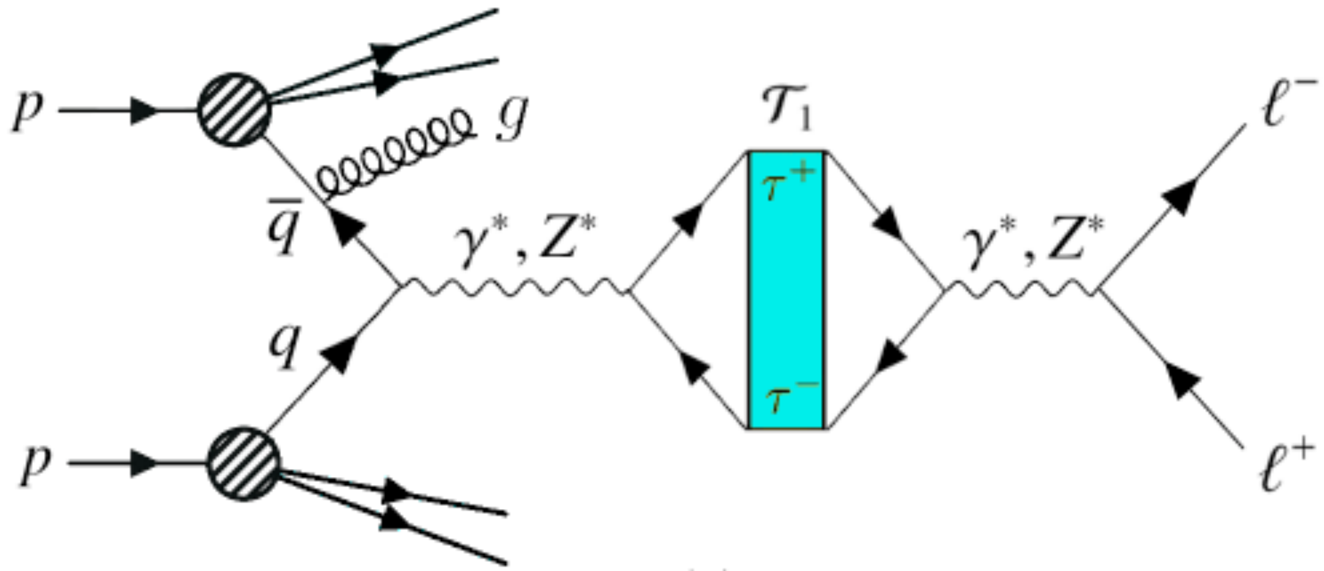
How to observe true tauonium ?

- **The spin-triplet state (ortho-true tauonium $\mathcal{T}_1$)**
  - Exploring other possibilities

d'Enterria, HSS (PLB'23)

$$pp \to \mathcal{T}_1 + j \text{ @ LHC}$$

Using displaced vertex to kill huge backgrounds



Remaining challenge: combinatorial backgrounds (mainly from heavy quark decays)

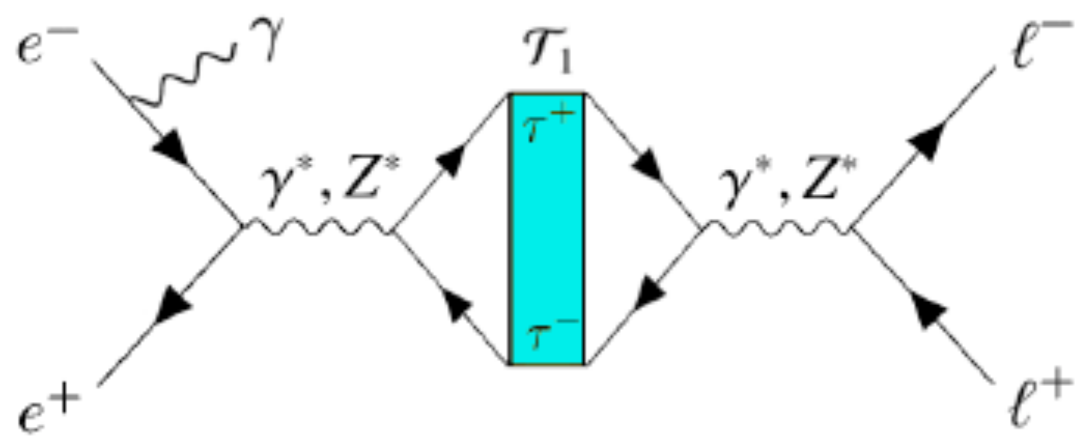
Colliding system, $\sqrt{s}$, $\mathcal{L}_{\text{int}}$, detector	σ_{NLO}		$N(\mathcal{T}_1 + j)$		with $L_{xy} > 30\ (100)\ \mu$m	
	$\mathcal{T}_1 + X$	$\mathcal{T}_1 + j$	$\mathcal{T}_1 \to e^+e^-$	$\mathcal{T}_1 \to \mu^+\mu^-$	$\mathcal{T}_1 \to \ell^+\ell^-$	$\mathcal{T}_1 \to \mu^+\mu^-$
p-p at 14 TeV, 3 ab^{-1}, ATLAS/CMS	42^{+11}_{-19} fb	18 ± 9 fb	1100	1100	130 (10)	130 (10)
p-p at 14 TeV, 300 fb^{-1}, LHCb	42^{+11}_{-19} fb	18 ± 9 fb	110	110	5 (–)	5 (–)
p-p at 114.6 GeV, 10 fb^{-1}, ALICE/LHCb	$2.2^{+0.3}_{-0.4}$ fb	1 ± 0.5 fb	<10	<10	–	–

How to observe true tauonium ?

- **The spin-triplet state (ortho-true tauonium $\mathcal{T}_1$)**
  - Exploring other possibilities

d'Enterria, HSS (PLB'23)

$$e^+e^- \to \mathcal{T}_1 + \gamma \text{ @ Belle II}$$



Using displaced vertex to kill huge backgrounds

If L_{xy} > 15 (20) μm, the total expected number of events would be 8 (4).

Remaining challenge: instrumental effects.

$$10^\circ < \theta^{\rm lab}_{\ell^\pm,\gamma} < 170^\circ \qquad m_{\ell^+\ell^-} \in (m_{\mathcal{T}} \pm 5 \text{ MeV})$$

Colliding system, $\sqrt{s}$, $\mathcal{L}_{\rm int}$, detector	σ			$N(\mathcal{T}_1+\gamma)$		with L_{xy} > 30 μm	
	$\mathcal{T}_1+\gamma$	$\mu^+\mu^-+\gamma$	$e^+e^-+\gamma$	$\mu^+\mu^-$	e^+e^-	$\mu^+\mu^-$	e^+e^-
e^+e^- at 10.6 GeV, 50 ab^{-1}, Belle II	7.1 ab	51 fb	1.9 pb	73	73	0.5	0.5

How to observe true tauonium ?

- **The spin-triplet state (ortho-true tauonium $\mathcal{T}_1$)**
  - Exploring other possibilities

    The rest are hopeless either plagued with huge backgrounds or too-few signal events ! d'Enterria, HSS (PLB'23)

  - Rare decays ?
    - B decay Fael & Mannel (NPB'18)

      $\mathrm{Br}(B \to K\mathcal{T}_1) = 1.4 \times 10^{-13}$ $\mathrm{Br}(B \to K^*\mathcal{T}_1) = 3.0 \times 10^{-13}$

    - Z decay d'Enterria, HSS (PLB'23); d'Enterria, Dung Le (J. Phys.G'25)

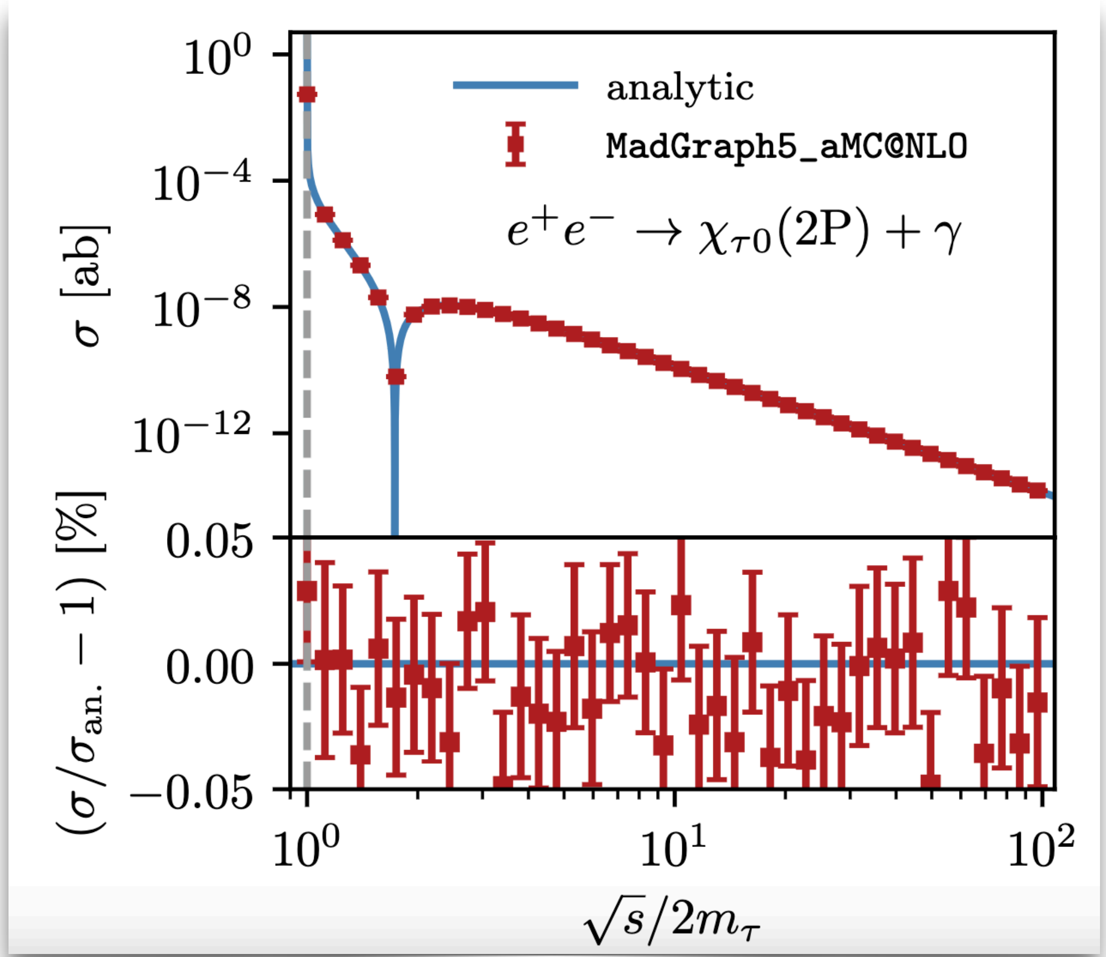
      $\mathrm{Br}(Z \to \gamma\mathcal{T}_1) = 8.9 \times 10^{-14}$

    - H decay d'Enterria, Dung Le (J. Phys. G'25)

      $\mathrm{Br}(H \to \gamma\mathcal{T}_1) = 1.2 \times 10^{-12}$ $\mathrm{Br}(H \to Z\mathcal{T}_1) = 1.4 \times 10^{-11}$

- **MadSONS module in MadGraph5_aMC@NLO**
  - S-wave states — Companies Serri et al. (JHEP'26, 2510.26773)
  - P-wave states — Maxia, HSS, Simon (2607.26739)

```
./bin/mg5_aMC
MG5_aMC> import model sm_onia-lepton_masses
MG5_aMC> generate e+ e- > tata(2|3P0) a
MG5_aMC> output; launch
```



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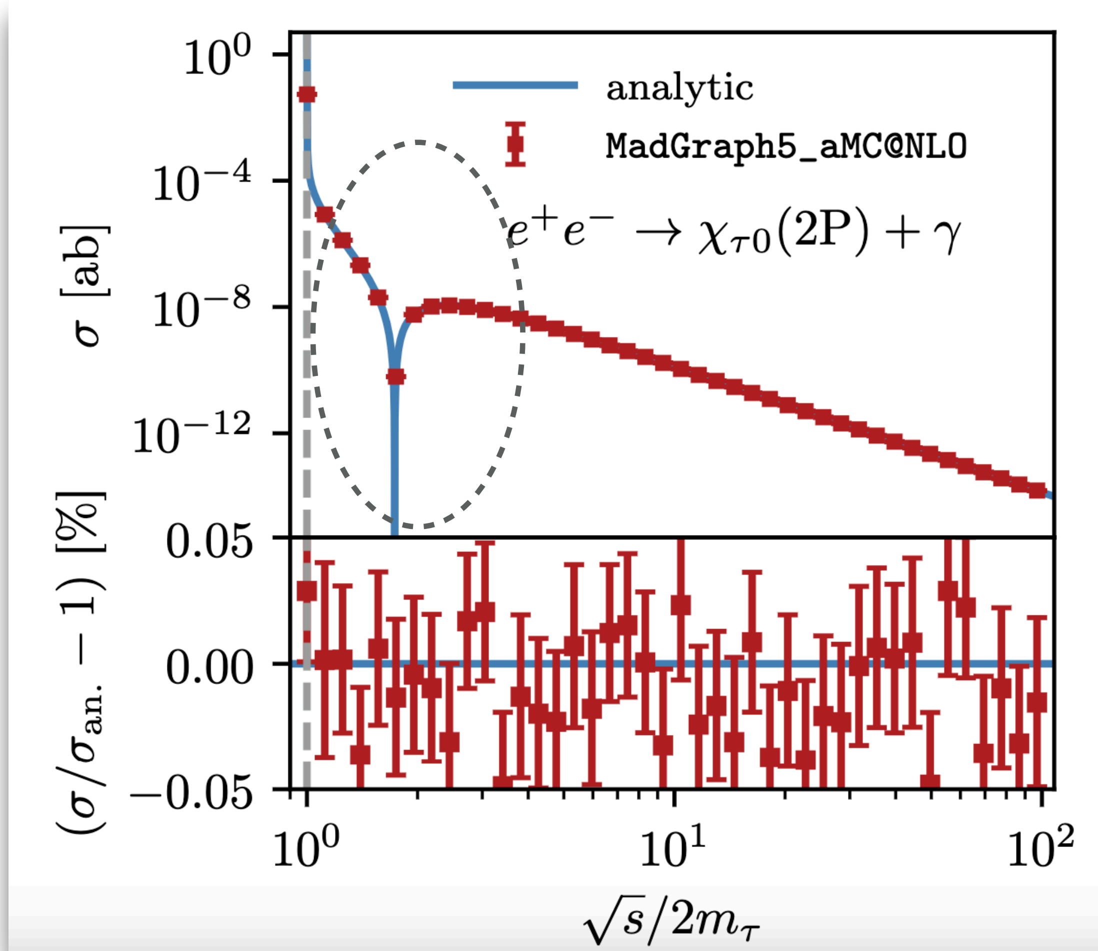

Dip around $\sqrt{s} = \sqrt{3}M_{\chi_{\tau 0}(2\mathrm{P})}$ arises from a dynamical mechanism in which different terms cancel at the amplitude level

It happens in $e^+e^- \to \chi_{c0} + \gamma$, too

This feature should be observed at STCF from energy scan.

Conclusions

- True tauonium can provide an interesting probe to study several aspects of particle physics.
- In QED, we can predict its energy levels and decay channels precisely.
- Its observation feasibilities at colliders have been explored.

$$e^+e^- \to \mathcal{T}_1 \to \mu^+\mu^- \text{ @ STCF}$$

$$pp \to \mathcal{T}_1 j \to \mu^+\mu^- j \text{ @ LHC}$$

$$\gamma\gamma \to \mathcal{T}_0 \to \gamma\gamma \text{ @ FCC} - \text{ee \& Belle II}$$

(See backup slides which I have no time to cover)

- With beam monochromatization, ortho-true tauonium at STCF can in fact allow a precise determination of the mass of tau lepton at O(25 keV)
- Please try MadSONS in MadGraph5_aMC@NLO to test your own ideas about leptonium (and/or quarkonium) !

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Thank you for your attention !

Backup Slides

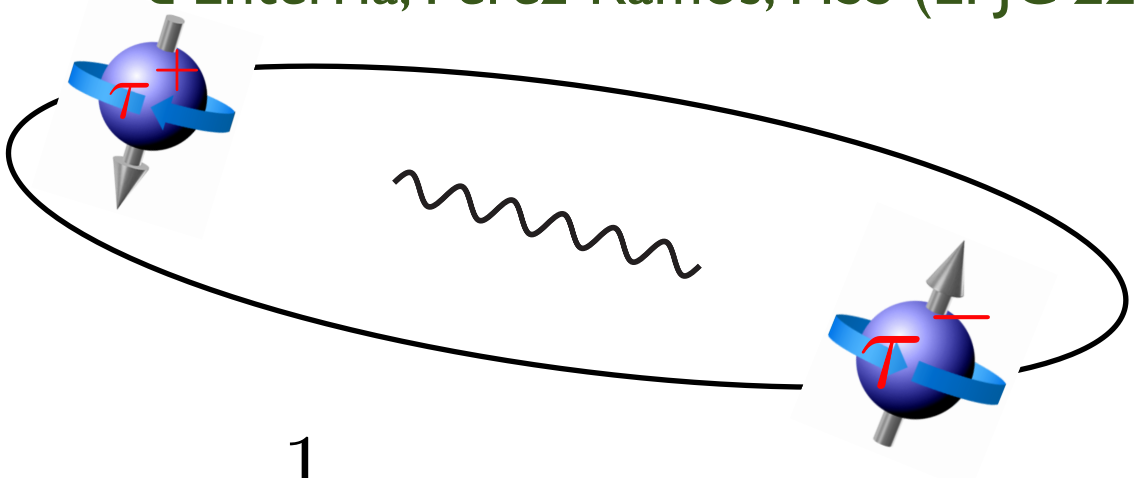
Decay channels of true tauonium

- **The spin-singlet state (para-true tauonium $\mathcal{T}_0$)**

d'Enterria, Perez-Ramos, HSS (EPJC'22)

$J^{PC} = 0^{-+}$

  - Mainly to di-photon
  - Percent-level Dalitz decay
  - O(10-4) double-Dalitz decay
  - 19% tau weak decays



$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$\mathcal{T}$ state	m_X (MeV)	J^{PC}	Γ_{tot} (eV)	Lifetime (fs)	Decay mode	Γ_X (eV)	$\mathcal{B}_X$
1^1S_0	3553.696 ± 0.240	0^{-+}	0.02384	27.60	$\gamma\gamma$	0.018533	77.72%
					γe^+e^-	$4.28 \cdot 10^{-4}$	1.79%
					$\gamma\mu^+\mu^-$	$1.24 \cdot 10^{-4}$	0.52%
					$\gamma q\bar{q}$	$2.20 \cdot 10^{-4}$	0.92%
					$e^+e^-e^+e^-$	$2.32 \cdot 10^{-6}$	0.0094%
					$e^+e^-\mu^+\mu^-$	$1.38 \cdot 10^{-6}$	0.0058%
					$e^+e^-q\bar{q}$	$1.20 \cdot 10^{-6}$	0.0050%
					$\mu^+\mu^-\mu^+\mu^-$	$1.65 \cdot 10^{-7}$	0.00069%
					$\mu^+\mu^-q\bar{q}$	$2.72 \cdot 10^{-7}$	0.0011%
					$q\bar{q}q'\bar{q}'$	$8.23 \cdot 10^{-8}$	0.00035%
					$(2)\tau \to X$	0.004535	19.02%

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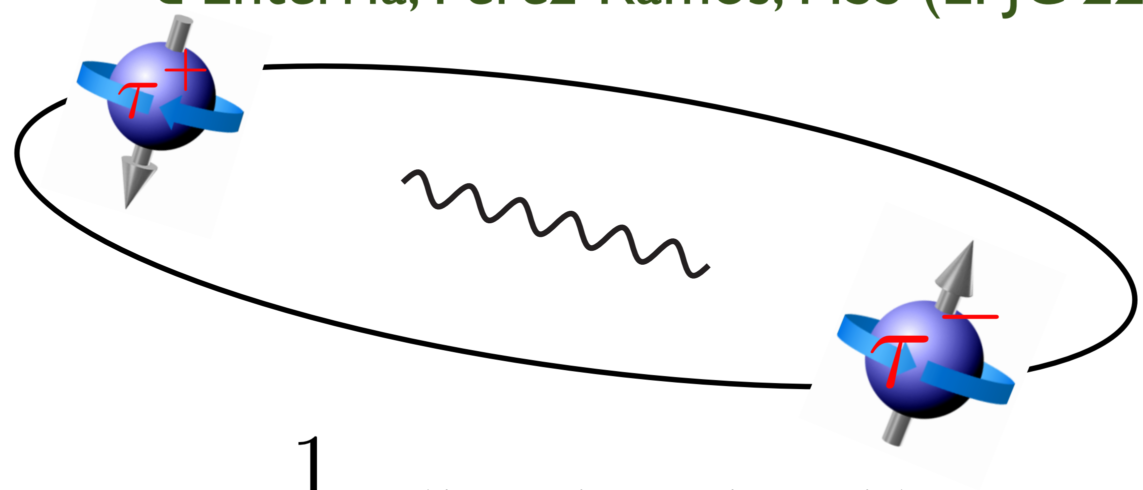

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  - Percent-level Dalitz decay
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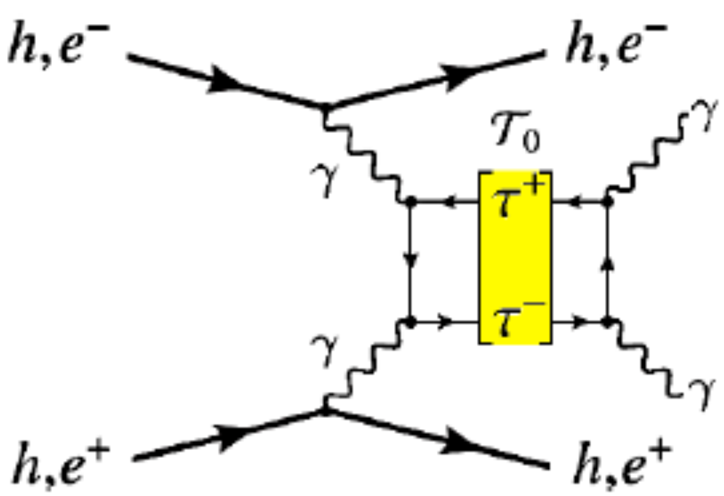
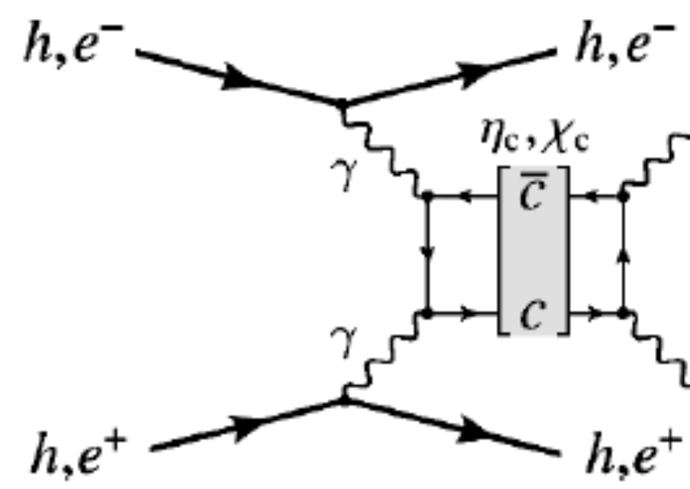
How to observe true tauonium ?

- **The spin-singlet state (para-true tauonium $\mathcal{T}_0$)** d'Enterria, HSS (PRD'22)

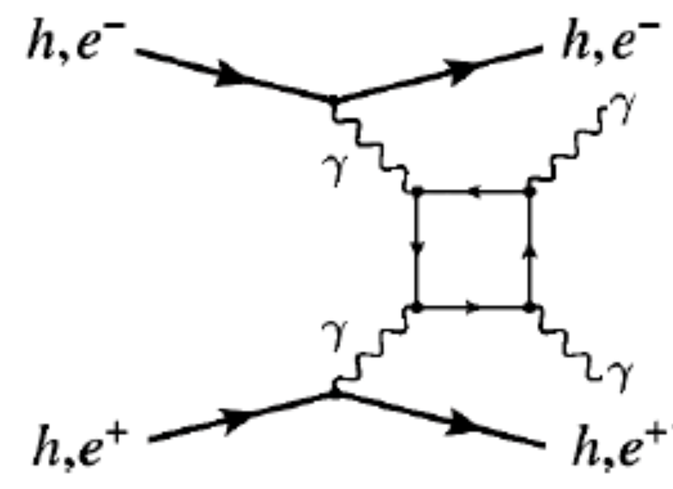
Signal



C-even charmonium



Light-by-Light



- Cross sections for signal and backgrounds are simulated with the equivalent-photon approximation (EPA)
- For e+e-, we used the well-known improved Weizsacker-Williams function
- For hadron-hadron, we considered the ultra-peripheral collisions, where the initial hadrons are intact

⇨ **gamma-UPC** d'Enterria, HSS (JHEP'22)

How to observe true tauonium ?

- **The spin-singlet state (para-true tauonium $\mathcal{T}_0$)** d'Enterria, HSS (PRD'22)
  - Cross sections for resonances are computed with HELAC-Onia
    HSS (CPC'13,CPC'16)
  - LbL cross sections are computed with MadGraph5_aMC@NLO
    Alwall et al. (JHEP'14)

Colliding system, c.m. energy, $\mathcal{L}_{\text{int}}$, exp.	$\sigma \times \mathcal{B}_{\gamma\gamma}$						$N \times \mathcal{B}_{\gamma\gamma}$	
	$\eta_c(1S)$	$\eta_c(2S)$	$\chi_{c,0}(1P)$	$\chi_{c,2}(1P)$	LbL	$\mathcal{T}_0$	$\mathcal{T}_0$	$\chi_{c,2}(1P)$
e^+e^- at 3.78 GeV, 20 fb^{-1}, BES III	120 fb	3.6 ab	15 ab	13 ab	30 ab	0.25 ab	–	–
e^+e^- at 10.6 GeV, 50 ab^{-1}, Belle II	1.7 fb	0.35 fb	0.52 fb	0.77 fb	1.7 fb	0.015 fb	750	38 500
e^+e^- at 91.2 GeV, 50 ab^{-1}, FCC-ee	11 fb	2.8 fb	3.9 fb	6.0 fb	12 fb	0.11 fb	5 600	$3 \cdot 10^5$
p-p at 14 TeV, 300 fb^{-1}, LHC	7.9 fb	2.0 fb	2.8 fb	4.3 fb	6.3 fb	0.08 fb	24	1290
p-Pb at 8.8 TeV, 0.6 pb^{-1}, LHC	25 pb	6.3 pb	8.7 pb	13 pb	21 pb	0.25 pb	0.15	8
Pb-Pb at 5.5 TeV, 2 nb^{-1}, LHC	61 nb	15 nb	21 nb	31 nb	62 nb	0.59 nb	1.2	62

- Cross sections increase with center-of-mass energy and Z^4
- Large charmonium background

$$\sigma(\eta_c(1S)) : \sigma(\chi_{c2}(1P)) : \sigma(\chi_{c0}(1P)) : \sigma(\eta_c(2S)) : \sigma(\mathcal{T}_0)$$

100 50 30 25 1

How to observe true tauonium ?

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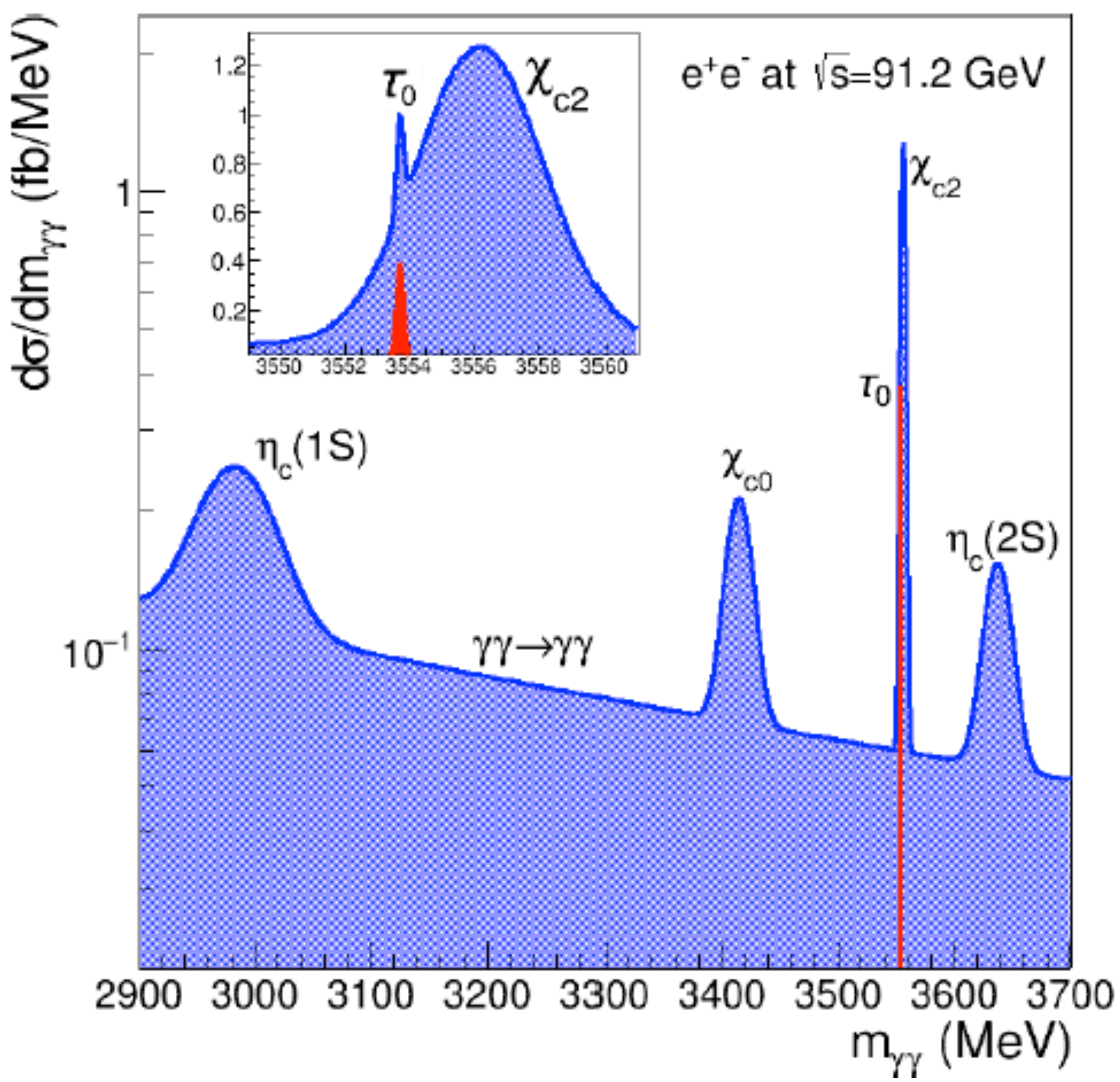
100 50 30 25 1

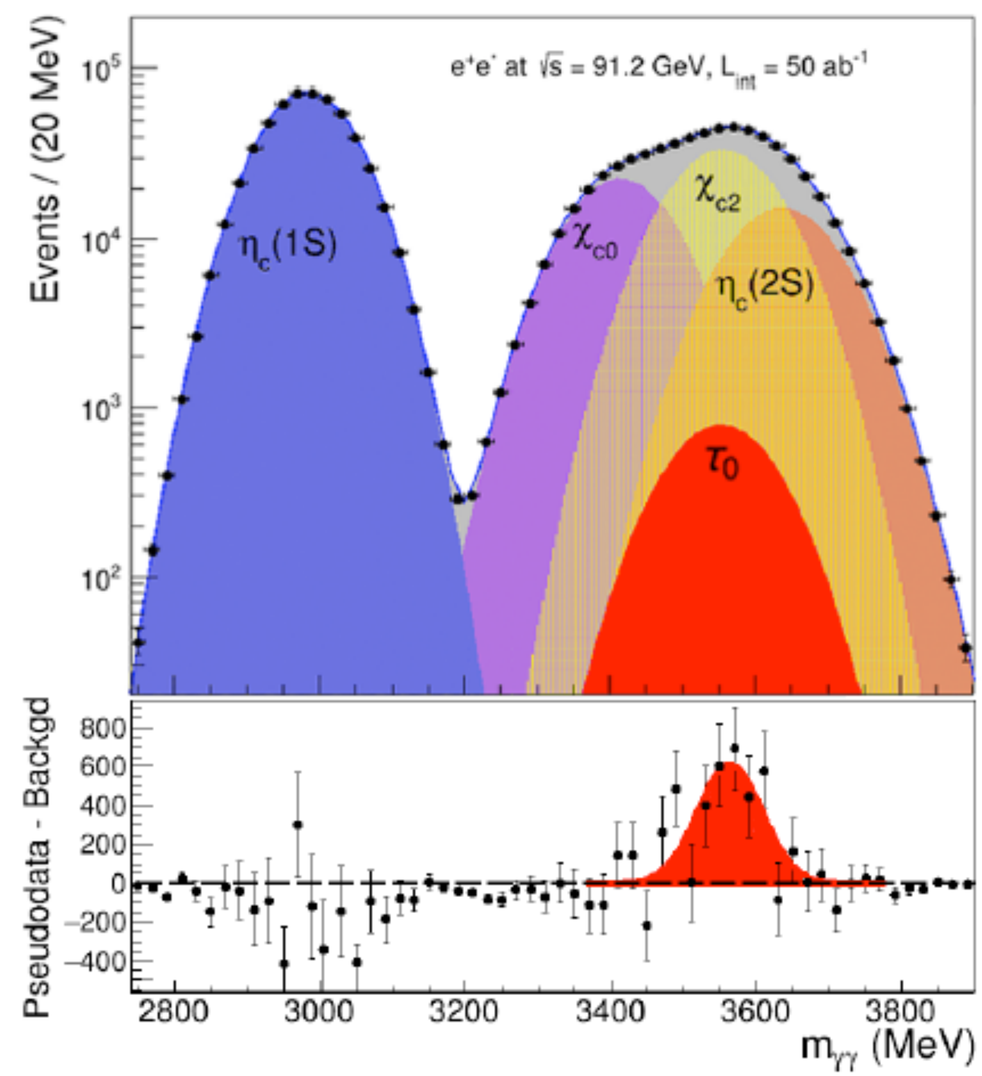
How to observe true tauonium ?

LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

- **The spin-singlet state (para-true tauonium $\mathcal{T}_0$)** d'Enterria, HSS (PRD'22)
  - Trigger: two back-to-back exclusive 1.5-2 GeV photons w/ $m_{\gamma\gamma} \approx m_{\mathcal{T}_0}$
  - Reconstruction performance (Belle-II type & high-resolution FCC-ee crystal calo)

Acceptance: $10^\circ < \theta_\gamma < 170^\circ$ *Mass resolution:* $\sim 2\%$ *Photon reco effic.:* $\sim 100\%$





How to observe true tauonium ?

d'Enterria, HSS (PRD'22)

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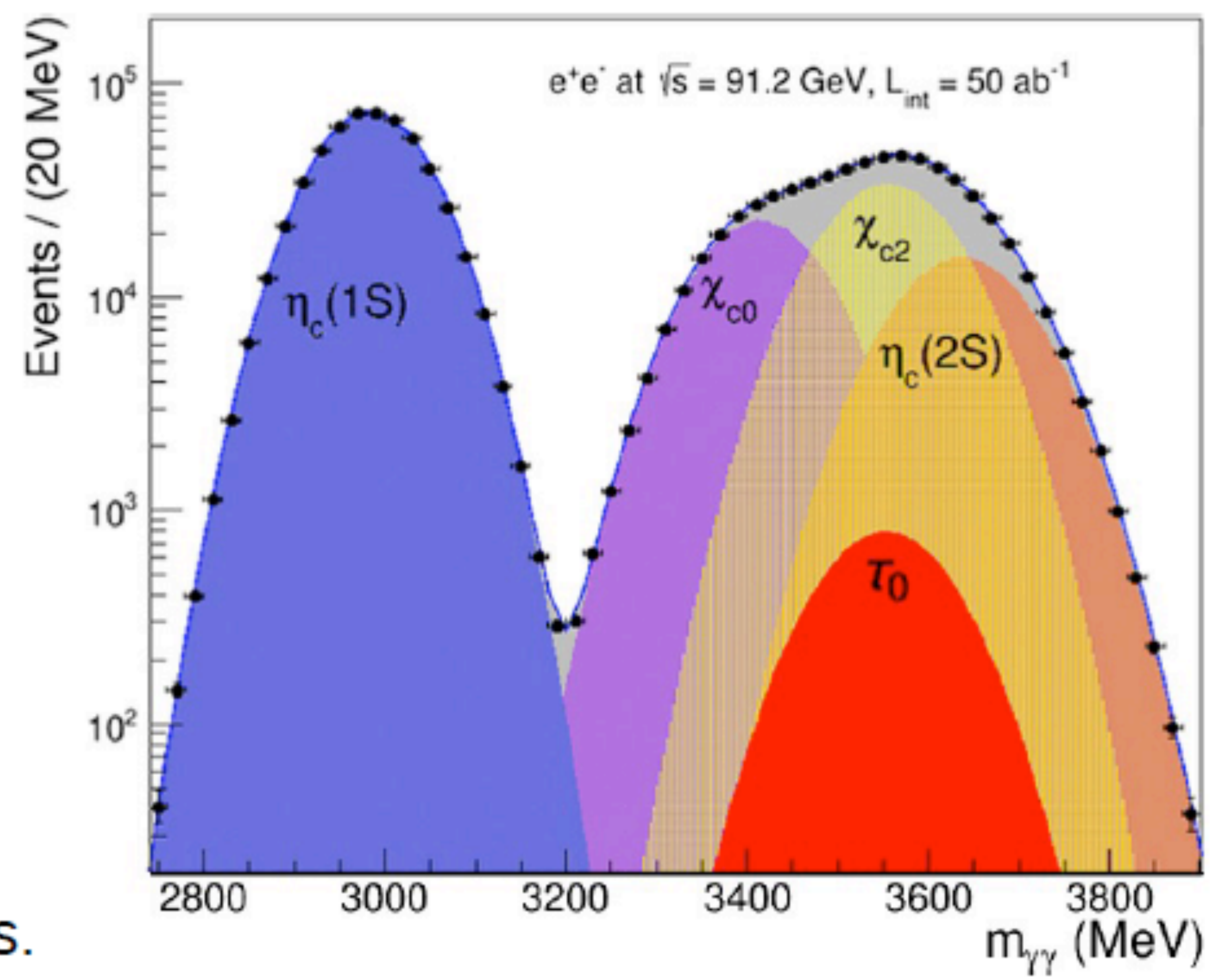
- 1-million events generated for signal & backgrounds. Run MVA (BDT) with 12 different single-γ and γ-pair kinematic variables for signal/backgds separation:
  (i) Strong discrimination power (factor of ~20) of LbL continuum from signal.
  (ii) No discrimination achieved for overlapping charmonia (decay γ angular modulation of tensor χ_{c2} different than scalar τ_0 signal, but $\times$50 suppressed yields)

- Signal extracted through multi-Gaussian $m_{\gamma\gamma}$ fit, by considering:
  → $\eta_c(1S)$: No overlap w/ signal ("std.candle"): 0.5M clean evts to fully control E_γ scale&res. plus exp. & theory uncertainties.
  → χ_{c0}, $\eta_c(2S)$: Partial overlap with signal. Exploit ~100M $\gamma\gamma\to\chi_{c0},\eta_c\to X$ decays with $\times$50 larger BRs (e.g. X=3- and 4-mesons) to fully remove their contamination.
  → χ_{c2}: Full overlap with signal! Exploit alternative $\gamma\gamma\to\chi_{c2}\to X$ decays (e.g. 11M evts. for X=4π) to determine its lineshape to within $\mathcal{O}$(0.2%).

Reconstructed yields (LbL subtracted)

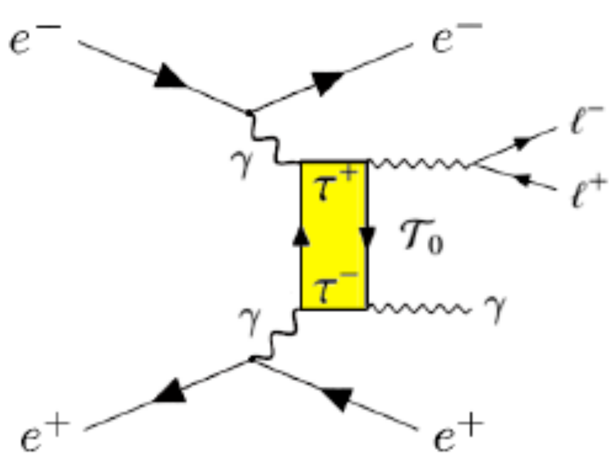


How to observe true tauonium ?

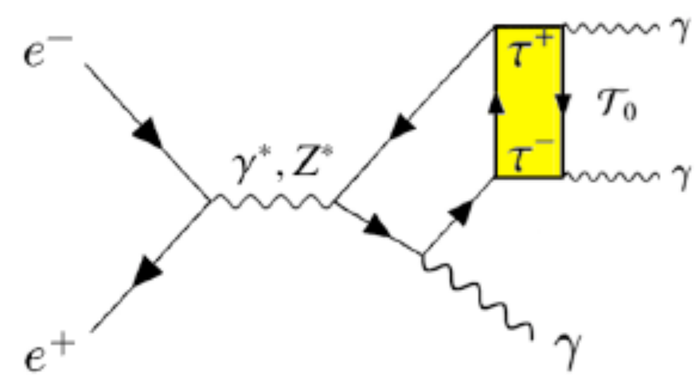
- **The spin-singlet state (para-true tauonium $\mathcal{T}_0$)** d'Enterria, HSS (PRD'22)
  - Significance
    FCC-ee 5σ
    Belle II 3σ
  - Main challenge

    Control the lineshapes of charmonia, in particular χ_{c2}
  - We consider other possibilities, but turn out to be challenging

Dalitz decay



FSR photon



	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$
$\mathrm{Br}(\Upsilon(nS)\to\gamma\mathcal{T}_0)=$	5.4×10^{-12}	3.7×10^{-12}	4.0×10^{-12}