Measurements of $\tau^- \to h^- h^- h^+ \nu_\tau$ branching fractions with Belle detector

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Introduction

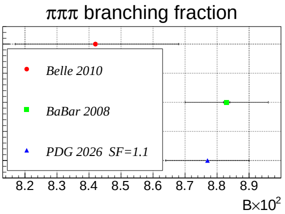
Physics motivation for $\tau^- \to h^- h^+ h^- \nu_\tau$ measurements

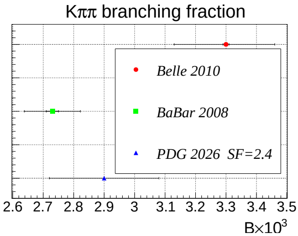
- Cabibbo-allowed and Cabibbo-suppressed decays
- Okubo-Zweig-Iizuka (OZI) rule
- V_{us} measurements in τ decays
- CP violation
- measurement of the m_s

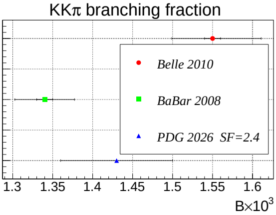
In this analysis, we measure the branching fractions for four modes in $h^- h^+ h^-$ system: $\pi^-\pi^+\pi^-$, $K^-\pi^+\pi^-$, $K^-K^+\pi^-$ and $K^-K^+K^-$.

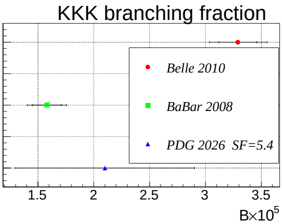
Introduction

Current status of branching fractions measurements for four $\tau^- \to h^- h^+ h^- \nu_\tau$ modes



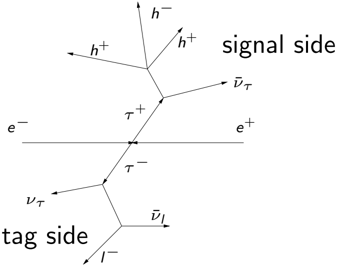






Results from Belle and BaBar contradict each other

Selection criteria



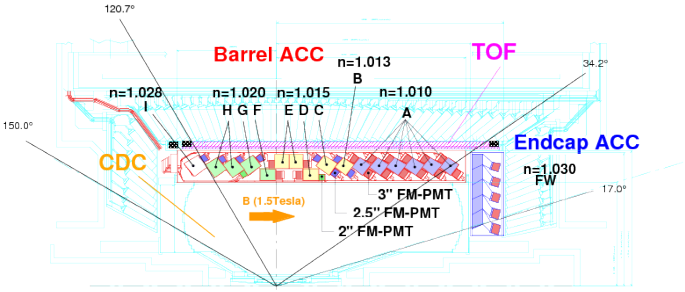
We select $e^+e^- \to \tau^+\tau^-$ events, with one τ-lepton decays to 3 hadrons $+ \nu_\tau$, and the other τ-lepton decays $l + \nu_l + \nu_\tau$

1. We select 4-track events from τ and hadron skims
2. $v_r < 0.5$ cm $|v_z| < 2.5$ cm
3. $Pr(K/\pi) > 0.9$ for K. Otherwise, we assign the tracks as π
4. Lepton (μ, e) $Pr(l) > 0.8$ and three hadrons, zero charge, K_S π^0 veto

1. $max\ p_t > 0.5 GeV/c$
2. $7 > \frac{m_{miss}}{GeV/c^2} > 1$
3. $E^{c.m.s.}_{extra} < 0.3 GeV$
4. $150^\circ > \Theta^{c.m.s.}_{miss} > 30^\circ$
5. $\Theta^{c.m.s.}_{hadrons/lepton} > 90^\circ$

Normalization on $e^+e^- \to \tau^+\tau^- \to \mu^\pm e^\mp \nu\bar{\nu}\nu\bar{\nu}$

$K^{\pm}/\pi^{\pm}$ identification at Belle detector



1. Experimental challenge for $\mathcal{B}(h^- h^+ h^-)$ measurements $\mathcal{B}(\pi^-\pi^+\pi^-) \sim 30 \times \mathcal{B}(K^-\pi^+\pi^-)$, $\mathcal{B}(K^-K^+\pi^-) \sim 70 \times \mathcal{B}(K^-K^+K^-)$
2. Typical fake rate that a pion is misidentified as a kaon is about 3 % in Belle.
3. For $K^-\pi^+\pi^-$, the cross-feed background from $\pi^-\pi^+\pi^-$ is the same order as the signal. For $K^-K^+K^-$, the cross-feed background from $K^-K^+\pi^-$ is the same order.
4. Need to suppress cross-feed background as small as possible (kinematic suppression)
5. Careful calibration of pion (mis)identification efficiency using data.

Kinematic supression

A simple kinematic condition to suppress the feed-down background due to $\pi \to K$ misID can be $m_{had} < m_\tau$. We found a more general condition using the method of the kinematic limits, that includes the above one $max(\mu_{had}^2) > 0$. P - is four-momentum of the $\tau^+\tau^-$, p_{had} - is four-momentum of the $h^-h^-h^+$, p_{lep} - is four-momentum of the charge lepton l^+, $p_{had}^2 = m_{had}^2, p_{lep}^2 = m_{lep}^2$.

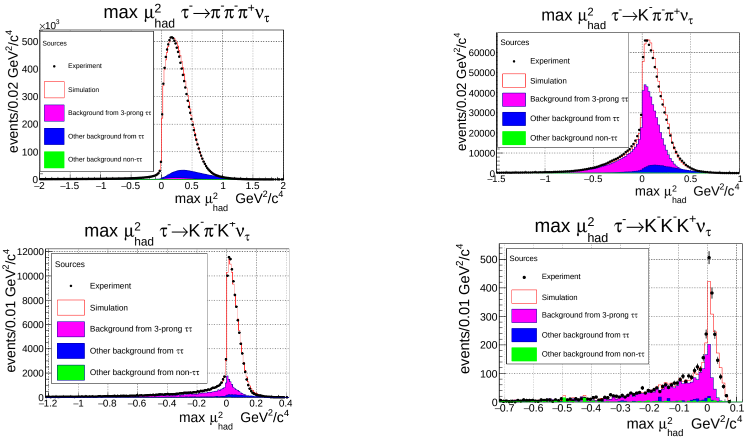
$$max(\mu_{had}^2) = \begin{cases} if \;\; m_\tau^2 + m_{lep}^2 - (Pp_{lep}) - \sqrt{P^2 - 4m_\tau^2}\dfrac{(Pp_{had})(Pp_{lep}) - (p_{had}p_{lep})P^2}{\sqrt{(Pp_{had})^2 - m_{had}^2 P^2}\sqrt{P^2}} \geq 0 \\ m_\tau^2 + m_{had}^2 - (Pp_{had}) + \sqrt{P^2 - 4m_\tau^2}\sqrt{(Pp_{had})^2/P^2 - m_{had}^2} \\ else\ if \;\; [P^2 - 4m_\tau^2][(Pp_{lep})^2/P^2 - m_{lep}^2] - [m_\tau^2 + m_{lep}^2 - (Pp_{lep})]^2 \geq 0 \\ m_\tau^2 + m_{had}^2 - (Pp_{had}) + \dfrac{1}{(Pp_{lep})^2/P^2 - m_{lep}^2} \times \\ \times \Big[[m_\tau^2 + m_{lep}^2 - (Pp_{lep})][(Pp_{had})(Pp_{lep})/P^2 - (p_{had}p_{lep})] + \\ + \sqrt{[(Pp_{had})^2/P^2 - m_{had}^2][(Pp_{lep})^2/P^2 - m_{lep}^2] - [(Pp_{had})(Pp_{lep})/P^2 - (p_{had}p_{lep})]} \\ \sqrt{[P^2 - 4m_\tau^2][(Pp_{lep})^2/P^2 - m_{lep}^2] - [m_\tau^2 + m_{lep}^2 - (Pp_{lep})]^2} \Big] \end{cases}$$

K. Menger. New foundation of Euclidean geometry. American Journal of Mathematics, **53(4)**:(1931) 721–745.

https://arxiv.org/abs/2606.04541

Kinematic suppression

$$max(\mu^2_{had}) > 0$$

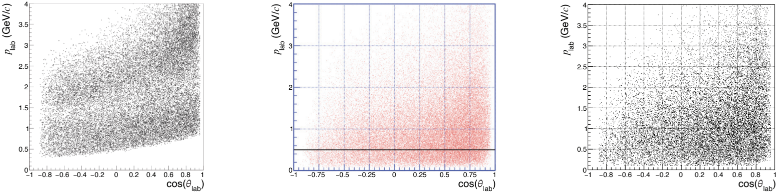


	$K^-\pi^-\pi^+$	$K^-K^+\pi^-$	$K^-K^+K^-$
Loss of the signal efficiency	5%	9%	13%
Reduction of cross-feed background	1.5	2.2	6.0
Reduction of systematic error due to cross-feed	2.0	2.3	4.3

PID correction

$$\frac{\mathcal{E}_{exp}}{\mathcal{E}_{MC}}(Q, p_{lab}, \cos(\theta_{lab}))$$

(a) (b) (c)



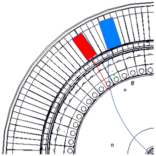
Two-dimension distribution momentum $p_{lab}(GeV/c)$ vs. cosine of the polar-angle $\cos(\theta_{lab})$ of the pions in the laboratory frame from: (a) $D^{*+} \to D^0\pi^+ \to (K^-\pi^+)\pi^+$ sample, (b) $K_S^0 \to \pi^+\pi^-$ sample, (c) $\tau^+\tau^-$ sample.

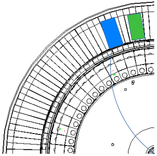
The distribution of K_S^0 sample is close to $\tau^+\tau^-$ sample.

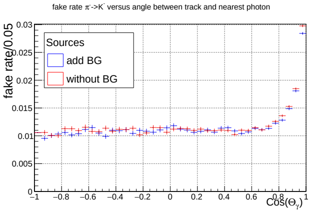
We calibrate pion (mis)ID efficiency by data using $D^{*+} \to D^0(K-\pi+)\pi$ and $K_S^0 \to \pi^+\pi^-$ samples

PID correction

An effect of the photon clusters nearby the charged track







Pion fake rate as a function of the opening angle between the track and the nearest cluster

If cluster is nearby the track ($\cos\Theta_\gamma > 0.75$), the pion-fake rate is increased.

The effect of the random beam-induced photon background is small (Since the red (w/o BG) and blue (w BG) points are almost indetical).

Branching fraction calculation

- Due to the cross-feed background between each other, four signal modes have been analyzed simultaneously. Branching fractions of 4 modes are obtained by solving the linear equation given below

$$N_k^{\mathrm{rec}} - N_k^{\mathrm{bg}} = \sum_{l=1}^{4} \mathcal{E}_{kl} N_{\mathrm{true}}^{l}$$

$$\mathcal{B}_l = N_{\mathrm{true}}^{l} / (2 N_{\tau\tau}^{e\mu})$$

N_k^{rec} - number of reconstructed events, N_k^{bg} -background.

- Wrong electric Charge Configuration (WCC) events $\tau^- \to K^+\pi^-\pi^-\nu_\tau$ and $\tau^- \to K^-K^-\pi^+\nu_\tau$ are due to the $\pi \to K$ mis-ID, since expected branching fractions are very small $O(10^{-12})$. These modes allow checking the correctness of the pion misID rate determined by the PID calibration. To check the correctness of the misID using WCC, we define the χ^2 using 6 modes (4 is signal and 2 is WCC)

$$\chi^2 = \sum_{k=1}^{6} [N_k^{\mathrm{rec}} - N_k^{\mathrm{bg}} - \sum_{l=1}^{4} \mathcal{E'}_{kl} N_{\mathrm{true}}^{l}]^2 / \sigma_k^2$$

Branching fraction calculation

Mode	N^{rec}	N_{sig}	$N^{\mathrm{bg}}_{\tau^+\tau^-}$	$N^{\mathrm{bg}}_{q\bar{q}}$	$N^{\mathrm{bg}}_{\mathrm{other}}$
$\pi^-\pi^-\pi^+$	1.200×10^7	1.093×10^7	1.06×10^6	5.6×10^3	1100
$K^-\pi^-\pi^+$	7.941×10^5	7.170×10^5	7.50×10^4	1.964×10^3	141
$K^-K^+\pi^-$	1.016×10^5	9.959×10^4	1.61×10^3	381	15
$K^-K^-K^+$	1478	1429	25.2	23.7	< 1
$K^+\pi^-\pi^-$	2.794×10^5	2.497×10^5	2.73×10^4	2.17×10^3	51.6
$K^-K^-\pi^+$	1.046×10^4	9.345×10^3	1.06×10^3	57.4	3.2

Number of reconstructed events in each category and estimated background

Rec. \ Gen.	$\pi^-\pi^-\pi^+$	$K^-\pi^-\pi^+$	$K^-K^+\pi^-$	$K^-K^-K^+$
$\pi^-\pi^-\pi^+$	22.84	7.51	2.38	0.92
$K^-\pi^-\pi^+$	0.843	16.45	4.73	2.51
$K^-K^+\pi^-$	0.0140	0.216	11.56	5.99
$K^-K^-K^+$	1.8×10^{-5}	1.5×10^{-3}	0.044	8.40
$K^+\pi^-\pi^-$	0.428	0.161	5.00	1.87
$K^-K^-\pi^+$	9.2×10^{-3}	0.215	0.088	2.59

Efficiency matrix $\mathcal{E}'_{kl}$ in %

PID calibration results

Rec. \ Gen.	$\pi^-\pi^-\pi^+$	$K^-\pi^-\pi^+$	$K^-K^+\pi^-$	$K^-K^-K^+$
$\pi^-\pi^-\pi^+$	0.94	0.98	0.99	1.01
$K^-\pi^-\pi^+$	2.27	0.96	1.00	1.01
$K^-K^+\pi^-$	5.47	2.31	0.98	1.00
$K^-K^-K^+$	14.2	5.83	2.35	1.00
$K^+\pi^-\pi^-$	2.26	2.35	0.98	1.01
$K^-K^-\pi^+$	5.52	2.28	2.34	1.00

The correction factor is about 2.3 for the $\pi \to K$ misID efficiency. We introduce the scale factor (SF) for this $\pi \to K$ misID efficiency. $\pi \to K$ misID correction $= 2.3 \times SF$.

$$n_{kl}^{\pi\to K} = 0,1,2,3$$

$$\hat{\mathcal{E}}_{kl} = \mathcal{E}'_{kl} SF^{n_{kl}^{\pi\to K}},$$

$$\sum_{m=1}^{m=6} \hat{\mathcal{E}}_{ml} = \sum_{m=1}^{m=6} \mathcal{E}'_{ml}$$

We determine this SF factor by minimizing the $\hat{\chi}^2$ defined using 6 modes. Obtained $SF = 1.032 \pm 0.003$ using K_S^0 sample for the calibration of $\pi \to K$ misID. The result is slightly different from the ideal value $SF = 1.0$. We have taken this difference from the ideal value into account in the systematic errors.

$$\hat{\chi}^2 = \sum_{k=1}^{6} [N_k^{\mathrm{rec}} - N_k^{\mathrm{bg}} - \sum_{l=1}^{4} \hat{\mathcal{E}}_{kl}(SF) N_{\mathrm{sig}}^{l}]^2 / \sigma_k^2$$

Systematic uncertainties

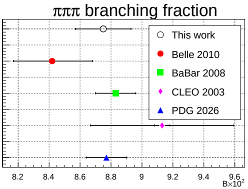
Source \ Mode	$\pi\pi\pi$	$K\pi\pi$	$KK\pi$	KKK
Hadron ID	0.7	4.3	1.7	2.0
Signal modeling	0.3	4.1	0.5	2.0
Neutral veto	1.5	0.5	0.5	1.8
Track reconstruction	0.7	0.7	0.7	0.7
Lepton ID	0.75	0.75	0.75	0.75
Background subtraction	0.2	0.4	0.4	0.45
Trigger	0.35	0.35	0.35	0.35
Normalization	0.16	0.16	0.16	0.16
Total	2.0	6.1	2.2	3.6

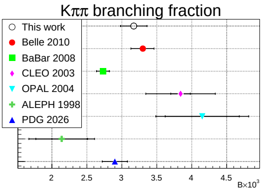
Relative uncertainties in %

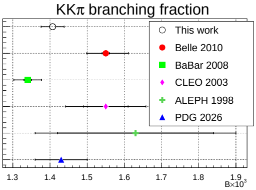
The largest systematic uncertainty for $K\pi\pi$ is from Hadron ID, and it is 4.3%.

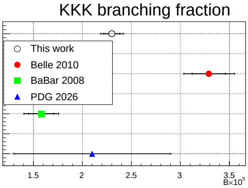
Uncertainty of the signal modeling is checked by model variation.

Results







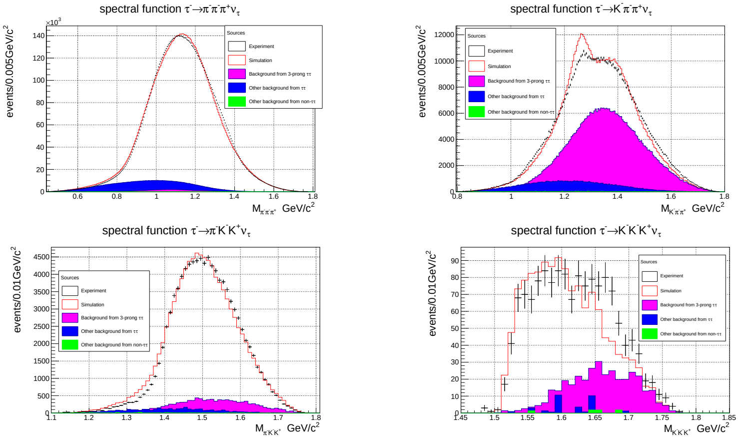


Mode	Units	This work	Belle	*BABAR*
$\pi^-\pi^-\pi^+$	10^{-2}	$8.753 \pm 0.003 \pm 0.176$	$8.42 \pm 0.003^{+0.26}_{-0.25}$	$8.83 \pm 0.01 \pm 0.13$
$K^-\pi^-\pi^+$	10^{-3}	$3.17 \pm 0.01 \pm 0.19$	$3.30 \pm 0.01^{+0.16}_{-0.17}$	$2.73 \pm 0.02 \pm 0.09$
$K^-K^+\pi^-$	10^{-3}	$1.407 \pm 0.005 \pm 0.031$	$1.55 \pm 0.01^{+0.06}_{-0.05}$	$1.346 \pm 0.01 \pm 0.036$
$K^-K^-K^+$	10^{-5}	$2.301 \pm 0.082 \pm 0.082$	$3.29 \pm 0.17^{+0.19}_{-0.20}$	$1.58 \pm 0.13 \pm 0.12$

Conclusions

1. We presented an update of the measurements branching fractions $\boldsymbol{\tau^- \to h^- h^- h^+ \nu_\tau}$ with Belle detector
2. The $\pi^\pm/K^\pm$ identification had been studed carefully:
   - Found correlation between photon clusters and track ID
   - The new correction from $K_S \to \pi^+\pi^-$ decay had been obtained from a low track multiplicity sample
   - Perform several validity tests
3. We introduce a new kinematic condition to suppress cross-feed background
4. Paper in preparation
5. Difference between Belle and Babar is reduced. For $K\pi\pi$, Belle and BaBar are consistent within 2.1σ. BF of KKK mode is 10^{-5} order. Belle and BaBar clearly identify it, but BF results are different by 3.4σ
6. Our measurement supersedes the previous Belle results

Spectral functions

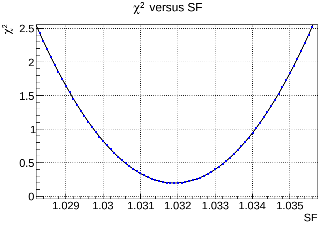


Validation tests

We introduce a scale factor for pion fake rate as:

$$\hat{\mathcal{E}}_{kl} = \mathcal{E}'_{kl} SF^{n_{kl}^{\pi\to K}},$$

$$\sum_{m=1}^{m=6} \hat{\mathcal{E}}_{ml} = \sum_{m=1}^{m=6} \mathcal{E}'_{ml}$$



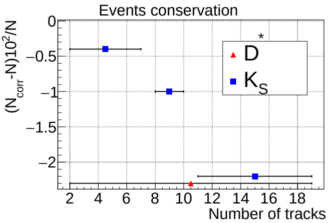
$$\hat{\chi}^2 = \sum_{k=1}^{6} [N_k^{\rm rec} - N_k^{\rm bg} - \sum_{l=1}^{4} \hat{\mathcal{E}}_{kl}(SF) N_{\rm sig}^{l}]^2 / \sigma_k^2$$

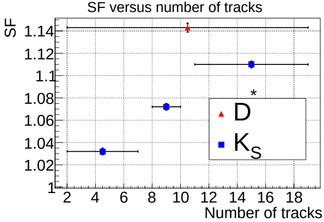
pion ID scale factor $SF = 1.14$ $D^\star$-sample
$SF = 1.032$ K_S^0-sample (final version)

$$\frac{\mathcal{B}(SF) - \mathcal{B}(SF=1)}{\mathcal{B}(SF=1)}$$

Mode	$\pi\pi\pi$	$K\pi\pi$	$KK\pi$	KKK
$D^\star$-sample	$+1.6\%$	-25%	-5%	-30%
K_S^0-sample	$+0.2\%$	-4.3%	-1.4%	-1.3%

Validation tests





$$N(P_{i/j} > P_{cut})^{\tau\tau}_{corr} = \frac{\int_{P_{cut}}^{1} F^{cal}_{exp}(t)dt}{\int_{P_{cut}}^{1} F^{cal}_{mc}(x)dx} \int_{P_{cut}}^{1} F^{\tau\tau}_{mc}(z)dz$$

$$N(P_{i/j} < P_{cut})^{\tau\tau}_{corr} = \frac{\int_{0}^{P_{cut}} F^{cal}_{exp}(t)dt}{\int_{0}^{P_{cut}} F^{cal}_{mc}(x)dx} \int_{0}^{P_{cut}} F^{\tau\tau}_{mc}(z)dz$$

$$\left[N(P_{i/j} > P_{cut})^{\tau\tau}_{corr} + N(P_{i/j} < P_{cut})^{\tau\tau}_{corr} - \int_{0}^{1} F^{\tau\tau}_{mc}(z)dz\right] / \int_{0}^{1} F^{\tau\tau}_{mc}(z)dz$$

$F(x)$ is the response function of PID. Correction only redistributed events! Signal events have low track multiplicity, and matched better with low track multiplicity K_S events.

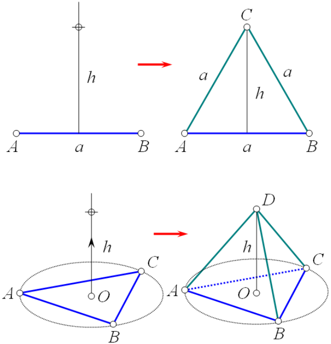
Kinematic suppression

square of the n-1 - dimensional simplex volume expressed via Cayley-Menger determinant, example for n=4.

$$\Delta_4 = \begin{vmatrix} 0 & d_{12}^2 & d_{13}^2 & d_{14}^2 & 1 \\ d_{21}^2 & 0 & d_{23}^2 & d_{24}^2 & 1 \\ d_{31}^2 & d_{32}^2 & 0 & d_{34}^2 & 1 \\ d_{41}^2 & d_{42}^2 & d_{43}^2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$$

d_{ij} distance between i-th and j-th points

$$V_n^2 = \frac{(-1)^{n-1}}{2^n (n!)^2} \Delta_n.$$



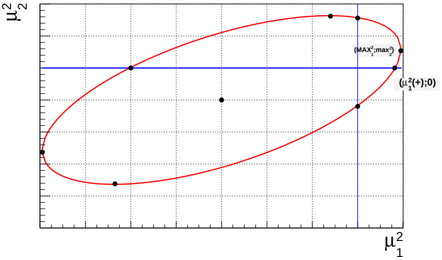
$$\Delta = \left\{ \sum_{i=0}^n t_i A_i : (\sum_{j=0}^n t_j = 1) \wedge (\forall i, t_i \geq 0) \right\}$$

3dim $\epsilon^{\alpha\beta\gamma} a_\alpha b_\beta c_\gamma \epsilon_{\alpha_1\beta_1\gamma_1} a^{\alpha_1} b^{\beta_1} c^{\gamma_1}$

4dim $\epsilon^{\alpha\beta\gamma\rho} a_\alpha b_\beta c_\gamma d_\rho \epsilon_{\alpha_1\beta_1\gamma_1\rho_1} a^{\alpha_1} b^{\beta_1} c^{\gamma_1} d^{\rho_1}$

K. Menger. New foundation of Euclidean geometry. American Journal of Mathematics, **53(4)**:(1931) 721–745.

Kinematic supression



$$\begin{aligned}
P &= p_{\tau^+} + p_{\tau^-} \\
Q &= p_{\tau^+} - p_{\tau^-} \\
Q^2 + P^2 &= 4m_\tau^2 \\
(QP) &= p_{\tau^+}^2 - p_{\tau^-}^2 = 0 \\
(p_{\tau^+} - p_1)^2 &= ((P+Q)/2 - p_1)^2 = \\
&= m_\tau^2 - (Pp_1) - (Qp_1) + m_1^2 = \mu_1^2 \\
&m_\tau^2 - (Pp_2) + (Qp_2) + m_2^2 = \mu_2^2 \\
(Qp_1) &= m_\tau^2 - (Pp_1) + m_1^2 - \mu_1^2 \\
(Qp_2) &= -m_\tau^2 + (Pp_2) - m_2^2 + \mu_2^2 \\
D &= \epsilon^{\alpha\beta\gamma\rho} Q_\alpha P_\beta p_{1\gamma} p_{2\rho}
\end{aligned}$$

$$max(\mu_1^2) =$$

$$= \begin{cases} MAX_1^2, \text{ if } max_2^2 \geq 0 \\ \mu_1^2(+), \text{ else if } (P^2 - 4m_\tau^2)[(Pp_2)^2/P^2 - m_2^2] - \\ -[m_\tau^2 + m_2^2 - (Pp_2)]^2 \geq 0 \end{cases}$$

$$\begin{aligned}
D^2 + [-P^2 g_{\nu\sigma} + P_\nu P_\sigma][[m_\tau^2 - \mu_1^2 + m_1^2 - (Pp_1)]p_2^\nu + [m_\tau^2 - \mu_2^2 + m_2^2 - (Pp_2)]p_1^\nu] \times \\
\times[[m_\tau^2 - \mu_1^2 + m_1^2 - (Pp_1)]p_2^\sigma + [m_\tau^2 - \mu_2^2 + m_2^2 - (Pp_2)]p_1^\sigma] = \\
= [4m_\tau^2 - P^2]\epsilon^{\alpha\beta\gamma\rho}\epsilon_{\alpha_1\beta_1\gamma_1\rho} P_\alpha p_{1\beta} p_{2\gamma} P^{\alpha_1} p_1^{\beta_1} p_2^{\gamma_1}
\end{aligned}$$