Cinvestav

Workshop on τ Physics at Belle II and SCTF
Changchun, China , 08/30-09/01 2026

Testing CVC in e^+e^- collisions and τ decays: the quest for precision

G. López Castro, Cinvestav, México
gabriel.lopez@cinvestav.mx

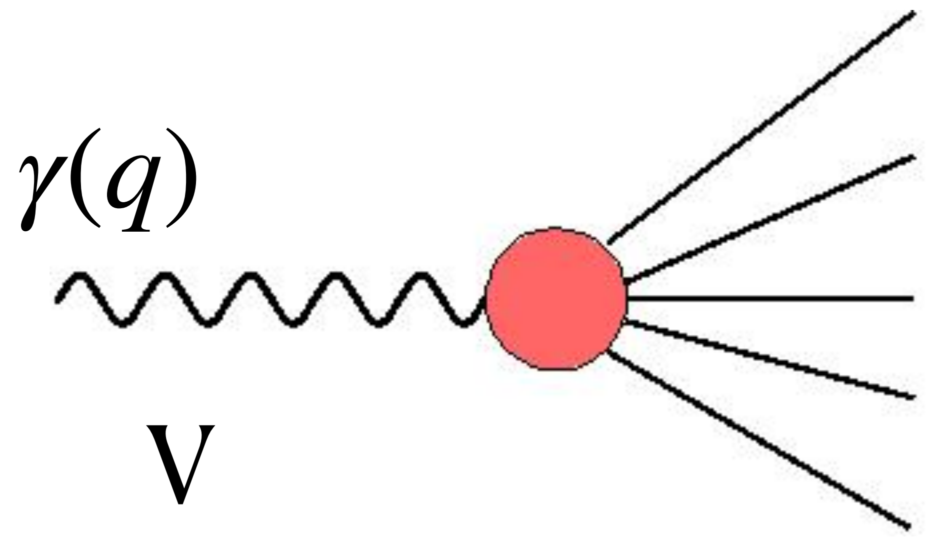
In collaboration with (2005-2026): M. Davier, A. Hoecker, F. Flores-Baez, A. Flores-Tlalpa, B. Malaescu, J. Miranda, X. Mo, K. Raya, P. Roig, G. Toledo, P. Wang, C. Z. Yuan, Z. Zhang

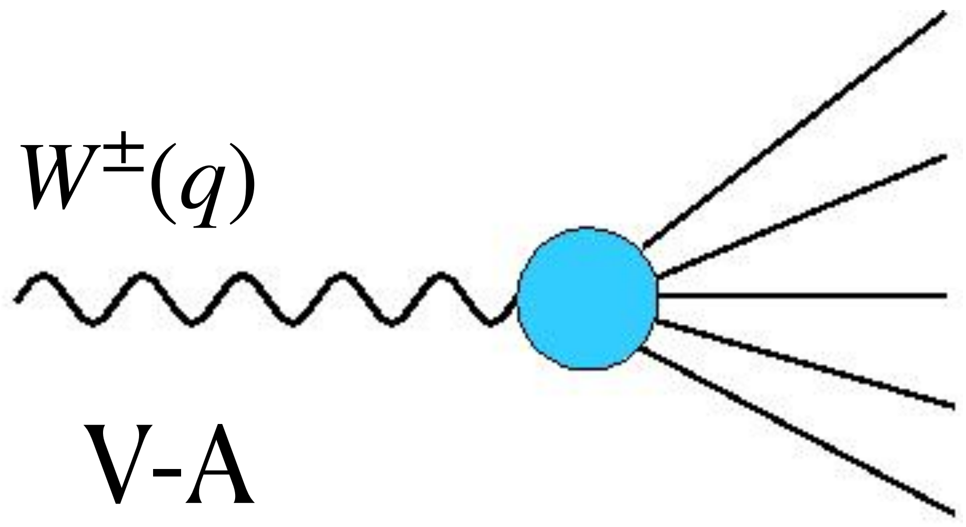
Outline

- Y.-S. Tsai, PRD (1971);
- H. Thacker & J.J Sakurai, PLB (1971)

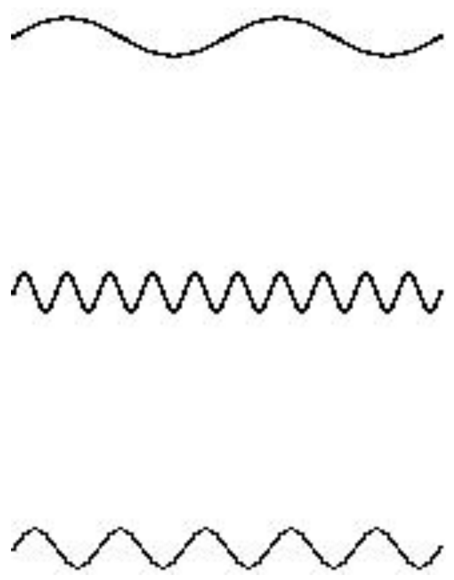
- CVC hypothesis and Isospin breaking,
- Some "loose" tests of CVC in e^+e^- vs τ data,
- Precision test of CVC in 2π channel and SM prediction of $(g-2)_\mu$
- Final comments

Clean hadron production in e^+e^- collisions and τ decays



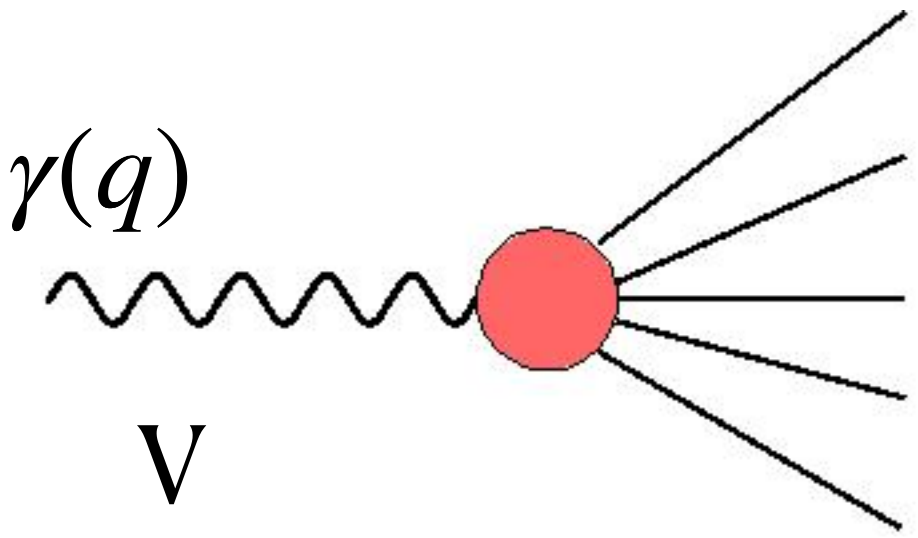


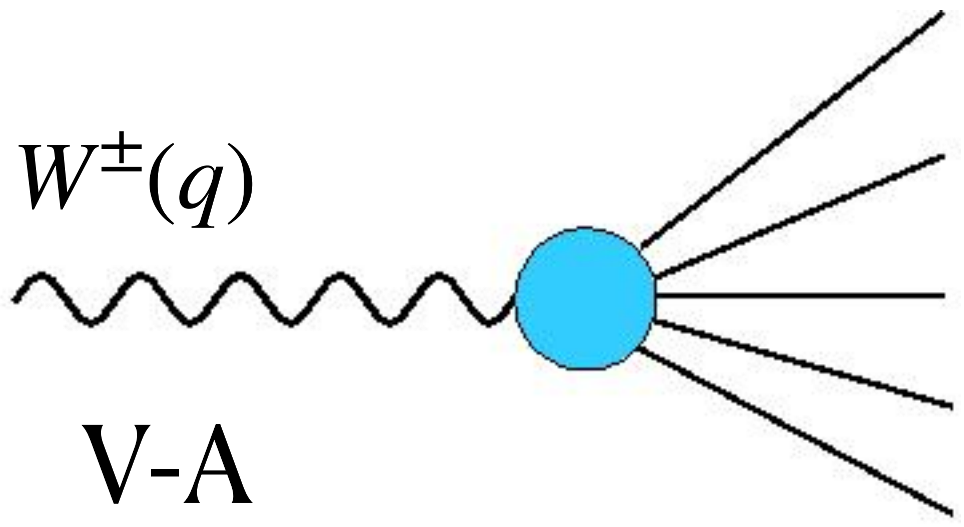
Form Factors



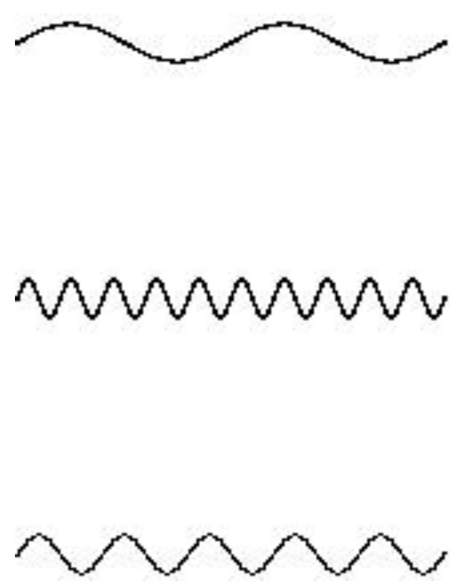
- Chiral symmetries at low q^2 and $q^2 = 0$
- Asymptotic behaviour at large q^2 predicted by QCD $(1/q^2)^N$
- Rich resonance structure in the intermediate region $q^2 \sim 1\ \text{GeV}^2$

Clean hadron production in e^+e^- collisions and τ decays





Form Factors



- Chiral symmetries at low q^2 and $q^2 = 0$
- Asymptotic behaviour at large q^2 predicted by QCD $(1/q^2)^N$
- Rich resonance structure in the intermediate region $q^2 \sim 1\ \mathrm{GeV}^2$

$\Longrightarrow$ Precision tests of the SM, spectroscopy, extraction SM parameters, hadron model predictions, tests of flavor symmetries, ...

Talks by Zhang, Li, Dai, Umeeda, Gonzalez-Solis, Epifanov, Guo, Bobrov, Abbas

Vector currents: isospin properties and CVC hypothesis

- Isospin properties **non-strange** currents: $\left[I_i,\ V_j^\mu(x)\right] = i\epsilon_{ijk}V_k^\mu(x)$, i,j,k=1,2,3

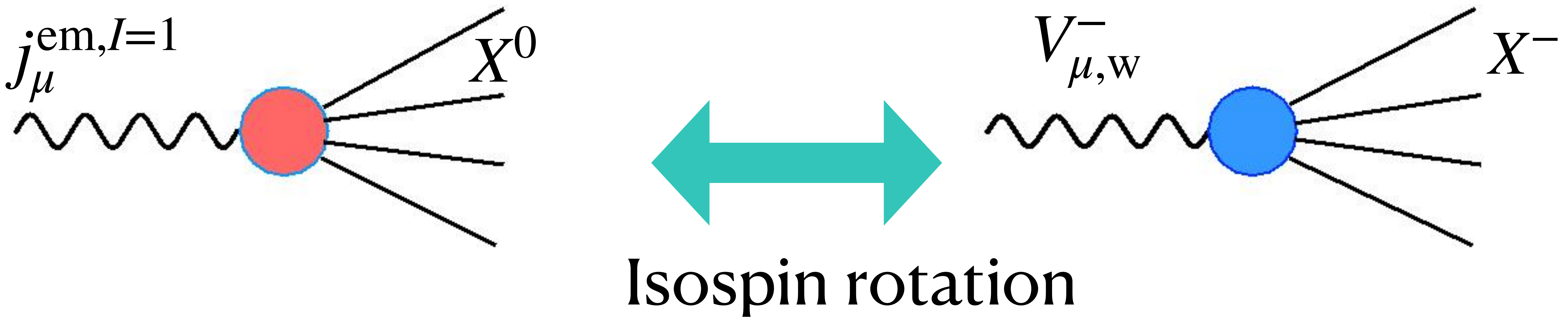
  with $J_{\rm em}^{\mu,I=1} = V_3^\mu,\ V_{\rm weak}^{\mu,\pm} = V_1^\mu \pm iV_2^\mu$

- Isospin symmetry:

  1) $\partial^\mu V_\mu^i = 0$, **CVC hypothesis**, exact relation HME/observables;

  2) Exact $G = Ce^{i\pi I_2}$ parity, $G\,|X^{0,-}\rangle = +\,|X^{0,-}\rangle$, $G[j_{\rm em}^{\mu,I=1},\ j_{\rm w}^{\mu,\pm}]G^{-1} = +\,[j^{\mu,I=1}, j_{\rm w}^{\mu,\pm}]$

Feynman-Gell-Mann '58; Gershtein-Zeldovich, '55



Independent of dynamics, for any q^2 value!

CVC related processes

Low-q

- $n \to p e^- \bar{\nu}_e \iff$ n, p charges and magnetic moments
- $\pi^+ \to \pi^0 e^+ \nu_e \iff$ pion charge
- $0^+ \to 0^+ e^+ \nu_e \iff$ charges of nuclei

High-q

- $\tau^- \to \pi^- \pi^0 \nu_\tau \iff e^+ e^- \to \pi^+ \pi^-$
- $\tau^- \to (2n\pi)^- \nu_\tau \iff e^+ e^- \to (2n\pi)^0, \; n \geq 2$
- $\tau^- \to \eta \pi^- \pi^0 \nu_\tau \iff e^+ e^- \to \eta \pi^+ \pi^-$
- $\tau^- \to (\omega, \phi) \pi^- \nu_\tau \iff e^+ e^- \to (\omega, \phi) \pi^0. \; \cdots$

Tree-level

$$\frac{\mathbf{d\Gamma^0}(\tau^- \to \mathbf{X}^- \nu_\tau)}{\mathbf{ds}} = \frac{s}{2m_\tau^3} \left(\frac{G_F |V_{ud}|}{8\pi^2 \alpha} \right)^2 (m_\tau^2 - s)^2 (m_\tau^2 + 2s) \sigma^{\mathbf{0}}(\mathbf{e}^+ \mathbf{e}^- \to \mathbf{X^0}) \, , \qquad I^G[X^{0,-}] = 1^+$$

$$s = q^2 = m_X^2, \qquad s_{\text{thres}} \leq s \leq m_\tau^2$$

- Old tests of CVC hypothesis (1980-2010)
- Consistency of BR predictions
- Isospin Breaking not required

$$\frac{d\Gamma_\tau^{\rm CVC}(X^-\nu_\tau)}{ds} = \mathcal{F}(s)\sigma_{ee}(X^0)(s)$$

$$\mathcal{F}(s) \equiv \frac{s}{2m_\tau^3}\left(\frac{G_F|V_{ud}|}{8\pi^2\alpha}\right)^2 (m_\tau^2 - s)^2(m_\tau^2 + 2s)$$
$$= \frac{3s}{2m_\tau^8\alpha^2}\Gamma^0(\tau \to e\nu\bar{\nu})|V_{ud}|^2(m_\tau^2 - s)^2(m_\tau^2 + 2s)$$

Better understanding of the CVC validity can be expected after very high data sample experiments at B-factories in the τ sector as well as the future improvement of the accuracy after two groups at VEPP-2M complete their analysis. Moreover, ex-

Branching ratio $B(\tau^- \to X^-\nu_\tau)$, %

Hadronic State X	CVC, 2000	World Average	WA - CVC
$\pi^-\pi^0$	24.94 ± 0.23	25.31 ± 0.18	0.37 ± 0.29
$\pi^-3\pi^0$	1.02 ± 0.05	1.08 ± 0.10	0.06 ± 0.11
$2\pi^-\pi^+\pi^0$	3.55 ± 0.20	4.19 ± 0.23	0.64 ± 0.30
$\omega\pi^-$	1.73 ± 0.06	1.92 ± 0.07	0.19 ± 0.09
$\eta\pi^-\pi^0$	0.13 ± 0.02	0.17 ± 0.02	0.04 ± 0.03
Others	0.37 ± 0.11	0.24 ± 0.02	-0.13 ± 0.11
Total	30.01 ± 0.33	30.99 ± 0.31	0.98 ± 0.45

S. Eidelman, Tau Workshop 2000

- Isospin is broken (IB): **1)** $m_u \neq m_d$ **and, 2)** $Q_u \neq Q_d$ $\qquad [\partial^\mu V_\mu^{\rm weak} \neq 0]$

  But still good symmetry, IB effects of $O((m_u - m_d)/\Lambda)$, $O(\alpha) \sim$ few %

- IB effects are calculable, model-dependent in general (except LQCD)

  $\longrightarrow$ precision comparison theory/experiment

$$\frac{d\Gamma(\tau^- \to X^- \nu_\tau)}{ds} = \frac{s}{2m_\tau^3}\left(\frac{G_F|V_{ud}|}{8\pi^2\alpha}\right)^2 (m_\tau^2 - s)^2(m_\tau^2 + 2s)\sigma(e^+e^- \to X^0)\,\frac{S_{EW}G_{EM}(s)|F^{X^-}(s)|^2}{\mathrm{FSR}(s)|F^{X^0}(s)|^2}\mathrm{PS}$$

short-distance Electroweak

Long-distance electromagnetic

Phase-space

FSR in e^+e^-

Hadronic structure

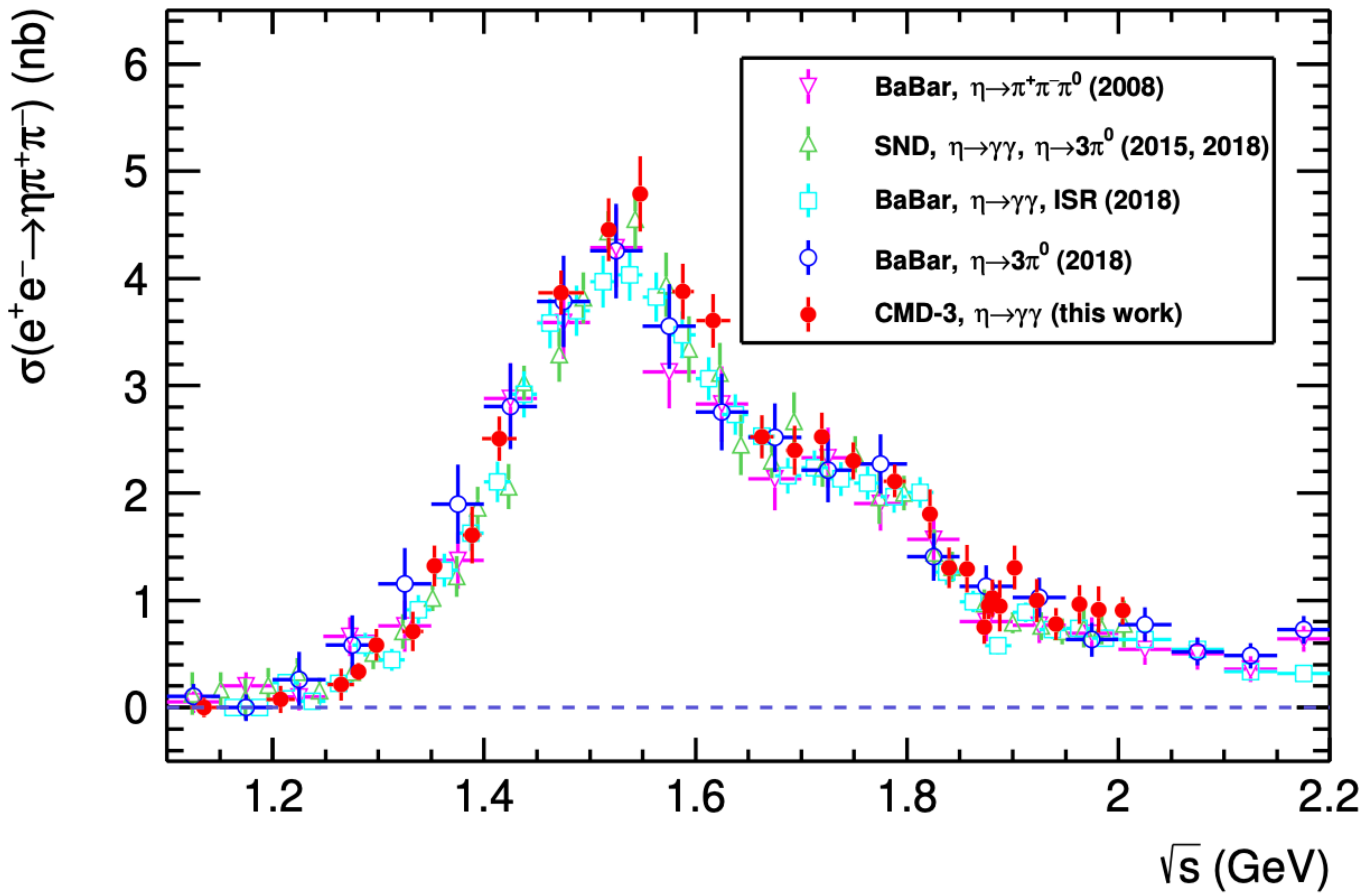
Measured!

1. Test of CVC in $e^+e^- \to \eta\pi^+\pi^-$ vs $\tau^- \to \eta\pi^-\pi^0\nu_\tau$

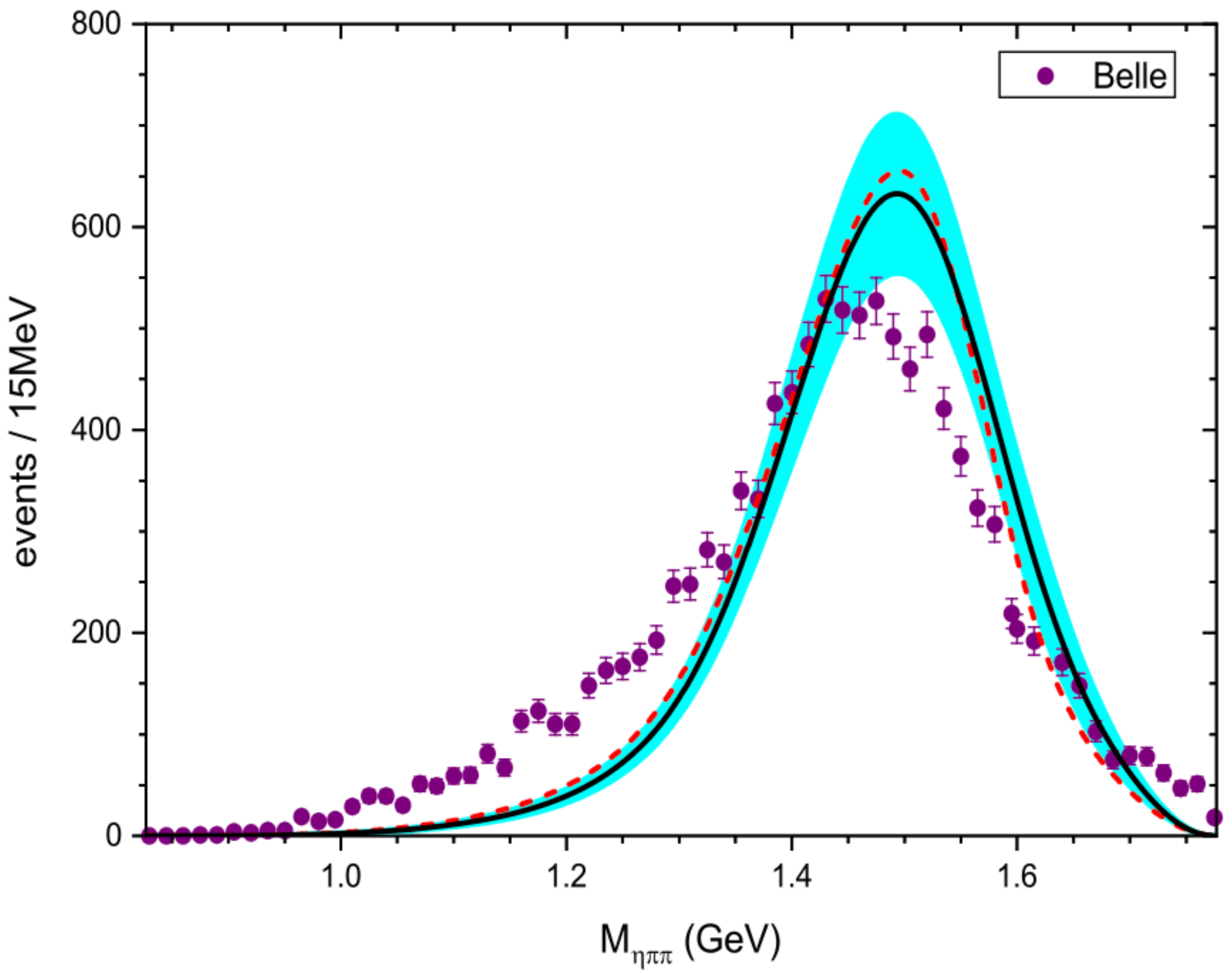
Recent cross section measurements; $\pi^+\pi^-$ spect. dominated by $\rho(770)$

- CMD-3, $1.1 \le \sqrt{s} \le 2.0$ GeV, **JHEP 01, (2020) 112**;
- BaBar, $1.15 \le \sqrt{s} \le 3.5$ GeV, **PRD 97, 052007 (2018)**;
- SND, $1.22 \le \sqrt{s} \le 2.0$ GeV, **PRD 91, 052013 (2015);**

5-10% uncertainty



BESIII, $2.0 \le \sqrt{s} \le 3.08$ GeV, $a_2^\pm(1320)\pi^\mp/\rho\eta$ observed, **PRD 108 (2023)**

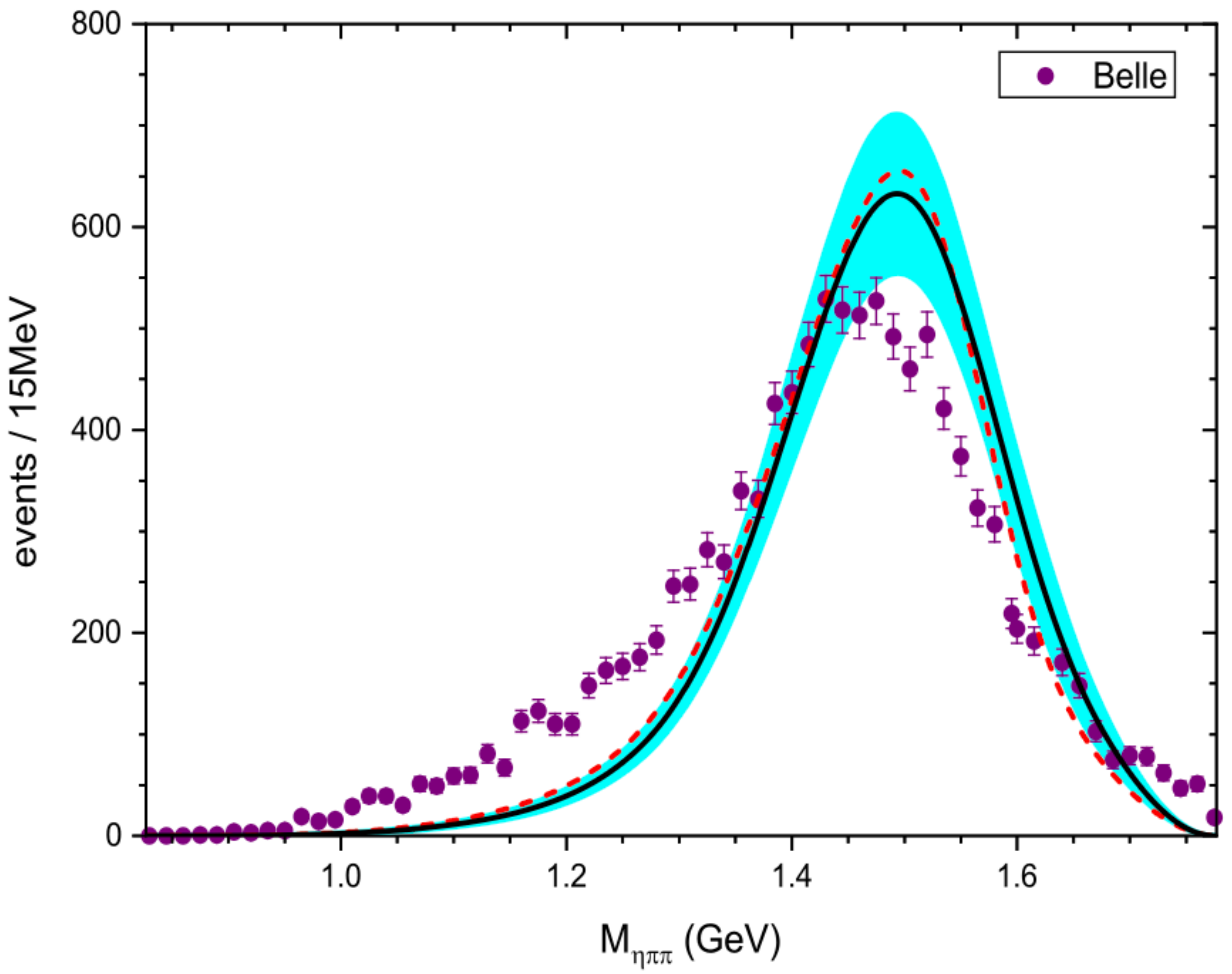


Comparison of $\eta\pi\pi$ spectra τ vs e^+e^-

Arteaga et al, PRD 106 (2022)

- Large departure from CVC
- New physics SMEFT not a solution

(see also Cherepanov+Eidelman, Tau2010)



Comparison of $\eta\pi\pi$ spectra τ vs e^+e^-

Arteaga et al, PRD 106 (2022)

- Large departure from CVC
- New physics SMEFT not a solution

(see also Cherepanov+Eidelman, Tau2010)

BR from CVC:

$$\frac{d\Gamma_\tau(\eta\pi^-\pi^0)}{ds} = S_{\rm EW}\mathcal{F}(s)\sigma_{ee}(\eta\pi^+\pi^-)$$

Source	$B(\tau\to\eta\pi\pi\nu)$ (%)	Reference
CMD-3 (e^+e^-)	$0.168\pm0.006\pm0.011$	JHEP 01,(2020)
BaBar (e^+e^-)	0.163 ± 0.008	PRD 97, (2018)
SND (e^+e^-)	$0.156\pm0.004\pm0.010$	PRD 91, (2015)
PDG Ave.	0.139 ± 0.007	* Belle (2009)

$(1.3\sim2)\sigma$

2. Test of CVC in $e^+e^- \to (4\pi)^0$ vs $\tau^- \to (4\pi)^-\nu_\tau$

Available data

- $e^+e^- \to 2\pi^+2\pi^-,\ \pi^+\pi^-2\pi^0;$
- $\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau,\ \pi^-3\pi^0\nu_\tau$

CVC (Pais) relations

$$\frac{d\Gamma_\tau(2\pi^-\pi^+\pi^0)}{ds} = S_{\mathrm{EW}}\mathcal{F}(s)\left[\frac{1}{2}\sigma_{ee}(2\pi^+2\pi^-) + \sigma_{ee}(\pi^+\pi^-2\pi^0)\right]$$

$$\frac{d\Gamma_\tau(\pi^-3\pi^0)}{ds} = S_{\mathrm{EW}}\mathcal{F}(s)\frac{1}{2}\sigma_{ee}(2\pi^+2\pi^-)$$

A. Pais, Ann. Phys. (1960);
R. Alemany et al, EPJC 2 (1998)

2. Test of CVC in $e^+e^- \to (4\pi)^0$ vs $\tau^- \to (4\pi)^-\nu_\tau$

Available data

- $e^+e^- \to 2\pi^+2\pi^-,\ \pi^+\pi^-2\pi^0;$
- $\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau,\ \pi^-3\pi^0\nu_\tau$

CVC (Pais) relations

$$\frac{d\Gamma_\tau(2\pi^-\pi^+\pi^0)}{ds} = S_{\rm EW}\mathcal{F}(s)\left[\frac{1}{2}\sigma_{ee}(2\pi^+2\pi^-) + \sigma_{ee}(\pi^+\pi^-2\pi^0)\right]$$

$$\frac{d\Gamma_\tau(\pi^-3\pi^0)}{ds} = S_{\rm EW}\mathcal{F}(s)\frac{1}{2}\sigma_{ee}(2\pi^+2\pi^-)$$

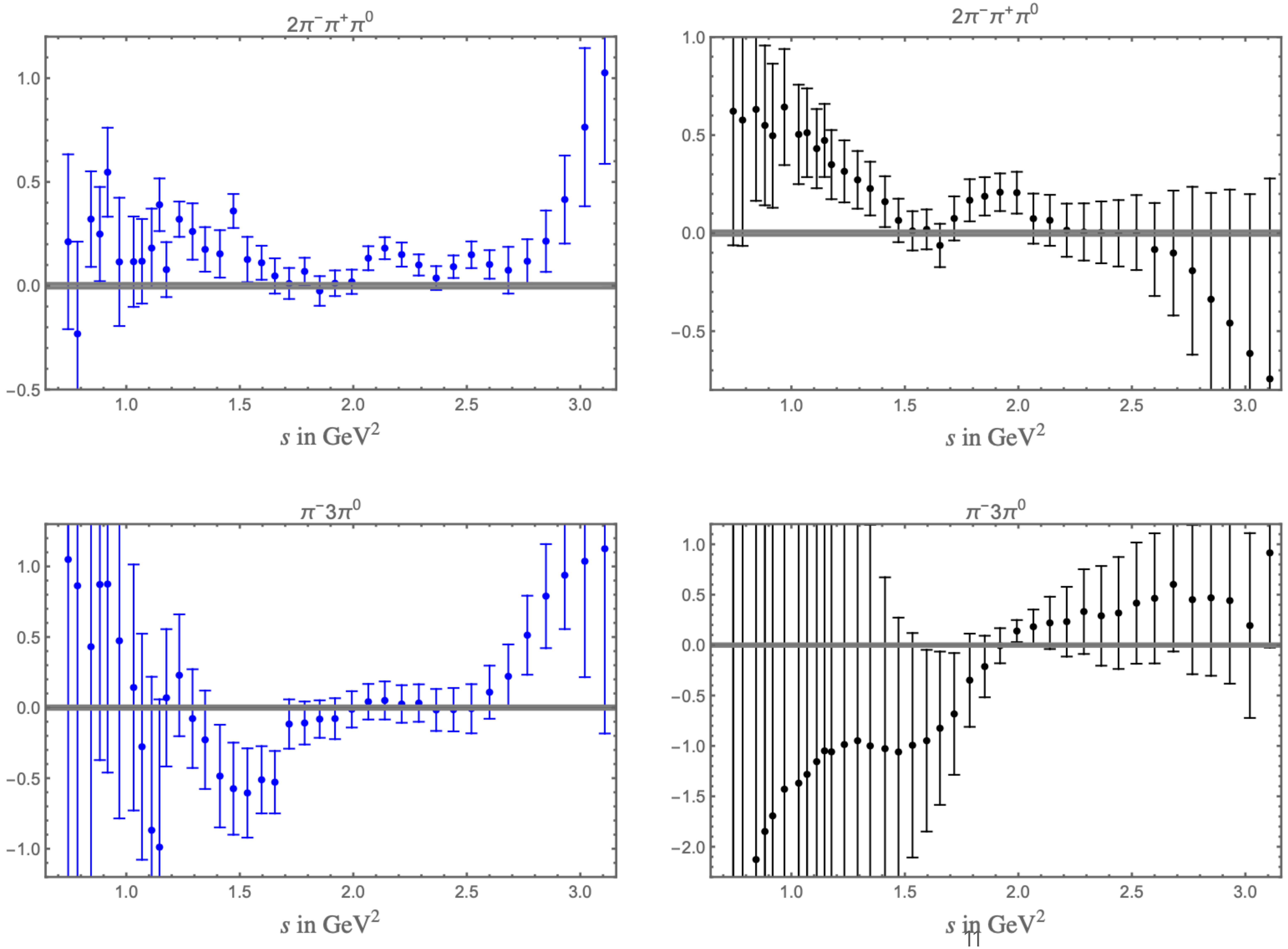
A. Pais, Ann. Phys. (1960);
R. Alemany et al, EPJC 2 (1998)

Test of CVC:

$$R(X^-) \equiv \frac{d\Gamma_\tau^{\rm exp}(X^-)/ds - d\Gamma_\tau^{\rm CVC}(X^-)/ds}{\frac{1}{2}\left[d\Gamma_\tau^{\rm exp}(X^-)/ds + d\Gamma_\tau^{\rm CVC}(X^-)/ds\right]}$$

$$X^- = 2\pi^-\pi^+\pi^0,\ \pi^-3\pi^0$$

N. Allen et al, PRD 114, (2026)



Tau data:

ALEPH, Phys. Rep (2005);

M. Davier et al, EPJC (2014);

OPAL, EPJC (1999)

Refs to e+e- data in

KNT19: A. Keshavarzi et al, PRD (2020)

Differences beyond few % difficult to explain with IB

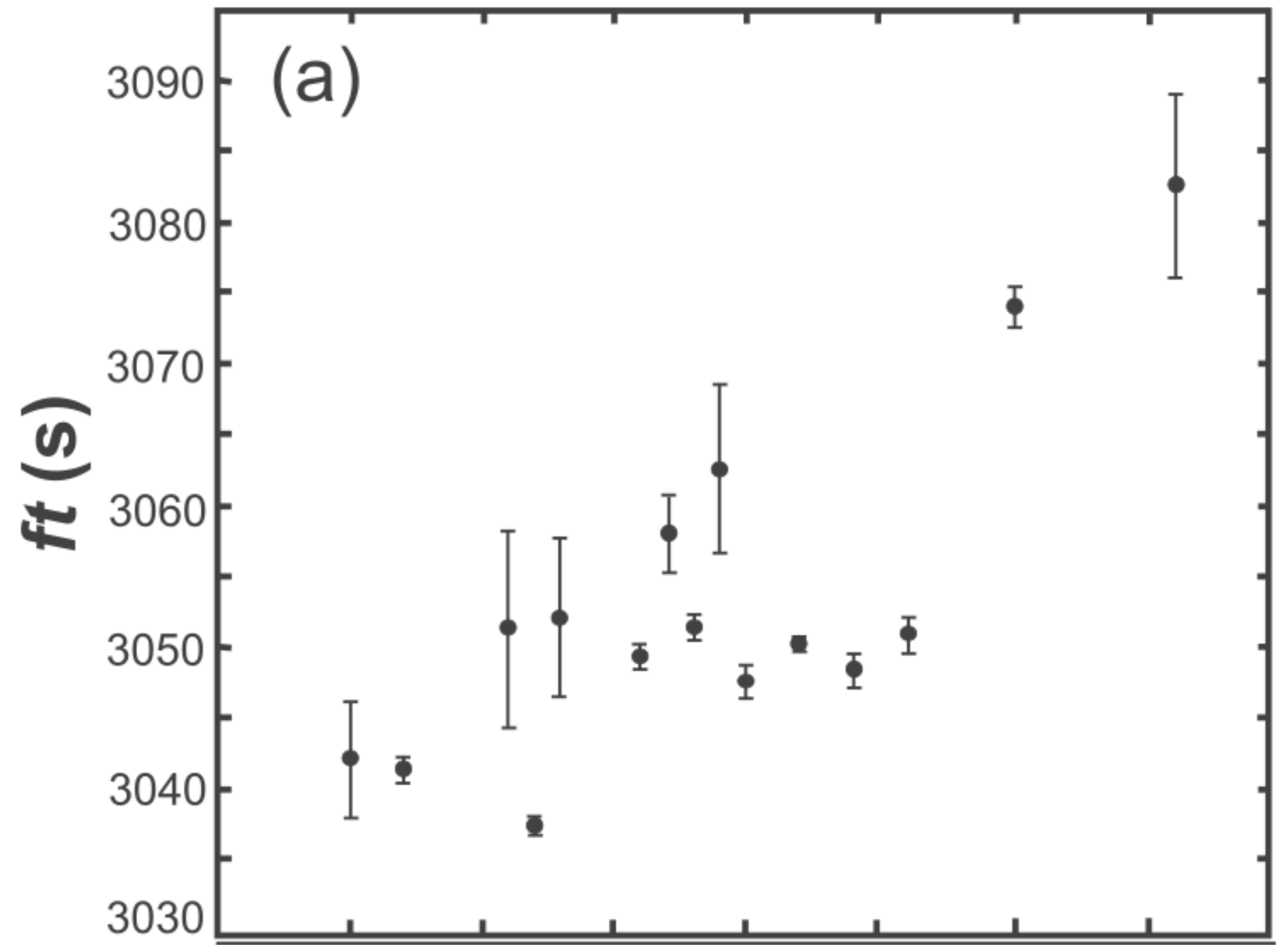
$|V_{ud}|$: The most precise test of CVC at low (vanishing) q^2

Hardy-Towner, PRC102, (2020)

15 transitions $0^+(Z,A) \to 0^+(Z-1,\, A)e^+\nu_e$, I=1

CVC → $= 2$

$$ft\,|F^{0^+\to 0^+}(0)|^2 = \frac{2\pi^3 \ln 2}{G_F^2 m_e^5 |V_{ud}|^2}$$



$|V_{ud}|$: The most precise test of CVC at low (vanishing) q^2

Hardy-Towner, PRC102, (2020)

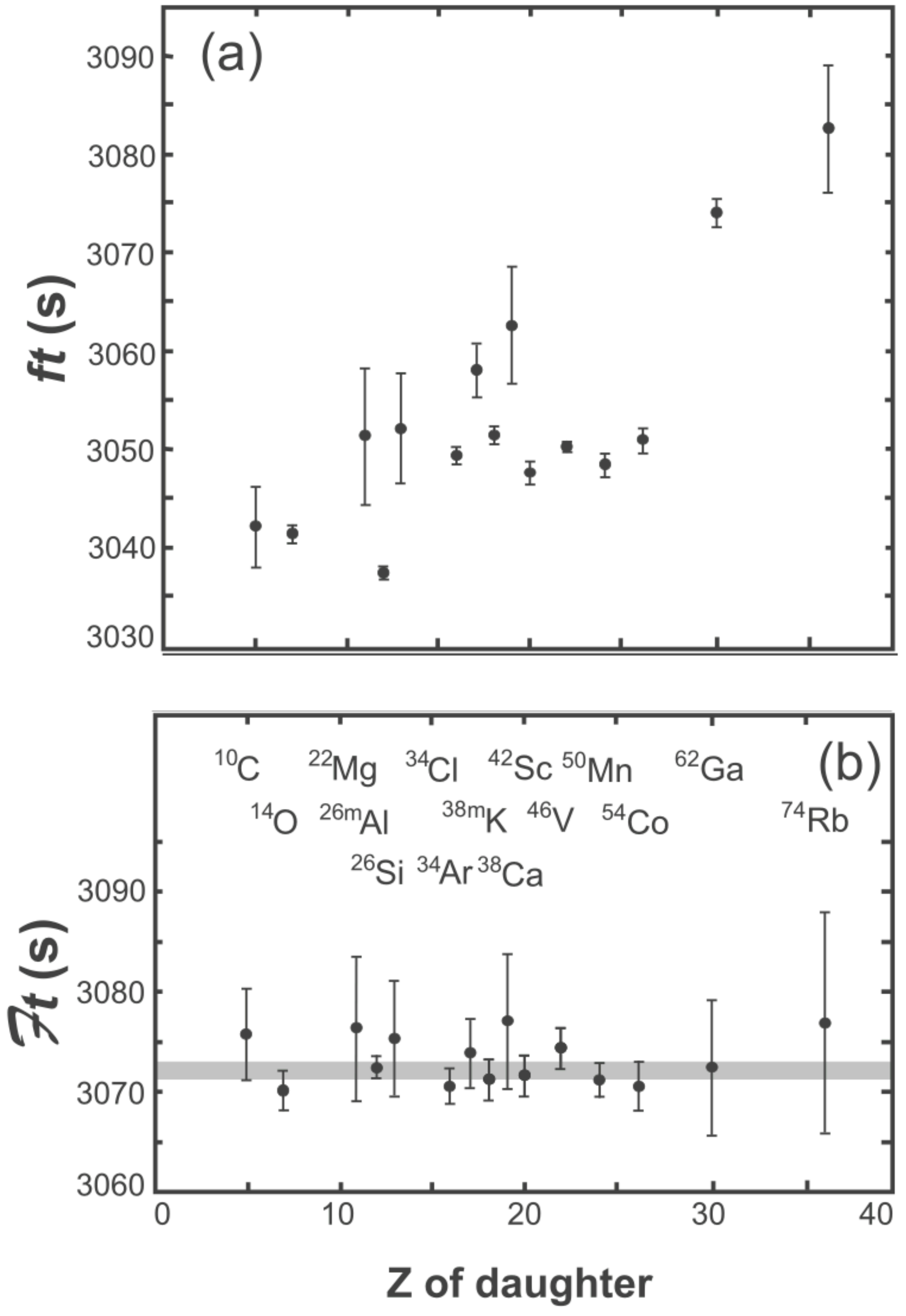
15 transitions $0^+(Z,A) \to 0^+(Z-1,\,A)e^+\nu_e$, I=1

CVC → $ft\,|F^{0^+\to 0^+}(0)|^2 = \frac{2\pi^3 \ln 2}{G_F^2 m_e^5 |V_{ud}|^2}$, with $|F^{0^+\to 0^+}(0)|^2 = 2$

$$\mathcal{F}t \equiv ft(1+\delta'_R)(1+\delta_{NS}-\delta_C)$$

$$= \frac{2\pi^3 \ln 2}{2G_F^2 m_e^5 |V_{ud}|^2 (1+\Delta_R^V)}$$

Test CVC $\Longrightarrow$ $|V_{ud}| = 0.97367(32)$



3. Muon g-2 in the dispersive approach using $e^+e^- \to \text{hadrons}$

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}}$$

LO NLO NNLO

$$[+692.78(2.42) \quad - 9.83(4) \quad + 1.24(1)] \times 10^{-10}$$

Keshavarzi et al PRD 101, (2020)

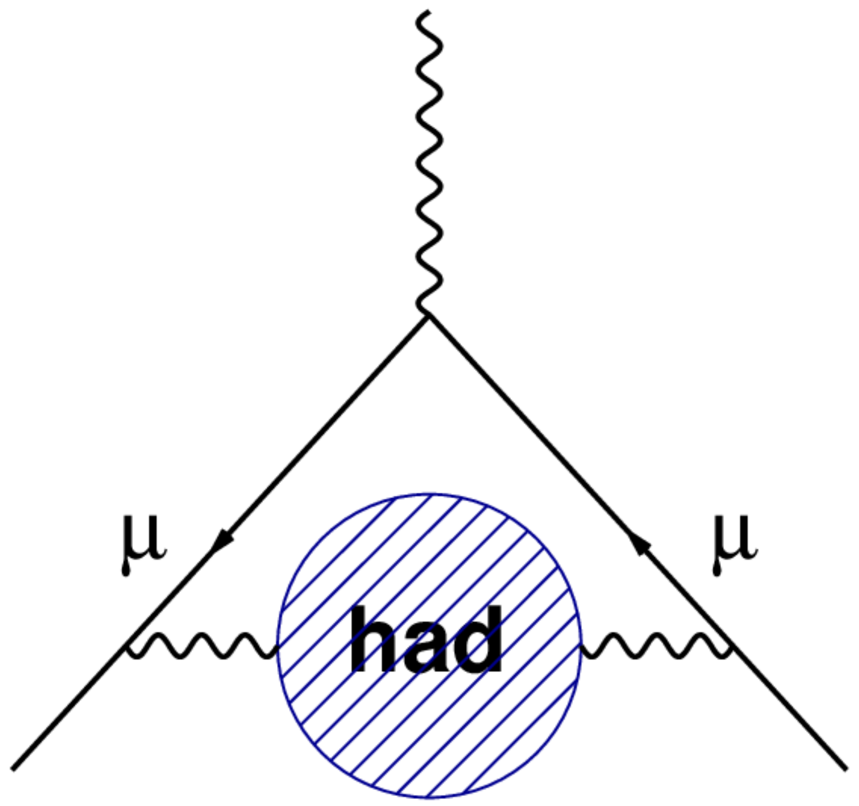
$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}}$$

LO NLO NNLO

$$[+692.78(2.42) \quad -9.83(4) \quad +1.24(1)] \times 10^{-10}$$

Keshavarzi et al PRD 101, (2020)

LO hadronic vacuum polarization in dispersive approach



$$a_\mu^{\text{HVP,LO}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\text{thr}}}^{\infty} ds \frac{K(s)}{s} \frac{\sigma[e^+e^- \to \text{had}(+\gamma)]}{4\pi\alpha^2/3s}$$

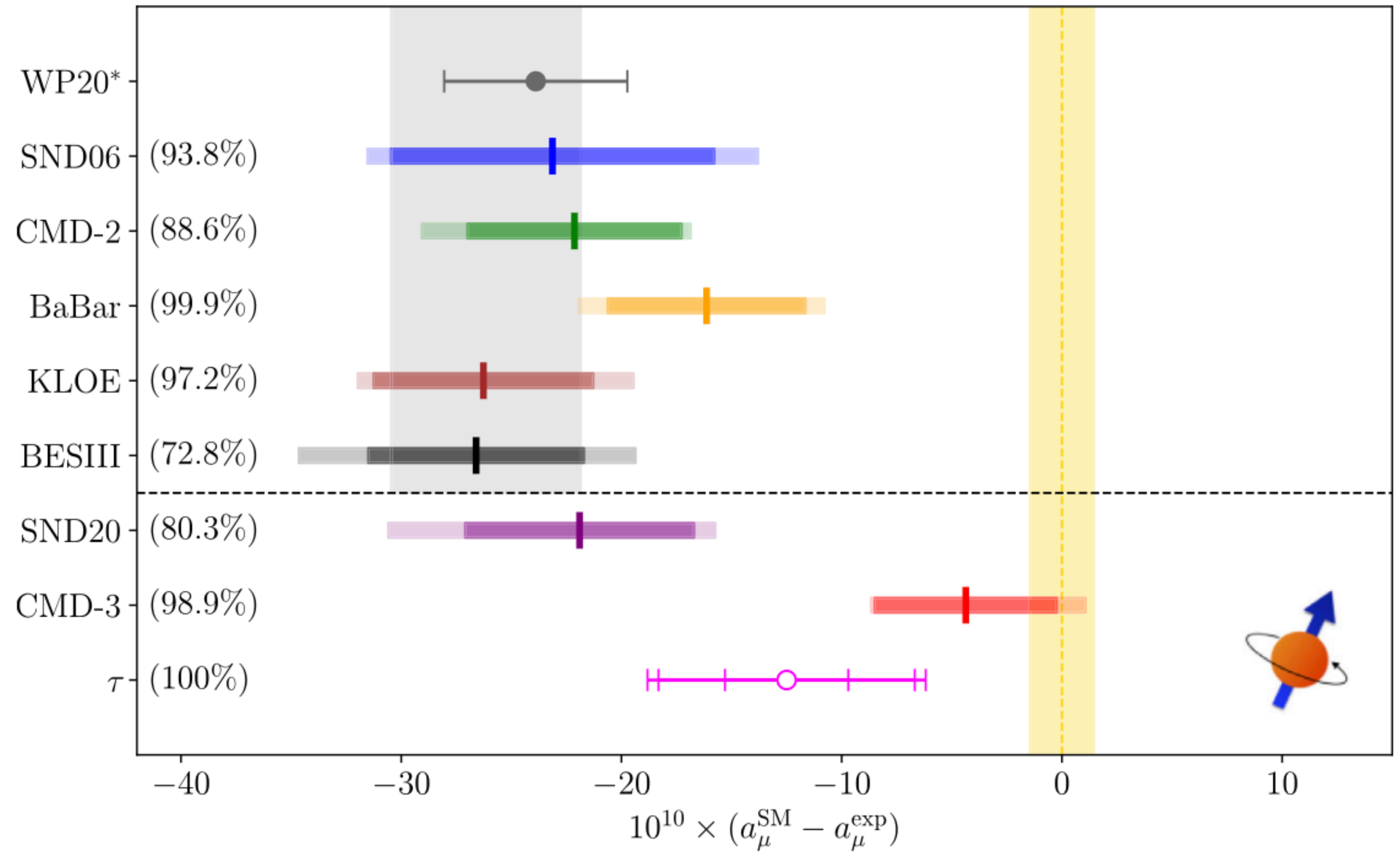
~ 73% from $\pi^+\pi^-(+\gamma)$ $\sqrt{s} < 2.0$ GeV, 80% of error in HVP LO

Tensions in data-driven predictions of a_μ using e^+e^-

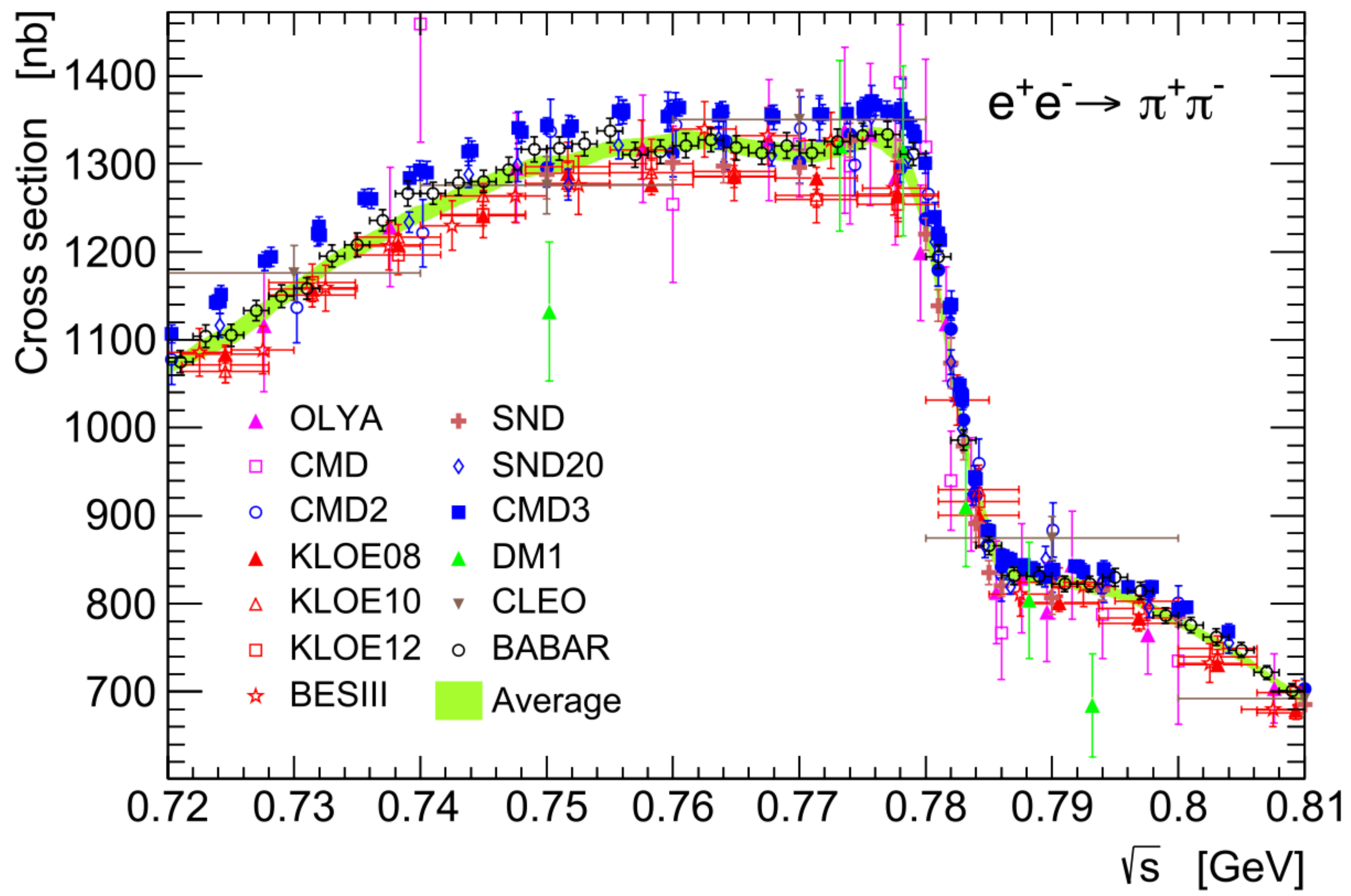
$$a_\mu^{\text{exp}} = 116\ 592\ 07.15(1.45) \times 10^{-10}$$

$$a_\mu^{\text{SM}}[e^+e^-] = 116\ 591\ 81.08(3.78) \times 10^{-10}$$ **KNT(2020)**

3.8σ



Tension in integrated data

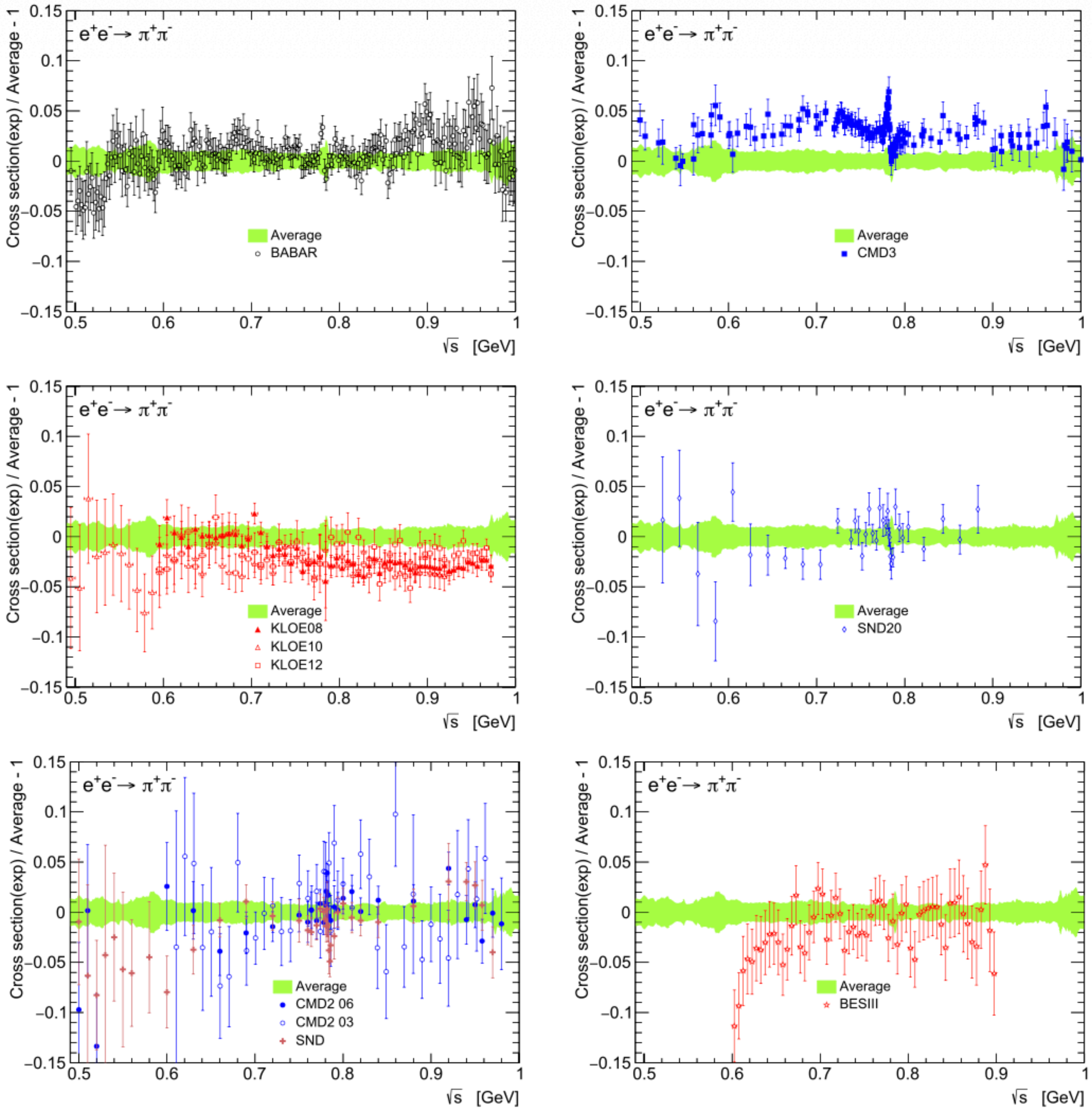


Tension in cross section $\sqrt{s} \sim m_\rho$

WP 2025: R. Aliberti et al, Phys Rep 1143 (2025)

Davier et al EPJ C84 (2024)

Comparison of $\pi^+\pi^-$ cross section measurements with their average close to the $\rho(770)$ resonance.



M. Davier et al, EPJC 84 (2020)

The pion form factor and CVC

$$s = (p_{\pi_1} + p_{\pi_2})^2$$

$$\langle \pi^-(p_-)\pi^+(p_+)|j_\mu^{\mathrm{em}}(0)|0\rangle = F_\pi^{\mathrm{V}}(s)(p_- - p_+)_\mu$$

$$\frac{1}{\sqrt{2}}\langle \pi^-(p_-)\pi^0(p_0)|V_\mu^+(0)|0\rangle = f_+(s)\left[(p_- - p_0)_\mu - \frac{\Delta}{s}(p_- + p_0)_\mu\right] + f_0(s)\frac{\Delta}{s}(p_- + p_0)_\mu$$

The pion form factor and CVC

$$s = (p_{\pi_1} + p_{\pi_2})^2$$

$$\langle \pi^-(p_-)\pi^+(p_+)|j_\mu^{\rm em}(0)|0\rangle = F_\pi^{\rm V}(s)(p_- - p_+)_\mu$$

$$\frac{1}{\sqrt{2}}\langle \pi^-(p_-)\pi^0(p_0)|V_\mu^+(0)|0\rangle = f_+(s)\left[(p_- - p_0)_\mu - \frac{\Delta}{s}(p_- + p_0)_\mu\right] + f_0(s)\frac{\Delta}{s}(p_- + p_0)_\mu$$

- **Isospin symmetry** (CVC), $\Delta = m_{\pi^+}^2 - m_{\pi^0}^2 = 0$, then
  - $F_\pi^{\rm V}(s) = f_+(s)$, for any value of s, indep. dynamics!
  - $f_+(0) = F_\pi^{\rm V}(0) = 1$, equal weak and em charges
- **Isospin breaking**,
  - $F_\pi^{\rm V}(s) \neq f_+(s)$, for $s \neq 0$, model-dependent
  - $F_\pi^{\rm V}(0) = 1, \quad f_+(0) = 1 - O[(m_u - m_d)^2]$

$< 10^{-5}$

- Behrends-Sirlin (1960);
- Ademollo-Gatto (1964)

Pion form factors parametrizations

IB

$$[F_\pi^V, f_+](s) = \mathrm{BW}_\rho(s) + (\rho', \rho'') + (\rho^0 - \omega \ \mathrm{mix.})$$

- $$\mathrm{BR}_\rho^{\mathrm{GS}}(s) = \frac{m_\rho^2[1 + d\Gamma_\rho/m_\rho]}{s - f(s, m_\rho, \Gamma_\rho) + i m_\rho \Gamma_\rho^{\mathrm{GS}}(s)}$$ Gounaris-Sakurai (GS), PRL (1968)
- $$\mathrm{BR}_\rho^{\mathrm{GS}}(s) = \frac{m_\rho^2}{s - m_\rho^2 + i m_\rho \Gamma_\rho^{\mathrm{KS}}(s)}$$ Kuhn-Santamaría (KS), Z.Phys. C (1990)
- $$\mathrm{BR}_\rho^{\mathrm{GP}}(s) = \frac{m_\rho^2}{s - m_\rho^2 + i m_\rho \Gamma_\rho^{\mathrm{GP}}(s)} \exp[2\tilde{H}_{\pi\pi}(s) + \tilde{H}_{KK}(s)] + \ldots$$ Guerrero-Pich (GP), PLB (1990)

Pion form factors parametrizations

IB

$$[F_\pi^V, f_+](s) = \mathrm{BW}_\rho(s) + (\rho', \rho'') + (\rho^0 - \omega \text{ mix.})$$

- $$\mathrm{BR}_\rho^{GS}(s) = \frac{m_\rho^2[1 + d\Gamma_\rho/m_\rho]}{s - f(s, m_\rho, \Gamma_\rho) + i m_\rho \Gamma_\rho^{GS}(s)}$$ **Gounaris-Sakurai (GS), PRL (1968)**
- $$\mathrm{BR}_\rho^{GS}(s) = \frac{m_\rho^2}{s - m_\rho^2 + i m_\rho \Gamma_\rho^{KS}(s)}$$ **Kuhn-Santamaría (KS), Z.Phys. C (1990)**
- $$\mathrm{BR}_\rho^{GP}(s) = \frac{m_\rho^2}{s - m_\rho^2 + i m_\rho \Gamma_\rho^{GP}(s)} \exp[2\tilde{H}_{\pi\pi}(s) + \tilde{H}_{KK}(s)] + \ldots$$ **Guerrero-Pich (GP), PLB (1990)**

$$\Gamma_\rho^{GS,KS}(s) = \Gamma_\rho \frac{s}{m_\rho^2}\left(\frac{\beta_{\pi\pi}(s)}{\beta_{\pi\pi}(m_\rho^2)}\right)^3 \theta_{\pi\pi}, \qquad \Gamma_\rho^{GP}(s) = \frac{m_\rho s}{96\pi f_\pi^2}\left[\beta_{\pi\pi}^3(s)\theta_{\pi\pi} + \frac{1}{2}\beta_{KK}^3(s)\theta_{KK}\right]$$

$\theta_{PP} = \theta(\sqrt{s} - 2m_\pi)$, $\qquad \beta_{PP}(s)$= pion velocity in rest rho frame

Data-driven evaluation using $\tau^- \to \pi^-\pi^0\nu_\tau(\gamma)$ data

Exp < 0.2%

Theory ~ 5%

$$a_\mu^{\mathrm{HVP,LO}}[\pi\pi] = a_\mu^{\mathrm{HVP,LO}}[\tau,\pi\pi] + \Delta a_\mu^{\mathrm{HVP,LO}}[\tau,\pi\pi]$$

CVC, Alemany et al EPJC, 1998

$$\Delta a_\mu^{\mathrm{HVP,\,LO}}[\tau,\pi\pi] = \frac{\alpha^2 m_\tau^2}{6|V_{ud}|^2\pi^2}\frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e}\int_{4m_\pi^2}^{m_\tau^2} ds\,\frac{K(s)}{s}\,\frac{dN_{\pi\pi^0}}{N_{\pi\pi^0}ds}\left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\left[\frac{R_{\mathrm{IB}}(s)}{S_{\mathrm{EW}}}-1\right]$$

Isospin breaking (QED and $m_d - m_u$) corrections

Cirigliano et al, 2002

$$\frac{R_{\mathrm{IB}}(s)}{S_{\mathrm{EW}}} = \frac{(\beta_0(s)/\beta_+(s))^3}{S_{\mathrm{EW}}G_{\mathrm{EM}}(s)}\mathrm{FSR}(s)\left|\frac{F_\pi^V(s)}{f_+(s)}\right|^2$$

$$\frac{F_\pi^V(s)}{f_+(s)} = 1 + a\Delta m_\rho + b\Delta\Gamma_\rho + c\delta_{\rho\omega}$$

SD electroweak corrections: S_{EW}

- Marciano, Sirlin, PRL, '86, '88, '93
- Braaten, Li, PRD, '90
- Erler, RMF, '04
- Cirigliano et al, PRL, '26

LD em corrections: $G_{\mathrm{EM}}(s)$

- Cirigliano et al PLB '01; JHEP '02
- Flores-Baez et al, PRD, '05
- Miranda, Roig, PRD, '20
- ZX Li, A Li, et al, PRD, '25
- Colangelo et al, JHEP '26; PRL '26

Talk by Zhi-Hui Guo

FSR(s) in $e^+e^- \to \pi^+\pi^-$

- J Schwinger, Particles,, Sources Book
- Drees, Hikasa, PLB, '90
- Flores-Baez et al, PRD, '26
- Hoferichter et al, PRL, '23

Isospin Breaking $\Delta\Gamma_\rho$ in $F_\pi^V(s)/f_+(s)$

- Alemany et al, EPJC, '98
- Davier et al, EPJC, '03
- Flores-Baez et al, PRD, '07
- Davier et al, EPJC, '10; '12, '24
- Flores-Baez et al, PRD, '26

Beyond sQED

Status IB corrections in Δa_μ(in 10^{-10} units): White Paper 2025

(Phys. Rep. '25)

		Refs. [166,194]	Ref. [209]	Refs. [237,247]	Our estimate
Phase space		-7.88	-7.52	–	$-7.7(2)$
S_{EW}		$-12.21(15)$	$-12.16(15)$	–	$-12.2(1.3)$
G_{EM}		$-1.92(90)$	$(-1.67)^{+0.60}_{-1.39}$	–	$-2.0(1.4)$
FSR		$4.67(47)$	$4.62(46)$	$4.42(4)$	$4.5(3)$
ρ–ω mixing		$4.0(4)$	$2.87(8)$	$3.79(19)$	$3.9(3)$
$\frac{F_\pi^V}{f_+}$ (w/o ρ–ω)	ΔM_ρ	$0.20(^{+27}_{-19})(9)$	$1.95^{+1.56}_{-1.55}$	–	
	$\Delta\Gamma_\rho(\Delta M_\pi)$	$4.09(0)(7)$	3.37	–	
	$\Delta\Gamma_\rho(\pi\pi\gamma)$	$-5.91(59)(48)$	$-6.66(73)$	–	
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	–	–	–	
	Total	$-1.62(65)(63)$	$(-1.34)^{+1.72}_{-1.71}$	–	$-1.5(4.7)$
Sum		$-14.9(1.9)$	$(-15.20)^{+2.26}_{-2.63}$	–	$-15.0(5.1)$

Exp: $\pm 1.45 \times 10^{-10}$

Largest uncertainties

- Matching and scheme-dependence in $S_{\rm EW}$, $G_{\rm EM}(s)$,
- Non-scalar QED corrections in $G_{\rm EM}(s)$, FSR(s) & $\Delta\Gamma_\rho[\pi\pi\gamma]$
- Largest uncertainty due to IB in $\rho\pi\pi$, $\left| g_{\rho^+\pi^+\pi^0}/g_{\rho^0\pi^+\pi^-} - 1 \right| \approx 2\alpha/\pi$

[166] Davier et al, EPJC, '24
[194] Davier et al, EPJC, '10
[209] Castro et al, PRD, '25
[237] Colangelo et al, JHEP, '22
[247] Hoferichter et al, PRL, '23

Some recent (after WP25) developments in IB corrections

- $G_{\rm EM}(s)$: Structure-dependent corrections, Colangelo et al, PRL 136 (2026); JHEP 02, (2026)
- $S_{\rm EW} \cdot G_{\rm EM}(s)$: Scheme-independence, Cirigliano et al, PRL 137 (2026)

$\Big\}\ \pm(1.8 \to \pm\, 0.5) \times 10^{-10}$

- $\mathrm{FSR}(s)$: Structure-dependent corrections, Flores-Baez et al, PRD 113 (2026) $\qquad \pm 10\,\% \to \pm\, 1\,\%$
- $F_V(s)/F_+(s)$: IB corrections in $\Delta\Gamma_\rho$, Flores-Baez et al 2026; GLC-KR ; Davier et al, 2504.13789 $\qquad \pm 30\,\% \to\ ?\,\%$

Some recent (after WP25) developments in IB corrections

- $G_{\rm EM}(s)$: Structure-dependent corrections, Colangelo et al, PRL 136 (2026); JHEP 02, (2026)
- $S_{\rm EW} \cdot G_{\rm EM}(s)$: Scheme-independence, Cirigliano et al, PRL 137 (2026)

$\Big\}\ \pm(1.8 \to \pm 0.5) \times 10^{-10}$

- $\mathrm{FSR}(s)$: Structure-dependent corrections, Flores-Baez et al, PRD 113 (2026) $\quad \pm 10\,\% \to \pm 1\,\%$
- $F_V(s)/F_+(s)$: IB corrections in $\Delta\Gamma_\rho$, Flores-Baez et al 2026; GLC-KR ; Davier et al, 2504.13789 $\quad \pm 30\,\% \to ?\,\%$

$$\frac{F_V(s)}{f_+(s)} = 1 + a\Delta m_\rho + b\Delta\Gamma_\rho$$

$$\Delta m_\rho = m_{\rho^+} - m_{\rho^0}, \quad \Delta\Gamma_\rho = \Gamma_{\rho^0} - \Gamma_{\rho^+}$$

- ± 1.3 MeV, PDG
- $< \pm 0.5$ MeV needed

Future: BESIII 10^8 $J/\psi \to \rho\pi$

Davier et al, 2504.13789

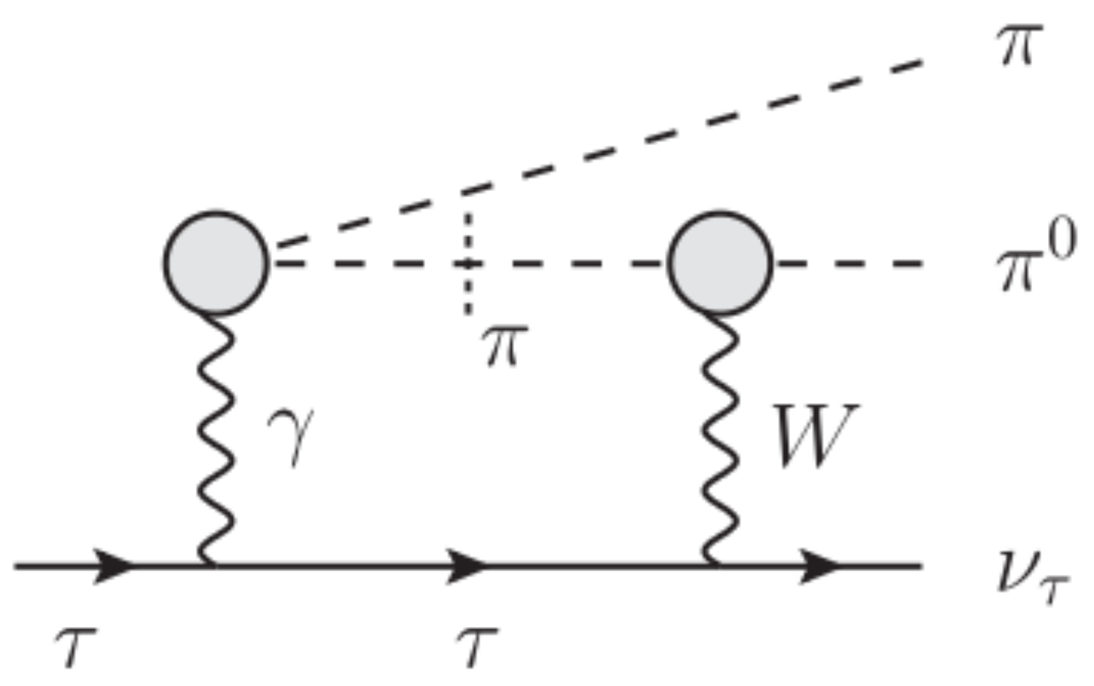
Theoretical input for $\Delta\Gamma_\rho$ needed!

Beyond scalar QED in virtual corrections

- $O(\alpha)$ QED corrections calculated assuming point-pions (sQED)

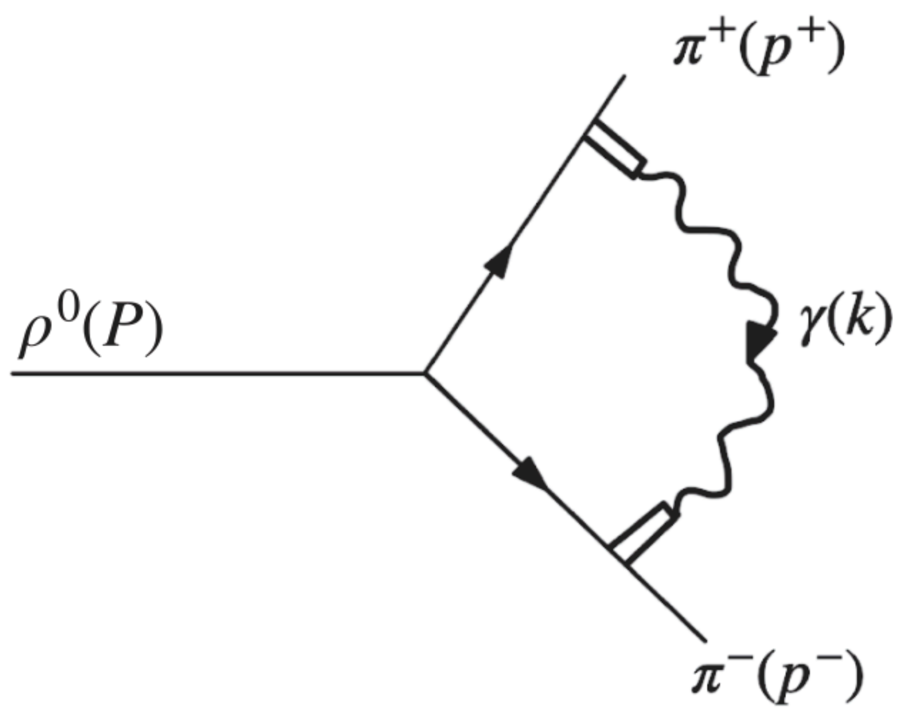
- pion structure-dependent virtual corrections to $\tau^- \to \pi^-\pi^0\nu_\tau \Longrightarrow G_{\rm EM}(s)$

  Colangelo et al PRL (2026); JHEP (2026)



- Structure-dependent corrections to $\rho^0 \to \pi^+\pi^-/\rho^+ \to \pi^+\pi^0 \Longrightarrow \Delta\Gamma_\rho$

  Flores-Baez et al, PRD (2026)



$$\frac{1}{k^2} \Longrightarrow \frac{1}{k^2}\left[\sum_V a_V F_V(k^2)\right]^2$$

Summary of IB in rho meson widths

$$
\begin{aligned}
\Delta\Gamma_\rho &= \Delta\Gamma_\rho[\pi\pi(\gamma)] + \Delta\Gamma_\rho(\mathrm{rest}) \\
&= \Gamma(\rho^0 \to \pi^+\pi^-) \left[\delta_0 - \delta_+ - 2\left(\frac{g_+}{g_0} - 1\right) - \frac{3}{2}\left(\frac{1-\beta_0^2}{\beta_0^2}\right)\frac{\delta m_\pi}{m_\pi}\right] + \Delta\Gamma_\rho(\mathrm{rest}) \\
&= [+0.690(6) + 0(1.4) - 1.104]\ \mathrm{MeV} + 0.085(17)\ \mathrm{MeV} \\
&= [-0.33 \pm 0.02\ (\pm 1.4)]\ \mathrm{MeV}
\end{aligned}
$$

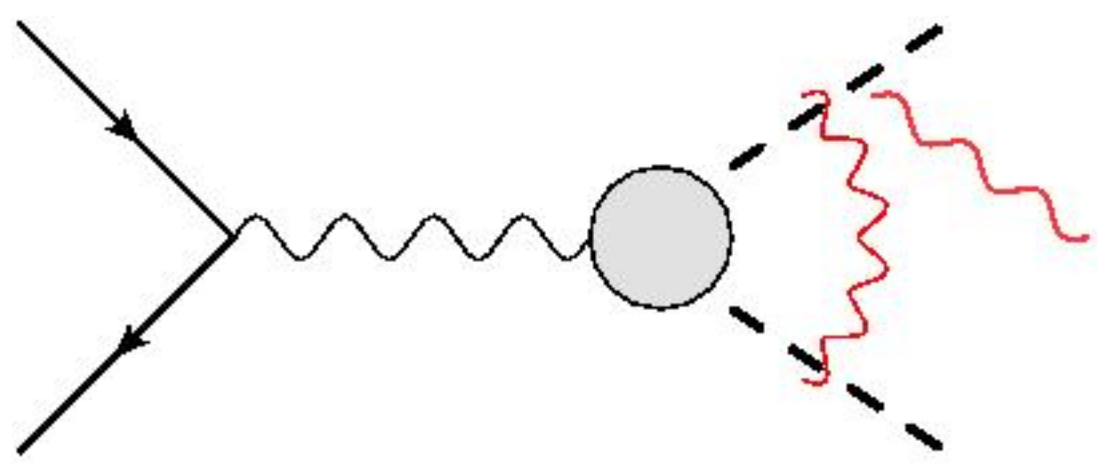
Compared to

- $(+0.03 \pm 1.3)$ MeV from **PDG2024**,
- (-0.58 ± 1.01) MeV (GS), from **Davier et al 2504.13789**

FSR corrections in $e^+e^- \to \pi^+\pi^-$

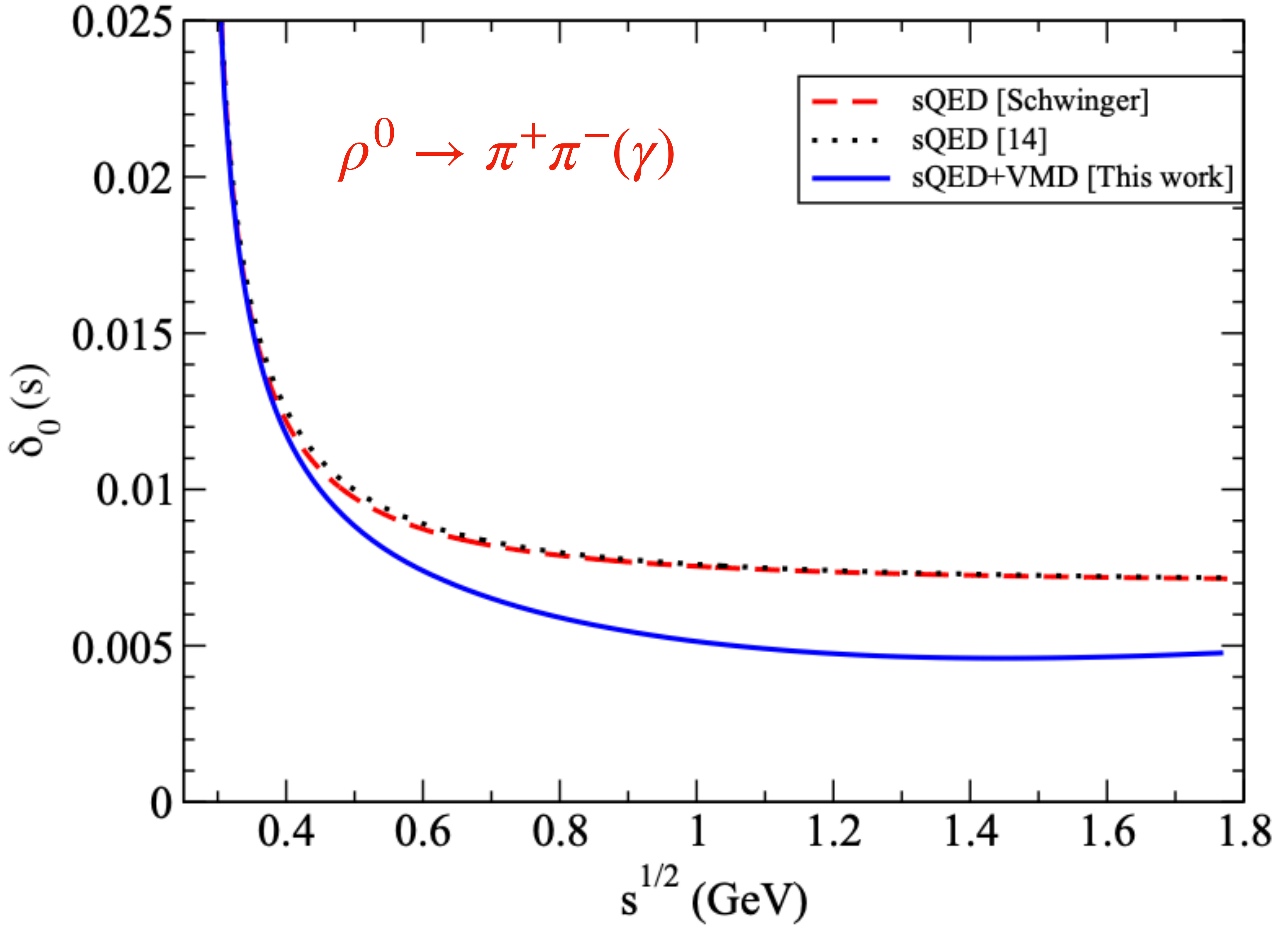
- Energy-dependence of QED radcorr

$$\mathrm{FSR}(s) = 1 + \delta_0(s)$$



- Scalar QED (sQED) corrections
    - Schwinger's Book;
    - Drees-Hikasa, PLB (1990)
    - Flores-Baez et al, PRD (2007).

sQED vs structure-dependent corrections



Flores-Báez et al, PRD 113(2026)

Summary: “Estimated” impact on $\Delta a_\mu^{\mathrm{HVP,LO}}[\tau, \pi\pi]$ (units 10^{-10})

		Refs. [a,b]	Ref. [c]	2026	
PS, S_{EW}, G_{EM}		$-22.01(91)$	$-21.4(^{+0.6}_{-1.4})$	$-24.9(5)$ [d,e]	
FSR		$+4.67(47)$	$+4.62(46)$	$+3.77(2)$ [f]	(*)
$\rho-\omega$ mixing		$+4.0(4)$	$+2.87(8)$	$+4.0(4)$	
$\frac{F_\pi^V}{f_+}$ (w/o $\rho-\omega$)	Δm_ρ	$+0.20(^{+27}_{-19})$	$+1.95(^{+1.56}_{-1.55})$	$+0.20(^{+27}_{-19})$	
	$\Delta\Gamma_\rho(\Delta m_\pi)$	$+4.09$	$+3.37$	$+4.09$	
	$\Delta\Gamma_\rho(\pi\pi\gamma)$	$-5.91(59)$	$-6.66(73)$	$-2.22(3)$ [f]	(*)
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	–	–	$0.0(4.7)$	
	Total	$-1.62(65)$	$-1.34(1.72)$	$+2.1(4.7)$	
$\Delta a_\mu^{\mathrm{HVP,LO}}[\tau, \pi\pi]$		$-14.9(1.9)$	$-15.2(^{+2.3}_{-2.6})$	$-15.1(4.8)$	

[a] Davier et al, EPJ C84, 721 (2024),
[b] Davier et al, EPJC C66, 127(2010)
[c] Castro et al, PRD 111, 073004 (2025)
[d] Colangelo et al PRL 136, 10(2026); JHEP 02, 181 (2026)
[e] Cirigliano et al, arXiv 2602.11253
[f] Flores-Baez et al, PRD 113 , 093002 (2026)
[g] GLC+K. Raya, preliminar

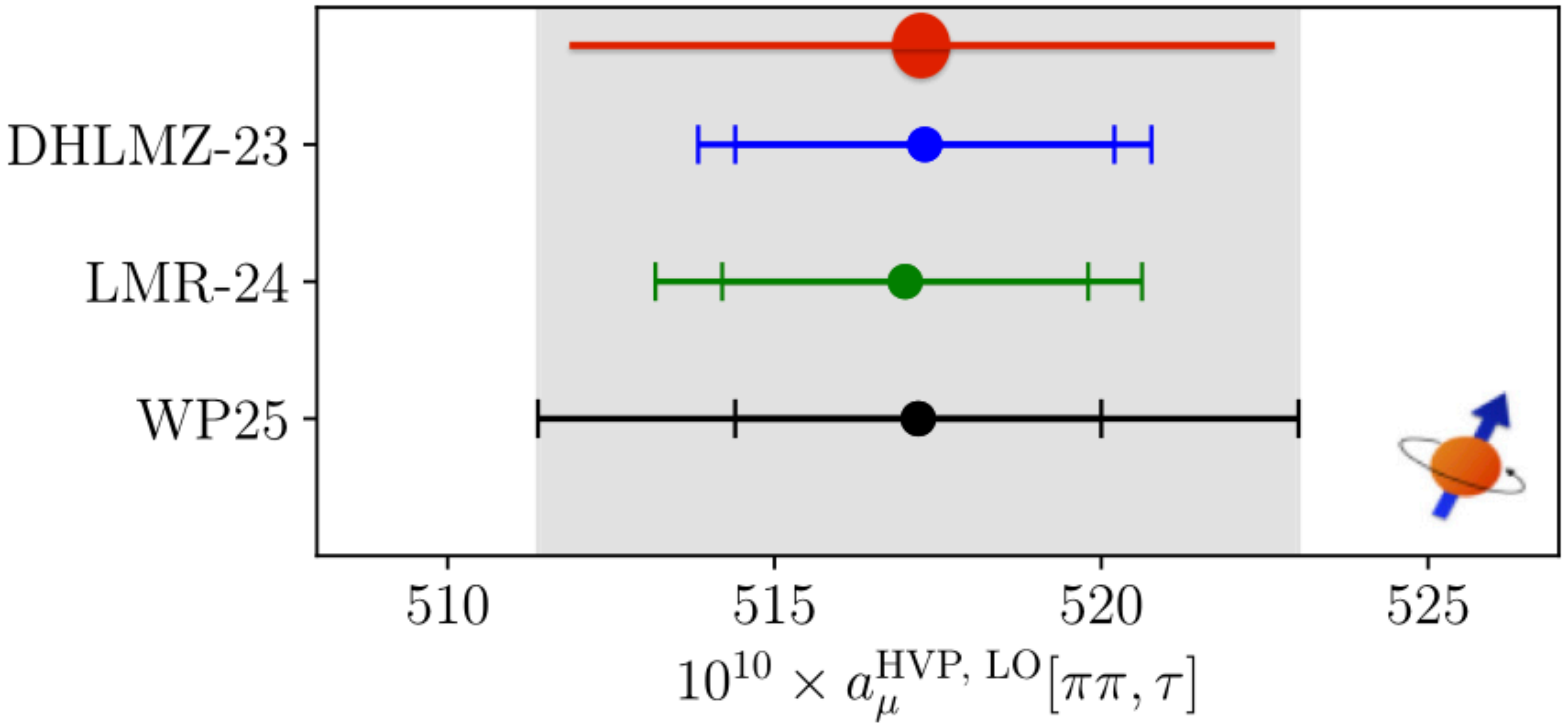
Summary: “Estimated” impact on $\Delta a_\mu^{\rm HVP,LO}[\tau,\pi\pi]$ (units 10^{-10})

		Refs. [a,b]	Ref. [c]	2026	
PS, $S_{\rm EW}$, $G_{\rm EM}$		$-22.01(91)$	$-21.4(^{+0.6}_{-1.4})$	$-24.9(5)$ [d,e]	
FSR		$+4.67(47)$	$+4.62(46)$	$+3.77(2)$ [f]	(*)
$\rho-\omega$ mixing		$+4.0(4)$	$+2.87(8)$	$+4.0(4)$	
$\frac{F_\pi^V}{f_+}$ (w/o $\rho-\omega$)	Δm_ρ	$+0.20(^{+27}_{-19})$	$+1.95(^{+1.56}_{-1.55})$	$+0.20(^{+27}_{-19})$	
	$\Delta\Gamma_\rho(\Delta m_\pi)$	$+4.09$	$+3.37$	$+4.09$	
	$\Delta\Gamma_\rho(\pi\pi\gamma)$	$-5.91(59)$	$-6.66(73)$	$-2.22(3)$ [f] (*)	
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	–	–	$0.0(4.7)$	
	Total	$-1.62(65)$	$-1.34(1.72)$	$+2.1(4.7)$	
$\Delta a_\mu^{\rm HVP,LO}[\tau,\pi\pi]$		$-14.9(1.9)$	$-15.2(^{+2.3}_{-2.6})$	$-15.1(4.8)$	

[a] Davier et al, EPJ C84, 721 (2024),
[b] Davier et al, EPJC C66, 127(2010)
[c] Castro et al, PRD 111, 073004 (2025)
[d] Colangelo et al PRL 136, 10(2026); JHEP 02, 181 (2026)
[e] Cirigliano et al, arXiv 2602.11253
[f] Flores-Baez et al, PRD 113 , 093002 (2026)
[g] GLC+K. Raya, preliminar

- Structure-dependent effects (*) in radcorr $\rho\to\pi\pi(\gamma)$
- Different results compatible within errors

Test of CVC in the 2-pion HVP-LO contribution to muon g-2



$a_\mu^{\mathrm{HVP,LO}}[\tau, \pi\pi] = 517.1(2.8)_{\mathrm{exp}}(4.8)_{\mathrm{th}} \times 10^{-10}$

CVC test $\Updownarrow$ 2.3σ

$a_\mu^{\mathrm{HVP,LO}}[e^+e^-, \pi\pi] = (503.46 \pm 1.91) \times 10^{-10}$

KNT(2020), wo/CMD-3

- Consistent determinations of $a_\mu^{\mathrm{HVP,LO}}[\tau, \pi\pi]$ using τ data
- Solving/understanding the tension of CVC remains a challenge
- Independent test: $\mathrm{BR}(\tau^- \to \pi^- \to \pi^-\pi^0\nu_\tau) = (25.49 \pm 0.09)\,\%$ **PDG(2026)**

2. Isospin breaking in $\rho\pi\pi$ couplings

- $|\delta_g| = \left|(g_+ - g_0)/g_0\right| \leq 2\alpha/\pi$, compromise guess , $|\Delta\Gamma_\rho| = 300|\delta_g|$ MeV ≤ 1.4 MeV
- $\Longrightarrow \Delta a_\mu^{\mathrm{LO,HVP}}[\tau, \pi\pi] = \pm\, 4.7 \times 10^{-10}$, **WP2025**
- Some model-dependence cancel in g_+/g_0 ratio (?)

---

2. Isospin breaking in $\rho\pi\pi$ couplings

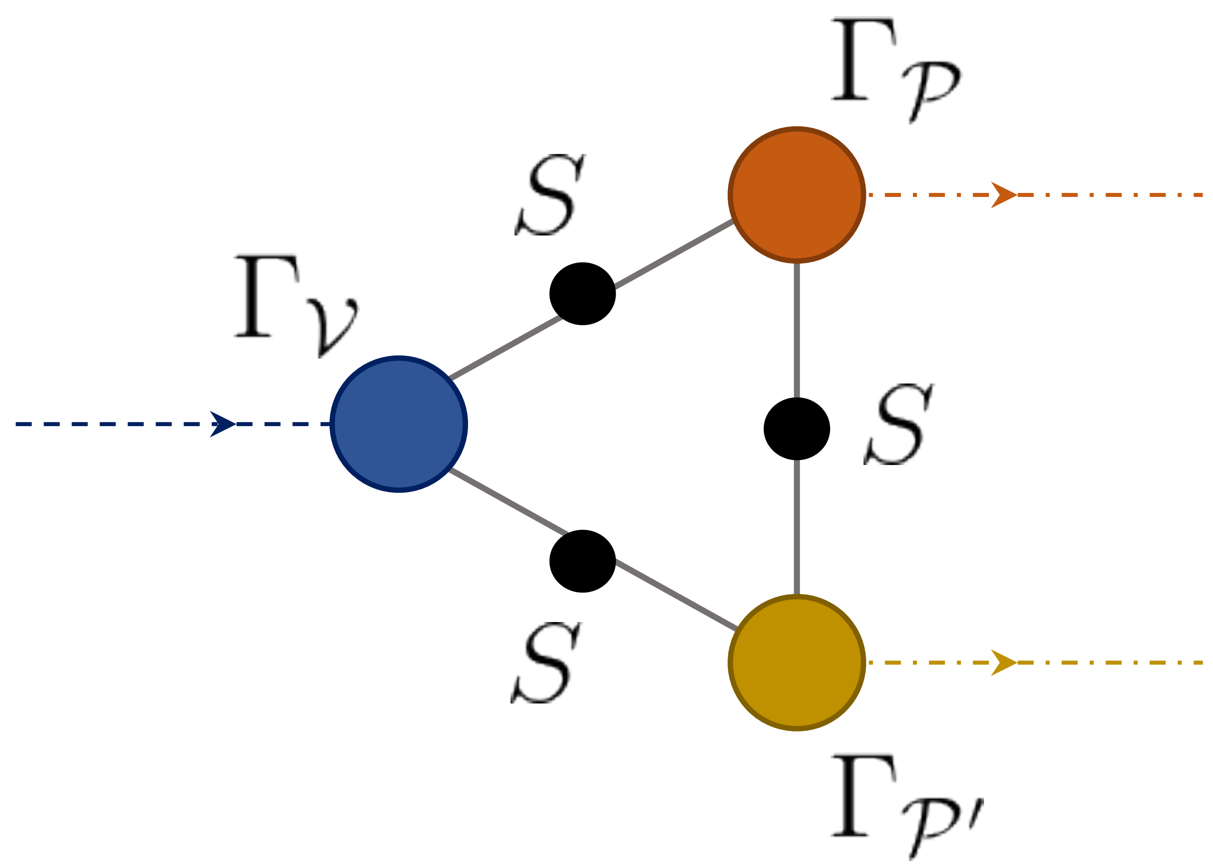
- $|\delta_g| = \left|(g_+ - g_0)/g_0\right| \leq 2\alpha/\pi$, compromise guess , $|\Delta\Gamma_\rho| = 300|\delta_g|$ MeV ≤ 1.4 MeV
- $\Longrightarrow \Delta a_\mu^{\rm LO,HVP}[\tau, \pi\pi] = \pm\, 4.7 \times 10^{-10}$, **WP2025**
- Some model-dependence cancel in g_+/g_0 ratio (?)

With Khepani Raya

The $g_{\rm VPP}$ strong coupling is computed via Schwinger-Dyson in the impulse approximation:

$$\rho(p_1, \epsilon) \to \pi(p_2)\pi(q)$$

$$g_{\mathcal{V}\mathcal{P}\mathcal{P}'}\epsilon^\lambda \cdot q = N_c {\rm tr} \int \frac{d^4k}{(2\pi)^4} S^{f_3}(k_3)\epsilon^\lambda \cdot \Gamma_{\mathcal{V}}(k_V; p_1) S^{f_1}(k_1) \times \bar{\Gamma}_{\mathcal{P}'}(k_{P'}, -p_2) S^{f_2}(k_2) \Gamma_{\mathcal{P}}(k_P, -q) \, .$$

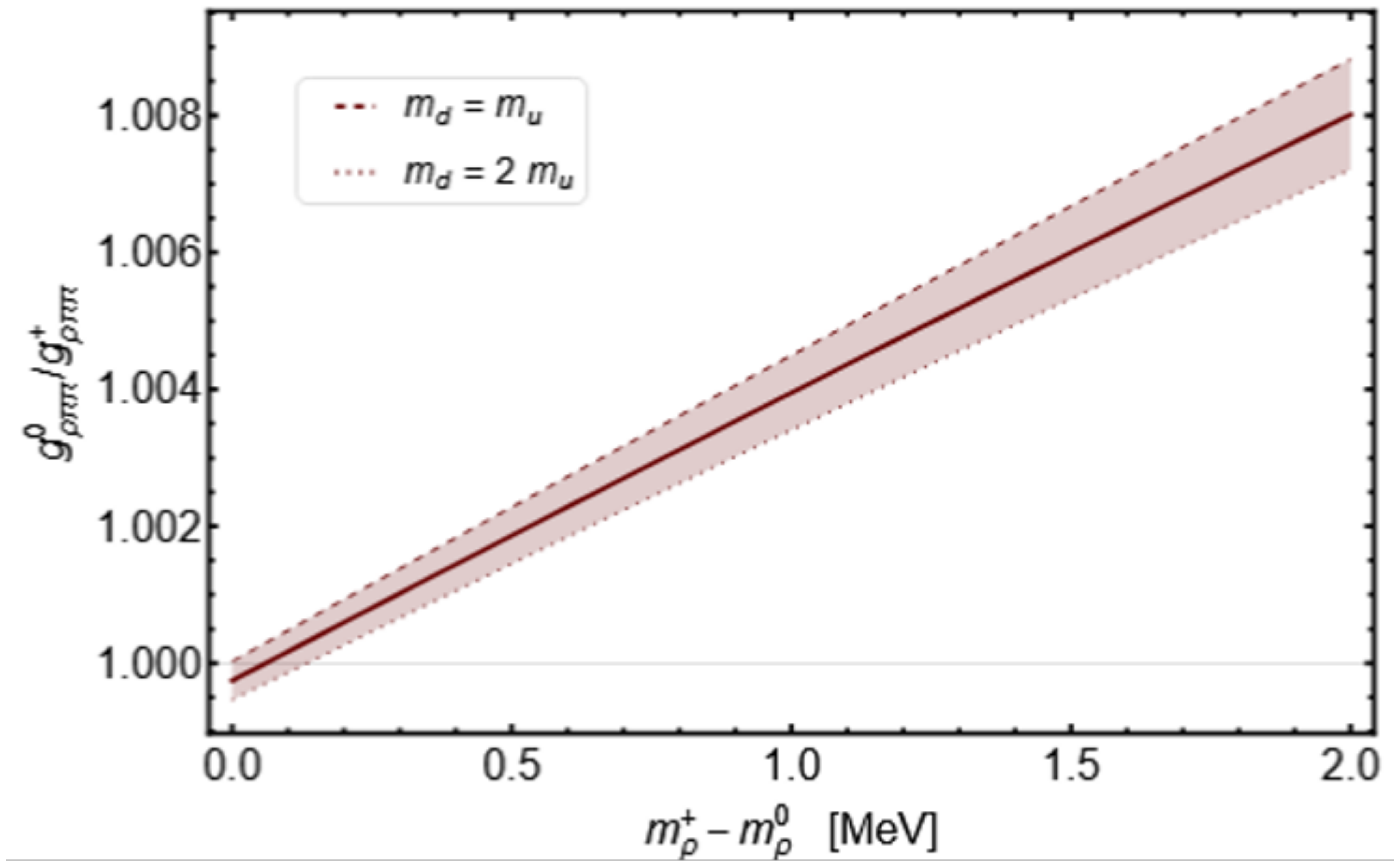


1) $m_\pi = 139.57$ MeV reproduced with

- $m_d = m_u = 7.00$ MeV, Isospin symmetry
- $m_d = 2m_u = 9.32$ MeV, Isospin breaking

2) with current quark masses fixed,

- $m_{\rho^+} = m_{\rho^0} = 775.19$ MeV, for $\xi = \xi_0 = 0.37$ in BSE,
- $m_{\rho^+} \neq m_{\rho^0}$, for varying $\xi_{u,d}$, keep $\xi = \sqrt{\xi_u \xi_d}$ fixed

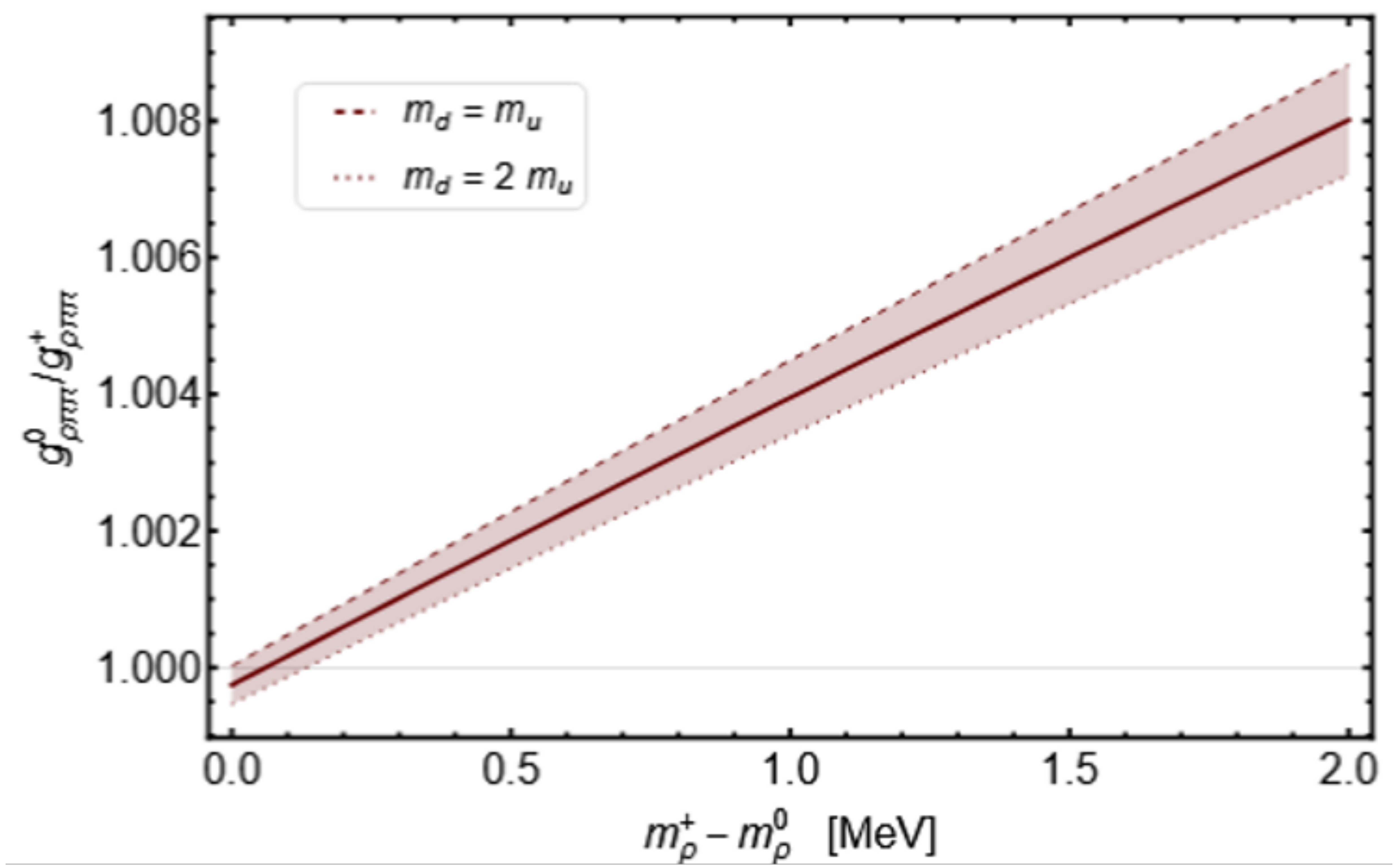


1) $m_\pi = 139.57$ MeV reproduced with

- $m_d = m_u = 7.00$ MeV, Isospin symmetry
- $m_d = 2m_u = 9.32$ MeV, Isospin breaking

2) with current quark masses fixed,

- $m_{\rho^+} = m_{\rho^0} = 775.19$ MeV, for $\xi = \xi_0 = 0.37$ in BSE,
- $m_{\rho^+} \neq m_{\rho^0}$, for varying $\xi_{u,d}$, keep $\xi = \sqrt{\xi_u \xi_d}$ fixed



Using $m_{\rho+} - m_{\rho^0} = +0.7(8)$ MeV **PDG,** one gets

$$\frac{g_{\rho^0\pi^+\pi^-}}{g_{\rho^+\pi^+\pi^0}} = 1.0024(17)$$

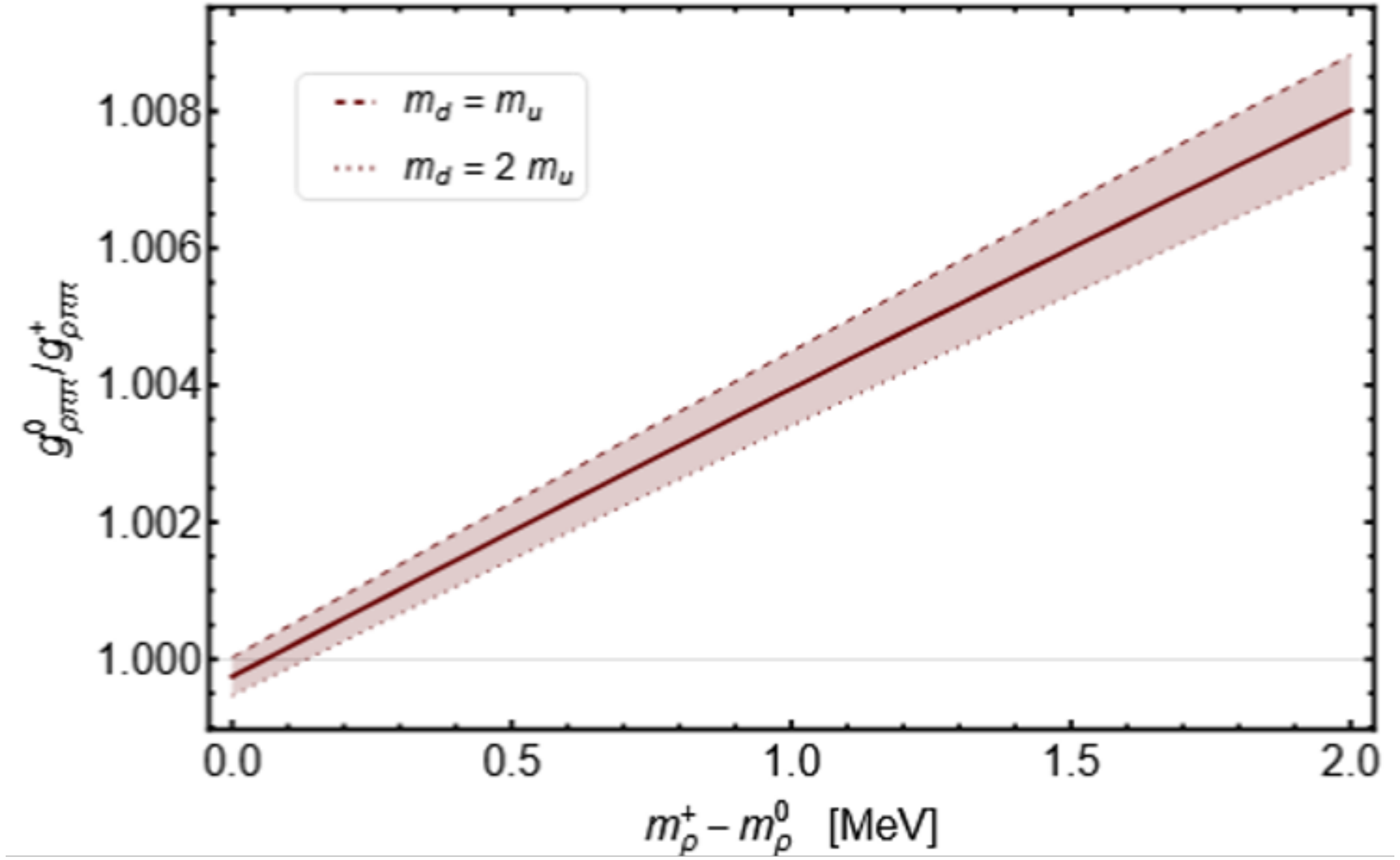
$\delta g = -0.0024(17)$, compared to $\delta g = 0(\frac{2\alpha}{\pi}) = 0.0000(46)$ **WP 2025**

1) $m_\pi = 139.57$ MeV reproduced with

- $m_d = m_u = 7.00$ MeV, Isospin symmetry
- $m_d = 2m_u = 9.32$ MeV, Isospin breaking

2) with current quark masses fixed,

- $m_{\rho^+} = m_{\rho^0} = 775.19$ MeV, for $\xi = \xi_0 = 0.37$ in BSE,
- $m_{\rho^+} \neq m_{\rho^0}$, for varying $\xi_{u,d}$, keep $\xi = \sqrt{\xi_u \xi_d}$ fixed



Using $m_{\rho+} - m_{\rho^0} = +0.7(8)$ MeV **PDG,** one gets

$$\frac{g_{\rho^0\pi^+\pi^-}}{g_{\rho^+\pi^+\pi^0}} = 1.0024(17)$$

$\delta g = -0.0024(17)$, compared to $\delta g = 0(\frac{2\alpha}{\pi}) = 0.0000(46)$ **WP 2025**

$$\Delta\Gamma_\rho = -0.33(2)(140)\ \text{MeV} \rightarrow +0.39(51) \quad \Delta a_\mu[\Delta\Gamma_\rho(g_{\rho\pi\pi})] : \left[0.0(4.7) \longrightarrow -2.3(1.6)\right] \times 10^{-10}$$

Final comments

- Consistent tests of CVC in τ vs e^+e^- hadron production with $I^G = 1^+$,
- More precision required in 4π weak and electromagnetic production,
- Intriguing discrepancies in $\eta\pi\pi$ channels; improvements required,
- Recent improvements in IB corrections to CVC,
- Channels of relevance for $(g-2)_\mu$ below 2 GeV

channel	(%) of $a_\mu^{\mathrm{HVP,LO}}$	(%) of $\delta a_\mu^{\mathrm{HVP,LO}}$
$\pi^+\pi^-$	72.6	62.3
$\pi^+\pi^-\pi^0$	6.7	15.1
$2\pi^+2\pi^-$	2.1	0.7
$\pi^+\pi^-2\pi^0$	2.8	10.4
K^+K^-	3.3	0.8
K_LK_S	1.9	0.6
Total	89.4	89.9

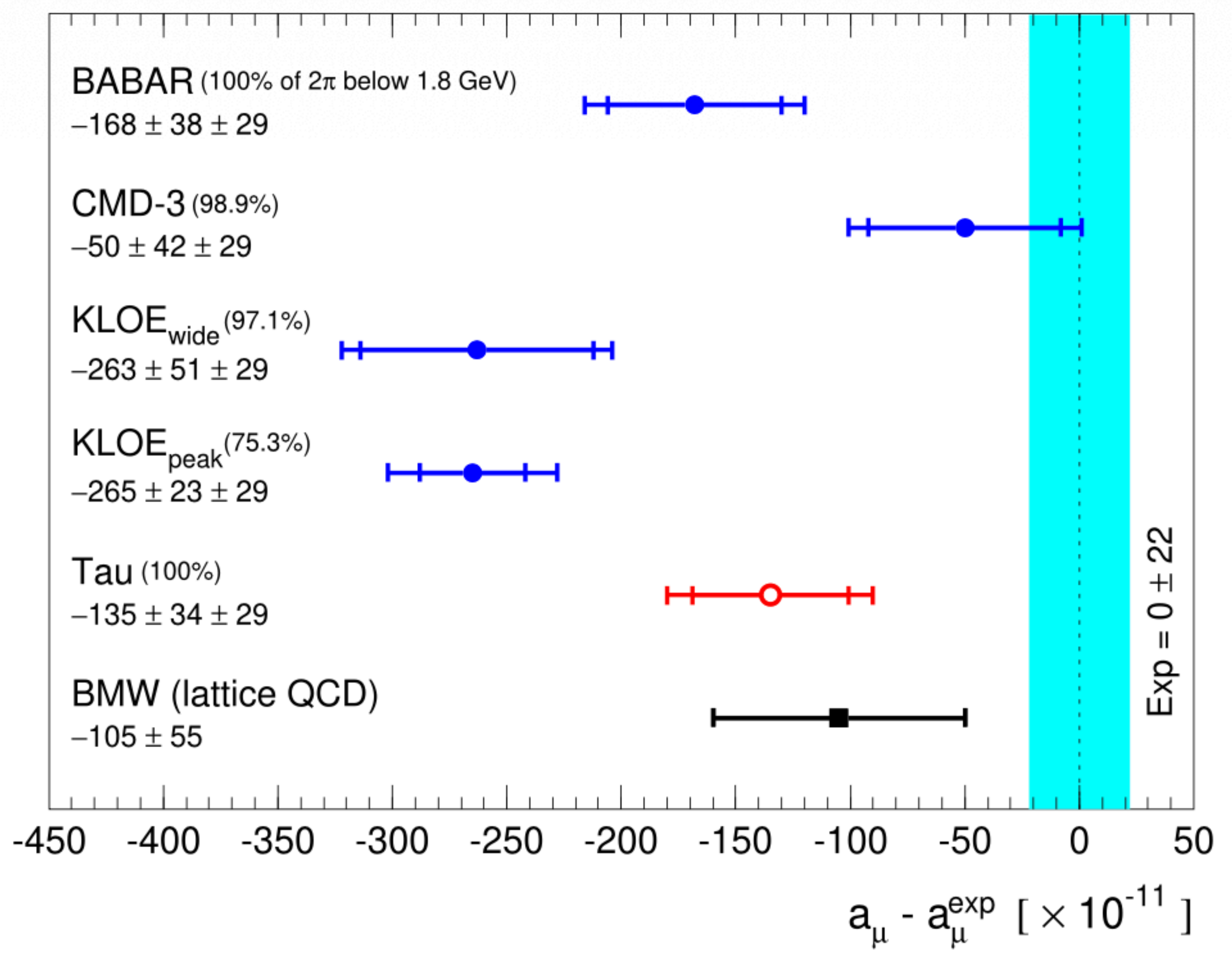
Consistency with CVC, reliable data-driven predictions

Thank you for your attention

Combining BaBar, CMD-3 and τ

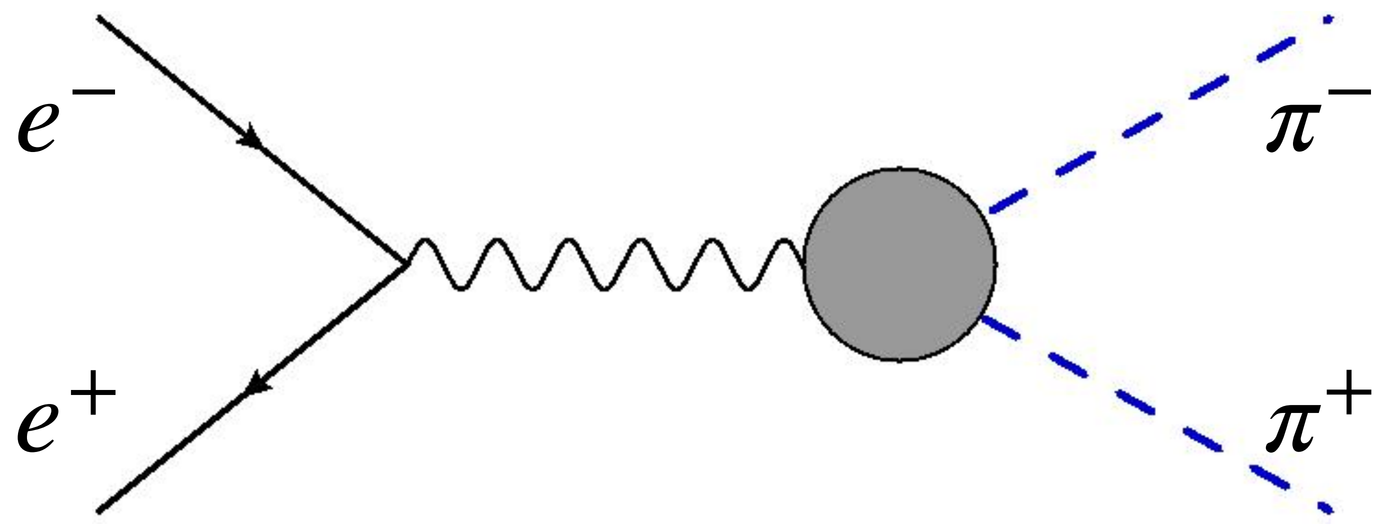
$$\Delta a_\mu = a_\mu^{\text{SM}} - a_\mu^{\text{exp}}$$
$$= -(12.3 \pm 3.3_{\pi\pi} \pm 2.9_{\text{rest}} \pm 2.2_{\text{exp}}) \times 10^{-10}$$

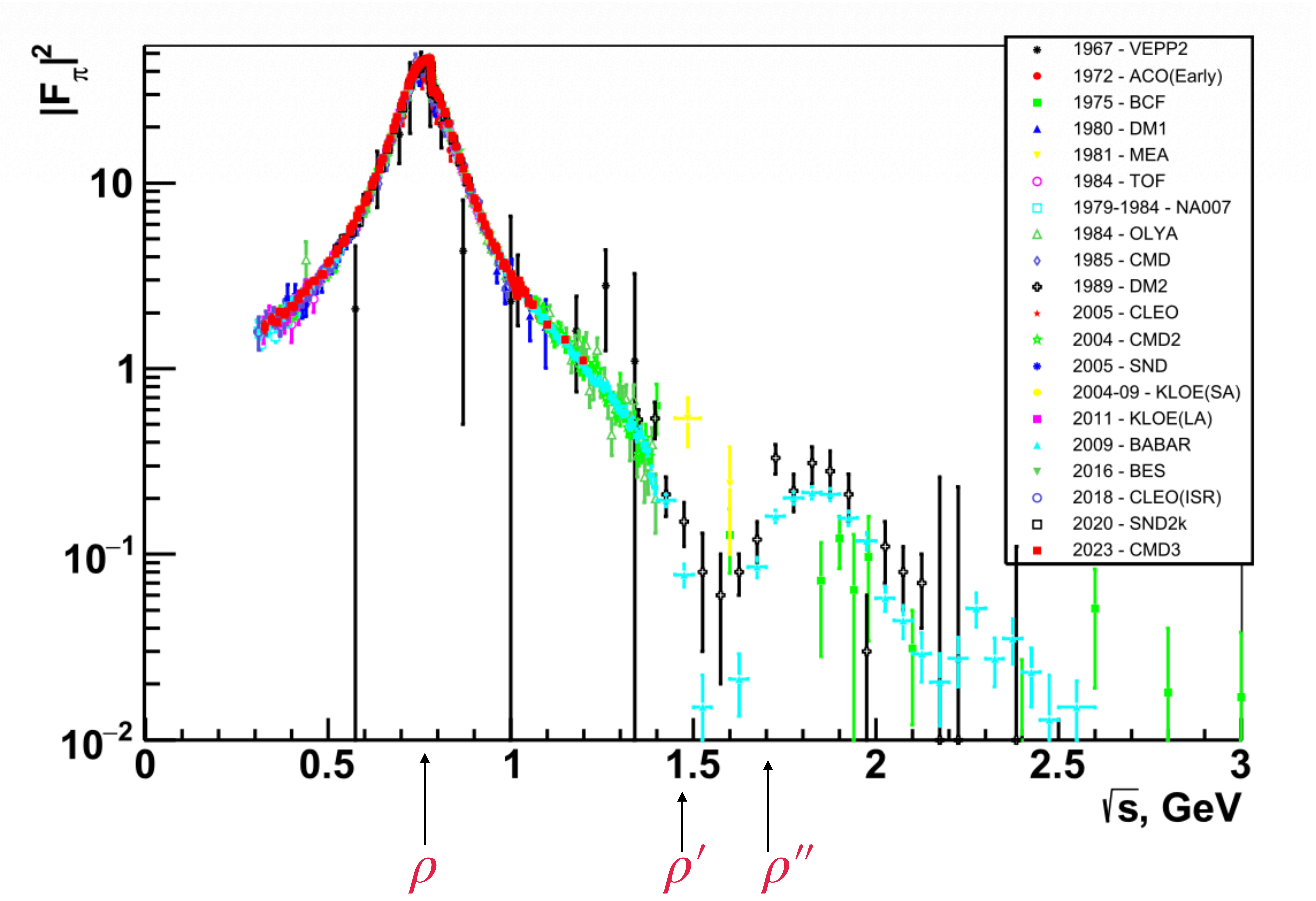
M. Davier et al EPJ C84 (2024)



$$\sigma(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2}{3s}|F_\pi^V(s)|^2\beta_\pi^3(s)$$

Measurements in last 60 years





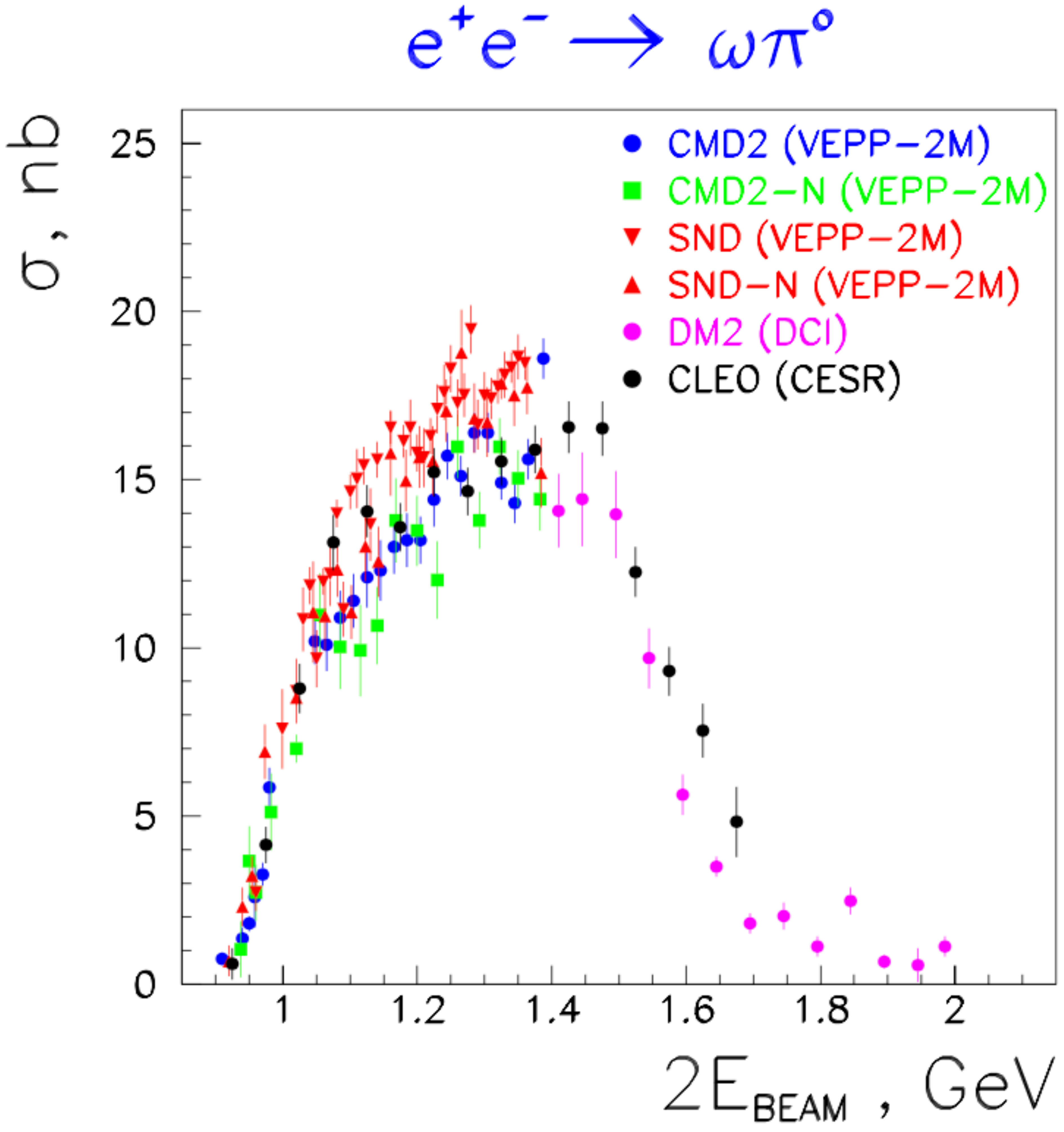
WP 2025, R. Aliberti, PR (2025)

Experiment	Percentage covered			
e^+e^-		CHKLS	DHMZ	KNTW
SND06	93.8%	508.2(7.3)	505.3(6.7)	506.2(6.8)
CMD-2	88.6%	506.2(5.0)	508.1(3.9)	508.4(3.7)
BaBar	99.9%	512.2(3.7)	514.1(3.8)	514.4(3.8)
KLOE	97.2%	501.3(2.6)	504.5(5.2)	504.5(5.1)
BESIII	72.8%	500.8(5.2)	505.9(3.7)	502.6(3.8)
SND20	80.3%	505.5(5.9)	509.6(3.6)	508.3(4.2)
CMD-3	98.9%	524.1(1.3)	525.9(4.2)	526.0(4.1)
τ			DHLMZ-23+LMR-24+WP25	
Belle+CLEO+ALEPH+OPAL	100%		517.2(5.8)	

CVC test in $\omega\pi$ channel (2000)

Tau data from CLEO,

K Edwards et al PRD61 (2000)



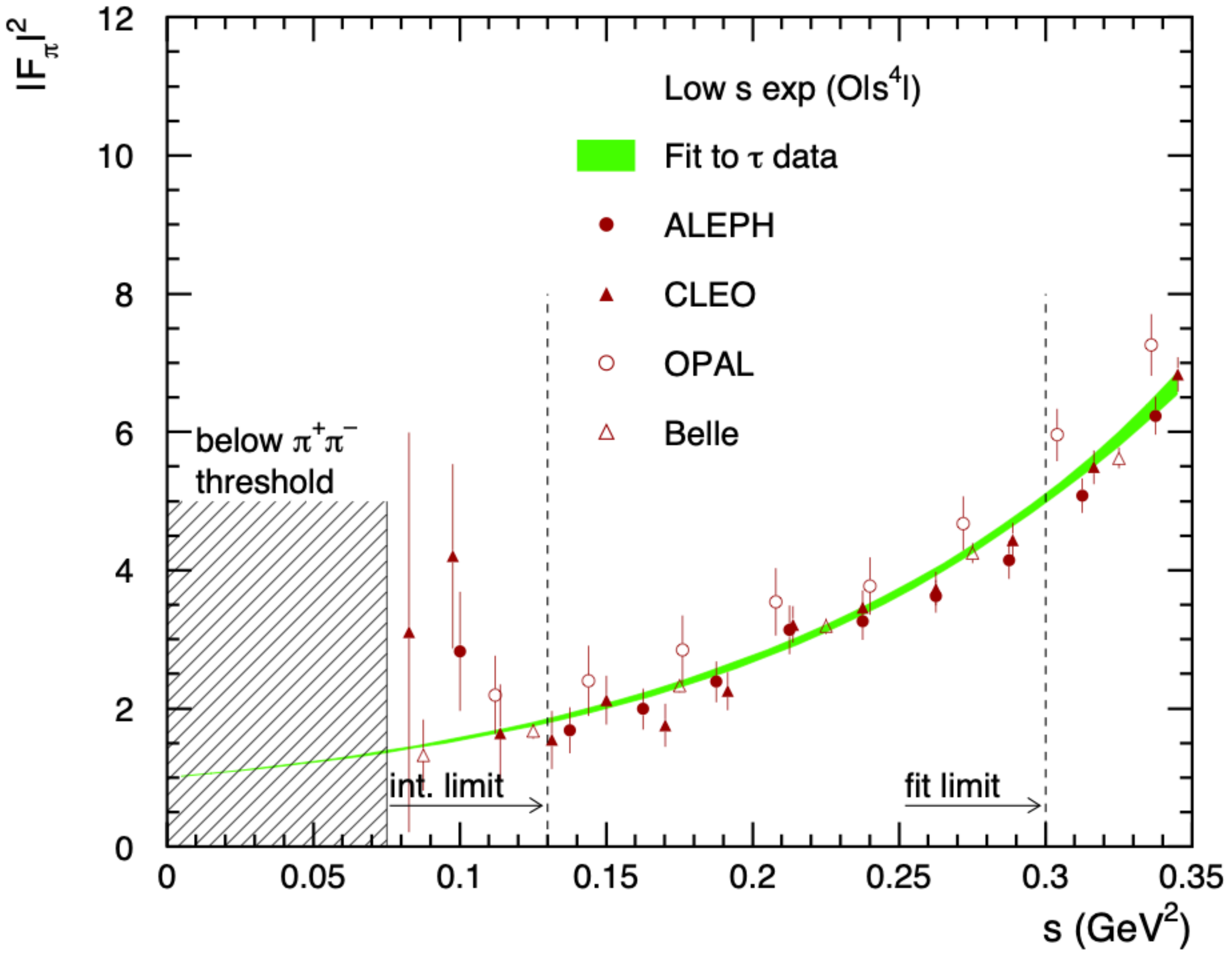
S. Eidelman, *eConf* C010430 (2001) M07

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{HVP}} + a_\mu^{\mathrm{HLbL}}$$

$$(\pm 0.2 \quad \pm 0.4 \quad \pm 61 \quad \pm 9.9) \times 10^{-11}$$ **Lattice WP 2025**

± 40 — e^+e^- **WP 2020**

± 62 — $\tau(\pi\pi) + e^+e^-$ **2025**



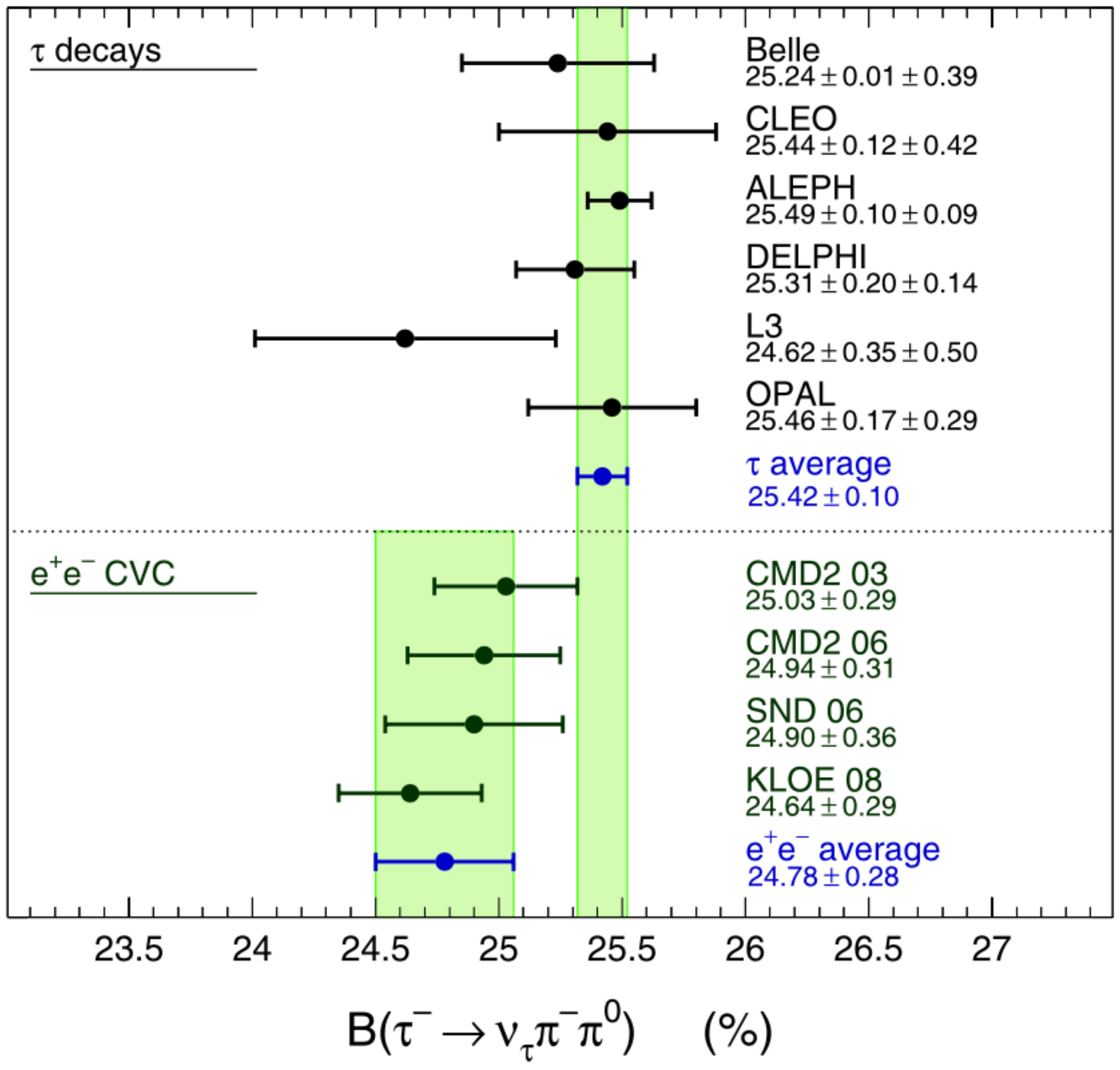
Low energy region compared to fit to tau data.

Experiment	$a_\mu^{\rm had,LO}[\pi\pi,\tau]$ (10^{-10}) $2m_{\pi^\pm} - 0.36$ GeV	$0.36 - 1.8$ GeV
ALEPH	$9.80 \pm 0.40 \pm 0.05 \pm 0.07$	$501.2 \pm 4.5 \pm 2.7 \pm 1.9$
CLEO	$9.65 \pm 0.42 \pm 0.17 \pm 0.07$	$504.5 \pm 5.4 \pm 8.8 \pm 1.9$
OPAL	$11.31 \pm 0.76 \pm 0.15 \pm 0.07$	$515.6 \pm 9.9 \pm 6.9 \pm 1.9$
Belle	$9.74 \pm 0.28 \pm 0.15 \pm 0.07$	$503.9 \pm 1.9 \pm 7.8 \pm 1.9$
Combined	$9.82 \pm 0.13 \pm 0.04 \pm 0.07$	$506.4 \pm 1.9 \pm 2.2 \pm 1.9$

IB corrections to CVC-based prediction

$$\Delta\mathcal{B}^{\rm CVC}_{\pi^-\pi^0} = \frac{3}{2}\frac{\mathcal{B}_e|V_{ud}|^2}{\pi\alpha^2 m_\tau^2}\int_{s_{\rm min}}^{m_\tau^2} ds\, s\, \sigma^0_{\pi^+\pi^-}(s) \times\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)\left[\frac{S_{\rm EW}}{R_{\rm IB}(s)}-1\right],$$

Davier et al, EPJC 66 (2010)

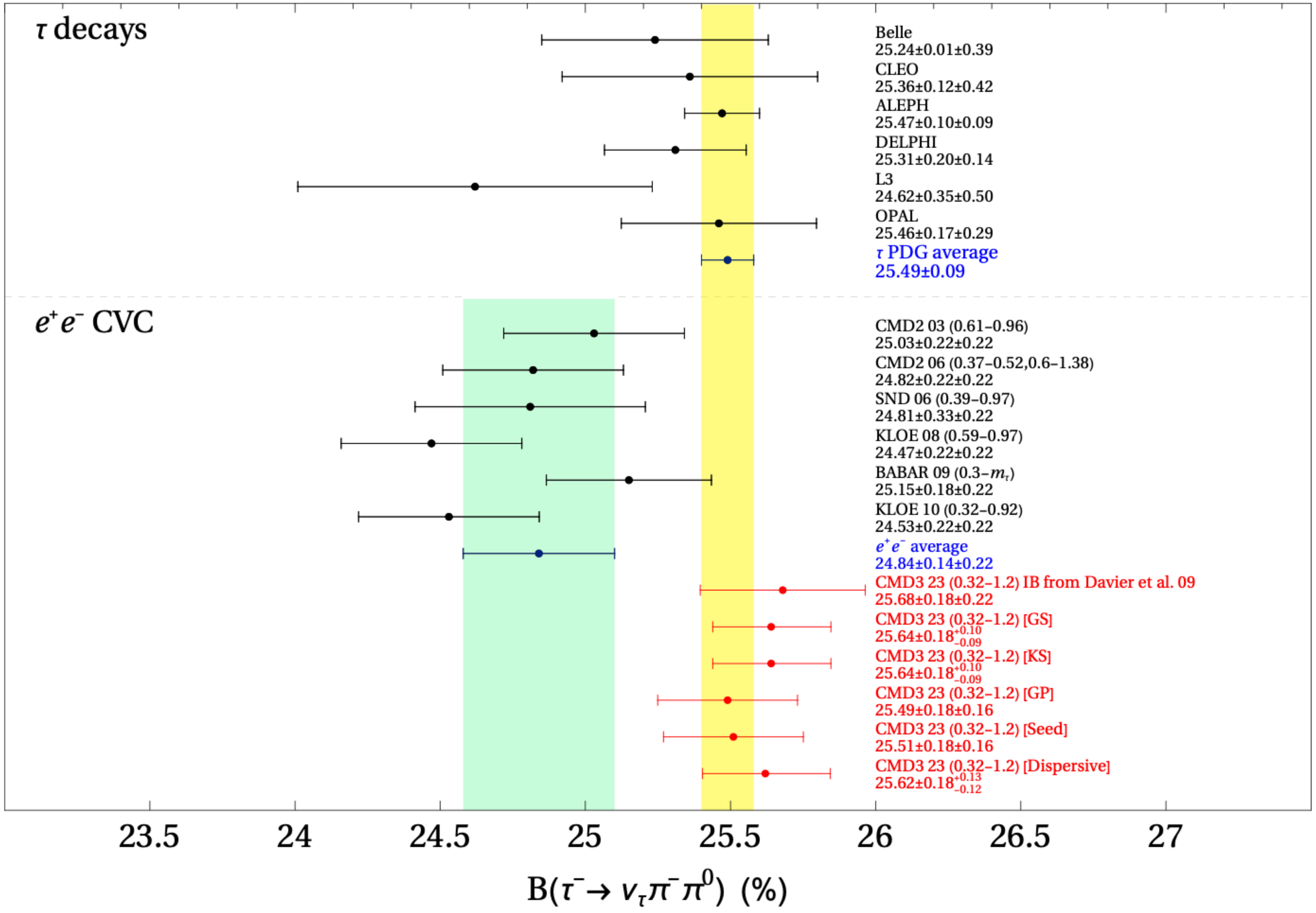


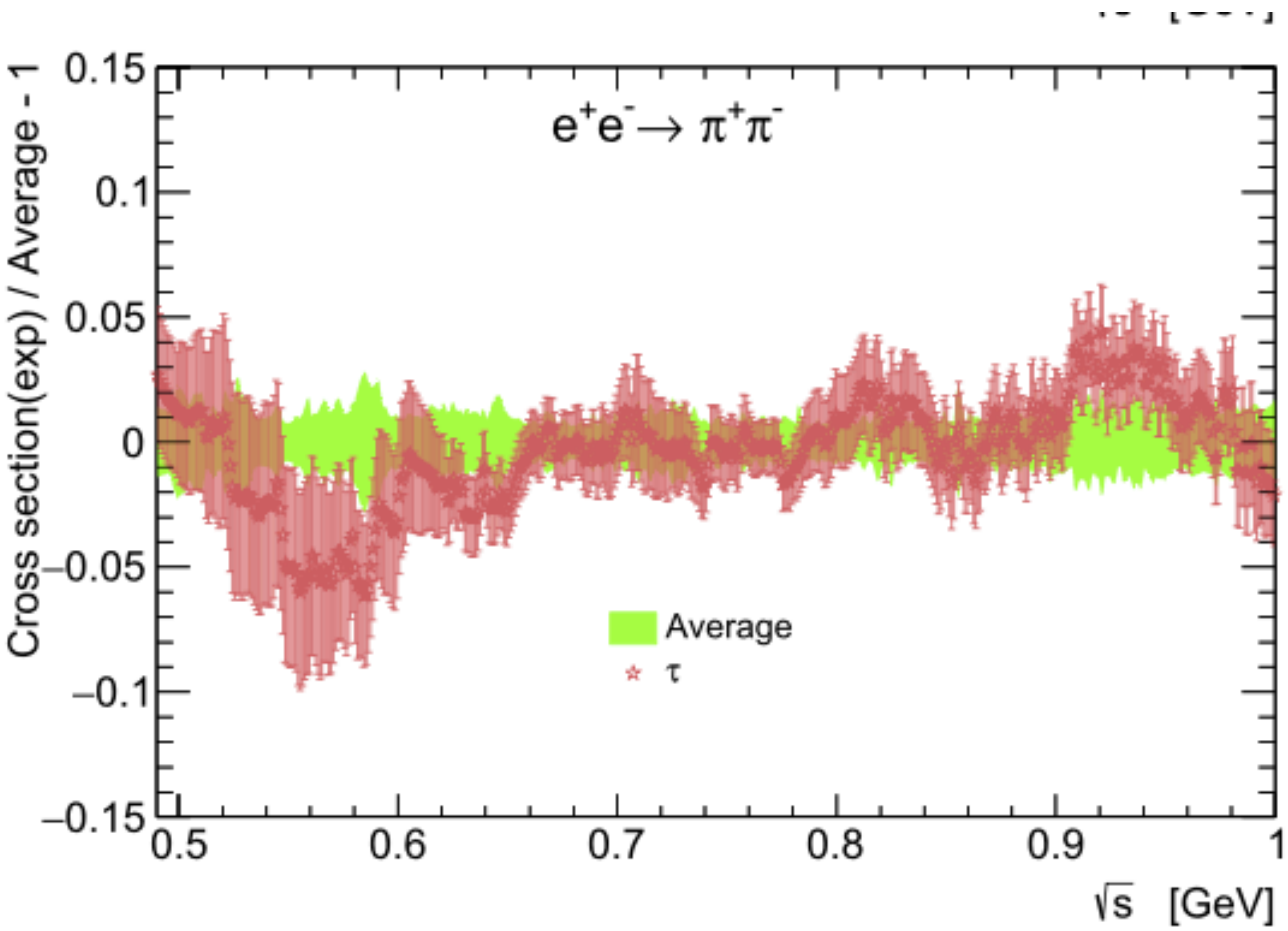
$$\mathrm{BR}(\tau^- \to \pi^-\pi^0\nu) = (24.78 \pm 0.17_{e^+e^-} \pm 0.22_{\rm IB})\,\% \;\; \text{w/KLOE}$$

$$\mathrm{BR}(\tau^- \to \pi^-\pi^0\nu) = (24.92 \pm 0.21_{e^+e^-} \pm 0.22_{\rm IB})\,\% \;\; \text{wo/KLOE}$$

IB corr= +0.69(22) %

Updated $BR^{CVC}(\tau \to \pi\pi\nu_\tau)$. **Castro-Miranda-Roig, PRD 111 (2025)**





$e^+e^- \to \pi^+\pi^-$
Cross section(exp) / Average - 1
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
0.5
0.6
0.7
0.8
0.9
1
$\sqrt{s}$ [GeV]
Average
τ