Tau physics at FCC-ee

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Tau lepton samples, past and future

tau pairs yields at past, present and future e^+e^- colliders

facility	Z [million]	$\tau^+\tau^-$ [million]	$\tau^+\tau^-$ relative sample size	
LEP	25	0.84		
BABAR	-	$0.5{\cdot}10^3$		
Belle	-	$1.0{\cdot}10^3$		
Belle II	-	$45{\cdot}10^3$		
STCF at 4.26 GeV, 10 years	-	$35{\cdot}10^3$		
CEPC	$4{\cdot}10^6$	$135{\cdot}10^3$	$1.6{\cdot}10^5\times$LEP	$3.0\times$Belle II
FCC-ee	$6{\cdot}10^6$	$200{\cdot}10^3$	$2.4{\cdot}10^5\times$LEP	$4.5\times$Belle II

- most tau measurements are best done at Z-peak than at lower energies

Tau flagship measurements in FCC Feasibility Report (Apr 2025)

- tau lifetime
- tau mass
- tau leptonic branching fractions
- tau decay LFV searches ($\tau\mu\gamma$, $tau \to \mu\mu\mu$)

documentation

- FCC FSR April 2025, [Future Circular Collider Feasibility Study Report Volume 1: Physics and Experiments, doi:10.17181/CERN.9DKX.TDH9]
- ESPPU 2026 FCC inputs, [Prospects in flavour physics at the FCC, doi:10.17181/jnzpp-1fw39]
- FCC Tau Physics note, [A.L., Tau Physics Prospects at FCC-ee, doi:10.17181/57pxj-6xd43]

Observable	present value	±	uncertainty	FCC-ee Stat.	FCC-ee Syst.	Comment and leading uncertainty
m_Z (keV)	91 187 600	±	2000	**4**	100	From Z line shape scan Beam energy calibration
Γ_Z (keV)	2 495 500	±	2300	**4**	12	From Z line shape scan Beam energy calibration
$\sin^2\theta_W^{eff}$ ($\times 10^6$)	231,480	±	160	**1.2**	1.2	From $A_{FB}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{QED}(m_Z^2)$ ($\times 10^3$)	128 952	±	14	**3.9** **0.8**	small tbc	From $A_{FB}^{\mu\mu}$ off peak From $A_{FB}^{\mu\mu}$ on peak QED&EW uncert. dominate
R_ℓ^Z ($\times 10^3$)	20 767	±	25	**0.05**	0.05	Ratio of hadrons to leptons Acceptance for leptons
$\alpha_S(m_Z^2)$ ($\times 10^4$)	1 196	±	30	**0.1**	1	Combined R_ℓ^Z, Γ_{tot}^Z, σ_{had}^0 fit
σ_{had}^0 ($\times 10^3$) (nb)	41 480.2	±	32.5	**0.03**	0.8	Peak hadronic cross section Luminosity measurement
N_ν($\times 10^3$)	2 996.3	±	7.4	**0.09**	0.12	Z peak cross sections Luminosity measurement
R_b ($\times 10^6$)	216 290	±	660	**0.25**	0.3	Ratio of $b\bar{b}$ to hadrons
$A_{FB}^{b,0}$ ($\times 10^4$)	992	±	16	**0.04**	0.04	b-quark asymmetry at Z pole From jet charge
$A_{FB}^{pol,\tau}$ ($\times 10^4$)	1 498	±	49	**0.07**	0.2	τ polarisation asymmetry τ decay physics
τ lifetime (fs)	290.3	±	0.5	**0.001**	0.005	ISR, τ mass
τ mass (MeV)	1 776.93	±	0.09	**0.002**	0.02	estimator bias, ISR, FSR
τ leptonic ($\mu\nu_\mu\nu_\tau$) BR (%)	17.38	±	0.04	**0.00007**	0.003	PID, π^0 efficiency
m_W (MeV)	80 360.2	±	9.9	**0.18**	0.16	From WW threshold scan Beam energy calibration
Γ_W (MeV)	2 085	±	42	**0.27**	0.2	From WW threshold scan Beam energy calibration
$\alpha_S(m_W^2)$ ($\times 10^4$)	1 010	±	270	**2**	2	Combined R_ℓ^W, Γ_{tot}^W fit
N_ν ($\times 10^3$)	2 920	±	50	**0.5**	small	Ratio of invis. to leptonic in radiative Z returns
m_{top} (MeV)	172 570	±	290	**4.2**	4.9	From $t\bar{t}$ threshold scan QCD uncert. dominate
Γ_{top} (MeV)	1 420	±	190	**10**	6	From $t\bar{t}$ threshold scan QCD uncert. dominate
$\lambda_{top}/\lambda_{top}^{SM}$	1.2	±	0.3	**0.015**	0.015	From $t\bar{t}$ threshold scan QCD uncert. dominate
ttZ couplings		±	30%	**0.5–1.5** %	small	From $\sqrt{s} = 365$ GeV run

FCC tau physics sensitivity estimates

Estrapolations with very limited use of MC simulations

- M.Dam, 2019–2021: m_τ, τ_τ, tau leptonic BRs, $\tau \to \mu\gamma$, $\tau \to \mu\mu\mu$
- A.L., 2022–2024: m_τ, τ_τ, tau leptonic BRs, $\tau \to \mu\gamma$, $\tau \to \mu\mu\mu$

New: FCC tau LFV searches sensitivity with Fastsim MC simulations

- M.Kazanecki (Krakow), A.L. (May-Jul 2026)
- 24 LFV tau decay modes
- Fastsim MC with Idea detector, KKMC generator for $\tau^+\tau^-$
- preliminary!!!

MC signal samples for tau LFV searches

- Delphes, detector IDEA
- Z-peak $e^+e^- \to \tau^+\tau^-$ with KKMC
- each signal tau LFV decay mode
  - 100k τ^- + 100k τ^+ decay to signal mode
  - the other τ decays according to SM
  - Pythia production, EvtGen decay
  - production: M. Kazanecki, 2026

Process
$Z \to \tau\tau$, with $\tau^- \to \mu^-\gamma$
$Z \to \tau\tau$, with $\tau^- \to e^-e^+e^-$
$Z \to \tau\tau$, with $\tau^- \to \mu^-e^+e^-$
$Z \to \tau\tau$, with $\tau^- \to \mu^+e^-e^-$
$Z \to \tau\tau$, with $\tau^- \to e^-\mu^+\mu^-$
$Z \to \tau\tau$, with $\tau^- \to e^+\mu^-\mu^-$
$Z \to \tau\tau$, with $\tau^- \to \mu^-\mu^+\mu^-$
$Z \to \tau\tau$, with $\tau^- \to \mu^-\pi^0$
$Z \to \tau\tau$, with $\tau^- \to \mu^-\rho$
$Z \to \tau\tau$, with $\tau^- \to \mu^-\eta$
$Z \to \tau\tau$, with $\tau^- \to \mu^-\omega$
$Z \to \tau\tau$, with $\tau^- \to \mu^-\eta'$
$Z \to \tau\tau$, with $\tau^- \to \mu^-\phi$
$Z \to \tau\tau$, with $\tau^- \to \mu^-K_S^0$
$Z \to \tau\tau$, with $\tau^- \to \mu^-\pi^+\pi^-$

MC background samples for tau LFV searches

- used FCC samples available at CERN
- Delphes, detector IDEA
- Pythia and KKMC generators

Process	n. events	Cross-section [pb]	Campaign
$Z \to \tau^+\tau^-$	100M	1477	winter2023
$Z \to \tau^+\tau^-$ with KKMC	100M	1477	M. Kazanecki, 2026
$Z \to e^+e^-$	100M	1462	winter2023
$Z \to \mu^+\mu^-$	100M	1462	winter2023
$Z \to q\bar{q},\ q = u, d, s$	1000M	18620	spring2021
$Z \to c\bar{c}$	500M	5215	winter2023
$Z \to b\bar{b}$	439M	6645	winter2023

Tau LFV searches analysis outline

- **Preselection**: select $Z \to \tau^+\tau^-$ suppressing backgrounds
- **PID Selection:** PID requirements implemented with PID (mis-)id weights
  - for background contributions through mis-id, weights increase effective MC sample size
- **Selection**: compute tau candidate mass and energy and require consistency within uncertainties
- **Background estimate**
  - count simulated events in signal region
  - when no background in signal region, count events in larger area and extrapolate to signal region
- **Sensitivity**: compute expected BR 90% CL upper limit using the estimated background

$\tau^+\tau^-$ preselection

- low charged-track multiplicity (1-1, 1-3) to suppress $q\bar{q}$;
- leading-track momentum and energy below beam energy to suppress e^+e^- and $\mu^+\mu^-$;
- require hemispheres momenta acollinearity to select $\tau^+\tau^-$ events with missing neutrinos;
- tighter momentum, energy, and acollinearity cuts in the forward region;
- non-zero event p_T to suppress two-photon and beam backgrounds;
- minimum visible energy to reject beam-gas and two-photon events;
- tracks from the IP to suppress cosmics, beam-gas

$\tau \to \mu\gamma$ specific preselection

- =1 charged track in the signal hemi; 1-3 charged tracks in tag hemi
- $p_\mu > 2\,\mathrm{GeV}/c$;
- require a hard photon candidate, $E_\gamma > 5\,\mathrm{GeV}$;
- no additional neutral candidate with $E_{\mathrm{neutral}} > 1\,\mathrm{GeV}$;
- require low tag-side visible energy, $E_{\mathrm{opp}} < 0.8\,E_{\mathrm{beam}}$, to suppress non-$\tau\tau$ backgrounds;

Estimation of expected background, upper limit calculation

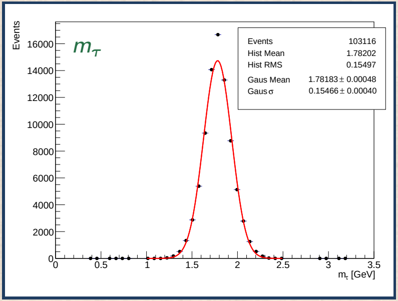
background estimation details

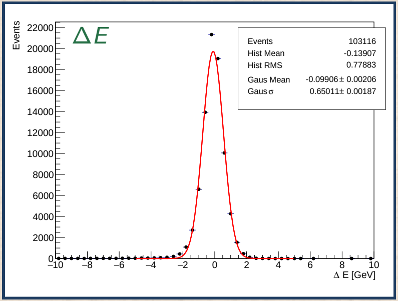
- signal candidates are counted for several MC background samples
- each sample i background count n_i regularized as $\hat{n}_i = (n_i + \frac{1}{2}) \pm (n_i + \frac{1}{2})$
  (mean and uncertainty of Bayesian posterior with Jeffreys prior)
  this procedure insures conservative background estimation when using small MC samples
- each $\hat{n}_i$ is scaled to the full FCC sample size according to:
  - PID weights w_i
  - ratio between signal region and (larger) background counting area
  - ratio between FCC design and MC sample integrated luminosities
- total estimated background $\hat{n} = \sum_i \hat{n}_i \cdot r_i \cdot w_i$

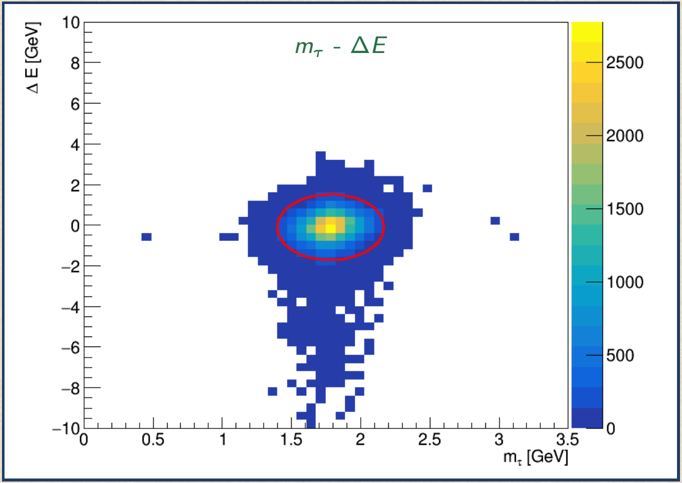
upper limit calculation for FCC $6{\cdot}10^{12}\,Z$

- if expected background $n = 0$, $\sigma_n = 0 \Rightarrow$ 90% CL upper limit $n^{\text{UL}} \simeq 2.3$ events
- if expected background $n \gg 1$, $\sigma_n = 0 \Rightarrow$ 90% CL upper limit $n^{\text{UL}} \simeq 1.28\sqrt{n}$ events
- code algorithm with
  - smooth approximate transition between the two regimes
  - accounting of expected background uncertainty σ_n
- $\mathcal{B}^{\text{UL}} = \dfrac{n^{\text{UL}}}{\varepsilon \cdot n_\tau}$

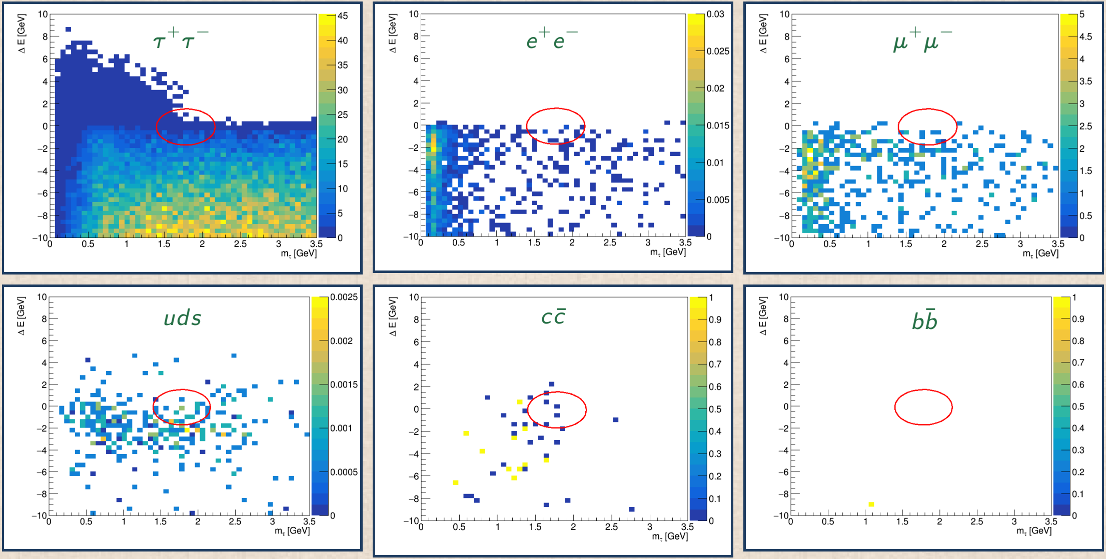
LFV search for $\tau \to \mu\gamma$, signal MC



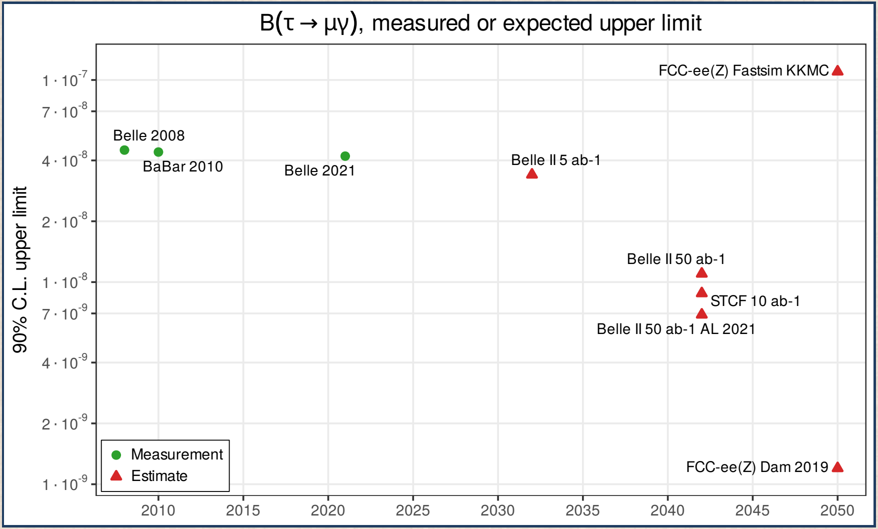




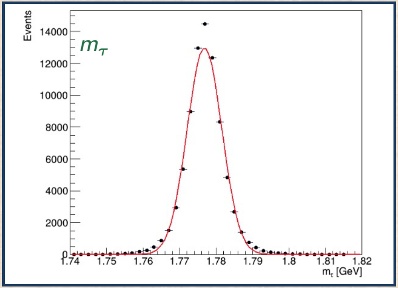
LFV search for $\tau \to \mu\gamma$, background MC

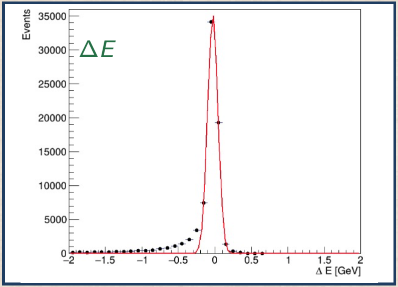


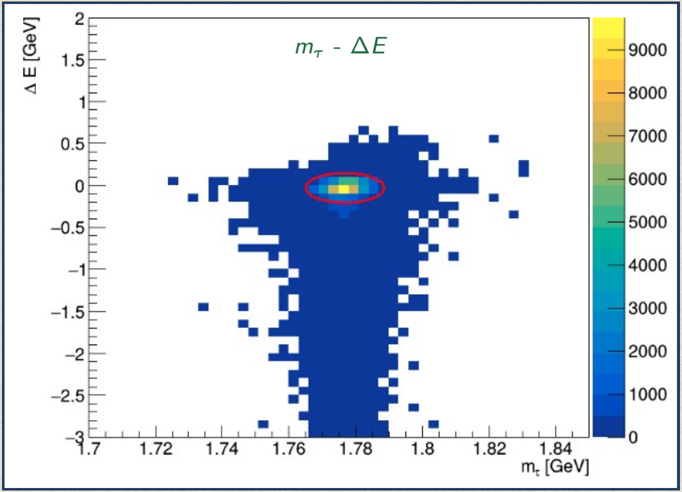
LFV search for $\tau \to \mu\gamma$ Fastsim estimate



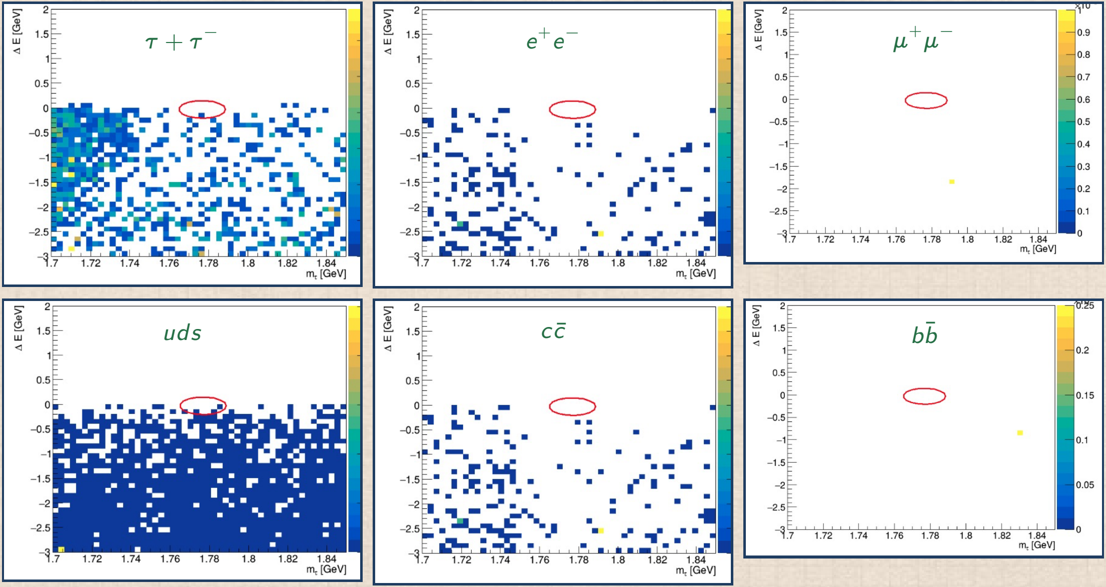
LFV search for $\tau \to \mu\mu\mu$, signal MC



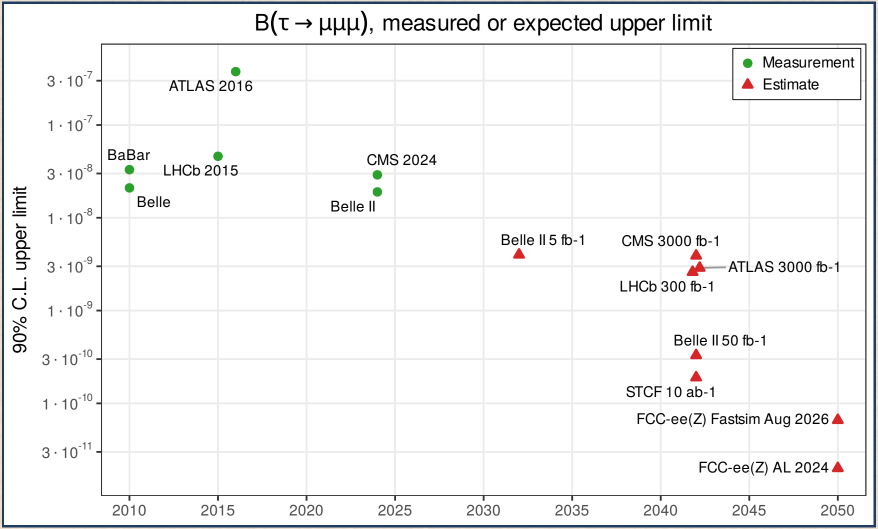




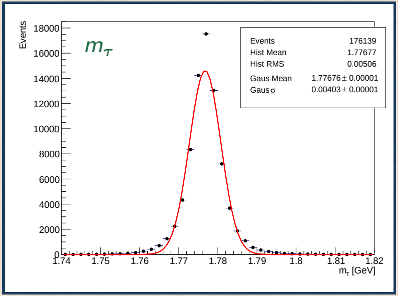
LFV search for $\tau \to \mu\mu\mu$, background MC

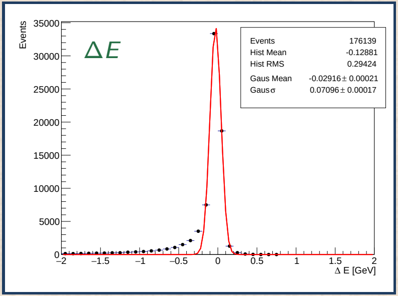


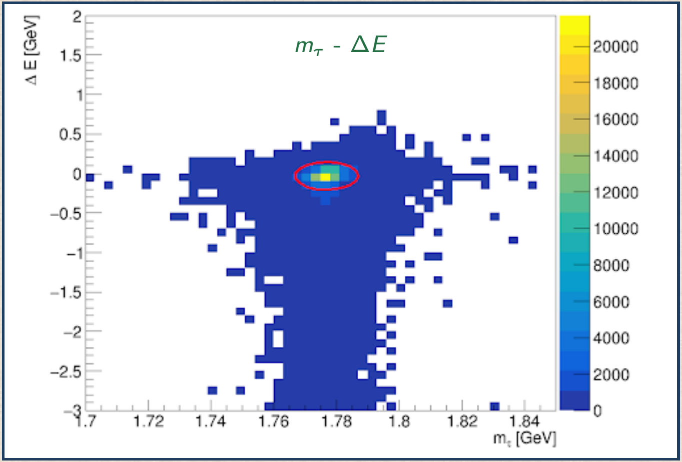
LFV search for $\tau \to \mu\mu\mu$



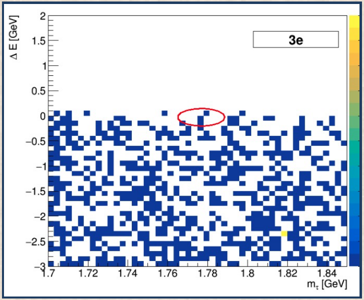
LFV search for $\tau \to eee$, signal MC

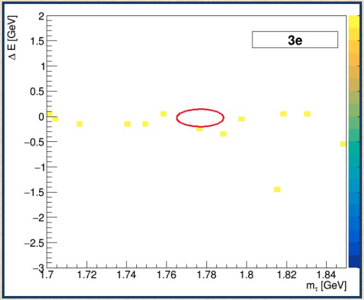


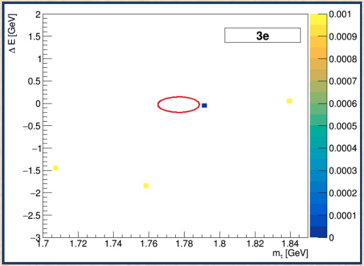


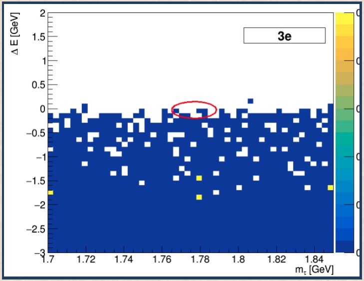


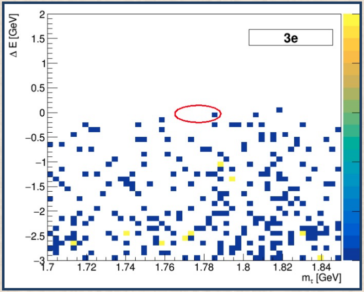
LFV search for $\tau \to eee$, background MC

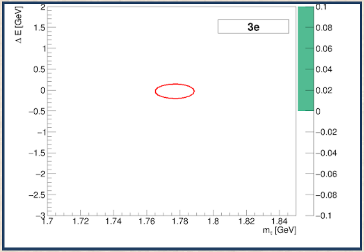




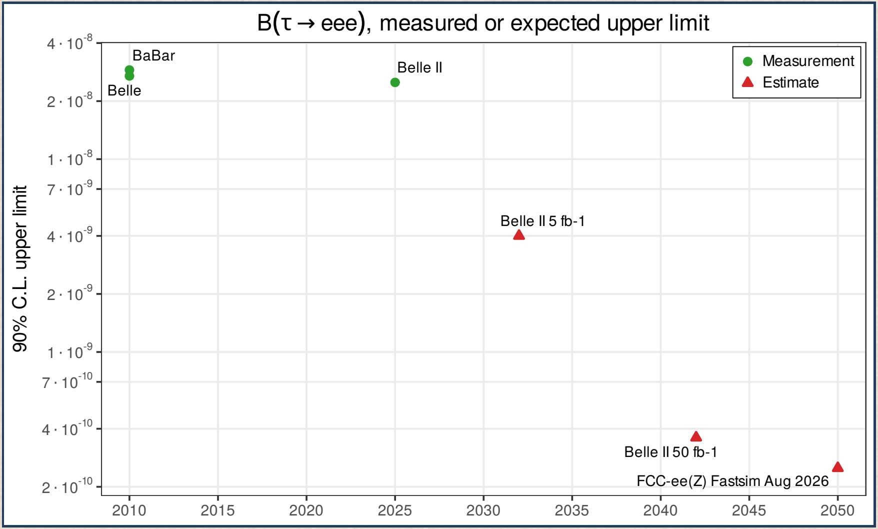








LFV search for $\tau \to eee$



LFV searches resolutions

m_τ

	σ [MeV]
$\tau \to \mu\gamma$	150
$\tau \to e\gamma$	150
$\tau \to \mu\mu\mu$	4
$\tau \to eee$	4

ΔE

	σ [MeV]
$\tau \to \mu\gamma$	650
$\tau \to e\gamma$	650
$\tau \to \mu\mu\mu$	71
$\tau \to eee$	71

- same resolutions for electrons and muons
- much worse resolution when a photon is involved

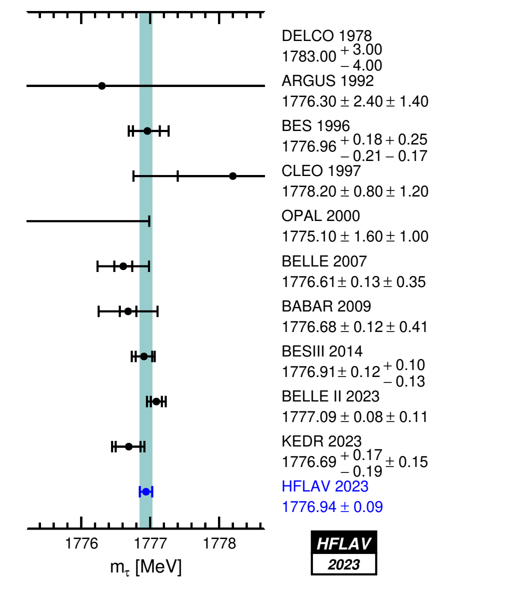
Expected tau LFV limits at FCC (preliminary) (1)

	UL 90% CL [10^{-11}]	b		b_{UL}
$\tau \to \mu\gamma$	11 000	9500	$\pm$ 3700	6100
$\tau \to e\gamma$	11 000	15 000	$\pm$ 4600	7600
$\tau \to \mu\pi^0$	11 000	6200	$\pm$ 2500	4100
$\tau \to e\pi^0$	11 000	6500	$\pm$ 2500	4100
$\tau \to 3\mu$	6.6	3.8	$\pm$ 3.1	6.5
$\tau \to 3e$	25	15	$\pm$ 12	22
$\tau \to \mu^- e^+ e^-$	1.9	0.057	$\pm$ 0.027	2.3
$\tau \to e^- \mu^+ \mu^-$	17	15	$\pm$ 12	22
$\tau \to \mu^- e^+ \mu^-$	2.5	0.0053	$\pm$ 0.0039	2.3
$\tau \to e^- \mu^+ e^-$	2.6	0.032	$\pm$ 0.02	2.3
$\tau \to \mu\phi$	5.0	0.47	$\pm$ 0.086	2.6
$\tau \to e\phi$	5.0	0.43	$\pm$ 0.083	2.5
$\tau \to \mu\rho$	7.4	15	$\pm$ 2.1	7.5
$\tau \to e\rho$	8.3	19	$\pm$ 2.0	8.3
$\tau \to \mu K^0_S$	59	18	$\pm$ 3.9	9.8
$\tau \to e K^0_S$	88	30	$\pm$ 6.9	15

Expected tau LFV limits at FCC (preliminary) (2)

	UL 90% CL $[10^{-11}]$	b	b_{UL}
$\tau \to \mu\pi\pi$	8.5	27 ± 2.6	9.7
$\tau \to e\pi\pi$	11	38 ± 3.8	12
$\tau \to \mu\eta$	180	73 ± 4.9	16
$\tau \to e\eta$	250	130 ± 7.7	23
$\tau \to \mu\omega$	140	240 ± 28	53
$\tau \to e\omega$	180	380 ± 35	65
$\tau \to \mu\pi^0$	11 000	6200 ± 2500	4100
$\tau \to e\pi^0$	11 000	6500 ± 2500	4100
$\tau \to \mu\eta'$	510	600 ± 40	78
$\tau \to e\eta'$	540	830 ± 41	82

Tau mass measurements



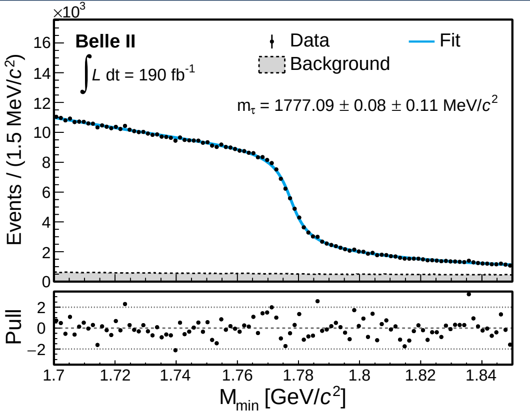
threshold scan technique

- measure onset of $\sigma(e^+e^- \to \tau^+\tau^-)(\sqrt{s})$
- DELCO, BES (92, 96), KEDR and BES III
- require precise E_{beam} using
  - either resonant depolarization of polarized $e^\pm$
  - or laser-beam Compton scattering

pseudo-mass technique

- select $e^+e^- \to \tau^+\tau^-$ events
- fit pseudo-mass using visible tau-decay products
  - empirical function, potential systematic
- ARGUS, OPAL, Belle, BABAR, Belle II (2023)
- require
  - good momentum scale calibration

Tau mass Belle II 2023 measurement



- empirical pseudo-mass distribution

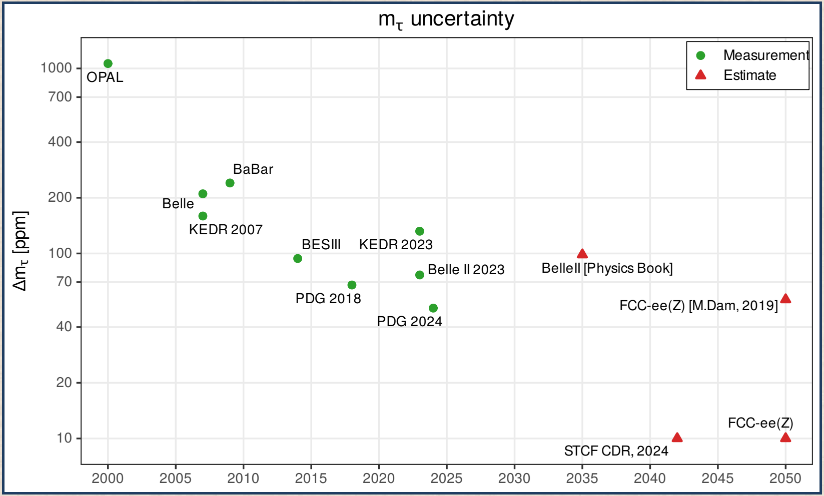
Source	Uncertainty (MeV/c^2)
Knowledge of the colliding beams:	
Beam-energy correction	0.07
Boost vector	< 0.01
Reconstruction of charged particles:	
Charged-particle momentum correction	0.06
Detector misalignment	0.03
Fit model:	
Estimator bias	0.03
Choice of the fit function	0.02
Mass dependence of the bias	< 0.01
Imperfections of the simulation:	
Detector material density	0.03
Modeling of ISR, FSR and τ decay	0.02
Neutral particle reconstruction efficiency	≤ 0.01
Momentum resolution	< 0.01
Tracking efficiency correction	< 0.01
Trigger efficiency	< 0.01
Background processes	< 0.01
Total	0.11

[Belle II, PRD 108 (2023) 032006]

Tau mass prospects at FCC-ee

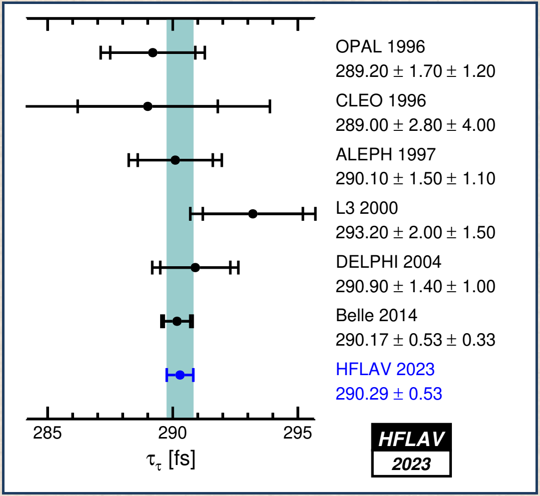
- $\Delta m_\tau^{\text{stat}} = 45\,\text{ppm}$ at Belle II with $190\,\text{fb}^{-1}$, 175 M tau pairs
- $\Delta m_\tau^{\text{stat}} = 1.3\,\text{ppm}$ at FCC-ee with $6{\cdot}10^{12}$ Z, $2.0{\cdot}10^{11}$ tau pairs
  - neglecting expected better FCC-ee efficiency
- $\Delta m_\tau^{\text{stat}} = 0.9\,\text{ppm}$ at FCC-ee when rescaling OPAL 2000 tau mass uncertainty
- Belle II dominant systematics expected to be significantly reduced at FCC-ee
  - beam energy (1 ppm at FCC-ee)
  - track momentum scale (2 ppm calibration maybe possible at FCC-ee with $m_{J/\psi}$)
- alignment systematics can be expected to scale with statistics
- Belle II limiting systematics 29 ppm
  - estimator bias ($0.03\,\text{MeV}/c^2$)
  - pseudo-mass fit function ($0.02\,\text{MeV}/c^2$)
  - detector material ($0.03\,\text{MeV}/c^2$)
  - modeling of ISR, FSR and tau decay ($0.02\,\text{MeV}/c^2$)
- may expect to reduce non-luminosity-scaling systematics by factor 3 at FCC-ee, to 10 ppm
- guestimate FCC-ee tau mass precision at 10 ppm
- no particular detector requirements are needed, a baseline realistic detector is sufficient

Tau mass measurement prospects



- FCC-ee(Z) CDR estimate [M. Dam, 2019]
- FCC-ee(Z) estimate [A.L., Tau Physics Prospects at FCC-ee, doi:10.17181/57pxj-6xd43]

Tau lifetime measurements



Belle 2014 tau lifetime measurement

analysis

- select 3-prong vs. 3-prong events
- kinematically reconstruct the $\tau^+\tau^-$ direction in the center-of-mass frame
- reconstruct signed decay length in lab frame
- fit decay length distribution, accounting for resolution
- compute tau lifetime using tau average velocity (momentum) from simulation
  - initial and final state radiation reduce average tau momentum

Tau lifetime uncertainties

Delphi 2004

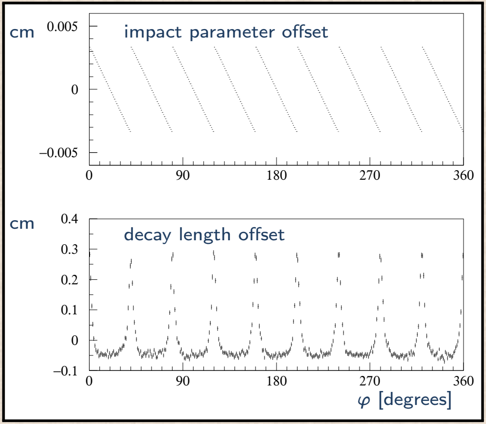
	ppm
statistical	8300
statistical, 3-3 prongs	17900
systematic	4500
Alignment	3500
Reconstruction Bias	2800
Background	690
Radiative Energy Loss	350

Belle 2014 (3-3 prongs)

	ppm
statistical	1830
systematic	1160
SVD alignment	1030
Asymmetry of R-function	350
Fit range	230
ISR and FSR description	210
Beam energy calibration	180
Background contribution	120
Error of the τ-lepton mass	100

Tau lifetime systematics from detector misalignment

- crude simulation of ALEPH vertex detector with all wafers radially shifted out by 100 μm
- impact parameter offset $= 100\,\mu\text{m} \cdot \sin\alpha$
  - $\alpha =$ track angle w.r.t. normal of wafer
- decay length offset (measured using 3-tracks vertex)
  - negative when 3 tracks hit same wafer
  - positive when 3 tracks hit two wafers



- decay length offset $\sim$ derivative of impact parameter offset
  $\Rightarrow$ decay length offset due to misalignment averages to zero over azymuth 2π
  - requires uniform acceptance (can be insured by event weighting)
  - approximately works over polar angle due to incomplete detector acceptance
- well confirmed with Monte Carlo full simulations
- [S.Wasserbaech, Nucl.Phys.Proc.Suppl. 76 (1999) 107]

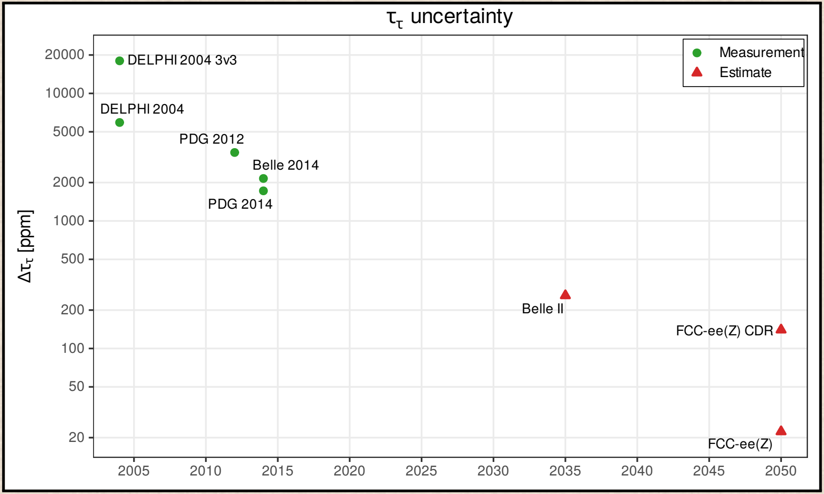
Tau Lifetime prospects at FCC-ee

- consider just tau pairs in 3-prong vs. 3-prong topology (3v3)
  - Belle 2014 best measurement uses these events
  - τ direction reconstruction using vertices reduces dependence from simulation
- extrapolate FCC-ee statistical precision starting from Delphi 2004 3v3 events statistical precision
  - expect no significant differences on selection efficiency
  - Delphi 2004 3v3 precision by rescaling 3v1+3v3 measurement to number of 3v3 candidates
  - τ_τ measurement is a measurement of transverse i.p. $\langle d_0 \sin\theta \rangle \approx 70\,\mu$m
  - Delphi 2004 3v3 precision consistent with a d_0 resolution $\approx 70\,\mu$m (tracking, beam spot)
  - assume FCC-ee has both transverse beam spot and can have d_0 resolution $\ll 70\mu$m
    $\Rightarrow$ resolution improvement factor $\sim 70\,\mu\text{m}/(70\,\mu\text{m} \oplus 70\,\mu\text{m}) \simeq 0.7$
- assume DELPHI systematics for background, reconstruction bias and alignment (total 1.3 fs) scale with luminosity to 3.9 ppm at FCC-ee (very optimistic)
- assume 30$\times$ less uncertain KKMC simulation on ISR+FSR energy loss in tau pair production to reduce the associated systematic contribution from 350 ppm to 11.5 ppm
- assume 10 ppm tau mass measurement at SCT/STCF or at FCC-ee
- assume 5 ppm vertex detector length scale (possible with optical methods proposed for MUonE)
  - requires studies, can only possibly monitor outer modules, which can serve as reference

Tau Lifetime measurement prospects at FCC-ee(Z)

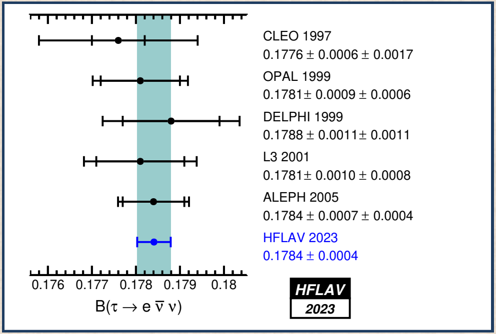
	DELPHI 2004 [fs]	DELPHI 2004 [ppm]	FCC-ee(Z) $6{\cdot}10^{12}$ Z [ppm]
statistical uncertainty	5.2	18000	15.0
luminosity-dependent systematics	1.3	4500	3.9
- background	0.2		
- reconstruction bias	0.8		
- vertex detector alignment	1.0		
luminosity-independent systematics			
- detector length scale	-	100	5.0
- average tau energy	-	-	1.0
- radiative energy loss	0.1	350	11.5
- tau mass	-	68	10.0
total systematics			15.9
total uncertainty			22.3

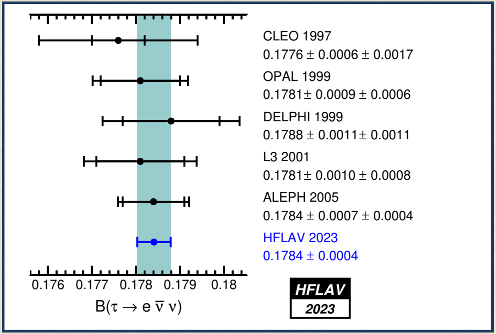
Tau Lifetime measurement prospects

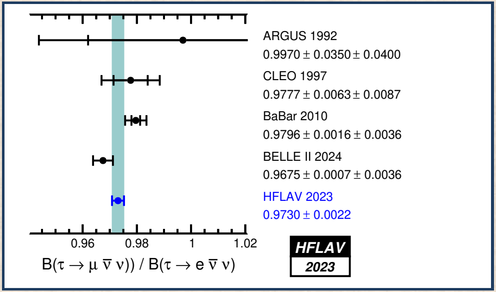


- FCC-ee(Z) CDR estimate [M. Dam, 1999]
- FCC-ee(Z) estimate [A.L., Tau Physics Prospects at FCC-ee, doi:10.17181/57pxj-6xd43]
- Belle II estimate [Belle II Physics Book]

Tau branching fractions measurements







Tau branching fractions measurements features

- branching fraction measurement: $\mathcal{B}(\tau \to X) = \dfrac{N(\tau \to X)}{N(\tau)}$

Z peak

- $\dfrac{N_{\text{obs}}(\tau \to X)/\epsilon(\tau \to X) - N_{\text{bkg}}(\tau \to X)}{N_{\text{obs}}(\tau)/\epsilon(\tau)}$
  - $N(\tau)$ from counting, not from luminosity
- $\epsilon(\tau \to X) \sim 100\%$
  small systematics from related MC modeling
- small non-tau N_{bkg}
  - easier to suppress hadrons, e^+e^-, $\mu^+\mu^-$
- $\Delta\mathcal{L} \sim 0.1\%$

$\Upsilon(4s)$ peak

- $\dfrac{N_{\text{obs}}(\tau \to X)/\epsilon(\tau \to X) - N_{\text{bkg}}(\tau \to X)}{2\sigma(e^+e^- \to \tau^+\tau^-)\mathcal{L}}$
  - $N(\tau)$ from from integrated luminosity
- $\epsilon(\tau \to X) \ll 100\%$
  large systematics from related MC modeling
- large non-tau N_{bkg}
  - more contamination of hadrons, e^+e^-, $\mu^+\mu^-$
- $\Delta\mathcal{L} \sim 1\%$

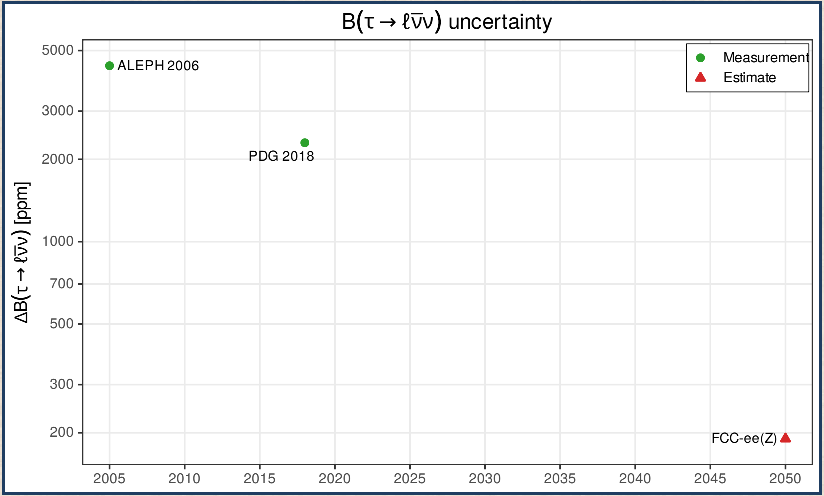
ALEPH at Z peak

- single analysis for all topological $\mathcal{B}$s
  (by n. of tracks and n. of π^0)
- most bkg from cross-feed of other channels
  i.e., fit on data rather than simulated

Tau leptonic branching fractions measurement prospects at FCC-ee(Z)

- ALEPH 2006 measurement precision: $4400\,\text{ppm} = [4000(\text{stat.}) \oplus 1900(\text{syst.})]\,\text{ppm}$ (average of the two similar electron and muon decays branching fractions)
  - complex simultaneous measurement of 12 tau branching fractions
  - many systematic uncertainties, no reliable extrapolations to FCC-ee statistics
  - several systematics related to photon and $\pi^0 \to \gamma\gamma$ reconstruction
- $\Delta^{\text{FCC}}_{\text{stat}}\mathcal{B}(\tau \to \ell\bar{\nu}\nu) = 4000\,\text{ppm} \cdot \sqrt{\dfrac{6.2{\cdot}10^6\,(\text{n. of ALEPH analysis Z bosons})}{6{\cdot}10^{12}(\text{n. of FCC Z bosons})}} = 4.1\,\text{ppm}$
- $\Delta^{\text{FCC}}_{\text{syst}}\mathcal{B}(\tau \to \ell\bar{\nu}\nu) = \dfrac{1}{10} \cdot 1900\,\text{ppm} = 190\,\text{ppm}$
  [assume also 100% correlated between $\mathcal{B}(\tau \to e\bar{\nu}\nu)$ and $\mathcal{B}(\tau \to \mu\bar{\nu}\nu)$ for LFU tests]
- $\Delta^{\text{FCC}}\mathcal{B}(\tau \to \ell\bar{\nu}\nu) \simeq 190\,\text{ppm}$ systematically dominated

Tau leptonic Branching fractions prospects at FCC-ee and other facilities



- FCC-ee(Z) with $6 \cdot 10^{12}$ Z
- Belle II is working on these measurements

Impact of radiative corrections on LFU tests with tau leptonic BFs

$$\Gamma[\mathcal{L} \to \nu_{\mathcal{L}} \ell \bar{\nu}_\ell (\gamma)] = \Gamma_{\mathcal{L}\ell} = \Gamma_{\mathcal{L}} \mathcal{B}_{\mathcal{L}\ell} = \frac{\mathcal{B}_{\mathcal{L}\ell}}{\tau_{\mathcal{L}}} = \frac{G_{\mathcal{L}} G_\ell m_{\mathcal{L}}^5}{192\pi^3} f^{\mathcal{L}\ell} (1 + \delta R_\gamma^{\mathcal{L}\ell})(1 + \delta R_W^{\mathcal{L}\ell})$$

$$G_{\mathcal{L}} = \frac{g_{\mathcal{L}}^2}{4\sqrt{2} M_W^2} \; ; \qquad f^{\mathcal{L}\ell} = 1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln\rho \quad \text{with} \quad \rho = \rho^{\mathcal{L}\ell} = m_\ell^2 / m_{\mathcal{L}}^2$$

$$\delta R_\gamma^{\mathcal{L}\ell} = \frac{\alpha(m_{\mathcal{L}})}{2\pi} \left(\frac{25}{4} - \pi^2 \right)$$

QED radiative correction, 1st order

$$\delta R_W^{\mathcal{L}\ell} = \frac{3}{5} \frac{m_{\mathcal{L}}^2}{M_W^2} + \frac{9}{5} \frac{m_\ell^2}{M_W^2}$$

EW radiative correction

[Marciano, 1988], [Pich, Precision Tau Physics, 2014]

$$\delta R_\gamma^{\mathcal{L}\ell} = H_1^{\mathcal{L}\ell} \frac{\alpha(m_{\mathcal{L}})}{\pi} + H_2^{\mathcal{L}\ell} \frac{\alpha^2(m_{\mathcal{L}})}{\pi^3} + H_3^{\mathcal{L}\ell} \frac{\alpha^3(m_{\mathcal{L}})}{\pi^3}$$

QED radiative correction, 3rd order

$$H_1^{\mathcal{L}\ell} = \frac{25}{8} - \frac{\pi^2}{2} - \left(9 + 4\pi^2 + 12 \ln\rho\right) \rho + 16\pi^2 \rho^{3/2} \quad \text{with} \quad \rho = \rho^{\mathcal{L}\ell} = m_\ell^2 / m_{\mathcal{L}}^2$$

$$H_2^{\mathcal{L}\ell} = \frac{156815}{5184} - \frac{518}{81}\pi^2 - \frac{895}{36}\zeta(3) + \frac{67}{720}\pi^4 + \frac{53}{6}\pi^2 \ln 2 + H_{2,\text{had}}^{\mathcal{L}\ell} - \frac{5}{4}\pi^2 \sqrt{\rho} \quad \text{with} \quad \rho = \rho^{\mathcal{L}\ell} = m_\ell^2 / m_{\mathcal{L}}^2$$

$H_{2,\text{had}}^{\mu e} = -0.042 \pm 0.002$ $\quad H_{2,\text{had}}^{\tau\mu} = ?$ $\quad H_{2,\text{had}}^{\tau e} = ?$ $\quad$ missing values set to zero in the following

$H_3^{\mu e} = -15.3 \pm 2.3$ $\quad H_3^{\tau\mu} = ?$ $\quad H_3^{\tau e} = ?$ $\quad$ missing values set to zero in the following

[J.Erler & A.Freitas, PDG EW review, 2022] [using modified minimal subtraction scheme ($\overline{\text{MS}}$)]

Uncertainties due to QED radiative corrections for tau LFU tests

uncertainties when using LO QED corrections instead of $\delta R_\gamma^{\tau\ell}$ up to α^2

- up to 140 ppm difference

uncertainties when using $\delta R_\gamma^{\tau\ell}$ up to α^2

- omission of unknown $H_{2,\text{had}}^{\tau(\mu,e)}$, $H_3^{\tau(\mu,e)}$
  - assume $H_{2,\text{had}}^{\tau(\mu,e)} \sim 20 \times H_{2,\text{had}}^{\mu e}$, $H_3^{\tau(\mu,e)} \sim 20 \times H_3^{\mu e}$ $\Rightarrow$ $\Delta(g_\alpha/g_\beta) \leq 2.4\,\text{ppm}$
- tau mass uncertainty on phase space factors $\Rightarrow$ $\Delta(g_\alpha/g_\beta) \leq 1.3\,\text{ppm}$
- uncertainty on $\alpha(m_\tau)$ $\Rightarrow$ $\Delta(g_\alpha/g_\beta) \leq 0.3\,\text{ppm}$
  - $\alpha(m_\tau)$ computed using [F.Jegerlehner 2023 code alphaQEDr23.f]
- up-to-2nd-order $\delta R_\gamma^{\mathcal{L}\ell}$ uncertainty gross estimate small enough also for FCC precision
  - approximate evaluation of $H_{2,\text{had}}^{\tau(\mu,e)}$, $H_3^{\tau(\mu,e)}$ desirable to check above estimates

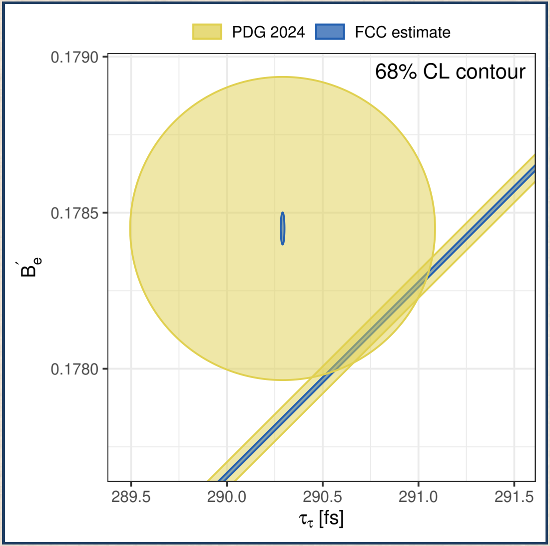
conclusions

- should start using up-to-α^2 corrections
- when up-to-α^2 corrections are used, their uncertainty should be fine also for FCC precision

Lepton universality tests updated to $\delta R_\gamma^{\mathcal{L}\ell}$ up to α^2

	QED rad. corr. 1st order	QED rad. corr. 2nd order
$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f^{\mu e}(1+\delta R_\gamma^{\mu e})(1+\delta R_W^{\mu e})}{\tau_\tau m_\tau^5 f^{\tau e}(1+\delta R_\gamma^{\tau e})(1+\delta R_W^{\tau e})}}$	1.0016 ± 0.0014	1.0016 ± 0.0014
$\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau\mu}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f^{\mu e}(1+\delta R_\gamma^{\mu e})(1+\delta R_W^{\mu e})}{\tau_\tau m_\tau^5 f^{\tau\mu}(1+\delta R_\gamma^{\tau\mu})(1+\delta R_W^{\tau\mu})}}$	1.0018 ± 0.0014	1.0017 ± 0.0014
$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau\mu}}{\mathcal{B}_{\tau e}} \frac{f^{\tau e}(1+\delta R_\gamma^{\tau e})(1+\delta R_W^{\tau e})}{f^{\tau\mu}(1+\delta R_\gamma^{\tau\mu})(1+\delta R_W^{\tau\mu})}}$	1.0002 ± 0.0011	1.0001 ± 0.0011

Canonical tau lepton universality plot extrapolation at FCC-ee



FCC-ee(Z) with $6{\cdot}10^{12}$ Z

Conclusion

- preliminary FCC sensitivities to tau LFV obtained with Fastsim simulations
  - some significant revisions of some former optimistic sensitivity estimates

End

Backup slides

Lepton universality tests

2013 [A.Pich, Precision Tau Physics (2014)]

$\lvert g_\mu/g_e\rvert$	$\Gamma_{\tau\to\mu}/\Gamma_{\tau\to e}$ 1.0018(14)	$\Gamma_{\pi\to\mu}/\Gamma_{\pi\to e}$ 1.0021(16)	$\Gamma_{K\to\mu}/\Gamma_{K\to e}$ 0.9978(20)	$\Gamma_{K\to\pi\mu}/\Gamma_{K\to\pi e}$ 1.0010(25)	$\Gamma_{W\to\mu}/\Gamma_{W\to e}$ 0.996(10)
$\lvert g_\tau/g_\mu\rvert$	$\Gamma_{\tau\to e}/\Gamma_{\mu\to e}$ 1.0011(15)	$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$ 0.9962(27)	$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$ 0.9858(70)		$\Gamma_{W\to\tau}/\Gamma_{W\to\mu}$ 1.034(13)
$\lvert g_\tau/g_e\rvert$	$\Gamma_{\tau\to\mu}/\Gamma_{\mu\to e}$ 1.0030(15)				$\Gamma_{W\to\tau}/\Gamma_{W\to e}$ 1.031(13)

2024 [V.Cirigliano *et al.*, 2022] [HFLAV 2023 report] [PDG 2024]

$\lvert g_\mu/g_e\rvert$	$\Gamma_{\tau\to\mu}/\Gamma_{\tau\to e}$ 1.002(11)	$\Gamma_{\pi\to\mu}/\Gamma_{\pi\to e}$ 1.0010(9)	$\Gamma_{K\to\mu}/\Gamma_{K\to e}$ 0.9978(18)	$\Gamma_{K\to\pi\mu}/\Gamma_{K\to\pi e}$ 1.0009(18)	$\Gamma_{W\to\mu}/\Gamma_{W\to e}$ 1.001(3)
$\lvert g_\tau/g_\mu\rvert$	$\Gamma_{\tau\to e}/\Gamma_{\mu\to e}$ 1.0016(14)	$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$ 0.9958(38)	$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$ 0.9856(75)		$\Gamma_{W\to\tau}/\Gamma_{W\to\mu}$ 1.007(10)
$\lvert g_\tau/g_e\rvert$	$\Gamma_{\tau\to\mu}/\Gamma_{\mu\to e}$ 1.0018(14)				$\Gamma_{W\to\tau}/\Gamma_{W\to e}$ 1.008(10)

- in most cases, tau measurements produce most precise LFU tests
- most precise LFU test using $\Gamma_{\pi\to\mu}/\Gamma_{\pi\to e}$, limited by $\mathcal{B}(\pi\to e\nu)$, best by [PIENU(TRIUMF) 2015]

μ/e universality from $\mathcal{B}(\pi \to e\nu(\gamma))/\mathcal{B}(\pi \to \mu\nu(\gamma))$

- $R^{\pi}_{e/\mu,\ \text{SM}} = 1.23524(15)\cdot 10^{-4}$
- $R^{\pi}_{e/\mu,\ \text{PIENU(2015)}} = 1.2344(23)\text{stat}(19)\text{syst}\cdot 10^{-4}$ PRL 115 (2015) 071801
- $R^{\pi}_{e/\mu,\ \text{EXP}} = 1.2327(23)\cdot 10^{-4}$ PDG 2024
- on-going PEN experiment at PSI also aims to measure $R^{\pi}_{e/\mu}$
- future: PIONEER (PSI) uncertainty goal equal to SM prediction precision

Belle II improvements on tau LFV searches

new Belle II limit on $\tau^- \to \mu^-\mu^+\mu^-$

- $\mathcal{B}(\tau^- \to \mu^-\mu^+\mu^-) < 1.9{\cdot}10^{-8}$ JHEP 09 (2024) 062, 424 fb^{-1}
- previous $\mathcal{B}(\tau^- \to \mu^-\mu^+\mu^-) < 2.1{\cdot}10^{-8}$ 90% CL, Phys.Lett.B687 (2010) 139-143, 782 fb^{-1}