Workshop on Tau Physics at Belle II and STCF
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Electromagnetic correction of hadronic tau decays and its implication for muon g-2

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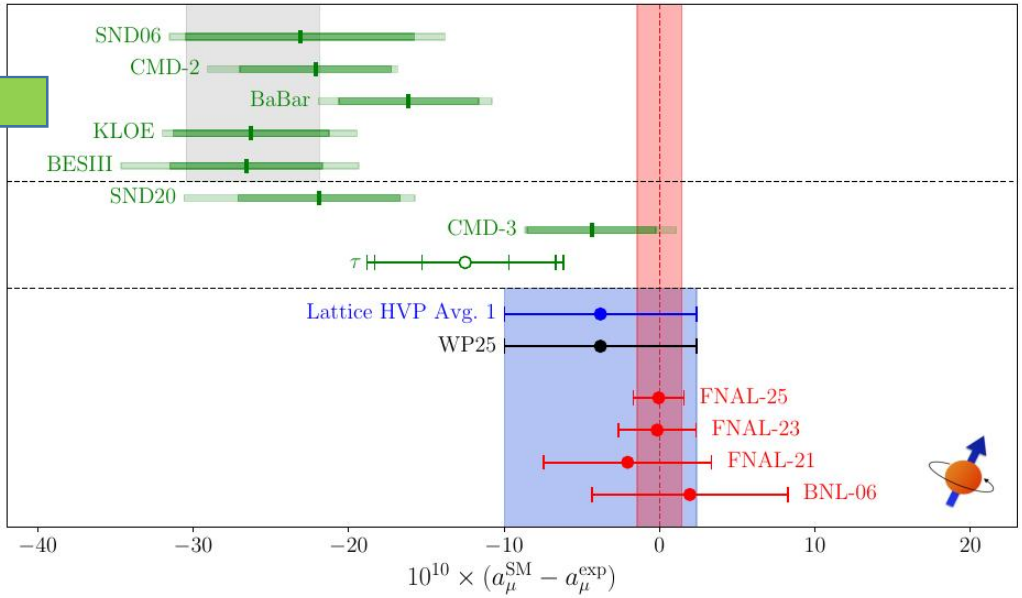
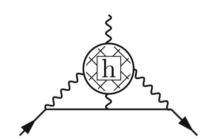
Current status on muon g-2

[2505.21476, White Paper 25]

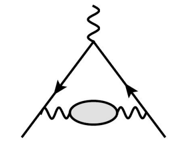


- **HLbL**



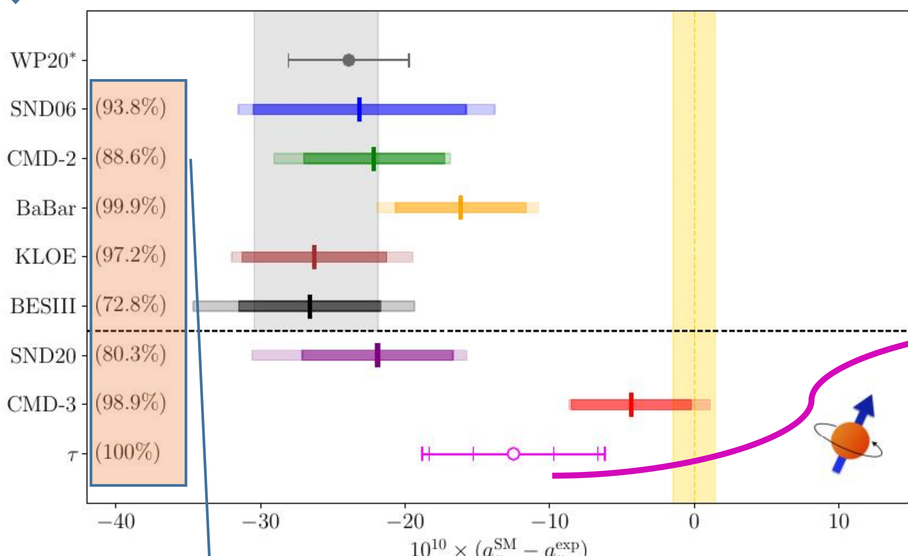
Greatly improved since WP20

LO HVP: most problematic !

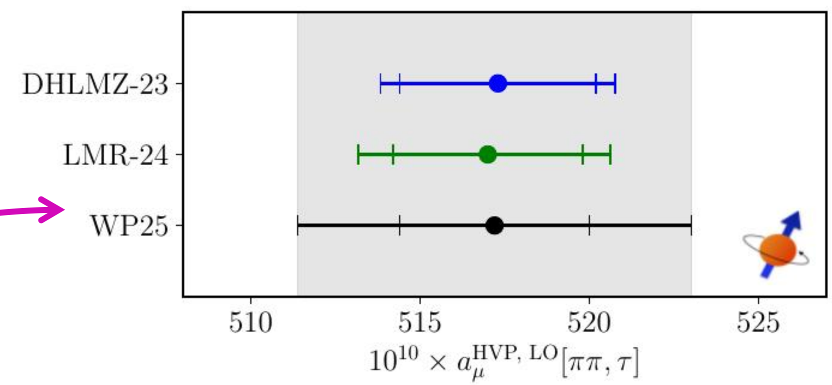


$\pi\pi$: (>70%) (>60%)

- **$e^+e^- \to \pi^+\pi^-$ from each indicated Exp shown in the plots**



$\pi\pi$ contribution from each Exp to HVP integral



[Davier, et al., (DHLMZ), EPJC'24]
[Lopez Castro, et al., (LMR) PRD'25]

Alternative way to address HVP from $\pi\pi$

$$a_\mu^{\mathrm{HVP,LO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} dt\, K(t)\, \sigma^0_{e^+e^-\to\mathrm{hadrons}}(t)$$

Known kernel function (enhanced contribution from energy below 1GeV)

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \frac{\pi\alpha^2}{3t}\beta_{\pi^+\pi^-}\left|F^{(0)}_{\pi\pi}(t)\right|^2$$

$$\frac{d\Gamma(\tau_{2\pi})}{dt} = \frac{G_F^2\,|V_{ud}|^2\, m_\tau^3 S_{\mathrm{EW}}}{384\pi^3}\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+\frac{2t}{m_\tau^2}\right)\beta_{\pi^-\pi^0}\left|F^{(-)}_{\pi\pi}(t)\right|^2$$

$\tau^- \to \pi^-\pi^0\nu_\tau$: $\langle\pi^-\pi^0|\bar{d}\gamma_\mu u|0\rangle \sim F^{(-)}_{\pi\pi}(t)$ [I=1, I_3=-1]

$e^+e^- \to \pi^+\pi^-$: $\langle\pi^+\pi^-|\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d|0\rangle \sim F^{(0)}_{\pi\pi}(t)$ [I=1, I_3=0]

Isospin limit

Conserved vector current (CVC)

$$F^{(0)}_{\pi\pi}(t) = F^{(-)}_{\pi\pi}(t)$$

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \frac{K_\sigma(t)}{K_\Gamma(t)}\,\frac{\beta_{\pi^+\pi^-}}{S_{\mathrm{EW}}\beta_{\pi^-\pi^0}}\,\frac{d\Gamma(\tau_{2\pi})}{dt}$$

$$K_\sigma(t) = \frac{\pi\alpha^2}{3t},\; K_\Gamma(t) = \frac{G_F^2\,|V_{ud}|^2\, m_\tau^3}{384\pi^3}\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+2\frac{t}{m_\tau^2}\right)$$

- **Isospin breaking (IB) effects become CRUCIAL at the sub-percent level.**
- **Full control of all the IB terms is yet to be reached.**

❖ **Key problem:** **isospin breaking (IB) effects**

$$a_\mu^{\mathrm{HVP,LO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} dt\, K(t)\, \sigma^0_{e^+e^-\to\mathrm{hadrons}}(t)$$

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \left[\frac{K_\sigma(t)}{K_\Gamma(t)}\frac{d\Gamma(\tau_{2\pi[\gamma]})}{dt}\right] \times \left(\frac{R_{\mathrm{IB}}(t)}{S_{\mathrm{EW}}}\right)$$

$$R_{\mathrm{IB}}(t) = \frac{FSR(t)}{G_{\mathrm{EM}}(t)}\frac{\beta^3_{\pi^+\pi^-}}{\beta^3_{\pi^-\pi^0}}\frac{\left|F_\pi^{(0)}(t)\right|^2}{\left|F_\pi^{(-)}(t)\right|^2}$$

[Cirigliano, et al., JHEP'02]

[Davier, et al., EPJC'10]

- **FSR: final-state radition**
- **$\beta_{\pi+\pi-}$ / $\beta_{\pi-\pi0}$: kinematics, significant deviation from unity near threshold**
- $F_{\pi\pi}^{(0)}(t)$, $F_{\pi\pi}^{(-)}(t)$ **: vector form factors of $\pi^+\pi^-$ and $\pi^-\pi^0$**
- **G_{EM}: long distance radiative correction to $\tau^- \to \pi^-\pi^0\nu_\tau$**

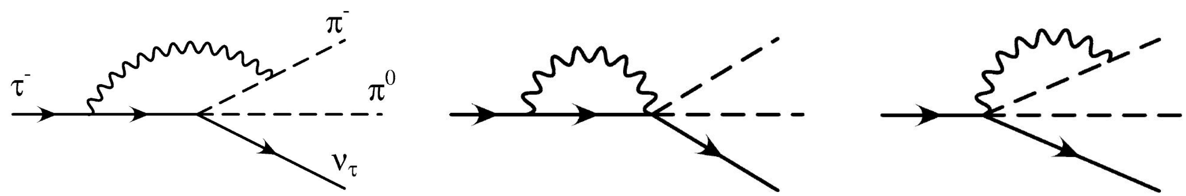
➢ $G_{EM}(t)$: long-distance EM corrections to $\tau^- \to \nu_\tau \pi^0 \pi^-$

$$\frac{d\Gamma(\tau_{2\pi[\gamma]})}{dt} = \underbrace{\frac{G_F^2 |V_{ud}|^2 m_\tau^3 S_{\rm EW}}{384\pi^3}\left(1-\frac{4m_\pi^2}{t}\right)\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+\frac{2t}{m_\tau^2}\right)\left|F_{\pi\pi}^{(-)}(t)\right|^2}_{\boxed{d\Gamma_{\tau\to\pi\pi\nu}/dt}} \boxed{G_{\rm EM}(t)}$$

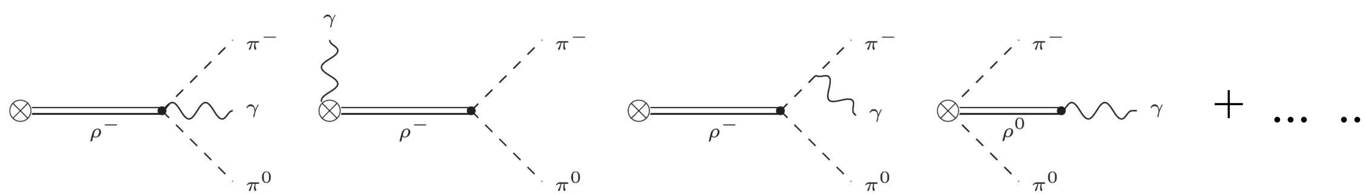
$G_{EM}(t) \sim$ virtual photon + real photon

virtual photon (loops)
[Cirigliano, et al.,PLB'01]

recent developments by including the pion FF in the loops [Colangelo, et al.,PRL'26]



real photon (radiative decay, hadronic modeling needed)

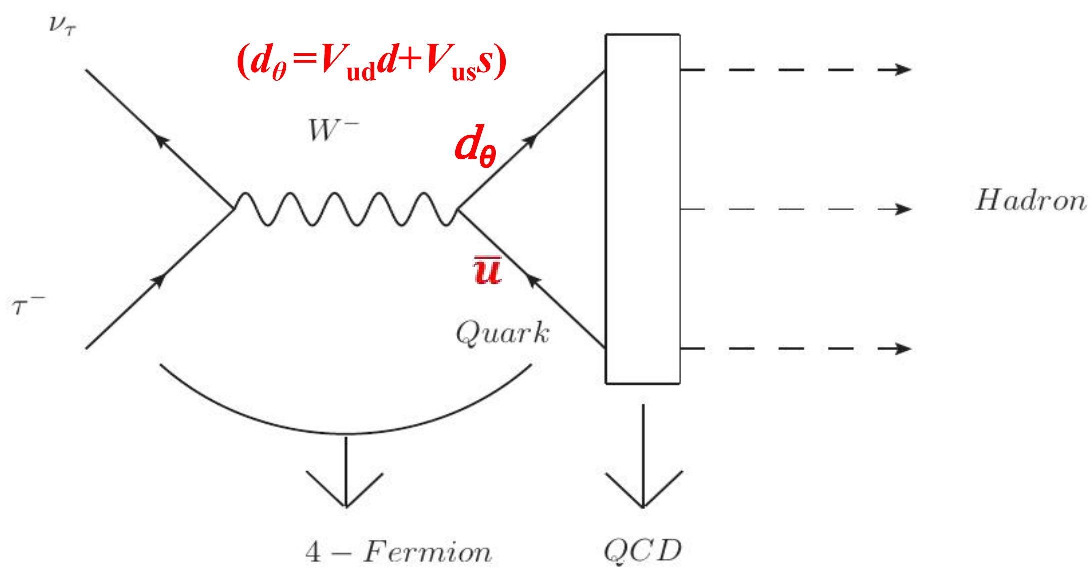


- ➢ **G_{EM} is infrared finite: cancellation between photon loop and bremsstrahlung of the real photon.**
- ➢ **Experimental measurement of $\tau \to \pi\pi\gamma\nu_\tau$ is absent: theoretical estimation needed.**

➢ Rich dynamics involving light-flavor hadrons:

(1) $\pi\pi$ vector form factor (2) $\rho^{(',")} \to \pi\gamma$, $\omega^{(',")} \to \pi\gamma$ (3) $a_1^{(',")} \to \rho\pi, \pi\gamma$

Resonance chiral theory & $\tau \to \pi\pi\gamma\nu_\tau$ decay



Hadronic V-A currents

$$\mathrm{H}_\mu = \langle H^- | \bar{u}\gamma_\mu (1-\gamma_5) d_\theta \, e^{iL_{QCD}} | 0 \rangle$$

Chiral EFT is the low energy realization of QCD:

$$e^{i\mathcal{Z}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}G_\mu \; e^{i\int d^4x \mathcal{L}_{QCD}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}u \; e^{i\int d^4x \mathcal{L}_{EFT}(v_\mu, a_\mu, s, p)}$$

$$\mathcal{L}^{QCD} = \mathcal{L}_0^{QCD} + \bar{q}\gamma^\mu (v_\mu + a_\mu \gamma_5) q - \bar{q}(s - i\gamma_5 p) q$$

v_μ, a_μ, s, p **are the external source fields .**

Amplitude of $\tau \to \pi\pi\gamma\nu_\tau$

$$\mathcal{M} = eG_F V_{ud}^{*} \varepsilon^{u}(k)^{*} \left\{ F_\nu \bar{u}(q)\gamma^\nu (1-\gamma_5)(m_\tau + \not{P} - \not{k})\gamma_\mu u (V_{\mu\nu} - A_{\mu\nu})\bar{u}(q)\gamma^\nu(1-\gamma_5)u(P) \right\}$$

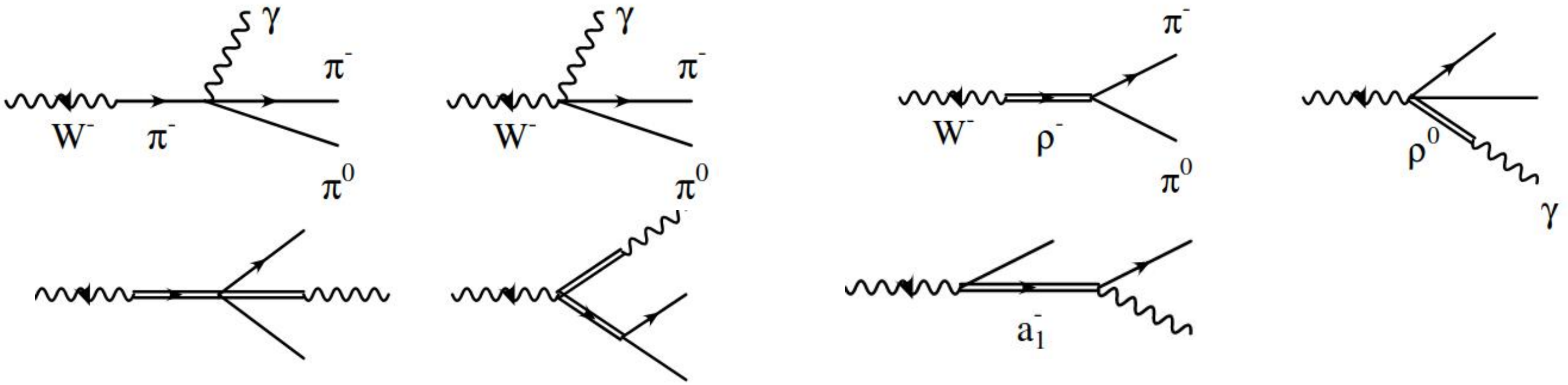
$$F_\nu = (p_2 - p_1)_\nu F_{\pi\pi}^{(-)}(t)/\left(2P\cdot k\right)$$

$$\begin{aligned} V_{\mu\nu} =& F_{\pi\pi}^{(-)}(u)\frac{p_{1\mu}}{p_1\cdot k}(p_1+k-p_2)_\nu - F_{\pi\pi}^{(-)}(u)g_{\mu\nu} + \frac{F_{\pi\pi}^{(-)}(u)-F_{\pi\pi}^{(-)}(t)}{(p_1+p_2)\cdot k}(p_1+p_2)_\mu (p_2-p_1)_\nu + \\ &+ v_1\left(g_{\mu\nu}p_1\cdot k - p_{1\mu}k_\nu\right) + v_2\left(g_{\mu\nu}p_2\cdot k - p_{2\mu}k_\nu\right) + v_3\left(p_{1\mu}p_2\cdot k - p_{2\mu}p_1\cdot k\right)p_{1\nu} + \\ &+ v_4\left(p_{1\mu}p_2\cdot k - p_{2\mu}p_1\cdot k\right)(p_1+p_2+k)_\nu \ , \end{aligned}$$

$$A_{\mu\nu} = i\left(a_1\epsilon_{\mu\nu\rho\sigma}p_1^\rho k^\sigma + a_2\epsilon_{\mu\nu\rho\sigma}p_2^\rho k^\sigma + a_3 p_{1\nu}\epsilon_{\mu\rho\beta\sigma}k^\rho p_1^\beta p_2^\sigma + a_4 p_{2\nu}\epsilon_{\mu\rho\beta\sigma}k^\rho p_1^\beta p_2^\sigma\right)$$

- $F_{\pi\pi}^{(-)}(t)$: **structure independent FF**
- v_i & a_i: **strucutre dependent FFs**

- **Minimal RChT contributions to $\tau \to \pi\pi\gamma\nu_\tau$** [Cirigliano et al., JHEP'02]



- **Other extentions by including anomalous vertices, such as the ρωπ types, and even-parity vertices of the $a_1\rho\pi$, are also studied.**

[Flores-Tlalpa, et al., PRD'05] [Miranda, Roig, PRD'20]

- **Dedicated study of the isospin-breaing effect is considered to calibrate the tau data in the estimation of muon g-2.**

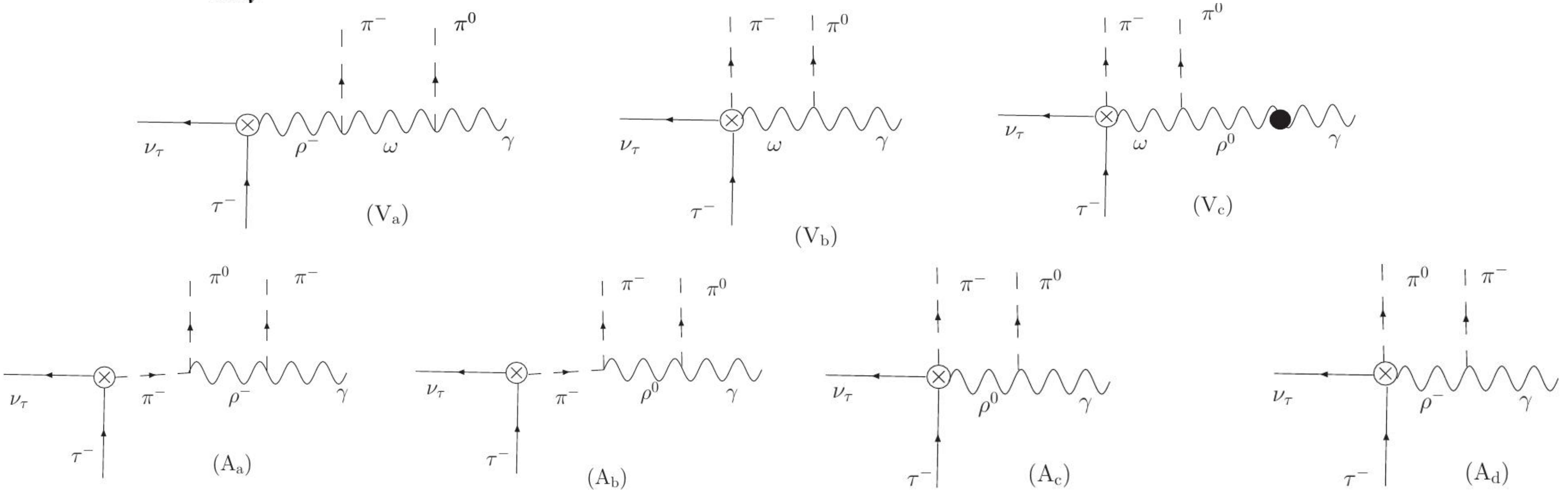
[Cirigliano,et al.,JHEP'02][Flores-Baez,et al.,PRD'06][Davier,et al.,EPJC'10][Miranda,Roig,PRD'20]

Contributions from VVP and VJP operators in RChT

[Ruiz-Femenia, Pich, Portoles, JHEP'03]

$$\begin{aligned}\mathcal{L}_{VVP} = \quad & d_1 \varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + i d_2 \varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\sigma}\}\chi_-\rangle \\ & + d_3 \varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu}, V^{\rho\alpha}\}u^\sigma\rangle + d_4 \varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\}u_\alpha\rangle\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{VJP} = \quad & \frac{c_1}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \frac{c_2}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\alpha}, f_+^{\rho\sigma}\}\nabla_\alpha u^\nu\rangle \\ & + \frac{ic_3}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\sigma}\}\chi_-\rangle + \frac{ic_4}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle V^{\mu\nu}[f_-^{\rho\sigma}, \chi_+]\rangle \\ & + \frac{c_5}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu}, f_+^{\rho\alpha}\}u^\sigma\rangle + \frac{c_6}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\alpha}, f_+^{\rho\sigma}\}u^\nu\rangle \\ & + \frac{c_7}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, f_+^{\rho\alpha}\}u_\alpha\rangle\,.\end{aligned}$$



[Chen, Duan, ZHG , JHEP'22]

High energy contraints to the resonance couplings

$$\int d^4x \int d^4y e^{i(p\cdot x+q\cdot y)}\langle 0|T[V_\mu^a(x)V_\nu^b(y)P^c(0)]|0\rangle$$
$$= d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \Pi_{\mathrm{VVP}}(p^2,q^2,r^2),$$

$$\lim_{\lambda\to\infty}\Pi^{(8)}_{\mathrm{VVP}}[(\lambda p)^2,(\lambda q)^2,(\lambda p+\lambda q)^2]$$
$$=\lim_{\lambda\to\infty}\Pi^{(0)}_{\mathrm{VVP}}[(\lambda p)^2,(\lambda q)^2,(\lambda p+\lambda q)^2]$$
$$=-\frac{\langle\bar\psi\psi\rangle_0}{2\lambda^4}\frac{p^2+q^2+r^2}{p^2q^2r^2}[1+\mathcal{O}(\alpha_S)]+O\left(\frac{1}{\lambda^6}\right).$$

$$c_1+4c_3=0 \qquad c_1-c_2+c_5=0 \qquad c_5-c_6=\frac{N_C M_V}{64\sqrt{2}\pi^2 F_V}$$

$$d_1+8d_2=-\frac{N_c M_V^2}{(8\pi F_V)^2}+\frac{F^2}{4F_V^2} \qquad d_3=-\frac{N_c M_V^2}{(8\pi F_V)^2}+\frac{F^2}{8F_V^2}$$

Other constraints from scattering and form factors

$$F_A=F_\pi,\quad F_V=\sqrt{2}F_\pi,\quad G_V=F_\pi/\sqrt{2}.$$

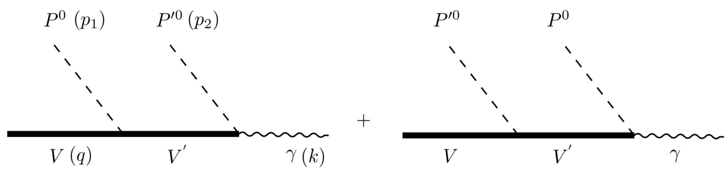
Or

$$F_A=\sqrt{2}F_\pi,\quad F_V=\sqrt{3}F_\pi,\quad G_V=F_\pi/\sqrt{3}$$

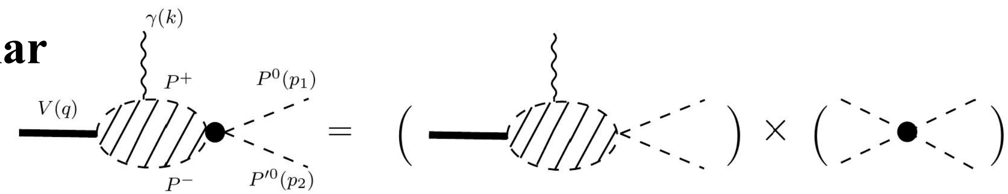
On-shell approxiamtion to the $J\omega\pi$ vertex and additional input from the $\omega \to \pi^0\pi^0\gamma$ decay width

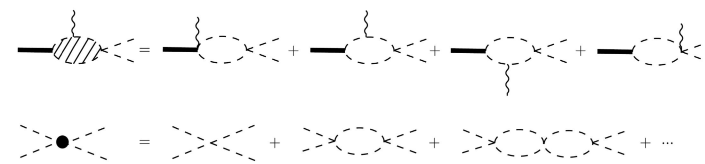
[Z.X.Li, A.Li, J.Hao, C.G.Duan, ZHG, PRD'26]

Vector



Scalar





Condition for the cancellation of UV divergences of the loop functions:

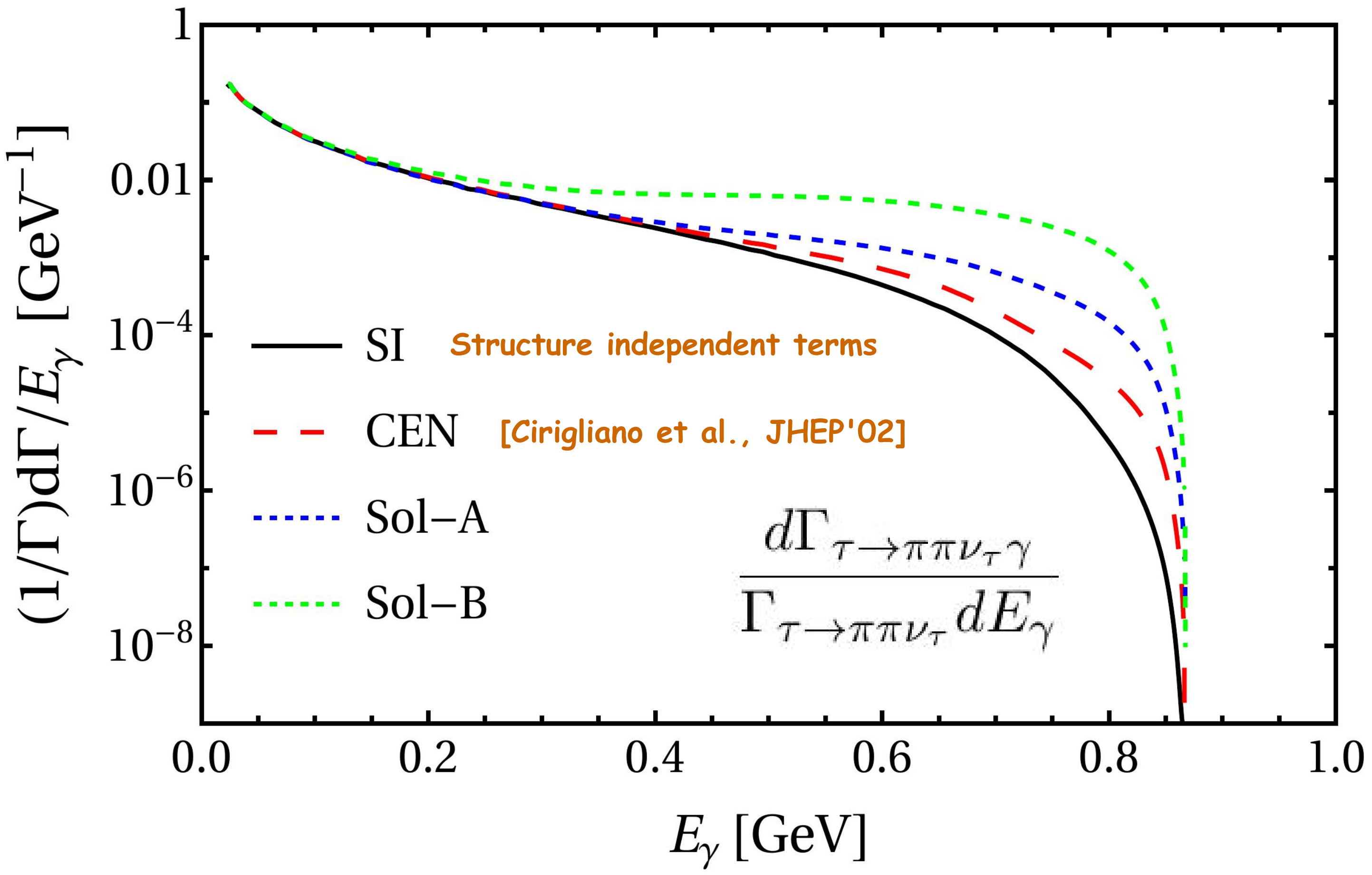
$$F_V = 2G_V$$

This gives preference to: $F_A = F_\pi, \quad F_V = \sqrt{2}F_\pi, \quad G_V = F_\pi/\sqrt{2}\,.$

$$\Gamma^{\mathrm{Exp}}_{\omega\to\pi^0\pi^0\gamma} = (5.8 \pm 1.0) \times 10^{-4}\ \mathrm{MeV} \quad \Rightarrow \quad \begin{aligned} d_4 &= -0.42 \pm 0.07\,, && \textbf{(Sol-A)} \\ d_4 &= 1.01 \pm 0.07\,. && \textbf{(Sol-B)} \end{aligned}$$

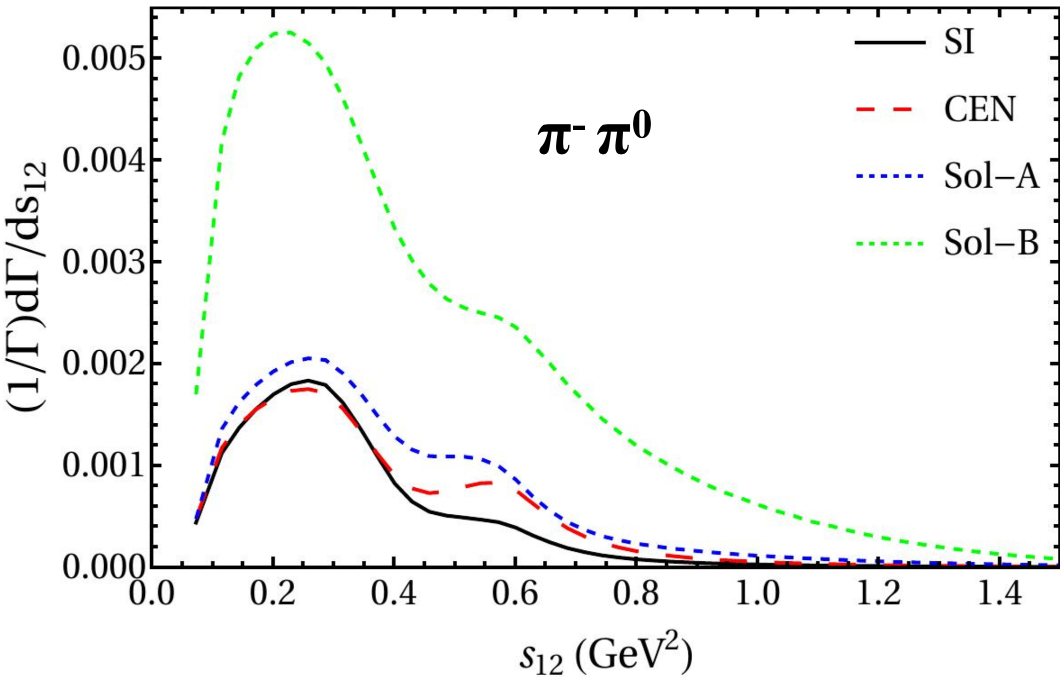
Important: we are left with a parameter free theoretical amplitude for the $\tau \to \pi\pi\gamma\nu_\tau$ process !

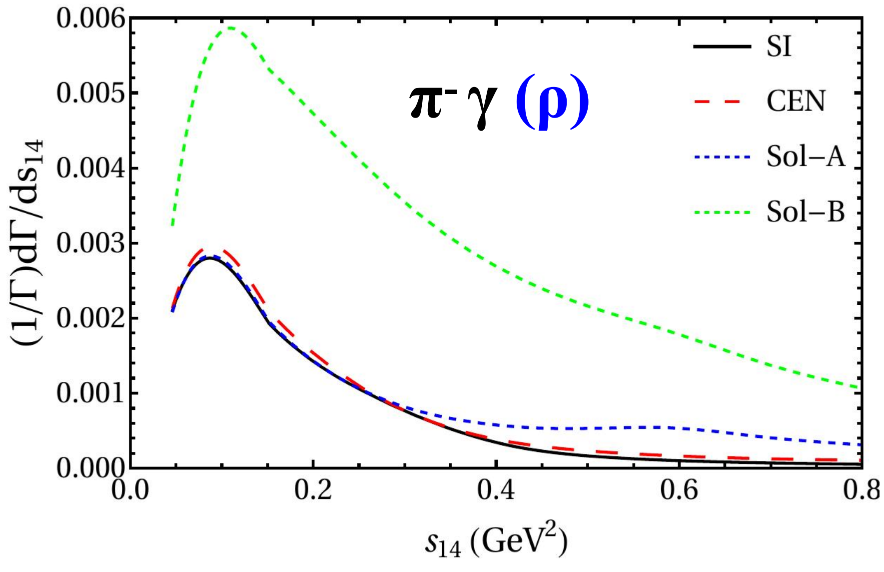
❖ **Differential decay widths as a function of photon energies**

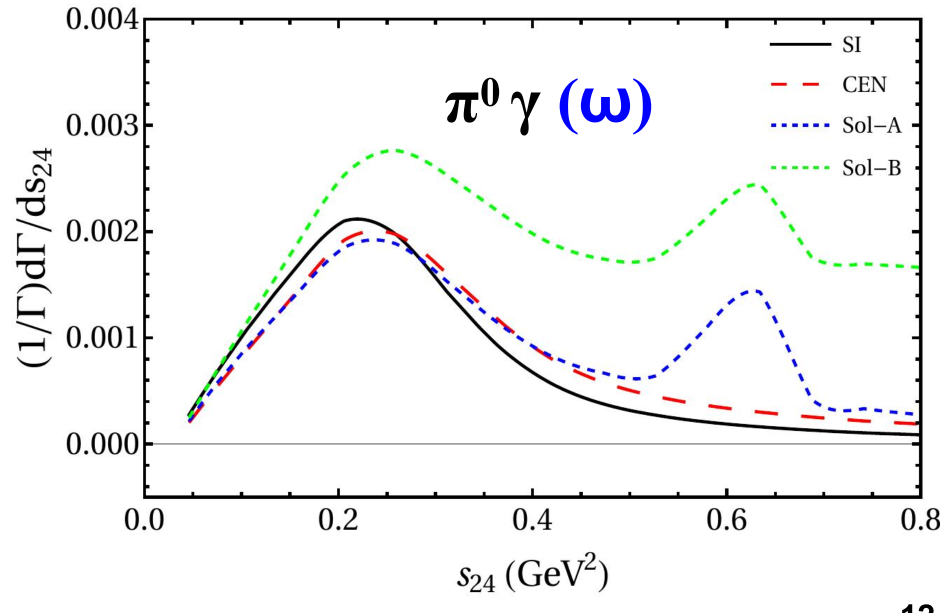


- ➢ **When the photon energy cutoff is around 300 MeV, the absolute branching ratio is predicted to be around 10^{-4}.**
- ➢ **$\tau \to \pi\pi\gamma\nu_\tau$ has good chance to be well measured with reasonable photon energy cuts, probably @ Belle-II, STCF, CEPC, ...**

❖ Invariant-mass distributions of the $\pi\pi$, $\pi^-\gamma$ and $\pi^0\gamma$ systems







❖ Impact on the muon g-2 [Z.X.Li, A.Li, J.Hao, C.G.Duan, ZHG, PRD'26]

$$\frac{d\Gamma_{\tau\pi\pi[\gamma]}}{dt} = \frac{d\Gamma^{(0)}_{\tau\pi\pi}}{dt} G_{\mathrm{EM}}(t)$$

$$\frac{d\Gamma^{(0)}_{\tau\pi\pi}}{dt} = \frac{G_F^2 \left|V_{ud}\right|^2 m_\tau^3 S_{\mathrm{EW}}}{384\pi^3} \left(1 - \frac{4m_\pi^2}{t}\right)^{\frac{3}{2}} \left(1 - \frac{t}{m_\tau^2}\right)^2 \left(1 + \frac{2t}{m_\tau^2}\right) \left|F^{(-)}_{\pi\pi}(t)\right|^2$$

(Non-radiative two-pion tau decay)

$$G_{\mathrm{EM}}(t) = 1 + G^{(v)}_{\mathrm{EM}}(t) + G^{(r)}_{\mathrm{EM}}(t)$$

Photon loops:

(virtual correction)

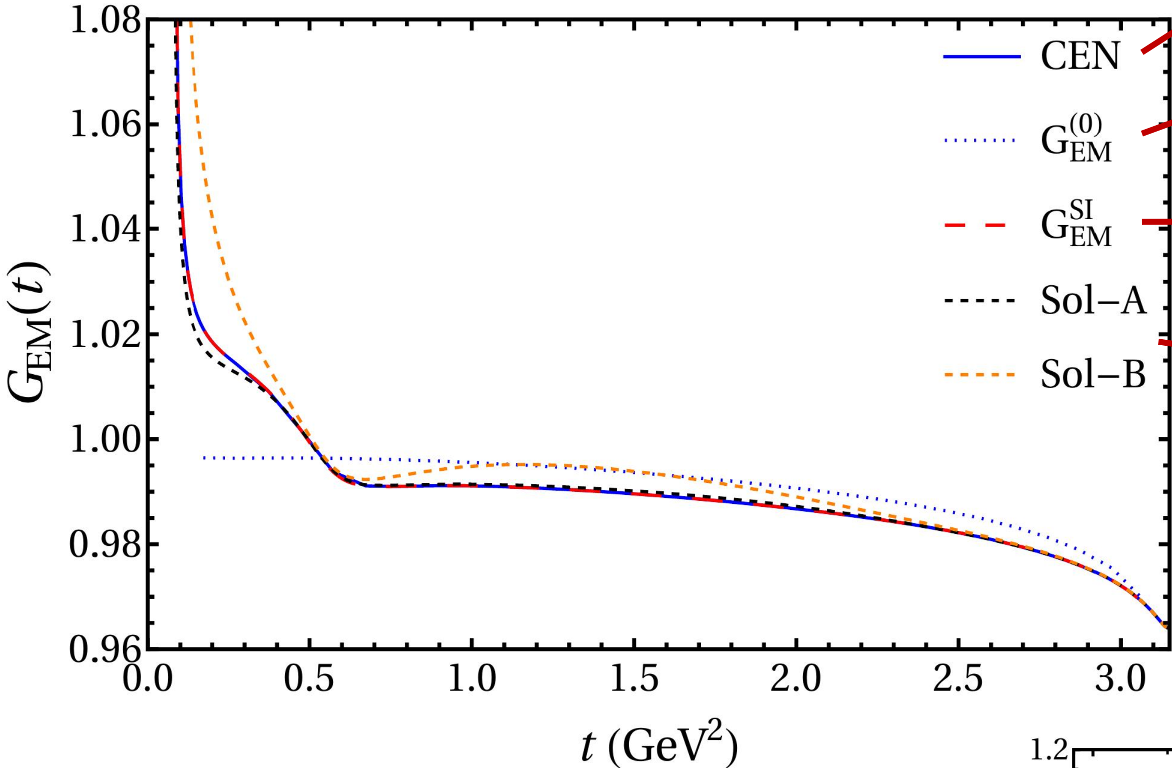
$$G^{(v)}_{\mathrm{EM}}(t) = \frac{12 \int_{u_{\min}}^{u_{\max}} D(t,u)\, f^{\mathrm{elm}}_{\mathrm{loop}}(u, M_\gamma) du}{m_\tau^6 \left(1 - \frac{4m_\pi^2}{t}\right)^{3/2} \left(1 - \frac{t}{m_\tau^2}\right)^2 \left(1 + \frac{2t}{m_\tau^2}\right)}$$

Radiative decay(real correction) from $\tau \to \pi\pi\gamma\nu_\tau$

$$G^{(r)}_{\mathrm{EM}}(t) = \frac{1}{\frac{d\Gamma^{(0)}_{\tau\pi\pi}}{dt}} \frac{\pi^2}{32(2\pi)^{12} m_\tau^2} \int_{S^-_{\pi\pi\nu}}^{S^+_{\pi\pi\nu}} dS_{\pi\pi\nu} \int_{S^-_{\nu\gamma}}^{S^+_{\nu\gamma}} dS_{\nu\gamma} \int_{S^-_{\pi\nu}}^{S^+_{\pi\nu}} dS_{\pi\nu} \int_{S^-_{\pi\nu\gamma}}^{S^+_{\pi\nu\gamma}} \frac{dS_{\pi\nu\gamma}}{\sqrt{-\Delta_4}} \overline{|\mathcal{M}|}^2_{\tau\to\pi\pi\nu_\tau\gamma}$$

- **Infraed divergences cancelled by the virtual and real corrections**
- **In practice, a tiny value of the photon mass is kept in numerical calculations**

Results for G_{EM} & R_{IB}



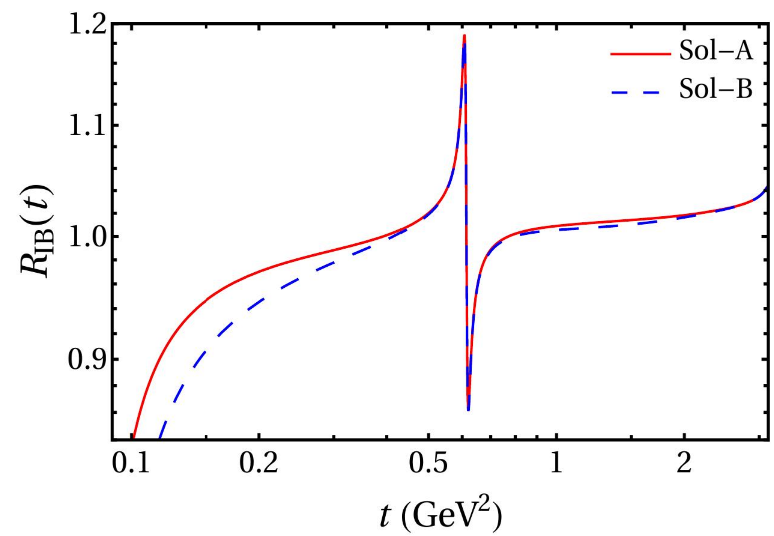
Minimal RChT

Leading Low Approximation

Structure independent

Ours with negative and positve values of d_4

$$R_{\mathrm{IB}}(t)=\frac{\mathrm{FSR}(t)}{G_{\mathrm{EM}}(t)}\,\frac{\beta^3_{\pi^+\pi^-}(t)}{\beta^3_{\pi^-\pi^0}(t)}\,\frac{\left|F^{(0)}_{\pi\pi}(t)\right|^2}{\left|F^{(-)}_{\pi\pi}(t)\right|^2}$$



❖ Impact on the muon g-2

Relative shifts to isospin symmetric case:

$$\Delta a_\mu^{\rm HVP,LO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{t_{max}} dt\, K(t)\left[\frac{K_\sigma(t)}{K_\Gamma(t)}\frac{d\Gamma_{\tau_{\pi\pi[\gamma]}}}{dt}\right]\times\left(\frac{R_{\rm IB}(t)}{S_{\rm EW}}-1\right)$$

Shifts caused by G_{EM} for different parameter inputs (in units of 10^{-11}):

	$\Delta a_\mu^{\rm HVP,LO}[\pi\pi]$ from $G_{\rm EM}(t)$					
$[t_{min}, t_{max}]$	Sol-A	Sol-B	CEN	MR$[\mathcal{O}(p^4)]$		MR$[\mathcal{O}(p^6)]$
$[4m_\pi^2, 1\,\mathrm{GeV}^2]$	-7.1	-44.9	-10.6	-10.4	-15.9	-63.2 ± 16.5
$[4m_\pi^2, 2\,\mathrm{GeV}^2]$	-6.4	-44.5	-9.8	-9.6	-15.2	-58.1 ± 12.2
$[4m_\pi^2, 3\,\mathrm{GeV}^2]$	-6.3	-44.4	-9.7	-9.5	-15.1	-67.8 ± 17.5
$[4m_\pi^2, m_\tau^2]$	-6.3	-44.4	-9.7	-9.5	-15.1	-64.9 ± 13.4

[Cirigliano et al., JHEP'02] [Miranda, Roig, PRD'20]

➢ **The shift of a_μ caused by G_{EM} reaches the current Exp uncertainty: 14.5×10^{-11} !**

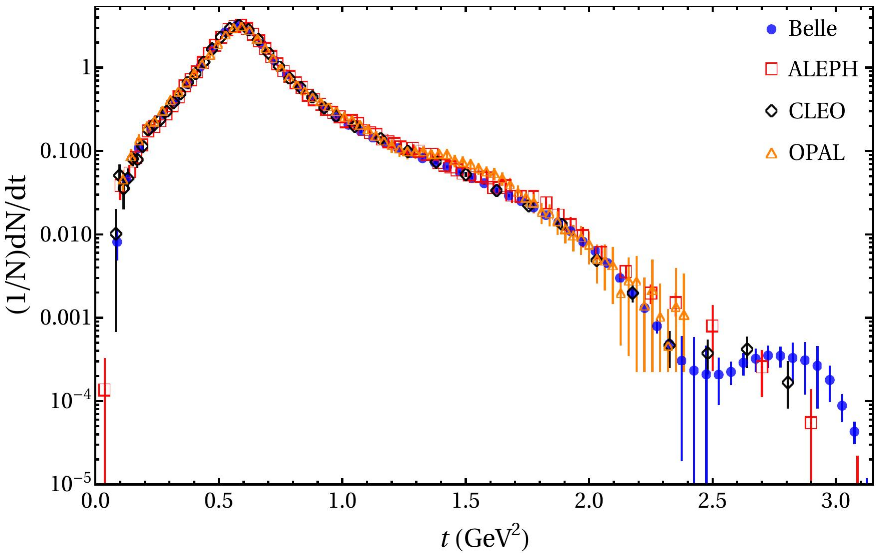
❖ Impact on the muon g-2

Evaluation of $e^+e^- \rightarrow \pi^+\pi^-$ cross section by using tau data with IB corrections

$$\sigma^0_{e^+e^-\to\pi^+\pi^-}(t) = \frac{2\pi\alpha^2 m_\tau^2}{3|V_{ud}|^2 t}\frac{1}{\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+\frac{2t}{m_\tau^2}\right)}\frac{\mathcal{B}_{\tau\pi\pi}}{\mathcal{B}_{\tau e}}\boxed{\frac{1}{N_{\tau\pi\pi}}\frac{dN_{\tau\pi\pi}}{dt}}\frac{R_{\mathrm{IB}}(t)}{S_{\mathrm{EW}}}$$

$$a_\mu^{\mathrm{HVP,LO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} dt\, K(t)\, \sigma^0_{e^+e^-\to\mathrm{hadrons}}(t)$$

Parameters	Experiments	$a_\mu^{\mathrm{HVP,LO}}\|_{\pi\pi,\tau\mathrm{data}}$
Sol-A	Belle	$516.7 \pm 2.1 \pm 7.9 \pm 2.2$
	ALEPH	$513.3 \pm 4.3 \pm 2.8 \pm 2.1$
	CLEO	$516.9 \pm 3.2 \pm 8.8 \pm 2.2$
	OPAL	$527.2 \pm 9.8 \pm 6.8 \pm 2.1$
Sol-B	Belle	$513.4 \pm 2.0 \pm 7.9 \pm 2.2$
	ALEPH	$510.2 \pm 4.2 \pm 2.8 \pm 2.1$
	CLEO	$513.7 \pm 3.2 \pm 8.8 \pm 2.2$
	OPAL	$523.5 \pm 9.5 \pm 6.8 \pm 2.1$



$$a_\mu^{\mathrm{HVP,LO}|_{\pi\pi,\tau\mathrm{data,\ Belle}}} = 516.7 \pm 2.1_{\mathrm{Spec}} \pm 7.9_{\mathrm{BR}} \pm 2.2_{\mathrm{IB}} \pm 3.3_{\mathrm{Sys}}$$

$$a_\mu^{\mathrm{HVP,LO}|_{\pi\pi,\tau\mathrm{data,\ ALEPH}}} = 513.3 \pm 4.3_{\mathrm{Spec}} \pm 2.8_{\mathrm{BR}} \pm 2.1_{\mathrm{IB}} \pm 3.2_{\mathrm{Sys}}$$

$$a_\mu^{\mathrm{HVP,LO}|_{\pi\pi,\tau\mathrm{data,\ CLEO}}} = 516.9 \pm 3.2_{\mathrm{Spec}} \pm 8.9_{\mathrm{BR}} \pm 2.2_{\mathrm{IB}} \pm 3.3_{\mathrm{Sys}}$$

$$a_\mu^{\mathrm{HVP,LO}|_{\pi\pi,\tau\mathrm{data,\ OPAL}}} = 527.2 \pm 9.8_{\mathrm{Spec}} \pm 6.8_{\mathrm{BR}} \pm 2.1_{\mathrm{IB}} \pm 3.7_{\mathrm{Sys}}$$

$$10^{10} \cdot a_\mu^{\mathrm{HVP,LO}|_{\pi\pi,\tau\mathrm{data}}} = 516.0 \pm 2.9_{\mathrm{Spec+BR}} \pm 4.0_{\mathrm{IB+Sys}}$$

❖ Impact on the muon g-2 from tau data

- To combine the ππ (evaluted from tau data) and others from WP20 (πππ, KK, πγ, ...), we have the full result for the LO HVP

$$10^{10} \cdot a_\mu^{\mathrm{HVP,LO}|_{\tau \mathrm{data}}} = 702.1 \pm 2.9_{\mathrm{Spec+BR}} \pm 4.0_{\mathrm{IB+Sys}} \pm 2.1_{\mathrm{Others}}$$

- To further combine the contributions from QED, EW, hadronic light-by-light from WP25 and compare with the newest world average of muon g-2, we have

$$\Delta a_\mu \equiv a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = (14.9 \pm 5.6) \times 10^{-10}$$

$\boxed{\mathbf{2.7\sigma}}$

[Z.X.Li,A.Li,J.Hao,C.G.Duan,ZHG, PRD'26]

❖ $\tau \to \pi\pi\gamma\nu_\tau$: A new place to look for T-odd asymmetry

A typical T-odd kinematical variable:

$$\xi \equiv \varepsilon_{\mu\nu\rho\sigma} a^\mu b^\nu c^\rho d^\sigma \xlongequal[\text{of particle } a]{\text{rest frame}} \vec{b} \cdot (\vec{c} \times \vec{d})\, m_a / s_a$$

a, b, c, d*: either *momentum* or *spin

❖ **When focusing on the situation with four momenta**

$$\xi = \varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma \xlongequal[\text{of particle } 1]{\text{rest frame}} \vec{p_2} \cdot (\vec{p_3} \times \vec{p_4})\, m_1 \qquad A_\xi = \frac{N_+ - N_-}{N_+ + N_-} \quad \text{with} \quad N_\pm = \int_{\xi \gtrless 0} \mathrm{d}\Gamma$$

- **Early proposals to search T-odd triple-product asym in $K_{l3\gamma}$, i.e. $K \to \pi\gamma l\nu_l$**

[Braguta et al., PRD'02 '03] [Muller et al., EPJC'06][Rudenko, PRD'11] ......

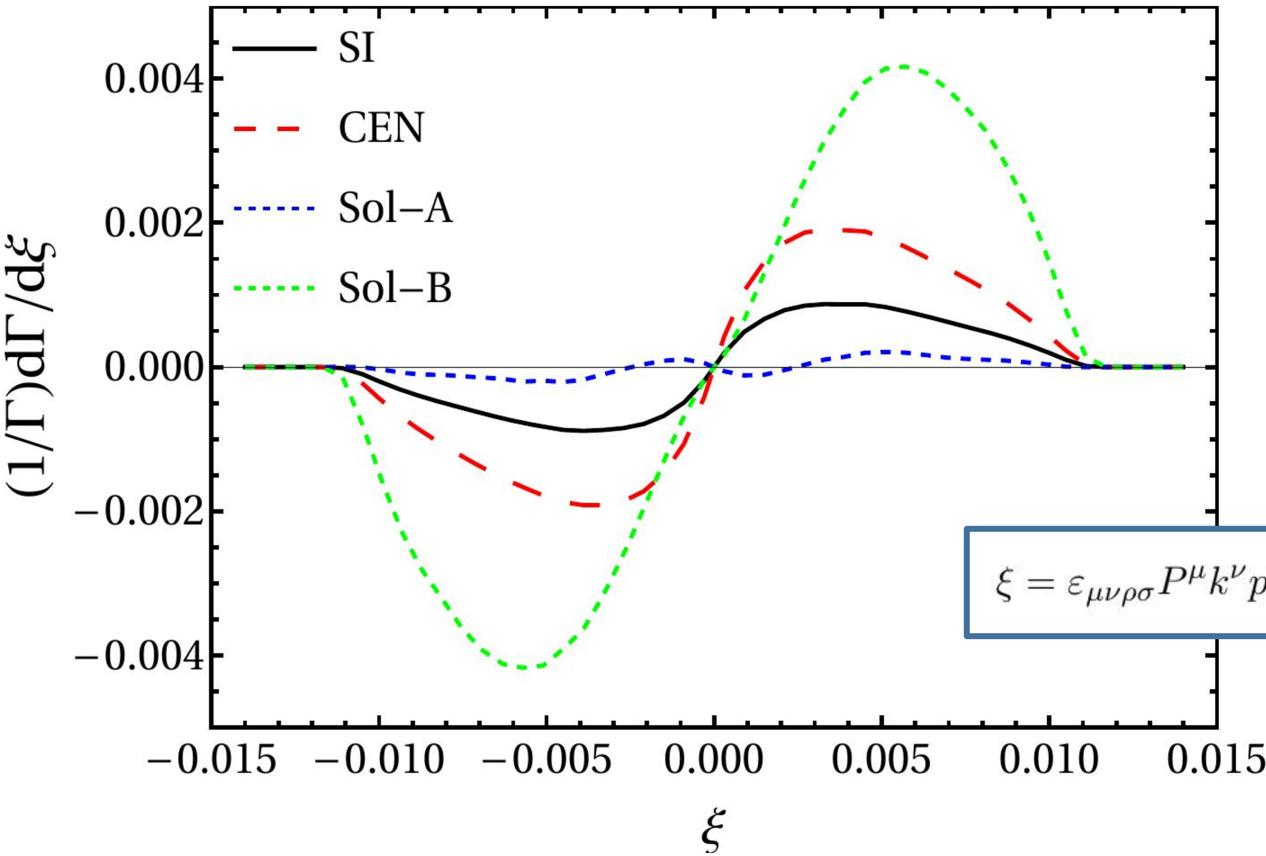
Strong interactions are suppressed for the T-odd asymmetry in $K_{l3\gamma}$: $A_\xi \sim 10^{-4}$

- **Enhancement in $\tau \to \pi\pi\gamma\nu_\tau$** [Chen, Duan, ZHG , JHEP'22]

Hadronic interactions are not suppressed !

$\tau \to \pi\pi\gamma\nu_\tau$: <u>good place to probe T-odd triple-product asymmety</u>

Predicitons of the T-odd asymmetry distributions with respect to ξ



$$\tau^-(P) \to \pi^-(p_1)\pi^0(p_2)\nu_\tau(q)\gamma(k)$$

$$\xi = \varepsilon_{\mu\nu\rho\sigma}P^\mu k^\nu p_1^\rho p_2^\sigma/m_\tau^4 \xlongequal[\text{of }\tau]{\text{rest frame}} \vec{k}\cdot(\vec{p}_1\times\vec{p}_2)/m_\tau^3$$

$$A_\xi = \frac{N_+ - N_-}{N_+ + N_-} \quad \text{with} \quad N_\pm = \int_{\xi\gtrless 0} \mathrm{d}\Gamma$$

- **The magnitudes of A_ξ for $\tau \to \pi\pi\gamma\nu_\tau$ are around 10^{-2}, that is roughly two orders larger than those in $K_{l3\gamma}$. It has the good chance to be measured in Belle-II and super tau-charm facilities.**

Summary

- **Rich phenomenologies in $\tau^- \to \pi^-\pi^0\gamma\nu_\tau$: photon spectrum (useful inputs for the estimation of muon g-2 from tau data), different resonance interactions in the $\pi^-\pi^0$, $\pi^-\gamma$, $\pi^0\gamma$ spectra.**
- **Tension of muon g-2 between SM and Fermilab Exp is reduced to 2.7σ, by relying on the current $\tau^- \to \pi^-\pi^0\nu_\tau$ data and our estimation of radiative corrections.**
- **We give a promising prediction of the triple-product T-odd asymmetry in this tau decay: a useful guide for future experiments, suhc as Belle-II and STCF.**

谢谢大家！