Theoretical study of tau lepton decaying to two vector/pseduscalar mesons

L. R. Dai

Huzhou Normal University, Zhejiang, China

Workshop on Tau Physics at Belle II and STCF

September 1, 2026, Jilin University, China

湖州师范大学
HUZHOU NORMAL UNIVERSITY

Outline

Part 1 Build the theory in tau decay using the algebra method

arXiv:1805.04573 finding G-parity is important

Part 2 Some predictions in tau decay $\tau \to \nu_\tau VP$

G-parity can filter different reactions

arXiv:1809.02510 polarization

arXiv:1809.11007 scalar mesons

arXiv:1811.06875 arXiv: 2005.02653 axial-vector mesons

We build the theory in tau decay using the algebra method and find that the G-parity plays an important role in these reactions.

1. $\tau \to \nu_\tau M_1 M_2$, with M_1, M_2 pseudoscalar or vector mesons
LRD, Pavao, Sakai, Oset, EPJA55(2019)20 arXiv:1805.04573

2. Polarization amplitudes in $\tau \to \nu_\tau VP$ decay beyond the Standard Model (BSM)
LRD, Oset, EPJA54(2018)219 arXiv:1809.02510

3. Triangle singularity in $\tau \to \nu_\tau \pi f_0(980)$ ($a_0(980)$) decays
LRD, Yu, Oset, PRD99(2019)016021 arXiv:1809.11007

4. τ decay into a pseudoscalar and an axial-vector meson
LRD, Roca, Oset, PRD99(2019)096003 arXiv:1811.06875

5. Tau decay into ν_τ and $a_1(1260)$, $b_1(1235)$, and two $K_1(1270)$
LRD, Roca, Oset, EPJC80(2020)673 arXiv: 2005.02653

$$G = (-1)^{L+S+I}$$

Part 1 Build the theory in tau decay using the algebra method

original motivation

Tau decays have been instrumental to learn about weak interaction as well as strong interaction affecting the hadrons produced on tau hadronic decay. τ decays into ν_τ and a pair of mesons make up for a sizeable fraction of the tau decay width in pdg (updated pdg in Int. J. Mod. Phys. A41 (2026) 2630011) .

One may **wonder** whether there is some fundamental reason for this experimental fact, because the pseudoscalar and vector mesons differ only by the spin arrangement of the quarks, it should be possible to relate the rates of decay for two pseudoscalar mesons and the related pseudoscalar-vector or vector modes, for instance,
$\tau^- \to \nu_\tau K^0 K^-, \nu_\tau K^0 K^{*-}, \nu_\tau K^{*0} K^-, \nu_\tau K^{*0} K^{*-}$.

by looking at the tau decays, we find that

- Several modes are well measured, $\tau \to \nu_\tau PP$ and $\tau \to \nu_\tau PV$.
- Surprisingly, there are no $\tau \to \nu_\tau VV$.

we hope to find why
$\Rightarrow$ a thorough study of all possible Cabibbo-favored and Cabibbo-suppressed reactions and compare with present available data and with results of other theoretical approaches.

1. $\tau \to \nu_\tau M_1 M_2$, with M_1, M_2 pseudoscalar or vector mesons, LRD, Pavao, Sakai, Oset, EPJA55(2019)20, arXiv:1805.04573

meson		J
pseudoscalar	P	1
vector	V	0

algebra method we use the basic weak interaction and angular momentum algebra to relate the different processes.

we find that the *G*-parity plays an important role in these reactions.

Eur. Phys. J. A (2019) **55**: 20
DOI 10.1140/epja/i2019-12690-9

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

$\tau^- \longrightarrow \nu_\tau M_1 M_2$, with M_1, M_2 pseudoscalar or vector mesons

L.R. Dai[1,2,a], R. Pavao[2,b], S. Sakai[2,3,c], and E. Oset[2,d]

1 Department of Physics, Liaoning Normal University, Dalian 116029, China
2 Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Institutos de Investigación de Paterna, Apartado 22085, 46071 Valencia, Spain
3 Institute of Theoretical Physics, CAS, Zhong Guan Cun East Street 55, 100190 Beijing, China

Received: 10 October 2018 / Revised: 3 December 2018
Published online: 15 February 2019

Communicated by Shi-Lin Zhu

Abstract. We perform a calculation of the $\tau^- \to \nu_\tau M_1 M_2$, with M_1, M_2 either pseudoscalar or vector mesons using the basic weak interaction and angular momentum algebra to relate the different processes. The formalism also leads to a different interpretation of the role played by *G*-parity in these decays. We also observe that, while PP p-wave production is compatible with chiral perturbation theory and experiment, VP and VV p-wave production is clearly incompatible with experiment and we develop the formalism also in this case, producing the VP or VV pairs in s-wave. We compare our results with experiment and other theoretical approaches for rates and invariant mass distributions and make predictions for unmeasured decays. We show the value of these reactions, particularly if the $M_1 M_2$ mass distribution is measured, as a tool to learn about the meson-meson interaction and the nature of some resonances, coupling to two mesons, which are produced in such decays.

The G-parity plays an important role in these reactions. we offer a new perspective into this issue arXiv:1805.04573

- charge symmetry

  Steven Weinberg discussed the issue [S. Weinberg, Phys. Rev. 112, 1375 (1958)] and the classification of the weak interaction into first and second class currents.

- G-parity

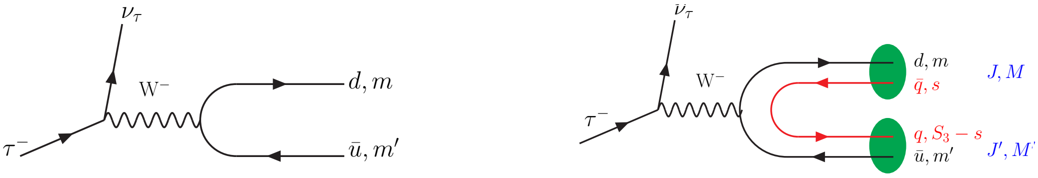
  one interesting reaction is the $\tau \to \nu_\tau \pi \eta(\eta')$, which according to that classification is forbidden by G-parity.

in arXiv:1805.04573
we make a thorough study of all possible Cabibbo-favored and Cabibbo-suppressed reactions and compare with present available data.

- G-parity for the non strange mesons plays an important role and the rules are different for pseudoscalar-pseudoscalar (PP) pseudoscalar-vector (PV) or vector-vector (VV) production.
- an extension of these rules appears also in the strange sector for the $\tau \to \nu_\tau K\eta(\eta')$ and $\nu_\tau K^{*-}\eta(\eta')$ reactions.

arXiv:1805.04573 what we did? establish a relationship

we use the basic dynamics of the weak interaction at the quark level, producing a primary $q\bar{q}$ pair. The hadronization of this pair into two mesons is done using the $^3P_0$ model. $\Rightarrow$ It is possible to relate the rates of decay for $\tau \to \nu_\tau PP, PV, VP, VV$ reactions.



L. Micu, Nucl. Phys. B 10, 521 (1969).
A. Le Yaouanc, L. Oliver, O. P ne, and J. -C. Raynal, Phys. Rev. D 8, 2223 (1973).
F. E. Close, An Introduction to Quark and Partons, Academic Press, 1979.

- The method allows us to correlate many different processes.
- we carry out an elaborate and detailed angular momentum algebra
  $\Rightarrow$ that allows us to write amplitudes analytically in a very simple form.

Algebra method

The derivation requires some patience, but we succeed using Racah Algebra

no any free parameter
relate the different processes

relevant form factors would be the same
the structures can be very different for the produced P or V

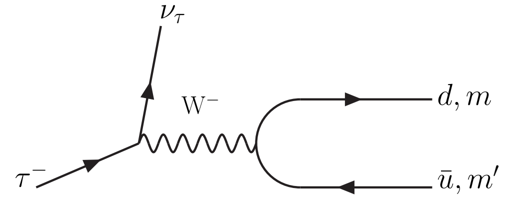
we obtained the analytical amplitudes for each reaction

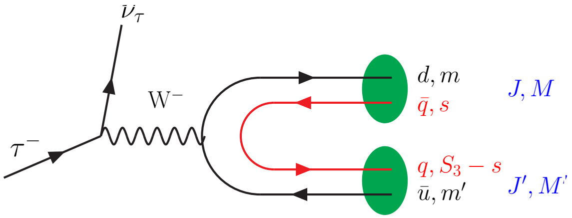
- The evaluation of the invariant mass distributions and branching ratios of rates for PP, PV&VV cases
- good agreement with experimental data
- make some predictions

LRD, Pavao, Sakai, Oset, EPJA55(2019)20 arXiv:1805.04573 $JJ' = 00, 01, 10, 11$

Appendix A. Evaluation of the matrix elements for the operators "1" and σ_i
Appendix B. Evaluation of $\overline{\sum}\sum|t|^2$

The derivation - Cabibbo-favored $d\bar{u}$ production and Hadronization





$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

$$d\bar{u} \to \sum_{i=1}^{3} d\ \bar{q}_i\ q_i\ \bar{u} = M_{2i}\ M_{i1} = (M \cdot M)_{21}$$

$$M \Rightarrow P \text{ or } V$$

$$
P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{6}} \end{pmatrix},
$$

$$
V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_\mu
$$

For the hadronization, we use the $^3P_0$ model, which has been widely used in the literature and recently it has been found very instrumental to address different problems in hadron physics.

$^3P_0$ model

1) L. Micu, NPB10(1969)521
2) A. Le Yaouanc, et. al., PRD8(1973)2223
3) F. E. Close, An Introduction to Quark and Partons, Academic Press, 1979

Cabibbo-favored $d\bar{u}$ productions and hadronization $\Rightarrow$ h_i and $\overline{h}_i$

$$d\bar{u} \to \sum_{i=1}^{3} d\ \bar{q}_i\ q_i\ \bar{u} = M_{2i}\ M_{i1} = (M \cdot M)_{21}$$

$$\begin{aligned}
(P \cdot P)_{21} &= \frac{1}{\sqrt{2}}(\pi^-\pi^0 - \pi^0\pi^-) + \frac{1}{\sqrt{3}}(\pi^-\eta + \eta\pi^-) + \frac{1}{\sqrt{6}}(\pi^-\eta' + \eta'\pi^-) + K^0K^-\,, \\
(P \cdot V)_{21} &= \frac{1}{\sqrt{2}}(\pi^-\rho^0 + \pi^-\omega) - \frac{\pi^0\rho^-}{\sqrt{2}} + \frac{\eta\rho^-}{\sqrt{3}} + \frac{\eta'\rho^-}{\sqrt{6}} + K^0K^{*-}\,, \\
(V \cdot P)_{21} &= \frac{\rho^-\pi^0}{\sqrt{2}} + \frac{\rho^-\eta}{\sqrt{3}} + \frac{\rho^-\eta'}{\sqrt{6}} + \frac{1}{\sqrt{2}}(-\rho^0\pi^- + \omega\pi^-) + K^{*0}K^-\,, \\
(V \cdot V)_{21} &= \frac{1}{\sqrt{2}}(\rho^-\rho^0 - \rho^0\rho^-) + \frac{1}{\sqrt{2}}(\rho^-\omega - \omega\rho^-) + K^{*0}K^{*-}\,.
\end{aligned} \tag{1}$$

Similarly for Cabibbo-suppressed $s\bar{u}$ production and hadronization

$$s\bar{u} \to \sum_{i=1}^{3} s\ \bar{q}_i\ q_i\ \bar{u} = M_{3i}\ M_{i1} = (M \cdot M)_{31}$$

$$\begin{aligned}
(P \cdot P)_{31} &= K^- \frac{\pi^0}{\sqrt{2}} + \bar{K}^0 \pi^- + \left(K^- \frac{\eta}{\sqrt{3}} - \frac{\eta}{\sqrt{3}} K^- \right) + \left(K^- \frac{\eta'}{\sqrt{6}} + \frac{2\eta'}{\sqrt{6}} K^- \right), \\
(P \cdot V)_{31} &= K^- \left(\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \right) + \bar{K}^0 \rho^- + \left(-\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{6}} \right) K^{*-}, \\
(V \cdot P)_{31} &= K^{*-} \left(\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} \right) + \bar{K}^{*0} \pi^- + \phi K^-, \\
(V \cdot V)_{31} &= K^{*-} \left(\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \right) + \bar{K}^{*0} \rho^- + \phi K^{*-}. \qquad (2)
\end{aligned}$$

weak matrix elements in Standard Model (SM)

$$H = \mathcal{C} L^{\mu} Q_{\mu}$$

where $\mathcal{C}$ containing weak interaction constants and radial matrix elements.

L^{μ} is the leptonic current

$$L^{\mu} = \langle \bar{u}_{\nu} | \gamma^{\mu} - \gamma^{\mu}\gamma_5 | u_{\tau} \rangle$$

Q^{μ} is the quark current

$$Q^{\mu} = \langle \bar{u}_d | \gamma^{\mu} - \gamma^{\mu}\gamma_5 | v_{\bar{u}} \rangle$$

In the evaluation of Q_{μ} matrix element we assume that quark spinors are at rest in that frame

$$u_r = \left(\begin{array}{c} \chi_r \\ 0 \end{array} \right), v_r = \left(\begin{array}{c} 0 \\ \chi_r \end{array} \right)$$

The amplitudes

$$Q_0 = \langle \chi' | 1 | \chi \rangle \equiv M_0$$
$$Q_i = \langle \chi' | \sigma_i | \chi \rangle \equiv N_i$$

for simplicity:

$$\overline{L}^{\mu\nu} = \overline{\sum} \sum L^{\mu} L^{\nu\dagger}$$

which can be easily evaluated in [Navarra, Nielsen, Oset, Sekihara, PRD92(2015)014031]

$$\begin{aligned}
\overline{\sum} \sum |t|^2 &= \overline{\sum} \sum L^{\mu} L^{\nu\dagger} Q_{\mu} Q^*_{\nu} \\
&= \overline{L}^{00} M_0 \; M_0^* + \overline{L}^{0i} M_0 \; N_i^* + \overline{L}^{i0} N_i \; M_0^* + \overline{L}^{ij} N_i \; N_j^*
\end{aligned}$$

where we sum over the final polarizations of the mesons produced.

Matrix elements in p-wave for M_0 and N_μ

PP	$J=0, J'=0$	$M_0 = 0$
PV	$J=0, J'=1$	$M_0 = (-1)^{-M-M'}\frac{1}{\sqrt{6}}\, q\, Y_{1,-(M+M')}(\hat{\boldsymbol{q}})\,\delta_{M0}$
VP	$J=1, J'=0$	$M_0 = (-1)^{-M-M'}\frac{1}{\sqrt{6}}\, q\, Y_{1,-(M+M')}(\hat{\boldsymbol{q}})\,\delta_{M'0}$
VV	$J=1, J'=1$	$M_0 = (-1)^{-M-M'}\frac{1}{\sqrt{3}}\mathcal{C}(111; M, M', M+M')$
		$\times\, q\, Y_{1,-(M+M')}(\hat{\boldsymbol{q}})$

PP	$J=0, J'=0$	$N_\mu = \frac{1}{\sqrt{6}}\, q\, Y_{1,\mu}(\hat{\boldsymbol{q}})\,\delta_{M0}\,\delta_{M'0}$
PV	$J=0, J'=1$	$N_\mu = (-1)^{1-M'}\frac{1}{\sqrt{3}}\, q\, Y_{1,\mu-M'}(\hat{\boldsymbol{q}})\mathcal{C}(111; M', -\mu, M'-\mu)\,\delta_{M0}$
VP	$J=1, J'=0$	$N_\mu = (-1)^{-M}\frac{1}{\sqrt{3}}\, q\, Y_{1,\mu-M}(\hat{\boldsymbol{q}})\mathcal{C}(111; M, -\mu, M-\mu)\,\delta_{M'0}$
VV	$J=1, J'=1$	$N_\mu = \frac{1}{\sqrt{6}}\, q\, Y_{1,\mu-M-M'}(\hat{\boldsymbol{q}})\{(-1)^{-M'}\delta_{\mu M} + 2\,(-1)^{-M}$
		$\times\mathcal{C}(111; M, -\mu, M-\mu)\mathcal{C}(111; M', -M-M'+\mu, -M+\mu)\}$

signs change of amplitudes by permuting the order of the mesons

in p-wave production				
	PP	PV	VP	VV
M_0	0	$-$	$-$	$+$
N_μ	$-$	$+$	$+$	$-$

from Cabibbo-favored $d\bar{u}$ productions and hadronization $\Rightarrow$

$$(P \cdot P)_{21} = \frac{1}{\sqrt{2}}(\pi^-\pi^0 - \pi^0\pi^-) + \frac{1}{\sqrt{3}}(\pi^-\eta + \eta\pi^-) + \frac{1}{\sqrt{6}}(\pi^-\eta' + \eta'\pi^-) + K^0 K^-$$

Weights for the different channels and Contributions

Channels	h_i (for M_0)	$\overline{h}_i$ (for N_μ)	M_0	N_μ	G-parity
$\pi^0\pi^-$	0	$\sqrt{2}$	0	$\times$	$+$
$\pi^-\eta$	0	0	0	0	$-$
$\pi^-\eta'$	0	0	0	0	$-$

we offer a different perspective that why $\pi^-\eta(\eta')$ are forbidden by G-parity from algebra method

$$(P\cdot P)_{21} \;=\; \frac{1}{\sqrt{2}}(\pi^-\pi^0-\pi^0\pi^-)+\frac{1}{\sqrt{3}}(\pi^-\eta+\eta\pi^-)+\frac{1}{\sqrt{6}}(\pi^-\eta'+\eta'\pi^-)+K^0K^-$$

- for the $\pi^-\pi^0$ channel

  It comes with the combination $\pi^-\pi^0-\pi^0\pi^-$. As a consequence N_μ adds for the two terms and we have a weight $2\frac{1}{\sqrt{2}}=\sqrt{2}$ for the $\pi^-\pi^0$ channel

- for $\pi^-\eta$ channel

  It comes with the combinations $\pi^-\eta+\eta\pi^-$, and then the combination of the two terms cancels $\Rightarrow$ do not have $\pi^-\eta$ production forbidden by G-parity

Matrix elements in s-wave for M_0 and N_μ and signs change

PP	$J=0, J'=0$	$M_0 = 0$
PV	$J=0, J'=1$	$M_0 = \frac{1}{\sqrt{6}}\frac{1}{4\pi}$
VP	$J=1, J'=0$	$M_0 = \frac{1}{\sqrt{6}}\frac{1}{4\pi}$
VV	$J=1, J'=1$	$M_0 = \frac{1}{\sqrt{3}}\frac{1}{4\pi}\mathcal{C}(111; M, M', M+M')$

PP	$J=0, J'=0$	$N_\mu = \frac{1}{\sqrt{6}}\frac{1}{4\pi}\,\delta_{M0}\,\delta_{M'0}\,(-1)^{-\mu}$
PV	$J=0, J'=1$	$N_\mu = -(-1)^{-\mu}\frac{1}{\sqrt{3}}\frac{1}{4\pi}\mathcal{C}(111; M', -\mu, M'-\mu)\,\delta_{M0}$
VP	$J=1, J'=0$	$N_\mu = (-1)^{-\mu}\frac{1}{\sqrt{3}}\frac{1}{4\pi}\mathcal{C}(111; M, -\mu, M-\mu)\,\delta_{M'0}$
VV	$J=1, J'=1$	$N_\mu = \frac{1}{\sqrt{6}}\frac{1}{4\pi}\Big\{\delta_{M\mu} + 2\,(-1)^{-\mu-M'}\mathcal{C}\,(111; M, -\mu, M-\mu)$ $\times\mathcal{C}\,(111; M', -M-M'+\mu, -M+\mu)\Big\}$

signs change of amplitudes by permuting the order of the mesons in s-wave production

	PP	PV	VP	VV
M_0	0	$+$	$+$	$-$
N_μ	$+$	$-$	$-$	$+$

The analytical amplitudes in s-wave production

1) $PP(J=0, J'=0)$

$$\overline{\sum}\sum|t|^2 = \frac{1}{m_\tau m_\nu}\left(\frac{1}{4\pi}\right)^2\left(E_\tau E_\nu - \frac{\boldsymbol{p}^2}{3}\right)\frac{1}{2}\overline{h}_i^2 \tag{2-a}$$

2) $PV(J=0, J'=1)$; $VP(J=1, J'=0)$

$$\overline{\sum}\sum|t|^2 = \frac{1}{m_\tau m_\nu}\left(\frac{1}{4\pi}\right)^2\left[\left(E_\tau E_\nu + \boldsymbol{p}^2\right)\frac{1}{2}h_i^2 + \left(E_\tau E_\nu - \frac{\boldsymbol{p}^2}{3}\right)\overline{h}_i^2\right] \tag{2-b}$$

3) $VV(J=1, J'=1)$

$$\overline{\sum}\sum|t|^2 = \frac{1}{m_\tau m_\nu}\left(\frac{1}{4\pi}\right)^2\left[\left(E_\tau E_\nu + \boldsymbol{p}^2\right)h_i^2 + \frac{7}{2}\left(E_\tau E_\nu - \frac{\boldsymbol{p}^2}{3}\right)\overline{h}_i^2\right] \tag{2-c}$$

Similarly, we also obtain the analytical amplitudes in p-wave production.

The final differential mass distribution

$$\frac{d\Gamma}{dM_{\text{inv}}(M_1 M_2)} = \frac{2\, m_\tau 2\, m_\nu}{(2\pi)^3} \frac{1}{4m_\tau^2}\, p_\nu \tilde{p}_1 \overline{\sum} \sum |t|^2 \tag{3}$$

where p_ν is the neutrino momentum in the tau rest frame, and $\tilde{p}_1$ the momentum of M_1 in the M_1, M_2 rest frame.

$$p_\nu = \frac{\lambda^{1/2}(m_\tau^2, m_\nu^2, M_{\text{inv}}^2(M_1 M_2))}{2M_\tau}\,, \qquad \tilde{p}_1 = \frac{\lambda^{1/2}(M_{\text{inv}}^2(M_1 M_2), m_{M_1}^2, m_{M_2}^2)}{2M_{\text{inv}}(M_1 M_2)}$$

Then by integrating Eq. (3) over the $M_1 M_2$ invariant mass, we obtain the width.

h_i and $\overline{h}_i$ coefficients in s-wave production

channels	h_i (for M_0)	$\overline{h}_i$ (for N_μ)	
$\pi^-\rho^0$	0	$\sqrt{2}$	
$\pi^-\omega$	$\sqrt{2}$	0	
$\pi^0\rho^-$	0	$-\sqrt{2}$	
$\eta\rho^-$	$\frac{2}{\sqrt{3}}$	0	
$\eta'\rho^-$	$\frac{2}{\sqrt{6}}$	0	
$K^{*0}K^-$	1	1	Scalar resonances
K^0K^{*-}	1	1	
$\rho^-\omega$	0	$\sqrt{2}$	
$K^{*0}K^{*-}$	1	1	Axial-vector resonances
$\rho^-\rho^0$	$\sqrt{2}$	0	
ηK^{*-}	0	$-\frac{2}{\sqrt{3}}\tan\theta_c$	
$\eta' K^{*-}$	$\frac{3}{\sqrt{6}}\tan\theta_c$	$\frac{1}{\sqrt{6}}\tan\theta_c$	

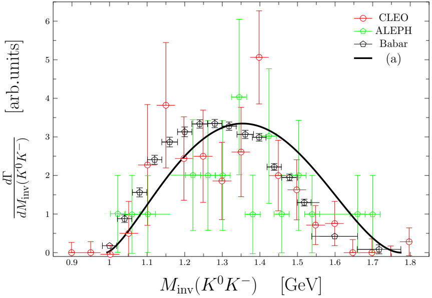
Comparisons with experiments for rates and invariant mass distributions

We find (many tables for decay rates in the paper [EPJA55,20], we do not list here)

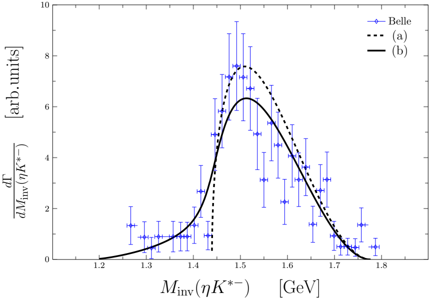
a) PP case, p-wave production

b) PV and VV cases, s-wave production

PP case



PV case



our predictions are in line with the results of other theoretical approaches

for examples:

Kühn, Santamaria, ZPC48(1990)445;

Barish, troynowski, Phys Rep 157(1988) 1 ;

Li, PRD52(1995)5165; Li, PRD52(1995)5184 (vector meson dominance)

Volkov, Kostunin, Phys Part Nucl Lett 10 (2013)7

detailed discussions in section 7 (comparison with other approaches):
LRD, Pavao, Sakai, Oset, EPJA55(2019)20

Part 2: some predictions in tau decay

a) Polarization amplitudes $\tau \to \nu_\tau V P$

LRD, Oset, EPJA54(2018)219 arXiv:1809.02510

- $\tau \to \nu_\tau K^{*0} K^-$
- project over spin components

M, M' are the third components of the K^{*0} and K^-, respectively

K^{*0}	$J = 1$	$M = 0, \pm 1$
K^-	$J' = 0$	$M' = 0$

The quantization axis is taken along the direction of the neutrino in the tau rest frame.

we obtain the amplitude for each M component and differential width

$$\frac{d\Gamma}{dM_{\text{inv}}^{(K^{*0}K^-)}} = \frac{2\,m_\tau 2\,m_\nu}{(2\pi)^3}\frac{1}{4m_\tau^2}\,p_\nu \widetilde{p}_1 \overline{\sum}\sum|t|^2$$

where p_ν momentum of neutrino in tau rest frame, $\widetilde{p}_1$ of K^{*0} in the $K^{*0}K^-$ rest frame.

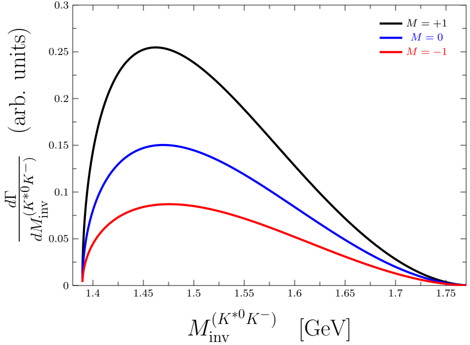
$$R = \frac{d\Gamma}{dM_{\text{inv}}^{(K^{*0}K^-)}}\Big|_{M=+1} + \frac{d\Gamma}{dM_{\text{inv}}^{(K^{*0}K^-)}}\Big|_{M=0} + \frac{d\Gamma}{dM_{\text{inv}}^{(K^{*0}K^-)}}\Big|_{M=-1}$$

we obtain the ratio for difference of $M = +1$ and $M = -1$

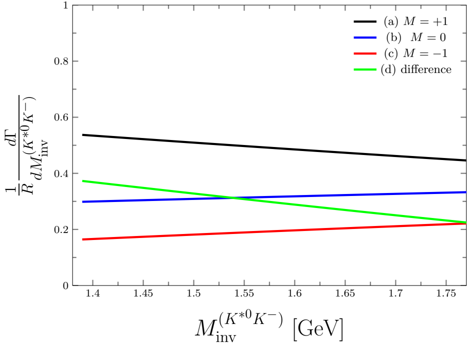
$$\frac{1}{R}\left(\frac{d\Gamma}{dM_{\text{inv}}^{(K^{*0}K^-)}}\Big|_{M=+1} - \frac{d\Gamma}{dM_{\text{inv}}^{(K^{*0}K^-)}}\Big|_{M=+1}\right)$$

we propose to measure the difference

The differential width for each different M



ratio for difference



a big sensitivity of magnitude

$\Rightarrow$ we propose to measure the difference

Extension to the consideration of right-handed quark currents and new differential widths (BSM)

$$a(\gamma^\mu - \gamma^\mu\gamma_5) + b(\gamma^\mu + \gamma^\mu\gamma_5) = \gamma^\mu - \alpha\gamma^\mu\gamma_5 \,.$$

we will study the distributions for different M' as a function of α.

1) $M = 0$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\frac{1}{6}\frac{1}{(4\pi)^2}\left\{\left(E_\tau E_\nu + p^2\right) + 2\alpha^2\left(E_\tau E_\nu - p^2\right)\right\}$$

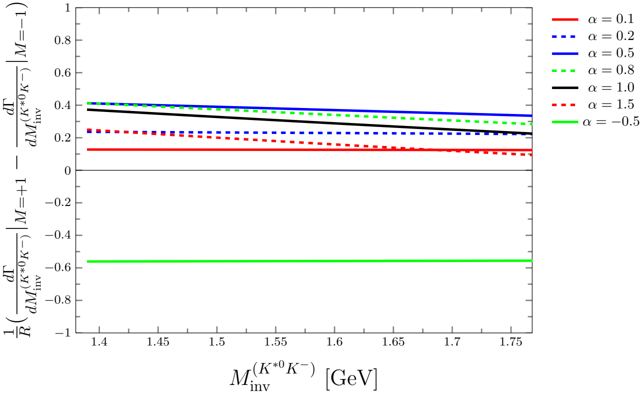
2) $M = 1$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\frac{1}{6}\frac{1}{(4\pi)^2}\left\{(E_\tau E_\nu + p^2) + 2\alpha(E_\nu + E_\tau)p + \left[2E_\tau E_\nu + (E_\nu - E_\tau)p\right]\alpha^2\right\}$$

3) $M = -1$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\frac{1}{6}\frac{1}{(4\pi)^2}\left\{(E_\tau E_\nu + p^2) - 2\alpha(E_\nu + E_\tau)p + \left[2E_\tau E_\nu - (E_\nu - E_\tau)p\right]\alpha^2\right\}$$

suggest to measure the difference experimentally



It is seen that the difference of M=+1 and M=-1 contributions depend strongly on different α

$\Rightarrow$ a big sensitivity of magnitude

This magnitude should be easy to differentiate experimentally.

b) G-parity can filter different reactions for scalar meson

Triangle singularity in $\tau \to \nu_\tau \pi f_0(980)$ ($a_0(980)$) decays

LRD, Yu, Oset, PRD99(2019)016021 arXiv:1809.11007

$\tau \to \nu_\tau K^{*0} K^-$ for $J=1, J'=0$ case

the experimental branching ratio $\mathcal{B}(\tau \to \nu_\tau K^{*0} K^-) = (2.1 \pm 0.4) \times 10^{-3}$

Signs resulting in the amplitudes by permuting the order of the mesons in s-wave production

	PP	PV	VP	VV
M_0	$-$	$+$	$+$	$-$
N_μ	$+$	$-$	$-$	$+$

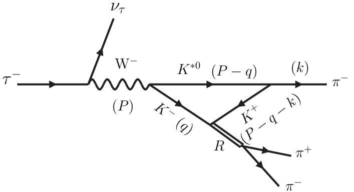
G-Parity

π	$f_0(980)$	$a_0(980)$
$-$	$+$	$-$

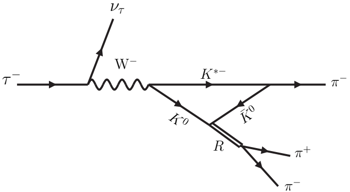
- while M_0 is the same for VP and PV productions, N_i changes sign which is essential for the conservation of G-parity in the reaction.
- there is no simultaneous contribution of the two terms in these reactions
  $\pi^- f_0(980)$ will proceed with the N_i amplitude while $\pi^- a_0(980)$ proceeds with the M_0 term

Diagram for the $\tau \to \nu_\tau K^{*0} K^-$ decay

$\tau \to \nu_\tau \pi^- \pi^+ \pi^-$ for $f_0(980)$

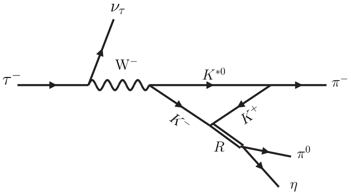


(a)

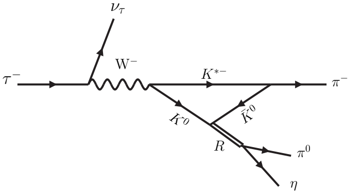


(b)

$\tau \to \nu_\tau \pi^- \pi^0 \eta$ for $a_0(980)$



(a)



(b)

hugeExplicit filter of G-parity states

For the production of $\pi^- f_0(980)$ $\Rightarrow$ negative G-parity

$$\overline{\sum}\sum |t|^2 = \bar{L}^{ij}\widetilde{N}_i\ \widetilde{N}_j^*\, g^2\,|\,2\,t_{K^+K^-,\pi^+\pi^-}|^2$$
$$= \frac{\mathcal{C}^2}{m_\tau m_\nu}\left(E_\tau E_\nu - \frac{1}{3}p^2\right)\frac{1}{3}\frac{1}{(4\pi)^2}\,k^2|t_L|^2\,g^2\,|\,2\,t_{K^+K^-,\pi^+\pi^-}|^2$$

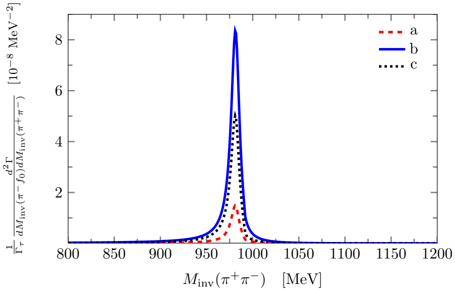
For the production of $\pi^- a_0(980)$ $\Rightarrow$ positive G-parity

$$\overline{\sum}\sum |t|^2 = \bar{L}^{00}\widetilde{M}_0\ \widetilde{M}_0^*\, g^2\,|\,2\,t_{K^+K^-,\pi^0\eta}|^2$$
$$= \frac{\mathcal{C}^2}{m_\tau m_\nu}\left(E_\tau E_\nu + p^2\right)\frac{1}{6}\frac{1}{(4\pi)^2}\,k^2|t_L|^2\,g^2\,|\,2\,t_{K^+K^-,\pi^0\eta}|^2$$

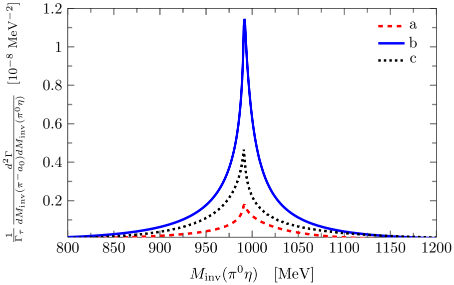
double differential mass distribution as a function of $M_{\mathrm{inv}}(R)$

Chiral Unitary Approach + Triangle Singularity

$\tau \to \nu_\tau \pi^- \pi^+ \pi^-$ for $f_0(980)$



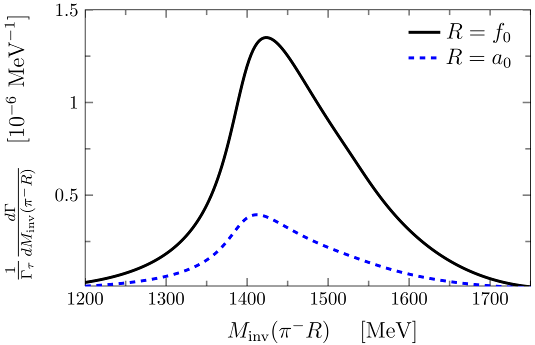
$\tau \to \nu_\tau \pi^- \pi^0 \eta$ for $a_0(980)$



$M_{\mathrm{inv}}(\pi^- R)$ at 1317 MeV (Line a), 1417 MeV (Line b), and 1517 MeV (Line c)

- The distribution with largest strength is near $M_{\mathrm{inv}}(\pi^- R)$=1417 MeV
- A strong peak in the $\pi^+\pi^-$ mass distribution around 980 MeV corresponding to the $f_0(980)$
- The distinctive cusp in the $\pi^0\eta$ mass distribution around 990 MeV corresponding to the $a_0(980)$

branching fractions by integrating $\frac{d\Gamma}{dM_{\text{inv}}(\pi^- R)}$ over $M_{\text{inv}}(\pi^- R)$



$$\mathcal{B}(\tau^- \to \nu_\tau \pi^- f_0(980);\, f_0(980) \to \pi^+\pi^-) = (2.6 \pm 0.5) \times 10^{-4}$$

$$\mathcal{B}(\tau^- \to \nu_\tau \pi^- a_0(980);\, a_0(980) \to \pi^0\eta) = (7.1 \pm 1.4) \times 10^{-5}$$

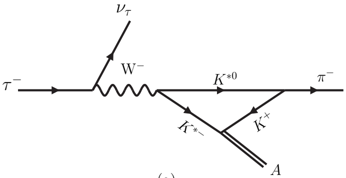
It is found that these numbers are within measurable range in the tau decay.

c) G-parity can filter different reactions for axial-vector meson

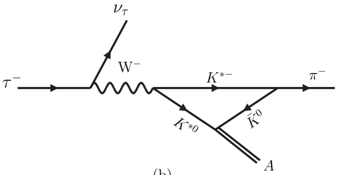
LRD, Roca, Oset, PRD99(2019)096003 arXiv:1811.06875

Diagrams for the decay of $\tau^- \to \nu_\tau \pi^- A$ (A axial vectors)

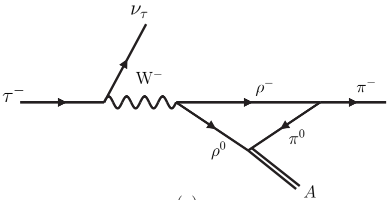
G-Parity		
π	$f_1(1285)$	$b_1(1235)$
$-$	$+$	$+$
$h_1(1170)$	$h_1(1385)$	$a_1(1260)$
$-$	$-$	$-$



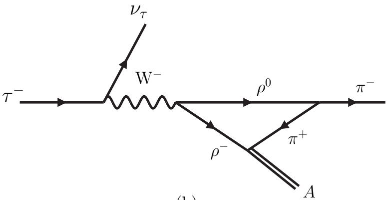
(a)



(b)



(a)



(b)

Explicit filter of G-parity states

	G-Parity	
π	$f_1(1285)$	$b_1(1235)$
$-$	$+$	$+$
$h_1(1170)$	$h_1(1385)$	$a_1(1260)$
$-$	$-$	$-$

There is no simultaneous contribution in these reactions

$\pi^- f_1(1285)$ and $\pi^- b_1(1235)$ will proceed with the N_i amplitude.

$\pi^- h_1(1170)$, $\pi^- h_1(1380)$ and $\pi^- a_1(1260)$ proceed with the M_0 term.

- for G-parity negative axial states

$$\overline{\sum}\sum |t|^2 = \frac{\mathcal{C}^2}{m_\tau m_\nu}\frac{1}{(4\pi)^2}\frac{1}{3}\left(E_\tau E_\nu + p^2\right) g^2\, k^2\, |(-1) g_{A,K^*\bar{K}}\, t_L(K^*\bar{K}^*) - 2\,D(-1) g_{A,\rho\pi}\, t_L(\rho\rho)|^2$$

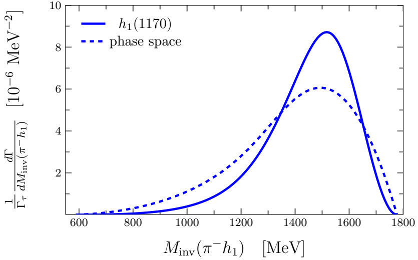
- for G-parity positive axial states

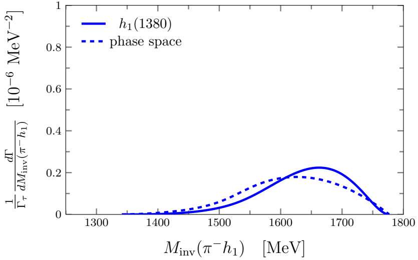
$$\overline{\sum}\sum |t|^2 = \frac{\mathcal{C}^2}{m_\tau m_\nu}\frac{1}{(4\pi)^2}\frac{7}{6}(E_\tau E_\nu - \frac{1}{3}p^2) g^2 k^2 |g_{A,K^*\bar{K}}|^2\, |t_L(K^*\bar{K}^*)|^2$$

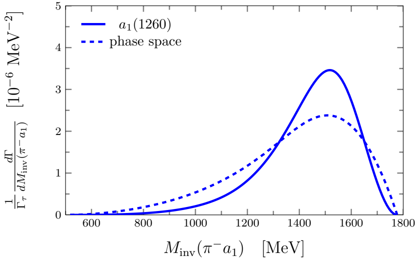
- (only input) from the experimental branching ratio to obtain $\frac{\mathcal{C}^2}{\Gamma_\tau}$

$$\mathcal{B}(\tau \to \nu_\tau K^{*0} K^{*-}) = \frac{1}{\Gamma_\tau}\Gamma(\tau \to \nu_\tau K^{*0} K^{*-}) = (2.1 \pm 0.5)\times 10^{-3} \;\Rightarrow\; \frac{\mathcal{C}^2}{\Gamma_\tau} = (5.0)\times 10^{-4}\,\text{MeV}^{-1}.$$

The invariant mass distributions and branching ratios for positive G-parity



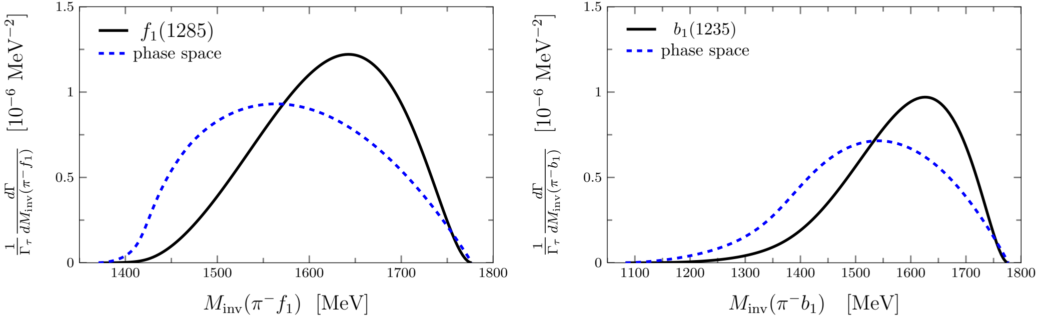




A	$\mathcal{B}(\tau \to \nu_\tau \pi A)$
$h_1(1170)$	3.1×10^{-3}
$a_1(1260)$	1.3×10^{-3}
$b_1(1235)$	2.4×10^{-4}

These numbers are within measurable range.

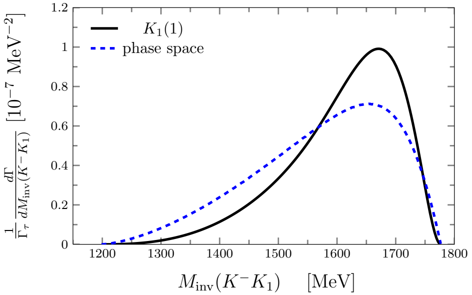
The invariant mass distributions and branching ratios for negative G-parity

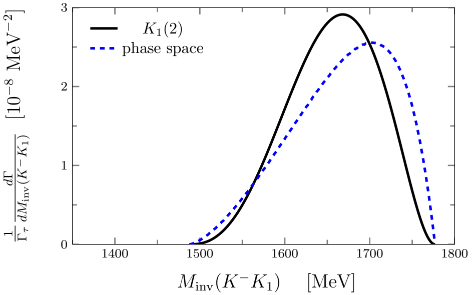


A	$\mathcal{B}(\tau \to \nu_\tau \pi A)$
$f_1(1285)$	2.4×10^{-4}
$h_1(1380)$	3.8×10^{-5}

These numbers are within measurable range.

The invariant mass distributions and branching ratios for two $K_1(1270)$ states





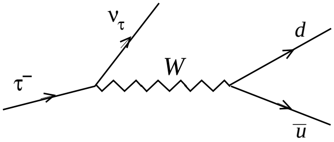
	$\mathcal{B}(\tau^- \to \nu_\tau K^- K_1)$
$K_1(1)$	2.1×10^{-5}
$K_1(2)$	4.1×10^{-6}

These numbers are within measurable range.

d) Tau decay into ν_τ and $a_1(1260)$, $b_1(1235)$, and two $K_1(1270)$

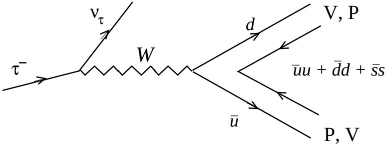
LRD, Roca, Oset, EPJC80(2020)673 arXiv:2005.02653

Cabibbo-favored τ decay to quark-antiquark



$$d\bar{u} \to \sum_{i=1}^{3} d\bar{q}_i q_i \bar{u} = \sum_{i=1}^{3} M_{2i} M_{i1} = (M^2)_{21}$$

Hadronization of the primary $q\bar{q}$ pair (with the quantum numbers of the vacuum) to produce a vector and pseudoscalar meson.



$$(P \cdot V)_{21} = \pi^- \left(\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \right) + \left(-\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} \right) \rho^- + K^0 K^{*-} \,,$$

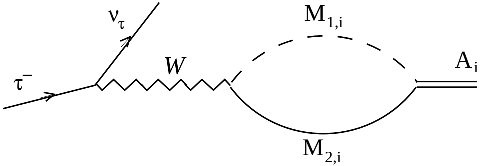
$$(V \cdot P)_{21} = \rho^- \left(\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} \right) + \left(-\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \right) \pi^- + K^{*0} K^- \,.$$

Cabibbo-suppressed: proceed analogously but the hadronization of $s\bar{u}$ gives rise to the matrix element $(M^2)_{31}$ two $K_1(1270)$ with $I = \frac{1}{2}$

$$(P \cdot V)_{31} = K^- \left(\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \right) + \bar{K}^0 \rho^- + \left(-\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{6}} \right) K^{*-} ,$$

$$(V \cdot P)_{31} = K^{*-} \left(\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} \right) + \bar{K}^{*0} \pi^- + \phi K^- .$$

final state interaction



Mechanism for the production of the dynamically generated axial-vector resonance A_i through the coupled channels $M_{1,i}$, $M_{2,i}$, of pseudoscalar and vector mesons

axial-vector resonance A_i in s-wave in the chiral unitary approach

Roca, Oset, Singh, PRD 72(2005)014002; Geng, Oset, Roca, Oller, PRD75(2007)014017

$\tau \to \nu_\tau a_1(1260)$ as an example

Table: Weights h'_i ($\bar{h}'_i$) of the different $I=1$, $G=-1$, VP components for the M_0 (N_i) amplitudes.

	$\rho^0\pi^-$	$\rho^-\pi^0$	$K^{*0}K^-$	$K^{*-}K^0$
h'_i	0	0	1	1
$\bar{h}'_i$	$\sqrt{2}$	$-\sqrt{2}$	-1	1

Table: Couplings of the $I=1$, $G=-1$, axial-vector resonance $a_1(1260)$ to the different VP channels

	$\rho^0\pi^-$	$\rho^-\pi^0$	$K^{*0}K^-$	$K^{*-}K^0$
g_i	$\frac{1}{\sqrt{2}}g_{a_1,\rho\pi}$	$-\frac{1}{\sqrt{2}}g_{a_1,\rho\pi}$	$\frac{1}{\sqrt{2}}g_{a_1,K^*\bar{K}}$	$-\frac{1}{\sqrt{2}}g_{a_1,K^*\bar{K}}$

The amplitude LRD, Pavao, Sakai, Oset, EPJA55(2019)20 for the (final state interaction) mechanism is readily obtained simply by substituting

$$\begin{aligned} h'_j \to h''_j &= \sum_i h'_i G_i(M_{A_j}) g_{A_j,i} \\ \bar{h}'_j \to \bar{h}''_j &= \sum_i \bar{h}'_i G_i(M_{A_j}) g_{A_j,i} \end{aligned}$$

here we use $a_1(1260)$ for A_j and G_i the loop function.

Results of $\Gamma(\tau \to \nu A)$

$$\Gamma(\tau \to \nu A) = \frac{2\, m_\tau 2\, m_\nu}{8\pi} \frac{1}{m_\tau^2}\, p_\nu \overline{\sum} \sum |t|^2$$

Table: Branching ratios for creation of the different axial-vector resonances.

mode	Decay channel	$\mathcal{B}$ (%)
Cabibbo-favored	$\tau^- \to \nu_\tau a_1(1260)$	18 ± 7 (exp)
	$\tau^- \to \nu_\tau b_1(1235)$	10 ± 4
Cabibbo-suppressed	$\tau^- \to \nu_\tau K_1(A)$	0.63 ± 0.25
	$\tau^- \to \nu_\tau K_1(B$	0.65 ± 0.26

for Cabibbo-favored mode, we find that the unmeasured rates for $b_1(1235)$ production are similar to those of the $a_1(1260)$.

for Cabibbo-suppressed mode, two $K_1(1270)$ states are produced with smaller rates, since they are Cabibbo suppressed by a factor about $\tan\theta_c^2 \simeq 1/20$, but they have very distinct decay modes, which we propose to differentiate.

Suggestions for the two K_1 states

So far experiments look at the $\bar{K}\pi\pi$ invariant mass distribution, which contains both $K^*\pi$ and ρK.

$$\Gamma(\tau \to \nu_\tau A \to \nu_\tau VP) = \Gamma(\tau \to \nu_\tau A)\frac{\Gamma(A \to VP)}{\Gamma_A}, \qquad \Gamma(A \to VP) = \frac{|g_{A,VP}|^2}{8\pi M_A^2} q$$

Table: Branching ratios (in %) for the $\Gamma(\tau \to \nu_\tau A \to \nu_\tau VP)$ process for the $I = 1/2$ intermediate axial-vector resonances. The results have an uncertainty of 40%.

Decay channel	$K^{*-}\pi^0$	$\bar{K}^{*0}\pi^-$	$\rho^0 K^-$	$\rho^- \bar{K}^0$	ωK^-	ϕK^-	$K^{*-}\eta$
$\tau^- \to \nu_\tau K_1(1) \to \nu_\tau VP$	0.12	0.23	0.005	0.010	0.007	0	0
$\tau^- \to \nu_\tau K_1(2) \to \nu_\tau VP$	0.019	0.037	0.085	0.17	0.012	0	0.007

we suggest to measure: (the channels to which this resonance couple most strongly)

- the $K^*\pi$ decay mode to see the $K_1(1)$ state
- the ρK decay mode to see the $K_1(2)$ state

These measurements should shed light on the existence of these two K_1 states.

Summary

Part 1 Build the theory in tau decay using algebra method

relate $\Rightarrow$ only one parameter decided by experiment

offer a different perspective to explain why $\pi^-\eta(\eta')$ are forbidden by G-parity

G-parity plays an important role in these reactions

Part 2 Some predictions in tau decay

arXiv:1809.02510 Polarization amplitudes and test BSM $\tau \to \nu_\tau VP$

G-parity can filter of different *G*-parity states $\tau \to \nu_\tau VP$

arXiv:1809.11007 scalar mesons
arXiv:1811.06875 arXiv: 2005.0265 axial-vector mesons

We predict the branching ratios and these numbers are within measurable range.

Thanks!