Michel parameters of τ at Belle/Belle II

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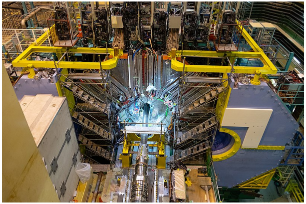
6. Summary

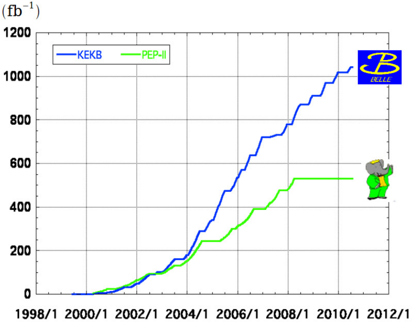


Introduction: τ physics

- In the SM τ decays due to the charged weak interaction described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions. There are two main classes of tau decays:
  - Decays with leptons, like: $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$, $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma$, $\tau^- \to \ell^- \ell'^+ \ell'^- \bar{\nu}_\ell \nu_\tau$; $\ell, \ell' = e, \mu$. They provide very clean laboratory to probe electroweak couplings, which is complementary/competitive to precision studies with muon (in experiments with muon beam). Plenty of New Physics models can be tested/constrained in the precision studies of the dynamics of decays with leptons.
  - Hadronic decays of τ offer unique tools for the precision study of low energy QCD.
- The world largest statistics of τ leptons collected by e^+e^- B factories (Belle II, Belle and *BABAR*) opens new era in the precision tests of the SM.

Introduction: Belle and *BABAR*

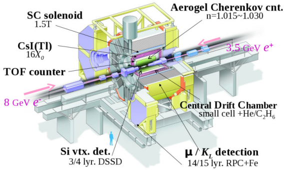
Integrated luminosity of B factories



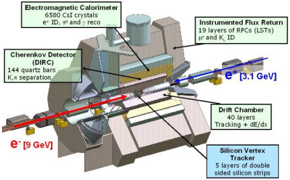
> 1 ab^{-1}
On resonance:
$\Upsilon(5S)$: 121 fb^{-1}
$\Upsilon(4S)$: 711 fb^{-1}
$\Upsilon(3S)$: 3 fb^{-1}
$\Upsilon(2S)$: 25 fb^{-1}
$\Upsilon(1S)$: 6 fb^{-1}
Off reson./scan:
~ 100 fb^{-1}

~ 550 fb^{-1}
On resonance:
$\Upsilon(4S)$: 433 fb^{-1}
$\Upsilon(3S)$: 30 fb^{-1}
$\Upsilon(2S)$: 14 fb^{-1}
Off resonance:
~ 54 fb^{-1}

Belle Detector



BABAR detector

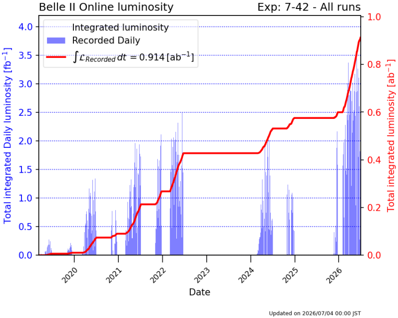


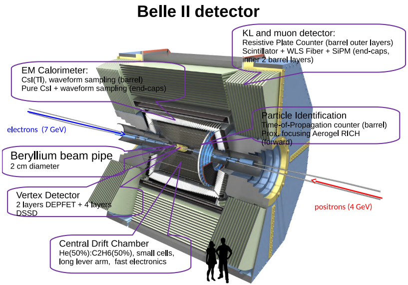
$$\sigma(b\bar{b}) = 1.05 \text{ nb} \quad N_{b\bar{b}} = 1.2 \times 10^9$$
$$\sigma(c\bar{c}) = 1.30 \text{ nb} \quad N_{c\bar{c}} = 2.0 \times 10^9$$
$$\sigma(\tau\tau) = 0.92 \text{ nb} \quad N_{\tau\tau} = 1.4 \times 10^9$$

B-factories are also charm- and τ-factories !

Introduction: Belle II

Next generation e^+e^- B-factory at the HEP intensity frontier





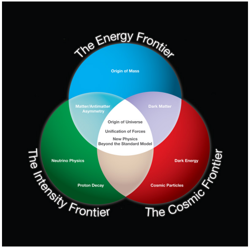
SuperKEKB luminosity record (June 2026):
5.3×10^{34} cm^{-2}s^{-1}

The project SuperKEKB luminosity: 6×10^{35} cm^{-2}s^{-1}
Planned integrated luminosity is 50 ab^{-1}

$\sigma(b\bar{b}) = 1.05$ **nb** $\quad N_{b\bar{b}} = 53 \times 10^9$

$\sigma(c\bar{c}) = 1.30$ **nb** $\quad N_{c\bar{c}} = 65 \times 10^9$

$\boldsymbol{\sigma(\tau\tau) = 0.92}$ **nb** $\quad \boldsymbol{N_{\tau\tau} = 46 \times 10^9}$



Michel parameters in τ decays

In the SM, charged weak interaction is described by the exchange of $W^\pm$ with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics. $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ ($\ell = e, \mu$) decays provide clean laboratory to probe electroweak couplings.
The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g^N_{ij} \Big[\bar{u}_i(l^-)\Gamma^N v_n(\bar{\nu}_l)\Big] \Big[\bar{u}_m(\nu_\tau)\Gamma_N u_j(\tau^-)\Big],$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu),\ \text{chiralities } n,\ m \text{ are fixed by } i,\ j$$

Ten couplings g^N_{ij}, in the SM the only non-zero constant is $g^V_{LL} = 1$

Four bilinear combinations of g^N_{ij}, which are called as Michel parameters (MP): $\boldsymbol{\rho}$, $\boldsymbol{\eta}$, $\boldsymbol{\xi}$ and $\boldsymbol{\delta}$ appear in the energy spectrum of the outgoing lepton:

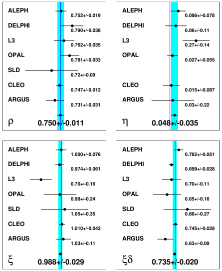
$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2 M_\tau E^4_{\rm max}}{(2\pi)^4}\sqrt{x^2 - x_0^2}\Bigg(x(1-x) + \frac{2}{9}\boldsymbol{\rho}(4x^2 - 3x - x_0^2) + \boldsymbol{\eta} x_0(1-x)$$

$$\mp\frac{1}{3}P_\tau cos\theta_\ell \boldsymbol{\xi}\sqrt{x^2 - x_0^2}\Big[1 - x + \frac{2}{3}\boldsymbol{\delta}\Big(4x - 4 + \sqrt{1 - x_0^2}\Big)\Big]\Bigg),\ x = \frac{E_\ell}{E_{\rm max}},\ x_0 = \frac{m_\ell}{E_{\rm max}}$$

In the SM: $\boldsymbol{\rho} = \frac{3}{4}$, $\boldsymbol{\eta} = 0$, $\boldsymbol{\xi} = 1$, $\boldsymbol{\delta} = \frac{3}{4}$

Michel parameters of τ, current status, NP

Michel par.	Measured value	Experiment	SM value
ρ (e or μ)	$0.747 \pm 0.010 \pm 0.006$ 1.2%	CLEO-97	3/4
η (e or μ)	$0.012 \pm 0.026 \pm 0.004$ 2.6%	ALEPH-01	0
ξ (e or μ)	$1.007 \pm 0.040 \pm 0.015$ 4.3%	CLEO-97	1
$\xi\delta$ (e or μ)	$0.745 \pm 0.026 \pm 0.009$ 2.8%	CLEO-97	3/4
$\xi_{\rm h}$ (all hadr.)	$0.992 \pm 0.007 \pm 0.008$ 1.1%	ALEPH-01	1



In BSM models the couplings to τ are expected to be enhanced in comparison with μ.

- **Type II 2HDM:** $\eta_\mu(\tau) = \frac{m_\mu M_\tau}{2}\left(\frac{tan^2\beta}{M^2_{H^\pm}}\right)^2$; $\frac{\eta_\mu(\tau)}{\eta_e(\mu)} = \frac{M_\tau}{m_e} \approx 3500$
- **Tensor interaction:** $\mathcal{L} = \frac{g}{2\sqrt{2}} W^\mu \left\{ \bar{\nu}\gamma_\mu(1-\gamma^5)\tau + \frac{\kappa^W_\tau}{2m_\tau}\partial^\nu \left(\bar{\nu}\sigma_{\mu\nu}(1-\gamma^5)\tau\right)\right\}$,
  $-0.096 < \kappa^W_\tau < 0.037$: DELPHI Abreu EPJ C16 (2000) 229.
- **Unparticles:** Moyotl PRD 84 (2011) 073010, Choudhury PLB 658 (2008) 148.
- **Lorentz and CPTV:** Hollenberg PLB **701** (2011) 89
- **Heavy Majorana neutrino:** M. Doi *et al.*, Prog. Theor. Phys. **118** (2007) 1069.

Effect of the e^- beam polarization at the STCF

At the STCF with polarized electron beam the average polarization of single τ is nonzero, hence the differential decay probability will contain both, τ spin-dependent and spin-independent parts.

$$\frac{d\sigma(\vec{\zeta}^-,\vec{\zeta}^+)}{d\Omega_\tau} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i^-\zeta_j^+ + \mathcal{P}_e(F_i^-\zeta_i^- + F_j^+\zeta_j^+))$$

$$D_0 = 1+\cos^2\theta + \frac{1}{\gamma_\tau^2}\sin^2\theta,\ \mathcal{P}_e = \frac{N_e(+)-N_e(-)}{N_e(+)+N_e(-)}$$

$$D_{ij} = \begin{pmatrix} (1+\frac{1}{\gamma_\tau^2})\sin^2\theta & 0 & \frac{1}{\gamma_\tau}\sin 2\theta \\ 0 & -\beta_\tau^2\sin^2\theta & 0 \\ \frac{1}{\gamma_\tau}\sin 2\theta & 0 & 1+\cos^2\theta - \frac{1}{\gamma_\tau^2}\sin^2\theta \end{pmatrix}$$

Single τ studies at the STCF:

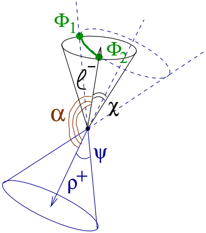
$$\frac{d\sigma(\vec{\zeta}^-)}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2}\beta_\tau(D_0 + \mathcal{P}_e F_i^-\zeta_i^-)$$

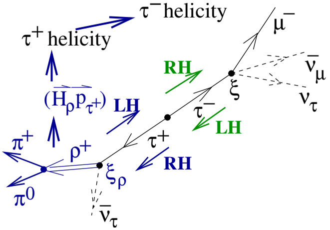
As a result, there are two methods to measure MP:

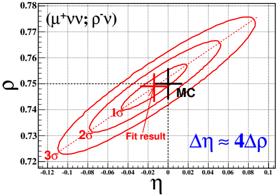
- **(I) Unbinned fit of the $(\ell,\ \rho)$ events in 9D phase space (spin-spin correlations + polarized e^- beam)**
- **(II) Unbinned fit of the $(\ell,\ \mathrm{all})$ events in 3D lepton phase space (only polarized e^- beam)**

Method at e^+e^- factory with unpolarized beams

Effect of τ spin-spin correlation is used to measure ξ and δ MP.
Events of the $(\tau^\mp \to \ell^\mp \nu\nu;\, \tau^\pm \to \rho^\pm \nu)$ topology are used to measure: ρ, η, $\xi_\rho \xi$ and $\xi_\rho \xi \delta$, while $(\tau^\mp \to \rho^\mp \nu;\, \tau^\pm \to \rho^\pm \nu)$ events are used to extract ξ_ρ^2.







$$\frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dE^*_\ell\, d\Omega^*_\ell\, d\Omega^*_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi\, d\Omega_\tau} = A_0 + \rho A_1 + \eta A_2 + \xi_\rho \xi A_3 + \xi_\rho \xi \delta A_4 = \sum_{i=0}^{4} A_i \Theta_i$$

$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dp_\ell\, d\Omega_\ell\, dp_\rho\, d\Omega_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dE^*_\ell\, d\Omega^*_\ell\, d\Omega^*_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi\, d\Omega_\tau} \left| \frac{\partial(E^*_\ell, \Omega^*_\ell, \Omega^*_\rho, \Omega_\tau)}{\partial(p_\ell, \Omega_\ell, p_\rho, \Omega_\rho, \Phi_\tau)} \right| d\Phi_\tau$$

$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\; \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)})/\mathcal{N}(\vec{\Theta}),\; \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z},\; \vec{\Theta} = (1, \rho, \eta, \xi_\rho\xi_\ell, \xi_\rho\xi_\ell\delta_\ell)$$

$$\boldsymbol{\mathcal{P}}_{\textbf{total}} = \left(1 - \sum_{i=1}^{4} \lambda_i\right) \boldsymbol{\mathcal{P}}^{\ell-\rho}_{signal} + \lambda_1 \mathcal{P}^{\ell-3\pi}_{bg} + \lambda_2 \mathcal{P}^{\pi-\rho}_{bg} + \lambda_3 \mathcal{P}^{\rho-\rho}_{bg} + \lambda_4 \boldsymbol{\mathcal{P}}^{other}_{bg}\,\textbf{(MC)}$$

MP are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space $\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m^2_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$ in CMS.

Michel par. at Belle, systematic uncertainties

Belle-CONF-1402 arXiv:1409.4969

Source	$\Delta(\rho)$, %	$\Delta(\eta)$, %	$\Delta(\xi_\rho\xi)$, %	$\Delta(\xi_\rho\xi\delta)$, %
Physical corrections				
ISR+$\mathcal{O}(\alpha^3)$	0.10	0.30	0.20	0.15
$\tau \to \ell\nu\nu\gamma$	0.03	0.10	0.09	0.08
$\tau \to \rho\nu\gamma$	0.06	0.16	0.11	0.02
Background	0.20	0.60	0.20	0.20
Apparatus corrections				
Resolution $\oplus$ brems.	0.10	0.33	0.11	0.19
$\sigma(E_{\text{beam}})$	0.07	0.25	0.03	0.15
Normalization				
$\Delta\mathcal{N}$	0.11	0.50	0.17	0.13
without Data/MC corr.	**0.29**	**0.95**	**0.38**	**0.38**
trigger eff. corr.	$\sim$ **1**	$\sim$ **2**	$\sim$ **3**	$\sim$ **3**

At Belle, various EXP/MC (L1, L3/L4 software trigger, track and π^0 rec., πID and ℓID) efficiency corrections produce the systematic uncertainties in MP of about few percent.

Analysis of $(\ell,\ \mathrm{all})$ events in 3D

$$\frac{d\sigma(\vec{\zeta})}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2}\beta_\tau(D_0 + \mathcal{P}_e F_i\zeta_i)$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \to \ell^\mp\nu\nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell(A(x^*) \mp \xi_\ell \vec{n}_\ell^* \vec{\zeta}^* B(x^*)),\ x^* = E_\ell^*/E_{\ell max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\sigma(\ell^\mp)}{dE_\ell^* d\Omega_\ell^* d\Omega_\tau} = \kappa_\ell \frac{\alpha^2\beta_\tau}{32E_\tau^2}(D_0 A(E_\ell^*) \mp \mathcal{P}_e\xi_\ell F_i n_{\ell i}^* B(E_\ell^*))$$

$$\frac{d\sigma(\ell^\mp)}{dp_\ell d\Omega_\ell} = \int\limits_{\Omega_\tau-\mathrm{sector}} \frac{d\sigma(\ell^\mp)}{dE_\ell^* d\Omega_\ell^* d\Omega_\tau}\left|\frac{\partial(E_\ell^*, \Omega_\ell^*)}{\partial(p_\ell, \Omega_\ell)}\right| d\Omega_\tau$$

Ω_τ-sector is determined by the kinematical constraint $m_{\nu\nu} > 0$

- All Michel parameters (ρ, η, $\mathcal{P}_e\xi$, $\mathcal{P}_e\xi\delta$) are measured in the unbinned maximum likelihood fit of $(\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau;\ \tau^+ \to \mathrm{all})$ events in the **3D** phase space.
- The reduced 3D phase space allows one to tabulate various EXP/MC corrections to the detection efficiency more precisely.
- **The crucial point in this method is to have high-efficiency 1-track trigger and precise $\mathcal{P}_e$ monitor.**

$\tau^- \to \pi^-/\rho^-\nu_\tau$ decay to monitor $\mathcal{P}_e$

$$\frac{d\sigma(\vec{\zeta})}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2}\beta_\tau(D_0 + \mathcal{P}_e F_i\zeta_i)$$

$$\frac{d\Gamma(\tau^\mp \to \pi^\mp\nu)}{d\Omega_\pi^*} = \kappa_\pi(1 \pm \xi_\pi\vec{\zeta}\vec{n}_\pi^*), \quad \frac{d\Gamma(\tau^\mp \to \rho^\mp\nu)}{dm_{\pi\pi}^2 d\Omega_\rho^*\tilde{\Omega}_\pi} = f(\vec{k}_1, \vec{k}_2)(1 \pm \xi_\rho\vec{\zeta}\vec{H}_\rho^*)$$

$$\frac{d\sigma(\pi^\mp)}{d\Omega_\pi^* d\Omega_\tau} = \kappa_\pi\frac{\alpha^2\beta_\tau}{32E_\tau^2}(D_0 \pm \mathcal{P}_e\xi_\pi F_i n_{\pi i}^*)$$

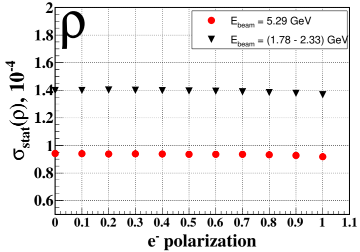
$$\frac{d\sigma(\rho^\mp)}{d\Omega_\rho^* dm_{\pi\pi}^2\tilde{\Omega}_\pi d\Omega_\tau} = f(\vec{k}_1, \vec{k}_2)\frac{\alpha^2\beta_\tau}{32E_\tau^2}(D_0 \pm \mathcal{P}_e\xi_\rho F_i H_{\rho i}^*)$$

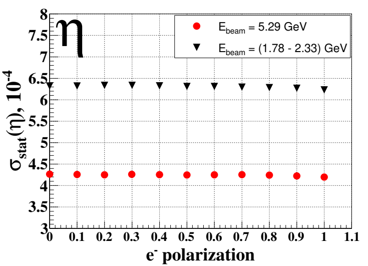
$$\frac{d\sigma(\pi^\mp)}{dp_\pi d\Omega_\pi} = \int_0^{2\pi}\frac{d\sigma(\pi^\mp)}{d\Omega_\pi^* d\Omega_\tau}\left|\frac{\partial(\Omega_\pi^*, \Omega_\tau)}{\partial(p_\pi, \Omega_\pi, \Phi_\tau)}\right| d\Phi_\tau$$

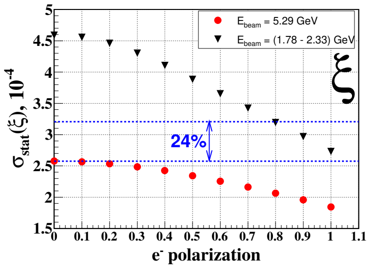
$$\frac{d\sigma(\rho^\mp)}{dp_\rho d\Omega_\rho dm_{\pi\pi}^2\tilde{\Omega}_\pi} = \int_0^{2\pi}\frac{d\sigma(\rho^\mp)}{d\Omega_\rho^* dm_{\pi\pi}^2\tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(\Omega_\rho^*, \Omega_\tau)}{\partial(p_\rho, \Omega_\rho, \Phi_\tau)}\right| d\Phi_\tau$$

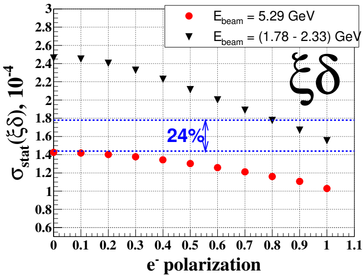
Parameters ($\mathcal{P}_e\xi_\pi$, $\mathcal{P}_e\xi_\rho$) are measured in the unbinned maximum likelihood fit of the ($\tau^- \to \pi^-/\rho^-\nu_\tau$; $\tau^+ \to \text{all}$) events. These decays can be used to monitor $\mathcal{P}_e$ with high precision.

Fit of (ℓ, ρ) in 9D at Belle II and STCF



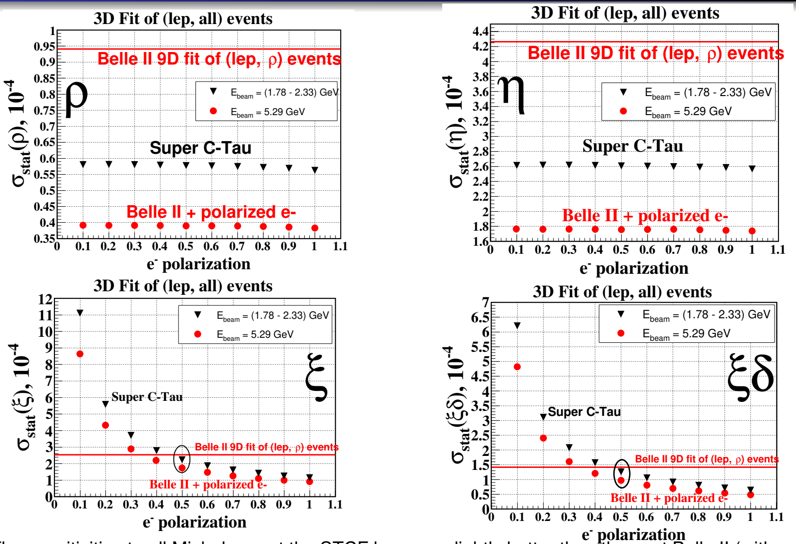






Sensitivities to Michel par. at Belle II (with unpolarized e^- beam) are slightly better (by a factor of 1.2–1.5) than those at the STCF.

Fit of $(\ell,\ \text{all})$ in 3D at Belle II and STCF

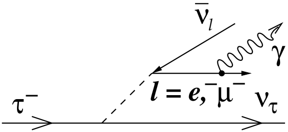


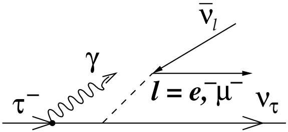
The sensitivities to all Michel par. at the STCF become slightly better than those at Belle II (with unpolarized e^- beam) for $\mathcal{P}_e > 0.5$.

Expected MP stat. uncertainties are $\sim 10^{-4}$, to reach the same level systematic uncertainty, the NNLO corrections ($\mathcal{O}(\alpha^4)$) to the differential $e^+e^- \to \tau^+\tau^-$ cross section are mandatory.

Michel parameters in $\tau \to \ell\nu\nu\gamma$ at Belle (I)

A. B. Arbuzov and T. V. Kopylova, JHEP **1609** (2016) 109. ($m_\ell \neq 0$)





Photon carries information about spin state of outgoing lepton, as a result two additional parameters, $\bar{\eta}$ and $\xi\kappa$, can be extracted.
These parameters were measured in τ decays at Belle for the first time.

$$\frac{d\Gamma(\tau^\mp \to \ell^\mp \nu_\ell \nu_\tau \gamma)}{dx\, dy\, d\Omega_\ell\, d\Omega_\gamma} = \Gamma_0 \frac{\alpha}{64\pi^3} \frac{\beta_\ell}{y} \Big[F(x,y,d) \pm P_\tau \left(\beta_\ell \cos\theta_\ell G(x,y,d) + \cos\theta_\gamma H(x,y,d) \right) \Big],$$

$$\Gamma_0 = G_F^2 m_\tau^5 / 192\pi^3, \;\; \beta_\ell = \sqrt{1 - m_\ell^2/E_\ell^2}, \;\; x = 2E_\ell/m_\tau, \;\; y = 2E_\gamma/m_\tau, \;\; d = 1 - \beta_\ell \cos\theta_{\ell\gamma}$$

$$F = F_0 + \bar{\eta} F_1, \;\; G = G_0 + \xi\kappa G_1, \;\; H = H_0 + \xi\kappa H_1, \;\; \frac{d\sigma(\ell^\mp \nu\nu\gamma, \rho^\pm \nu)}{dE_\ell^*\, d\Omega_\ell^*\, dE_\gamma^*\, d\Omega_\gamma^*\, d\Omega_\rho^*\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi\, d\Omega_\tau} = A_0 + \bar{\eta} A_1 + \xi\kappa A_2$$

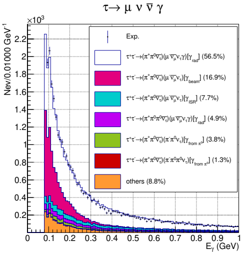
$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp \nu\nu\gamma, \rho^\pm \nu)}{dp_\ell\, d\Omega_\ell\, dp_\gamma\, d\Omega_\gamma\, dp_\rho\, d\Omega_\rho\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp \nu\nu\gamma, \rho^\pm \nu)}{dE_\ell^*\, d\Omega_\ell^*\, dE_\gamma^*\, d\Omega_\gamma^*\, d\Omega_\rho^*\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi\, d\Omega_\tau} |\mathrm{JACOBIAN}|\, d\Phi_\tau$$

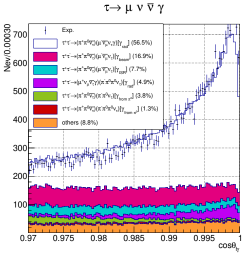
$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)}, \;\; \mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})} = \frac{\mathcal{F}_0 + \mathcal{F}_1 \bar{\eta} + \mathcal{F}_2 \xi\kappa}{\mathcal{N}_0 + \mathcal{N}_1 \bar{\eta} + \mathcal{N}_2 \xi\kappa}, \;\; \mathcal{N}_k = \int \mathcal{F}_k(\vec{z}) d\vec{z}, \;\; (k = 0, 1, 2)$$

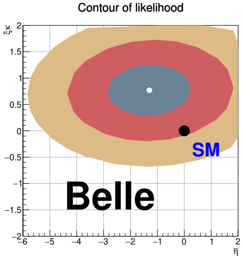
$\bar{\eta}$ and $\xi\kappa$ are extracted in the unbinned maximum likelihood fit of ($\ell\nu\nu\gamma$; $\rho\nu$) events in the 12D phase space in CMS.

Michel parameters in $\tau \to \ell\nu\nu\gamma$ at Belle (II)

$N_{\tau\tau} = 646 \times 10^6$, **selected: 71171** $(\mu\nu\nu\gamma;\ \rho\nu)$ **and 776834** $(e\nu\nu\gamma;\ \rho\nu)$ **events**







Source	$\sigma^e_{\bar\eta}$	$\sigma^e_{\xi\kappa}$	$\sigma^\mu_{\bar\eta}$	$\sigma^\mu_{\xi\kappa}$
Normalization	4.3	0.94	0.15	0.04
Background PDF	2.5	0.24	0.67	0.22
Branching ratios	3.8	0.05	0.25	0.01
Cluster merge in ECL	2.2	0.46	0.02	0.06
Detector resolution	0.74	0.20	0.22	0.02
Data/MC eff. corr.	1.9	0.14	0.04	0.04
Total	7.0	1.1	0.76	0.24

Belle result

$$\bar\eta = -\mathbf{1.3} \pm \mathbf{1.5} \pm \mathbf{0.8}$$

$$\xi\kappa = \mathbf{0.5} \pm \mathbf{0.4} \pm \mathbf{0.2}$$

N. Shimizu *et al.* [Belle Collab.], PTEP **2018** (2018) no.2, 023C01.

Measurement of ξ' at Belle (I)

Decay-in-flight muon polarization measurement ($\tau^- \to \mu^-\nu\nu$, $\mu^- \to e^-\nu\nu$) allows one to extract ξ' Michel parameter ($\xi'_{\rm SM} = 1$):

D. Bodrov and P. Pakhlov, JHEP 10 (2022) 035

D. Bodrov et al., PRL 131 (2023) 021801; PRD 108 (2023) 012003

$$\frac{d^2\Gamma(\tau\to\mu\nu\nu)}{dx\, d\cos\theta} = \frac{m_\tau}{4\pi^3} W_{\ell\tau}^4\, G_F^2 \sqrt{x^2 - x_0^2}\, \left[G_0 + (\vec{G}\cdot\vec{\zeta})\right]$$

$$G_0 = F_{IS}(x) \pm F_{AS}(x) P_\tau \cos\theta,$$

$$\vec{G} = \left(F_{T_1}(x) P_\tau \sin\theta,\ F_{T_2}(x) P_\tau \sin\theta,\ \pm F_{IP}(x) + F_{AP}(x) P_\tau \cos\theta\right),$$

$$F_{IP}(x) = \frac{1}{54}\sqrt{x^2 - x_0^2}\left[-9\xi'\left(2x - 3 + \frac{x_0^2}{2}\right) + 4\xi\left(\delta - \frac{3}{4}\right)\left(4x - 3 - \frac{x_0^2}{2}\right)\right],$$

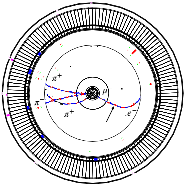
$$\frac{d^2\Gamma(\mu\to e\nu\nu)}{dy\, d\Omega_e} = \frac{G_F^2 m_\mu^5}{384\pi^4} y^2 \left[(3-2y) \pm (2y-1)(\vec{n}_e\cdot\vec{\zeta})\right].$$

$$\frac{d^5\Gamma}{dx\, d\cos\theta\, dy\, d\Omega'_e\, dt} = \mathcal{B}_{\mu\to e\nu\nu} \frac{\Gamma_{\tau\to\mu\nu\nu}}{1-3x_0^2} \frac{3}{\pi} y^2 \sqrt{x^2 - x_0^2}\left[(3-2y)G_0 \pm (2y-1)(\vec{n}'_e\cdot\vec{G})\right] \frac{1}{\tau_\mu} e^{-t/\tau_\mu},$$

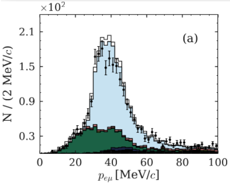
$$\frac{d^3\Gamma}{dx\, dy\, d\cos\theta'_e} = \mathcal{B}_{\mu\to e\nu\nu} \frac{12\Gamma_{\tau\to\mu\nu\nu}}{1-3x_0^2} y^2 \sqrt{x^2 - x_0^2}\left[(3-2y)F_{IS}(x) + (2y-1)F_{IP}(x)\cos\theta'_e\right]$$

Measurement of ξ' at Belle (II)

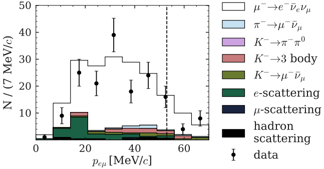
The idea is to select events with ordinary leptonic $\tau^- \to \mu^-\nu\nu$ decays in which we have additional out-of-beam track from the consequent $\mu^- \to e^-\nu\nu$ decay, which appears as a kink in the original muon track. BDT based selection allows one to suppress background substantially keeping the signal efficiency at the level of 78%.

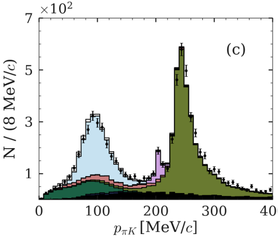


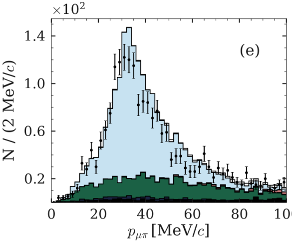
before BDT



after BDT







Measurement of ξ' at Belle (III)

ξ' is extracted in the 2D $((y,\, \cos\theta_e) \equiv (y,\, c))$ unbinned maximum likelihood fit.

$$\mathcal{L} = \prod_{i=1}^{n} \mathcal{P}(y_i,\, c_i;\, \xi'),\; \mathcal{P}(y,\, c;\, \xi') = p\, \mathcal{P}_{\rm sig}(y,\, c;\, \xi') + (1-p)\, \mathcal{P}_{\rm bkg}(y,\, c),$$

$p = N_{\rm sig}/(N_{\rm sig} + N_{\rm bckg}) = 0.74$. Both, signal PDF, $\mathcal{P}_{\rm sig}(y,\, c;\, \xi')$, and background PDF, $\mathcal{P}_{\rm bkg}(y,\, c)$, are obtained from the MC simulation.

$$\mathcal{P}_{\rm sig}(y,\, c;\, \xi') = \frac{1}{2}\left\{\mathcal{P}_{+1}(y,\, c) + \mathcal{P}_{-1}(y,\, c) + \xi'\left[\mathcal{P}_{+1}(y,\, c) - \mathcal{P}_{-1}(y,\, c)\right]\right\}$$

$$\xi' = 0.22 \pm 0.94 \pm 0.42$$

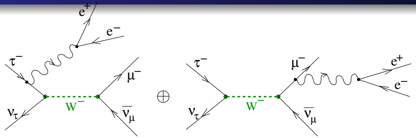
As $\xi\kappa = -\frac{1}{12}(3\xi - 8\xi\delta + 3\xi') \approx (1-\xi')/4$, from radiative leptonic decays $\xi' \approx 1 - 4\xi\kappa = -1.0 \pm 1.8$, i.e. new Belle result on ξ' is almost 2 times more precise. However $\bar{\eta} = \frac{1}{4}(7 - 8\rho - \xi'')$ is related to ξ'', so still radiative leptonic decay provides additional Michel parameter constraint.

At Belle II and STCF with the improved track rec. algorithm in the drift chamber of the detector and higher statistics it will be possible to improve the accuracy.

Belle II developed the Kink Finder tool for the new ξ' measurement:

D. Bodrov et al. NIMA 1087 (2026) 171438

Tau decays into 5 leptons (I)



D. A. Dicus and R. Vega, Phys. Lett. B **338** (1994) 341.

M. S. Alam *et al.* [CLEO Collaboration], Phys. Rev. Lett. **76** (1996) 2637.

A. Flores-Tlalpa, G. Lopez Castro and P. Roig, JHEP 1604 (2016) 185.

Mode	$\mathcal{B}_{\rm theory}$	$\mathcal{B}_{\rm CLEO}$
$e^\mp e^+ e^- 2\nu$	$(4.21 \pm 0.01) \times 10^{-5}$	$(2.7^{+1.6}_{-1.2}) \times 10^{-5}$
$\mu^\mp e^+ e^- 2\nu$	$(1.984 \pm 0.004) \times 10^{-5}$	$< 3.2 \times 10^{-5}$(90% CL)
$e^\mp \mu^+ \mu^- 2\nu$	$(1.247 \pm 0.001) \times 10^{-7}$	
$\mu^\mp \mu^+ \mu^- 2\nu$	$(1.183 \pm 0.001) \times 10^{-7}$	

A. Kersch, N. Kraus and R. Engfer [SINDRUM], Nucl. Phys. A **485** (1988) 606.

$$\frac{d\Gamma(\tau)}{d\mathcal{PS}} = Q_{LL} d_1 + Q_{LR} d_2 + Q_{RL} d_3 + Q_{RR} d_4 + B_{RL} d_5 + B_{LR} d_6$$

Up to now Q_{LL}, Q_{LR}, Q_{RL}, Q_{RR}, B_{RL}, B_{LR} were measured only in muon decays ($\mu^- \to e^- e^- e^+ \nu_\mu \bar{\nu}_e$) with the accuracy of about $10 \div 20\%$.
Michel parameters can be measured in two ways: in the study of the dynamics and from the measurement of the branching fraction:

$$\mathcal{B}_{\rm exp}/\mathcal{B}_{\rm SM} = Q_{LL} + \alpha_{LR} Q_{LR} + \alpha_{RL} Q_{RL} + \alpha_{RR} Q_{RR} + \beta_{RL} B_{RL} + \beta_{LR} B_{LR}$$

Tau decays into 5 leptons (II)

J.Sasaki (Belle Collaboration) J. Phys. Conf. Ser. 912 (2017) 012002.

$$\frac{d\Gamma_5}{d^3p_1 d^3p_2 d^3p_3} \propto \frac{1}{E_1E_2E_3}[(Q_{LL}T^Q_{LL} + Q_{RL}T^Q_{RL} + B_{RL}T^B_{RL} + L \leftrightarrow R) + I_\alpha \Re(T^I_\alpha) + I_\beta \Re(T^I_\beta)]$$

$$BR^{\tau^-\to e^-e^+e^-\bar\nu_e\nu_\tau}_{\rm exp} = BR^{\tau^-\to e^-e^+e^-\bar\nu_e\nu_\tau}_{\rm SM}\{[Q_{LL} + (1.051\pm0.036)Q_{LR} + (-0.2053\pm0.1431)B_{LR} + L\leftrightarrow R] + (0.2416\pm0.0002)I_\alpha + (0.8606\pm0.0001)I_\beta\}.$$

$$BR^{\tau^-\to \mu^-e^+e^-\bar\nu_\mu\nu_\tau}_{\rm exp} = BR^{\tau^-\to \mu^-e^+e^-\bar\nu_\mu\nu_\tau}_{\rm SM}\{[Q_{LL} + (1.220\pm0.049)Q_{LR} + (-0.8717\pm0.1957)B_{LR} + L\leftrightarrow R] + (181.3\pm0.1)I_\alpha + (104.4\pm0.1)I_\beta\}.$$

$$BR^{\tau^-\to e^-\mu^+\mu^-\bar\nu_e\nu_\tau}_{\rm exp} = BR^{\tau^-\to e^-\mu^+\mu^-\bar\nu_e\nu_\tau}_{\rm SM}\{[Q_{LL} + (1.226\pm0.001)Q_{LR} + (-0.8456\pm0.0001)B_{LR} + L\leftrightarrow R] + (0.2253\pm0.0001)I_\alpha + (0.5231\pm0.0001)I_\beta\}.$$

$$BR^{\tau^-\to \mu^-\mu^+\mu^-\bar\nu_\mu\nu_\tau}_{\rm exp} = BR^{\tau^-\to \mu^-\mu^+\mu^-\bar\nu_\mu\nu_\tau}_{\rm SM}\{[Q_{LL} + (1.216\pm0.005)Q_{LR} + (-0.8459\pm0.0005)B_{LR} + L\leftrightarrow R] - (18.00\pm0.01)I_\alpha + (197.3\pm0.1)I_\beta\}.$$

τ^- decay mode	$e^-e^+e^-\bar\nu_e\nu_\tau$	$\mu^-e^+e^-\bar\nu_\mu\nu_\tau$	$e^-\mu^+\mu^-\bar\nu_e\nu_\tau$	$\mu^-\mu^+\mu^-\bar\nu_\mu\nu_\tau$
Detection efficiency	(1.769±0.004)%	(1.204±0.003)%	(3.561±0.006)%	(1.674±0.004)%
Main backgrounds	$e^-\bar\nu_e\nu_\tau\gamma$ $\to e^-\bar\nu_e\nu_\tau(e^+e^-)$ $\pi^-\pi^0\nu_\tau$ $\to\pi^-(\gamma\gamma)\nu_\tau$ $\to\pi^-((e^+e^-)\gamma)\nu_\tau$ (mis-ID π as e) $e^-\bar\nu_e\nu_\tau$	$\mu^-\bar\nu_\mu\nu_\tau\gamma$ $\to\mu^-\bar\nu_\mu\nu_\tau(e^+e^-)$ $\pi^-\pi^0\nu_\tau$ $\to\pi^-(e^+e^-\gamma)\nu_\tau$ $\pi^-\pi^0\pi^0\nu_\tau$ $\to\pi^-(\gamma\gamma)(\gamma\gamma)\nu_\tau$ $\to\pi^-((e^+e^-)\gamma)(\gamma\gamma)\nu_\tau$ (mis-ID π as μ)	$\pi^-\pi^0\nu_\tau$ $\to\pi^-(\gamma\gamma)\nu_\tau$ $\to\pi^-((e^+e^-)\gamma)\nu_\tau$ $\pi^-\pi^+\pi^-\nu_\tau$ (mis-ID π as μ, e)	$\pi^-\pi^0\nu_\tau$ $\to\pi^-(\gamma\gamma)\nu_\tau$ $\to\pi^-((e^+e^-)\gamma)\nu_\tau$ $\pi^-\pi^+\pi^-\nu_\tau$ (mis-ID π as μ)
Expected number of signal events	1300	430	8	4
Fraction of the signal	47%	50%	37%	16%

If we measure $\mathcal{B}(\tau^-\to\mu^-e^+e^-\nu_\tau)$ with 10% relative accuracy it will be possible to constrain I_α and I_β on the level of 10^{-3} for the first time.

Belle II Collaboration started this analysis.

Summary

- **Belle II is the main player in τ studies in the nearest future.**
  Nonzero average polarization of single τ at the STCF with polarized e^- beam makes STCF to be competitive with Belle II in τ-lepton studies.
- Measurement of the main Michel parameters (ρ, η, ξ, $\xi\delta$) in ordinary leptonic decays at Belle is pending, systematics of 3% should be improved.
  At Belle II with much better detector and DAQ performance it is possible to reach the world best accuracy.
- Measurement of Michel parameters in the radiative leptonic τ decays:
  A. B. Arbuzov and T. V. Kopylova, JHEP **1609** (2016) 109. ($m_\ell \neq 0$)
  First experimental result from Belle:
  N. Shimizu et al., PTEP **2018** (2018) no.2, 023C01.
  Statistical improvement at Belle II and STCF. At the STCF the polarized τ allows one further to inrease sensitivity to $\xi\kappa$ parameter.
- Decay-in-flight muon polarization measurement ($\tau \to \mu\nu\nu$, $\mu \to e\nu\nu$) to measure ξ' Michel parameter:
  D. Bodrov et al., PRL 131 (2023) 021801; PRD 108 (2023) 012003
  The Belle result $\xi' = 0.22 \pm 0.94 \pm 0.42$ has almost twice better accuracy than that from the $\xi\kappa$ from radiative leptonic decays.
  At Belle II and STCF with the improved track rec. algorithm in the drift chamber of the detector and higher statistics it will be possible to improve the accuracy. With the whole expected Belle II statistics the relative accuracy of ξ' will approach the level of 0.5%.
- Precise study of the five-body leptonic τ decays $\tau \to \ell\ell'^+\ell'^-\nu\nu$:
  A. Flores-Tlalpa, G. Lopez Castro and P. Roig, JHEP **1604** (2016) 185.
  While $\tau^- \to e^-e^+e^-2\nu$ and $\tau^- \to \mu^-e^+e^-2\nu$ modes can be studies already at B factories, the $\tau^- \to e^-\mu^+\mu^-2\nu$ and $\tau^- \to \mu^-\mu^+\mu^-2\nu$ modes can be discovered at Belle II and STCF. The analysis has been started at Belle II.
  Possibility to measure $\boldsymbol{I}_\alpha$ and $\boldsymbol{I}_\beta$ Michel parameters with the accuracy of 10^{-3} with already available statistics at Belle and Belle II, and to search for T-odd correlations for the first time in the purely leptonic decays.

Backup slides

Michel parameters

$$\rho = \frac{3}{4} - \frac{3}{4}\left(|g^V_{LR}|^2 + |g^V_{RL}|^2 + 2|g^T_{LR}|^2 + 2|g^T_{RL}|^2 + \Re(g^S_{LR}g^{T*}_{LR} + g^S_{RL}g^{T*}_{RL})\right)$$

$$\eta = \frac{1}{2}\Re\left(6g^V_{RL}g^{T*}_{LR} + 6g^V_{LR}g^{T*}_{RL} + g^S_{RR}g^{V*}_{LL} + g^S_{RL}g^{V*}_{LR} + g^S_{LR}g^{V*}_{RL} + g^S_{LL}g^{V*}_{RR}\right)$$

$$\xi = 4\Re(g^S_{LR}g^{T*}_{LR}) - 4\Re(g^S_{RL}g^{T*}_{RL}) + |g^V_{LL}|^2 + 3|g^V_{LR}|^2 - 3|g^V_{RL}|^2 - |g^V_{RR}|^2 +$$

$$+5|g^T_{LR}|^2 - 5|g^T_{RL}|^2 + \frac{1}{4}|g^S_{LL}|^2 - \frac{1}{4}|g^S_{LR}|^2 + \frac{1}{4}|g^S_{RL}|^2 - \frac{1}{4}|g^S_{RR}|^2$$

$$\xi\delta = \frac{3}{16}|g^S_{LL}|^2 - \frac{3}{16}|g^S_{LR}|^2 + \frac{3}{16}|g^S_{RL}|^2 - \frac{3}{16}|g^S_{RR}|^2 - \frac{3}{4}|g^T_{LR}|^2 + \frac{3}{4}|g^T_{RL}|^2 +$$

$$+\frac{3}{4}|g^V_{LL}|^2 - \frac{3}{4}|g^V_{RR}|^2 + \frac{3}{4}\Re(g^S_{LR}g^{T*}_{LR}) - \frac{3}{4}\Re(g^S_{RL}g^{T*}_{RL})$$

$$\bar{\eta} = \left|g^V_{RL}\right|^2 + \left|g^V_{LR}\right|^2 + \frac{1}{8}\left(\left|g^S_{RL} + 2g^T_{RL}\right|^2 + \left|g^S_{LR} + 2g^T_{LR}\right|^2\right) + 2\left(\left|g^T_{RL}\right|^2 + \left|g^T_{LR}\right|^2\right)$$

$$\xi\kappa = \left|g^V_{RL}\right|^2 - \left|g^V_{LR}\right|^2 + \frac{1}{8}\left(\left|g^S_{RL} + 2g^T_{RL}\right|^2 - \left|g^S_{LR} + 2g^T_{LR}\right|^2\right) + 2\left(\left|g^T_{RL}\right|^2 - \left|g^T_{LR}\right|^2\right)$$

Method: theoretical framework

W. Fetscher, Phys. Rev. D **42** (1990) 1544. K. Tamai, Nucl. Phys. B **668** (2003) 385.

$$\frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i\zeta_j')$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \to \ell^\mp\nu\nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell(A(x^*) \mp \xi\vec{n}_\ell^*\vec{\zeta}^* B(x^*)),\ x^* = E_\ell^*/E_{\ell max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\Gamma(\tau^\pm(\vec{\zeta'}^*) \to \rho^\pm\nu)}{dm_{\pi\pi}^2 d\Omega_\rho^* d\tilde{\Omega}_\pi} = \kappa_\rho(A' \mp \xi_\rho\vec{B'}\vec{\zeta'}^*)W(m_{\pi\pi}^2) = \kappa_\rho A'(1 \mp \xi_\rho\vec{H}_\rho\vec{\zeta'}^*)W(m_{\pi\pi}^2)$$

$$\vec{H}_\rho = \frac{\vec{B'}}{A'} - \text{polarimeter vector},\ \xi_\rho = -\frac{2Re(c_V^* c_A)}{|c_V|^2 + |c_A|^2} = -h_{\nu_\tau}\ (h_{\nu_\tau} = -1 \text{ in the SM})$$

$$A' = 2(q,Q)Q_0^* - Q^2 q_0^*,\ \vec{B'} = Q^2\vec{K}^* + 2(q,Q)\vec{Q}^*,\ W = |F_\pi(m_{\pi\pi}^2)|^2\frac{p_\rho(m_{\pi\pi}^2)\tilde{p}_\pi(m_{\pi\pi}^2)}{M_\tau m_{\pi\pi}}$$

$$\frac{d\sigma(\ell^\mp,\rho^\pm)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = \kappa_\ell\kappa_\rho\frac{\alpha^2\beta_\tau}{64E_\tau^2}(D_0 A' A(E_\ell^*) + \xi_\rho\xi_\ell D_{ij} n_{\ell i}^* B_j' B(E_\ell^*))\,W(m_{\pi\pi}^2)$$

$$\frac{d\sigma(\ell^\mp,\rho^\pm)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2}\frac{d\sigma(\ell^\mp,\rho^\pm)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(E_\ell^*,\Omega_\ell^*,\Omega_\rho^*,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)}\right|d\Phi_\tau$$

Multidimensional unbinned maximum likelihood fit

4 Michel parameters ($\vec{\Theta} = (1,\ \rho,\ \eta,\ \xi_\rho\xi_\ell,\ \xi_\rho\xi_\ell\delta_\ell)$) are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space in CMS,
$\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$. The PDF for individual k-th event is written in the form:

$$\mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})},\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z}$$

Likelihood function for N events:

$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{L} = -\ln L = N\ln\mathcal{N}(\vec{\Theta}) - \sum_{k=1}^{N} \ln\mathcal{F}^{(k)},\ \mathcal{F}^{(k)} = \mathcal{F}(\vec{z}^{(k)})$$

$$\mathcal{F}^{(k)} = A_0^{(k)}\Theta_0 + A_1^{(k)}\Theta_1 + A_2^{(k)}\Theta_2 + A_3^{(k)}\Theta_3 + A_4^{(k)}\Theta_4 = \sum_{i=0}^{4} A_i^{(k)}\Theta_i$$

$$\mathcal{N} = C_0\Theta_0 + C_1\Theta_1 + C_2\Theta_2 + C_3\Theta_3 + C_4\Theta_4,\ C_j = \frac{1}{N}\sum_{k=1}^{N} C_j^{(k)},\ C_j^{(k)} = \frac{A_j^{(k)}}{\sum_{i=0}^{4} A_i^{(k)}\Theta_i^{MC}}$$

$$\vec{\Theta}^{MC} = (1,\ 0.75,\ 0,\ 1,\ 0.75),\ \mathcal{L} = N\ln\left(\sum_{j=0}^{4} C_j\Theta_j\right) - \sum_{k=1}^{N}\ln\left(\sum_{i=0}^{4} A_i^{(k)}\Theta_i\right)$$

As a result fitted statistics is represented by a set of $5 \times N$ values of $A_i^{(k)}$ $(k = 1 \div N,\ i = 0 \div 4)$, which is calculated only once.

C_i $(i = 0 \div 4)$ are calculated using MC simulation.

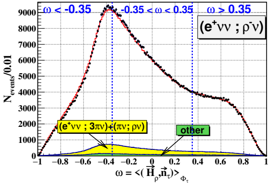
In ideal case (no rad. corr., $\varepsilon = 100\%$): $C_0 = 1$, $C_2 = 4m_\ell/m_\tau$, $C_{1,3,4} = 0$

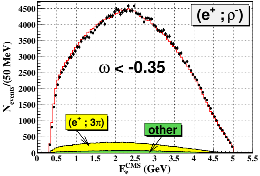
Method: helicity sensitive variable

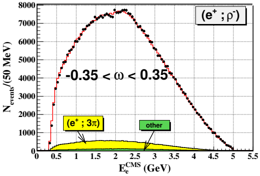
M. Davier *et. al* Phys. Lett. B **306** (1993) 411.

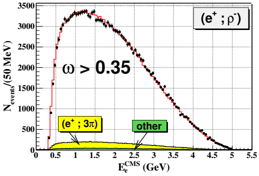
Helicity sensitive variable ω is introduced as:

$$\omega = \frac{1}{\Phi_2 - \Phi_1} \int_{\Phi_1}^{\Phi_2} (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) d\Phi = < (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) >_{\Phi_\tau}$$









Spin-spin correlation manifests itself through momentum-momentum correlations of final lepton and pions.

Method: corrections, detector effects, background

Physical corrections:

- All $\mathcal{O}(\alpha^3)$ QED and electroweak higher order corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$ are included
- Radiative leptonic decays $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$
- Radiative decay $\tau^- \to \pi^-\pi^0\nu_\tau\gamma$

Detector effects:

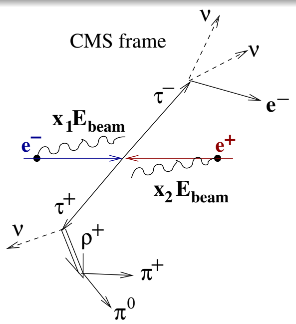
- Track momentum resolution
- γ energy and angular resolution
- Effect of external bremsstrahlung for $e-\rho$ events
- Beam energy spread
- EXP/MC efficiency corrections (trigger, track rec., π^0 rec., ℓID, πID)

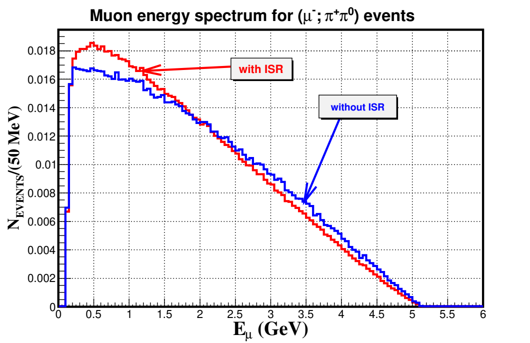
Background at Belle:

The main background comes from $(\ell\nu\nu;\ \pi 2\pi^0\nu)(\sim 10\%)$, $(\pi\nu;\ \pi\pi^0\nu)(\sim 1.5\%)$ and $(\rho^+\nu;\ \rho^-\nu)(\sim 0.5\%)$ events, it is included in PDF analytically. The remaining background($\sim$2.0%) is taken into account using MC-based approach.

Background from the non-$\tau\tau$ events is $\lesssim 0.1\%$.

Initial state radiation (ISR)



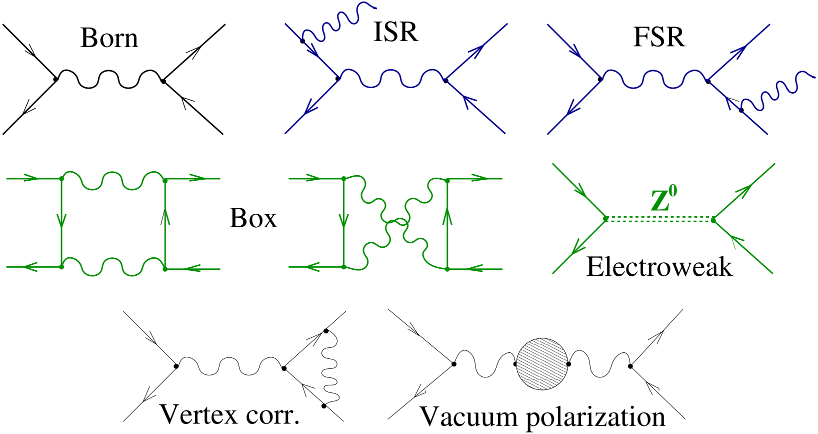


$$\frac{d\sigma_{\rm vis}(s)}{dp_\ell\, d\Omega_\ell\, dp_\rho\, d\Omega_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi} = \iint_0^1 dx_1\, dx_2\, D(x_1) D(x_2)\, \frac{d\sigma(s(1-x_1)(1-x_2))}{dp'_\ell\, d\Omega'_\ell\, dp'_\rho\, d\Omega'_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi} \left|\frac{\partial(p'_\ell, \Omega'_\ell)}{\partial(p_\ell, \Omega_\ell)}\right| \left|\frac{\partial(p'_\rho, \Omega'_\rho)}{\partial(p_\rho, \Omega_\rho)}\right|$$

- $D(x) = x^{\beta/2-1} h(x)$ - probability function for initial $e^\mp$ to emit a γ-quantum jet carrying $x_{1,2}$ part of $e^\mp$ energy $E_{\rm beam} = \sqrt{s}/2$. $\beta = \frac{2\alpha}{\pi}(\ln \frac{s}{m^2} - 1)$, $h(x)$ - smooth limited function.
- $\left|\frac{\partial(p'_i, \Omega'_i)}{\partial(p_i, \Omega_i)}\right|$ $(i = \ell,\ \rho)$ - Jacobian of transformation from the $\tau^+\tau^-$ rest frame to the Belle CMS.

At the Super Charm-Tau factory the impact of the ISR is expected to be essentially smaller.

$\mathcal{O}(\alpha^3)$ corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$



S. Jadach and Z. Was, Acta Phys. Polon. B **15** (1984) 1151 [Erratum-ibid. B **16** (1985) 483].
A. B. Arbuzov *et al* JHEP **9710** (1997) 001.

Charge-odd part of the cross section comes from the interference of the ISR and FSR diagrams as well as box and Born diagrams, and Z^0-exchange and Born diagrams.

Description of background at Belle

Total PDF

$$\mathcal{P}(x) = \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\Bigg((1-\sum_i \lambda_i)\frac{S(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} S(x)dx} + \lambda_{3\pi}\frac{\tilde{B}_{3\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_{3\pi}(x)dx} + \lambda_{\pi}\frac{\tilde{B}_{\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_{\pi}(x)dx} + \lambda_{\rho}\frac{\tilde{B}_{\rho}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_{\rho}(x)dx} +$$

$$+(1-\sum_i \lambda_i)\frac{N^{\rm sel}_{\rm rest}(x)}{N^{\rm sel}_{\rm sig}(x)} S_{\rm SM}(x)\Bigg)$$

$$\tilde{B}_{3\pi}(x) = \int 2(1-\varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y)B_{3\pi}(x,y)dy,\ \tilde{B}_{\pi}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)}B_\pi(x)$$

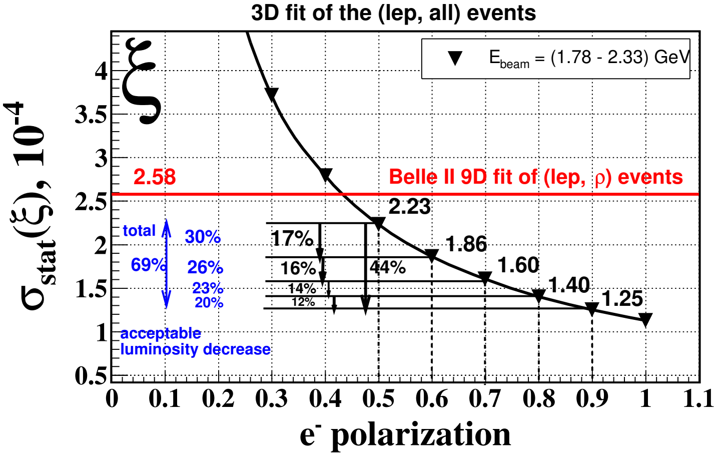
$$\tilde{B}_{\rho}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)}\int (1-\varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y)B_{\rho}(x,y)dy,\ \overline{\varepsilon(x)} = \epsilon^{\rm EXP}_{\rm corr}(x)\varepsilon(x)$$

- $x = (p_\ell,\ \Omega_\ell,\ p_\rho,\ \Omega_\rho,\ m^2_{\pi\pi},\ \tilde{\Omega}_\pi);\ y = (p_{\pi^0},\ \Omega_{\pi^0})$;
- $S(x)$ - theoretical density of signal ($\ell^\mp\nu\nu,\ \rho^\pm\nu$) events;
- $B_{3\pi}(x,y)$ - theoretical density of background ($\ell^\mp\nu\nu,\ \pi^\pm 2\pi^0\nu$) events;
- $B_\pi(x)$ - theoretical density of background ($\pi^\mp\nu,\ \rho^\pm\nu$) events;
- $B_\rho(x)$ - theoretical density of background ($\rho^\mp\nu,\ \rho^\pm\nu$) events;
- $\varepsilon(x)$ - detection efficiency for signal events (**common multiplier**);
- $N^{\rm sel}_{\rm rest}(x)/N^{\rm sel}_{\rm sig}(x)$ - number of the selected (remaining/signal) MC events in the multidimensional cell around "x". Admixture of the remaining background is $(1 \div 2)$%.
- λ_i - i-th background fraction (from MC)
- $\varepsilon_{\pi^0}(y)$ - π^0 detection efficiency (tabulated from MC);
- $\varepsilon_{\rm add}(y) = \varepsilon^{3\pi}_{\rm add}(y)/\varepsilon^{\rm sig}_{\rm add}$ - ratio of the $E^{\rm LAB}_{\gamma\rm rest}$ cut efficiencies (tabulated from MC);
- $\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)/\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)$ is tabulated from MC;
- $\epsilon^{\rm EXP}_{\rm corr}(x)$ - EXP/MC efficiency correction.

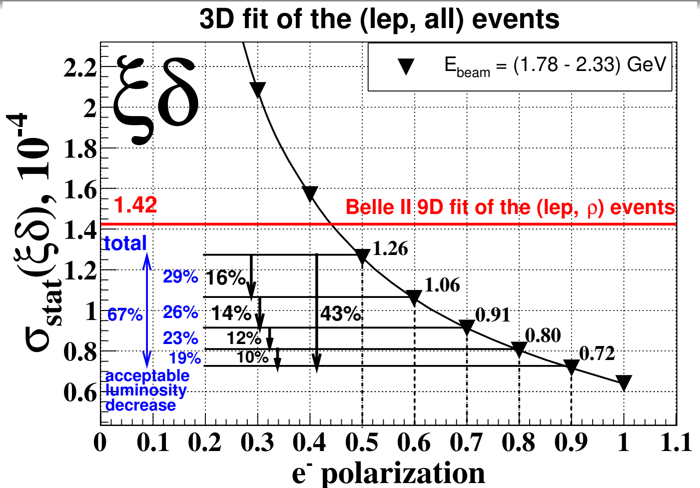
Toy MC studies of the effect of polarized e^- beam

- 66 10M $(\mu,\ \rho)$ samples, at 6 center-of-mass (c.m.s.) energies (according to Table 1.1 in STCF CDR part I) : $2E = 3.554$ GeV ($\tau^+\tau^-$ production threshold), $2E = 3.686$ GeV ($\psi(2S)$), $2E = 3.770$ GeV ($\psi(3770)$), $2E = 4.170$ GeV ($\psi(4160)$), $2E = 4.650$ GeV (maximum of the $\sigma(e^+e^- \to \Lambda_c^+\Lambda_c^-)$), $2E = 10.58$ GeV (Belle II), for 11 values of e^- beam polarization: 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, were generated for the calculation of the normalizations. 66 statistically independent 1M samples at the same energies and polarizations were generated for the fit.
- To evaluate MP sensitivities (rescaling the sensitivities obtained in the fits of 1M samples) we took the detection efficiency of $(\mu,\ \rho)$ events to be 20% (to be compared with 12% efficiency obtained at Belle, where the π^0 rec. efficiency is only 40%). The detection efficiency of (μ, all) events was taken to be 30%.
- To measure ρ, ξ and $\xi\delta$ MP, samples with $\ell = e,\ \mu$ were taken into account, while η MP is measured in samples with $\ell = \mu$ only.

ξ from the fit of $(\ell, \text{ all})$ in 3D at Super C-Tau



For the increases of the e^- beam polarizations, 0.5 $\to$ 0.6, 0.6 $\to$ 0.7, 0.7 $\to$ 0.8, 0.8 $\to$ 0.9, the corresponding improvements in the sensitivities to the ξ parameter, 17%, 16%, 14%, 12%, respectively. If we move from the polarization of 0.5 to the higher polarizations: 0.5 $\to$ 0.6, 0.5 $\to$ 0.7, 0.5 $\to$ 0.8, 0.5 $\to$ 0.9, the acceptable luminosity decrease factors (to keep the sensitivity at the level of that we have for polarization 0.5) are: $(1.86/2.23)^2 = 0.70$, $(1.60/2.23)^2 = 0.51$, $(1.40/2.23)^2 = 0.39$, $(1.25/2.23)^2 = 0.31$, respectively.



For the increases of the e^- beam polarizations, 0.5 $\to$ 0.6, 0.6 $\to$ 0.7, 0.7 $\to$ 0.8, 0.8 $\to$ 0.9, the corresponding improvements in the sensitivities to the $\xi\delta$ parameter, 16%, 14%, 12%, 10%, respectively. If we move from the polarization of 0.5 to the higher polarizations: 0.5 $\to$ 0.6, 0.5 $\to$ 0.7, 0.5 $\to$ 0.8, 0.5 $\to$ 0.9, the acceptable luminosity decrease factors (to keep the sensitivity at the level of that we have for polarization 0.5) are:
$(1.06/1.26)^2 = 0.71$, $(0.91/1.26)^2 = 0.52$, $(0.80/1.26)^2 = 0.40$, $(0.72/1.26)^2 = 0.33$,

respectively.