

Prospect of Tau Decays



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By reviewing the current status of tau decays, I shall try to point out where improvements are needed

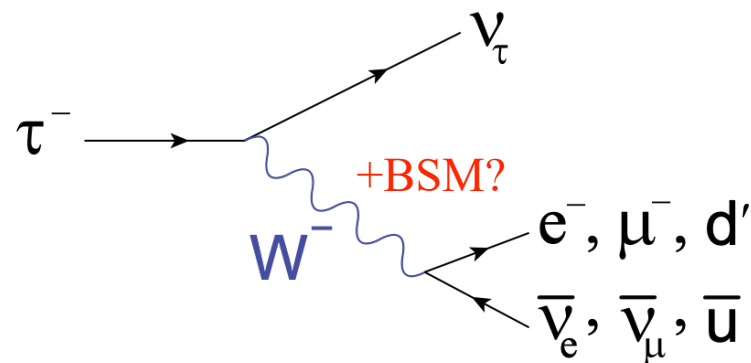
Based whenever possible on materials in my personal publications in close collaboration with Davier (D), Eidelman (E), Hoecker (H), Malaescu (M), Yuan (Y) et al.

Tau Decays

Tau (τ) lepton has a mass around $1776.9 \text{ MeV}/c^2$

The only lepton heavy enough to have both leptonic and hadronic* decay modes

$$|d'\rangle = V_{ud} |d\rangle + V_{us} |s\rangle$$



Many decay modes:

- Lifetime much shorter than μ
- Challenge & opportunity for searches & precision measurements wrt e & μ

An interesting mass scale:

- For studying the transition from non-perturbative QCD (non-pQCD) hadron dynamics to pQCD quark-like regime

* Strictly speaking, it should be semileptonic given the ν_τ in the final state

Tau Decays vs Physics Topics

Rare or forbidden decays

Direct observation of new physics:
e.g. Lepton Flavor Violation (LFV)

Leptonic decays

Hadronic decays

Nonstrange modes ($|S|=0$)

Strange modes ($|S|=1$)

In terms of

- Branching Fractions (BFs)
- Spectral Functions (SFs)

Precision measurements for probing
deviations data / SM-predictions:

- Testing flavor universalities
- Evaluating Hadronic Vacuum Polarization (HVP) contribution
- Hadronic contribution $\Delta\alpha_{\text{had}}(s)$
- Determining strong coupling constant α_s
- Studying V_{us} puzzle
- Searching for CPV in tau
- Probing non-SM interactions via Michel parameters

Tau Production

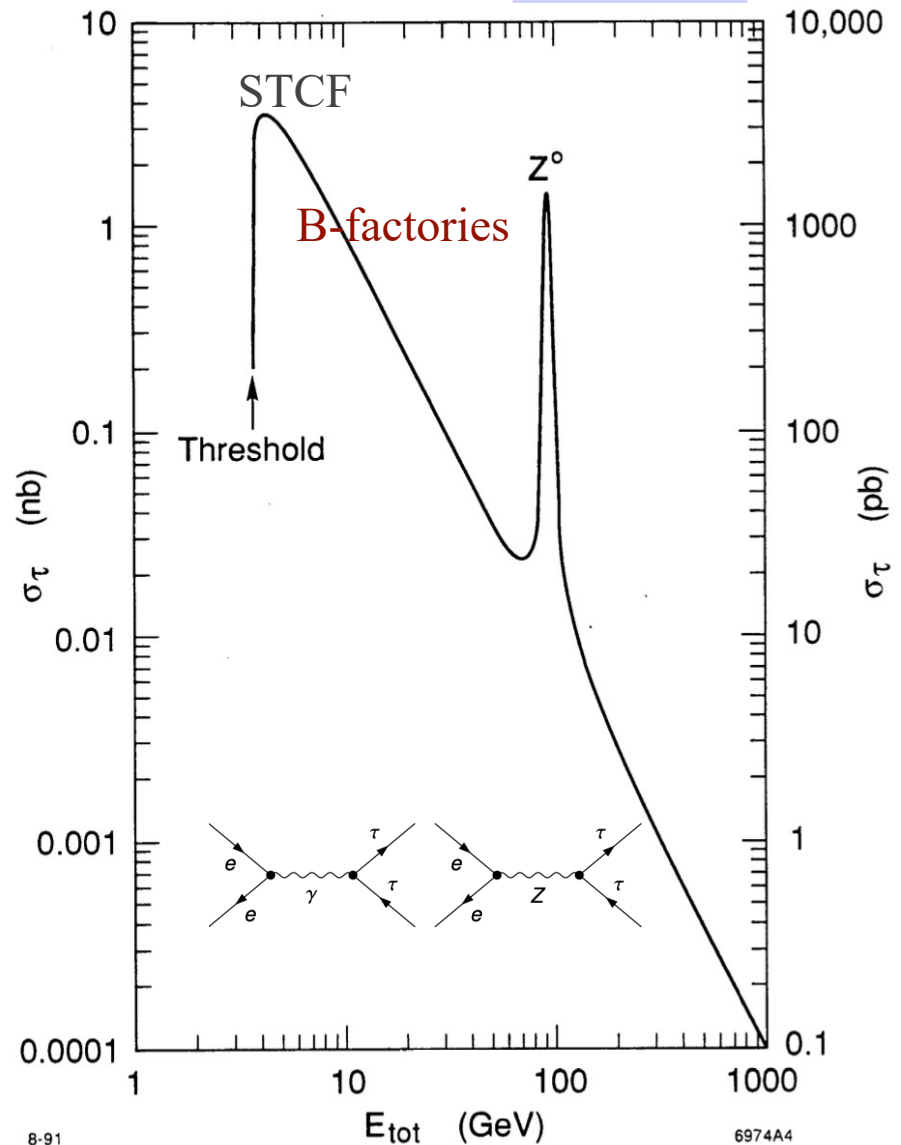
M. Perl 1991

For both new physics discovery and precision measurements, we need high luminosity and large tau production cross section

Improved event selections and analysis techniques are equally important

SuperKEKB/Belle II and STCF both provide such opportunities

- Talk of Petar Rados (Belle II)
- Talk of Mingyi Liu (STCF)

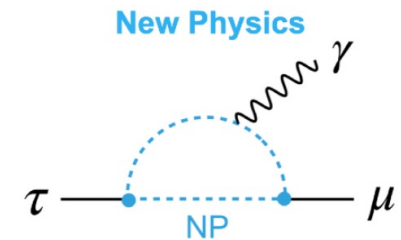
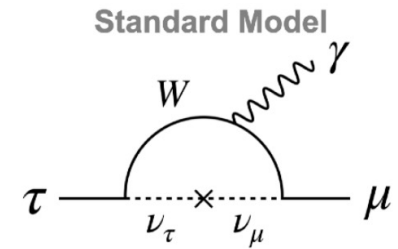


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Search for New Physics (NP) via LFV Decays

- SM predicts $\text{BF} \propto \left(\frac{\Delta m_\nu^2}{m_W^2}\right)^2 < 10^{-50}$ thus negligible
- NP models predict much larger BF's up to 10^{-8}
- Therefore any observation such a LFV decay would be a sign of NP
- There are many competing experiments (+ those (Mu2e@FermiLab, Mu3e@PSI, MEG II@PSI, COMET@J-PARC) for non tau processes)

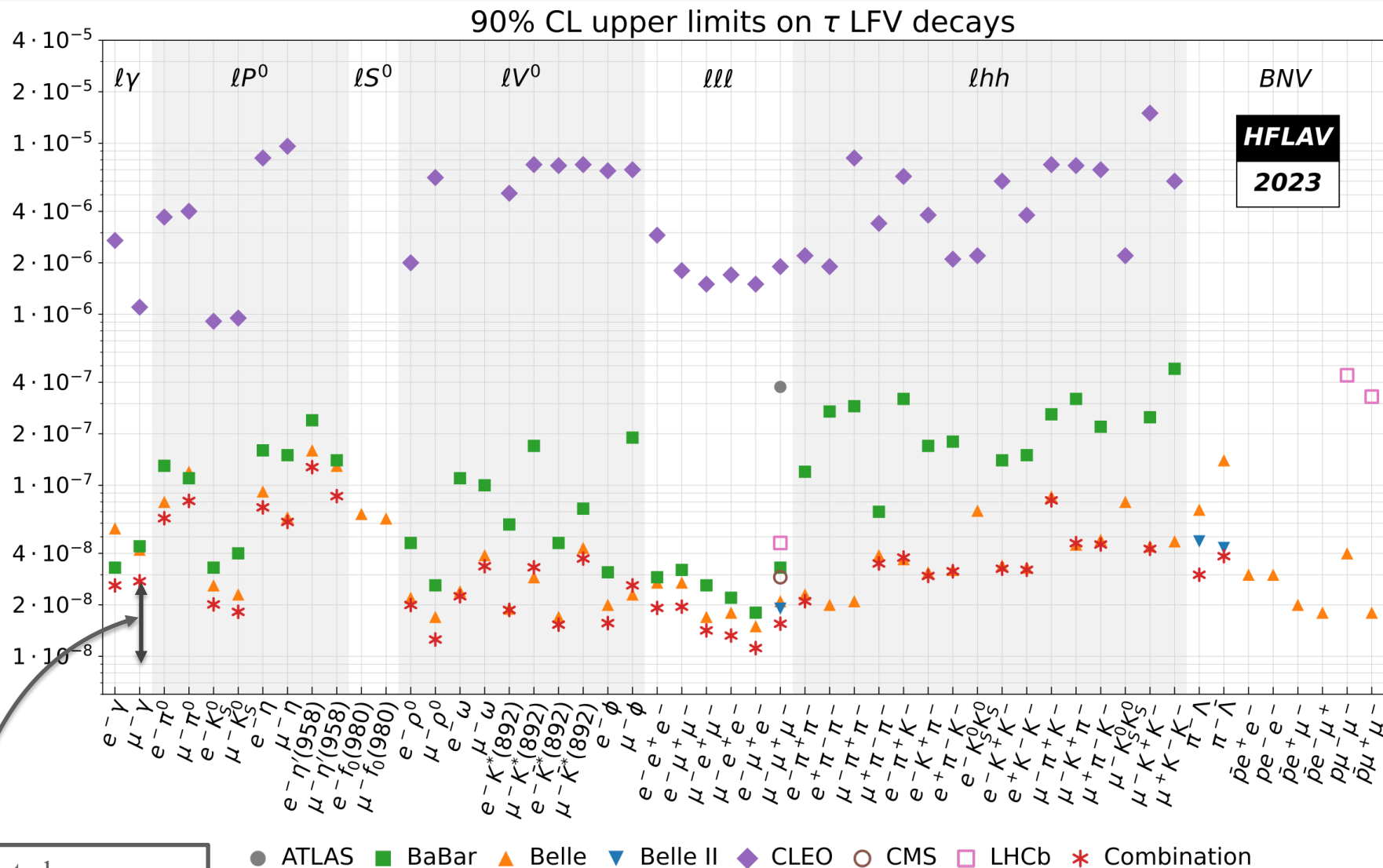


@10.58 GeV, $\sigma(\tau^+\tau^-) \sim 0.919$ nb

An integrated luminosity of 1 ab^{-1} provides 0.919×10^9 tau pairs

→ Higher luminosity and good selection efficiency are the key

Rare and LFV Decays



Plot not up to date, Belle II has several published and ongoing analyses

→ Talk of Laura Zani

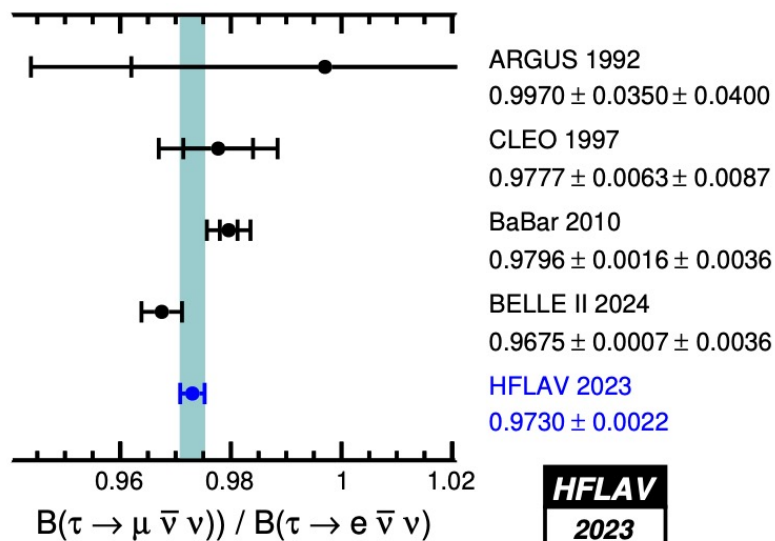
STCF study:
Xiang-Shi-Wang-Zhou
[EPJC 83 \(2023\) 908](#)

Various Flavor Universality Tests

BFs related universality tests:

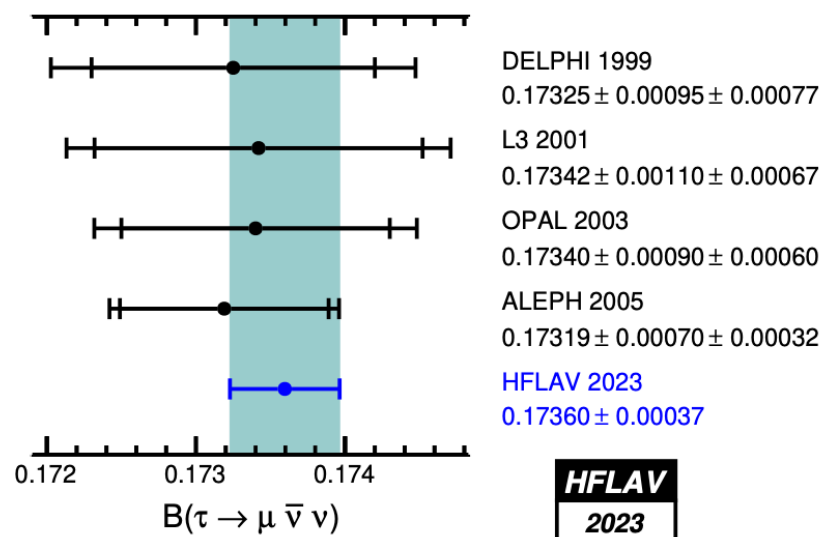
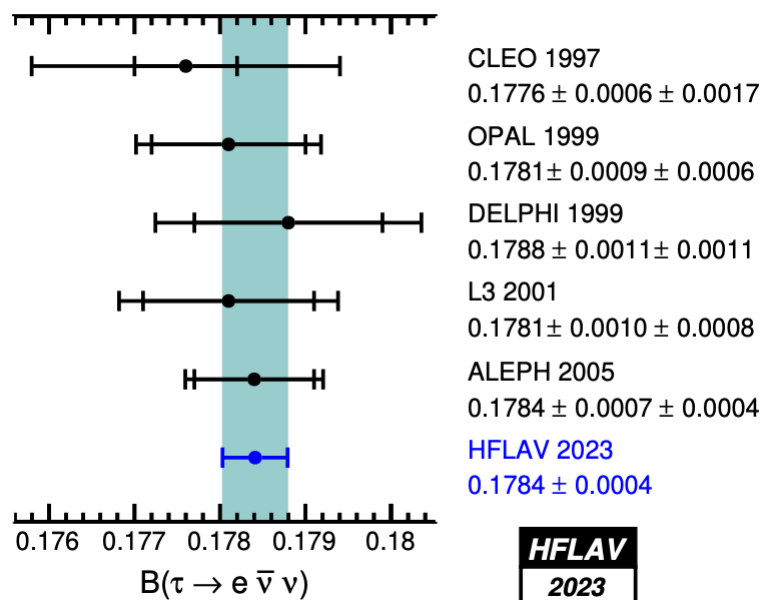
- $B(\tau \rightarrow e \nu_\tau \nu_e)$ vs $B(\tau \rightarrow \mu \nu_\tau \nu_\mu)$
- $B(\tau \rightarrow e \nu_\tau \nu_e)$, tau lifetime (τ_τ) and tau mass (m_τ)
- $B(\tau \rightarrow \pi \nu_\tau)$ vs $B(\pi \rightarrow \mu \nu_\mu)$ and their lifetimes
- $B(\tau \rightarrow K \nu_\tau)$ vs $B(K \rightarrow \mu \nu_\mu)$ and their lifetimes

Leptonic BFs



Belle II's relative measurement (0.38%)

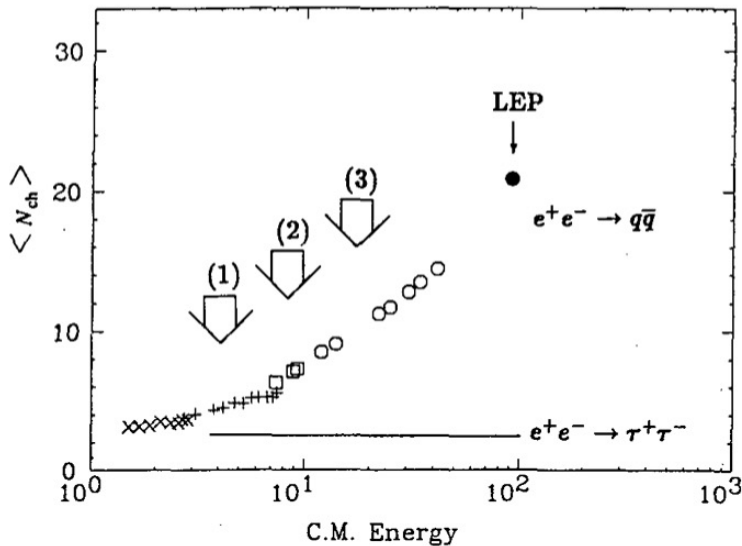
The best absolute BF measurements (0.45%) are still from ALEPH more than 20 years ago!



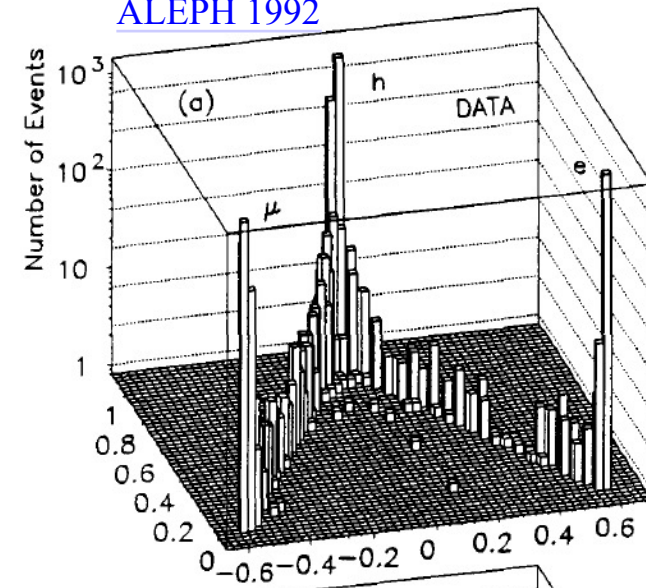
Why could ALEPH do so well?

- Very high tau-pair selection efficiency (over 90% within the acceptance)
- Hadronic background (<1%, easy signal / background separation)
- Excellent identification for $e/\mu/\pi/K$ separation, γ detection and π^0 reconstruction allowing to classify all main decays in 13 modes + 1 unclassified mode
→ The best way to optimize efficiency and cross-feed among different modes & to control missing modes

Zhang, PhD thesis 1992



ALEPH 1992



Final BFs Measured by ALEPH in 2005

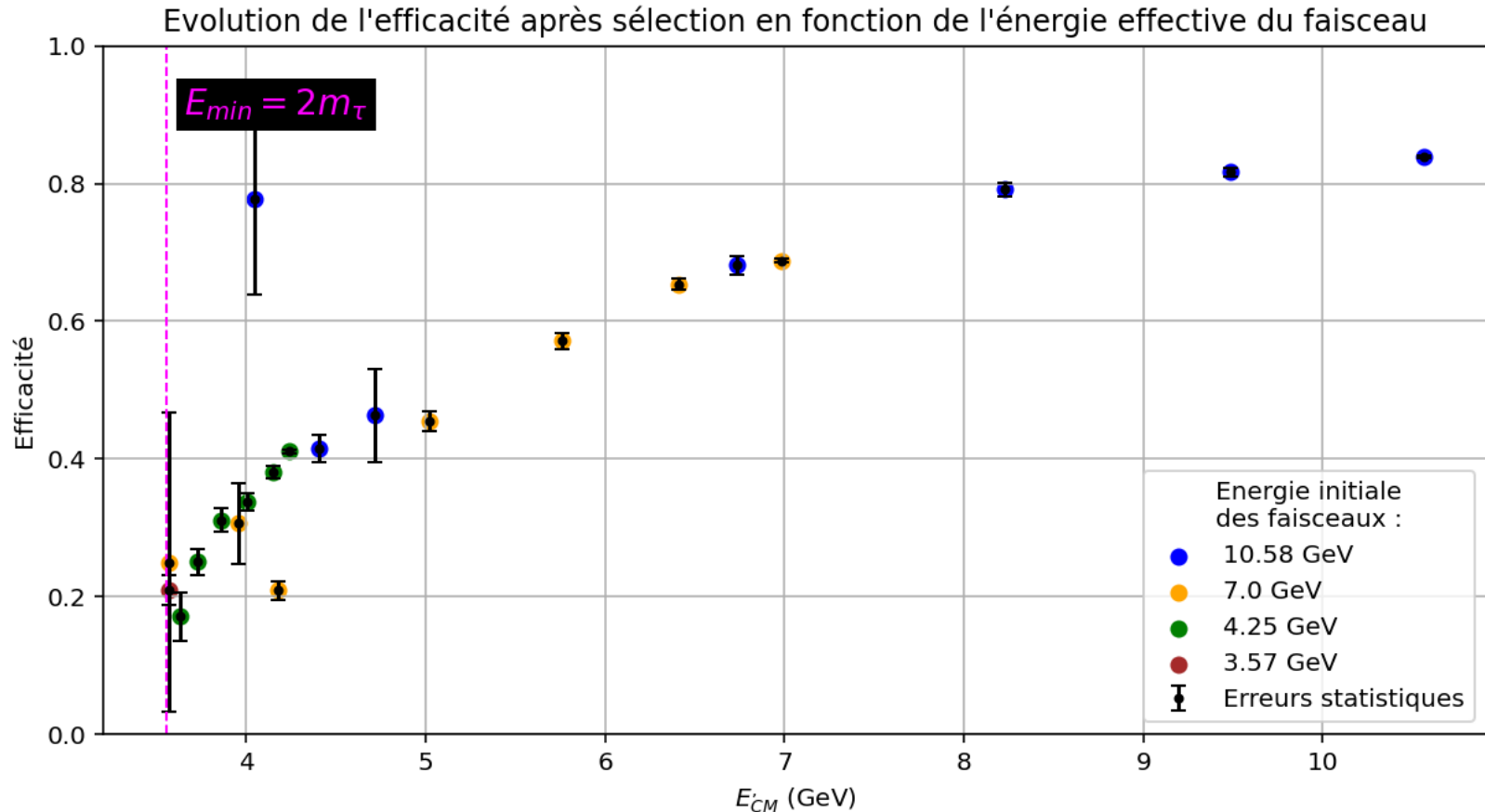
ALEPH 2005

mode	$B \pm \sigma_{\text{stat}} \pm \sigma_{\text{syst}} [\%]$	
e	$17.837 \pm 0.072 \pm 0.036$	
μ	$17.319 \pm 0.070 \pm 0.032$	
π^-	$10.828 \pm 0.070 \pm 0.078$	The contribution of the kaon modes (ALEPH '99) having a BF of $(2.87 \pm 0.12)\%$ has been subtracted from the relevant decay modes
$\pi^- \pi^0$	$25.471 \pm 0.097 \pm 0.085$	
$\pi^- 2\pi^0$	$9.239 \pm 0.086 \pm 0.090$	
$\pi^- 3\pi^0$	$0.977 \pm 0.069 \pm 0.058$	
$\pi^- 4\pi^0$	$0.112 \pm 0.037 \pm 0.035$	
$\pi^- \pi^- \pi^+$	$9.041 \pm 0.060 \pm 0.076$	
$\pi^- \pi^- \pi^+ \pi^0$	$4.590 \pm 0.057 \pm 0.064$	
$\pi^- \pi^- \pi^+ 2\pi^0$	$0.392 \pm 0.030 \pm 0.035$	
$\pi^- \pi^- \pi^+ 3\pi^0$	$0.013 \pm 0.000 \pm 0.010$	estimate
$3\pi^- 2\pi^+$	$0.072 \pm 0.009 \pm 0.012$	
$3\pi^- 2\pi^+ \pi^0$	$0.014 \pm 0.007 \pm 0.006$	
$\pi^- \pi^0 \eta$	$0.180 \pm 0.040 \pm 0.020$	ALEPH [13]
$\pi^- 2\pi^0 \eta$	$0.015 \pm 0.004 \pm 0.003$	CLEO [27]
$\pi^- \pi^- \pi^+ \eta$	$0.024 \pm 0.003 \pm 0.004$	CLEO [27]
$a_1^- (\rightarrow \pi^- \gamma)$	$0.040 \pm 0.000 \pm 0.020$	estimate
$\pi^- \omega (\rightarrow \pi^0 \gamma, \pi^+ \pi^-)$	$0.253 \pm 0.005 \pm 0.017$	ALEPH [13]
$\pi^- \pi^0 \omega (\rightarrow \pi^0 \gamma, \pi^+ \pi^-)$	$0.048 \pm 0.006 \pm 0.007$	ALEPH [13] + CLEO [26]
$\pi^- 2\pi^0 \omega (\rightarrow \pi^0 \gamma, \pi^+ \pi^-)$	$0.002 \pm 0.001 \pm 0.001$	CLEO [27]
$\pi^- \pi^- \pi^+ \omega (\rightarrow \pi^0 \gamma, \pi^+ \pi^-)$	$0.001 \pm 0.001 \pm 0.001$	CLEO [27]

Due to limited data sample, ALEPH measurements were limited to modes with $\text{BF} > 0.1\%$

Thrust Axis Method at Belle II and STCF?

The efficiency of the method drops towards lower (effective) center-of-mass energies



Outcome of a short recent internship study by a student under my supervision

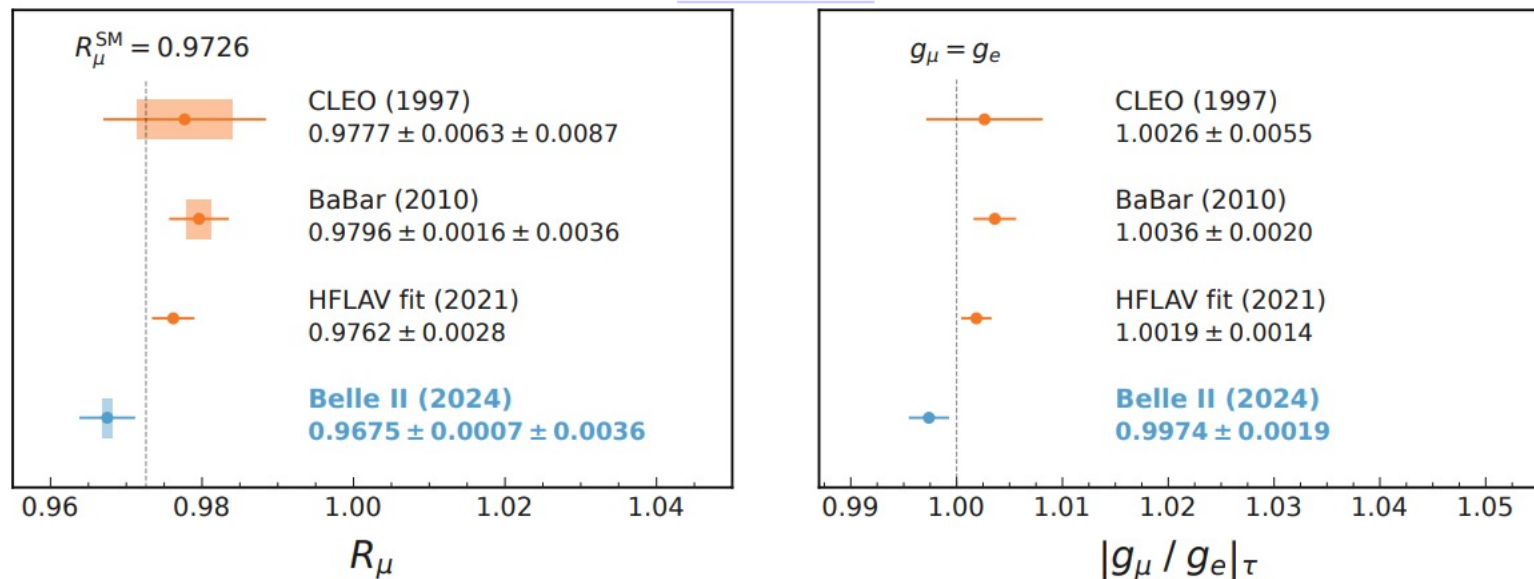
Lepton Flavor Universality (LFU) Test - B_e vs B_μ

The BFs of the two modes differ by a phase space factor due to their mass difference:

$$\Gamma(\tau \rightarrow \nu_\tau \ell \bar{\nu}_\ell) \propto f\left(\frac{m_\ell^2}{m_\tau^2}\right) \quad \text{with} \quad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

→ SM prediction: 0.9726 [known $\mathcal{O}(10^{-6})$]

Belle II 2024



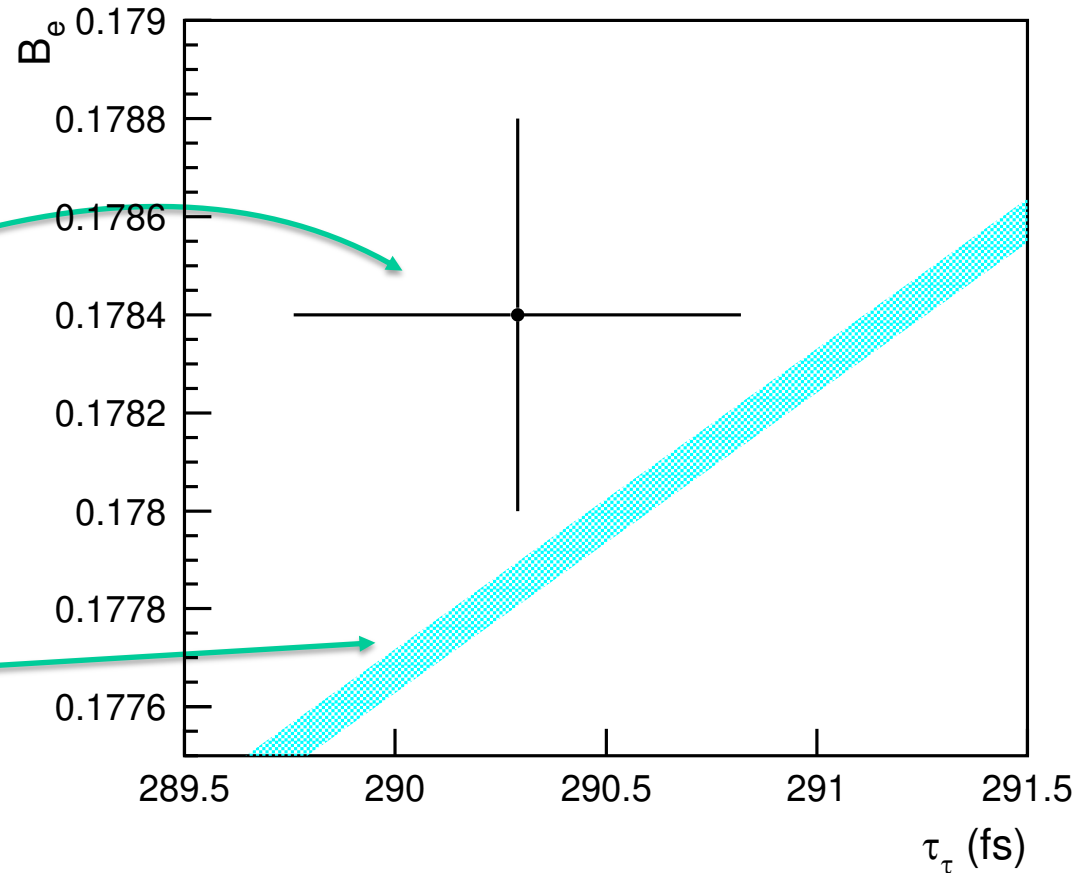
Belle II has a precision comparable to that of BABAR, but not yet precise enough for a stringent test (-1.4σ)

LFU Test with B_e , τ_τ and m_τ

The test is currently limited by the precision of B_e (0.45%) and τ_τ (0.22%) measurements

and by m_τ (0.008%) on the prediction side using

$$\frac{B_{\tau \rightarrow \nu_\tau e \bar{\nu}_e}}{\tau_\tau} = \frac{B_{\mu \rightarrow \nu_\mu e \bar{\nu}_e}}{\tau_\mu} \left(\frac{m_\tau}{m_\mu} \right)^5$$



The ongoing Belle-II analysis will improve the precision of τ_τ by a factor ~ 4

→ BF_e will be the only limiting factor

→ Talk of Fabian Becherer

Pion BF and π - μ Universality Test

The best BF measurement $B(\tau \rightarrow \pi \nu_\tau)$ is still from ALEPH:
(10.828 $\pm$ 0.070 $\pm$ 0.078 [0.105])%

To be compared with the prediction based on the π - μ universality:

$$\begin{aligned} \mathcal{B}_\pi^{\text{uni}} &= \frac{\tau_\tau \mathcal{B}_{\pi \rightarrow \mu \nu}}{\tau_\pi} \frac{m_\tau^3}{2m_\pi m_\mu^2} \left(\frac{1 - m_\pi^2/m_\tau^2}{1 - m_\mu^2/m_\pi^2} \right)^2 \delta_{\tau/\pi} && \text{Davier-Hoecker-Zhang,} \\ &&& \text{RMP 78 (2006) 1043} \\ &= \frac{\tau_\tau}{(2\,663.7 \pm 4.0) \text{ fs}} = (10.910 \pm 0.044)\% && \rightarrow (10.900 \pm 0.024)\% \\ &&& \text{2005} \qquad \qquad \rightarrow \qquad \qquad \text{2026} \end{aligned}$$

The error reduction is
mainly from tau

→ In good agreement but need a factor of 4 improvement or more
on $B(\tau \rightarrow \pi \nu_\tau)$ from the measurement side

K BF and $K\text{-}\mu$ Universality Test

Current world average $B(\tau \rightarrow K \nu_\tau)$:

$$(0.685 \pm 0.023)\%$$

dominated by the best K BF measurement from ALEPH:

$$(0.696 \pm 0.025 \pm 0.014 [0.029])\%$$

To be compared with the prediction based on the $\pi\text{-}\mu$ universality:

$$\begin{aligned} \mathcal{B}_K^{\text{uni}} &= \frac{\tau_\tau \mathcal{B}_{K \rightarrow \mu \nu}}{\tau_K} \frac{m_\tau^3}{2m_K m_\mu^2} \left(\frac{1 - m_K^2/m_\tau^2}{1 - m_\mu^2/m_K^2} \right)^2 \delta_{\tau/K} && \text{Davier-Hoecker-Zhang,} \\ &&& \text{RMP 78 (2006) 1043} \\ &= \frac{\tau_\tau}{(40\,636 \pm 162) \text{ fs}} = (0.715 \pm 0.003)\% \quad \rightarrow (0.716 \pm 0.003)\% \\ &&& \text{2005} \qquad \qquad \qquad \rightarrow \qquad \qquad \text{2026} \end{aligned}$$

A tension of -1.3σ despite the measurement uncertainty is a factor of ~ 8 larger than that of the expectation

→ A much more precise measurement $B(\tau \rightarrow K \nu_\tau)$ is needed

Hadronic Decays

The inclusive hadronic BF may be obtained by subtracting the leptonic ones:

$$\mathcal{B}(\tau \rightarrow \nu_\tau \text{ hadrons}) = 1 - \mathcal{B}_e - \mathcal{B}_\mu = 0.6478 \pm 0.0007$$

[Measurements in exclusive channels are of course also needed]

It can be further decomposed by

nonstrange ($|S|=0$) decay modes $\sim 95.5\%$

strange ($|S|=1$) decay modes $\sim 4.5\%$

The nonstrange component may be further decomposed into

Vector (V) with $G = +1$: even number pions $\sim 97.5\%$

Axial-vector (A) with $G = -1$: odd number pions $\sim 2.5\%$

[with some V/A ambiguity involving even number kaon modes]

$$\rightarrow \frac{\mathcal{B}(V - A)}{\mathcal{B}(V + A)} \sim 0.026$$

This ratio is a measure of relative non-pQCD contributions.

Its smallness represents a proof of the remarkable property of quark-hadron duality.

It is the basis of the QCD analyses of hadronic τ decays.

Spectral Functions (SFs)

The SFs are nothing but (scaled and unfolded) invariant mass distributions

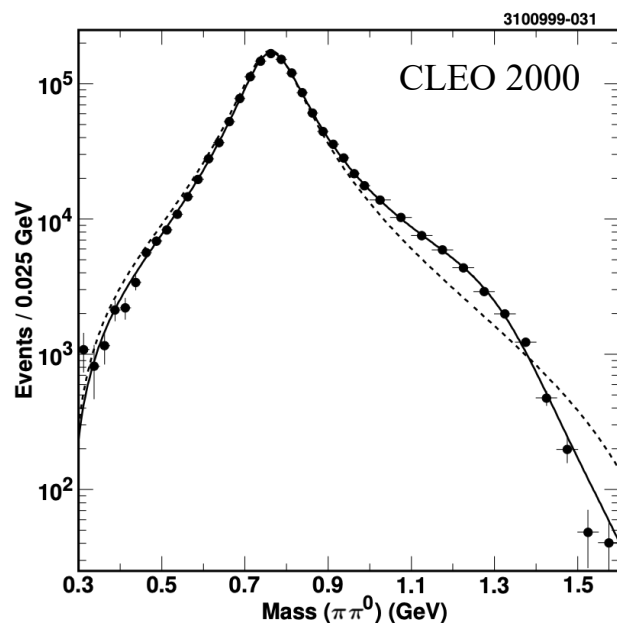
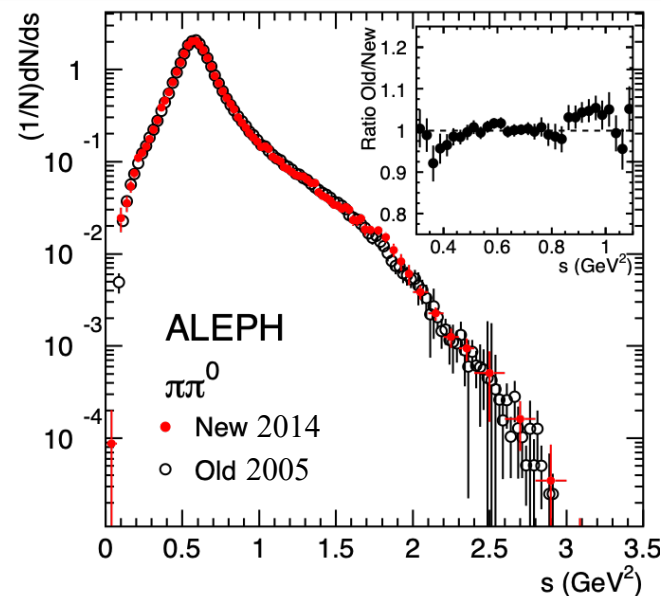
$$v_1(s)/a_1(s) = \frac{m_\tau^2}{6 |V_{\text{CKM}}|^2 S_{\text{EW}}} \frac{\mathcal{B}(\tau^- \rightarrow V^- / A^- \nu_\tau)}{\mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \\ \times \frac{dN_{V/A}}{N_{V/A} ds} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \right]^{-1},$$
$$a_0(s) = \frac{m_\tau^2}{6 |V_{\text{CKM}}|^2 S_{\text{EW}}} \frac{\mathcal{B}(\tau^- \rightarrow \pi^- (K^-) \nu_\tau)}{\mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{dN_A}{N_A ds} \left(1 - \frac{s}{m_\tau^2}\right)^{-2},$$

with

- subscripts 1/0: spin J of the hadronic system
- V_{CKM} : V_{ud} or V_{us} for $|S|=0$ and $|S|=1$, respectively
- S_{EW} : electroweak (EW) radiative corrections

For π/K decays, the SFs a_0 are simply a delta function

Two-Pion Measurements Performed Long Time Ago



[DHMYZ 2014](#)

SFs measured under very different exp. conditions

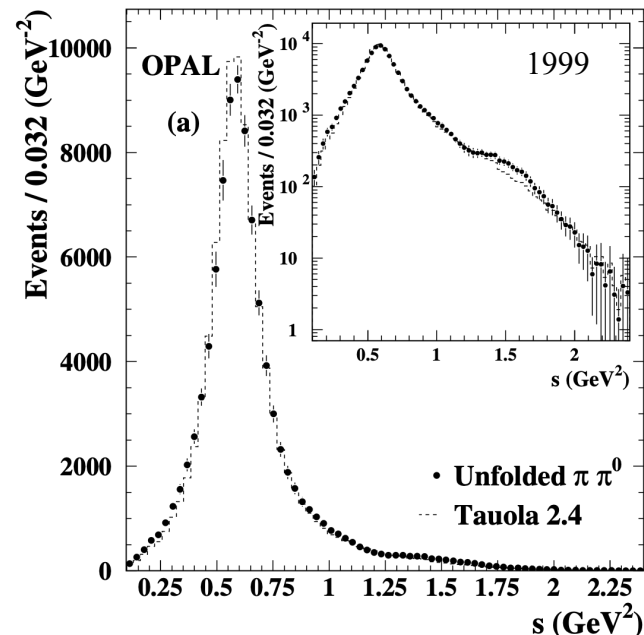
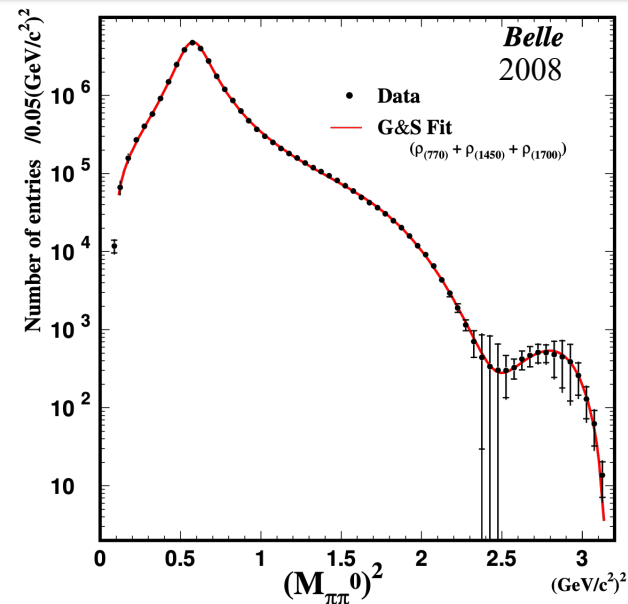
ALEPH had the best BF (normalisation) measurement, while Belle had the best shape measurement

ALEPH: [Phys. Rept. 421 \(2005\) 191](#), [Eur. Phys. J. C 74 \(2014\) 2803](#)

Belle: [Phys. Rev. D 78, 072006 \(2008\)](#)

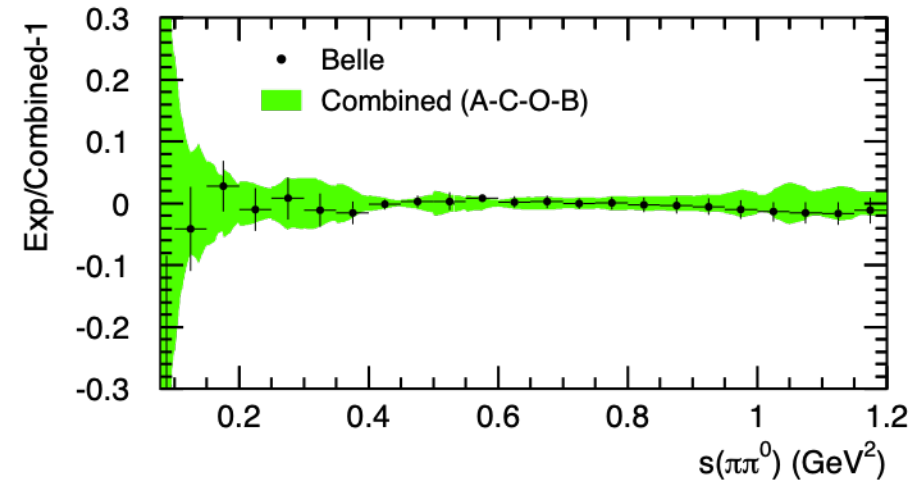
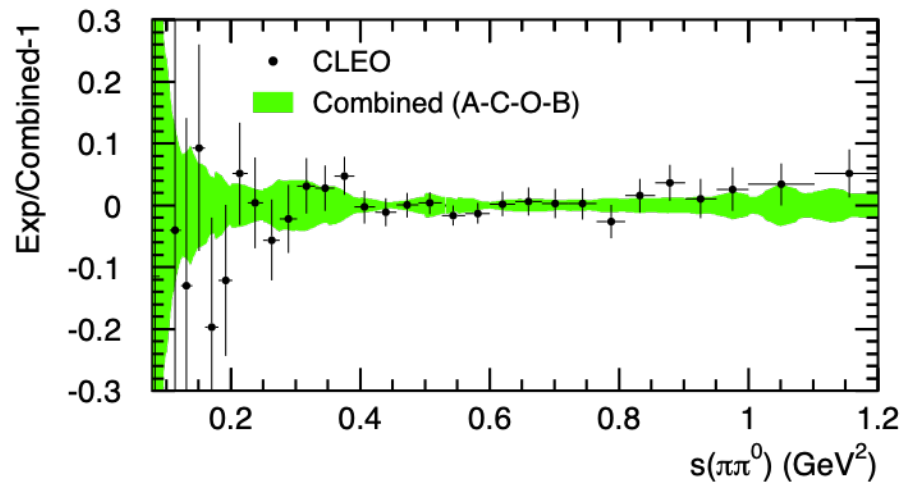
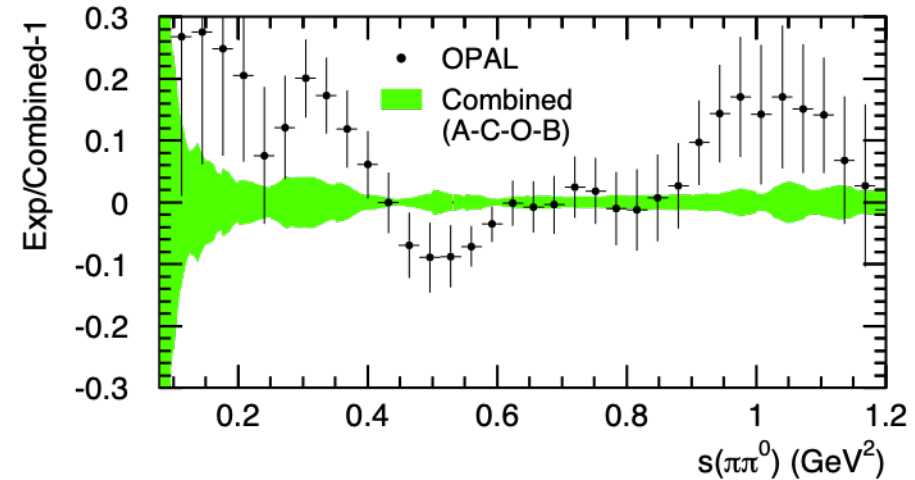
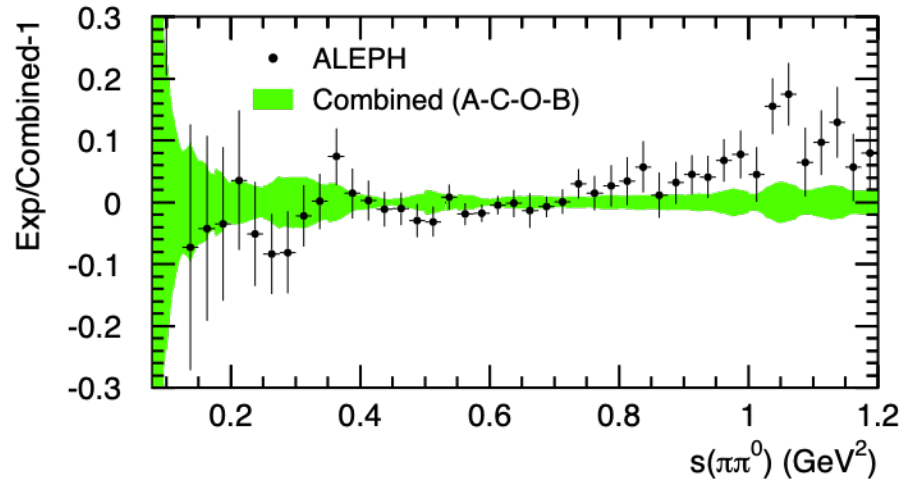
CLEO: [Phys. Rev. D 61, 112002 \(2000\)](#)

OPAL: [Eur. Phys. J. C 7, 571 \(1999\)](#)



Data Combination* for the $\pi\pi^0$ Channel

Figures from [DHMYZ 2014](#)



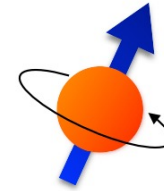
The normalization is constrained dominantly by ALEPH while the shape by Belle

* The combination performed using HVPTools [[DHMYZ, EPJC66 \(2010\) 1](#)]

Application to Muon Anomalous Magnetic Moment

For a charged lepton with charge q_ℓ and mass m_ℓ , its magnetic moment is connected to its spin by a g_ℓ factor:

$$\vec{\mu}_\ell = g_\ell \left(\frac{q_\ell}{2m_\ell} \right) \vec{S}_\ell$$



Dirac predicted $g_\ell = 2$ for electrons in 1928 [Proc. Roy. Soc. Lond. A 118, 351 (1928)]

Quantum corrections of the SM from all three sectors (QED, strong and weak) modify the value of g_ℓ by ~ 0.002

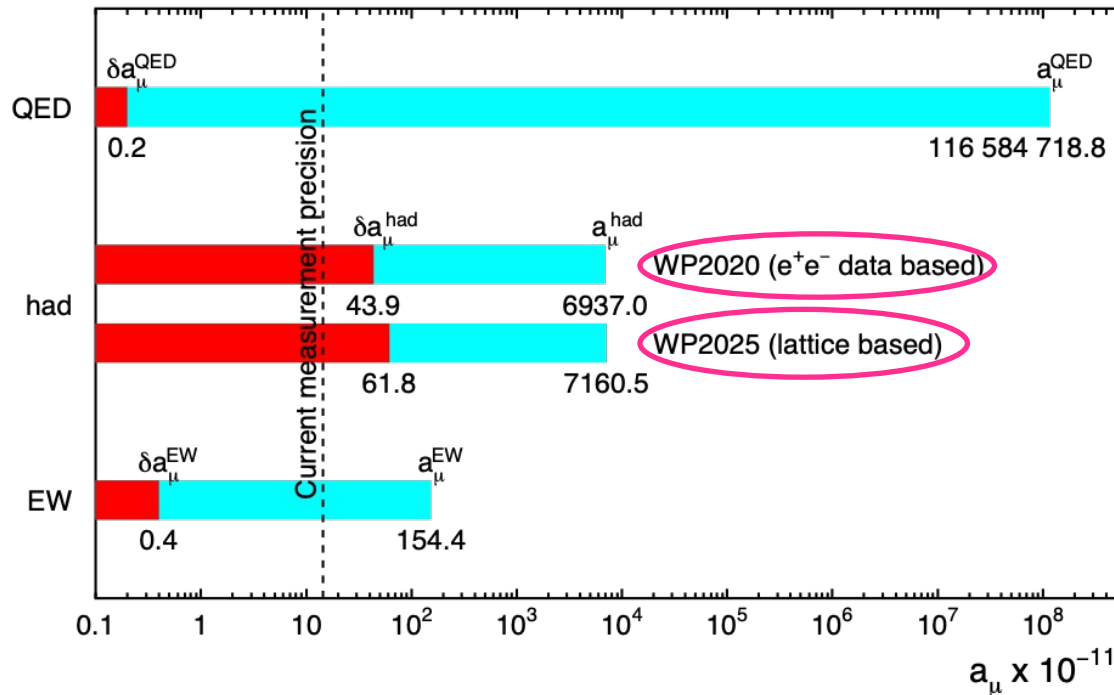
The anomalous magnetic moment a_ℓ was introduced to quantify the deviation from 2:

$$a_\ell = \frac{g_\ell - 2}{2} \sim 0.001$$

a_μ one of the most precisely predicted and measured quantities in particle physics, providing a precision probe for SM and BSM physics

SM Predictions

$$a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{had}} + a_{\mu}^{\text{EW}} \sim 0.001$$



$$a_{\mu}^{\text{had}} = a_{\mu}^{\text{HVP LO}} + a_{\mu}^{\text{HVP HO}} + a_{\mu}^{\text{HLbL}}$$

had	HVP LO	HVP HO	HLbL
	$a_{\mu}[10^{-11}]$ (unc)		
WP2020	6931 (40)	-85.9 (0.7)	92 (18)
WP2025	7132 (61)	-87.2 (1.3)	115.5 (9.9)

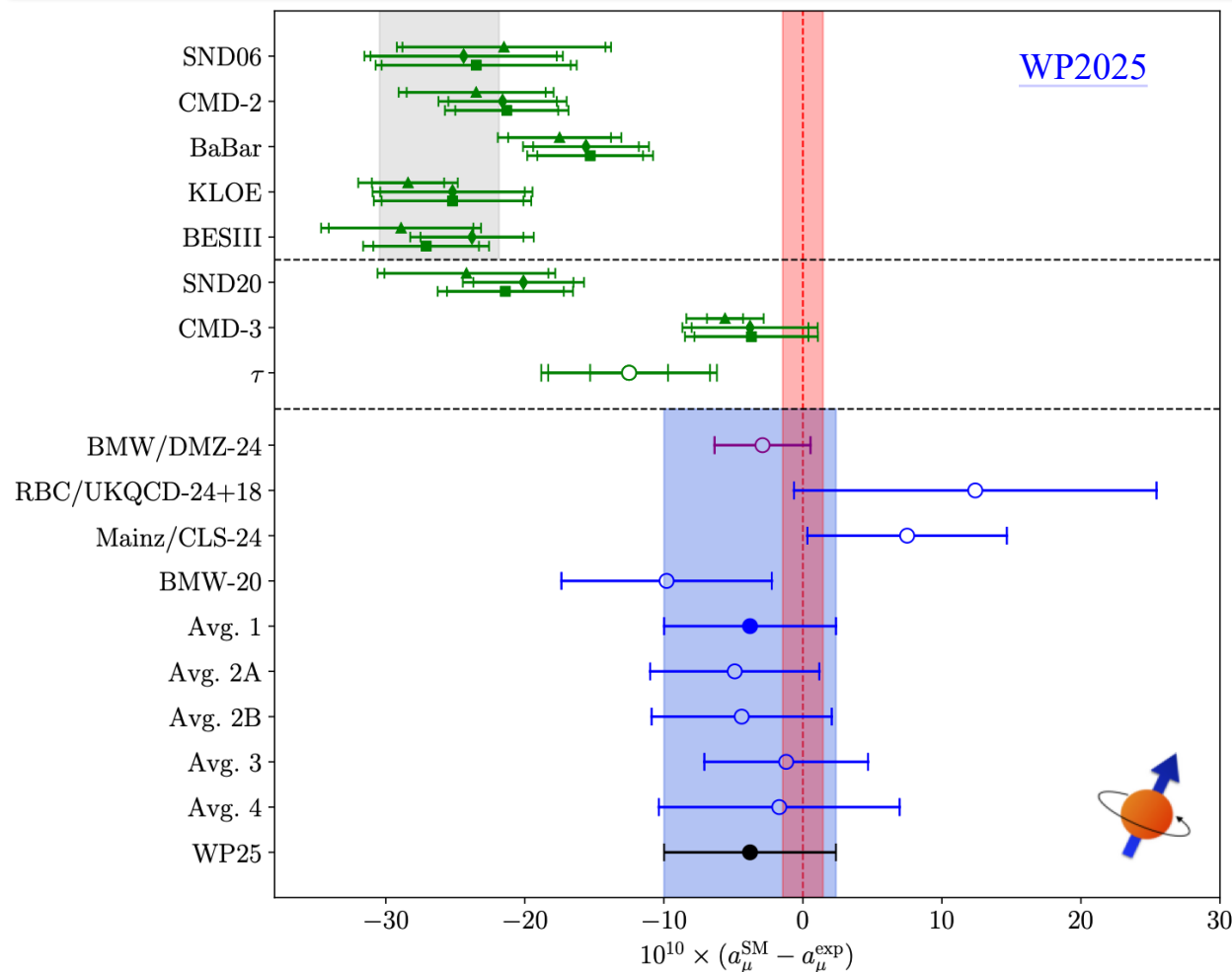
WP2020: [Phys. Rept. 887 \(2020\) 1](#)

WP2025: [arXiv:2505.21476](#)

WP stands for the white paper of the [muon g-2 theory initiative](#)

- QED has by far the largest contribution
- QED and EW both calculable with high precision within the SM
- HVP LO, of non-perturbative nature at low energies, has the largest uncertainty → the focus of studies since decades

Current Status of the Muon Magnetic Anomaly



➤ The e^+e^- based prediction shows a discrepancy of $>5\sigma$ with the measurement

➤ The lattice based prediction, though less precise, agrees with the measurement

➔ How is the e^+e^- / τ data-based HVP calculation performed?

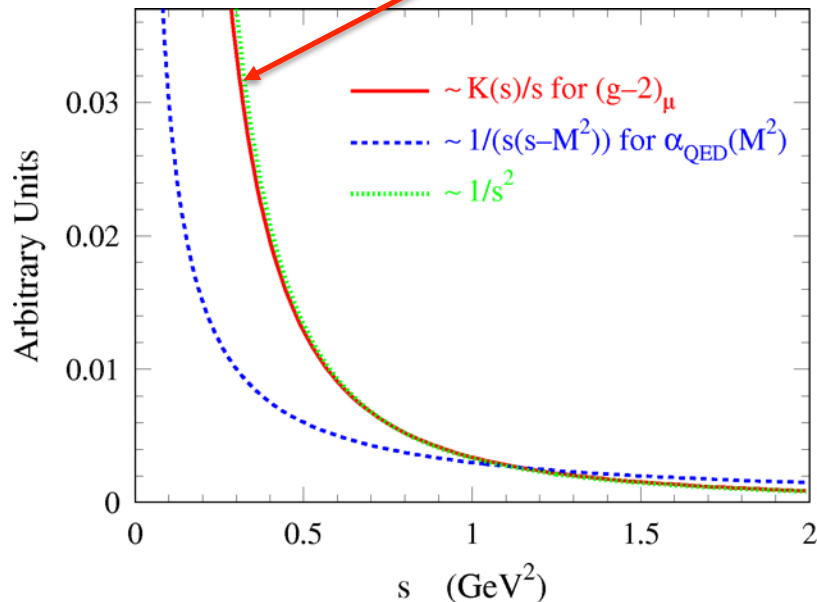
➔ What are the issues with the e^+e^- / τ data?

- Grey band: the e^+e^- based WP2020 SM precision
- Blue band: the lattice based WP2025 SM precision
- Red band: the measurement precision
- Green error bars: three slightly different evaluations

e^+e^- Data-Based HVP Calculation

Based on analyticity and unitarity, the LO HVP contribution can be calculated using the dispersion relation [1] over $e^+e^- \rightarrow \text{hadrons}$ cross sections

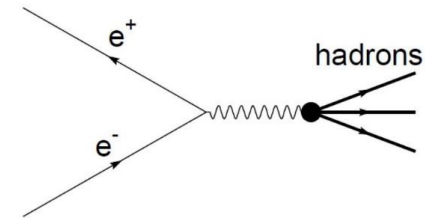
$$a_\mu^{\text{HVP LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$



Bare: $\sigma^{(0)}(s) = \sigma(s) (\alpha / \alpha(s))^2$

$$12\pi \text{Im}\Pi_\gamma(s) = \frac{\sigma^0 [e^+e^- \rightarrow \text{hadrons} (\gamma_{\text{FSR}})]}{\sigma_{pt}} \equiv R(s)$$

$$\text{Im}[\text{diagram}] \propto |\text{diagram} \rightarrow \text{hadrons}|^2$$



The QED kernel $K(s)$ [2] has such an s dependence that low energy data contribute most:

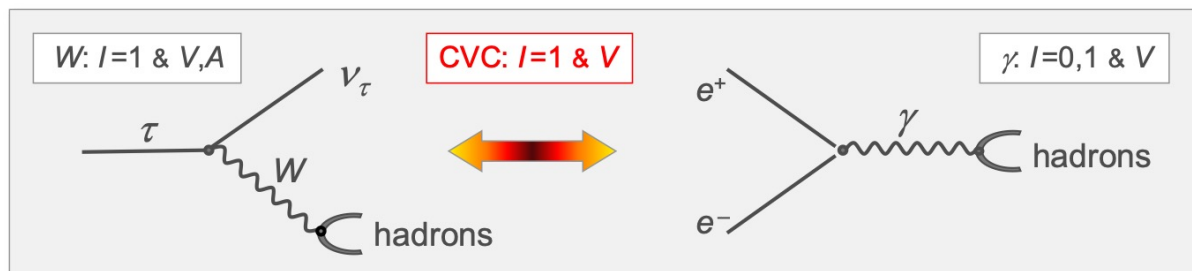
→ The precision is driven by that of $e^+e^- \rightarrow \text{hadrons}$
 ~73% is contributed by the two-pion channel

[1] Bouchiat and Michel, 1961

[2] Brodsky, de Rafael, 1968

Use tau SFs

- The use of tau SFs for the leading-order (LO) HVP evaluation was originally proposed by Alemany-Davier-Hoecker [ADH 1997]
 - Based on CVC (conserved vector current) relations:



$$\sigma_{e^+e^- \rightarrow \pi^+\pi^-}^{I=1} = \frac{4\pi\alpha^2}{s} v_{1,\pi^-\pi^0\nu_\tau}$$

$$\sigma_{e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-}^{I=1} = 2 \cdot \frac{4\pi\alpha^2}{s} v_{1,\pi^-3\pi^0\nu_\tau}$$

$$\sigma_{e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0}^{I=1} = \frac{4\pi\alpha^2}{s} \left[v_{1,2\pi^-\pi^+\pi^0\nu_\tau} - v_{1,\pi^-3\pi^0\nu_\tau} \right]$$

New precise Belle II $\pi\pi^0$ [1] will help to clarify the situation

We also need better $3\pi\pi^0$ and $\pi 3\pi^0$ [2] SFs!

where v are the relevant spectral functions in the 2π and 4π tau decay channels

- Isospin-breaking corrections are needed to take into account
 - EW radiative corrections
 - Mass difference between charged and neutral pions
 - Mass and width differences between charged and neutral rho resonances

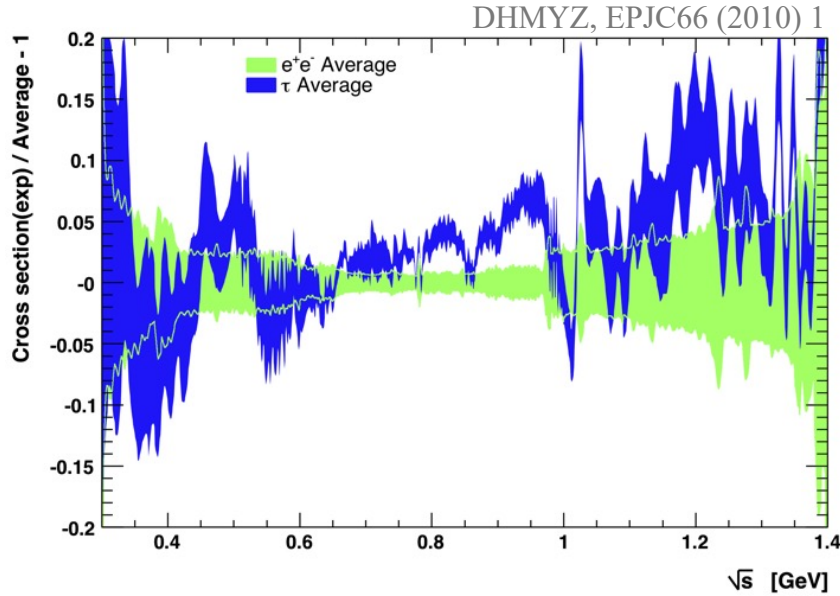
➔ Talk of Gabriel Lopez Castro

[1] Belle II analysis ongoing by Yipu Liao et al.

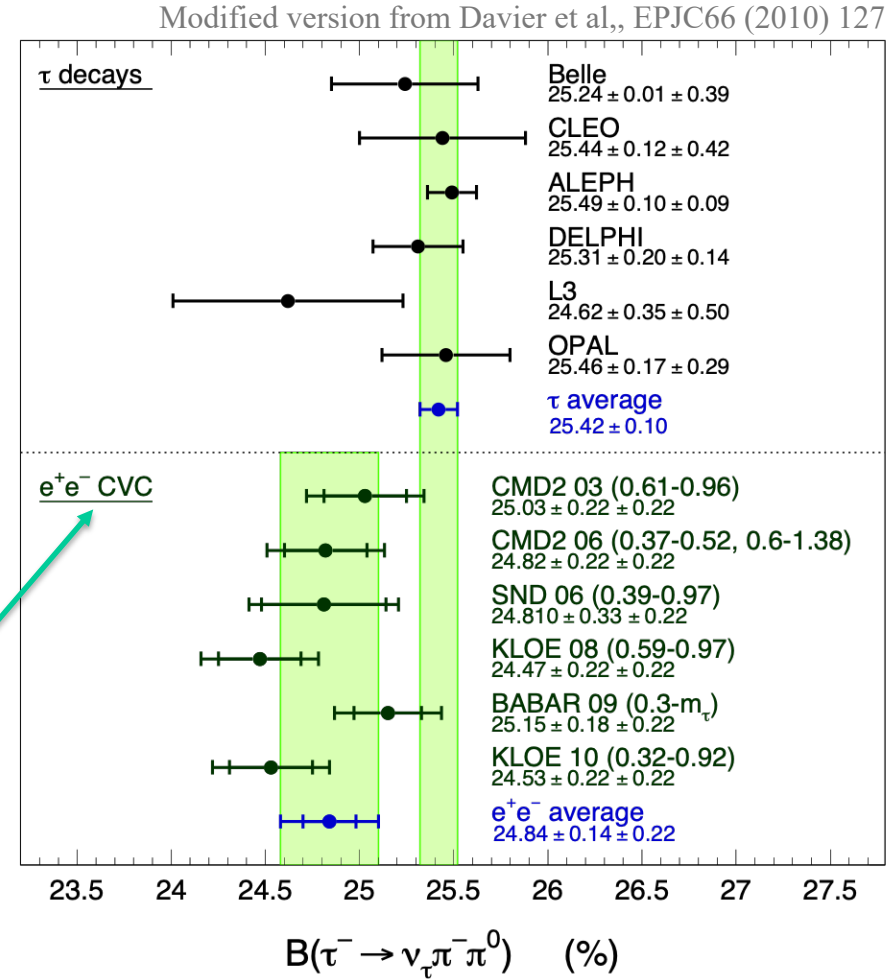
[2] Belle II analysis ongoing by Yijia Zeng et al.

Tau and e^+e^- data comparison in two-pion channel

Take into account all known isospin breaking corrections except for the ρ - γ mixing correction



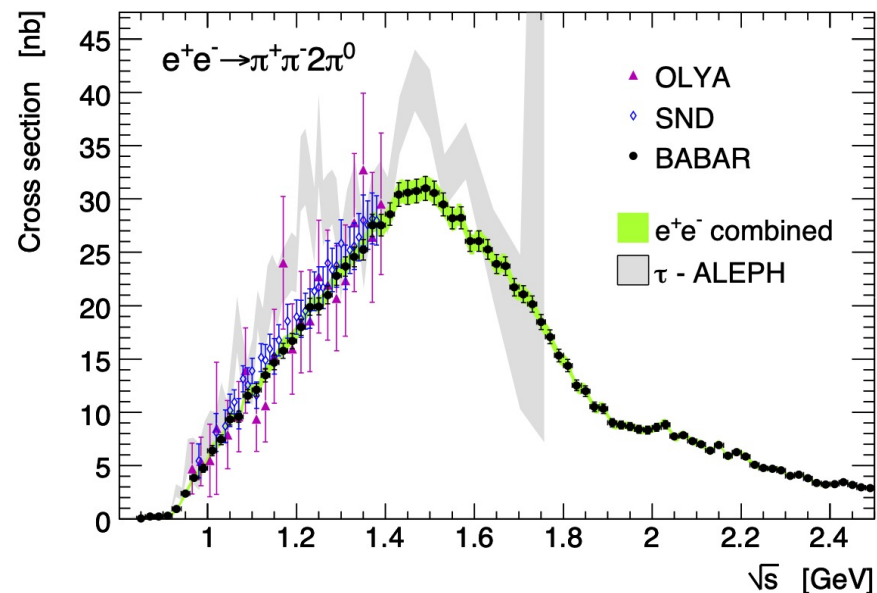
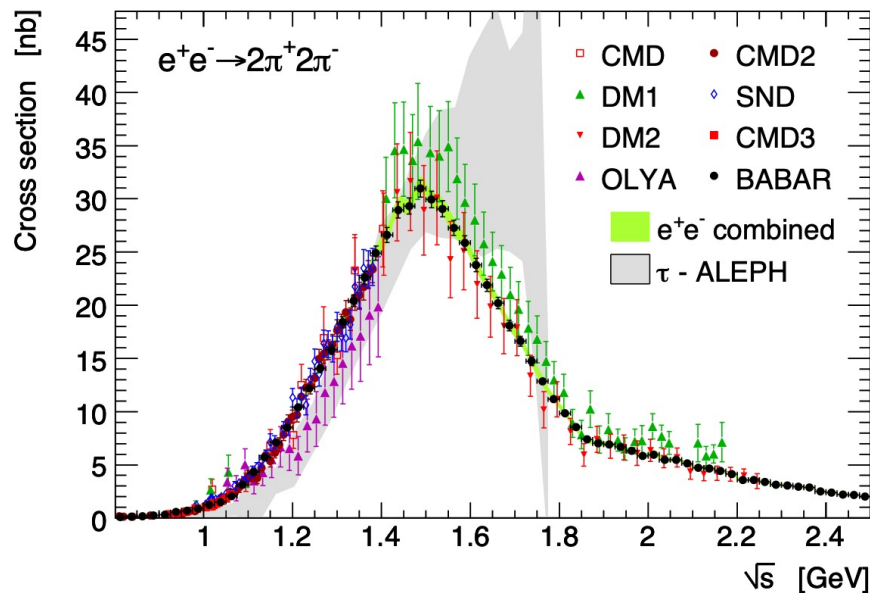
$$B_X^{\text{CVC}} = \frac{3}{2} \frac{B_e |V_{ud}|^2}{\pi \alpha^2 m_\tau^2} \int_{s_{\min}}^{m_\tau^2} ds s \sigma_{X^0} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)$$



Clear difference in shape and in BR between e^+e^- and τ average

Current Situation of Four-Pion Modes

Figure from [DHMZ, EPJC 77 \(2017\) 827](#)



Only ALEPH measured SFs in these τ decay modes. The precision was very limited

→ Big room for Belle II to have a significant impact here

HVP Contribution to Running QED $\alpha(s)$

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha_{\text{lep}}(s) - \Delta\alpha_{\text{had}}(s)}$$

$$\Delta\alpha_{\text{had}}(s) = -\frac{\alpha(0)}{3\pi} \text{Re} \int_{4m_\pi^2}^{\infty} ds' \frac{R(s')}{s'(s' - s - i\epsilon)}$$

$R(s)$: the same input as for a_μ LO HVP except that the higher energy part receives relatively larger weights

The current precision for $\Delta\alpha_{\text{had}}(s)$ is a limiting factor for stringent SM consistency tests:

$$\Delta\alpha_{\text{lep}}(M_Z^2) = 314.979(2) \times 10^{-4}$$

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = 275.7(1.0) \times 10^{-4}$$

$$\Delta\alpha_{\text{had}}^{\text{top}}(M_Z^2) = -0.72(1) \times 10^{-4}$$

Indeed, using our 2011 evaluation increases the most probable Higgs boson mass by +7 GeV from 84 to 91 GeV with an uncertainty of $^{+30}_{-23}$ GeV, in excellent agreement with the discovery of the Higgs boson in 2012 at ~ 125 GeV

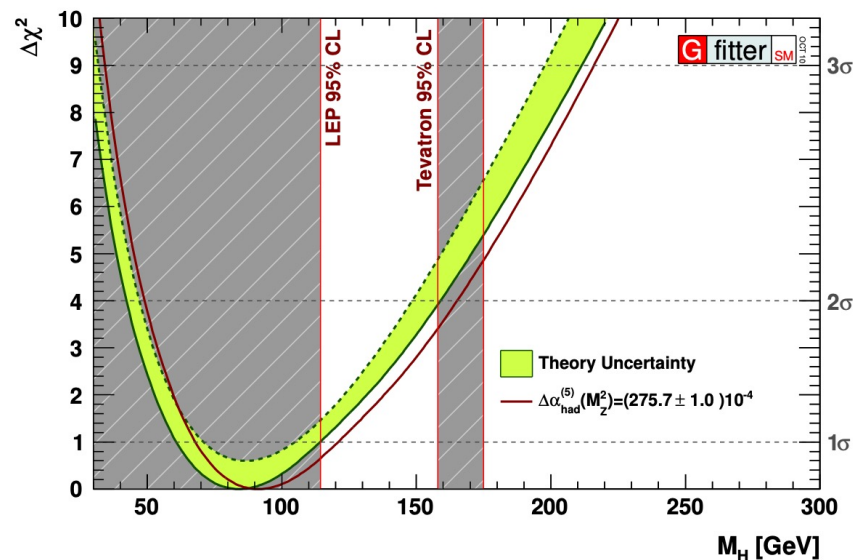
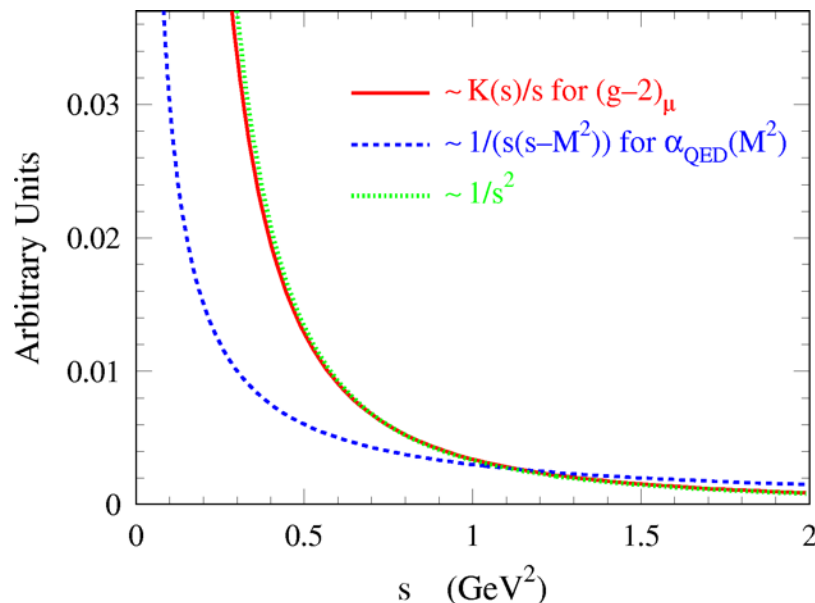


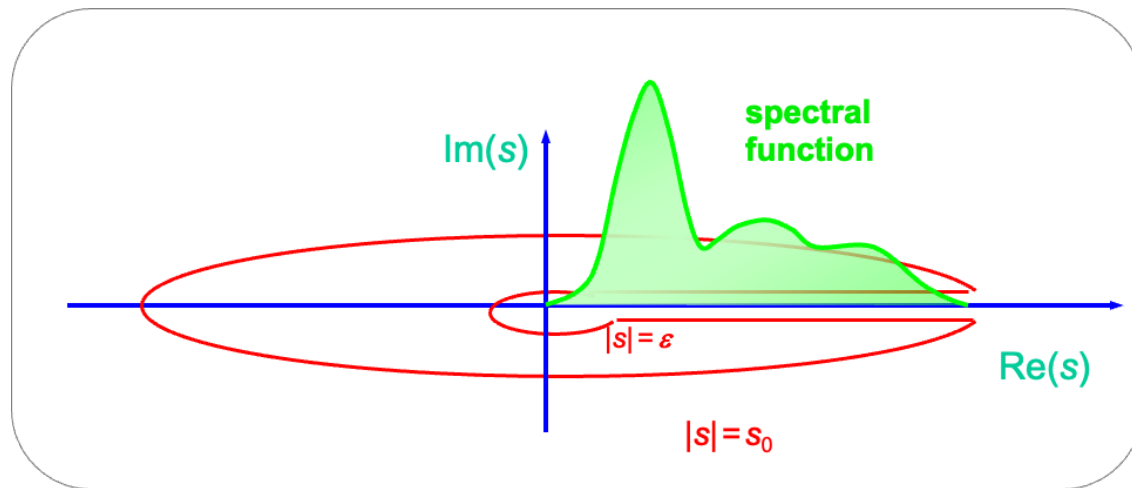
Figure from DHMZ, [EPJC71 \(2011\) 1515](#)

Application to the determination of α_s

Measured SFs can be connected to theoretical prediction with

$$R(s_0) \propto \int_0^{s_0} ds w(s) \text{Im}\Pi(s + i\epsilon) \iff -\frac{1}{2i} \oint_{s=s_0} ds w(s) \Pi(s)$$

$$\propto |V_{\text{CKM}}|^2 S_{\text{EW}} \left(1 + \delta_{\text{pQCD}}^{(0)}(\alpha_s) + \delta'_{\text{EW}} + \delta^{(2)}(m_q) + \sum_{D=4,6,\dots} \delta_{\text{non-pQCD}}^{(D)} \right)$$



SF: Fundamental ingredient relating long distance (resonances) to short distance description (QCD)

$$\delta_{\text{pQCD}}^{(0)}(\alpha_s)$$

Dominant pQCD contribution, known up to 5th order

$$\delta'_{\text{EW}} \simeq 0.0010$$

Non-logarithmic EW correction ([Braaten-Li, 1990](#))

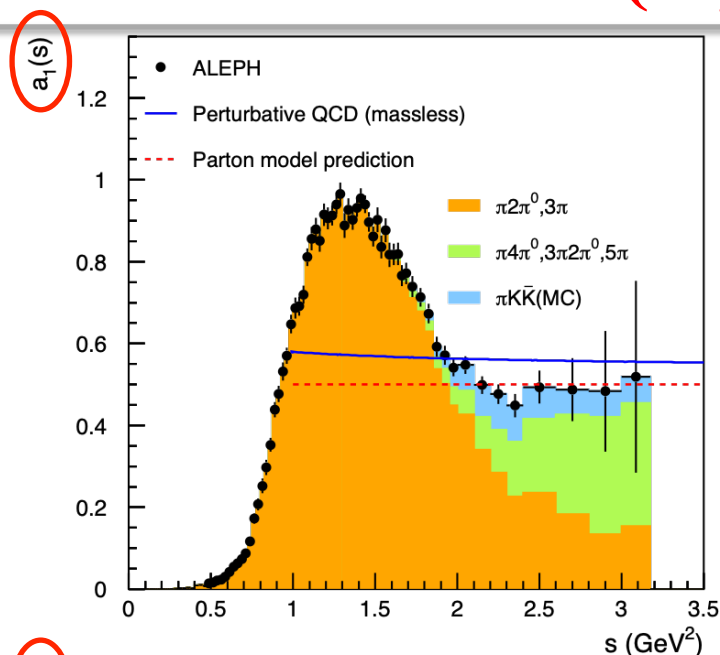
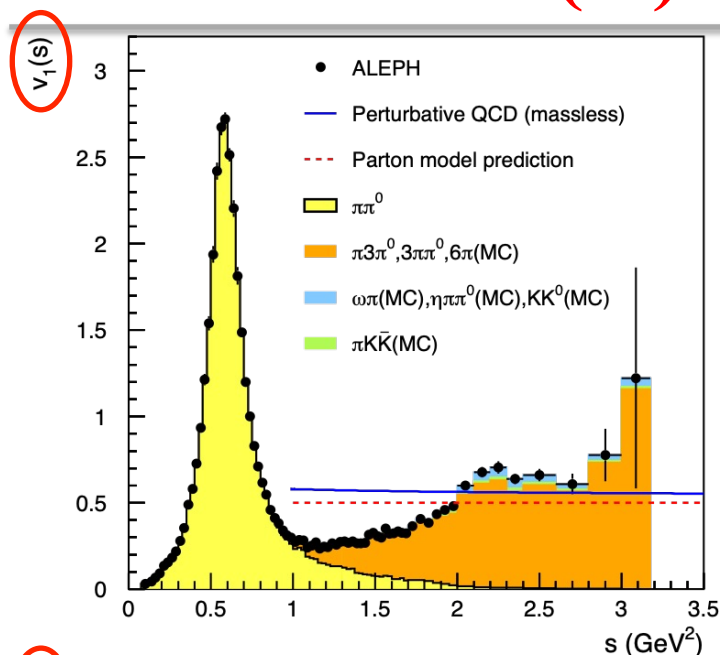
$$\delta^{(2)}(m_q)$$

Dimension 2 pQCD contribution from quark mass (<0.1% for u, d quarks)

$$\delta_{\text{non-pQCD}}^{(D)}$$

Dimension D non-pQCD contributions

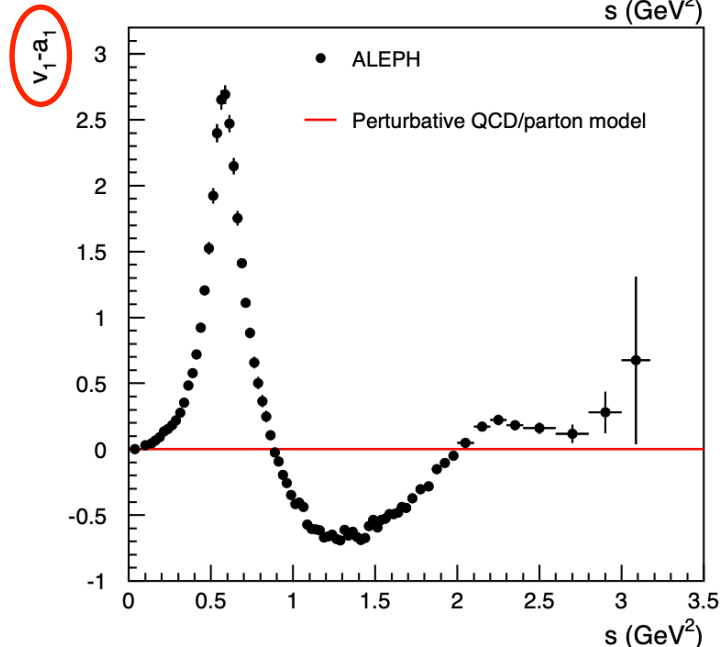
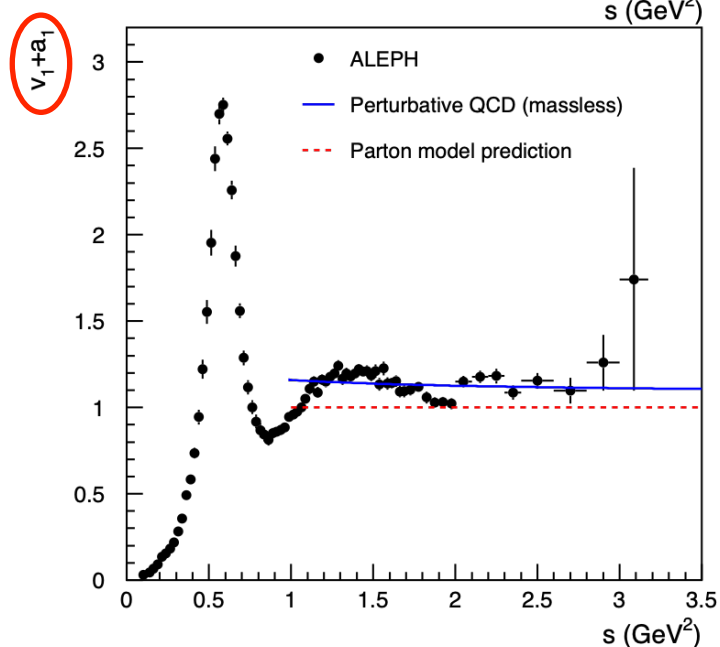
Vector (v_1) and Axial-Vector (a_1) SFs



DHMYZ 2014

Limited precision at high energy tails

Unmeasured modes based on MC



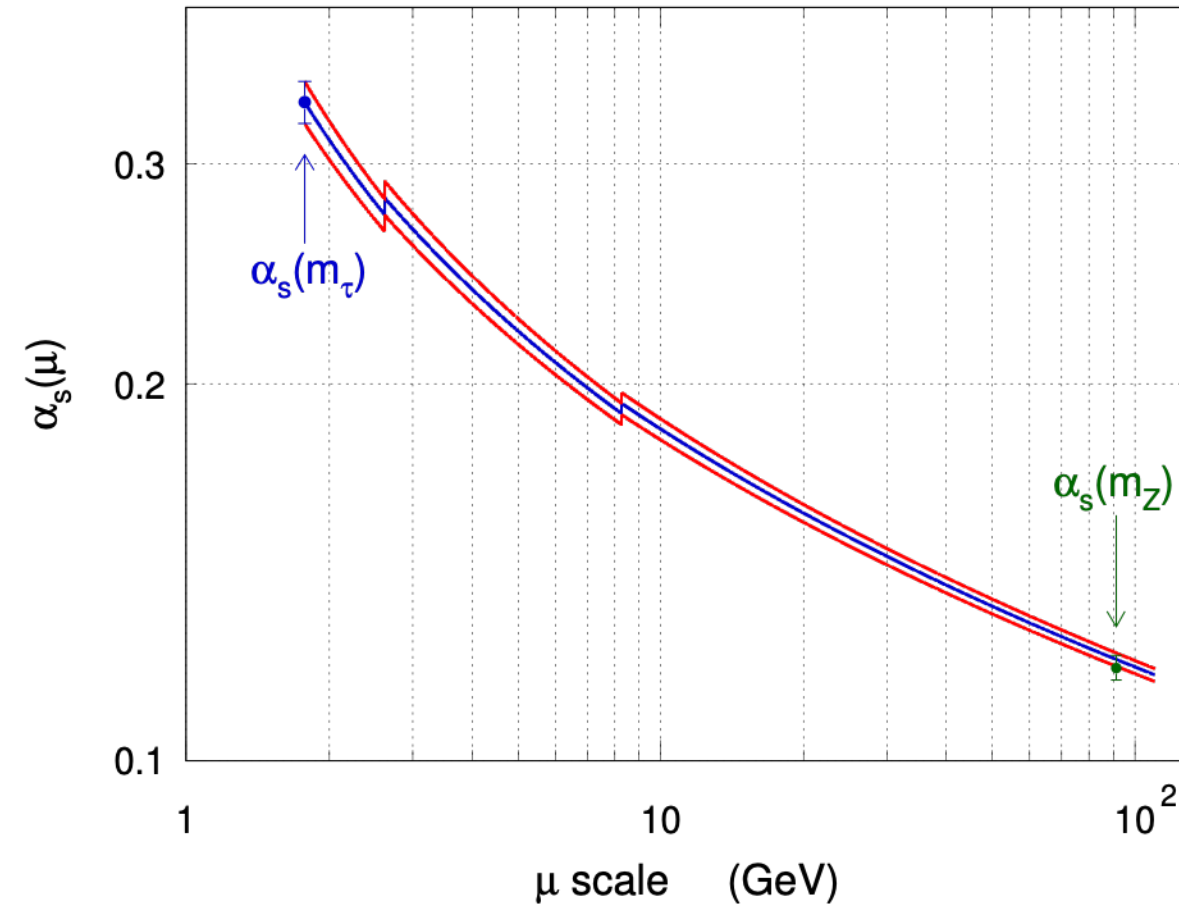
$v_1 + a_1$ at high energy tail reaches the asymptotic limit in agreement with pQCD prediction
 $\rightarrow \alpha_s$ determination

$v_1 - a_1$ purely non-perturbative contribution

Running Strong Coupling Constant

Tau α_s determination evolved to M_Z : $\alpha_s(M_Z^2) = 0.1209 \pm 0.0006_{\text{exp}} \pm 0.0016_{\text{th}} \pm 0.0005_{\text{evol}}$
 $= 0.1209 \pm 0.0018$.

ALEPH 2005

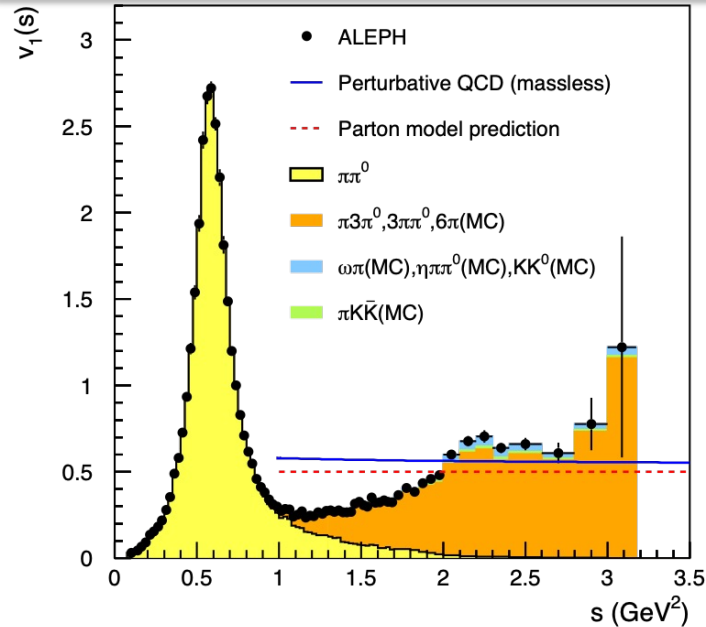


to be compared with

$$\alpha_s(M_Z^2)_{\text{Z width}} = 0.1186 \pm 0.0027$$

provides a powerful test of the evolution of α_s over a large energy range

Belle II can/should play an important role here

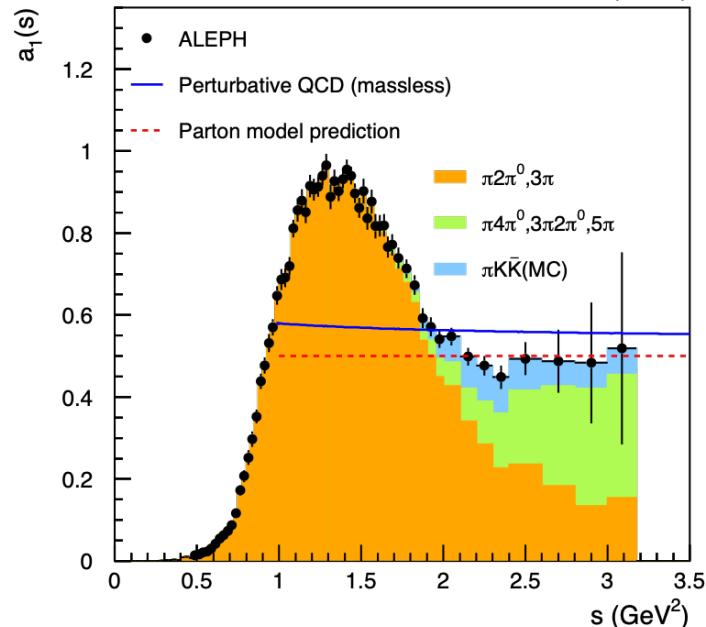


For the vector component:

- $2\pi^{[1]}/4\pi^{[2]}$ channel(s) have the largest contribution at low/high energies
- Several channels unmeasured previously

[1] $\pi\pi^0$: Yipu Liao et al.

[2] $\pi 3\pi^0$: Yijia Zeng et al.



For the axial-vector component:

- Both 3π and $5\pi^{[3]}$ channels have large contribution towards m_τ
- $\pi K K^{[4]}$ channels previously unmeasured, it has also large uncertainty in the v/a separation

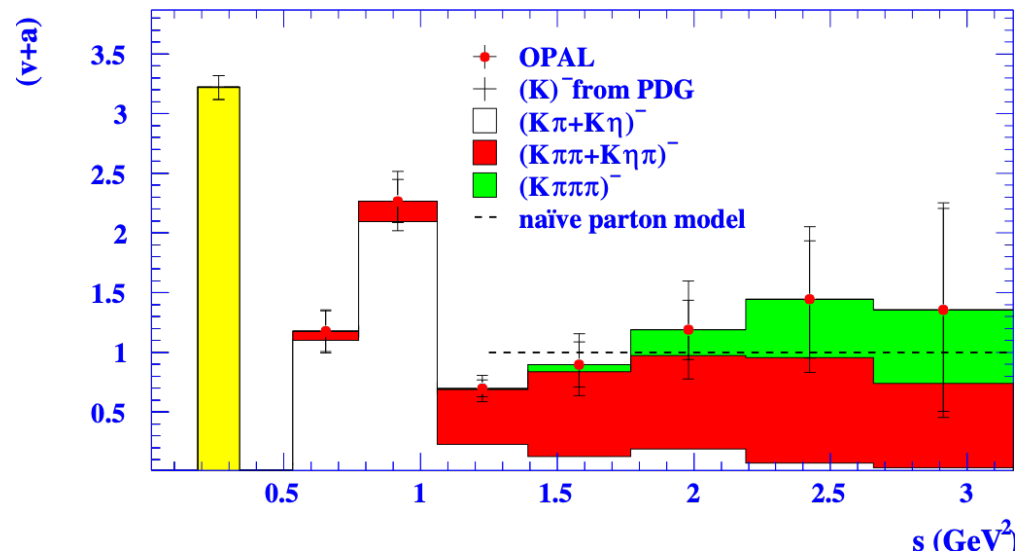
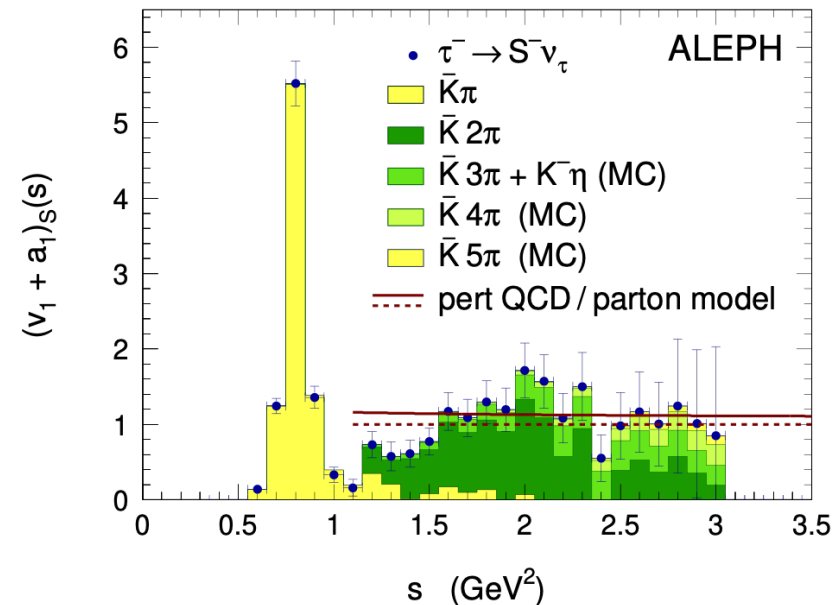
[3] $3\pi 2\pi^0$: Xu Gao et al.

[4] $KK(\pi)$: Xinping Xu et al.

Strange SFs and its Applications

Measured strange SFs

DHZ, RMP 78 (2006) 1043



OPAL normalized each channel with BF from PDG, dominated by ALEPH
[Belle](#) also measured $Ks\pi$ in 2007

Very limited precision in the strange SFs
 There is large room for improvement

Connection to Theoretical Predictions

$$R_\tau(m_\tau^2) = 12\pi S_{\text{EW}} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im}\Pi^{(1+0)}(s + i\epsilon) - 2\frac{s}{m_\tau^2} \text{Im}\Pi^{(0)}(s + i\epsilon) \right]$$

$$R_{\tau,S} = 3|V_{us}|^2 S_{\text{EW}} \left(1 + \delta^{(0)} + \delta'_{\text{EW}} + \delta_S^{(2,m_q)} + \sum_{D=4,6,\dots} \delta_S^{(D)} \right)$$

$\delta^{(0)}$ perturbative part of mass dimension $D = 0$

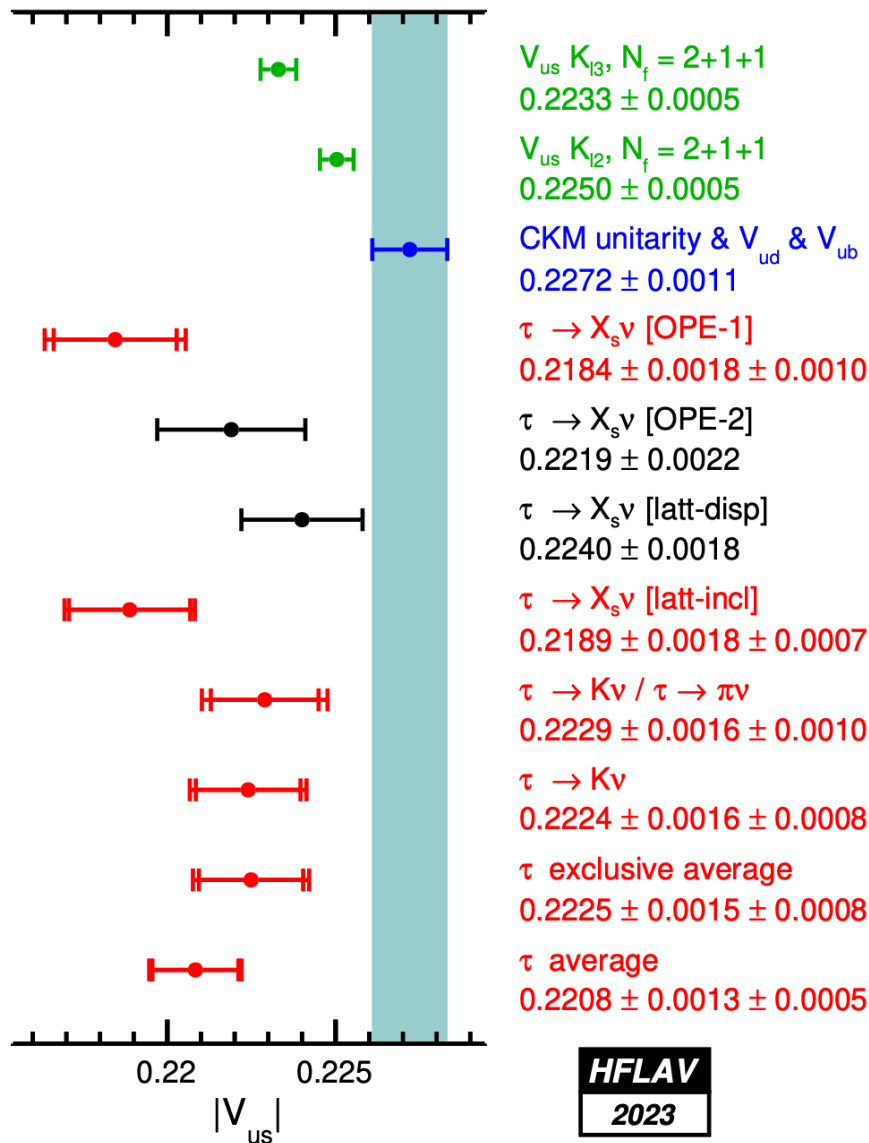
$$\delta'_{\text{EW}} \simeq 0.0010$$

$$\delta_S^{(2,m_s)} = -8 \frac{m_s^2(m_\tau^2)}{m_\tau^2} \left[1 + \frac{16}{3} a_s + 46.00 a_s^2 + \left(283.6 + \frac{3}{4} x_3^{(1+0)} \right) a_s^3 + \dots \right]$$

The dimension-2 term is sensitive to strange quark mass

V_{us} , and/or m_s can be extracted from strange SFs and their moments

Current Status of V_{us} Determinations



We really need high precision BF's and SF's measurements in the strange decay modes to clarify the discrepancies!

Many ongoing analyses in Belle II try to improve the relevant BF and SF measurements inclusively and exclusively:

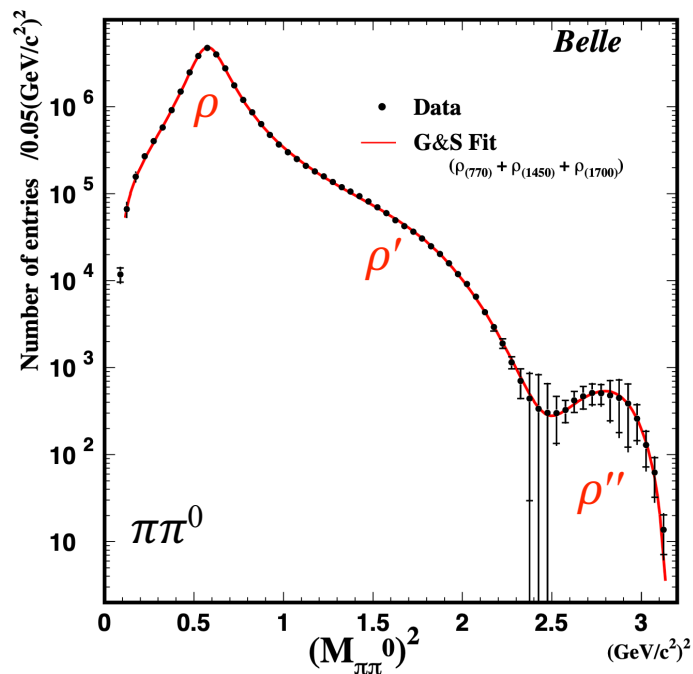
- Two inclusive analyses:
 - One by Arthur Thaller et al.
 - Another by Xiantao Hou et al.
- Several exclusive analyses:
 - $K\pi^0$: Qiming Li $\rightarrow$ Yanking Li et al.
 - $K\eta$: Wenzhe Li et al.
 - $K\pi\pi$: Denis Bodrov et al.
 - $K\eta\pi^0$: Xinping Xu et al.
 - $Ks\pi$: Junli Ma et al.
 - etc

$\rightarrow$ Talk of Michele Mantovano

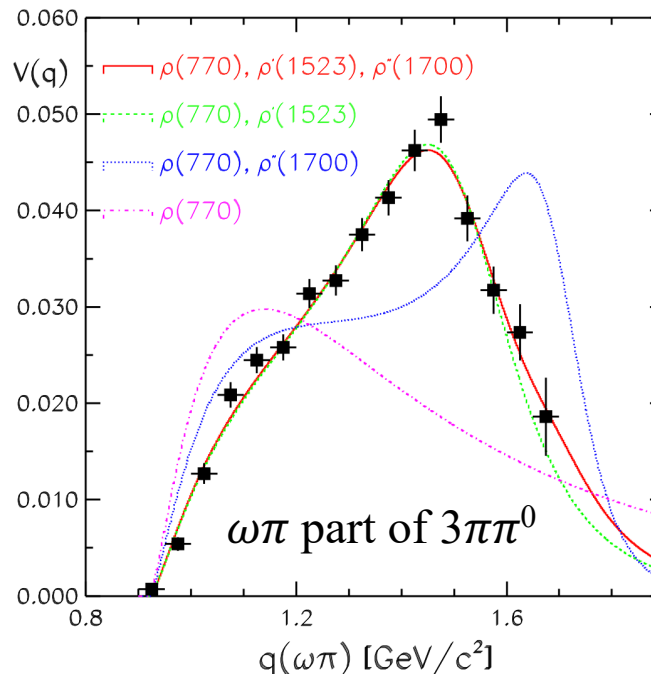
Analyses for understanding Hadron Production Dynamics

- We have seen SFs are the key ingredient for connecting non-pQCD and pQCD
- However our current knowledge of hadron production dynamics is very limited
 - ➔ big discrepancy between measurements and TAUOLA expectations
- To make progress, we need more differential measurements to understand better the hadron production dynamics / structure of hadronic currents

[Belle, PRD78 \(2008\) 072006](#)



[CLEO, PRD61 \(2000\) 072003](#)



➔ Joint analyses in different decay modes?

See also partial-wave analysis (PWA) of CLEO to $\pi 2\pi^0$ mode:

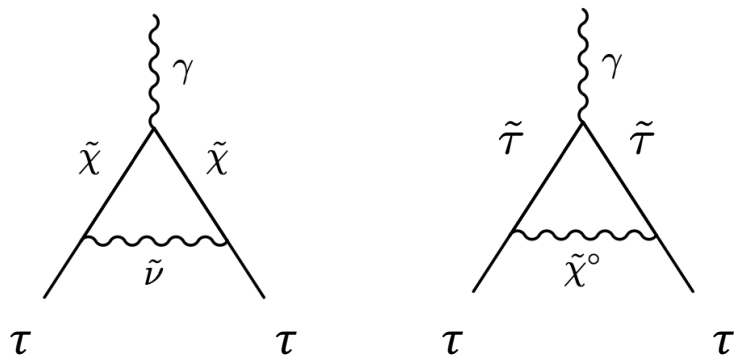
[PRD61 \(2000\) 012002](#)

Electric Dipole and Anomalous Magnetic Moments

Joël Gogniat, Tau2025

		
$d_\ell^{\text{exp}} \lesssim 10^{-30}$ Roussy et al. 2023	$\lesssim 10^{-19}$ BNL 2009	$\lesssim 10^{-17}$ Belle 2022
$\Delta a_\ell^{\text{exp}} \simeq 10^{-13}$ Northwestern 2022	$\simeq 10^{-10}$ Fermilab 25	$\simeq 10^{-3}$ CMS 2024
$\Delta a_\ell^{\text{SM}} \simeq 10^{-12}$ Parker et al. 2018 Morel et al. 2020	$\lesssim 10^{-9}$ Muon g-2 TI 25	$\simeq 10^{-7}$ Keshavarzi et al. 2024

Tau lepton expected to be more sensitive to NP contributions: $a_\tau \propto (m_\tau / \Lambda)^2$
 Future longitudinally polarized beam(s) would increase the sensitivity



Recent STCF study:
 Sun-Wu-Zhou
[Chin. Phys. C 49 \(2025\) 113001](#)
 $|d_\tau| < 3.89 \times 10^{-18} \text{ ecm @ 68\%CL}$

→ Talk of Yongcheng Wu

CP Violation in Tau Decays?

CP violation observed in the quark sector cannot explain the baryon asymmetry of the Universe

Tau leptons decaying into hadrons provide a unique opportunity to study and search for new CP violation in the lepton sector

CP asymmetry in $\tau \rightarrow \nu_\tau \pi K_S (\rightarrow 2\pi)$

→ Talk of Xin-Qiang Li

$$\mathcal{A}_\tau = \frac{\Gamma(\tau^+ \rightarrow \pi^+ K_S^0 \bar{\nu}_\tau) - \Gamma(\tau^- \rightarrow \pi^- K_S^0 \nu_\tau)}{\Gamma(\tau^+ \rightarrow \pi^+ K_S^0 \bar{\nu}_\tau) + \Gamma(\tau^- \rightarrow \pi^- K_S^0 \nu_\tau)}$$

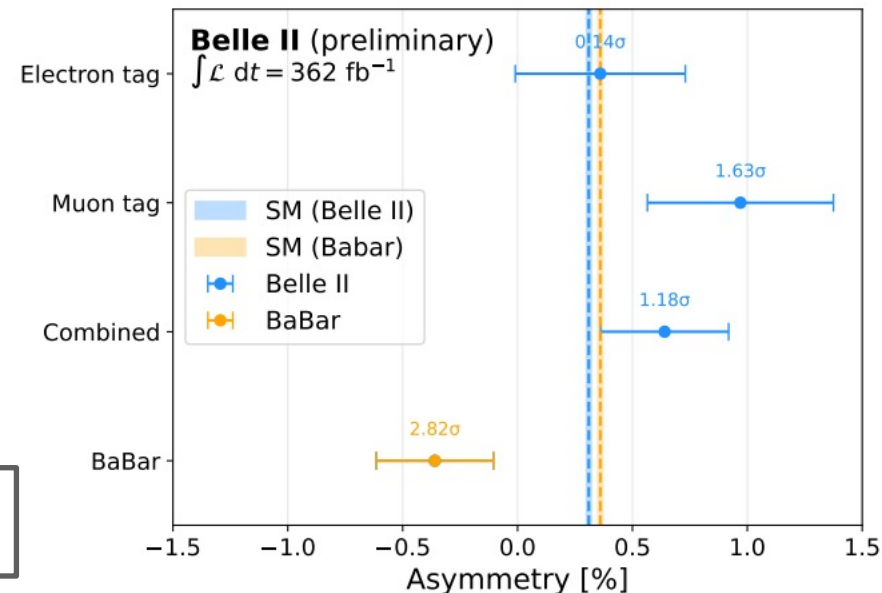
is expected due to the K-Kbar mixing and predicted to be $+0.36(0.01)\%$ [BS 2005, GN 2012]

This process can be served as a calibration for searching for additional CP violation in τ decays

BABAR's measurement: $-0.36(0.23)(0.11)\%$ [BABAR 2011]

New Belle II measurement is closer to the SM expectation

STCF study: Xiang-Shi-Zhou-Kang-Liu, [Chin. Phys. C 45 \(2021\) 053003](#), See also H.Y. Cheng et al., [arXiv:2502.08907](#)



Other Topics

- Dark sector searches for dark matter or axion-like candidates → Talk of Dmitri Liventsev
- Michel parameters - tau decay parameters (study if the Lorentz structure of charged weak currents is SM like) → Talk of Denis Epifanov
- Quantum entanglement → Talk of Chentao Bao
- Search for new physics using effective field theory (EFT) approaches

Summary

- Continue to search for rare or forbidden decay modes
- Measure with best precision
 - Absolute and/or relative branching fractions (BFs) of all tau decay modes
 - More differential distributions
- All need dedication, imagination, innovative analysis techniques & good coordination between relevant analyses
- Comment: It is good/important to have the best measurement for each decay mode. However, one needs to keep all the relevant analysis information so that one can handle correctly the correlation (normalisation and shape) between different decay modes