



Measurements of $|V_{us}|$ at Belle II

Workshop on Tau Physics at Belle II and STCF

31st August, Jilin University, Changchun, China

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HELMHOLTZ

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The $|V_{us}|$ element of the CKM matrix

SM parameters:

- 3 gauge couplings
- 2 Higgs parameters
- 3 charged lepton masses
- 6 quark masses
- **3 mixing angles + 1 phase**

Quark flavor

$$\text{CKM} = \begin{matrix} & \begin{matrix} d & s & b \end{matrix} \\ \begin{matrix} u \\ c \\ t \end{matrix} & \begin{pmatrix} 1 & \lambda & A\lambda^3\rho e^{i\phi} \\ -\lambda & 1 & A\lambda^2 \\ -A\lambda^3\rho e^{i\phi} & -A\lambda^2 & 1 \end{pmatrix} \end{matrix}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- $|V_{ud}| = 0.97384 \pm 0.00021$ (from nuclear β & neutron decays)
- $|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$ (from $B \rightarrow X_u \ell \nu$ decays)
- $|V_{us}| = 0.2272 \pm 0.0011$ [see next slides]

Precision measurement of $|V_{us}|$ is a stringent test of CKM unitarity

$|V_{us}|$ determination

HFLAV23

Kl3 decays:

$$K^0 \begin{array}{c} \bar{s} \\ d \end{array} \begin{array}{c} \nu \\ \ell^+ \end{array} \begin{array}{c} \bar{u} \\ d \end{array} \pi^- \quad K^+ \begin{array}{c} \bar{s} \\ u \end{array} \begin{array}{c} \nu \\ \ell^+ \end{array} \begin{array}{c} \bar{u} \\ u \end{array} \pi^0 \longrightarrow |V_{us}| f_+(0)$$

Kl2 decays:

$$K^+ \begin{array}{c} \bar{s} \\ u \end{array} \begin{array}{c} \nu \\ \ell^+ \end{array} \quad \pi^+ \begin{array}{c} \bar{d} \\ u \end{array} \begin{array}{c} \nu \\ \ell^+ \end{array} \longrightarrow \frac{|V_{us}|}{|V_{ud}|} \frac{F_K}{F_\pi}$$

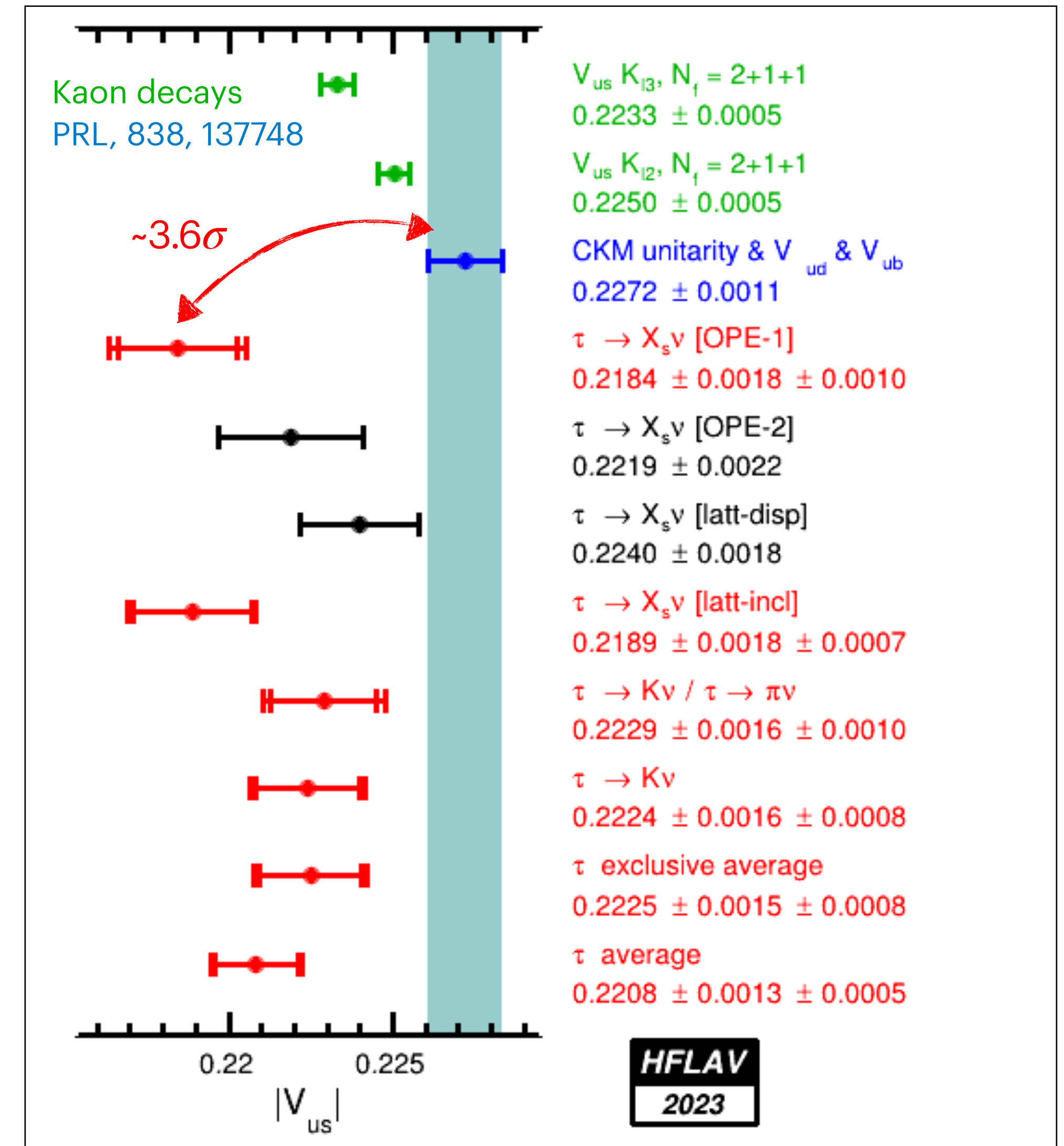
Hyperon decays:

$$\Xi^0 \begin{array}{c} s \\ u \\ s \end{array} \begin{array}{c} \nu \\ \ell^- \end{array} \begin{array}{c} u \\ u \\ s \end{array} \Sigma^+ \longrightarrow |V_{us}| f_1(0)$$

τ decays:

$$\tau^- \begin{array}{c} d' \\ \bar{u} \end{array} = V_{ud}d + V_{us}s \longrightarrow \alpha_s, m_s, |V_{us}|$$

Focus of this talk



SuperKEKB and Belle II experiment

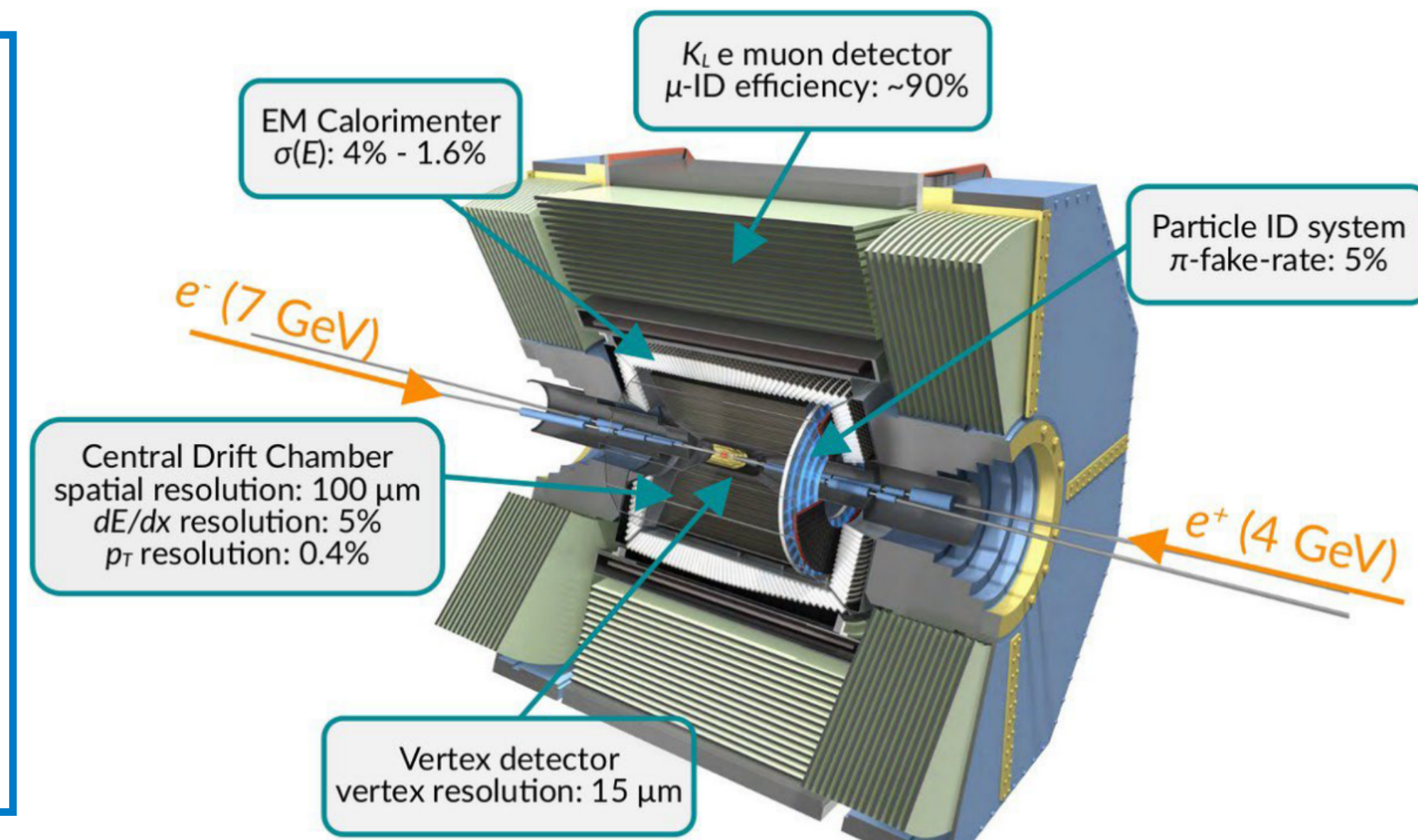
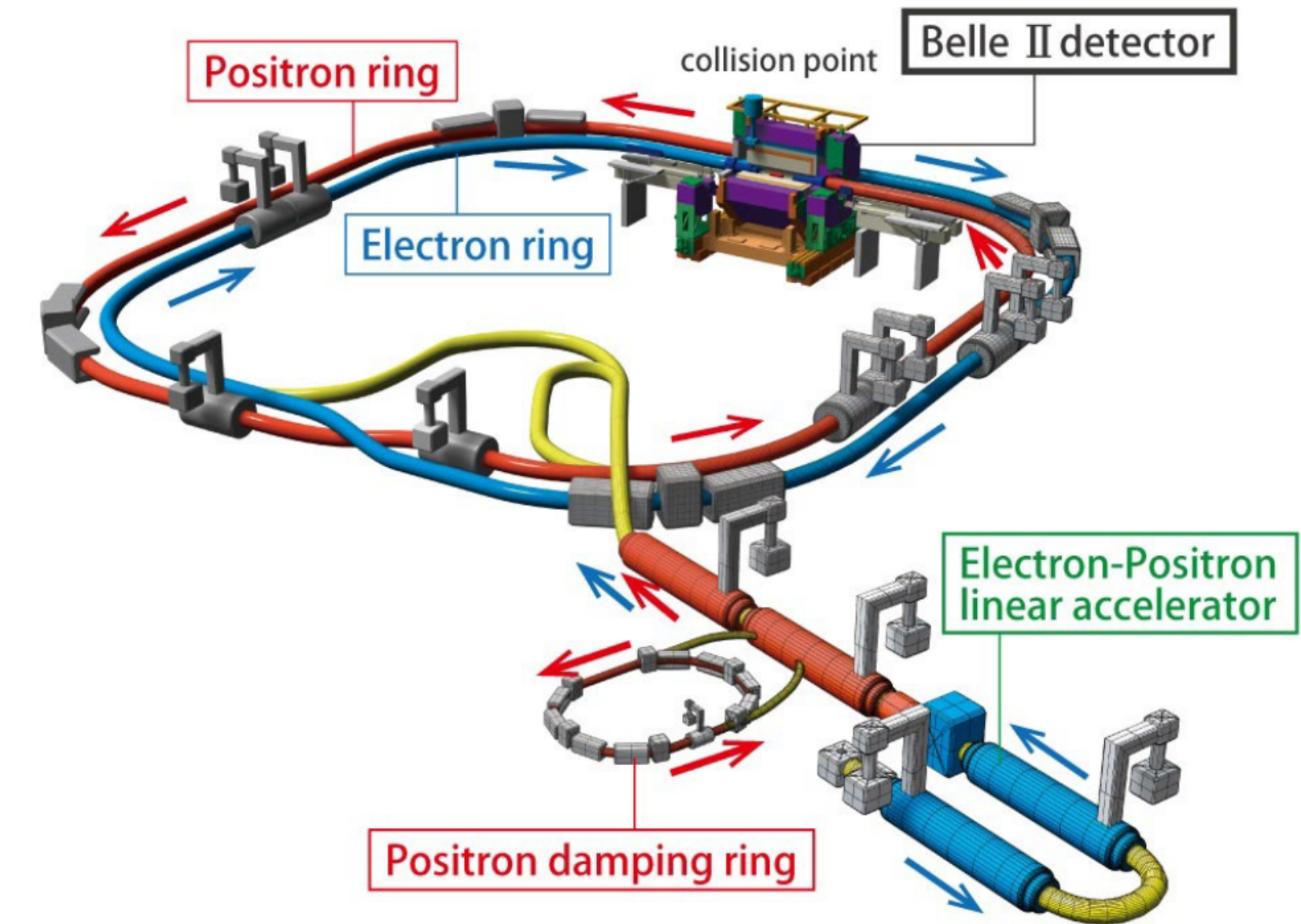
More than just *B*-factories

- Mostly operating at the $\Upsilon(4S)$ resonance ($\sqrt{s} = 10.58 \text{ GeV}$)
- **World record instantaneous luminosity of $5.32 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$**
- Belle II explores the intensity frontier at SuperKEKB:
 - Accumulated world's largest $\Upsilon(4S)$ dataset, surpassing Belle:
 - 808 fb^{-1} at $\Upsilon(4S)$ and 914 fb^{-1} overall

Ideal for τ physics

- Clean experimental environment with high trigger efficiency
- Hermetic detector with well-known initial state → **strong in searches with missing energy**
- Efficient reconstruction of neutrals (γ, π^0, η) & excellent charged PID capabilities (e, μ, π, K)
- Excellent track momentum & vertex resolution (e.g. τ lifetime)

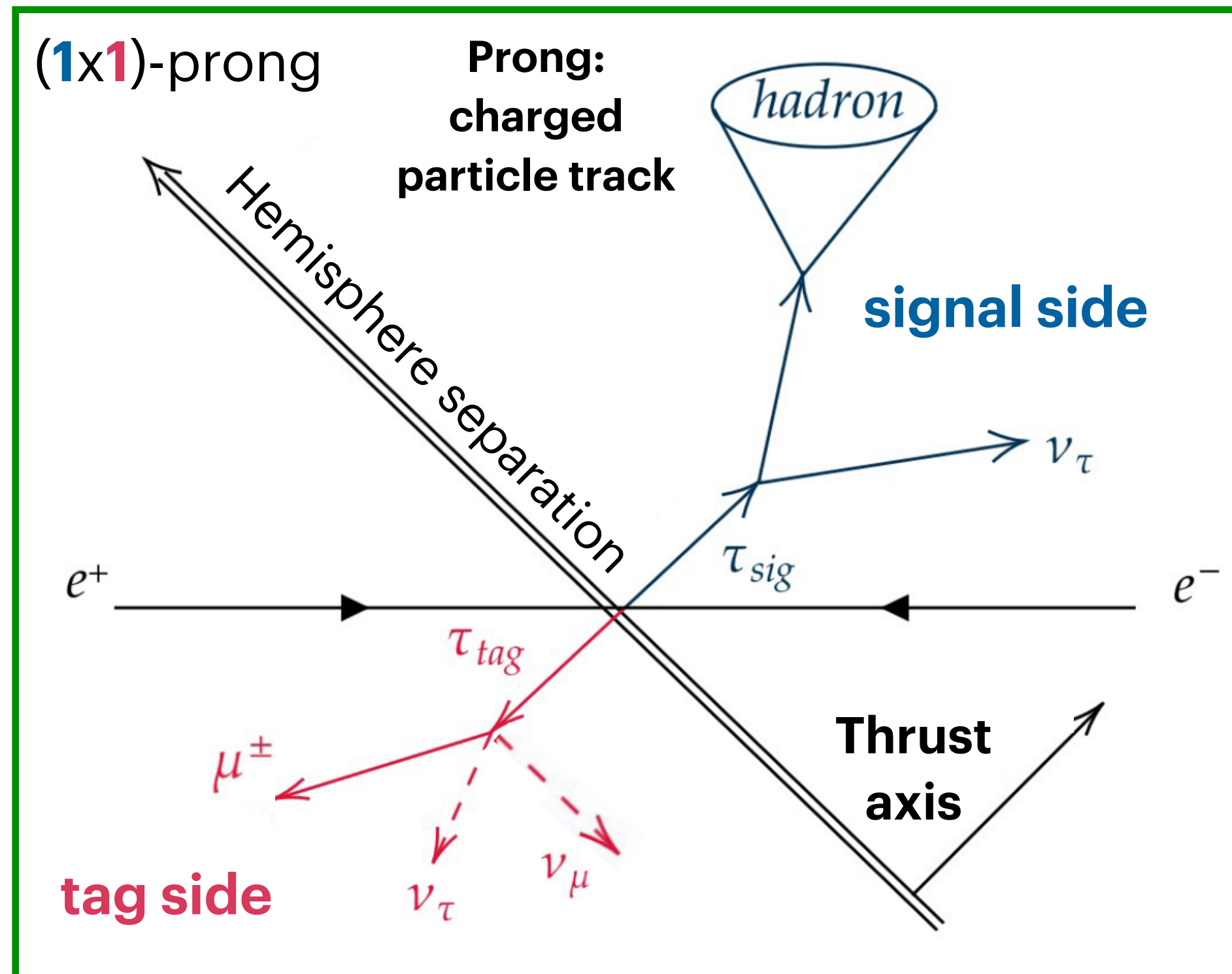
See Fabian's talk



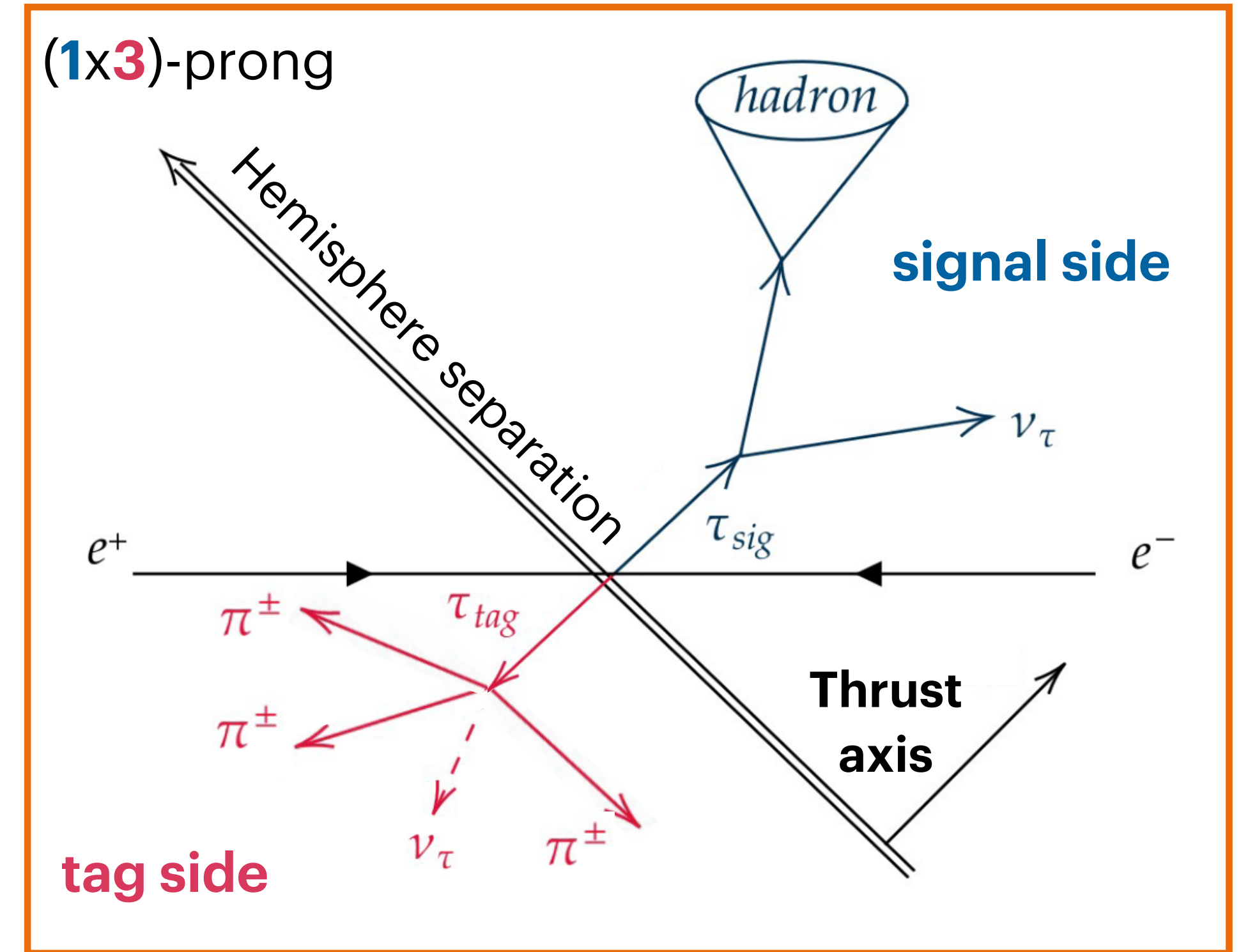
τ topologies and signatures @ Belle II

- Large cross section for $e^+e^- \rightarrow \tau^+\tau^-$. SM τ cannot be fully reconstructed due to the missing neutrino(s)
- Identify $e^+e^- \rightarrow \tau^+\tau^-$ events using the thrust direction of the event (**thrust axis**)
- Two hemispheres defined in the centre-of mass system by a plane perpendicular to the thrust axis

$$\sigma(e^+e^- \rightarrow b\bar{b}) = 1.05 \text{ nb}$$
$$\sigma(e^+e^- \rightarrow \tau^+\tau^-) = 0.92 \text{ nb}$$



Large QED background
Small continuum background



Small QED background
Large continuum background

Overview

Multiple ongoing analyses to measure both exclusive and inclusive $|V_{us}|$ at Belle II

Exclusive determination



Exclusive $|V_{us}|$ measurement using 1x1 topology



Exclusive $|V_{us}|$ measurement using 1x3 topology

Overview

Multiple ongoing analyses to measure both exclusive and inclusive $|V_{us}|$ at Belle II

Exclusive determination



Exclusive $|V_{us}|$ measurement using 1x1 topology



Exclusive $|V_{us}|$ measurement using 1x3 topology

Main topic
of this talk

Inclusive determination



First inclusive $|V_{us}|$ measurement using a
1-prong tag and all possible signal-side topologies

Overview

Multiple ongoing analyses to measure both exclusive and inclusive $|V_{us}|$ at Belle II

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Exclusive $|V_{us}|$ measurement using 1x1 topology



Exclusive $|V_{us}|$ measurement using 1x3 topology

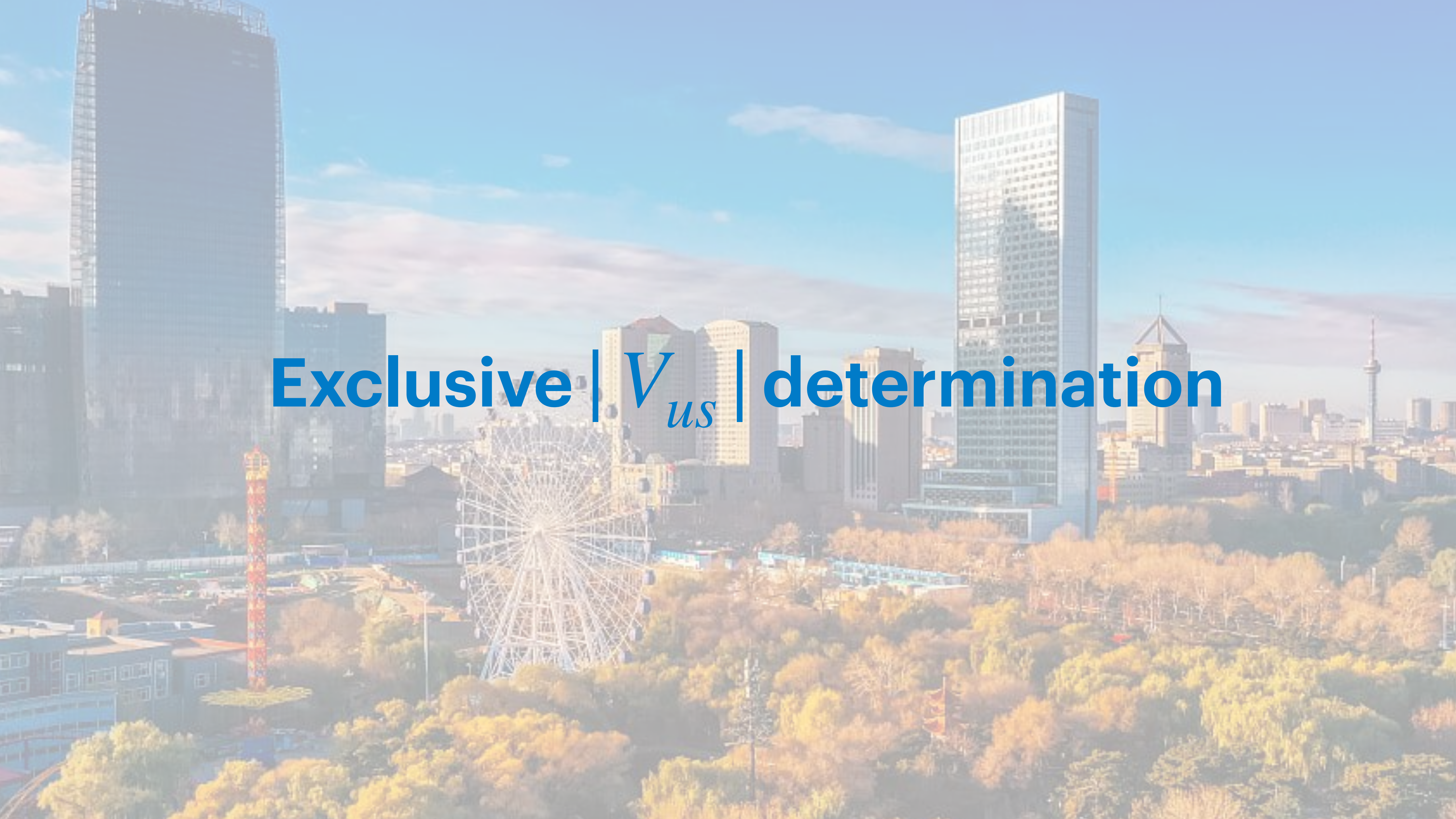
Main topic
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Inclusive determination



First inclusive $|V_{us}|$ measurement using a
1-prong tag and all possible signal-side topologies

+ additional ongoing studies providing useful inputs to $|V_{us}|$: e.g. BR and spectral functions of the decay channels with open strangeness $\tau \rightarrow KX\nu$ [see backup and Petar's and Alexander's talks]

An aerial photograph of a city skyline. In the foreground, there is a dense forest of trees with yellow and orange autumn foliage. Behind the trees, a large Ferris wheel is visible. In the background, several tall skyscrapers rise against a blue sky with light clouds. The text "Exclusive | V_{us} | determination" is overlaid in the center of the image.

Exclusive | V_{us} | determination

Exclusive $|V_{us}|$ determination

Compare the $\mathcal{B}(\tau \rightarrow \pi\nu)$ and $\mathcal{B}(\tau \rightarrow K\nu)$ [best measurement from BaBar [PRL 105,051602](#)]

Exclusive $|V_{us}|$ determination

Compare the $\mathcal{B}(\tau \rightarrow \pi\nu)$ and $\mathcal{B}(\tau \rightarrow K\nu)$ [best measurement from BaBar [PRL 105,051602](#)]

- Ratio method

$$\frac{\mathcal{B}(\tau^- \rightarrow K^- \nu_\tau)}{\mathcal{B}(\tau^- \rightarrow \pi^- \nu_\tau)} = \frac{f_K^2 |V_{us}|^2}{f_\pi^2 |V_{ud}|^2} \frac{(m_\tau^2 - m_K^2)^2}{(m_\tau^2 - m_\pi^2)^2} (1 + \delta_{LD})$$

decay constant electroweak corrections

$|V_{us}| = 0.2255 \pm 0.0024$
consistent with CKM unitarity

Exclusive $|V_{us}|$ determination

Compare the $\mathcal{B}(\tau \rightarrow \pi \nu)$ and $\mathcal{B}(\tau \rightarrow K \nu)$ [best measurement from BaBar [PRL 105,051602](#)]

- Ratio method

$$\frac{\mathcal{B}(\tau^- \rightarrow K^- \nu_\tau)}{\mathcal{B}(\tau^- \rightarrow \pi^- \nu_\tau)} = \frac{f_K^2 |V_{us}|^2}{f_\pi^2 |V_{ud}|^2} \frac{(m_\tau^2 - m_K^2)^2}{(m_\tau^2 - m_\pi^2)^2} (1 + \delta_{LD})$$

decay constant
electroweak corrections

$$|V_{us}| = 0.2255 \pm 0.0024$$

consistent with CKM unitarity

- Absolute decay

$$\mathcal{B}(\tau^- \rightarrow K^- \nu_\tau) = \frac{G_F^2 f_K^2 |V_{us}|^2 m_\tau^3 \tau_\tau}{16\pi} \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}$$

Fermi constant
electroweak corrections

$$|V_{us}| = 0.2193 \pm 0.0032$$

within 2σ of the value predicted by the CKM unitarity

Exclusive $|V_{us}|$ determination @ Belle II

Aim precision:

Measure of $R_{K/\pi} = \frac{\mathcal{B}(\tau \rightarrow K\nu)}{\mathcal{B}(\tau \rightarrow \pi\nu)} \propto \frac{|V_{us}|^2}{|V_{ud}|^2}$ with a total systematic uncertainty $< 1\%$

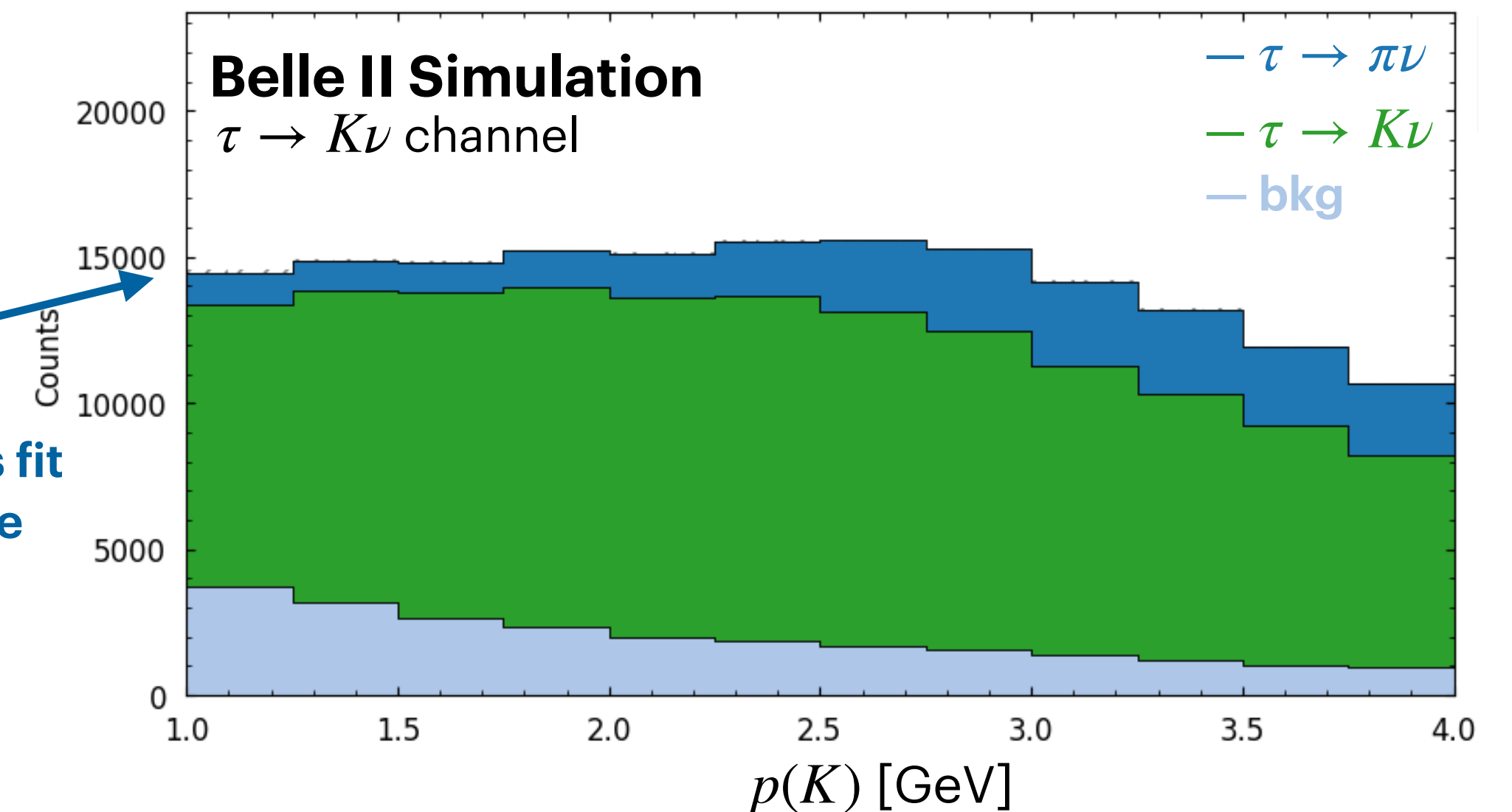
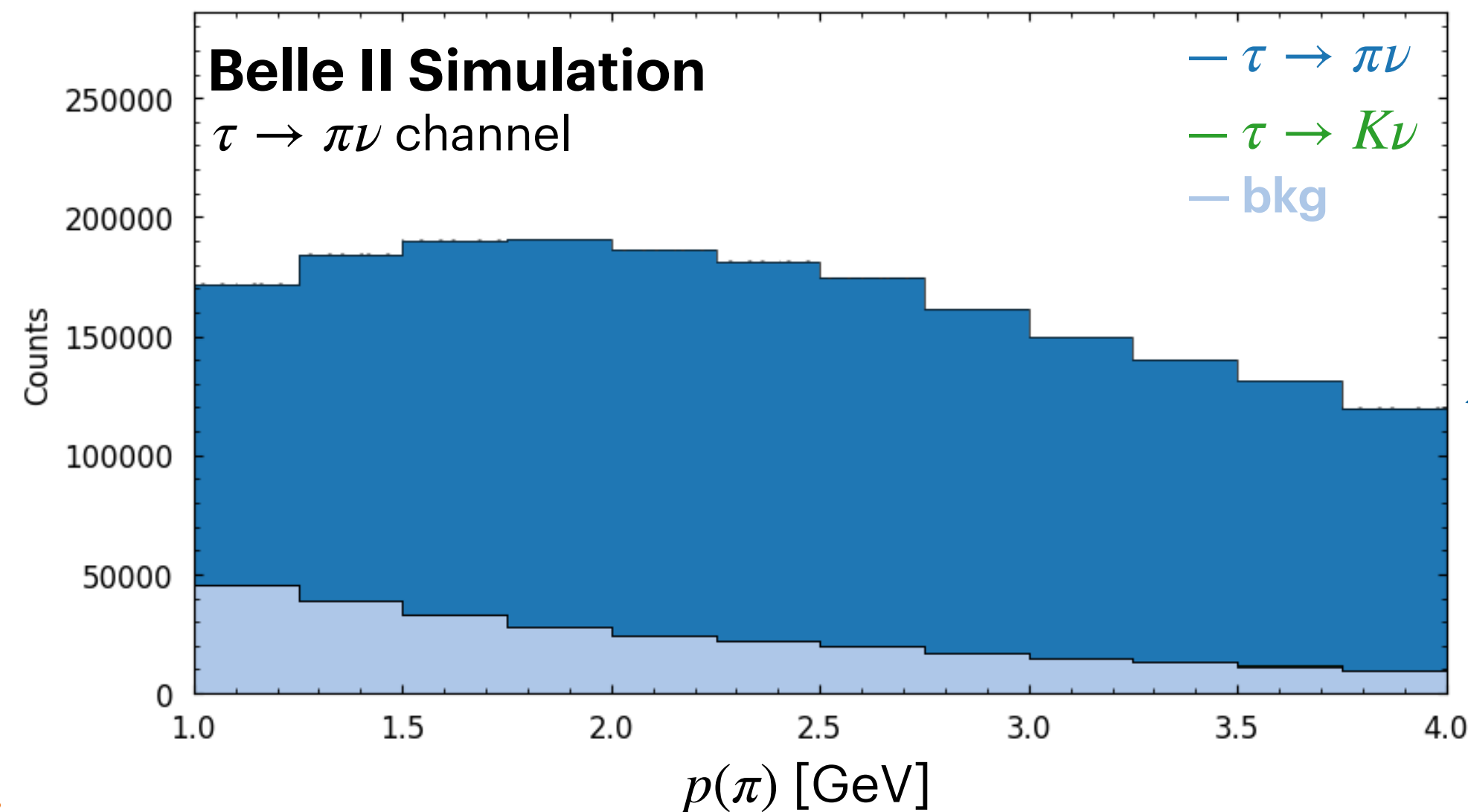
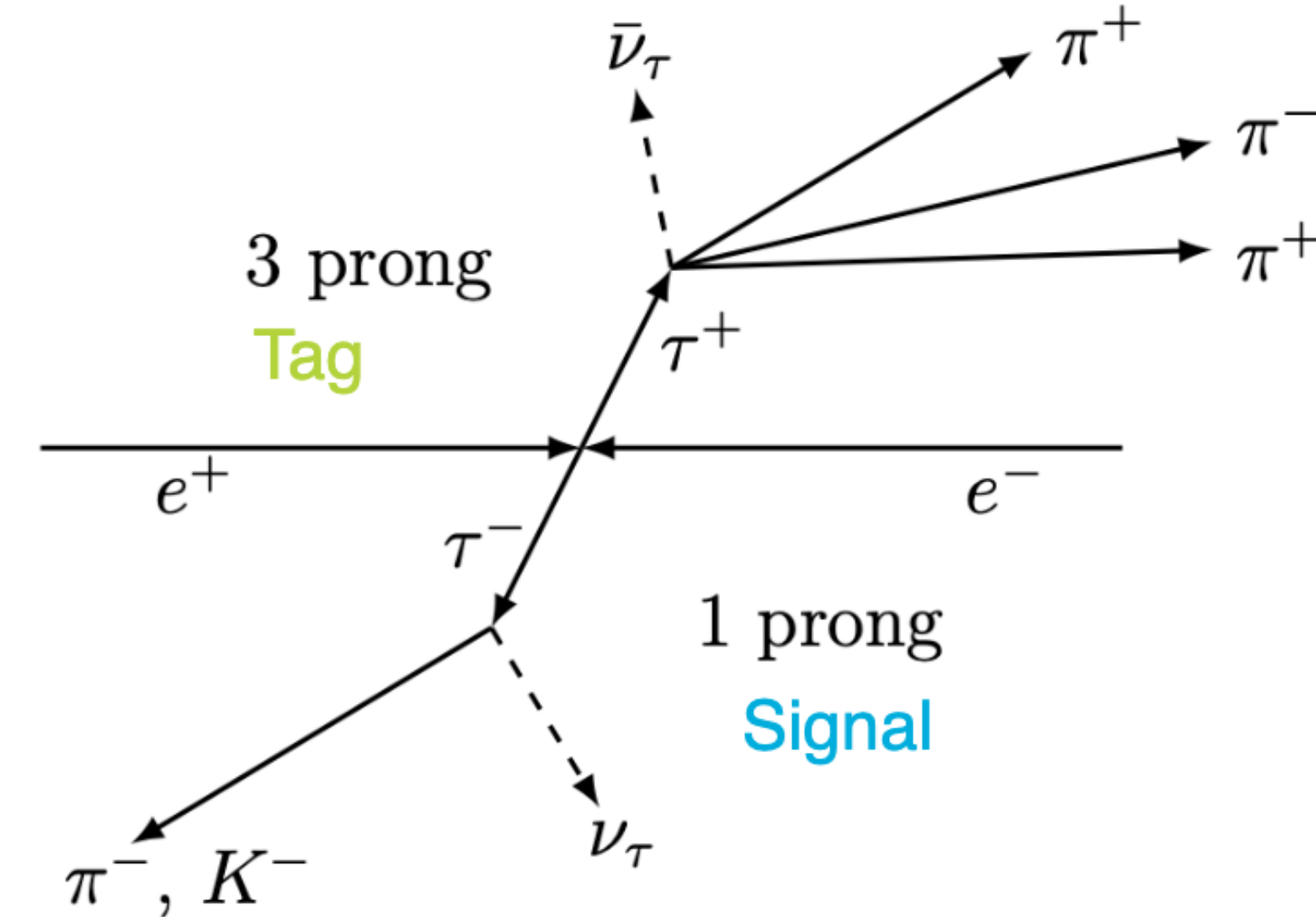
Strategy:

Signal side: $\tau \rightarrow h\nu$, tag side: $\tau \rightarrow \ell\nu\nu$, $\tau \rightarrow n(h)\nu$ where $h = \pi, K$

Simultaneous template fit to both the pion and kaon channels

Main backgrounds:

- Pion channel: 60% from $\tau \rightarrow \pi\pi^0\nu$, 20% from $\tau \rightarrow \mu\nu\nu$
- Kaon channel: 28% from $\tau \rightarrow \mu\nu\nu$, 22% from $\tau \rightarrow KK_L\nu$, 14% from $\tau \rightarrow K\pi^0\nu$, 9% from $\tau \rightarrow \pi\pi^0\nu$



simultaneous fit
between the
channels

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Inclusive | V_{us} | determination

Inclusive $|V_{us}|$ determination

Compare the $\mathcal{B}(\tau \rightarrow (\bar{u}d) \nu)$ and $\mathcal{B}(\tau \rightarrow (\bar{u}s) \nu)$



fundamental parameters of SM

$$(\alpha_s, m_s, |V_{us}|)$$

Inclusive $|V_{us}|$ determination

Compare the $\mathcal{B}(\tau \rightarrow (\bar{u}d) \nu)$ and $\mathcal{B}(\tau \rightarrow (\bar{u}s) \nu)$

Theory:

- OPE-1: uses perturbative QCD and experimental low energy scattering measurements

$$|V_{us}|_{\tau\text{-OPE-1}} = \sqrt{R_{us} / \left[\frac{R_{ud}}{|V_{ud}|^2} - \delta R_{\tau\text{-}SU(3)\text{-break}} \right]} = 0.2184 \pm 0.0021$$

Annotations for the equation:

- hadrons with S=1 (points to R_{us})
- BR w.r.t. $\mathcal{B}(\tau \rightarrow e\nu\nu)$ (points to R_{us})
- hadrons with S=0 (points to R_{ud})
- SU(3)-breaking effects (points to $\delta R_{\tau\text{-}SU(3)\text{-break}}$)

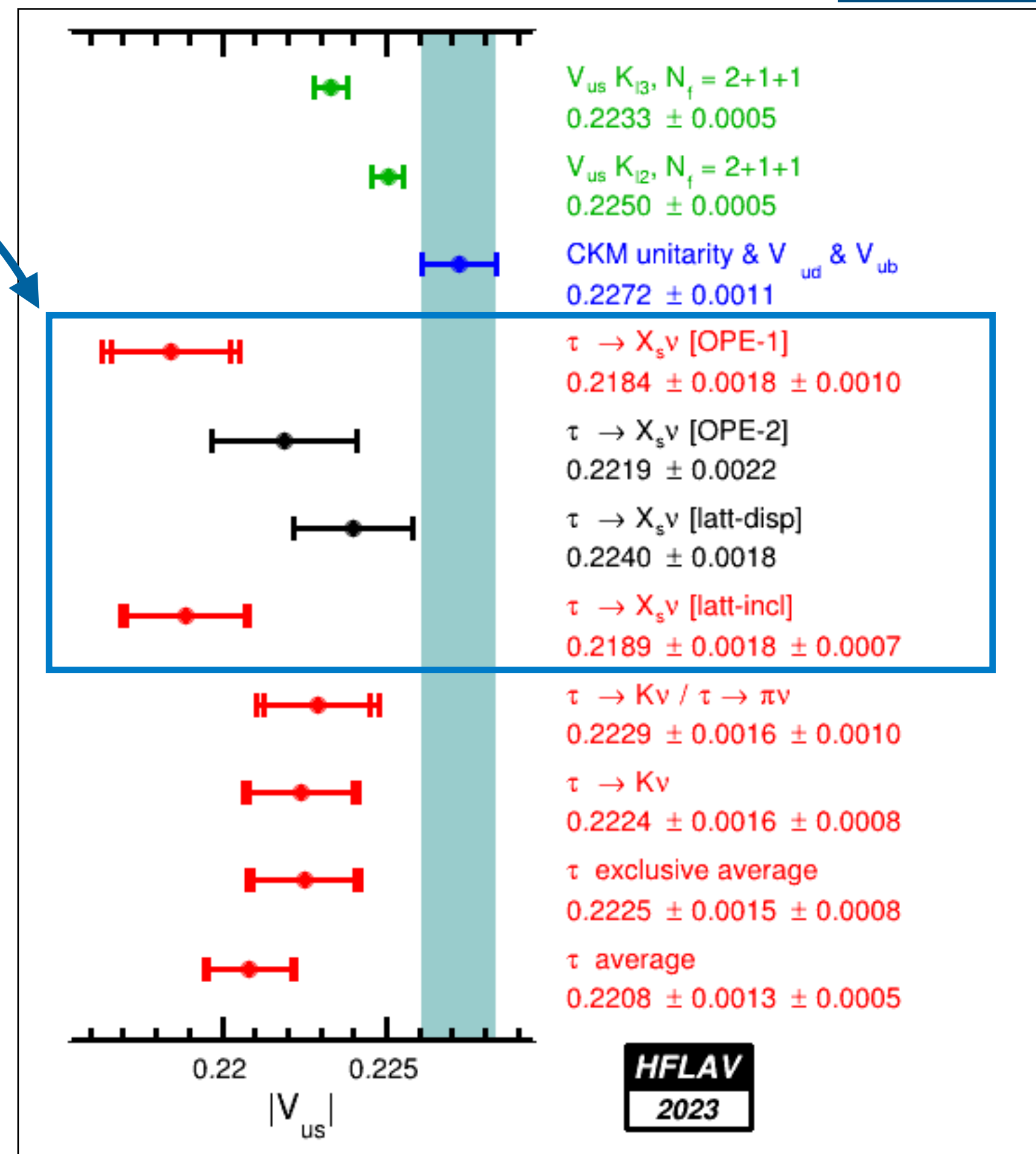
- OPE-2 and latt-disp: alternative procedures using τ spectral functions and lattice QCD methods

Latt-incl: $|V_{us}|_{\tau\text{-latt-incl}} = \sqrt{\left(\frac{|V_{us}|^2}{R_{us}} \right)_{\text{latt-incl}}} \cdot R_{us}$

fundamental parameters of SM

$$(\alpha_s, m_s, |V_{us}|)$$

HFLAV23



Inclusive $|V_{us}|$ determination

Compare the $\mathcal{B}(\tau \rightarrow (\bar{u}d) \nu)$ and $\mathcal{B}(\tau \rightarrow (\bar{u}s) \nu)$

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- OPE-1: uses perturbative QCD and experimental low energy scattering measurements

$$|V_{us}|_{\tau\text{-OPE-1}} = \sqrt{R_{us} / \left[\frac{R_{ud}}{|V_{ud}|^2} - \delta R_{\tau\text{-}SU(3)\text{-break}} \right]} = 0.2184 \pm 0.0021$$

Annotations for the equation:

- R_{us} : hadrons with S=1
- R_{ud} : hadrons with S=0
- $\mathcal{B}(\tau \rightarrow e \nu \nu)$: BR w.r.t. $\mathcal{B}(\tau \rightarrow e \nu \nu)$
- $\delta R_{\tau\text{-}SU(3)\text{-break}}$: SU(3)-breaking effects

- OPE-2 and latt-disp: alternative procedures using τ spectral functions and lattice QCD methods

$$\text{Latt-incl: } |V_{us}|_{\tau\text{-latt-incl}} = \sqrt{\left(\frac{|V_{us}|^2}{R_{us}} \right)_{\text{latt-incl}}} \cdot R_{us}$$

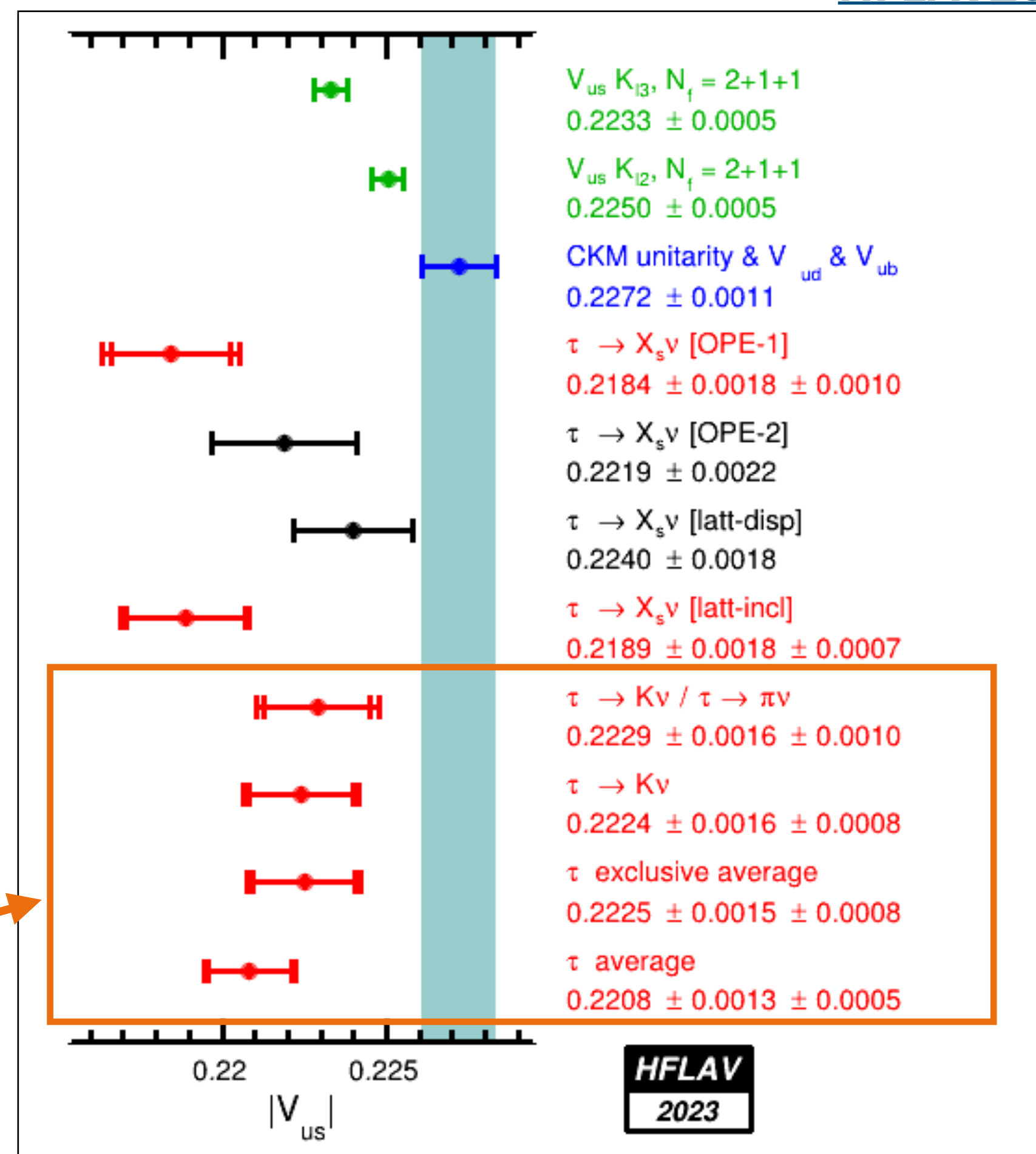
Experiment:

- No $\mathcal{B}(\tau \rightarrow X_s \nu_\tau)$
- Sum of exclusive modes

fundamental parameters of SM

$$(\alpha_s, m_s, |V_{us}|)$$

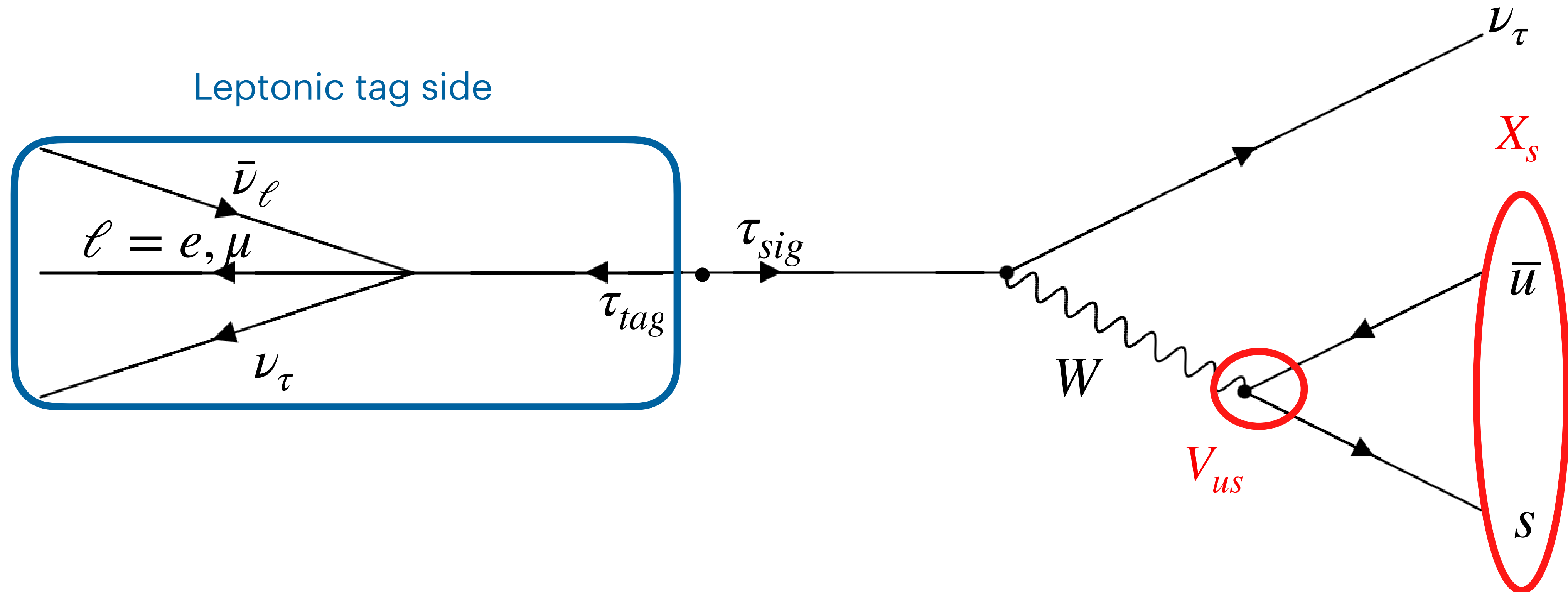
HFLAV23



$$X = n(\pi^0), \pi^+\pi^-, K^0, \bar{K}^0, K^+\pi^-$$

Inclusive $|V_{us}|$ determination @ Belle II

Goal: first inclusive $|V_{us}|$ measurement by measuring the inclusive $\mathcal{B}(\tau \rightarrow X_s \nu_\tau)$



$$V_{us}^{incl.} \propto \mathcal{B}(\tau \rightarrow X_s \nu_\tau) \rightarrow \mathcal{B}(\tau \rightarrow K^+ X \nu_\tau) + \mathcal{B}(\tau \rightarrow \pi K^0 X \nu_\tau)$$

Challenges vs advantages

Main focus of today

Challenge 1: access $\tau^+ \rightarrow K^+ X \nu$ and $\tau^+ \rightarrow \pi^+ K^0 X \nu$ from unfolded yields

- Decide how many modes to subtract, how many to unfold

Challenge 2: be maximally model independent

- Unfolding need MC for efficiencies
- Independent of modelling: perform the measurement relying only on phase-space MC

Challenge 3: need to measure $\mathcal{B}(\tau^+ \rightarrow \pi^+ K_L^0 K_S^0 n(\pi^0) \nu)$

- No isospin relations or assumptions
- Challenging K_L^0 detection at Belle II

Challenge 4: measure $|V_{us}|$ with a target precision of ~1% (systematics limited)

Challenges vs advantages

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Challenge 1: access $\tau^+ \rightarrow K^+ X \nu$ and $\tau^+ \rightarrow \pi^+ K_S X \nu$ from unfolded yields

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Challenge 4: measure $|V_{us}|$ with a target precision of ~1% (systematics limited)

Advantage 1: independent of the sum of exclusive modes from different experiments

Advantage 2: insight into the historical shift of $\mathcal{B}(\tau \rightarrow X_S \nu_\tau)$ towards lower values

.....

Challenge 1

Access $\tau^+ \rightarrow K^+ X \nu$ and $\tau^+ \rightarrow \pi^+ K^0 X \nu$: strategy

$$V_{us}^{incl.} \propto \mathcal{B}(\tau \rightarrow K^+ X \nu_\tau) + \mathcal{B}(\tau \rightarrow \pi K_S^0 X \nu_\tau) + \mathcal{B}(\tau \rightarrow \pi K_L^0 X \nu_\tau)$$

Decay mode			Contribution
1	$\tau^+ \rightarrow K^+ X \nu$		$V_{us} + V_{ud}(K)$
1.1		$\tau^+ \rightarrow K^+ n(\pi^0) \nu$	V_{us}
1.2		$\tau^+ \rightarrow K^+ K_S n(\pi^0) \nu$	$V_{ud}(K)$
1.3		$\tau^+ \rightarrow K^+ K^- \pi^+ n(\pi^0) \nu$	$V_{ud}(K)$

Decay mode			Contribution
2	$\tau^+ \rightarrow \pi^+ K_S X \nu$		$V_{us} + V_{ud}(K)$
2.1		$\tau^+ \rightarrow \pi^+ K_S n(\pi^0) \nu$	V_{us}
2.2		$\tau^+ \rightarrow \pi^+ K_S K_S n(\pi^0) \nu$	$V_{ud}(K)$
2.3		$\tau^+ \rightarrow \pi^+ K_L K_S n(\pi^0) \nu$	$V_{ud}(K)$

Even more challenging

- Alternative:
 $\mathcal{B}(\tau \rightarrow \pi K_L n(\pi^0) \nu_\tau) = \mathcal{B}(\tau \rightarrow \pi K_S n(\pi^0) \nu_\tau)$
- Add systematic uncertainty

NB: assume a negligible impact from the modes with 3K ($\mathcal{B}(3K) < 10^{-7}$)

X-talk between the channels due to $\pi \leftrightarrow K$ misID

$$N_1 + 2N_2 = \underbrace{N_{V_{us},1.1} + 2N_{V_{us},2.1}}_{V_{us}} + \underbrace{2N_{1.2} + N_{1.3} + 2N_{2.2} + N_{2.3}}_{V_{ud}}$$

Extract the inclusive $|V_{us}|$ and the $|V_{ud}|$ contributions using the **unfolding procedure**

Challenge 1

Access $\tau^+ \rightarrow K^+ X \nu$ and $\tau^+ \rightarrow \pi^+ K^0 X \nu$: sample composition

Reconstruct the $\tau \rightarrow \pi K_S n(\pi^0) \nu$, $\tau \rightarrow \pi K_S K_S n(\pi^0) \nu$ and $\tau \rightarrow \pi K_S K_L n(\pi^0) \nu$ channels, with a leptonic tag side.

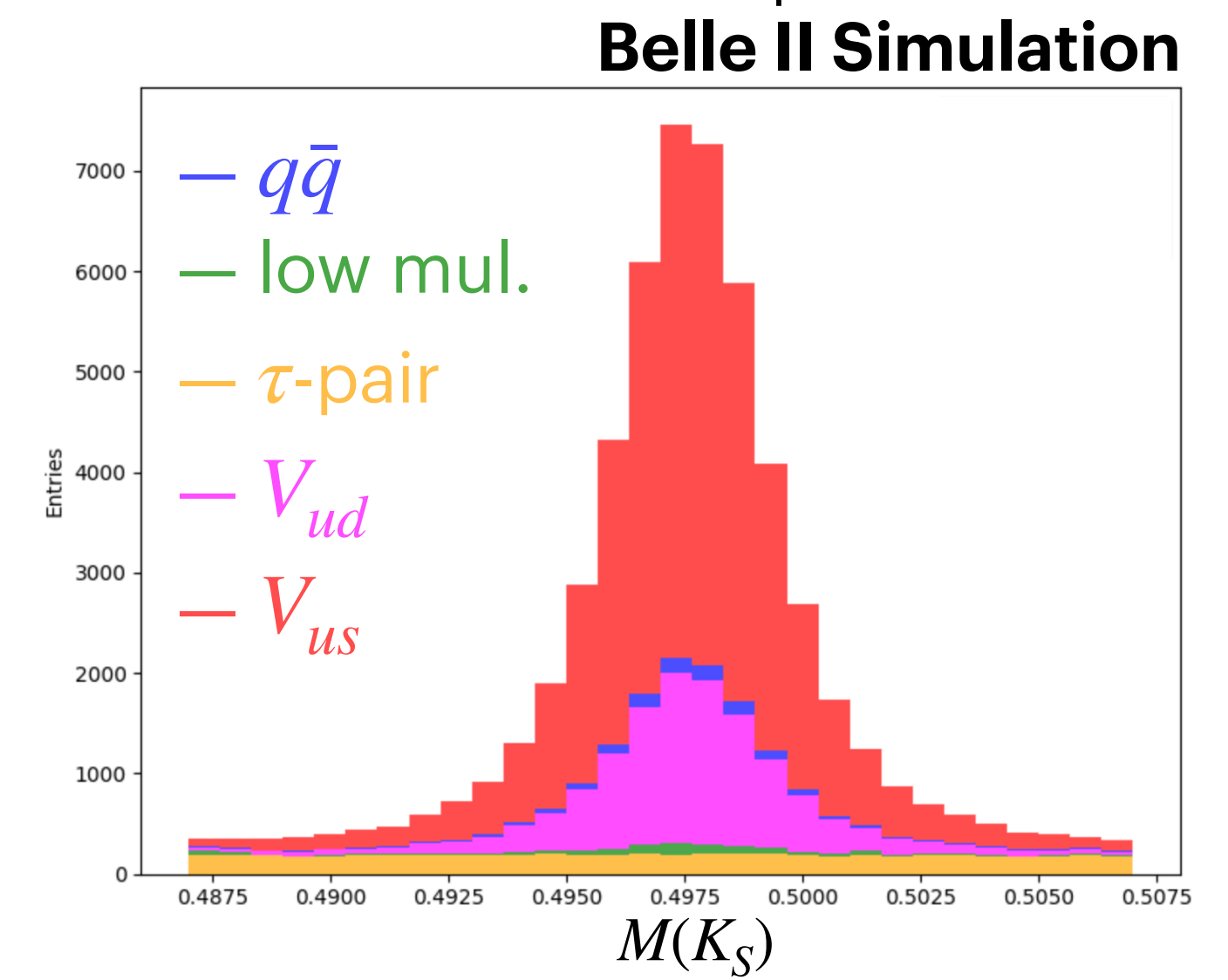
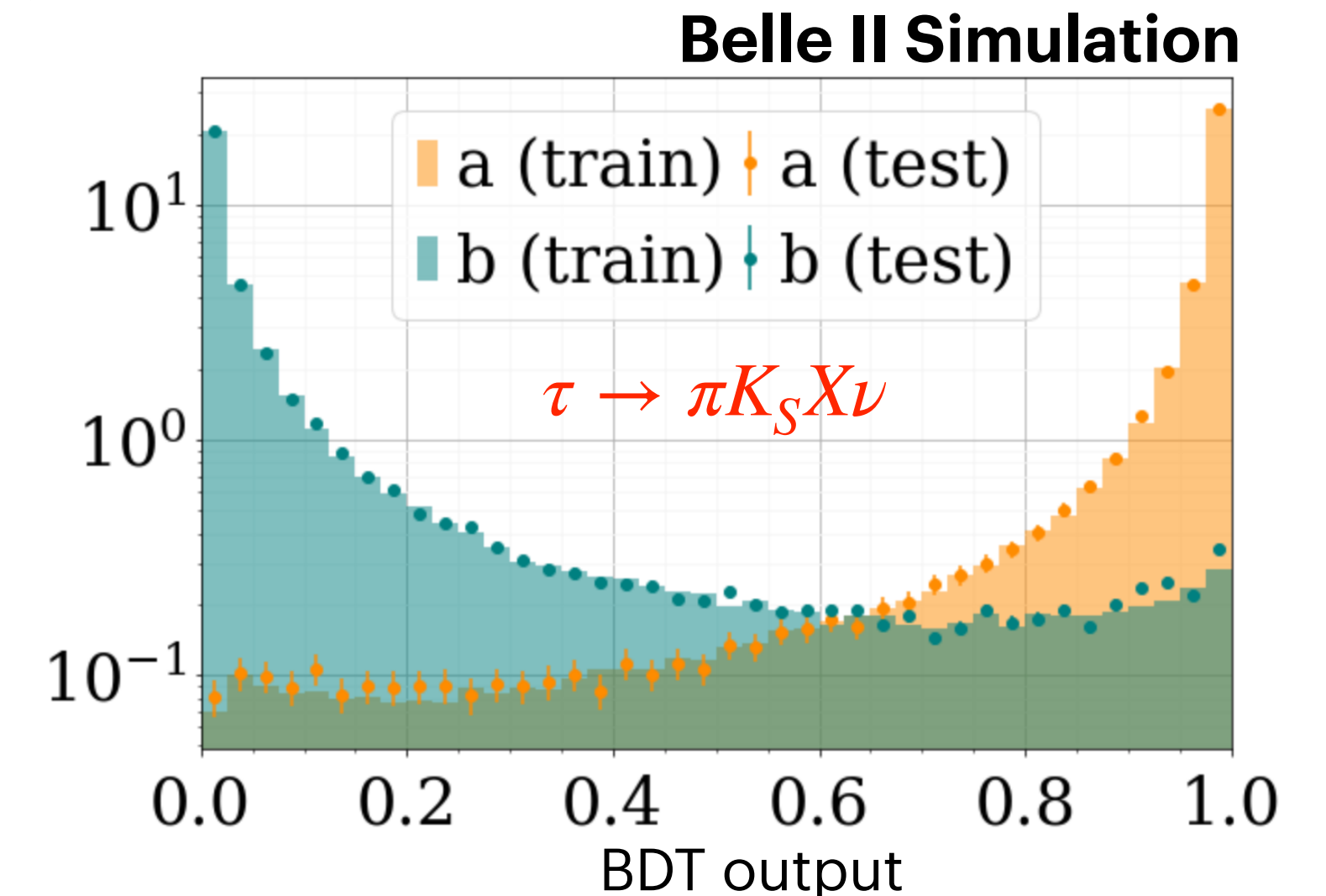
1. Suppress continuum and low-multiplicity events using a BDT

2. After the BDT, the samples are dominated by $|V_{us}|$, $|V_{ud}|$ and remaining τ -pair events

3. Suppress the remaining τ -pair events using a fit to $M(K_S)$

4. After 2. and 3., the sample are dominated by only $|V_{us}|$, $|V_{ud}|$
 $\rightarrow$ ready for the unfolding

Exploring different strategies to suppress background for the $\tau^+ \rightarrow K^+ n(\pi^0) \nu$ channel. For the $\tau \rightarrow K K_S n(\pi^0) \nu$, we can apply the same methodology described above.



Challenge 1

Access $\tau^+ \rightarrow K^+ X \nu$ and $\tau^+ \rightarrow \pi^+ K^0 X \nu$: unfolding

To extract the inclusive $|V_{us}|$ and separate the $|V_{ud}|$ contributions, we aim to apply the unfolding procedure to account for the migration between channels due to misidentification.

Define the migration matrix $S(i, j)$ as:

$$S_{(i,j)} = \frac{n_{(i,j)}^{reco}}{n_j^{gen}}$$

where:

$n_{(i,j)}^{reco}$ = number of events generated in the channel j
and reconstructed in the channel i .

n_j^{gen} = number of events generated in the channel j .

Reco \ Gen	$\tau^+ \rightarrow \pi^+ K_S n(\pi^0) \nu$ $\tau^+ \rightarrow \pi^+ K_S K_S n(\pi^0) \nu$ $\tau^+ \rightarrow \pi^+ K_S K_L n(\pi^0) \nu$			
	$\tau^+ \rightarrow \pi^+ K_S n(\pi^0) \nu$	$\tau^+ \rightarrow \pi^+ K_S K_S n(\pi^0) \nu$	$\tau^+ \rightarrow \pi^+ K_S K_L n(\pi^0) \nu$	
$\tau^+ \rightarrow \pi^+ K_S n(\pi^0) \nu$	ϵ_1	$S(1,2)$	$S(1,3)$	$\dots S(1,j)$
$\tau^+ \rightarrow \pi^+ K_S K_S n(\pi^0) \nu$	$S(2,1)$	ϵ_2	$S(2,3)$	
$\tau^+ \rightarrow \pi^+ K_S K_L n(\pi^0) \nu$	$S(3,1)$	$S(3,2)$	ϵ_3	
$\vdots$				
$S(i,1)$				ϵ_i

By using the inverted $S(i, j)$ matrix, we can extract the true number of events for each channel from the reconstructed ones

Challenge 2

Model independence: strategy

Rely on MC only for the efficiencies and remain as model-independent as possible

Idea: perform the measurement using only phase-space MC and test whether other generation models can be reproduced by reweighting the main kinematic variables

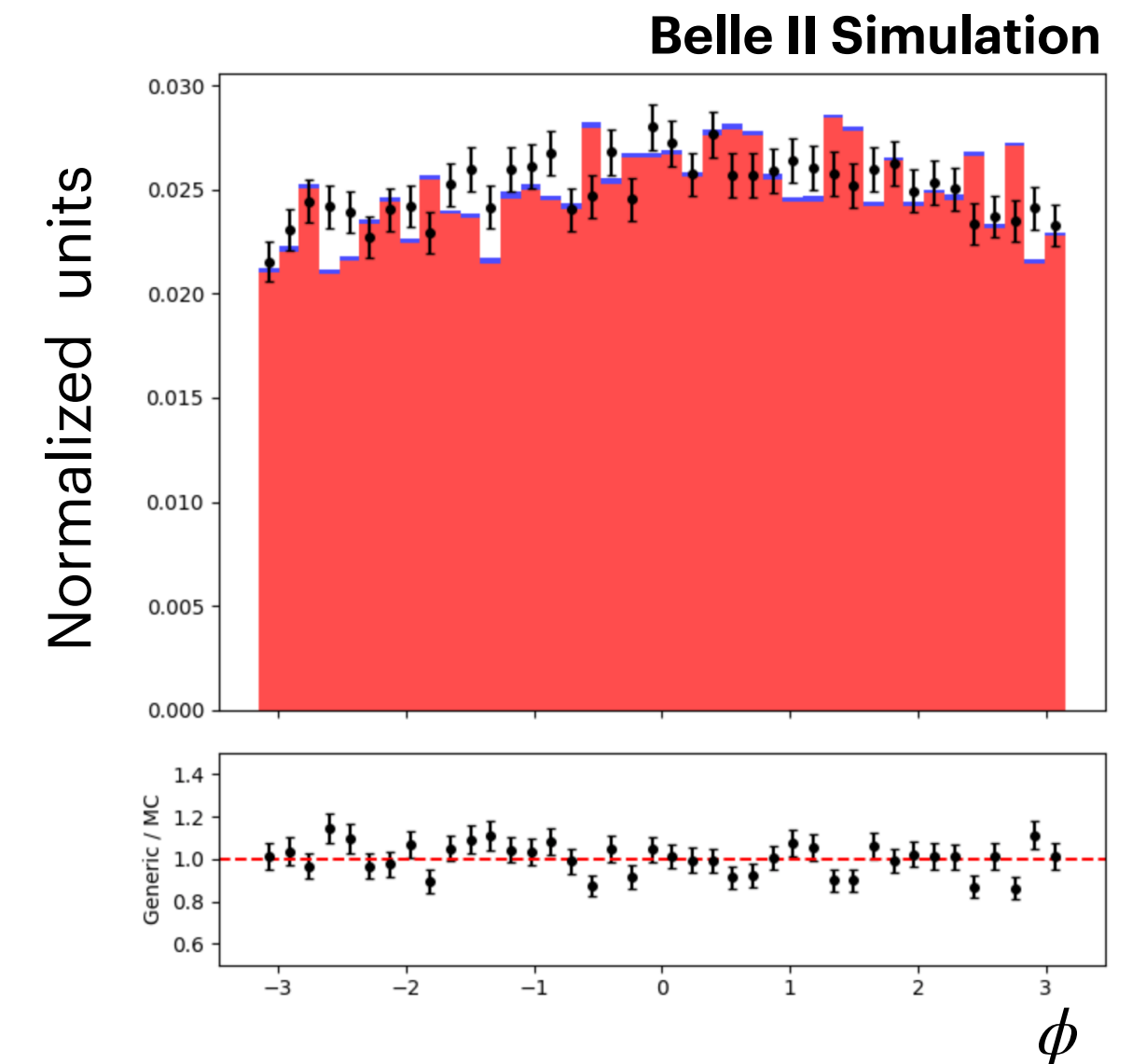
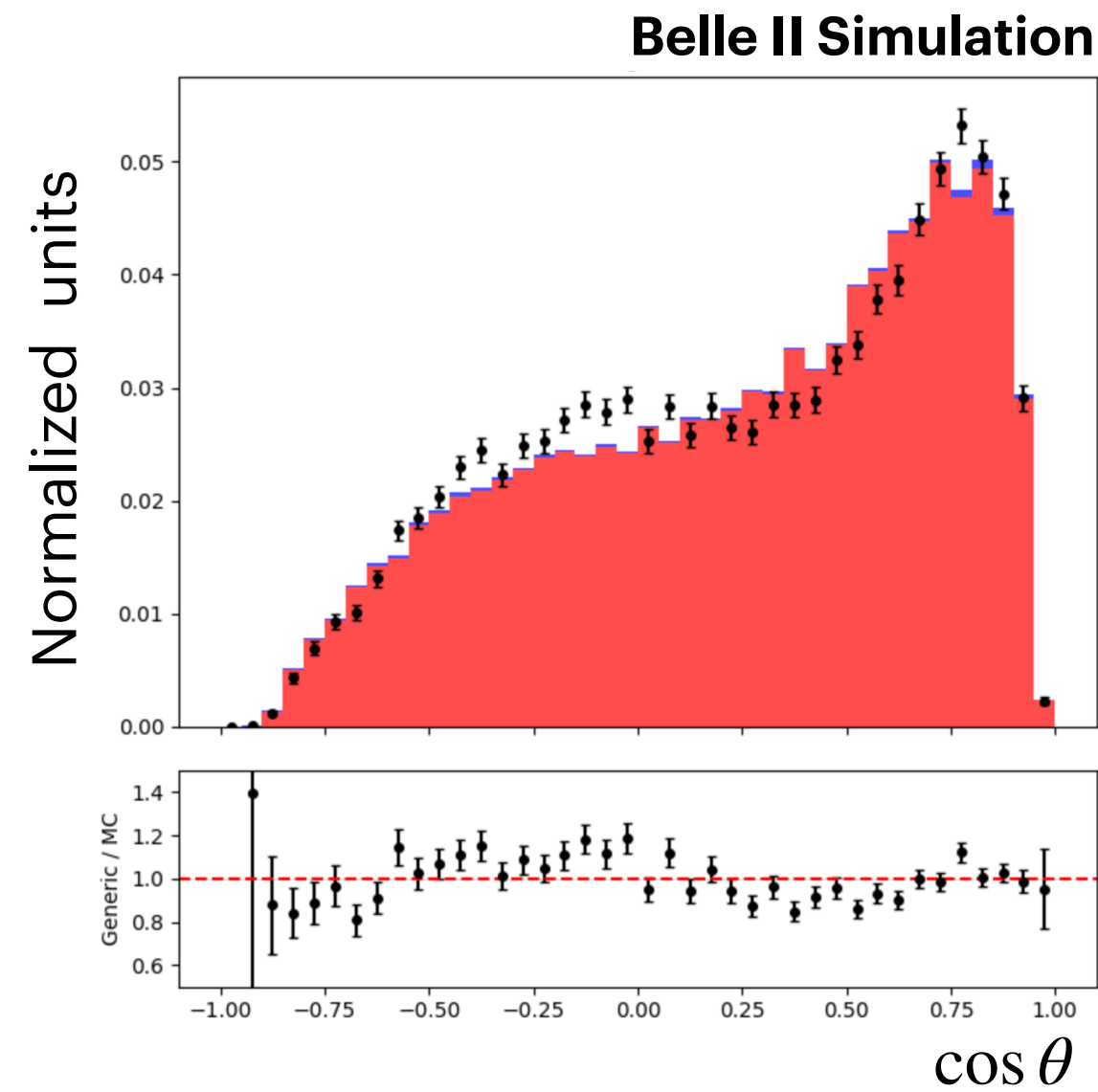
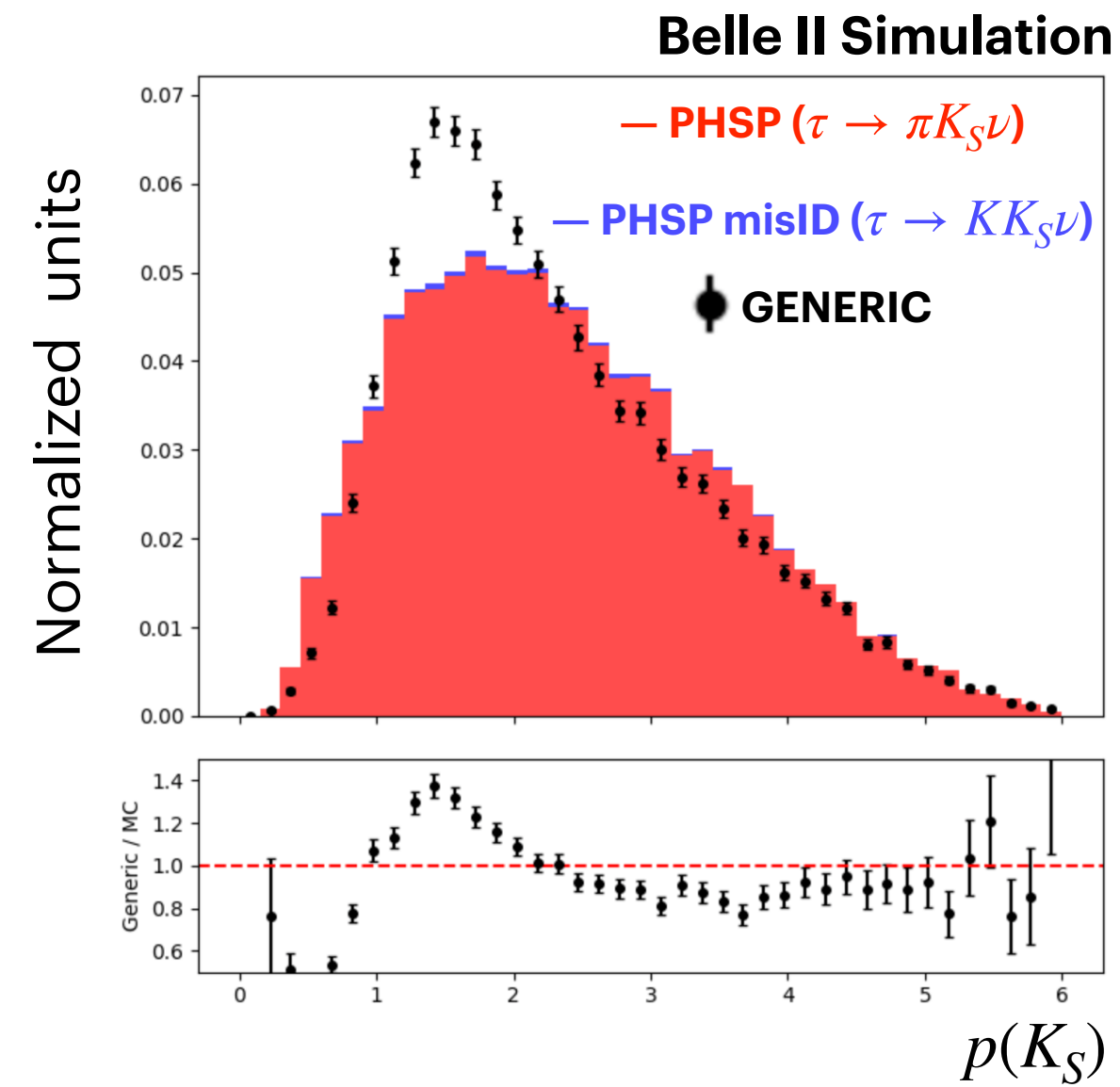
Methodology

- Let's consider two decay channels $\tau \rightarrow \pi K_S \nu$ and $\tau \rightarrow K K_S \nu$ (without additional π^0 s)
- Generate both channels using two different MC configurations: PHSP and GENERIC
- Compare the main kinematic variables (momentum, polar and azimuthal angles, ...) and identify those showing the largest discrepancies
- Reweight the discrepant kinematic distributions and check whether the reweighted PHSP distributions reproduce the GENERIC ones
- Evaluate the impact on the unfolding using the reweighed samples

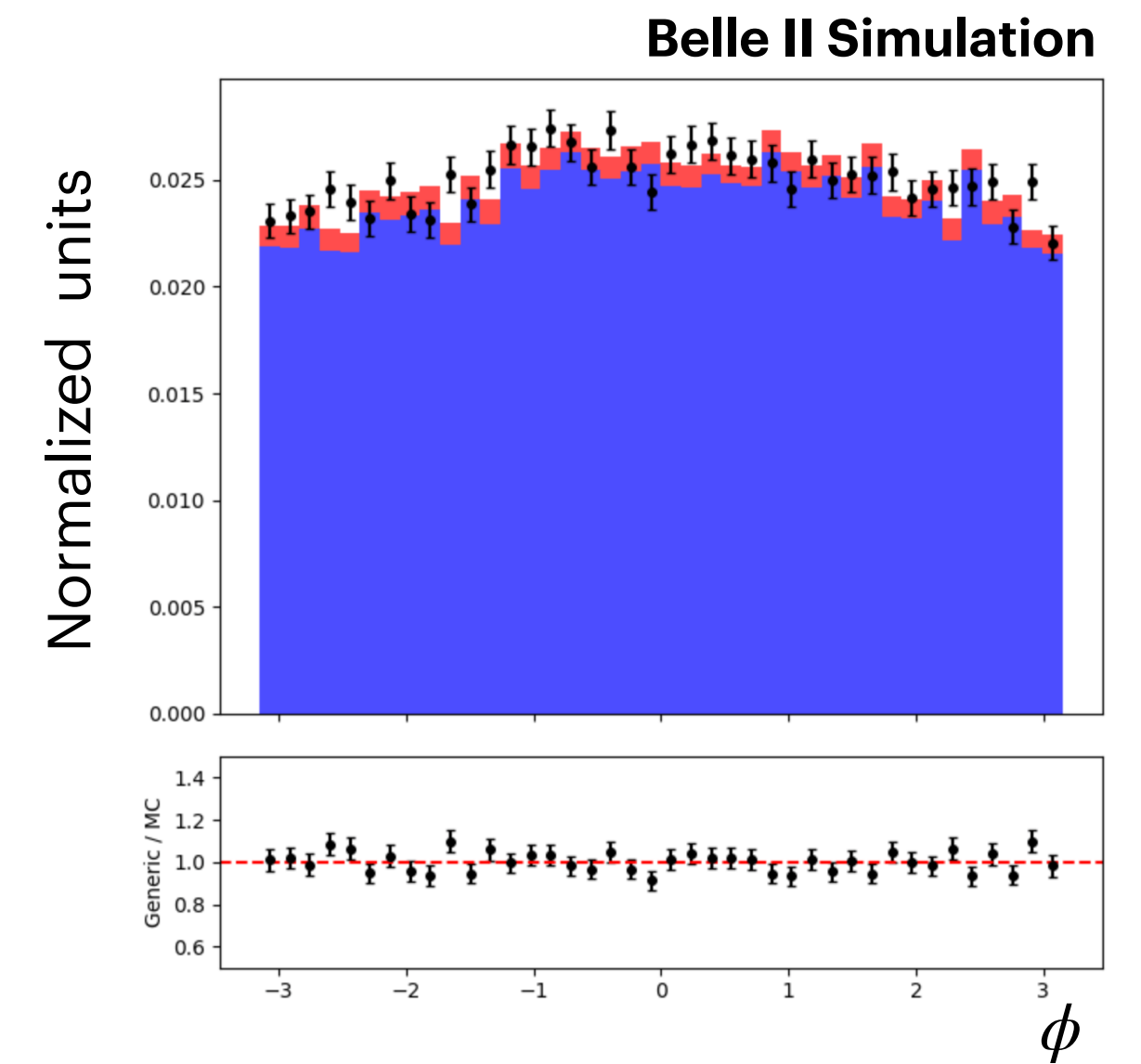
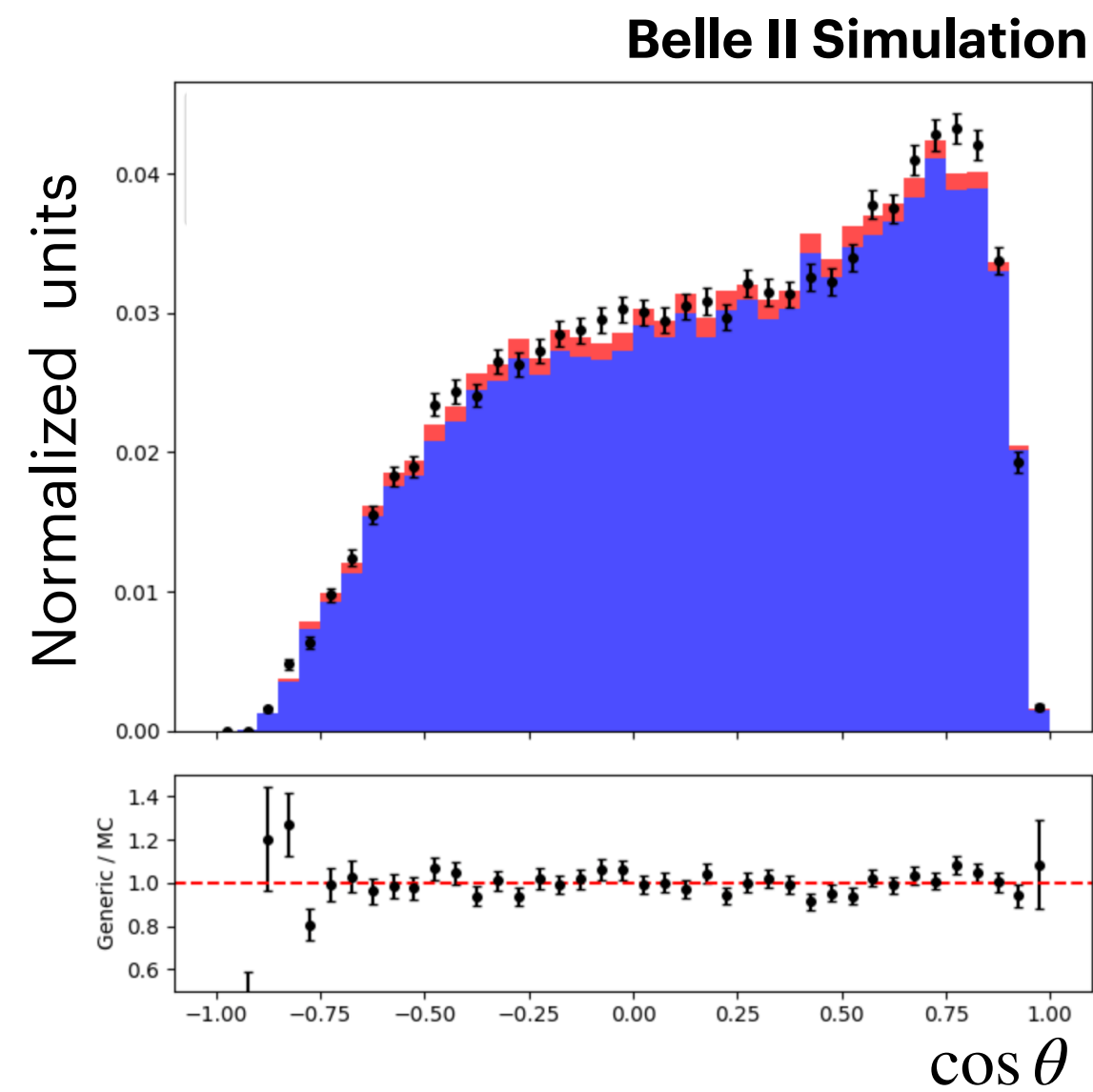
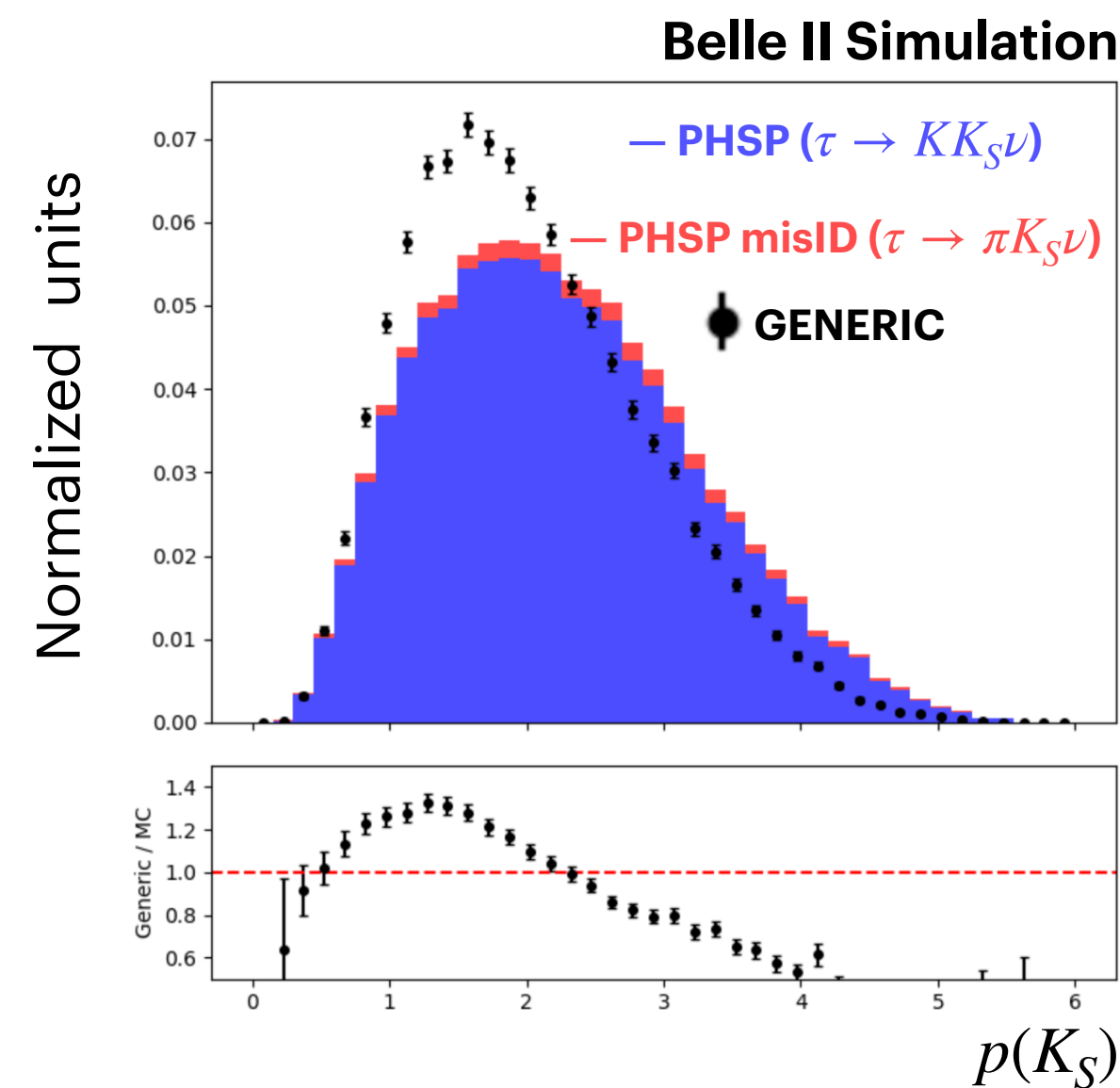
Challenge 2

Model independence: kinematic distributions

$\tau \rightarrow \pi K_S \nu$ channel



$\tau \rightarrow K K_S \nu$ channel



Challenge 2

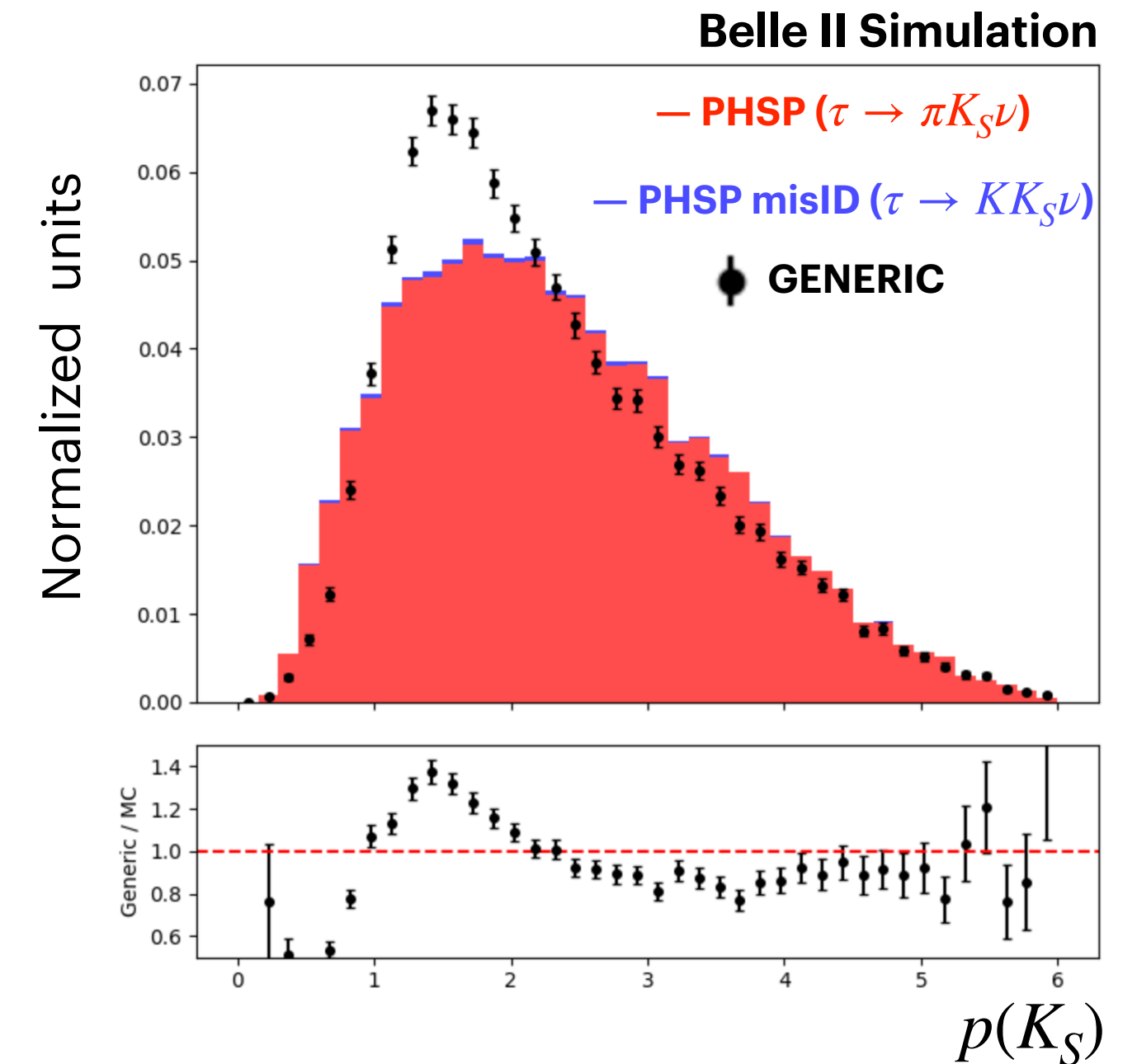
Model independence: reweighting

1. Correct the $\tau \rightarrow \pi K_S \nu$ component using weights derived from the ratio of the PHSP and GENERIC distributions (e.g. p_{K_S}) in the $\tau \rightarrow \pi K_S \nu$ channel, assuming that the disagreement is mostly due to $\tau \rightarrow \pi K_S \nu$
2. Apply the weights evaluated in 1. to the misID component $\tau \rightarrow \pi K_S \nu$ in the $\tau \rightarrow KK_S \nu$ channel
3. Calculate additional weights to account for the remaining disagreement between PHSP and GENERIC in the $\tau \rightarrow KK_S \nu$ channel
4. Apply the new weights only to the $\tau \rightarrow KK_S \nu$ component in the $\tau \rightarrow KK_S \nu$ channel

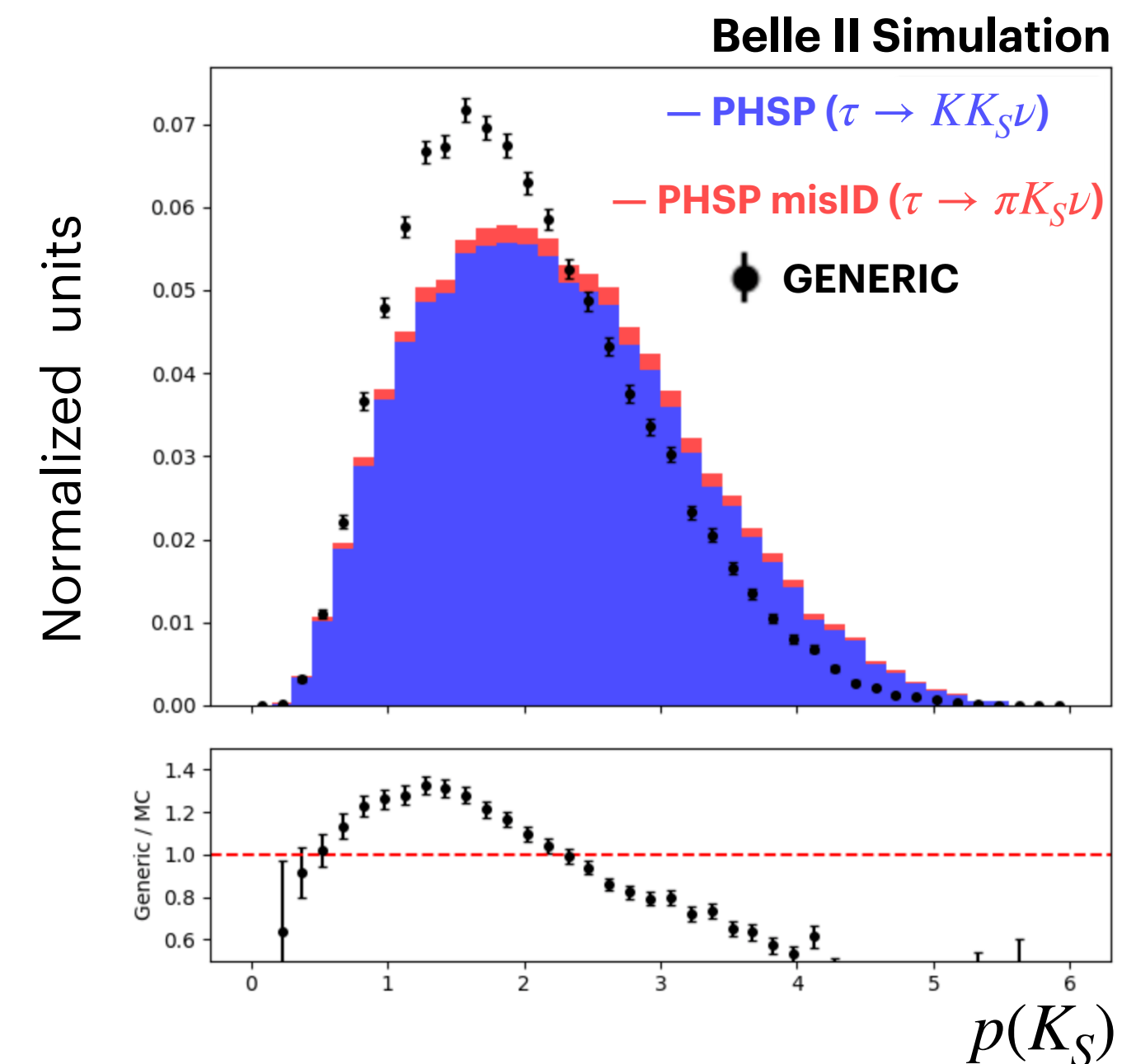
Iterate the procedure until good agreement is achieved

**Ongoing work to extend the procedure to more complex cases
(e.g. multiple channels and/or multiple variables to reweigh)**

$\tau \rightarrow \pi K_S \nu$ channel



$\tau \rightarrow KK_S \nu$ channel



Challenge 2

Model independence: impact on the unfolding

Use unfolding procedure to obtain the true number of events needed to measure $|V_{us}|$

τ^+

$\times$

$\rightarrow$

$\pi^+ K_S \nu$

τ^+

$\times$

$\rightarrow$

$K^+ K_S \nu$

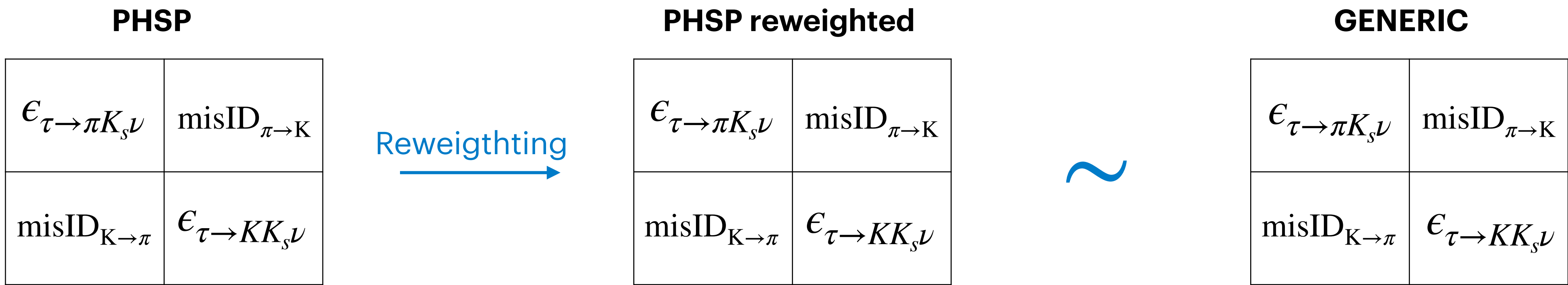
Generated

Reconstructed

$\epsilon_{\tau \rightarrow \pi K_S \nu}$	$\text{misID}_{\pi \rightarrow K}$
$\text{misID}_{K \rightarrow \pi}$	$\epsilon_{\tau \rightarrow K K_S \nu}$

$\begin{pmatrix} N_{true}(\tau \rightarrow \pi K_S \nu) \\ N_{true}(\tau \rightarrow K K_S \nu) \end{pmatrix} = \begin{pmatrix} \epsilon_{\tau \rightarrow \pi K_S \nu} & \text{misID}_{\pi \rightarrow K} \\ \text{misID}_{K \rightarrow \pi} & \epsilon_{\tau \rightarrow K K_S \nu} \end{pmatrix}^{-1} \cdot \begin{pmatrix} N_{reco}(\tau \rightarrow \pi K_S \nu) \\ N_{reco}(\tau \rightarrow K K_S \nu) \end{pmatrix}$

Check the reweighted PHSP against the GENERIC unfolding matrix (and reconstructed events)



Preliminary studies show that the method works well

Challenge 3

Strategy on $\tau^+ \rightarrow \pi^+ K_S K_L n(\pi^0) \nu$: strategy

$$\mathcal{B}(\tau^+ \rightarrow \pi^+ K_L K_S \nu) = (1.08 \pm 0.24) \times 10^{-3}$$

$$\mathcal{B}(\tau^+ \rightarrow \pi^+ K_L K_S \pi^0 \nu) = (0.32 \pm 0.12) \times 10^{-3}$$

LEP measurements

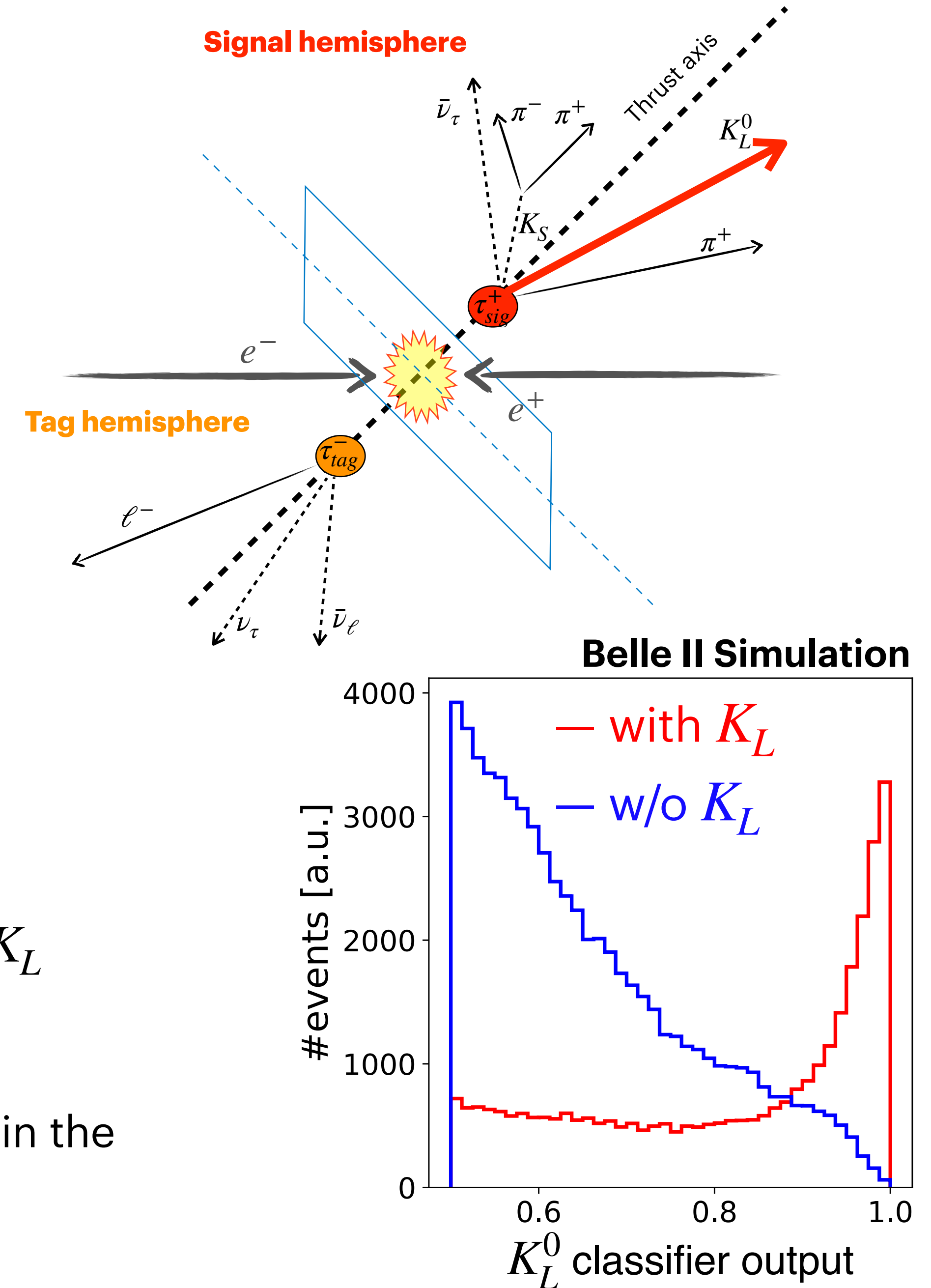
Belle II cannot measure K_L energy:

- EM calorimeter has $< 1 \lambda_{\text{had. int.}}$
- KLM system is more of a muon system rather than HCAL

How we can select a true K_L ?

- Apply same selection of $\tau^+ \rightarrow \pi^+ K_S n(\pi^0) \nu$
- Specific K_L selection exploiting the ECL + KLM clusters information
- Dedicated BDT to identify ECL + KLM clusters compatible with a true K_L
- Visible peak of the true K_L in the BDT output.

Final step: fit BDT output to count the number of events with a true K_L in the $\tau^+ \rightarrow \pi^+ K_S K_L n(\pi^0) \nu$ channel.



Challenge 3

Strategy on $\tau^+ \rightarrow \pi^+ K_S K_L n(\pi^0) \nu$: control channels

Use two independent control channels to validate the K_L BDT:

- $D^{+*} \rightarrow \pi^+ D^0(\rightarrow \pi^- \pi^+ K_L)$ [$p_{K_L} < 1.5$ GeV]
- $e^+ e^- \rightarrow \gamma \Phi(\rightarrow K_L K_S)$ [$p_{K_L} > 1.5$ GeV]

Can control:

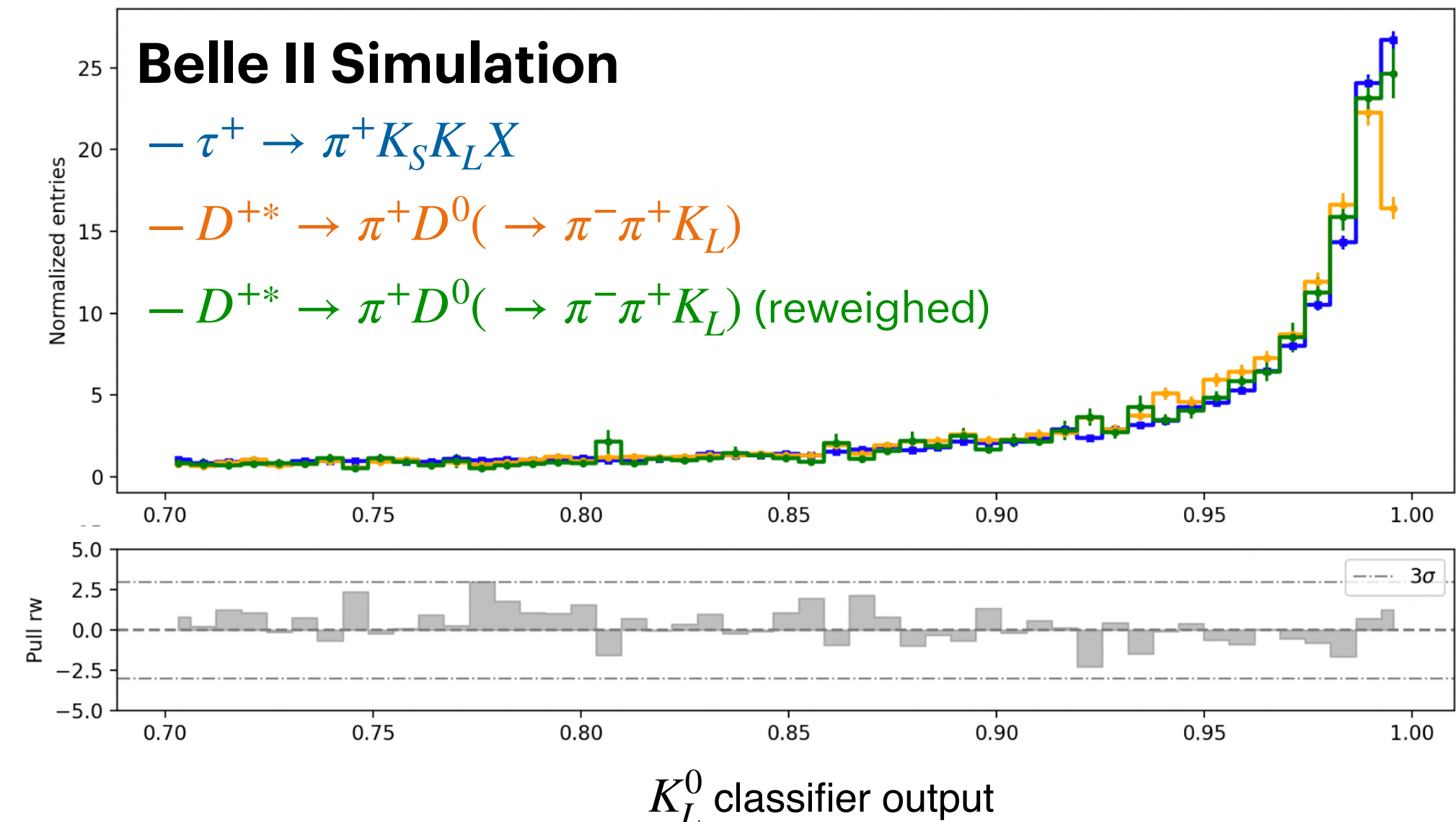
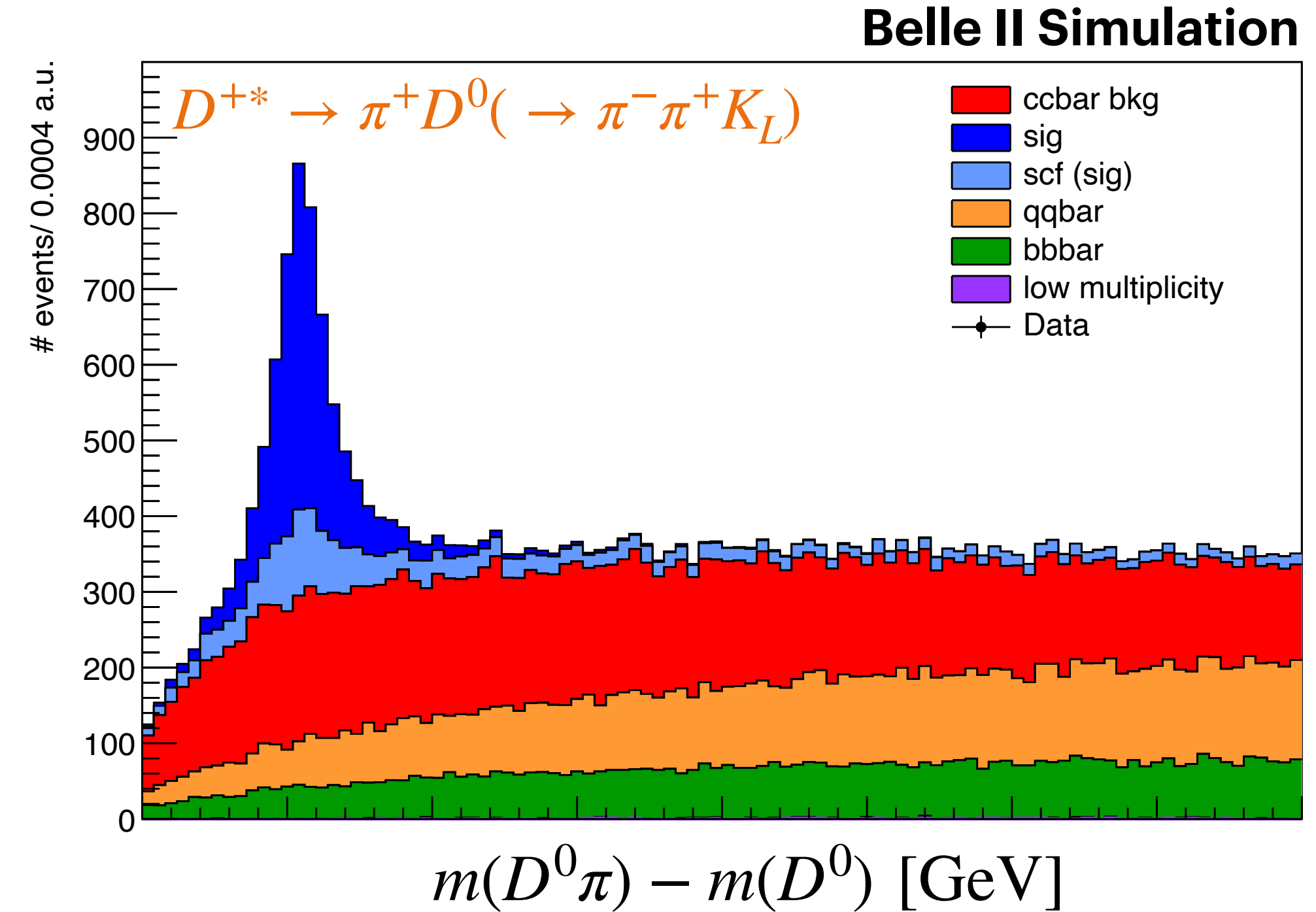
- Shape of the multivariate output for signal after reweighting the K_L variables to match those in the control channel
- K_L reco efficiency, e.g.:

$$\frac{N(D^0 \rightarrow K_S^0 \pi^+ \pi^-)}{\varepsilon(D^0 \rightarrow K_S^0 \pi^+ \pi^-)} = \frac{N(D^0 \rightarrow K_L^0 \pi^+ \pi^-)}{\varepsilon(D^0 \rightarrow K_L^0 \pi^+ \pi^-)}$$

Work in progress, but ~10% of precision on the BR seems feasible

→ plan to validate by checking

$$\mathcal{B}(\tau \rightarrow K_S^0 \pi^+ n(\pi^0) \nu) = \mathcal{B}(\tau \rightarrow K_L^0 \pi^+ n(\pi^0) \nu)$$



Challenge 4

Expected systematic uncertainty on $|V_{us}|$

Ongoing work to evaluate the total systematic uncertainty on $|V_{us}|$

Systematic uncertainties evaluated so far:

- Luminosity and cross section $\rightarrow$ possibility to get rid of using a normalization channel
- Particle ID efficiencies and misID
- CP-violation effects for the assumption $\mathcal{B}(\tau \rightarrow \pi K_L n(\pi^0)\nu) = \mathcal{B}(\tau \rightarrow \pi K_S n(\pi^0)\nu)$

$\rightarrow$ **all found to be well below 1%**

Remaining systematic uncertainties:

- Model dependence of the unfolding
- Kinematic dependence of the efficiencies

.....

Conclusion

Multiple ongoing analyses to measure both exclusive and inclusive $|V_{us}|$ at Belle II

Competitive results expected from exclusive $|V_{us}|$ measurements using both 1x1 and 1x3 topologies

First-ever inclusive $|V_{us}|$ measurement expected to be highly competitive and provide interesting insight into the tension with the CKM unitary

An aerial photograph of a city skyline. In the foreground, there is a dense forest of trees with yellow and orange autumn foliage. Behind the trees, a large Ferris wheel is visible. To the left of the Ferris wheel, there is a tall, colorful tower. In the background, several skyscrapers are visible, including a very tall, dark one on the left and a tall, light-colored one on the right. The sky is blue with some light clouds.

Backup

Inclusive $|V_{us}|$ determination @ Belle II

BR and $|V_{us}|$ extraction

Extract the true number of events (N_{sig}^{true}) for the $\tau \rightarrow X_s \nu$ decay from the unfolding procedure

$BR(\tau \rightarrow X_s)$ can be measured by using

$$BR(\tau \rightarrow X_s) = \frac{N_{sig}^{true}}{2 \cdot \epsilon \cdot \mathcal{L} \cdot \sigma_{\tau\tau} \cdot BR(X_s) \cdot BR(\tau \rightarrow \ell \nu \nu)}$$

$$BR(\tau \rightarrow \ell \nu \nu) = (0.3521 \pm 0.04) \%$$

$BR(X_s)$ = depends on the specific channel

$$\mathcal{L} \cdot \sigma_{\tau\tau} = N_{\tau\tau} \sim 4.59 \cdot 10^8 \text{ in } 500 \text{ fb}^{-1}$$

ϵ = reconstruction efficiency

$|V_{us}|$ can be extracted from the measurement of $BR(\tau \rightarrow X_s)$

$$|V_{us}| = \sqrt{\frac{R_{\tau,s}}{\frac{R_{\tau,no s}}{|V_{ud}|^2} - \delta R_{QCD}}}$$

$$R_{\tau,s} = BR(\tau \rightarrow X_s) / BR(\tau \rightarrow e \nu \nu) \quad \delta R_{QCD} \sim (0.240 \pm 0.032)$$

$$R_{\tau,no s} = \sum_i BR_i(X_{no s})$$

$$|V_{ud}| \sim 0.974$$

Inclusive $|V_{us}|$ determination @ Belle II

Current systematic uncertainties on BR and $|V_{us}|$

Different sources of systematic uncertainties for BR and $|V_{us}|$:

1. **PID:** efficiency and fake-rate correction for hadrons and leptons. Systematic uncertainties come from Data/MC corrections derived from different control channels.
2. **Luminosity and cross section:** the relative uncertainty on $\mathcal{L} = (498.41 \pm 2.5)\text{fb}^{-1}$ [0.5%] and $\sigma_{tt} = (919 \pm 3)\text{pb}$ [0.33%] contribute equally to the uncertainties on BR and $|V_{us}|$.
Possibility to get rid of using a normalization channel.
3. **CP breaking effect:** to take into account the small difference in CP violation between K_S and K_L when assuming $BR(K_S) = BR(K_L)$. Conservative uncertainty of 0.3% is assigned, of the same order as ϵ_K .

Other possible systematic uncertainties are currently being evaluated

Precise measurement of τ hadronic decays @ Belle II

BR and spectral functions (SF) of τ hadronic decays provide important input for $|V_{us}|$ and a_μ^{had} :

- Persistent $>3\sigma$ discrepancy exists in $|V_{us}|$ between τ measurements and unitary expectation
- Persistent discrepancy exists between $a_\mu^{e^+e^-}$ and a_μ^{exp} , and $\tau \rightarrow n(\pi)\nu$ can contribute

Precise results on BR from LEP (ALEPH mainly)

→ Belle II can perform an independent measurement!

Precise results on SF from BaBar and Belle

→ Belle II can provide much larger statistics!

Ongoing studies at Belle II to measure:

- BR of τ hadronic decays normalised to $\tau \rightarrow \ell\nu\nu$ decays
- SF measurement via unfolding

