

CP asymmetries in $\tau \rightarrow K_S \pi \nu_\tau$ decays

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based on:

Feng-Zhi Chen, **XQLI**, Shi-Can Peng, Ya-Dong Yang, Hong-Hao Zhang, JHEP 01 (2022) 108

Feng-Zhi Chen, **XQLI**, Ya-Dong Yang, JHEP 05 (2020) 151

Feng-Zhi Chen, **XQLI**, Ya-Dong Yang, Xin Zhang, PRD 100 (2019) 113006

Outline

□ Introduction

□ CP asymmetries in $\tau \rightarrow K_S \pi \nu_\tau$ decays within the SM

□ CP asymmetries in $\tau \rightarrow K_S \pi \nu_\tau$ decays in a general EFT framework

□ Summary

- commonly discussed **decay-rate asymmetry**
- CP asymmetry in the **angular distribution**

$$A_{\text{CP}}^{\text{rate}} \equiv \frac{\Gamma(\tau^+ \rightarrow [\pi^+ \pi^-] "K_S" \pi^+ \bar{\nu}_\tau) - \Gamma(\tau^- \rightarrow [\pi^+ \pi^-] "K_S" \pi^- \nu_\tau)}{\Gamma(\tau^+ \rightarrow [\pi^+ \pi^-] "K_S" \pi^+ \bar{\nu}_\tau) + \Gamma(\tau^- \rightarrow [\pi^+ \pi^-] "K_S" \pi^- \nu_\tau)}$$

BaBar Collaboration, PRD 85 (2012) 031102 & Belle II, 2604.23757

$$A_{\text{CP}}^i = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^1 \cos \alpha \left[\frac{d^2 \Gamma(\tau^- \rightarrow K_S \pi^- \nu_\tau)}{ds d \cos \alpha} - \frac{d^2 \Gamma(\tau^+ \rightarrow K_S \pi^+ \bar{\nu}_\tau)}{ds d \cos \alpha} \right] ds d \cos \alpha}{\frac{1}{2} \int_{s_{1,i}}^{s_{2,i}} \int_{-1}^1 \left[\frac{d^2 \Gamma(\tau^- \rightarrow K_S \pi^- \nu_\tau)}{ds d \cos \alpha} + \frac{d^2 \Gamma(\tau^+ \rightarrow K_S \pi^+ \bar{\nu}_\tau)}{ds d \cos \alpha} \right] ds d \cos \alpha}$$

Belle Collaboration, PRL 107 (2011) 131801

Tau lepton physics

□ τ : discovered in 1975 by Martin Perl *et al.* (SLAC-LBL)

- Mass: $m_\tau = 1776.93 \pm 0.09$ MeV;
- Lifetime: $\tau_\tau = (2.903 \pm 0.005) \times 10^{-13}$ s

PDG 2026

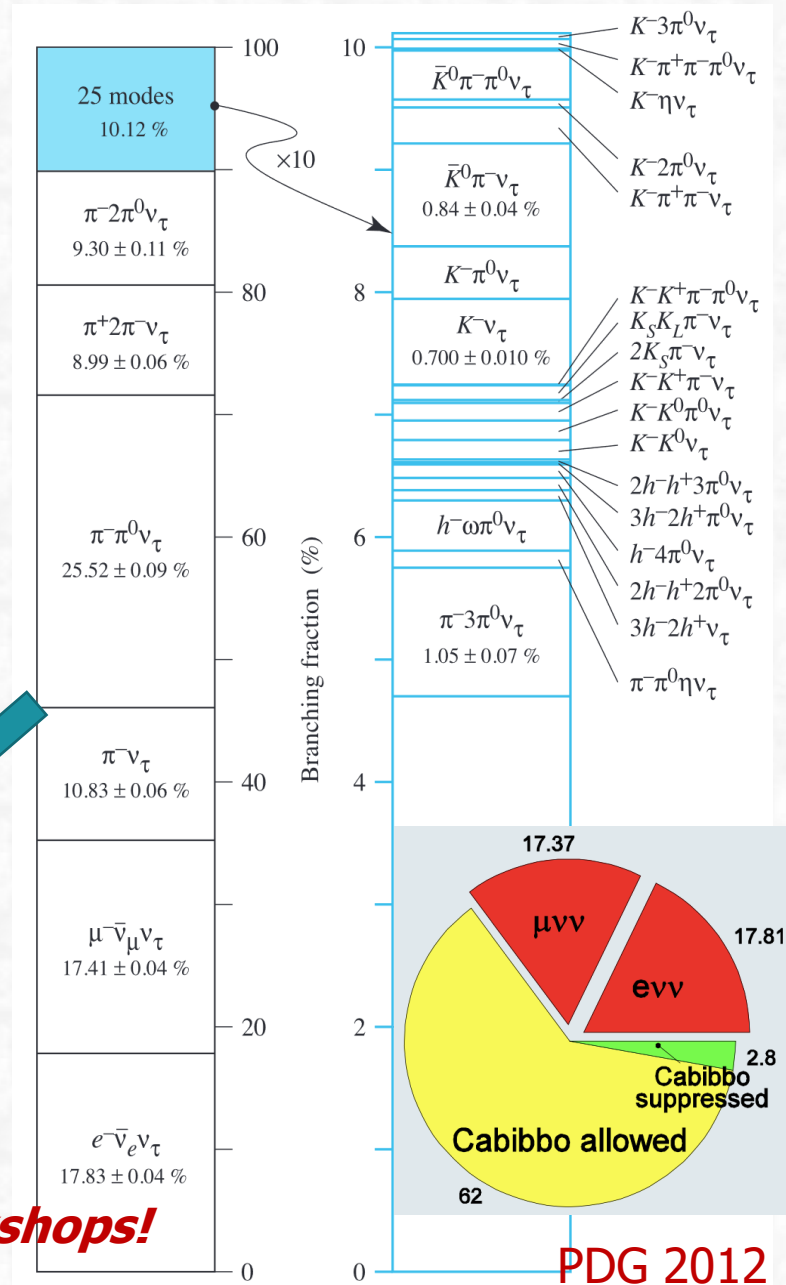
□ In SM, τ decays via **charged-current weak interaction**:

- purely (radiative) leptonic: $\tau \rightarrow \nu_\tau \ell \bar{\nu}_\ell$, $\tau \rightarrow \nu_\tau \ell \bar{\nu}_\ell \gamma$, ...
- semi-leptonic (hadronic): $\tau \rightarrow \nu_\tau \pi$, $\tau \rightarrow \nu_\tau K \pi$, ...
- rare and forbidden LFV, LNV, BNV, ...: $\tau \rightarrow \mu \gamma, \mu \mu \mu, \dots$

6 modes account for 90%, and 25 modes for the last 10%

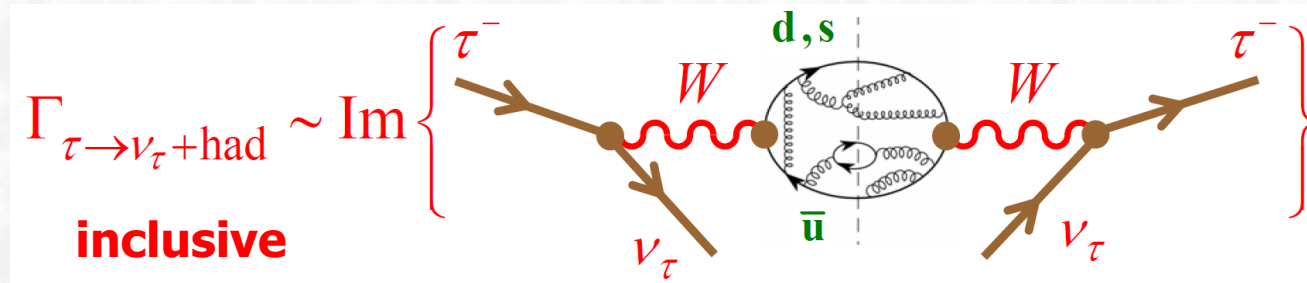
□ τ : the only lepton that is heavy enough to decay into hadrons with a branching fraction of $\text{Br} \simeq 65\%$

□ Tau lepton: very rich phenomenology; **biennial tau workshops!**



Semi-leptonic (hadronic) tau decays

- Can be used to extract the **fundamental SM parameters**: $\alpha_s(m_\tau)$, m_s , $|V_{us}|$, ...

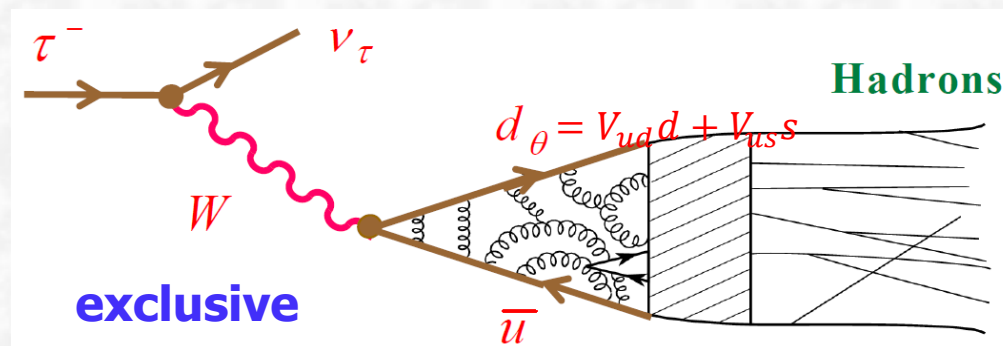


$$\alpha_s(m_\tau^2) = 0.2983 \pm 0.0101$$

RGE ↓ Boito et al., 2502.08147

$$\alpha_s(m_Z^2) = 0.1159 \pm 0.0014$$

- An ideal **low-energy QCD-testing laboratory**: how QCD currents are hadronized, and further information about the hadronic resonance parameters (M_R , Γ_R), ...



Decay channel
$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$
$\tau^- \rightarrow K^- K_S \nu_\tau$
$\tau^- \rightarrow K_S \pi^- \nu_\tau$
$\tau^- \rightarrow K^- \eta^{(\prime)} \nu_\tau$
$\tau^- \rightarrow \pi^- \eta^{(\prime)} \nu_\tau$

Resonances
$\rho(770), \rho(1450), \rho(1700)$
$\rho(770), \rho(1450), \rho(1700)$
$K^*(892), K^*(1410)$
$K^*(1410)$
$a_0(980)$

- Offer further possibilities for **CPV studies in the SM & beyond** [I. I. Bigi, 1210.2968; 2111.08126]

Semi-leptonic (hadronic) tau decays

□ Amplitude for a generic hadronic tau decay: $\mathcal{M}(\tau \rightarrow H \nu_\tau) = \frac{G_F}{\sqrt{2}} V_{CKM} [\bar{u}_{\nu_\tau} \gamma^\mu (1 - \gamma_5) u_\tau] H_\mu$

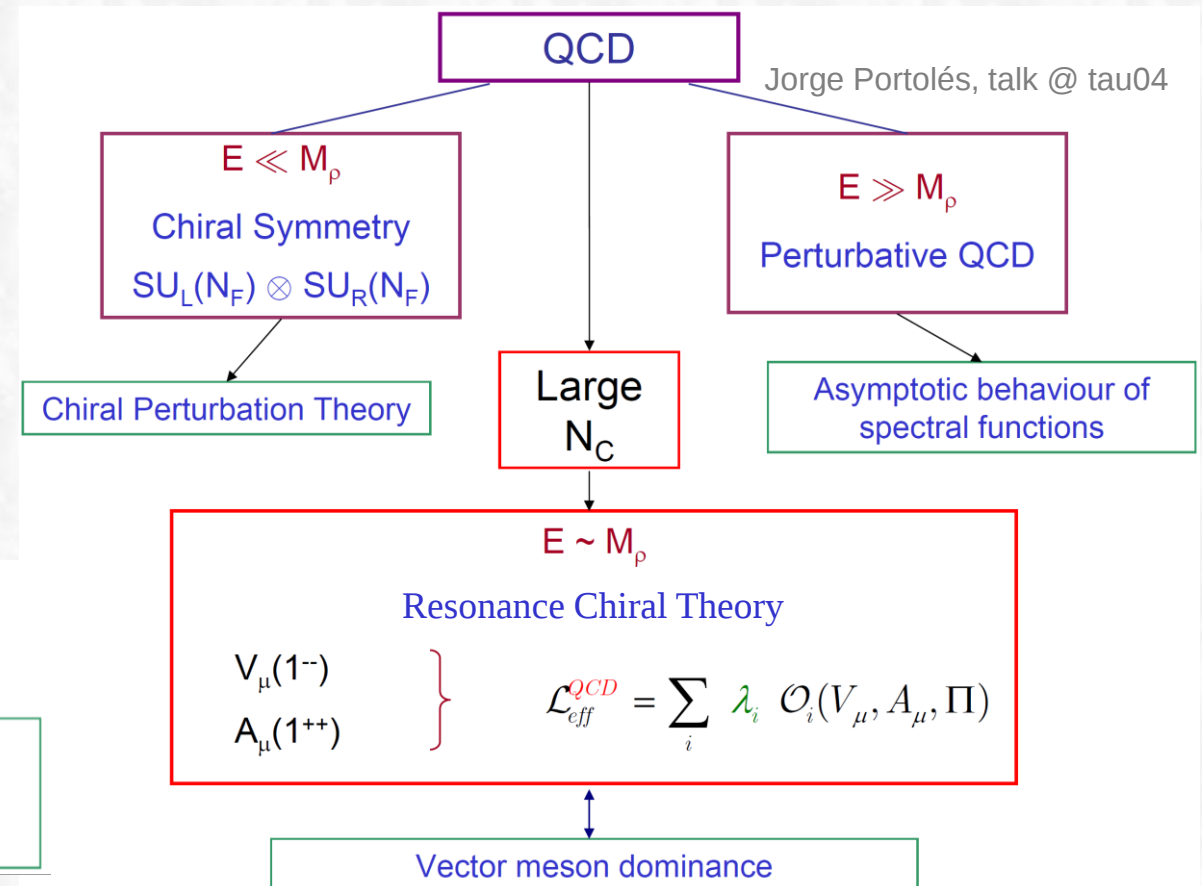
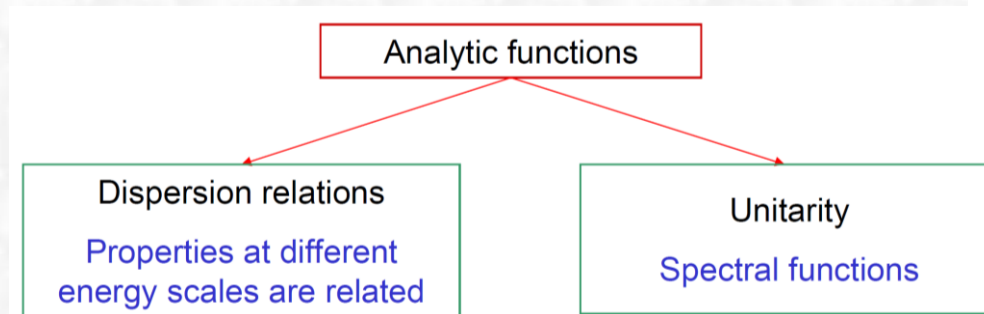
$$H_\mu = \langle H | (\mathcal{V}_\mu - \mathcal{A}_\mu) e^{i\mathcal{L}_{QCD}} | 0 \rangle$$

$$= \sum_i \underbrace{(\dots)}_{\text{Lorentz structure}}^i \underbrace{F_i(q^2, \dots)}_{\text{Form factor}}$$

Lorentz structure

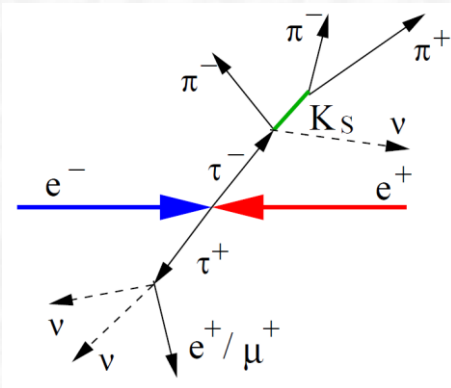
encoding strong interaction effects

□ Main task: evaluate these **FFs**



Why $\tau \rightarrow K_S \pi \nu_\tau$ decays

□ Have the **largest Br** among semi-lep. decays with 1 kaon [D. Epifanov et al. [Belle], PLB 654 (2007) 65]



$$\text{Br}(\tau \rightarrow K_S \pi \nu_\tau) = (0.404 \pm 0.002(\text{stat.}) \pm 0.013(\text{syst.}))\%$$

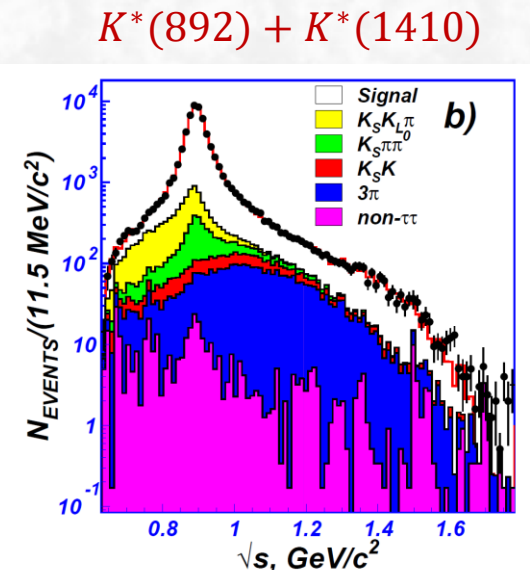
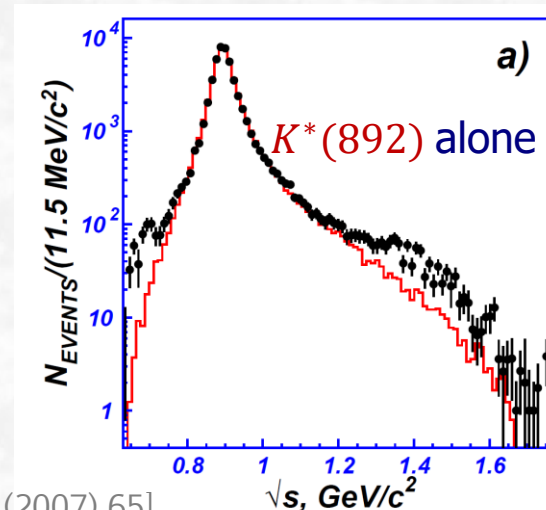
➤ Differential decay distribution within the SM:

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 |V_{us}|^2 M_\tau^3}{32\pi^3 s} S_{\text{EW}} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2 \frac{s}{M_\tau^2}\right) q_{K\pi}^3 |F_+^{K\pi}(s)|^2 + \frac{3\Delta_{K\pi}^2}{4s} q_{K\pi} |F_0^{K\pi}(s)|^2 \right]$$

➤ Hadronic currents parametrized by **two FFs** in SM:

$$J^\mu = F_+(q^2) \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) (q_1 - q_2)_\nu + F_0(q^2) q^\mu$$

- ✓ $K^*(892)$ alone not sufficient to describe the $K\pi$ spectrum
- ✓ Fitted result with $K^*(892) + K^*(1410)$ can reproduce data well [D. Epifanov et al. [Belle], PLB 654 (2007) 65]



Why $\tau \rightarrow K_S \pi \nu_\tau$ decays

□ Promising for CPV studies both within the SM and beyond

[I. I. Bigi, 1210.2968; 2111.08126]

➤ Decay-rate asymmetry in $\tau \rightarrow K_S \pi \nu_\tau$ decays:

$$A_{\text{CP}}^{\text{rate}} \equiv \frac{\Gamma(\tau^+ \rightarrow [\pi^+ \pi^-] "K_S" \pi^+ \bar{\nu}_\tau) - \Gamma(\tau^- \rightarrow [\pi^+ \pi^-] "K_S" \pi^- \nu_\tau)}{\Gamma(\tau^+ \rightarrow [\pi^+ \pi^-] "K_S" \pi^+ \bar{\nu}_\tau) + \Gamma(\tau^- \rightarrow [\pi^+ \pi^-] "K_S" \pi^- \nu_\tau)}$$

BaBar Collaboration, PRD 85 (2012) 031102

Belle II, 2604.23757

$$\begin{aligned} \text{2.8}\sigma \\ \text{deviation} \end{aligned} \quad \begin{cases} A_{\text{CP}}^{\text{Exp}} = (-3.6 \pm 2.3 \pm 1.1) \times 10^{-3} \\ A_{\text{CP}}^{\text{SM}} = (3.6 \pm 0.1) \times 10^{-3} \end{cases}$$

I. Bigi and A. I. Sanda PLB 625 (2005) 47

Y. Grossman and Y. Nir, JHEP 04 (2012) 002

➤ CP asymmetry in the angular distribution of $\tau \rightarrow K_S \pi \nu_\tau$ decays

$$A_{\text{CP}}^i = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^1 \cos \alpha \left[\frac{d^2 \Gamma(\tau^- \rightarrow K_S \pi^- \nu_\tau)}{ds d \cos \alpha} - \frac{d^2 \Gamma(\tau^+ \rightarrow K_S \pi^+ \bar{\nu}_\tau)}{ds d \cos \alpha} \right] ds d \cos \alpha}{\frac{1}{2} \int_{s_{1,i}}^{s_{2,i}} \int_{-1}^1 \left[\frac{d^2 \Gamma(\tau^- \rightarrow K_S \pi^- \nu_\tau)}{ds d \cos \alpha} + \frac{d^2 \Gamma(\tau^+ \rightarrow K_S \pi^+ \bar{\nu}_\tau)}{ds d \cos \alpha} \right] ds d \cos \alpha}$$

Belle Collaboration, PRL 107 (2011) 131801

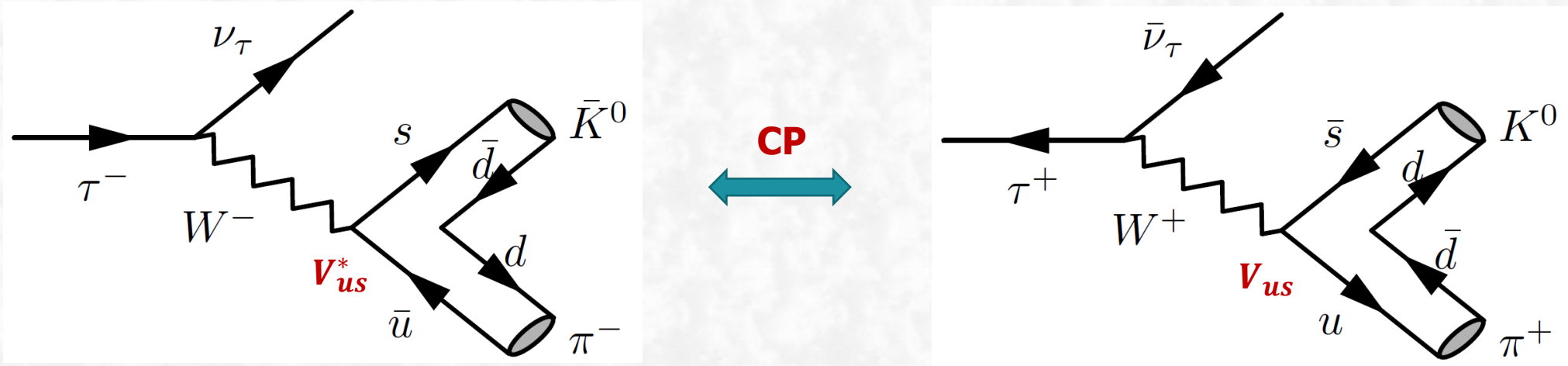
compatible with zero with a sensitivity of $\mathcal{O}(10^{-3})$

$\sqrt{s}$ [GeV]	$A_{\text{SM},i}^{\text{CP}} [10^{-3}]$	$A_{\text{exp},i}^{\text{CP}} [10^{-3}]$
0.625 – 0.890	0.39 ± 0.01	$7.9 \pm 3.0 \pm 2.8$
0.890 – 1.110	0.04 ± 0.01	$1.8 \pm 2.1 \pm 1.4$
1.110 – 1.420	0.12 ± 0.02	$-4.6 \pm 7.2 \pm 1.7$
1.420 – 1.775	0.27 ± 0.05	$-2.3 \pm 19.1 \pm 5.5$

□ Can be used to probe many BSM effects: $H^\pm$, leptoquark, ...

$\tau \rightarrow K_S \pi \nu_\tau$ decays within the SM

□ Tree-level Feynman diagrams in the SM: $\tau^\mp$ only decays into $\bar{K}^0(K^0)$ due to $\Delta S = \Delta Q$ rule



□ In the SM, V_{us} is real (i.e. no weak phase) and the two CP-related processes have the same strong phase



$$\mathcal{A}(\tau^+ \rightarrow K^0 \pi^+ \bar{\nu}_\tau) = \mathcal{A}(\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau)$$



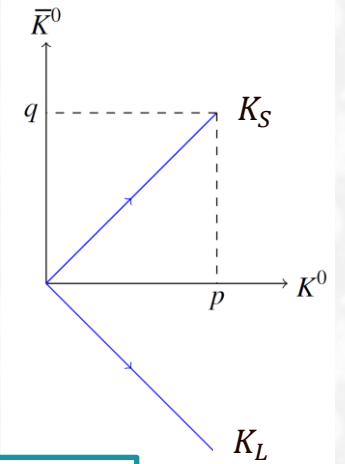
no CP asymmetries
in both **decay rate** &
angular distribution

$\tau \rightarrow K_S \pi \nu_\tau$ decays within the SM

❑ **Caveat:** due to $K^0 - \bar{K}^0$ mixing, the exp. reconstructed kaons are the **mass** ($|K_S\rangle, |K_L\rangle$) rather than the **flavor** ($|K^0\rangle, |\bar{K}^0\rangle$) eigenstates

$$\begin{aligned} |K_S^0\rangle &= \frac{(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle}{\sqrt{2(1+|\epsilon|^2)}}, & |K^0\rangle &= \frac{\sqrt{2(1+|\epsilon|^2)}}{2(1+\epsilon)} [|K_S^0\rangle + |K_L^0\rangle], \\ |K_L^0\rangle &= \frac{(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle}{\sqrt{2(1+|\epsilon|^2)}}, & |\bar{K}^0\rangle &= \frac{\sqrt{2(1+|\epsilon|^2)}}{2(1-\epsilon)} [|K_S^0\rangle - |K_L^0\rangle]. \end{aligned}$$

ϵ : characterizes the amount of CPV in the neutral kaon system



❑ In the **absence of CPV**, $\epsilon = 0$, we have in the SM that :

$$\mathcal{A}(\tau^+ \rightarrow K^0 \pi^+ \bar{\nu}_\tau) = \mathcal{A}(\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau)$$

$$|K_S\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}}$$

$$|K_L\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}$$



$$\begin{aligned} \frac{d^2\Gamma(\tau^- \rightarrow K_{S,L} \pi^- \nu_\tau)}{ds d\cos\alpha} &= \frac{1}{2} \frac{d^2\Gamma(\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau)}{ds d\cos\alpha} \\ \frac{d^2\Gamma(\tau^+ \rightarrow K_{S,L} \pi^+ \bar{\nu}_\tau)}{ds d\cos\alpha} &= \frac{1}{2} \frac{d^2\Gamma(\tau^+ \rightarrow K^0 \pi^+ \bar{\nu}_\tau)}{ds d\cos\alpha} \end{aligned}$$



no CP asymmetries
in both **decay rate & angular distribution**

❑ However, CPV in neutral kaon system is well established: $|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$

$\tau \rightarrow K_S \pi \nu_\tau$ decays within the SM

- Once **CPV in $K^0 - \bar{K}^0$ mixing** considered, **non-zero CP asymmetries** appear in the decays
- Experimentally, the K_S intermediate state is reconstructed via the **$\pi^+ \pi^-$ final state**
- When the kaon decay time is long enough, the **$\pi^+ \pi^-$ final state** can arise not only from K_S , but also from K_L , due to the **CPV in $K^0 - \bar{K}^0$ mixing**



the interference between K_S & K_L amplitudes is important for studying CPV!

- Time-dependent & doubly differential decay widths:

$$\frac{d^2\Gamma(\tau^- \rightarrow K_{S,L}\pi^-\nu_\tau \rightarrow [\pi^+\pi^-]\pi^-\nu_\tau)}{ds d\cos\alpha} = \frac{d^2\Gamma(\tau^- \rightarrow \bar{K}^0\pi^-\nu_\tau)}{ds d\cos\alpha} \Gamma(\bar{K}^0(t) \rightarrow \pi^+\pi^-),$$
$$\frac{d^2\Gamma(\tau^+ \rightarrow K_{S,L}\pi^+\bar{\nu}_\tau \rightarrow [\pi^+\pi^-]\pi^+\bar{\nu}_\tau)}{ds d\cos\alpha} = \frac{d^2\Gamma(\tau^+ \rightarrow K^0\pi^+\bar{\nu}_\tau)}{ds d\cos\alpha} \Gamma(K^0(t) \rightarrow \pi^+\pi^-),$$

→ s : the $K\pi$ invariant mass squared

→ α : the angle between the directions of K & τ seen in the $K\pi$ rest frame

Time-dep. CPA in the angular distribution

□ Time-dep. CP asymmetry in the angular distribution:

$$(d\omega = ds d\cos\alpha)$$

- bin choice $[s_{1,i}, s_{2,i}]$
- time interval $[t_1, t_2]$
- exp.-dep. effects parametrized by $F(t)$

$$A_K^{CP}(t_1, t_2) = \frac{\int_{t_1}^{t_2} F(t) [\bar{\Gamma}_{\pi^+\pi^-}(t) - \Gamma_{\pi^+\pi^-}(t)] dt}{\int_{t_1}^{t_2} F(t) [\bar{\Gamma}_{\pi^+\pi^-}(t) + \Gamma_{\pi^+\pi^-}(t)] dt}$$

→ CPA in kaon decay

$$A_{\tau,i}^{CP} = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^1 \left[\frac{d\Gamma^{\tau-}}{d\omega} - \frac{d\Gamma^{\tau+}}{d\omega} \right] d\omega}{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^1 \left[\frac{d\Gamma^{\tau-}}{d\omega} + \frac{d\Gamma^{\tau+}}{d\omega} \right] d\omega}$$

→ CPA in tau decay

$$A_i^{CP}(t_1, t_2) = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^1 \cos\alpha \left[\frac{d\Gamma^{\tau-}}{d\omega} \int_{t_1}^{t_2} F(t) \bar{\Gamma}_{\pi^+\pi^-}(t) dt - \frac{d\Gamma^{\tau+}}{d\omega} \int_{t_1}^{t_2} F(t) \Gamma_{\pi^+\pi^-}(t) dt \right] d\omega}{\frac{1}{2} \int_{s_{1,i}}^{s_{2,i}} \int_{-1}^1 \left[\frac{d\Gamma^{\tau-}}{d\omega} \int_{t_1}^{t_2} F(t) \bar{\Gamma}_{\pi^+\pi^-}(t) dt + \frac{d\Gamma^{\tau+}}{d\omega} \int_{t_1}^{t_2} F(t) \Gamma_{\pi^+\pi^-}(t) dt \right] d\omega}$$

$$= \frac{\left(\langle \cos\alpha \rangle_i^{\tau-} + \langle \cos\alpha \rangle_i^{\tau+} \right) A_K^{CP}(t_1, t_2) + \left(\langle \cos\alpha \rangle_i^{\tau-} - \langle \cos\alpha \rangle_i^{\tau+} \right)}{1 + A_K^{CP}(t_1, t_2) \cdot A_{\tau,i}^{CP}}$$

tau decay width weighted by $\cos\alpha$

$$\langle \cos\alpha \rangle_i^{\tau-} + \langle \cos\alpha \rangle_i^{\tau+} = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^1 \cos\alpha \left[\frac{d\Gamma^{\tau-}}{d\omega} + \frac{d\Gamma^{\tau+}}{d\omega} \right] d\omega}{\frac{1}{2} \int_{s_{1,i}}^{s_{2,i}} \int_{-1}^1 \left[\frac{d\Gamma^{\tau-}}{d\omega} + \frac{d\Gamma^{\tau+}}{d\omega} \right] d\omega}$$

$$\langle \cos\alpha \rangle_i^{\tau-} - \langle \cos\alpha \rangle_i^{\tau+} = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^1 \cos\alpha \left[\frac{d\Gamma^{\tau-}}{d\omega} - \frac{d\Gamma^{\tau+}}{d\omega} \right] d\omega}{\frac{1}{2} \int_{s_{1,i}}^{s_{2,i}} \int_{-1}^1 \left[\frac{d\Gamma^{\tau-}}{d\omega} + \frac{d\Gamma^{\tau+}}{d\omega} \right] d\omega}$$

□ Within the SM:

$$\mathcal{A}(\tau^+ \rightarrow K^0 \pi^+ \bar{\nu}_\tau) = \mathcal{A}(\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau)$$



$$\frac{d\Gamma^{\tau+}}{d\omega} = \frac{d\Gamma^{\tau-}}{d\omega} \implies A_{\tau,i}^{CP} = 0, \quad \langle \cos\alpha \rangle_i^{\tau-} = \langle \cos\alpha \rangle_i^{\tau+}$$

$$\Downarrow$$

$$A_i^{CP}(t_1, t_2) = 2 \langle \cos\alpha \rangle_i^{\tau-} A_K^{CP}(t_1, t_2)$$

Decay-rate asymmetry within the SM

□ Within the SM, since $A_{\tau}^{CP} = 0$, the **decay-rate asymmetry** is exactly equal to A_K^{CP} :

$$A_{CP}^{\text{rate}}(t_1, t_2) = \frac{\Gamma_{\tau^+} \int_{t_1}^{t_2} dt F(t) \Gamma(K^0(t) \rightarrow \pi^+ \pi^-) - \Gamma_{\tau^-} \int_{t_1}^{t_2} dt F(t) \Gamma(\bar{K}^0(t) \rightarrow \pi^+ \pi^-)}{\Gamma_{\tau^+} \int_{t_1}^{t_2} dt F(t) \Gamma(K^0(t) \rightarrow \pi^+ \pi^-) + \Gamma_{\tau^-} \int_{t_1}^{t_2} dt F(t) \Gamma(\bar{K}^0(t) \rightarrow \pi^+ \pi^-)}$$

$$= \frac{A_{\tau}^{CP} + A_K^{CP}(t_1, t_2)}{1 + A_{\tau}^{CP} A_K^{CP}(t_1, t_2)} = A_K^{CP}(t_1, t_2)$$

$$A_K^{CP}(t_1, t_2) = \frac{\int_{t_1}^{t_2} F(t) [\bar{\Gamma}_{\pi^+ \pi^-}(t) - \Gamma_{\pi^+ \pi^-}(t)] dt}{\int_{t_1}^{t_2} F(t) [\bar{\Gamma}_{\pi^+ \pi^-}(t) + \Gamma_{\pi^+ \pi^-}(t)] dt}$$

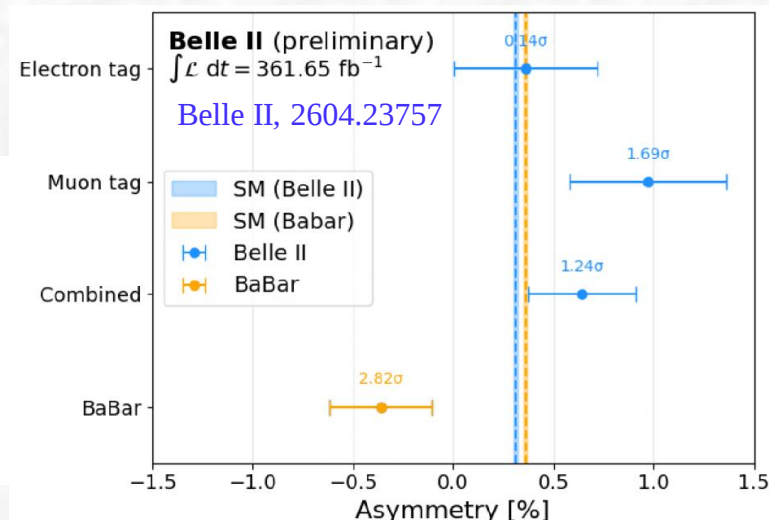
- $A_K^{CP}(t_1, t_2)$: CPV in $K^0 - \bar{K}^0$ mixing

$$A_K^{CP}(t_1 \ll \Gamma_S^{-1}, \Gamma_S^{-1} \ll t_2 \ll \Gamma_L^{-1}) \approx -2\text{Re}(\epsilon_K) = -(3.32 \pm 0.06) \times 10^{-3}$$

$$F(t) = \begin{cases} 1 & t_1 < t < t_2 \\ 0 & \text{otherwise.} \end{cases}$$

Y. Grossman and Y. Nir, JHEP 04 (2012) 002

↑
efficiency function $F(t)$ provided by BaBar



□ SM result :

$$A_{\text{SM}}^{CP} = (3.6 \pm 0.1) \times 10^{-3}$$

2.82σ

$$A_{\text{BaBar}}^{CP} = (-3.6 \pm 2.3 \pm 1.1) \times 10^{-3}$$

1.24σ

$$A_{\text{Belle II}}^{CP} = (7.1 \pm 2.6 \pm 0.6 \pm 1.5) \times 10^{-3}$$

Time-dep. CPA in the angular distribution

□ Within the SM, **CPA in angular distribution:**

$$A_i^{CP}(t_1, t_2) = 2 \langle \cos \alpha \rangle_i^{\tau^-} A_K^{CP}(t_1, t_2)$$

$$A_K^{CP}(t_1 \ll \Gamma_S^{-1}, \Gamma_S^{-1} \ll t_2 \ll \Gamma_L^{-1}) \approx -2\text{Re}(\epsilon_K) = -(3.32 \pm 0.06) \times 10^{-3} \quad \text{as input!}$$

□ For $\langle \cos \alpha \rangle_i^{\tau^\pm}$: depending on the two **normalized FFs**

$$\tilde{F}_{+,0}(s) = F_{+,0}(s) / F_+(0)$$

$$\begin{aligned} \langle \cos \alpha \rangle^{\tau^-}(s) &= \frac{\int_{-1}^1 \cos \alpha \left(\frac{d^2 \Gamma^{\tau^-}}{ds d \cos \alpha} \right) d \cos \alpha}{\int_{-1}^1 \left(\frac{d^2 \Gamma^{\tau^-}}{ds d \cos \alpha} \right) d \cos \alpha} \\ &= \frac{-2 \Delta_{K\pi} \text{Re}[\tilde{F}_+(s) \tilde{F}_0^*(s)] \lambda^{1/2}(s, M_K^2, M_\pi^2)}{|\tilde{F}_+(s)|^2 \left(1 + \frac{2s}{m_\tau^2} \right) \lambda(s, M_K^2, M_\pi^2) + 3 \Delta_{K\pi}^2 |\tilde{F}_0(s)|^2} \end{aligned}$$

$$\langle \bar{K}^0(p_K) \pi^-(p_\pi) | \bar{s} \gamma^\mu u | 0 \rangle = \left[(p_K - p_\pi)^\mu - \frac{\Delta_{K\pi}}{s} q^\mu \right] F_+(s) + \frac{\Delta_{K\pi}}{s} q^\mu F_0(s)$$

→ due to interference between **vector** & **scalar** FFs

□ **Hadronic FFs:**

- Vector form factor : **the thrice-subtracted dispersion representation**
D.R. Boito, R. Escribano and M. Jamin, Eur. Phys. J. C59 (2009) 821

$$F_+(s) = \exp \left\{ \lambda'_+ \frac{s}{M_{\pi^-}^2} + \frac{1}{2} (\lambda''_+ - \lambda'^2_+) \frac{s^2}{M_{\pi^-}^4} + \frac{s^3}{\pi} \int_{s_{K\pi}}^{s_{cut}} ds' \frac{\delta_+(s')}{(s')^3 (s' - s - i\epsilon)} \right\},$$

- Scalar form factor : **the coupled-channel dispersive representation**
M. Jamin, J.A. Oller and A. Pich, Nucl. Phys. B622 (2002) 279

$$F_0^1(s) = \frac{1}{\pi} \sum_{j=1}^3 \int_{s_j}^{\infty} ds' \frac{\sigma_j(s') F_0^j(s') t_0^{1 \rightarrow j}(s')^*}{s' - s - i\epsilon}, \quad (1 \equiv K\pi, 2 \equiv K\eta, \text{ and } 3 \equiv K\eta')$$

For CPV studies, the **Breit-Wigner** forms are **not applicable**, since they **violates the Watson's theorem & unphysical!**

Time-dep. CPA in the angular distribution

□ Numerical results for $A_i^{CP}(t_1, t_2)$ in **four mass bins**:

$\sqrt{s}$ [GeV]	$A_{SM,i}^{CP} [10^{-3}]$	$A_{exp,i}^{CP} [10^{-3}]$	n_i/N_s [%]
0.625 – 0.890	0.39 ± 0.01	$7.9 \pm 3.0 \pm 2.8$	36.53 ± 0.14
0.890 – 1.110	0.04 ± 0.01	$1.8 \pm 2.1 \pm 1.4$	57.85 ± 0.15
1.110 – 1.420	0.12 ± 0.02	$-4.6 \pm 7.2 \pm 1.7$	4.87 ± 0.04
1.420 – 1.775	0.27 ± 0.05	$-2.3 \pm 19.1 \pm 5.5$	0.75 ± 0.02

Our SM predictions still below the **Belle sensitivity of $\mathcal{O}(10^{-3})$** , but are expected to be detectable at **Belle II**, with $\sqrt{70}$ times more sensitive results!

Belle-II, *PTEP* 2019 (2019) 123C01

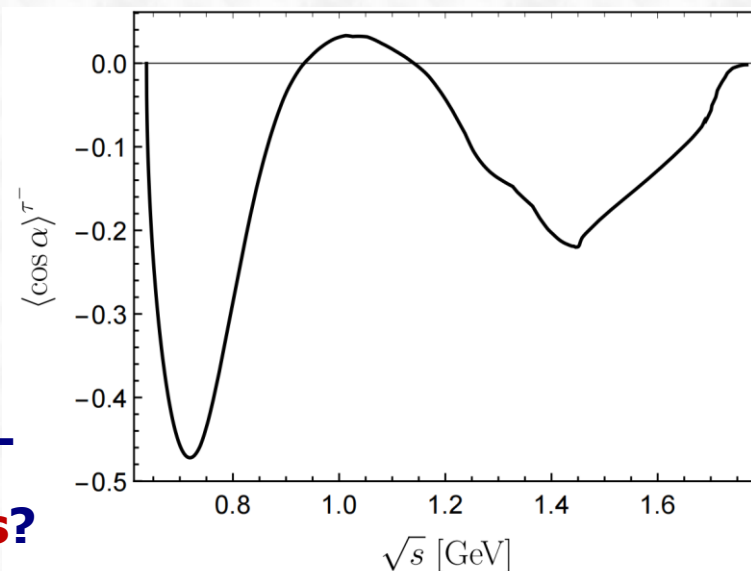
□ Two more predictions:

as large as the SM prediction for A_{CP}^{rate}

$$A_{SM}^{CP} = (3.6 \pm 0.1) \times 10^{-3}$$

$$A_i^{CP}(t_1, t_2) = \begin{cases} (3.06 \pm 0.06) \times 10^{-3}, & 0.70 \text{ GeV} < \sqrt{s} < 0.75 \text{ GeV} \\ (1.38 \pm 0.18) \times 10^{-3}, & 1.40 \text{ GeV} < \sqrt{s} < 1.50 \text{ GeV} \end{cases}$$

□ Interesting to see if **Belle II & STCF** can measure the CP-violating angular observables in **such two mass intervals**?



$\tau \rightarrow K_S \pi \nu_\tau$ in a general EFT framework

- When NP presents in tau decays:

$$\mathcal{A}(\tau^+ \rightarrow K^0 \pi^+ \bar{\nu}_\tau) \neq \mathcal{A}(\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau)$$

$$\hookrightarrow \frac{d\Gamma^{\tau^+}}{d\omega} \neq \frac{d\Gamma^{\tau^-}}{d\omega} \implies A_{\tau,i}^{CP} \neq 0, \quad \langle \cos \alpha \rangle_i^{\tau^-} \neq \langle \cos \alpha \rangle_i^{\tau^+}$$

- CP-violating observables:

$$A_i^{CP} \simeq \left(\langle \cos \alpha \rangle_i^{\tau^-} + \langle \cos \alpha \rangle_i^{\tau^+} \right) A_K^{CP} + \left(\langle \cos \alpha \rangle_i^{\tau^-} - \langle \cos \alpha \rangle_i^{\tau^+} \right)$$

- General low-energy effective Lagrangian invariant under $SU(3)_C \otimes U(1)_{em}$:

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & - \frac{G_F V_{us}}{\sqrt{2}} \left\{ \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau \cdot \bar{u} [\gamma^\mu - (1 - 2\hat{e}_R) \gamma^\mu \gamma_5] s \right. \\ & \left. + \bar{\tau} (1 - \gamma_5) \nu_\tau \cdot \bar{u} [\hat{e}_S - \hat{e}_P \gamma_5] s + 2\hat{e}_T \bar{\tau} \sigma_{\mu\nu} (1 - \gamma_5) \nu_\tau \cdot \bar{u} \sigma^{\mu\nu} s \right\} + \text{h.c.} \end{aligned}$$

- Decay amplitude for $\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau$:

scalar operator + tensor operator

$$\begin{aligned} \mathcal{M} &= \mathcal{M}_V + \mathcal{M}_S + \mathcal{M}_T \\ &= \frac{G_F V_{us}}{\sqrt{2}} [L_\mu H^\mu + \hat{e}_S^* L H + 2\hat{e}_T^* L_{\mu\nu} H^{\mu\nu}] \end{aligned}$$

Tensor form factors

□ Leptonic currents:

$$\begin{aligned} L &= \bar{u}(p_{\nu_\tau})(1 + \gamma_5)u(p_\tau), \\ L_\mu &= \bar{u}(p_{\nu_\tau})\gamma_\mu(1 - \gamma_5)u(p_\tau), \\ L_{\mu\nu} &= \bar{u}(p_{\nu_\tau})\sigma_{\mu\nu}(1 + \gamma_5)u(p_\tau), \end{aligned}$$

□ Hadronic matrix elements:

$$\begin{aligned} H &= \langle \pi^- \bar{K}^0 | \bar{s}u | 0 \rangle = F_S(s) \\ H^\mu &= \langle \pi^- \bar{K}^0 | \bar{s}\gamma^\mu u | 0 \rangle = Q^\mu F_+(s) + \frac{\Delta_{K\pi}}{s} q^\mu F_0(s) \\ H^{\mu\nu} &= \langle \pi^- \bar{K}^0 | \bar{s}\sigma^{\mu\nu} u | 0 \rangle = iF_T(s) (p_K^\mu p_\pi^\nu - p_\pi^\mu p_K^\nu) \end{aligned}$$

□ **$K\pi$ tensor FF:** due to lack of enough exp. data, the **once-subtracted dispersion representation**

$$F_T(s) = F_T(0) \exp \left\{ \frac{s}{\pi} \int_{s_{K\pi}}^{\infty} ds' \frac{\delta_T(s')}{s'(s' - s - i\epsilon)} \right\}$$

➤ **for $F_T(0)$:** obtained from the lowest-order χ PT with tensor source

$$\bullet \mathcal{L}_4^{\chi\text{PT}} = \Lambda_1 \langle t_+^{\mu\nu} f_{+\mu\nu} \rangle - i\Lambda_2 \langle t_+^{\mu\nu} u_\mu u_\nu \rangle + \Lambda_3 \langle t_+^{\mu\nu} t^{+\mu\nu} \rangle + \Lambda_4 \langle t_+^{\mu\nu} \rangle^2$$

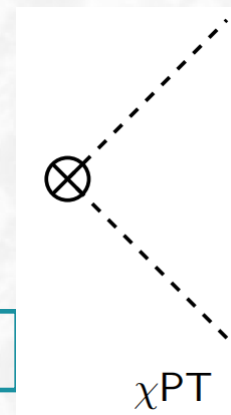


$$\left\langle \bar{K}^0(p_K) \pi^-(p_\pi) \left| \frac{\delta \mathcal{L}_4^{\chi\text{PT}}}{\delta \bar{t}_{\mu\nu}} \right| 0 \right\rangle = i \frac{\Lambda_2}{F_\pi^2} (p_K^\mu p_\pi^\nu - p_K^\nu p_\pi^\mu)$$

$$F_T(0) = \Lambda_2 / F_\pi^2, \text{ with } \Lambda_2 = (11.1 \pm 0.4) \text{ MeV}$$

O. Cata and V. Mateu, JHEP 09 (2007) 078

I. Baum *et al.*, PRD 84 (2011) 074503



Tensor form factors

□ To account for the **s -dependence** of these FFs, we have included **two spin-1 resonances** **$K^*(892)$ & $K^*(1410)$** ; both of them contribute dominantly to **$F_+(s)$ & $F_T(s)$**

➤ **$F_T(s)$: obtained with $R\chi T$ including these spin-1 resonances**

$$\bullet \mathcal{L}_6^{R\chi T} = \mathcal{L}_{kin}(\hat{V}_\mu) - \frac{1}{2\sqrt{2}} \left(f_V \langle \hat{V}_{\mu\nu} f_+^{\mu\nu} \rangle + ig_V \langle \hat{V}_{\mu\nu} [u^\mu, u^\nu] \rangle \right) - \bar{f}_V^T \langle \hat{V}_{\mu\nu} t_+^{\mu\nu} \rangle$$



Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. de Rafael, PLB 223 (1989) 425

$$F_T(s) = \frac{\Lambda_2}{F_\pi^2} \left[1 + \frac{\sqrt{2} f_V^T g_V}{\Lambda_2} \frac{s}{M_{K^*}^2 - s} + \frac{\sqrt{2} f_V^{T'} g_V'}{\Lambda_2} \frac{s}{M_{K^{*'}}^2 - s} \right]$$

$$= \frac{\Lambda_2}{F_\pi^2} \left[\frac{M_{K^*}^2 + \beta s}{M_{K^*}^2 - s} - \frac{\beta s}{M_{K^{*'}}^2 - s} \right]$$

energy-dep. width $\gamma_n(s)$

$$\tilde{F}_T(s) = F_T(s)/F_T(0)$$

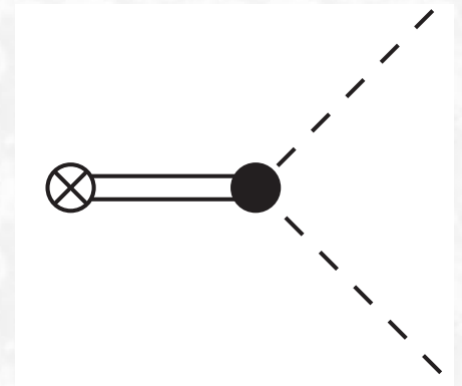
$$= \frac{m_{K^*}^2 - \kappa_{K^*} \tilde{H}_{K\pi}(0) + \beta s}{D(m_{K^*}, \gamma_{K^*})} - \frac{\beta s}{D(m_{K^{*'}}, \gamma_{K^{*'}})}$$

$\beta = \frac{\sqrt{2} f_V^T g_V}{\Lambda_2} - 1 \simeq \pm 0.75\gamma$: characterizes the **relative weight**

of the two resonances, and plays the same role as γ for $F_+(s)$

Feng-Zhi Chen, Xin-Qiang Li, Ya-Dong Yang,

Xin Zhang, PRD 100 (2019) 113006



Tensor form factors

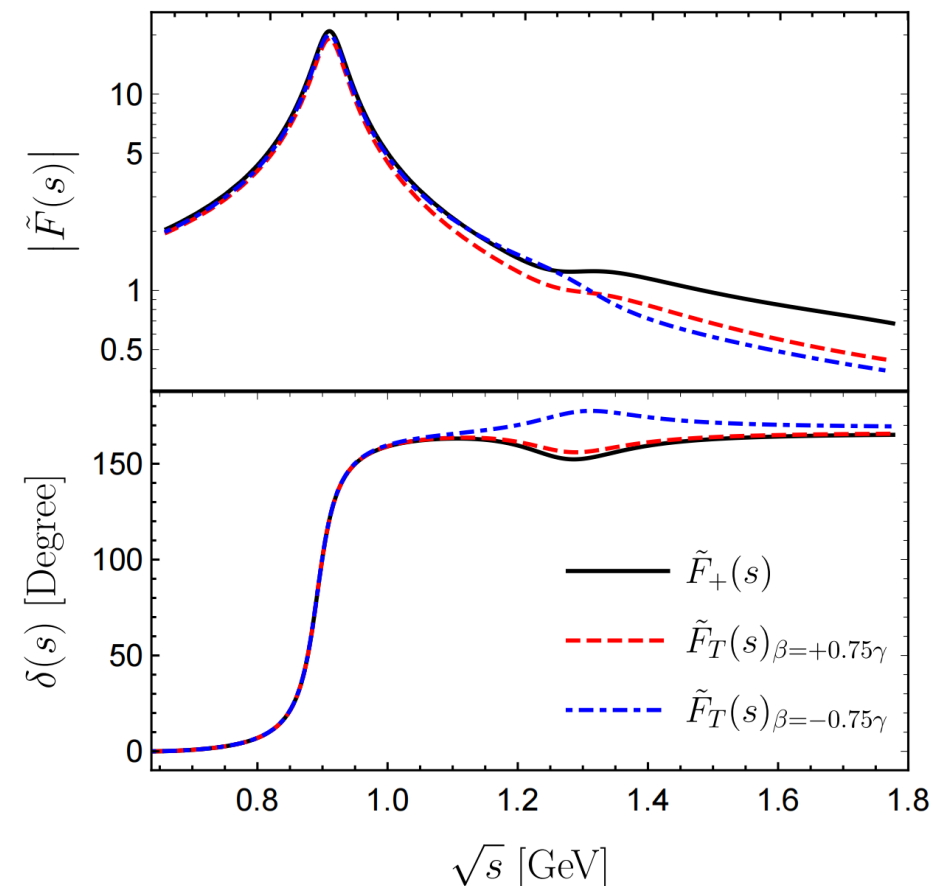
- Combining χ PT @ low s + R χ T @ intermediate s + asymptotic behaviors @ high s , we obtain the once-subtracted dispersion representation:

$$F_T(s) = F_T(0) \exp \left\{ \frac{s}{\pi} \int_{s_{K\pi}}^{\infty} ds' \frac{\delta_T(s')}{s'(s' - s - i\epsilon)} \right\}$$

$$\delta_T(s) = \begin{cases} \arctan \left[\frac{\Im \tilde{F}_T(s)}{\Re \tilde{F}_T(s)} \right], & s_{K\pi} < s < s_{\text{cut}} \\ n_T \pi, & s \geq s_{\text{cut}} \end{cases}$$

asymptotic $1/s$ as dictated by pQCD

- in elastic region (below ~ 1.2 GeV), $\delta_T(s) = \delta_+(s)$ as required by Watson's theorem [K. M. Watson, Phys. Rev. 95 (1954) 228]
- in inelastic region (above ~ 1.2 GeV), $\delta_T(s)$ and $\delta_+(s)$ start to behave differently due to different relative weights of the two resonances $K^*(892)$ & $K^*(1410)$



CP-violating observables in the general EFT

□ Decay-rate asymmetry:

only vector-tensor interference
as the only possible mechanism

$$A_{\text{CP}}^{\text{rate}}(\tau \rightarrow K \pi \nu_\tau) = \frac{\Gamma(\tau^+ \rightarrow K^0 \pi^+ \bar{\nu}_\tau) - \Gamma(\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau)}{\Gamma(\tau^+ \rightarrow K^0 \pi^+ \bar{\nu}_\tau) + \Gamma(\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau)}$$

$$= \frac{\text{Im}[\hat{e}_T] G_F^2 |V_{us}|^2 S_{\text{EW}}}{128 \pi^3 m_\tau^2 \Gamma(\tau \rightarrow K_S \pi \nu_\tau)} \int_{s_{K\pi}}^{m_\tau^2} ds \left(1 - \frac{m_\tau^2}{s}\right)^2 \lambda^{\frac{3}{2}}(s, M_K^2, M_\pi^2)$$

$$\times \underbrace{|F_T(s)| |F_+(s)| \sin[\delta_T(s) - \delta_+(s)]},$$

□ CPA in angular distribution:

from both scalar-vector and
scalar-tensor interferences

$$A_{\text{CP}}^i \simeq \Delta_{K\pi} S_{\text{EW}} \frac{N_s}{n_i} \int_{s_{1,i}}^{s_{2,i}} \left\{ - \frac{\text{Im}[\hat{e}_S]}{m_\tau(m_s - m_u)} \underbrace{\text{Im}[F_+(s)F_0^*(s)]} - \frac{2\text{Im}[\hat{e}_T]}{m_\tau} \underbrace{\text{Im}[F_T(s)F_0^*(s)]} \right.$$

$$\left. + \left[\left(\frac{1}{s} + \frac{\text{Re}[\hat{e}_S]}{m_\tau(m_s - m_u)} \right) \underbrace{\text{Re}[F_+(s)F_0^*(s)]} - \frac{2\text{Re}[\hat{e}_T]}{m_\tau} \underbrace{\text{Re}[F_T(s)F_0^*(s)]} \right] A_K^{\text{CP}} \right\} C(s) ds.$$

□ Constraints on $\text{Re}[\hat{e}_{S,T}]$: more stringent from decay rates of various exclusive τ decays

$$\text{Re}[\hat{e}_S] = (0.8_{-0.9}^{+0.8} \pm 0.3)\%, \quad \text{Re}[\hat{e}_T] = (0.9 \pm 0.7 \pm 0.4)\%$$

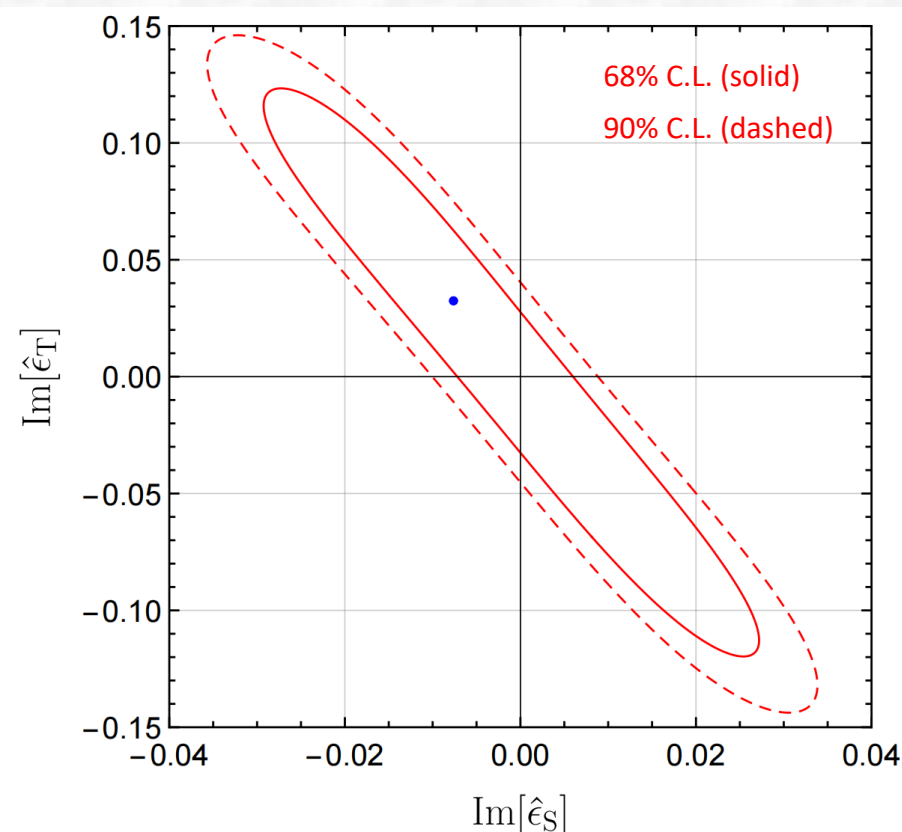
S. González-Solís, A. Miranda, J. Rendón and P. Roig, Phys. Lett. B 804 (2020) 135371

CP-violating observables in general EFT

□ Fit results on $\text{Im}[\hat{\epsilon}_{S,T}]$ from $\mathcal{B}_{\text{exp}}^{\tau^-}$ and $A_{\text{exp},i}^{CP}$ in four different bins from Belle:

$$\chi^2 = \sum_{i=1}^4 \left(\frac{A_{\text{exp},i}^{CP} - A_{\text{th},i}^{CP}}{\sigma_i} \right)^2 + \left(\frac{\mathcal{B}_{\text{exp}}^{\tau^-} - \mathcal{B}_{\text{th}}^{\tau^-}}{\sigma_{\mathcal{B}}} \right)^2$$

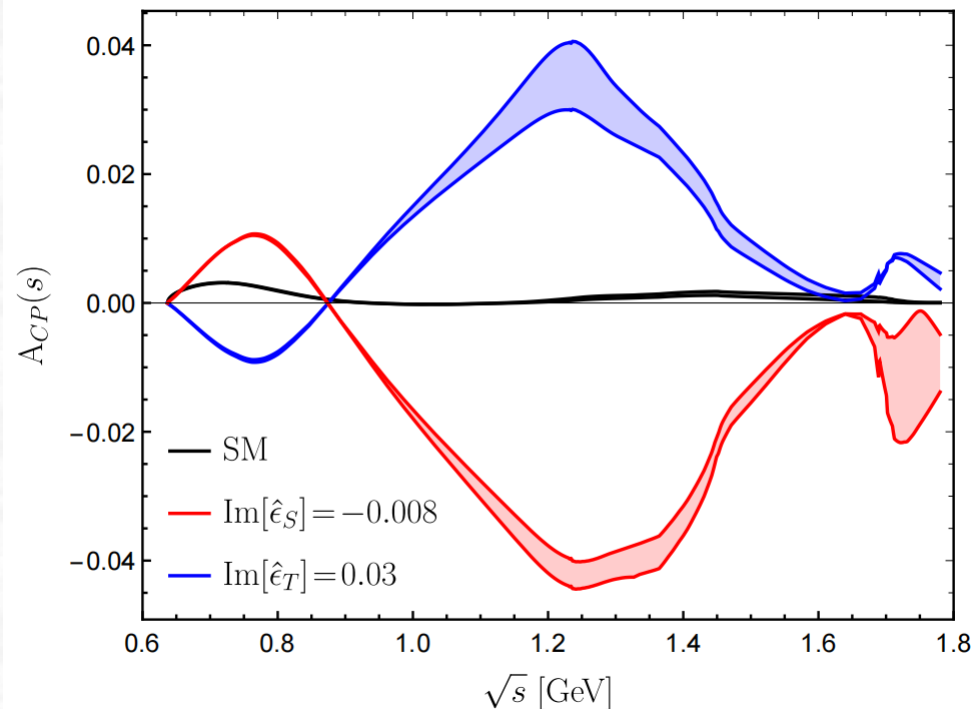
$$\text{Im}[\hat{\epsilon}_S] = -0.008 \pm 0.027, \quad \text{Im}[\hat{\epsilon}_T] = 0.03 \pm 0.12$$



- remarkably **negative correlation** between $\text{Im}[\hat{\epsilon}_S]$ & $\text{Im}[\hat{\epsilon}_T]$, as both vector & tensor FFs are dominated by $K^*(892)$ and $K^*(1410)$, and hence have almost the same phases, especially in **elastic region**
- bound on $\text{Im}[\hat{\epsilon}_S]$ consistent with $|\text{Im}(\eta_S)| < 0.026$ @ 90% C.L. obtained by Belle [PRL 107 (2011) 131801]
- upper bound on $\text{Im}[\hat{\epsilon}_T]$ only of $\mathcal{O}(10^{-1})$, which is much weaker than $2|\text{Im}[\hat{\epsilon}_T]| \leq 10^{-5}$ from neutron EDM & $D^0 - \bar{D}^0$ mixing [V. Cirigliano *et al.*, PRL 120 (2018) 141803]

CP-violating observables in general EFT

□ A_i^{CP} in the presence of non-standard **scalar & tensor** interactions:



- with best-fit values of $\text{Im}[\hat{\epsilon}_S]$ and $\text{Im}[\hat{\epsilon}_T]$, the CPA distributions have almost the **same magnitude** but **opposite in sign** in whole $K\pi$ invariant-mass region
- the non-standard scalar and tensor contributions are about **one order of magnitude larger** than that with the SM prediction

□ We strongly suggest **Belle II** & **STCF** to make more precise measurement of these **CP asymmetries in the angular distributions**

Belle-II, *PTEP* **2019** (2019) 123C01; H. Sang, X. Shi, X. Zhou, X. Kang and J. Liu, *CPC* **45** (2021) 053003

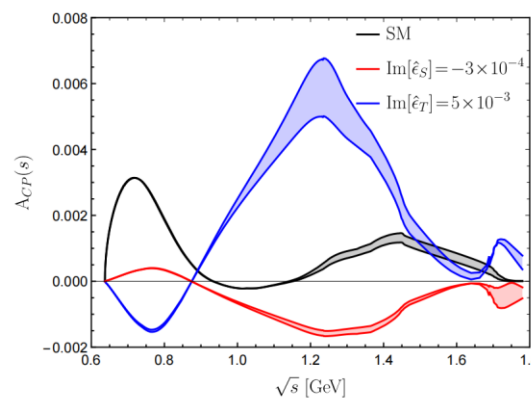
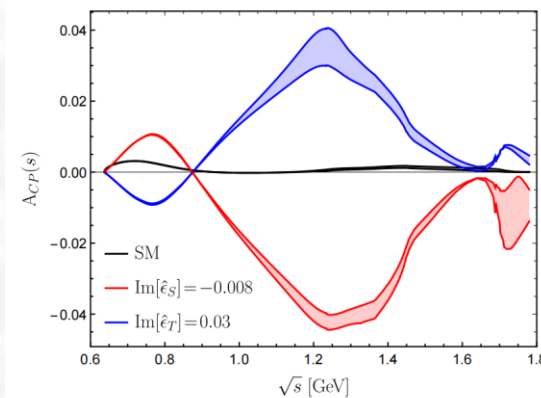
Summary

□ $\tau \rightarrow K_S \pi \nu_\tau$ decays: very promising for CPV studies

- In the SM, there exist both decay-rate asymmetry & CP asymmetry in angular distribution due to CPV in $K^0 - \bar{K}^0$ mixing, with results of $\mathcal{O}(10^{-3})$ and detectable @ Belle II & STCF
- Within a general EFT, only vector-tensor interference produces a direct decay-rate asymmetry, while both scalar-vector & scalar-tensor interferences possible for CPA in the angular distribution

□ With other bounds considered, 2.8σ deviation for A_{CP}^{rate} not easily explained by heavy NP

□ CP asymmetry in the angular distribution in three different cases:



sensitive to BSM scalar & tensor interactions!

□ Measurable @ Belle II & STCF?

Thank you for your attention!

Backup

Constraints on $\text{Im}[\hat{\epsilon}_{S,T}]$ from other processes

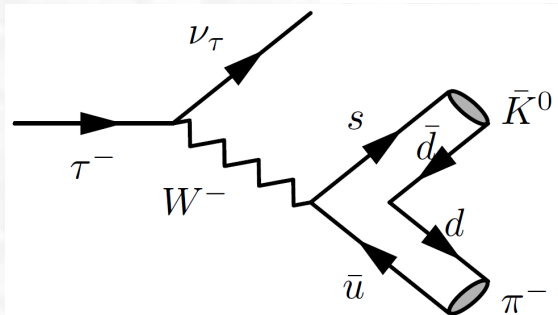
□ If BSM interactions originate from a **weakly-coupled heavy NP** well above the EW scale

↳ **$SU(2)_L$ invariance** of $\mathcal{L}_{eff}$ implies other processes to put further limits on **$\text{Im}[\hat{\epsilon}_{S,T}]$**

□ For the **tensor operator**: $\mathcal{L}_{\text{SMEFT}} \supset [C_{\ell equ}^{(3)}]_{klmn} (\bar{\ell}_L^i \sigma_{\mu\nu} e_{Rl}) \epsilon^{ij} (\bar{q}_{Lm}^j \sigma^{\mu\nu} u_{Rn}) + \text{h.c.}$

↓ dim-6 SMEFT operator

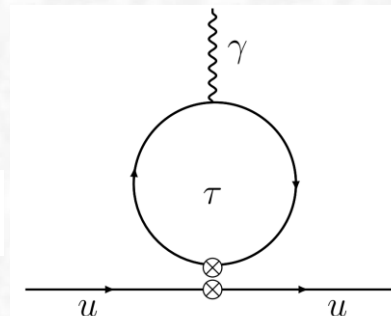
$$[C_{\ell equ}^{(3)}]_{klmn} [(\bar{\nu}_L^i \sigma_{\mu\nu} e_{Rl})(\bar{d}_{Lm}^j \sigma^{\mu\nu} u_{Rn}) - V_{am}(\bar{e}_L^i \sigma_{\mu\nu} e_{Rl})(\bar{u}_{La}^j \sigma^{\mu\nu} u_{Rn})] + \text{h.c.}$$



↳ **$\tau \rightarrow K_S \pi \nu_\tau$ & neutron EDM share the same WC:**

$$d_u(\mu) = -2\sqrt{2}G_F \frac{em_\tau}{\pi^2} V_{us}^2 \text{Im}[\hat{\epsilon}_T(\mu)] \log \frac{\Lambda}{\mu}$$

$$[C_{\ell equ}^{(3)}]_{3321} = -2\sqrt{2}G_F V_{us} \hat{\epsilon}_T^*$$



➤ Assuming the neutron EDM receives contribution **only from $\hat{\epsilon}_T$** , we get:

$$|d_n = g_T^u(\mu) d_u(\mu)| < 1.8 \times 10^{-26} e \text{ cm} \Rightarrow |\text{Im}[\hat{\epsilon}_T]| \leq \frac{1.5 \times 10^{-5}}{\ln(\frac{\Lambda}{\mu_\tau})} \lesssim 4 \times 10^{-6} \text{ for } \Lambda \gtrsim 100 \text{ GeV \& } \mu_\tau = 2 \text{ GeV}$$

nEDM collaboration, PRL 124 (2020) 081803

$$A_{CP}^i|_{\text{max}} \sim \mathcal{O}(10^{-6})$$

strongest limit obtained so far!

Constraints on $\text{Im}[\hat{\epsilon}_{S,T}]$ from other processes

❑ **Caveat:** tensor operator $(\bar{\tau}\sigma_{\mu\nu}\tau_R)(\bar{u}_L\sigma^{\mu\nu}u_R)$ can also associate with the WC combination

$$V_{ud} \text{Im}[C_{\ell equ}^{(3)}]_{3311} + V_{us} \text{Im}[C_{\ell equ}^{(3)}]_{3321} = 2\sqrt{2}G_F \left(V_{ud}^2 \text{Im}[\epsilon_T]_{3311} + V_{us}^2 \text{Im}[\epsilon_T]_{3321} \right)$$

$$d_u(\mu) = -2\sqrt{2}G_F \frac{em_\tau}{\pi^2} \left(V_{ud}^2 \text{Im}[\epsilon_T]_{3311} + V_{us}^2 \text{Im}[\epsilon_T]_{3321} \right) \log \frac{\Lambda}{\mu}$$

➤ when delicate cancellation exists between them, stringent limit on $\text{Im}[\hat{\epsilon}_T]$ can be diluted

❑ In this case, another interesting constraint on $(\bar{\tau}\sigma_{\mu\nu}\tau_R)(\bar{c}_L\sigma^{\mu\nu}u_R)$ from $D^0 - \bar{D}^0$ mixing:

$$V_{cd}[C_{\ell equ}^{(3)}]_{3311} + V_{cs}[C_{\ell equ}^{(3)}]_{3321} = V_{ud}V_{cd}[\epsilon_T]_{3311} + V_{us}V_{cs}[\epsilon_T]_{3321}$$

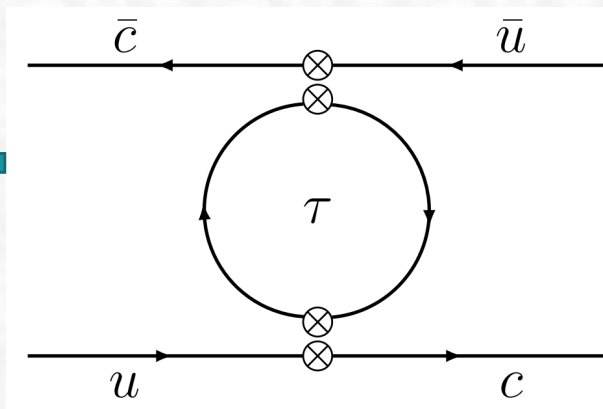
double insertion

$$M_{12}^{\text{NP}} = \frac{1}{2M_D} \left[C'_2(\mu) \langle D^0 | (\bar{c}_L^\alpha u_R^\alpha) (\bar{c}_L^\beta u_R^\beta) | \bar{D}^0 \rangle (\mu) + C'_3(\mu) \langle D^0 | (\bar{c}_L^\alpha u_R^\beta) (\bar{c}_L^\beta u_R^\alpha) | \bar{D}^0 \rangle (\mu) \right]$$

$$C'_2 = \frac{1}{2}C'_3 = 16G_F^2 \frac{m_\tau^2}{\pi^2} (V_{ud}V_{cd}[\epsilon_T]_{3311} + V_{us}V_{cs}[\epsilon_T]_{3321})^2 \log \frac{\Lambda}{\mu_\tau}$$

$$x_{12} = \frac{2|M_{12}|}{\Gamma_D} = (0.409 \pm 0.048)\%, \quad \phi_{12} = \arg\left(\frac{M_{12}}{\Gamma_{12}}\right) = (0.58^{+0.91}_{-0.90})^\circ$$

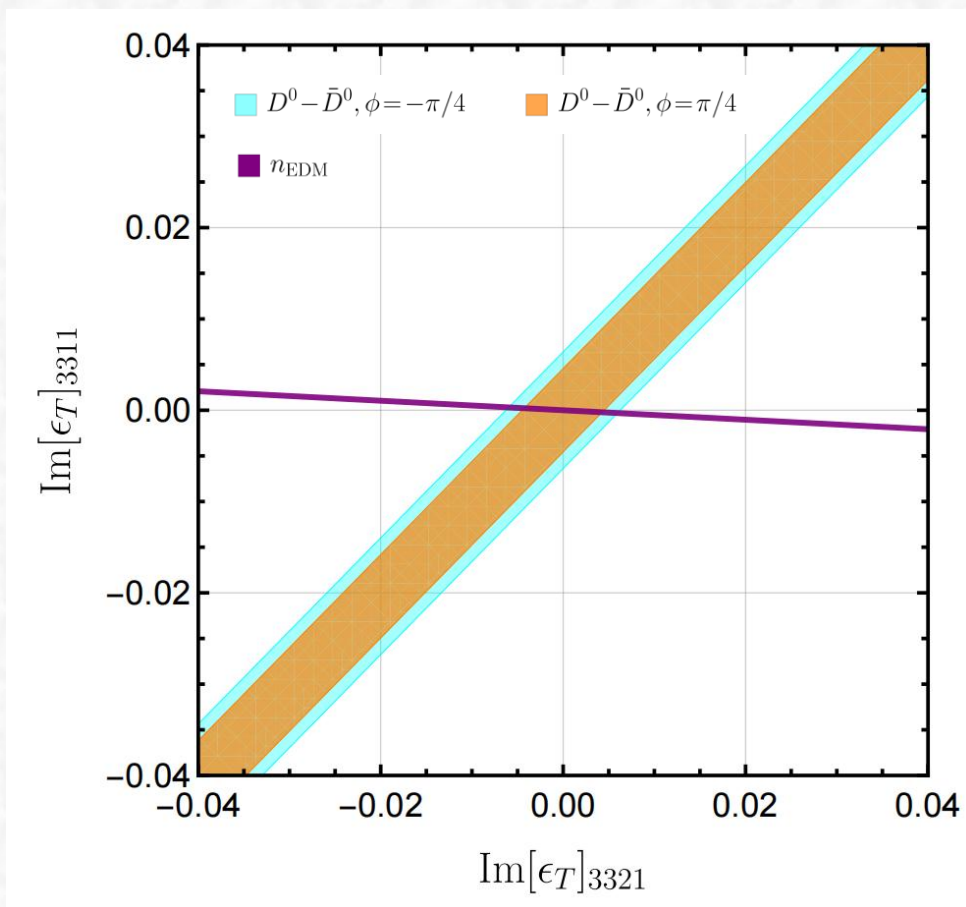
➤ we can get further constraint on $\text{Im}[\hat{\epsilon}_T]$!



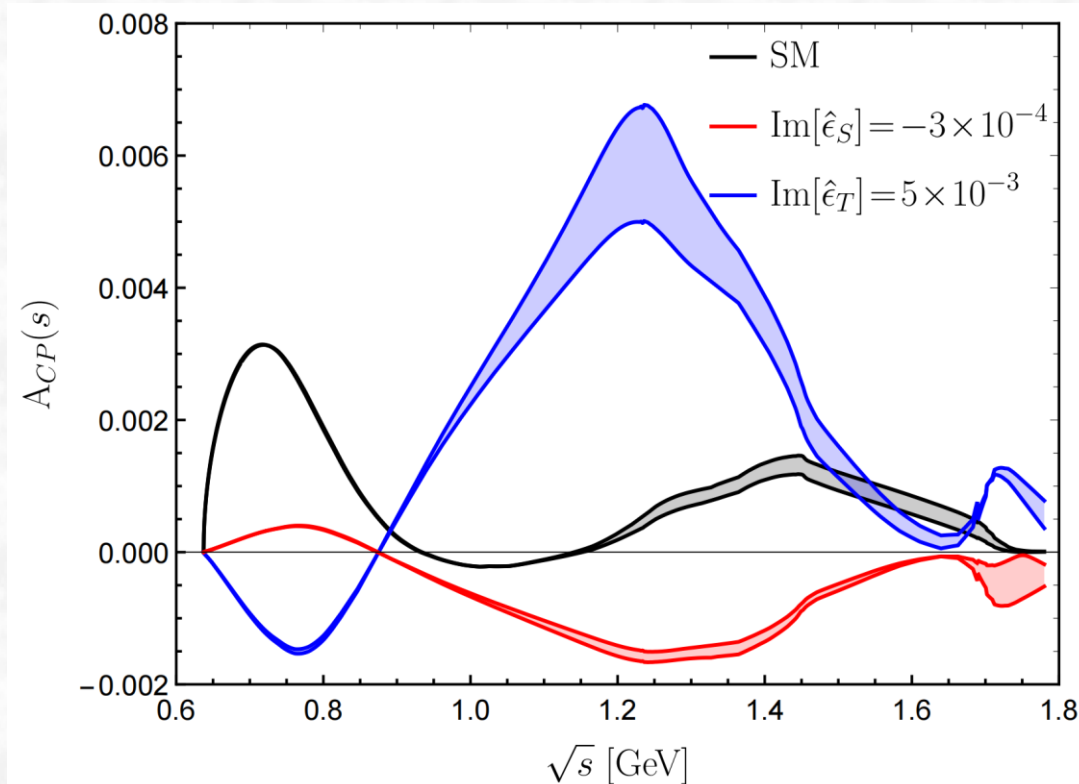
Constraints on $\text{Im}[\hat{\epsilon}_{S,T}]$ from other processes

□ Combined constraints from d_n and $D^0 - \bar{D}^0$ mixing:

$$|\text{Im}[\hat{\epsilon}_T]| \lesssim 4 \times 10^{-6}$$



➤ upper limit: $|\text{Im}[\epsilon_T]| \sim 5 \times 10^{-3}$, significantly diluted



□ Prediction for CPA with $|\text{Im}[\epsilon_T]| = 5 \times 10^{-3}$: *still has a significant impact on the CPA!*

Constraints on $\text{Im}[\hat{\epsilon}_{S,T}]$ from other processes

□ For the **scalar operator**: originate from the following **two SMEFT operators**

$$\mathcal{L}_{\text{SMEFT}} \supset [C_{\ell equ}^{(1)}]_{klmn} (\bar{\ell}_{Lk}^i e_{Rl}) \epsilon^{ij} (\bar{q}_{Lm}^j u_{Rn}) + [C_{\ell edq}]_{klmn} (\bar{\ell}_{Lk}^i e_{Rl}) (\bar{d}_{Rm} q_{Ln}^i) + \text{h.c.}$$

$$\supset [C_{\ell equ}^{(1)}]_{klmn} \left[(\bar{\nu}_{Lk} e_{Rl}) (\bar{d}_{Lm} u_{Rn}) - V_{am} (\bar{e}_{Lk} e_{Rl}) (\bar{u}_{La} u_{Rn}) \right]$$

$$+ [C_{\ell edq}]_{klmn} \left[V_{an}^* (\bar{\nu}_{Lk} e_{Rl}) (\bar{d}_{Rm} u_{La}) + (\bar{e}_{Lk} e_{Rl}) (\bar{d}_{Rm} d_{Ln}) \right] + \text{h.c.}$$

$$\longrightarrow (\bar{\nu}_\tau \tau_R) (\bar{s}_L u_R)$$

$$\longrightarrow (\bar{\nu}_\tau \tau_R) (\bar{s}_R u_L)$$

scalar operator

$$(\bar{\nu}_\tau \tau_R) (\bar{s} u)$$



$$-2\sqrt{2}G_F V_{us}^* \hat{\epsilon}_S^* = [C_{\ell equ}^{(1)}]_{3321} + V_{ud}^* [C_{\ell edq}]_{3321} + V_{us}^* [C_{\ell edq}]_{3322} + V_{ub}^* [C_{\ell edq}]_{3323}$$

➤ constraint on $\hat{\epsilon}_S$ can be obtained from other processes;

➤ when potential cancellations exist between $C_{\ell equ}^{(1)}$ & $C_{\ell edq}$, the allowed values of $\hat{\epsilon}_S$ can be diluted

□ **Mixing between scalar and tensor operators:**

$$\hat{\epsilon}_S = -\frac{[C_{\ell equ}^{(1)}]_{3321}^*}{2\sqrt{2}G_F V_{us}}, \quad \hat{\epsilon}_T = -\frac{[C_{\ell equ}^{(3)}]_{3321}^*}{2\sqrt{2}G_F V_{us}}$$

$$\begin{pmatrix} \hat{\epsilon}_S \\ \hat{\epsilon}_T \end{pmatrix}_{(\mu=2 \text{ GeV})} = \begin{pmatrix} 1.72 & -0.0242 \\ -2.17 \times 10^{-4} & 0.825 \end{pmatrix} \begin{pmatrix} \hat{\epsilon}_S \\ \hat{\epsilon}_T \end{pmatrix}_{(\mu=m_Z)}, \quad \begin{pmatrix} C_{\ell equ}^{(1)} \\ C_{\ell equ}^{(3)} \end{pmatrix}_{(\mu=m_Z)} = \begin{pmatrix} 1.20 & -0.185 \\ -0.00381 & 0.959 \end{pmatrix} \begin{pmatrix} C_{\ell equ}^{(1)} \\ C_{\ell equ}^{(3)} \end{pmatrix}_{(\mu=1 \text{ TeV})}$$

Constraints on $\text{Im}[\hat{\epsilon}_{S,T}]$ from other processes

□ For **scalar operator**: no direct contribution to d_n , but can mix into tensor operator via RGE

$$|\text{Im}[\hat{\epsilon}_T]| \lesssim 4 \times 10^{-6} \longrightarrow |\text{Im}[\hat{\epsilon}_S(\mu_\tau)]| \lesssim 2.3 \times 10^{-3}, \quad \text{comparable to } \text{Im}[\hat{\epsilon}_S] = -0.008 \pm 0.027$$

□ The scalar operator $(\bar{\tau}_L \tau_R)(\bar{c}_L u_R)$ contributes to $D^0 - \bar{D}^0$ mixing:

double insertion

$$M_{12}^{\text{NP}} = \frac{1}{2M_D} \left[C'_2(\mu) \langle D^0 | (\bar{c}_L^\alpha u_R^\alpha) (\bar{c}_L^\beta u_R^\beta) | \bar{D}^0 \rangle (\mu) \right]$$

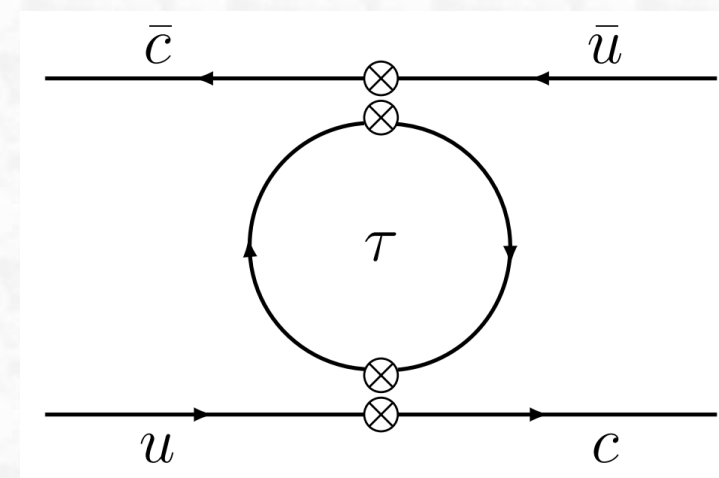
$$C'_2 = -G_F^2 \frac{m_\tau^2}{\pi^2} (V_{us} V_{cs} \hat{\epsilon}_S)^2 \log \frac{\Lambda}{\mu_\tau}$$

$$x_{12} = \frac{2|M_{12}|}{\Gamma_D} = (0.409 \pm 0.048)\%$$

$$\phi_{12} = \arg\left(\frac{M_{12}}{\Gamma_{12}}\right) = (0.58_{-0.90}^{+0.91})^\circ$$

HFLAV collaboration, EPJC 81 (2021) 226

$$\text{Im}[\hat{\epsilon}_S(\mu_\tau)] \in [-3.1, 1.6] \times 10^{-4} \text{ @ } 2\sigma$$



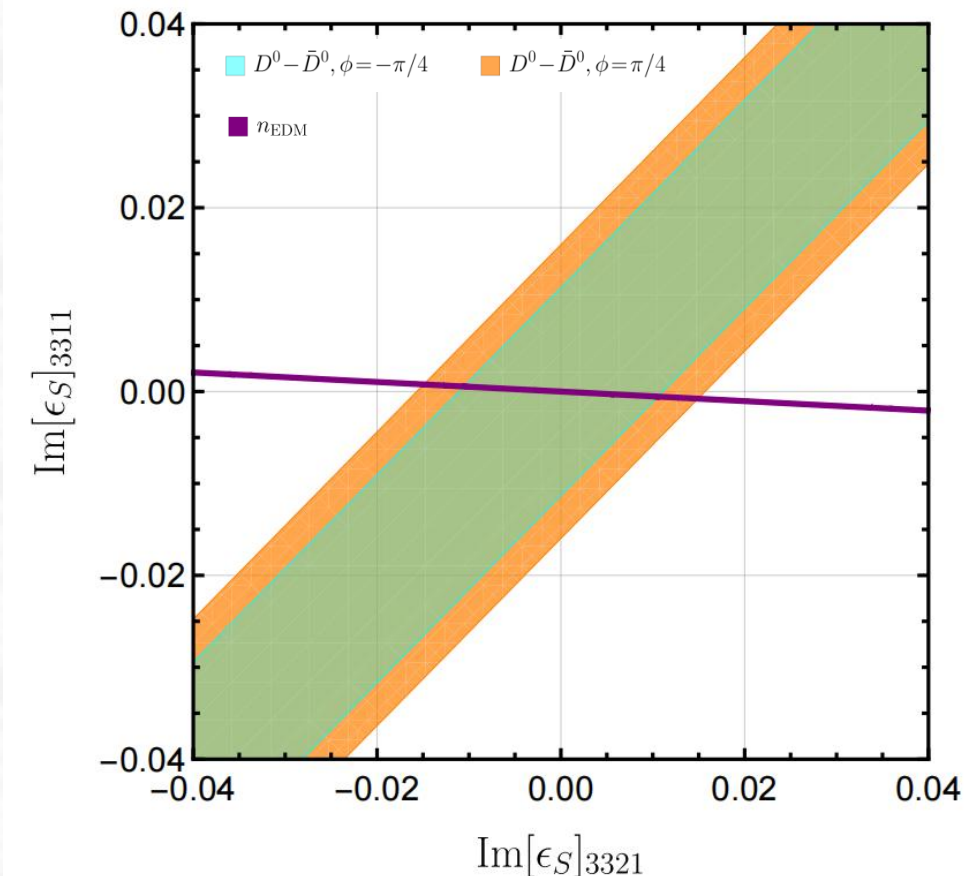
➤ Constraint from $D^0 - \bar{D}^0$ mixing is one order of magnitude stronger than from **neutron EDM**

➤ $A_{CP}^i|_{\text{max}} \sim \mathcal{O}(10^{-3})$: of the same order as A_{SM}^{CP}

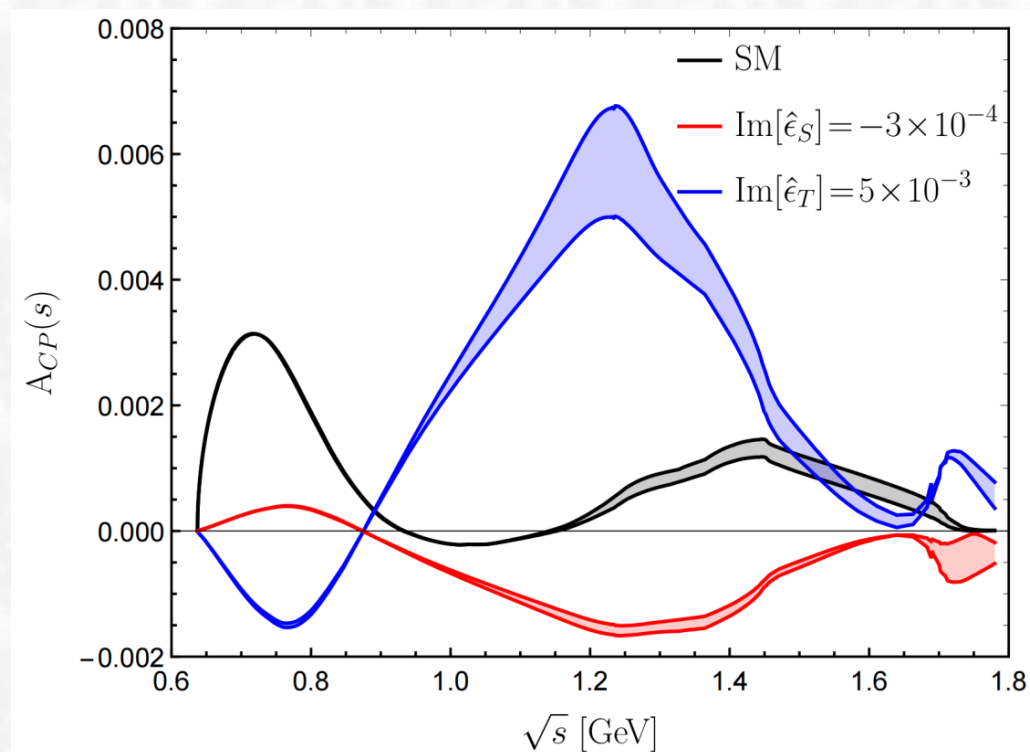
Constraints on $\text{Im}[\hat{\epsilon}_{S,T}]$ from other processes

❑ **Caveat:** once cancellations occur, bounds diluted

$$V_{ud}\text{Im}[C_{\ell equ}^{(1)}]_{3311} + V_{us}[C_{\ell equ}^{(1)}]_{3321} \quad (\text{for } d_n),$$



$$V_{cd}[C_{\ell equ}^{(1)}]_{3311} + V_{cs}[C_{\ell equ}^{(1)}]_{3321} \quad (\text{for } D^0 - \bar{D}^0 \text{ mixing})$$



❑ **Predictions for CPA with $\text{Im}[\hat{\epsilon}_S(\mu_\tau)] = -3 \times 10^{-4}$: slightly smaller than the SM prediction**