Polarization study of two vector mesons in $\tau \to VV\nu_\tau$

Hiroyuki Umeeda (Jilin University)

Collaborator: Vindhyawasini Prasad (Jilin University)

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Introduction: τ decays into mesons

Inclusive channels

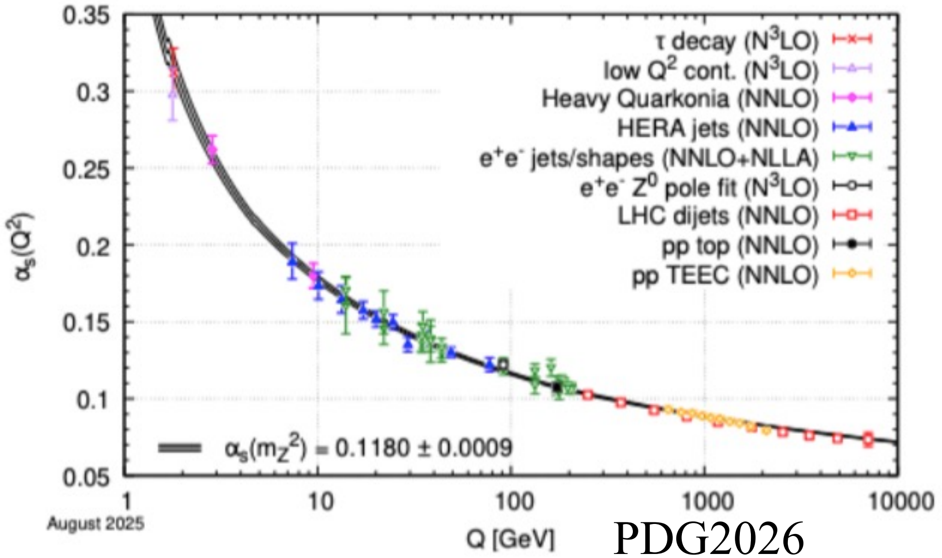
- $\tau \to X_q \nu_\tau$

$W^- \to \bar{u}d$ $\quad V_{ud}$ $\quad$ CF

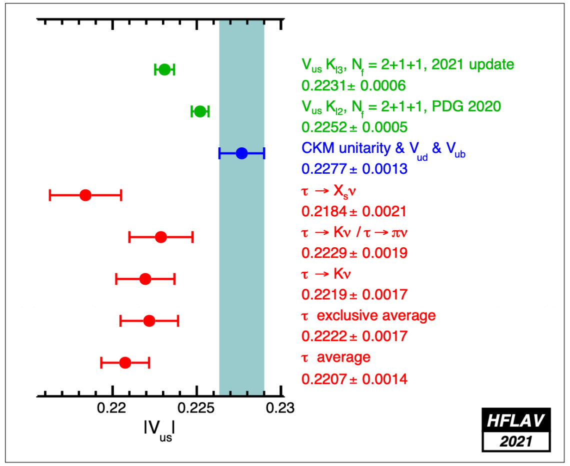
$W^- \to \bar{u}s$ $\quad V_{us}$ $\quad$ CS

Exclusive channels

Precision determinations of $\alpha_s(m_\tau)$ and V_{us}

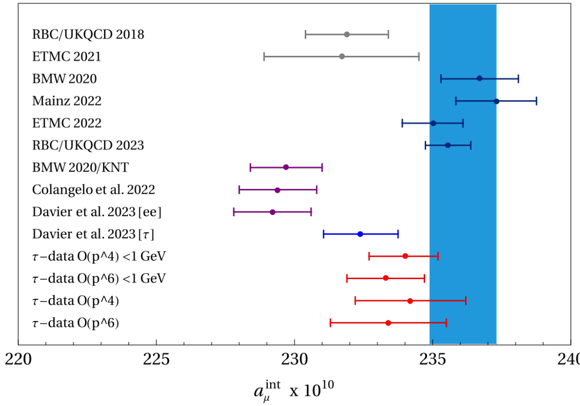


PDG2026



- $\tau \to \pi\pi\nu_\tau$ and similar $3\pi, 4\pi$ decays

Determination of invariant-mass distributions (spectral functions) $\xrightarrow{\text{Isospin symmetry}}$ Hadronic vacuum polarization (HVP) contribution to muon $g-2$



Masjuan, Miranda, and Roig [2305.20005]

✓ - $\tau \to VV\nu_\tau$: Smaller branching ratios due to limited phase space

— In light of measurements at Belle II, the relevant theoretical study is required.

— Final states are characterized by polarizations of vector mesons.

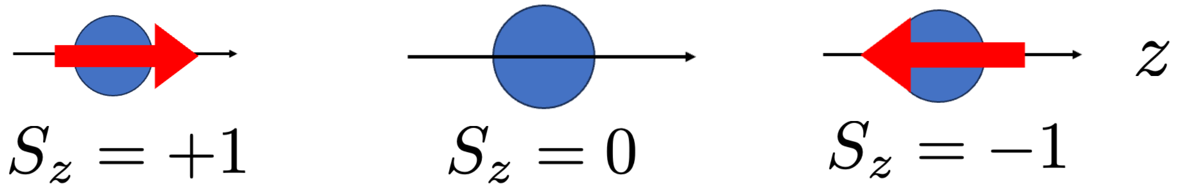
$m_\tau \approx 1.78$ GeV

$m_{\rho^+} + m_\omega \approx 1.56$ GeV

Spin-polarization studies

- For $P \to VV$ ($P = B, D$) decays, vector mesons have distinct helicity configurations.

Vector state ($J^{PC} = 1^{--}$)
2 transverse and 1 longitudinal modes
Arrows: spin-orientation direction



- Whether one polarization dominates the decay is determined by weak and QCD interactions.

- Partial amount of produced S_z=0 states are quantitatively defined by

longitudinal polarization fraction: $$f_L = \frac{\mathrm{Br}[P \to V_L V_L]}{\mathrm{Br}[P \to VV]} = \frac{|H(0)|^2}{|H(+)|^2 + |H(0)|^2 + |H(-)|^2}$$

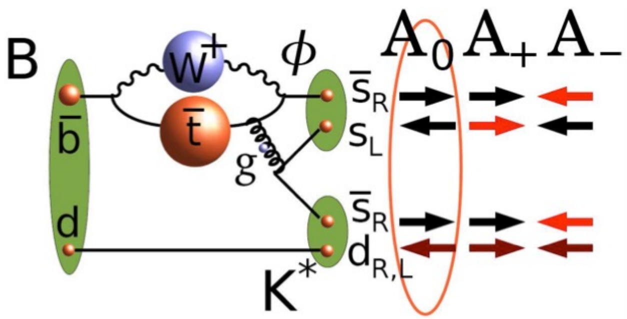
Helicity amplitude
$H(\lambda)$ $(\lambda = \pm, 0)$

- For B-meson decays, detailed values of f_{L} possibly enable us to search for new physics (NP) provided that uncertainties from QCD are controllable.

Polarization in $B \to VV$ decays

- In the SM, $f_L \simeq 1$ is expected due to W-boson interaction for $b \to q(q = u, d, s)$ transitions.

Gritsan [arXiv:0706.2030]



Helicity conservation

$$|A_0| \gg |A_+| \gg |A_-|$$

Ali, Korner and Kramer, Willrodt
Z. Phys. C 1 (1979) 269.

→ The longitudinal mode is dominant.

Channel	$B^0 \to \rho^+\rho^-$	$B^+ \to \rho^+\rho^0$	$B^+ \to \omega\rho^+$	$B^0 \to \omega\omega$
f_L (PDG)	0.970 ± 0.023	0.950 ± 0.016	0.90 ± 0.06	0.87 ± 0.18

— For the above modes, the fraction is close to f_L=1, supporting quark-level helicity argument.

Polarization puzzle $\quad B^+ \to \varphi K^{*+}$ (PDG) $f_L = 0.50 \pm 0.05$ $\quad B^0 \to \varphi K^{*0}$ (PDG) $f_L = 0.497 \pm 0.017$

Possibilities discussed in the literature:
(1) Underestimated uncertainties from QCD: penguin annihilation [0405134, 0411146] and rescattering [0401188, 0409317]
(2) New physics contributions: Simple Higgs model [0504145], MSSM [0511129], Z’ [0602140]

For the case of $D \to VV$ decays

Channel	$D^0 \to K^{*-}\rho^+$	$D^0 \to \rho^0\rho^0$	$D^0 \to \omega\phi$	$D^0 \to K^{*+}K^{*-}$
f_L (theo [1])	0.43	0.49	0.36	0.37
f_L (theo [2])	0.8480 ± 0.0576	0.7743 ± 0.0946	0.4858 ± 0.0725	0.7905 ± 0.0716
f_L (exp)	0.475 ± 0.271 MARK-III (1992)	$0.71 \pm 0.04 \pm 0.02$ FOCUS (2007)	$f_L < 0.24$ 95%C.L. BESIII (2022)	$0.468 \pm 0.046 \pm 0.011$ BESIII (2026)

Theo [1]: factorization Cheng and Chiang [2401.06316] Theo [2] FAT approach Yang, Zheng, Li and Zhou [2604.01008]

◉ A given theory method does not simultaneously explain all experimental results.

—For another theoretical method (NRCQM+final-state interactions [2303.00535]), the results are similar: $f_L(D^0 \to \rho^0\rho^0)$ is not reproduced while $f_L(D^0 \to K^{*-}\rho^+)$ and $f_L(D^0 \to \omega\phi)$ are accommodated.

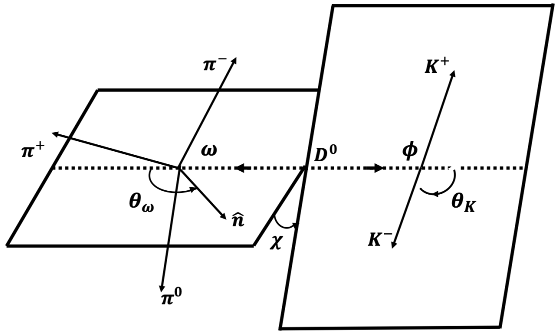
◉ Puzzling patterns are present for charm decays, as well as the B-meson case.

→ In this context, an independent study in $\tau \to VV\nu_\tau$ decays is highly desirable.

Polarized information from decay distributions

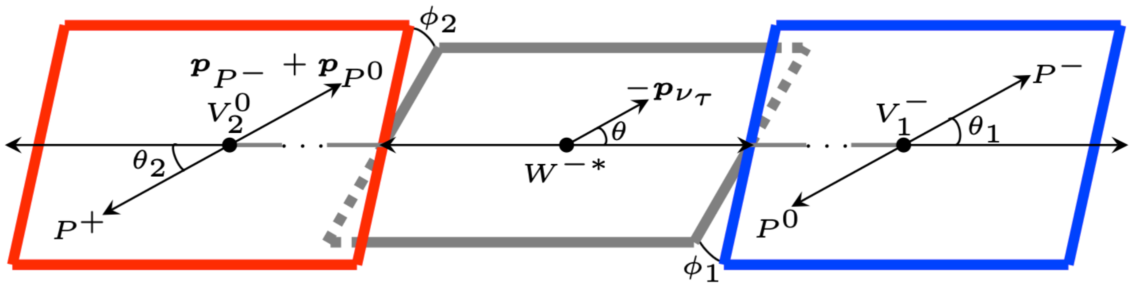
- Helicity amplitudes can be extracted from angular observables.

$\boxed{D^0 \to \omega\phi}$ $\omega \to 3\pi$ $\phi \to K\bar{K}$



BESIII [2108.02405]

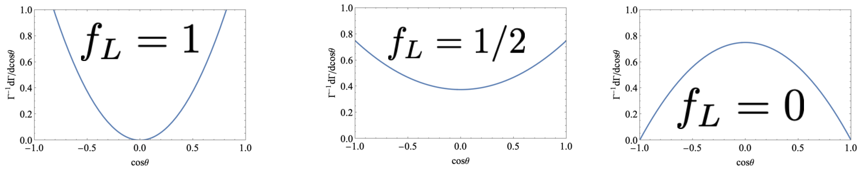
$\boxed{\tau^- \to V_1^- V_2^0 \nu_\tau}$ $V_1^- \to \pi^-\pi^0$ $V_2^0 \to 3\pi$



Helicity angles: θ, θ_1, and θ_2

- θ_i distributions: coefficients are sensitive to f_L.

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_i} = \frac{3}{4}\sin^2\theta_i\left[1 - f_L^{(V_i)}\right] + \frac{3}{2}\cos^2\theta_i f_L^{(V_i)}$$

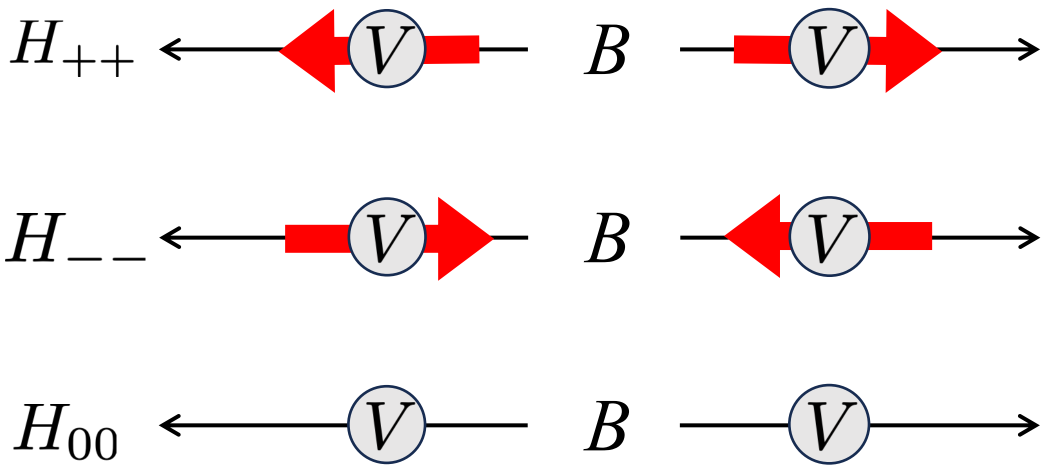


Analyzed channels: $(V_1^-, V_2^0) = (\rho^-, \rho^0), \underline{(\rho^-, \omega)}, \underline{(K^{*-}, \rho^0)}, (K^{*-}, \omega), (\rho^-, \bar{K}^{*0})$

appear in $\tau^- \to \pi^-\pi^0\pi^+\pi^-\pi^0\nu_\tau$ (experimental analysis ongoing) appear in $\tau^- \to K^-\pi^0\pi^+\pi^-\pi^0\nu_\tau$

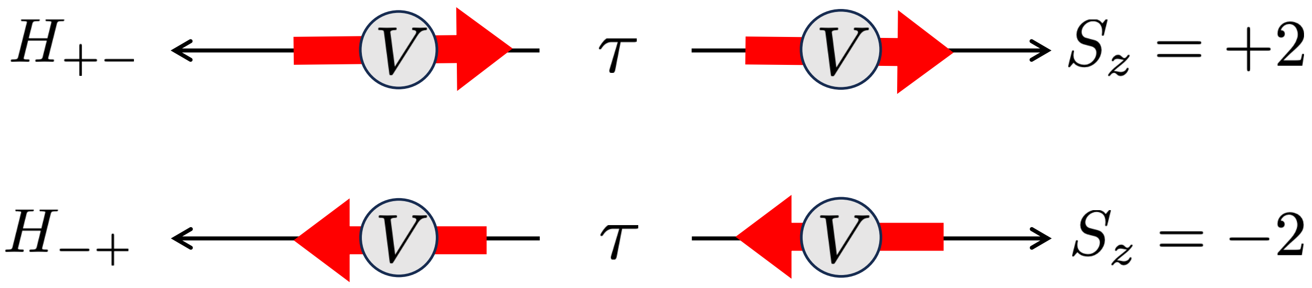
Number of independent polarizations of *VV* system

B-meson decays



In order to form *J*=0 (*J* is the angular momentum), consistent with the initial particle, the final *VV* state are reduced to only above three polarization modes.

τ decays



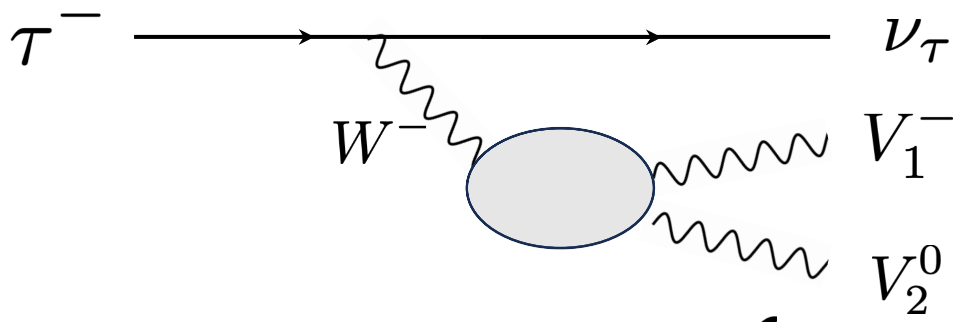
The above two helicity configurations do *not* occur, since a spin-2 state cannot be produced from an off-shell *W* boson.

Hence, in the standard model (or any others that do not contain spin-2 particles mediating τ decays) seven polarization configurations are possible.

Similar discussion for other processes gives:

	$\tau \to V\nu_\tau$	$\tau \to VP\nu_\tau$	$\tau \to VV\nu_\tau$	$D \to VP$	$D \to VV$
# of pols.	3	3	7	1	3

Analysis of $\tau \to V_1 V_2 \nu_\tau$ decays



Width formula $\quad q = d, s$

$$d\Gamma = \frac{G_F^2 |V_{uq}|^2}{4m_\tau} |L_\mu H^\mu_{\lambda_1\lambda_2}|^2 (\mathrm{LIPS})$$

Matrix elements of V-A currents:

$$\begin{cases} L_\mu = \langle \nu_\tau | \bar{\nu}_\tau \gamma_\mu (1-\gamma_5) \tau | \tau^- \rangle \\ H^\mu_{\lambda_1\lambda_2} = \langle V_1(\lambda_1) V_2(\lambda_2) | \bar{q} \gamma^\mu (1-\gamma_5) u | 0 \rangle \end{cases}$$

λ_i : helicity of V_i $(i = 1, 2)$ ● Polarized information of vector mesons can be extracted from $H^\mu_{\lambda_1\lambda_2}$.

Methods

(1) Resonance chiral theory (RχT): Ecker, Gasser, Pich and de Rafael, Nucl. Phys. B 321, 311 (1989) Ecker, Gasser, Leutwyler, Phys. Lett. B 223, 425 (1989)

— Successfully applied to tau decays, and used for the event generator (TAUOLA)

(2) Generalized hidden local symmetry (GHLS): Bando, Fujiwara and Yamawaki, PTP 79, 1140 (1988).

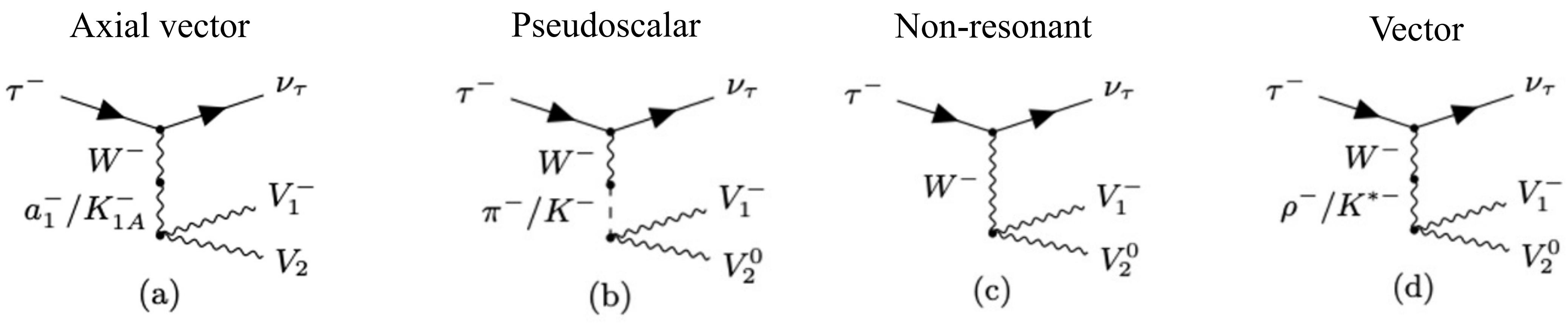
— Including both vector and axial vector mesons as dynamical gauge bosons (this work)

(3) Algebra of angular momenta: Dai, Pavao, Sakai, and Oset [1805.0457 [hep-ph]]

(4) Endpoint relations: Hiller and Zwicky, [arXiv:1312.1923 [hep-ph]] (discussed later)

— Using kinematical information for specific point on the phase space to predict f_L.

Contributions to $\tau \to V_1 V_2 \nu_\tau$ in the effective theory



◉ The helicity amplitude defined as a sum over the diagrams are

$$H^\mu_{\lambda_1\lambda_2} = (a) + (b) + (c) + (d)$$

◉ K_1 mixing (axial-vector meson)

$$\begin{cases} K_{1A}(1^{++}\ \text{nonet}) \\ K_{1B}(1^{+-}\ \text{nonet}) \end{cases} \qquad \begin{pmatrix} |K_1(1270)\rangle \\ |K_1(1400)\rangle \end{pmatrix} = \begin{pmatrix} \sin\theta_{K_1} & \cos\theta_{K_1} \\ \cos\theta_{K_1} & -\sin\theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_{1A}\rangle \\ |K_{1B}\rangle \end{pmatrix} \qquad \theta_{K_1} = 34^\circ, 57^\circ$$

Cheng [arXiv:1110.2249 [hep-ph]]

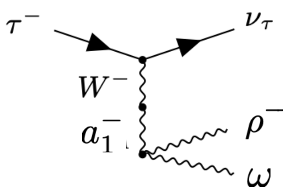
—Sensitivity of the choice of θ_{K_1} of the result is weak. → Setting $\theta_{K_1} = 34^\circ$ in what follows.

$a_1\rho\omega$ coupling in GHLS

$$\mathcal{L} = 2g^2\tilde{c}_8\epsilon^{\mu\nu\alpha\beta}(\partial_\mu\omega_\nu\rho^+_\alpha + \omega_\mu\partial_\nu\rho^+_\alpha)a^-_{1\beta}$$

vertex from Kaiser and Meissner, Nucl. Phys. A 519, 671 (1990) and also in Harvey, Hill and Hill [arXiv:0712.1230 [hep-th]]

$g,\ \tilde{c}_8$: couplings defined in GHLS



(a)

$$= c^{(a)}_{\mathrm{CF}}T^{(a)}$$

$\left\{ \begin{array}{l} c^{(a)}_{\mathrm{CF}} \text{ : coefficient defined by product of couplings} \\ T^{(a)} \text{ : tensor structure} \end{array} \right.$

definitions
$$\left\{ \begin{array}{l} \tilde{T}^{(b)} = -i\epsilon^{\mu\nu\alpha\beta}Q_\mu(p_1-p_2)_\nu\epsilon^*_{1\alpha}\epsilon^*_{2\beta} \\ \mathrm{BW}_X(Q^2) = \dfrac{m_X^2}{m_X^2 - Q^2 - im_X\Gamma_X} \\ T^{(c)} = -i\epsilon^{\mu\nu\alpha\beta}\epsilon_\mu(p_1-p_2)_\nu\epsilon^*_{1\alpha}\epsilon^*_{2\beta} \end{array} \right.$$

GHLS
$$\left\{ \begin{array}{l} c^{(a)}_{\mathrm{CF}} = 4\sqrt{2}\tilde{c}_8 g^3 F_\pi^2 \dfrac{\mathrm{BW}_{a_1}}{m^2_{a_1}}\delta_{Aa_1} \\ T^{(a)} = T^{(c)} - \dfrac{\epsilon\cdot Q}{m_A^2}\tilde{T}^{(b)} \end{array} \right.$$

Another result from TAUOLA $\tau \to 5\pi\nu_\tau$ analysis

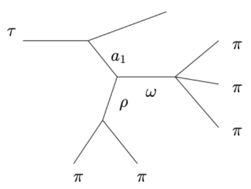
Kuhn and Was [arXiv:hep-ph/0602162]

$$\left\{ \begin{array}{l} c^{(a)}_{\rho^-\omega}\Big|_{\mathrm{TAUOLA}} = c_0\mathrm{BW}_{a_1} \\ T^{(a)}\Big|_{\mathrm{TAUOLA}} = -i\epsilon^{\mu\nu\alpha\beta}\epsilon_\mu\epsilon_{1\nu}\epsilon_{2\alpha}Q_\beta \end{array} \right.$$

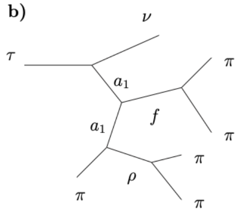
c_0=3

- The second expression is obtained from the process for which $\tau \to \rho\omega\nu_\tau$ is a subdiagram.

a)



b)



—The TAUOLA-based result gives much larger branching ratio than others.

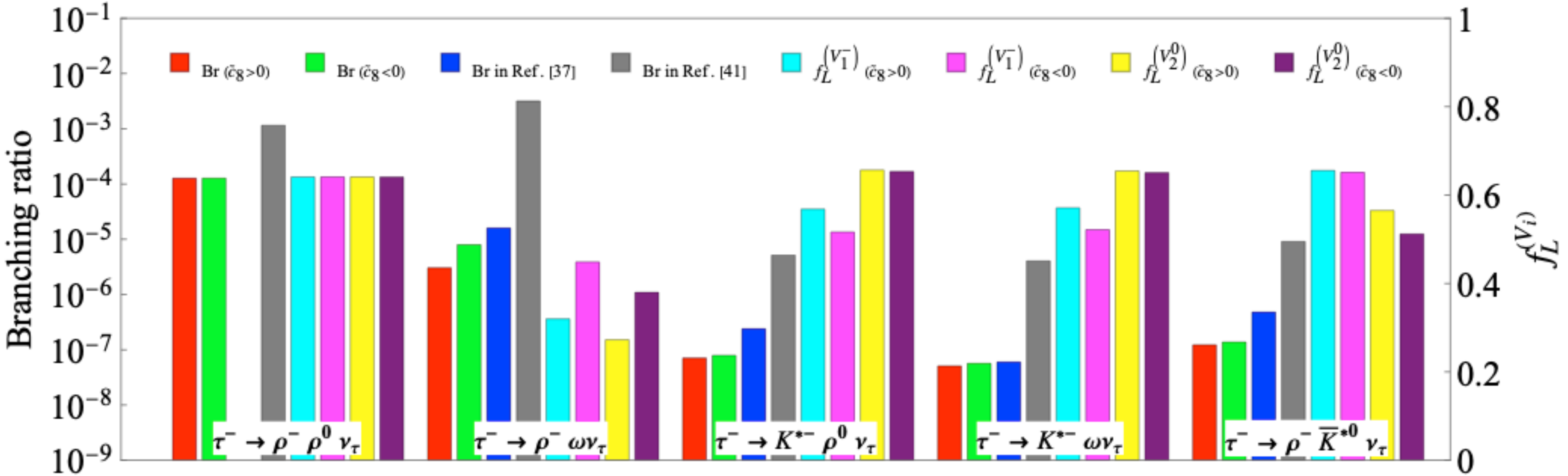
$\mathrm{Br}[\tau^- \to \rho^-\omega\nu_\tau] = \mathcal{O}(10^{-2})$	$\mathcal{O}(10^{-3})$	$\mathcal{O}(10^{-5})$
TAUOLA-based	Angular momentum algebra model [1805.0457 [hep-ph]]	GHLS Chiral effective model [hep-ph/960754]

- In GHLS, the interaction structure does not appear for parity and charge conjugation invariance with hermiticity.

→ The GHLS result is adopted for the axial vector contribution.

Numerical results

Numerical results



For branching ratios ● the results are comparable with ones in Ref. [37] while being smaller than Ref. [41].

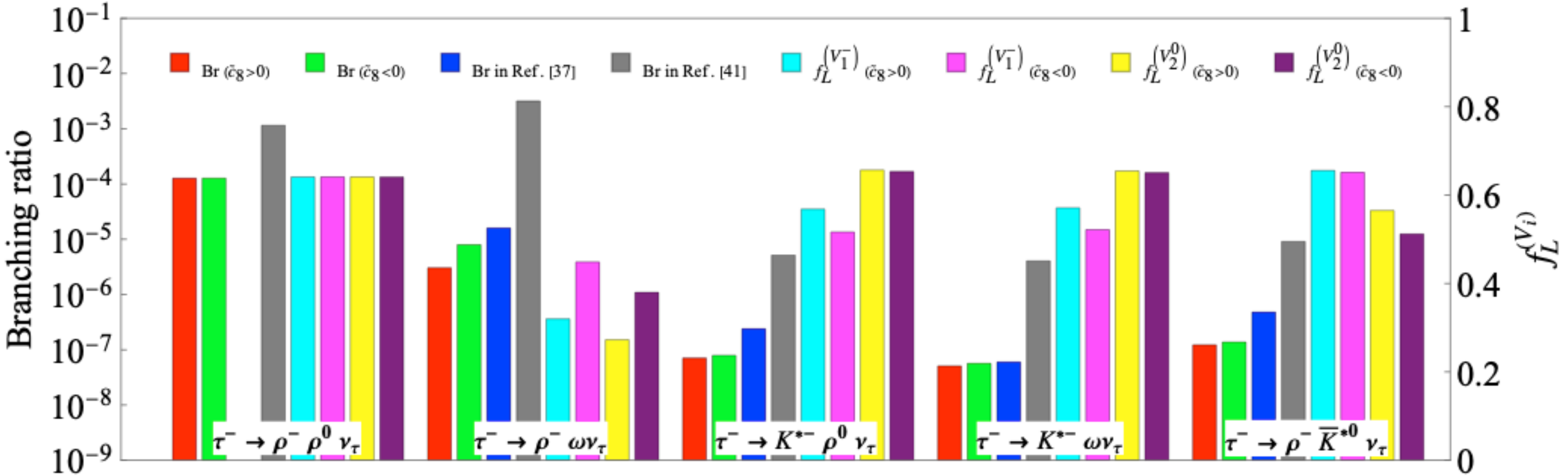
[37] Effective chiral model: Li [hep-ph/960754]. [41] Angular momentum algebra model: Dai, Pavao, Sakai, and Oset [1805.0457 [hep-ph]]

● Input-parameter dependence for $\tilde{c}_8$ (coupling for AVV, A is an axial-vector meson)

—From Br[$D_s^+ \to \rho^+\omega$]=0.99% (BESIII [2501.04451]), the coupling is fixed as $\tilde{c}_8 = \pm 0.030$

—Br[$\tau^- \to \rho^-\omega\nu_\tau$] is affected by sign of $\tilde{c}_8$. However, Br[$\tau^- \to \rho^-\rho^0\nu_\tau$] does not depend on that parameter, since axial vector contributions are prohibited by G-parity conservation.

Numerical results



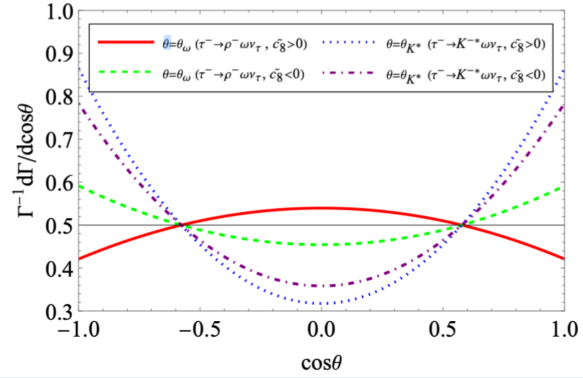
For f_L

● f_L in $\tau^- \to \rho^- \omega \nu_\tau$ (with no vector-meson contribution) is smaller than others.

$0.27 \leq f_L \leq 0.45$ $\qquad$ $0.51 \leq f_L \leq 0.66$

Angular distributions
$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_i} = \frac{3}{4}\sin^2\theta_i\left[1 - f_L^{(V_i)}\right] + \frac{3}{2}\cos^2\theta_i f_L^{(V_i)}$$

In accordance with the above figure, the distribution in $\tau^- \to \rho^- \omega \nu_\tau$ is distinguished from others, taking $\tau^- \to K^{*-} \omega \nu_\tau$ as an example.



Prediction for f_L from kinematical restrictions

- For $\tau \to VV\nu_\tau$ decays, the phase space region is small, and thus $|\boldsymbol{p}| \ll m_\tau$ is satisfied.

  $\boldsymbol{p}$: momentum of either vector meson (defined in the rest frame of VV system)

- If the approximation of $|\boldsymbol{p}| \simeq 0$ (or equivalently $m_\tau \simeq m_{V_1} + m_{V_2}$) is valid, f_L is subject to kinematical restriction from Lorentz symmetry. Hiller and Zwicky [1312.1923]

Example: $B \to K^* \ell^+ \ell^-$ $\quad f_L^{(K^*)}(q^2_{\max}) = 1/3$ $\quad q^2$: di-lepton invariant mass squared

—It is also applied to $\tau \to VV\nu_\tau$: $\quad f_L^{(V_1^-)}\Big|_{Q^2 \to Q^2_{\min}} = f_L^{(V_2^0)}\Big|_{Q^2 \to Q^2_{\min}} = 1/3$

$$Q^2 = (p_{V_1} + p_{V_2})^2$$
$$m_{V_1} + m_{V_2} \le \sqrt{Q^2} \le m_\tau$$

—At the endpoints, the longitudinal polarization fractions are reduced to the fixed value.

—However, deviations from $f_L = 1/3$ occur in general for the GHLS predictions.

$\tau^- \to \rho^- \rho^0 \nu_\tau$	$\tau^- \to \rho^- \omega \nu_\tau$	$\tau^- \to K^{*-} \rho^0 \nu_\tau$
$f_L^{(\rho^-)} = 0.64 \quad f_L^{(\rho^0)} = 0.64$	$f_L^{(\rho^-)} = 0.32 \quad f_L^{(\omega)} = 0.27$	$f_L^{(K^{*-})} = 0.57 \quad f_L^{(\rho^0)} = 0.66 \quad \tilde{c}_8 > 0$

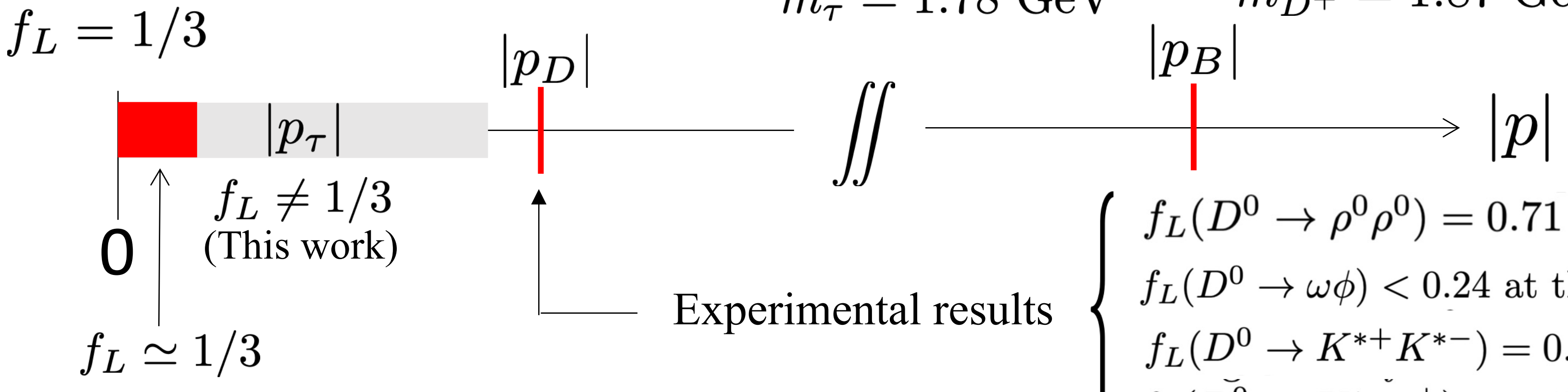
—The results indicate that the approximation $m_\tau \simeq m_{V_1} + m_{V_2}$ does not work adequately.

Comparison of f_L for $\tau \to VV\nu_\tau, D \to VV$, and $B \to VV$ decays

(in the rest frame of VV)

- The size of momentum of a vector meson follows the ordering: $|p_\tau| < |p_D| \ll |p_B|$

$m_\tau = 1.78$ GeV $\quad m_{D^+} = 1.87$ GeV $\quad m_{B^+} = 5.28$ GeV



$$\begin{cases} f_L(D^0 \to \rho^0\rho^0) = 0.71 \pm 0.04 \pm 0.02 & \text{FOCUS} \\ f_L(D^0 \to \omega\phi) < 0.24 \text{ at the 95\% confidence level} & \text{BESIII} \\ f_L(D^0 \to K^{*+}K^{*-}) = 0.468 \pm 0.046 \pm 0.011 & \text{BESIII} \\ f_L(D^0 \to K^{*-}\rho^+) = 0.475 \pm 0.271 & \text{Mark-III (large uncertainty)} \end{cases}$$

- There is no wonder that f_L in D-meson decays is deviated from 1/3.

—$|p_D|$ is more deviated from $|p|=0$ than $|p_\tau|$, while f_L in τ decays is *already* away from 1/3 .

- In this context, experimental verifications for f_L in τ decays are of certain relevance in the future study.

— providing notable information on polarization patterns at the low energy region, $E \sim m_\tau \sim m_D$

Conclusion

- We have studied $\tau \to VV\nu_\tau$ decays, especially focused on the longitudinal polarization fraction (f_L) for vector mesons in the final state.

- The branching fraction estimated by GHLS for $\tau^- \to \rho^-\omega\nu_\tau$ are comparable to one from the chiral effective model while being smaller than that for the angular momentum algebra model.

- The results give deviations from $f_L \simeq 1/3$, indicating that the endpoint approximation ($m_\tau \simeq m_{V_1} + m_{V_2}$) does not work accurately.

- Future measurement at Belle II constitutes a key step toward clarifying the puzzling polarization patterns in $D \to VV$ decays.

Back Up

Expressions for observables

Integrated decay rate: $\Gamma = \frac{G_F^2 |V_{uq}|^2 m_\tau}{192\pi^3} K$

Helicity-summed object

$$\begin{cases} K = \sum_{\lambda_1,\,\lambda_2} K_{\lambda_1\lambda_2} \\ I_{\lambda_1\lambda_2} = I^{(+1/2)}_{\lambda_1\lambda_2} + \frac{m_\tau^2}{Q^2} I^{(-1/2)}_{\lambda_1\lambda_2} \\ K_{\lambda_1\lambda_2} = \int dQ^2 \frac{|\boldsymbol{p}|}{\sqrt{Q^2}} \left(1 - \frac{Q^2}{m_\tau^2}\right)^2 I_{\lambda_1\lambda_2} \end{cases}$$

Longitudinal polarization fractions for

$V_i \ (i = 1, 2) \quad \tau^- \to V_1^- V_2^0 \nu_\tau$

$$\begin{cases} f_L^{(V_1)} = \sum_{\lambda_2} K_{0\lambda_2}/K \\ f_L^{(V_2)} = \sum_{\lambda_1} K_{\lambda_1 0}/K \end{cases}$$

Contribution from $\lambda_{\tau^-} = 1/2$ (tau helicity)

$$\begin{aligned} I^{(-1/2)}_{\lambda_1\lambda_2} &= S^{(++)}_{\lambda_1\lambda_2} + S^{(--)}_{\lambda_1\lambda_2} + S^{(00)}_{\lambda_1\lambda_2} + 3S^{(ss)}_{\lambda_1\lambda_2} \\ &- 2\mathrm{Re}(S^{(+-)}_{\lambda_1\lambda_2}) - \frac{3\sqrt{2}\pi}{4} \mathrm{Re}[S^{(+s)}_{\lambda_1\lambda_2} - S^{(-s)}_{\lambda_1\lambda_2}] \end{aligned}$$

$\lambda_{\tau^-} = -1/2$

$$\begin{aligned} I^{(+1/2)}_{\lambda_1\lambda_2} &= 2\left[S^{(++)}_{\lambda_1\lambda_2} + S^{(--)}_{\lambda_1\lambda_2} + S^{(00)}_{\lambda_1\lambda_2}\right] \\ &+ 2\mathrm{Re}(S^{(+-)}_{\lambda_1\lambda_2}) - \frac{3\sqrt{2}\pi}{4} \mathrm{Re}[S^{(+0)}_{\lambda_1\lambda_2} + S^{(-0)}_{\lambda_1\lambda_2}] \end{aligned}$$

Object quadratic in helicity amplitudes: $S^{(mm')}_{\lambda_1\lambda_2} = H^{(m)}_{\lambda_1\lambda_2} H^{(m')*}_{\lambda_1\lambda_2}$