Tau decays into two mesons

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Phys. Rev. D 94 (2016), 034008

JHEP 09 (2014) 042

JHEP 10 (2013) 039

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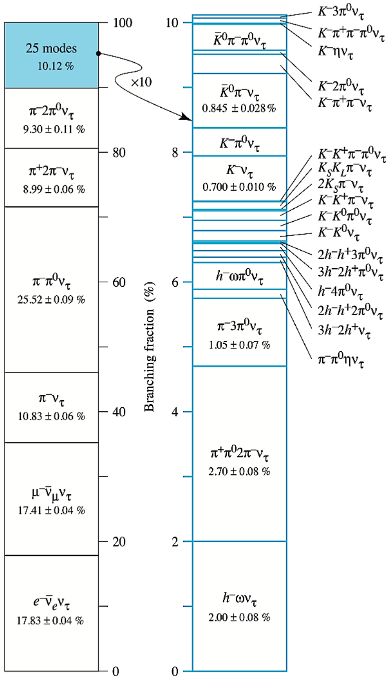
Workshop on Tau Physics at Belle II and STCF

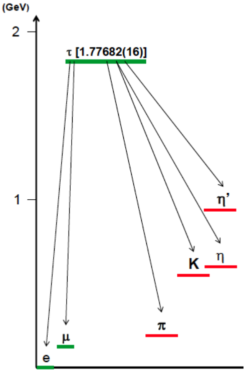
Jilin University, Changchun, China, Aug 30 - Sep 1, 2026

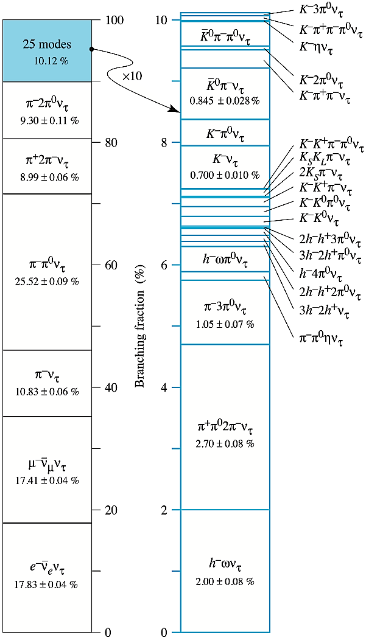
UNIVERSITAT DE BARCELONA



Introduction

- Tau **properties**:
  - Mass: $m_\tau = 1.77686(12)$ GeV
  - Lifetime: $\tau_\tau = 2.903(15) \times 10^{-13}$ s
- The only lepton heavy enough to **decay into hadrons**:
  - Very rich phenomenology
  - Test of QCD and EW interactions
- For the test:
  - Precise measurements needed
  - Hadronic uncertainties under control
- Tau decays $\longleftrightarrow$ New Physics searches



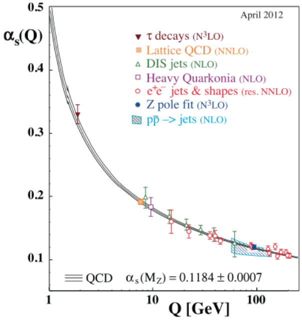


Test of QCD and EW Interactions

- **Inclusive** decays: $\tau^- \to (\bar{u}d, \bar{u}s)\nu_\tau$

  Full hadron spectra (precision physics)

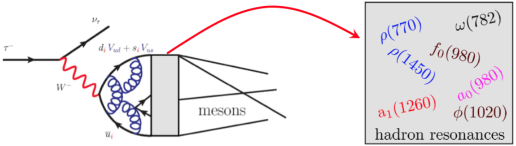
  Fundamental SM parameters:
  $\alpha_s(m_\tau), m_s, |V_{us}|$



- **Exclusive** decays: $\tau \to (PP, PPP, ...)\nu_\tau$

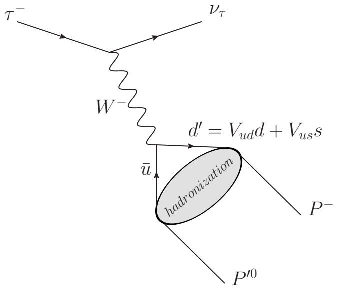
  specific hadron spectrum (approximate physics)

  Hadronization of QCD currents, study of Form Factors, resonance parameters (M_R, Γ_R)



Hadronic τ decays

- $\tau^- \to P^- \nu_\tau$: $F_{\pi,K}$ (decay constants)
- $\tau^- \to (2P)^- \nu_\tau$: form factors, good control (**this work**)
- $\tau^- \to (3P)^- \nu_\tau$: reasonable good control
- $\tau^- \to (> 3P)^- \nu_\tau$: poor knowledge

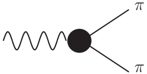


$$\frac{d\Gamma\left(\tau^- \to P^- P^0 \nu_\tau\right)}{ds} = \frac{G_F^2 |V_{ui}|^2 m_\tau^3}{768\pi^3} S_{EW}^{\text{had}} C_{PP'}^2 \left(1 - \frac{s}{m_\tau^2}\right)^2$$
$$\times \left\{ \left(1 + \frac{2s}{m_\tau^2}\right) \lambda_{P^-P^0}^{3/2}(s) |F_V^{P^-P^0}(s)|^2 + 3\frac{(m_{P^-}^2 - m_{P^0}^2)^2}{s^2} \lambda_{P^-P^0}^{1/2}(s) |F_S^{P^-P^0}(s)|^2 \right\}$$

Decay mode	BR	**Standard Model**	**Resonances**
$\tau^- \to \pi^- \pi^0 \nu_\tau$	$\sim 25\%$	Pion form factor, $(g-2)_\mu$	$\rho(770), \rho(1450), \rho(1700)$
$\tau^- \to K^- K_S \nu_\tau$	$\sim 10^{-4}$	Kaon form factor, $(g-2)_\mu$	$\rho(770), \rho(1450), \rho(1700)$
$\tau^- \to K_S \pi^- \nu_\tau$	$\sim 0.4\%$	$K\pi$ form factor, $K_{\ell 3}$, $\lvert V_{us}\rvert$	$K^*(892), K^*(1410)$
$\tau^- \to K^- \eta^{(\prime)} \nu_\tau$	$\sim 10^{-4}$	$K\pi$ form factor, $K_{\ell 3}$, $\lvert V_{us}\rvert$	$K^*(1410)$
$\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$	$< 9.9 \cdot 10^{-5}$	isospin violation, 2nd class currents	$a_0(980)$

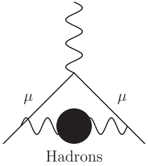
$\tau^- \to \pi^-\pi^0\nu_\tau$: the pion vector form factor $F_V^\pi(s)$

- photon conversion into two pions:



$$\longrightarrow \langle \pi^+(p)\pi^-(p')|J_\mu(0)|0\rangle = i(p-p')_\mu F_V^\pi(s)\,, \quad \left(s=(p+p')^2\right)$$

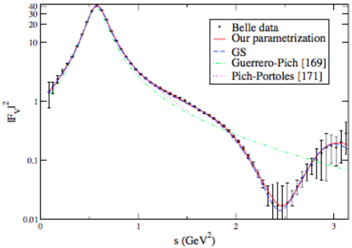
- $J_\mu(0) = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$ is the electromagnetic current
- $F_V^\pi(s)$ contain the energy (s) dependent response of the $\pi\pi$ system to $J_\mu(0)$
- Key object in many hadronic reactions, *e.g.* muon $(g-2)$

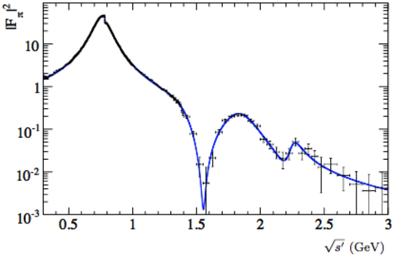


- Good pedagogic advent towards the challenges in the description of low-energy QCD

The pion vector form factor $F_V^\pi(s)$

- How to determine $F_V^\pi(s)$ experimentally?
  - $\tau^- \to \pi^-\pi^0\nu_\tau$ (Belle PRD 78 (2008) 072006) and $e^+e^- \to \pi^+\pi^-$ (BaBar PRD 86 (2012) 032013)

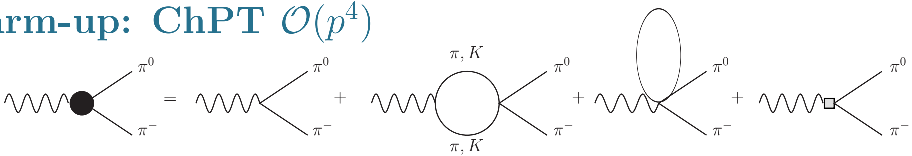




- What do we know theoretically on the form factor?
  - Its low-energy behaviour: given by ChPT (Gasser&Leutwyler'85)
  - Its high-energy behaviour ($\sim 1/s$): given by pQCD (Brodsky&Lepage'79)
  - For the intermediate energy region: models

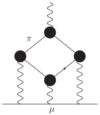
Gasser and Leutwyler Nucl.Phys.B 250, 517 (1985)

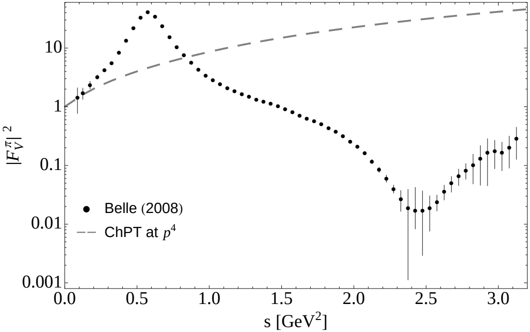
Warm-up: ChPT $\mathcal{O}(p^4)$



$$F_\pi(s)|_{\chi\mathrm{PT}}^{\mathcal{O}(p^4)} = 1 + \frac{2L_9^r(\mu)}{F_\pi^2}s - \frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right)$$

- **Drawbacks**:
  - limited energy range
  - $|F_V^\pi(s)|_{\chi\mathrm{PT}}^{\mathcal{O}(p^4)} \propto s$ vs $|F_V^\pi(s)|_{\mathrm{QCD}} \propto 1/s$
  - Non-predictive: divergent calculations

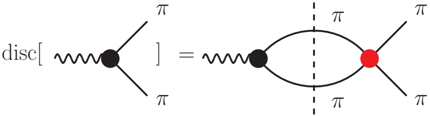




- **Solution**: Dispersion theory (no need of a Lagrangian)

Dispersive representation

- **Unitarity**:



$$\mathrm{disc}F_V^\pi(s) = 2i\sigma_\pi(s)F_V^\pi(s)\textcolor{red}{T_1^{1*}(s)} = 2iF_V^\pi(s)\textcolor{red}{\sin\delta_1^1(s)e^{-i\delta_1^1(s)}}\,,$$

$$F_V^\pi(s) = \frac{1}{2i\pi}\int_{4M_\pi^2}^{\infty} ds'\,\frac{\mathrm{disc}F_V^\pi(s')}{s'-s-i\varepsilon}\,,$$

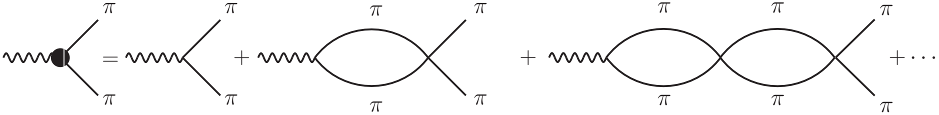
- **Analytic** solution, **Omnès equation** [Omnès, Nuovo Cimento 8, 316 (1958)]:

$$F_V^\pi(s) = P(s)\Omega(s)\,,\quad \Omega(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds'\,\frac{\textcolor{red}{\delta_1^1(s')}}{s'(s'-s-i\varepsilon)}\right\},$$

- Polynomial $P(s)$, not fixed by unitarity: matched to EFT or by fitted to data

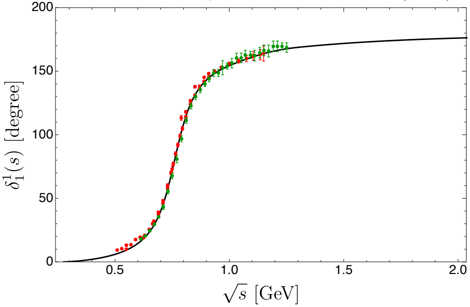
Omnès equation

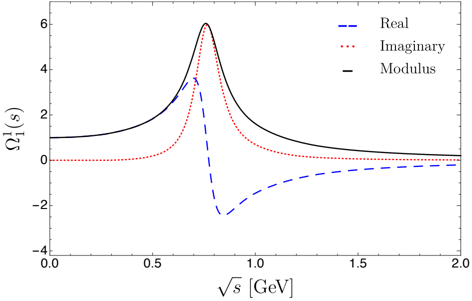
- Diagrammatic interpretation



- Solution: depends solely on the P-wave phase shift of $\pi\pi$

Garcia-Martin, *et.al.* Phys. Rev. D83, 074004 (2011)





Dispersive representation

- Dispersion relation with subtractions:

$$F_V^\pi(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\text{cut}}} ds' \frac{\phi(s')}{(s')^3(s'-s-i\varepsilon)}\right],$$

  — Low-energy observables:

$$F_V^\pi(s) = 1 + \frac{1}{6}\langle r^2\rangle_V^\pi s + c_V^\pi s^2 + d_V^\pi s^3 + \cdots,$$

$$\langle r^2\rangle_V^\pi\Big|_{\text{ChPT}}^{\mathcal{O}(p^4)} = \frac{12L_9^r(\mu)}{F_\pi^2} - \frac{1}{32\pi^2F_\pi^2}\left[2\log\left(\frac{M_\pi^2}{\mu^2}\right) + \log\left(\frac{M_K^2}{\mu^2}\right) + 3\right],$$

$$\langle r^2\rangle_V^\pi = 6\alpha_1\,, \quad c_V^\pi = \frac{1}{2}\left(\alpha_2 + \alpha_1^2\right)\,, \quad \alpha_k = \frac{k!}{\pi}\int_{4m_\pi^2}^{s_{\text{cut}}} ds' \frac{\phi(s')}{s'^{k+1}}\,.$$

  — s_{cut}: cut-off to check stability

Dispersive representation

- Dispersion relation with subtractions:

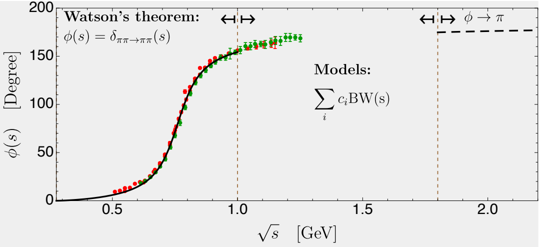
$$F_V^\pi(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\phi(s')}{(s')^3(s'-s-i\varepsilon)}\right],$$

- Form Factor phase $\phi(s)$:
  - $4m_\pi^2 < s < 1$ GeV: $\pi\pi$ phase $\delta_1^1(s)$
    (García-Martín et.al PRD 83, 074004 (2011))
  - $1 < s < m_\tau^2$: "pheno" phase:

$$\tan\phi(s) = \frac{\mathrm{Im}F_V^\pi(s)|^{3\,\rm res}_{\rm expo}}{\mathrm{Re}F_V^\pi(s)|^{3\,\rm res}_{\rm expo}}$$

  - $m_\tau^2 < s$: phase guided smoothly to π

González-Solís and Roig , Eur.Phys.J C79 (2019), 436



Model for the "pheno" phase at $1 < s < m_\tau^2$

- Incorporation of the $\rho'(1450), \rho''(1700)$

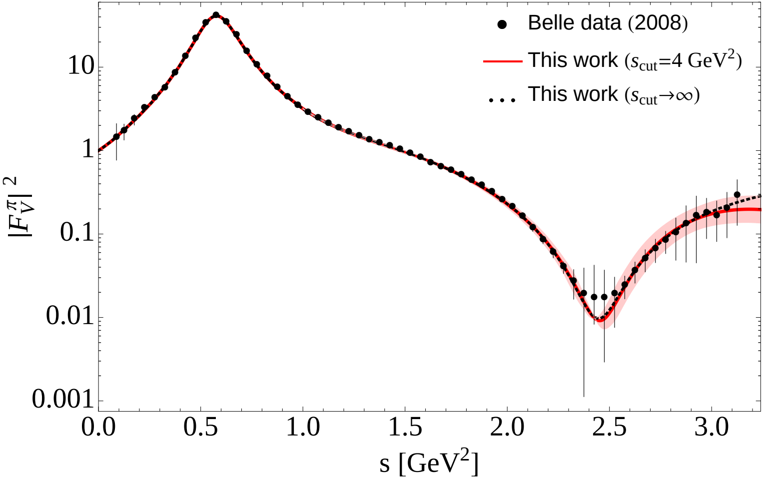
$$
\begin{aligned}
F_V^\pi(s)|^{3\,\mathrm{res}}_{\mathrm{expo}} \quad = \quad & \frac{M_\rho^2 + s\left(\gamma e^{i\phi_1} + \delta e^{i\phi_2}\right)}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\mathrm{Re}\left[-\frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right]\right\} \\
& -\gamma \frac{s\, e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left\{-\frac{s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)}\mathrm{Re}A_\pi(s)\right\} \\
& -\delta \frac{s\, e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left\{-\frac{s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)}\mathrm{Re}A_\pi(s)\right\},
\end{aligned}
$$

where

$$
\Gamma_{\rho',\rho''}(s) \quad = \quad \Gamma_{\rho',\rho''} \frac{M_{\rho',\rho''}}{\sqrt{s}} \frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_{\rho',\rho''}^2)} .
$$

Dispersive fits to $|F_V^\pi(s)|^2$ Belle experimental data

González-Solís and Roig , Eur.Phys.J C79 (2019), 436



- The results can be found as ancillary material in 1902.02273 [hep-ph]

Form factor phase output

- Form factor phase $\phi(s)$ for different values of s_{cut}

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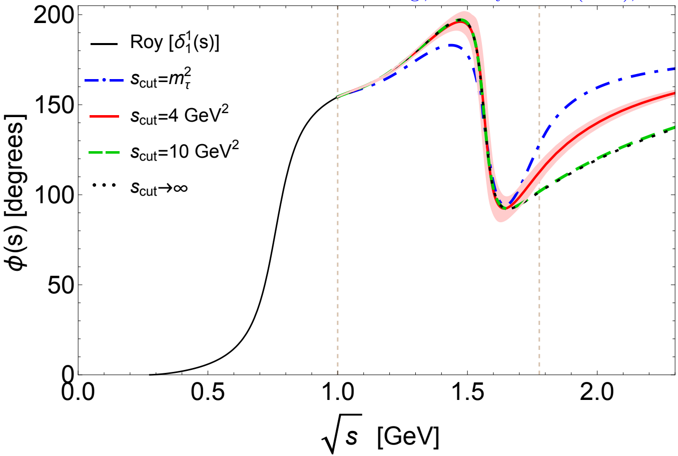

- The results can be found as ancillary material in 1902.02273 [hep-ph]

Dispersive fits to $|F_V^\pi(s)|^2$ Belle experimental data

- Fits for different values of $s_{\rm cut}$ and matching at 1 GeV

Fits	Parameter	$s_{\rm cut}$ [GeV^2]			
		m_τ^2	4	10	∞
Fit 1	α_1 [GeV^{-2}]	1.87(1)	1.88(1)	1.89(1)	1.89(1)
	α_2 [GeV^{-4}]	4.40(1)	4.34(1)	4.32(1)	4.32(1)
	m_ρ [MeV]	$=773.6(9)$	$=773.6(9)$	$=773.6(9)$	$=773.6(9)$
	M_ρ [MeV]	$=m_\rho$	$=m_\rho$	$=m_\rho$	$=m_\rho$
	$M_{\rho'}$ [MeV]	1365(15)	1376(6)	1313(15)	1311(5)
	$\Gamma_{\rho'}$ [MeV]	562(55)	603(22)	700(6)	701(28)
	$M_{\rho''}$[MeV]	1727(12)	1718(4)	1660(9)	1658(1)
	$\Gamma_{\rho''}$ [MeV]	278(1)	465(9)	601(39)	602(3)
	γ	0.12(2)	0.15(1)	0.16(1)	0.16(1)
	ϕ_1	$-0.69(1)$	$-0.66(1)$	$-1.36(10)$	$-1.39(1)$
	δ	$-0.09(1)$	$-0.13(1)$	$-0.16(1)$	$-0.17(1)$
	ϕ_2	$-0.17(5)$	$-0.44(3)$	$-1.01(5)$	$-1.03(2)$
	χ^2/d.o.f	1.47	0.70	0.64	0.64

Dispersive fits to $|F_V^\pi(s)|^2$ Belle experimental data

- Fits for different matching point and with $s_{\rm cut} = 4$ GeV

Fits	Parameter	Matching point [GeV]			
		0.85	0.9	0.95	1 (reference fit)
Fit I	α_1 [GeV^{-2}]	1.88(1)	1.88(1)	1.88(1)	1.88(1)
	α_2 [GeV^{-4}]	4.35(1)	4.35(1)	4.34(1)	4.34(1)
	m_ρ [MeV]	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$
	M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'}$ [MeV]	1394(6)	1374(8)	1351(5)	1376(6)
	$\Gamma_{\rho'}$ [MeV]	592(19)	583(27)	592(2)	603(22)
	$M_{\rho''}$[MeV]	1733(9)	1715(1)	1697(3)	1718(4)
	$\Gamma_{\rho''}$ [MeV]	562(3)	541(45)	486(7)	465(9)
	γ	0.12(1)	0.12(1)	0.13(1)	0.15(1)
	ϕ_1	$-0.44(3)$	$-0.60(1)$	$-0.80(1)$	$-0.66(1)$
	δ	$-0.13(1)$	$-0.13(1)$	$-0.13(1)$	$-0.13(1)$
	ϕ_2	$-0.38(3)$	$-0.51(2)$	$-0.62(1)$	$-0.44(3)$
	χ^2/d.o.f	0.75	0.74	0.68	0.70

Central results

- **Fit** results (central value ± stat fit error ± syst th. error)

$$\alpha_1 = 1.88(1)(1)\ \text{GeV}^{-2},\ \alpha_2 = 4.34(1)(3)\ \text{GeV}^{-4},$$

$$M_\rho \doteq 773.6 \pm 0.9 \pm 0.3\ \text{MeV}\,,$$

$$M_{\rho'} = 1376 \pm 6^{+18}_{-73}\ \text{MeV}\,, \quad \Gamma_{\rho'} = 603 \pm 22^{+236}_{-141}\ \text{MeV}\,,$$

$$M_{\rho''} = 1718 \pm 4^{+57}_{-94}\ \text{MeV}\,, \quad \Gamma_{\rho''} = 465 \pm 9^{+137}_{-53}\ \text{MeV}\,,$$

$$\gamma = 0.15 \pm 0.01^{+0.07}_{-0.03}\,, \quad \phi_1 = -0.66 \pm 0.01^{+0.22}_{-0.99}\,,$$

$$\delta = -0.13 \pm 0.01^{+0.00}_{-0.05}\,, \quad \phi_2 = -0.44 \pm 0.03^{+0.06}_{-0.90}\,,$$

- **Physical** pole mass and width

$$M_\rho^{\text{pole}} = 760.6 \pm 0.8\ \text{MeV}\,, \quad \Gamma_\rho^{\text{pole}} = 142.0 \pm 0.4\ \text{MeV}\,,$$

$$M_{\rho'}^{\text{pole}} = 1289 \pm 8^{+52}_{-143}\ \text{MeV}\,, \quad \Gamma_{\rho'}^{\text{pole}} = 540 \pm 16^{+151}_{-111}\ \text{MeV}\,,$$

$$M_{\rho''}^{\text{pole}} = 1673 \pm 4^{+68}_{-125}\ \text{MeV}\,, \quad \Gamma_{\rho''}^{\text{pole}} = 445 \pm 8^{+117}_{-49}\ \text{MeV}\,,$$

Low-energy observables

- $\langle r^2 \rangle_V^\pi = 6\alpha_1$

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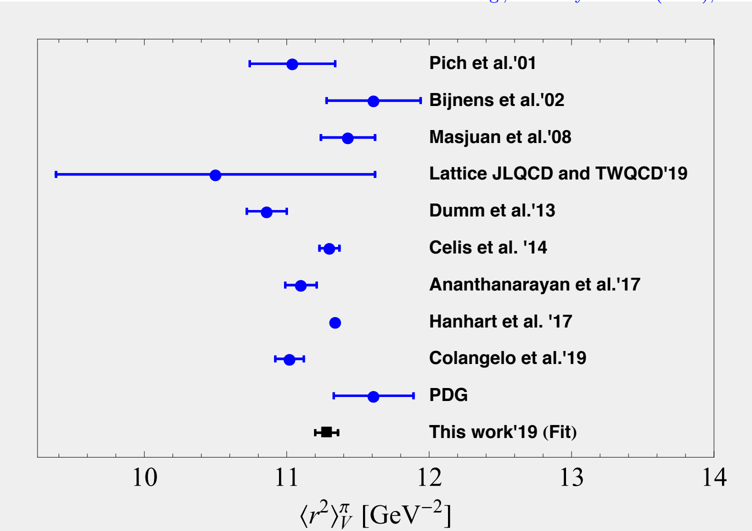
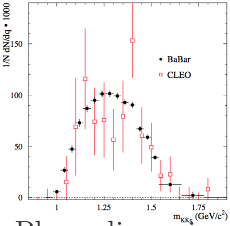


$\tau^- \to K^- K_S \nu_\tau$: Kaon vector form factor $F_V^K(s)$

- Motivation: BaBar measurement of $\tau^- \to K^- K_S \nu_\tau$ (PRD 98 (2018) no.3, 032010)



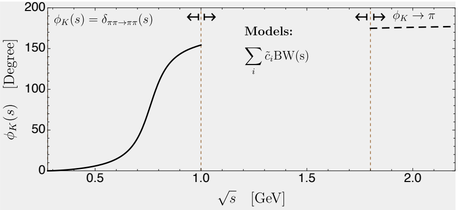
- good quality data
- sensitive to $\rho(1450)$ and $\rho(1700)$
- ChPT at $\mathcal{O}(p^4)$:

$$F_V^K(s)|_{\mathrm{ChPT}} = 1 + \frac{2L_9^r}{F_\pi^2} - \frac{s}{96\pi^2 F_\pi^2}\left[A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right] = F_V^\pi(s)$$

- Phase dispersive representation with $\alpha_{1,2}$ from $F_V^\pi(s)$

$$F_V^K(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\mathrm{cut}}} ds' \frac{\phi_K(s')}{(s')^3(s'-s-i\varepsilon)}\right],$$

- Form Factor phase $\phi_K(s)$:



Omnès exponential representation for $F_V^K(s)$

- Different resonance mixing contribution than $F_V^\pi(s)$

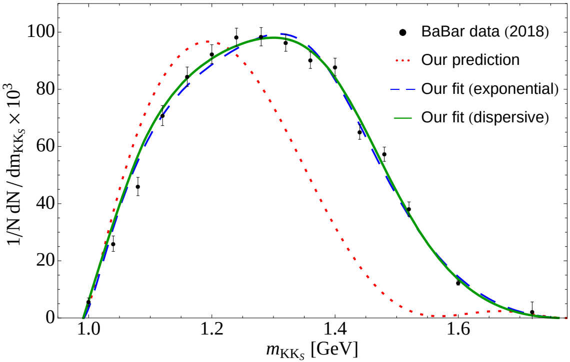
$$
\begin{aligned}
F_V^K(s) &= \frac{M_\rho^2 + s\left(\tilde{\gamma} e^{i\tilde{\phi}_1} + \tilde{\delta} e^{i\tilde{\phi}_2}\right)}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp\left\{ \mathrm{Re}\left[-\frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2} A_K(s) \right)\right]\right\} \\
&\quad -\tilde{\gamma} \frac{s\, e^{i\tilde{\phi}_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left\{ -\frac{s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \mathrm{Re} A_\pi(s) \right\} \\
&\quad -\tilde{\delta} \frac{s\, e^{i\tilde{\phi}_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left\{ -\frac{s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \mathrm{Re} A_\pi(s) \right\},
\end{aligned}
$$

$$
\Gamma_{\rho',\rho''}(s) = \Gamma_{\rho',\rho''} \frac{s}{M_{\rho',\rho''}^2} \frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_{\rho',\rho''}^2)} \theta(s - 4m_\pi^2) \,.
$$

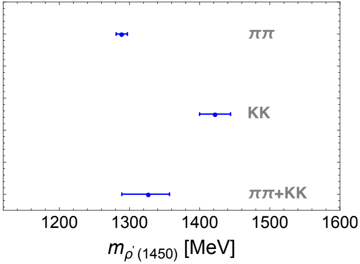
- Extract the phase $\tan\phi_{KK}(s) = \mathrm{Im} F_V^K(s) / \mathrm{Re} F_V^K(s)$
- Use a three-times subtracted dispersion relation

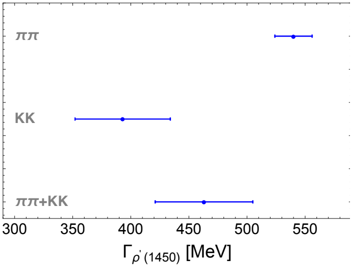
Fit results to BaBar $\tau^- \to K^- K_S \nu_\tau$ data

Parameter	Fit dispersive	Fit exponential
$\tilde{\alpha}_1$	$= 1.88(1)$	—
$\tilde{\alpha}_2$	$= 4.34(1)$	—
$M_{\rho'}$ [MeV]	1467(24)	1411(12)
$\Gamma_{\rho'}$ [MeV]	415(48)	394(35)
$\tilde{\gamma}$	0.10(2)	0.09(1)
$\tilde{\phi}_1$	$-1.19(16)$	$-1.88(9)$
χ^2/d.o.f.	2.9	3.3



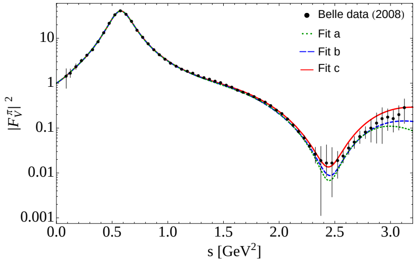
González-Solís and Roig , Eur.Phys.J C79 (2019), 436

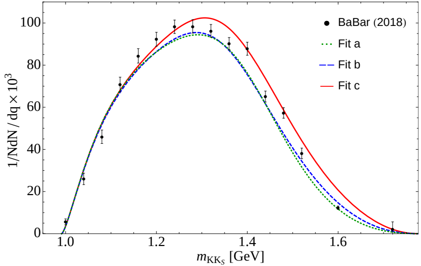




Combined analysis of $F_V^\pi(s)$ and $\tau^- \to K^- K_S \nu_\tau$

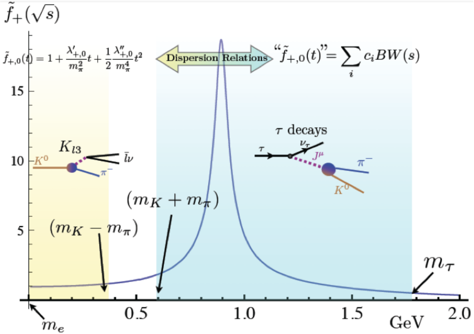
Parameter	$s_{\rm cut} = 4$ [GeV^2]		
	Fit a	Fit b	Fit c
α_1	1.88(1)	1.89(1)	1.87(1)
α_2	4.34(2)	4.31(2)	4.38(3)
$\tilde{\alpha}_1$	$=\alpha_1$	$=\alpha_1$	1.88(24)
$\tilde{\alpha}_2$	$=\alpha_2$	$=\alpha_2$	4.38(29)
m_ρ [MeV]	$=773.6(9)$	$=773.6(9)$	$=773.6(9)$
M_ρ [MeV]	$=m_\rho$	$=m_\rho$	$=m_\rho$
$M_{\rho'}$ [MeV]	1396(19)	1453(19)	1406(61)
$\Gamma_{\rho'}$ [MeV]	507(31)	499(51)	524(149)
$M_{\rho''}$ [MeV]	1724(41)	1712(32)	1746(1)
$\Gamma_{\rho''}$ [MeV]	399(126)	284(72)	413(362)
γ	0.12(3)	0.15(3)	0.11(11)
$\tilde{\gamma}$	0.11(2)	$=\gamma$	0.11(5)
ϕ_1	$-0.23(26)$	0.29(21)	$-0.27(42)$
$\tilde{\phi}_1$	$-1.83(14)$	$-1.48(13)$	$-1.90(67)$
δ	$-0.09(2)$	$-0.07(2)$	$-0.10(5)$
$\tilde{\delta}$	$=0$	$=0$	$-0.01(4)$
ϕ_2	$-0.20(31)$	0.27(29)	$-1.15(71)$
$\tilde{\phi}_2$	$=0$	$=0$	0.40(3)
χ^2/d.o.f	1.52	1.19	1.25





$\tau^- \to K_S \pi^- \nu_\tau$: $K\pi$ form factors

- $K\pi$ form factors suited to describe both $\tau \to K\pi\nu_\tau$ and $K_{\ell 3}$ decays:



- $K_{\ell 3}$ decays are the main route towards the determination of $|V_{us}|^2$

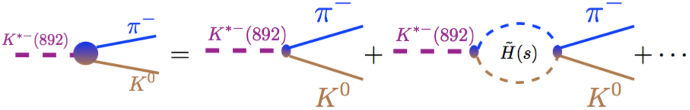
$$\Gamma_{K_{\ell 3}} \propto \underset{\substack{\downarrow \\ \text{ChPT, lattice}}}{|V_{us}|^2 |F_+(0)|^2} I_{K_{\ell 3}}\,, \quad I_{K_{\ell 3}} = \frac{1}{m_K^8}\int dt(p.s.)\left[\widetilde{F}_+(t)^2 + \eta(t,m_\ell)\widetilde{F}_0(t)^2\right]$$

ChPT, lattice

RChT+ dispersion relations

$\tau^- \to K_S \pi^- \nu_\tau$: $K\pi$ Vector form factor

- RχT with two resonances: $R = K^*(892), K^*(1410)$



$$\widetilde{F}_V^{K\pi}(s) = \frac{m_{K^\star}^2 - \kappa_{K^\star} \widetilde{H}_{K\pi}(0) + \gamma s}{D(m_{K^\star}, \gamma_{K^\star})} - \frac{\gamma s}{D(m_{K^{\star\prime}}, \gamma_{K^{\star\prime}})},$$

$$D(m_R, \gamma_R) \equiv m_R^2 - s - \kappa_R \mathrm{Re}\left[H_{K\pi}(s)\right] - i m_R \Gamma_R(s),$$

$$\kappa_R = \frac{192\pi F_K F_\pi}{\sigma_{K\pi}(m_{K^*}^2)} \frac{\gamma_{K^*}}{m_{K^*}}, \quad \Gamma_R(s) = \Gamma_R \frac{s}{m_R^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_R^2)}$$

- We then have a phase with two resonances

$$\delta^{K\pi}(s) = \tan^{-1}\left[\frac{\mathrm{Im} F_V^{K\pi}(s)}{\mathrm{Re} F_V^{K\pi}(s)}\right]$$

$K\pi$ Vector form factor: Dispersive representation

- Three subtractions: helps the convergence of the form factor and suppresses the the high-energy region of the integral

$$F_V^{K\pi}(s) \;=\; P(s)\exp\left[\alpha_1\frac{s}{m_{\pi^-}^2}+\frac{1}{2}\alpha_2\frac{s^2}{m_{\pi^-}^4}+\frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{\rm cut}}ds'\frac{\delta^{K\pi}(s')}{(s')^3(s'-s-i0)}\right]$$

- $\alpha_1 = \lambda_+^{'}$ and $\alpha_1^2+\alpha_2 = \lambda_+^{''}$ low energies parameters

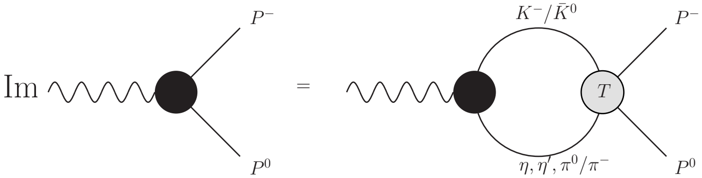
$$F_V^{K\pi}(t) = 1+\frac{\lambda_+^{'}}{M_{\pi^-}^2}t+\frac{1}{2}\frac{\lambda_+^{''}}{M_{\pi^-}^4}t^2$$

- $s_{\rm cut}$: cut-off to check stability
- Parameters to Fit: $\lambda'_+\,, \lambda''_+, \, m_{K^*}, \gamma_{K^*}, m_{K^{*\prime}}, \gamma_{K^{*\prime}}$

$K\pi$ scalar form factor

- Coupled channel dispersion relation

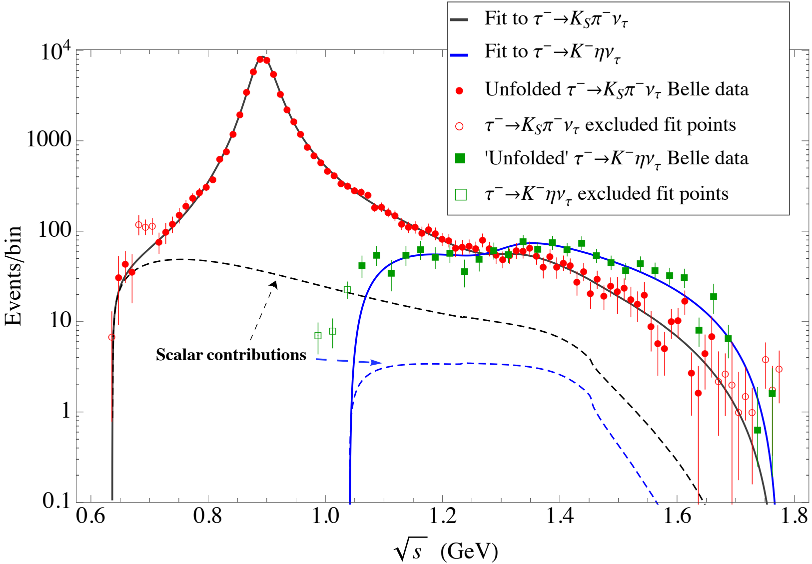
$$F_0^{P^-P^0}(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}F(s')}{s'(s'-s-i\epsilon)} = \frac{1}{\pi}\sum_{j=1}^{3}\int_{s_i}^{\infty} ds' \frac{\sigma_j(s')F_0^j(s')T_0^{i\to j}(s')^\star}{(s'-s-i0)}$$



(Jamin, Oller, Pich, Nucl.Phys.B 622 (2002))

Combined fit to $\tau^- \to K_S \pi^- \nu_\tau$ and $\tau^- \to K^- \eta \nu_\tau$

Escribano, Gonzàlez-Solís, Jamin and Roig , JHEP 1409 (2014) 042



Fit results

- Different choices regarding linear slopes and resonance mixing parameters ($s_{cut} = 4$ GeV^2)

Fitted value	Reference Fit	Fit A	Fit B	Fit C
$\bar{B}_{K\pi}(\%)$	0.404 ± 0.012	0.400 ± 0.012	0.404 ± 0.012	0.397 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.402)	(0.394)	(0.400)	(0.394)
M_{K^*}	892.03 ± 0.19	892.04 ± 0.19	892.03 ± 0.19	892.07 ± 0.19
Γ_{K^*}	46.18 ± 0.42	46.11 ± 0.42	46.15 ± 0.42	46.13 ± 0.42
$M_{K^{*\prime}}$	1305^{+15}_{-18}	1308^{+16}_{-19}	1305^{+15}_{-18}	1310^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	168^{+52}_{-44}	212^{+66}_{-54}	174^{+58}_{-47}	184^{+56}_{-46}
$\gamma_{K\pi} \times 10^2$	$= \gamma_{K\eta}$	$-3.6^{+1.1}_{-1.5}$	$-3.3^{+1.0}_{-1.3}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.9 ± 0.7	23.6 ± 0.7	23.8 ± 0.7	23.6 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2	11.6 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.58 ± 0.10	1.62 ± 0.10	1.57 ± 0.10	1.66 ± 0.09
$(B^{th}_{K\eta}) \times 10^4$	(1.45)	(1.51)	(1.44)	(1.58)
$\gamma_{K\eta} \times 10^2$	$-3.4^{+1.0}_{-1.3}$	$-5.4^{+1.8}_{-2.6}$	$-3.9^{+1.4}_{-2.1}$	$-3.7^{+1.0}_{-1.4}$
$\lambda'_{K\eta} \times 10^3$	20.9 ± 1.5	$= \lambda'_{K\pi}$	21.2 ± 1.7	$= \lambda'_{K\pi}$
$\lambda''_{K\eta} \times 10^4$	11.1 ± 0.4	11.7 ± 0.2	11.1 ± 0.4	11.8 ± 0.2
χ^2/n.d.f.	$108.1/105 \sim 1.03$	$109.9/105 \sim 1.05$	$107.8/104 \sim 1.04$	$111.9/106 \sim 1.06$

Results of the combined $K_S\pi^-$ and $K^-\eta$ analysis

- Reference fit results obtained for different values of s_{cut}

Parameter	3.24	4	9	∞
$B_{K\pi}(\%)$	0.402 ± 0.013	0.404 ± 0.012	0.405 ± 0.012	0.405 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.399)	(0.402)	(0.403)	(0.403)
M_{K^*}	892.01 ± 0.19	892.03 ± 0.19	892.05 ± 0.19	892.05 ± 0.19
Γ_{K^*}	46.04 ± 0.43	46.18 ± 0.42	46.27 ± 0.42	46.27 ± 0.41
$M_{K^{*\prime}}$	1301^{+17}_{-22}	1305^{+15}_{-18}	1306^{+14}_{-17}	1306^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	207^{+73}_{-58}	168^{+52}_{-44}	155^{+48}_{-41}	155^{+47}_{-40}
$\gamma_{K\pi}$	$=\gamma_{K\eta}$	$=\gamma_{K\eta}$	$=\gamma_{K\eta}$	$=\gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.3 ± 0.8	23.9 ± 0.7	24.3 ± 0.7	24.3 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2
$B_{K\eta} \times 10^4$	1.57 ± 0.10	1.58 ± 0.10	1.58 ± 0.10	1.58 ± 0.10
$(B^{th}_{K\eta}) \times 10^4$	(1.43)	(1.45)	(1.46)	(1.46)
$\gamma_{K\eta} \times 10^2$	$-4.0^{+1.3}_{-1.9}$	$-3.4^{+1.0}_{-1.3}$	$-3.2^{+0.9}_{-1.1}$	$-3.2^{+0.9}_{-1.1}$
$\lambda'_{K\eta} \times 10^3$	18.6 ± 1.7	20.9 ± 1.5	22.1 ± 1.4	22.1 ± 1.4
$\lambda''_{K\eta} \times 10^4$	10.8 ± 0.3	11.1 ± 0.4	11.2 ± 0.4	11.2 ± 0.4
χ^2/n.d.f.	105.8/105	108.1/105	111.0/105	111.1/105

Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

- Central results including the largest variation of s_{cut}

$$M_{K^*(892)} = 892.03 \pm 0.19 \text{ MeV}$$

$$\Gamma_{K^*(892)} = 46.18 \pm 0.44 \text{ MeV}$$

$$M_{K^*(1410)} = 1305^{+16}_{-18} \text{ MeV}$$

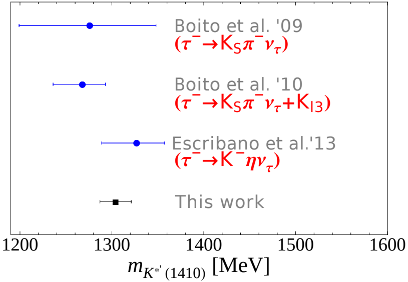
$$\Gamma_{K^*(1410)} = 168^{+65}_{-59} \text{ MeV}$$

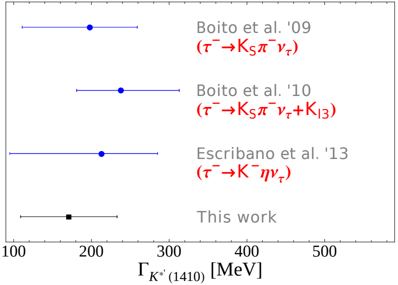
$$\gamma_{K\pi} = \gamma_{K\eta} = -3.4^{+1.2}_{-1.4} \cdot 10^{-2}$$

$$\bar{B}_{K\pi} = (0.0404 \pm 0.012)\%$$

$$\bar{B}_{K\eta} = (1.58 \pm 0.10) \cdot 10^{-4}$$

$$\chi^2/d.o.f = 108.1/105 = 1.03$$





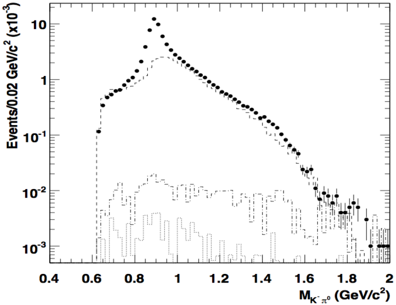
Low-energy parameters

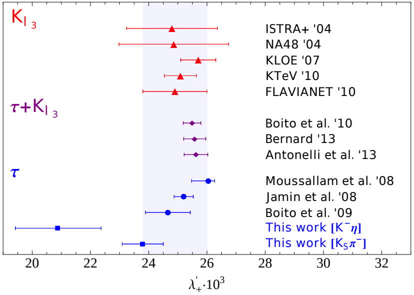
- Low-energy parameters

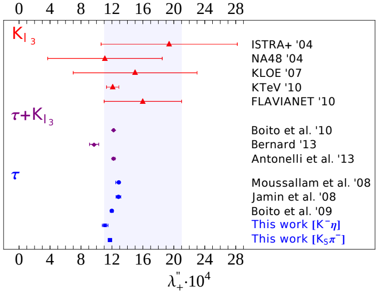
$$\left.\begin{aligned}\lambda'_{K\pi} &= (23.9 \pm 0.9)\cdot 10^{-3}\\ \lambda'_{K\eta} &= (20.9 \pm 2.7)\cdot 10^{-3}\end{aligned}\right\}\text{isospin violation?}$$

$$\left.\begin{aligned}\lambda''_{K\pi} &= (11.8 \pm 0.2)\cdot 10^{-4}\\ \lambda''_{K\eta} &= (11.1 \pm 0.5)\cdot 10^{-4}\end{aligned}\right\}\text{isospin violation?}$$

$\tau^- \to K^-\pi^0\nu_\tau$ (PRD 76 (2007) 051104)



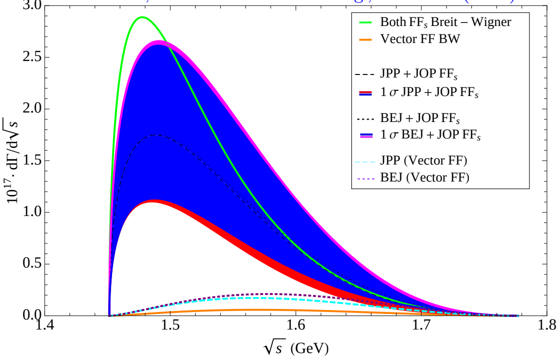




Predictions for the $\tau^- \to K^-\eta'\nu_\tau$ decay

- Decay dominated by the scalar Form Factor ($\sim 90\%$ of the BR)

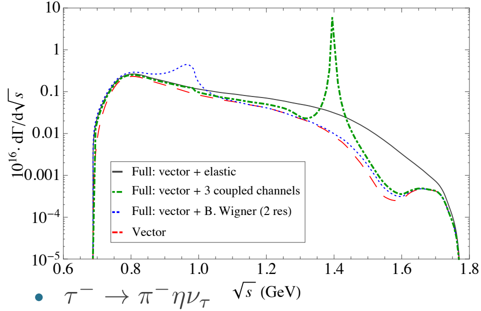
Escribano, Gonzàlez-Solís and Roig , JHEP 10 (2013) 039

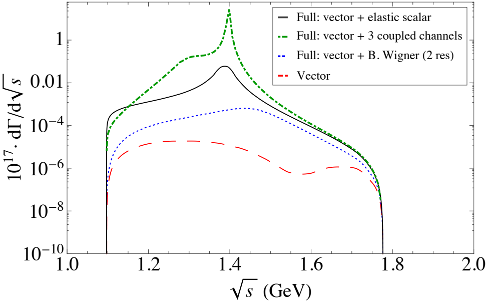


Source	Branchig ratio
Breit-Wigner	$(1.45^{+3.80}_{-0.87}) \cdot 10^{-6}$
Exponential representation	$(1.00^{+0.37}_{-0.29}) \cdot 10^{-6}$
Dispersion relation	$(1.03^{+0.37}_{-0.29}) \cdot 10^{-6}$
Experimental bound	$< 2.4 \cdot 10^{-6}$ at 90% C.L.

Predictions for the second-class currents $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$

Escribano, Gonzàlez-Solís, Roig PRD 94 (2016), 034008





- $\tau^- \to \pi^- \eta \nu_\tau$
  - Theory predictions: $BR \sim 1 \times 10^{-5}$ (Escribano'16, Moussallam'14)
  - BaBar: $BR < 9.9 \cdot 10^{-5}$ 95% CL , Belle: $BR < 7.3 \cdot 10^{-5}$ 90% CL
- $\tau^- \to \pi^- \eta' \nu_\tau$
  - Theory predictions: $BR \sim [10^{-7}, 10^{-6}]$ (Escribano'16)
  - BaBar: $BR < 4 \cdot 10^{-6}$ 90% CL

$\boxed{\text{Challenging for Belle II}}$

Summary

- Tau physics is a very rich field to test QCD and EW
- Important experimental activities: Belle (II), BaBar, LHCb, BESIII, STCF
- Hadronic τ decays as a privileged tool for the investigation of QCD currents:
  - Form factors from ChPT supplemented by subtracted dispersion relations
  - Determination of resonance parameters (mass, width)
  - low-energy observables (e.g. meson radius)
- A lot of interesting physics to be done in the tau sector