

Shower Shapes and Data-Driven Simulation Studies using a Highly Granular Crystal ECAL

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on behalf of the CEPC Working Group

Overview

Highly Granular Crystal ECAL

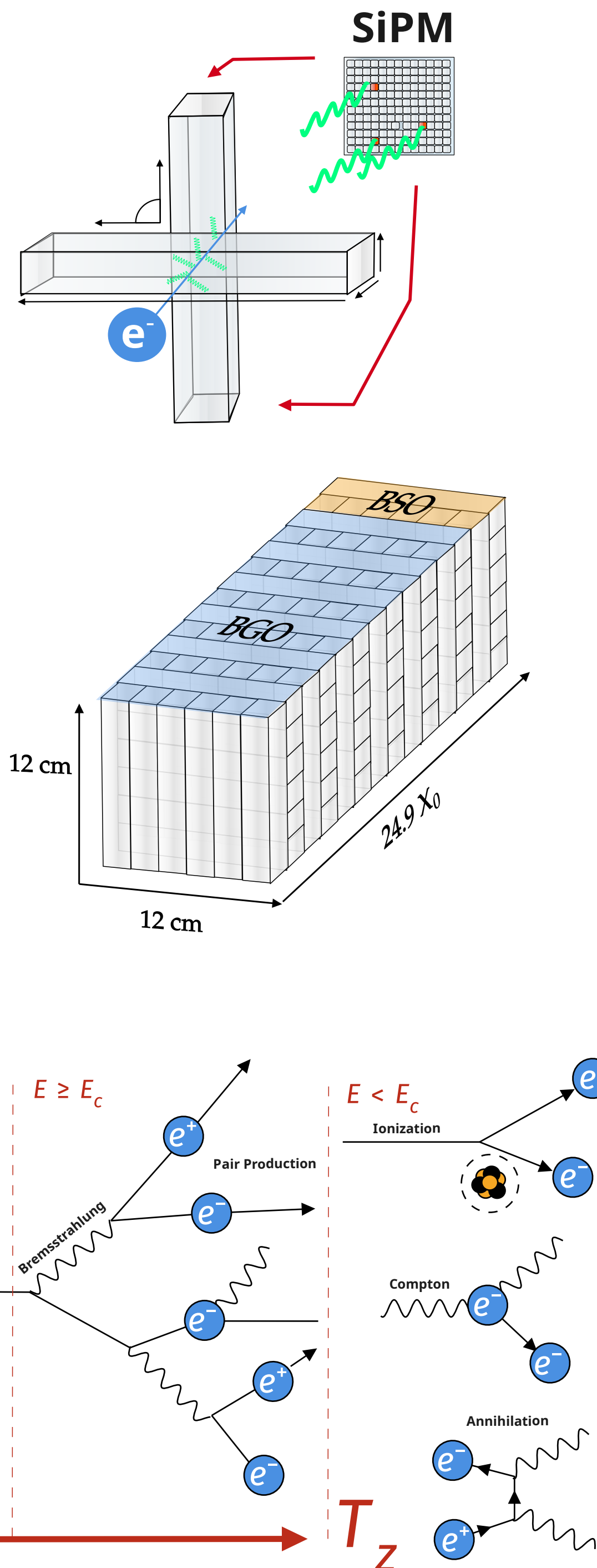
► criss-crossing scintillating crystal bars [1]:

- CEPC detector volume: $1.5 \times 1.5 \times 40 \text{ cm}^3$
- prototype volume: $2 \times 2 \times 12 \text{ cm}^3$
- homogeneous;
- crystal: BGO/BSO;

► granularity:

- particle separation capability
- energy density information;

► excellent EM energy resolution: $\sim 1\text{-}2\%/ \text{GeV}$



Shower Shapes

► Shower shapes can be used for:

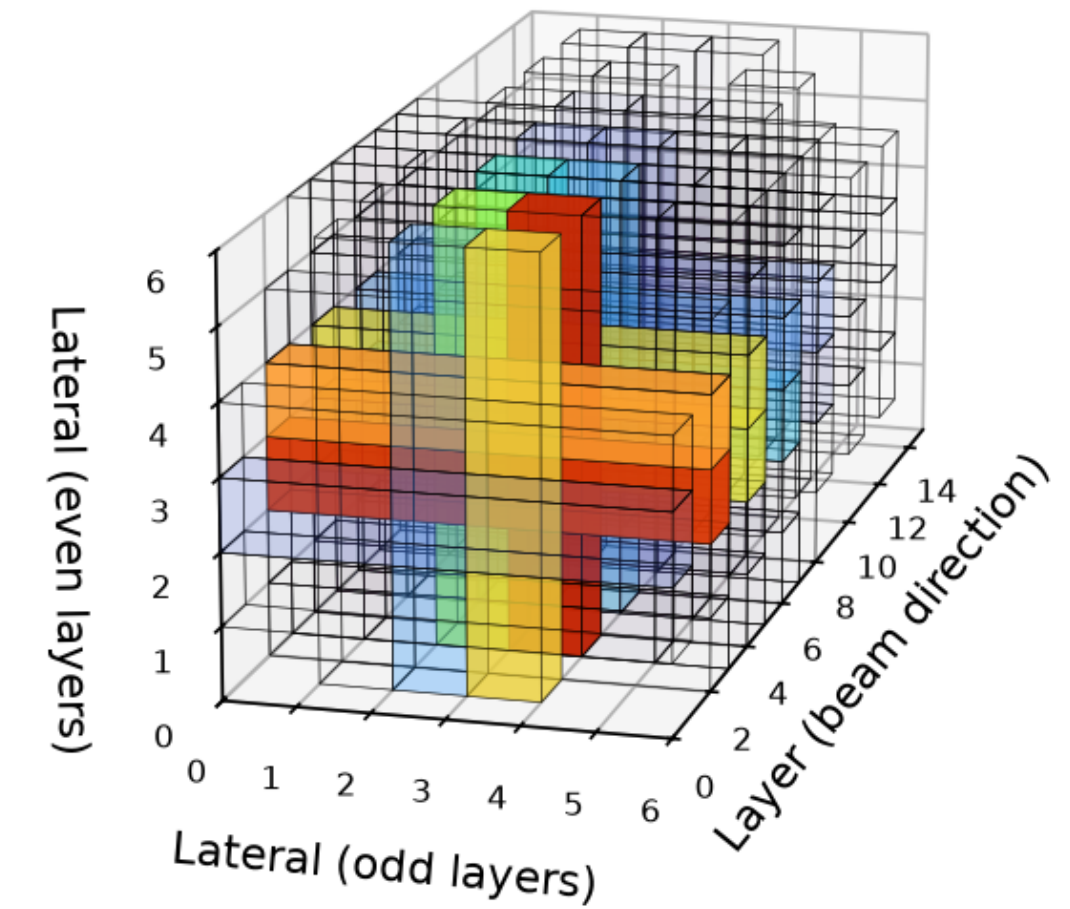
- studying shower fluctuations;
- measuring absorber properties;
- synthetic shower generation;

► Gap in literature for shower shapes $< 1 \text{ GeV}$.

Summary of Studies

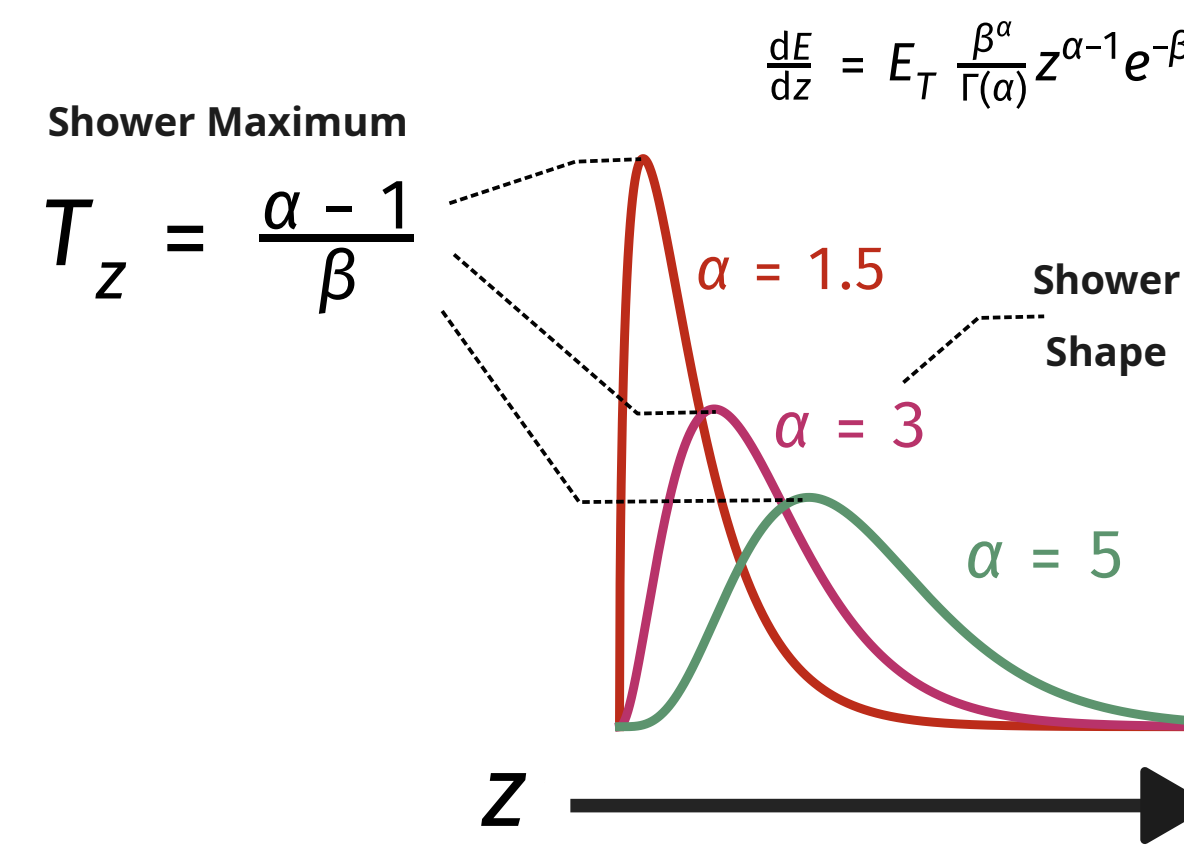
2025 Testbeam Data

- Taken at Proton Synchrotron (PS) T9 Beamline;
- Electrons, 0.2-6 GeV
- $\sim 2\text{M}$ electron events;
- PID used to select electrons;

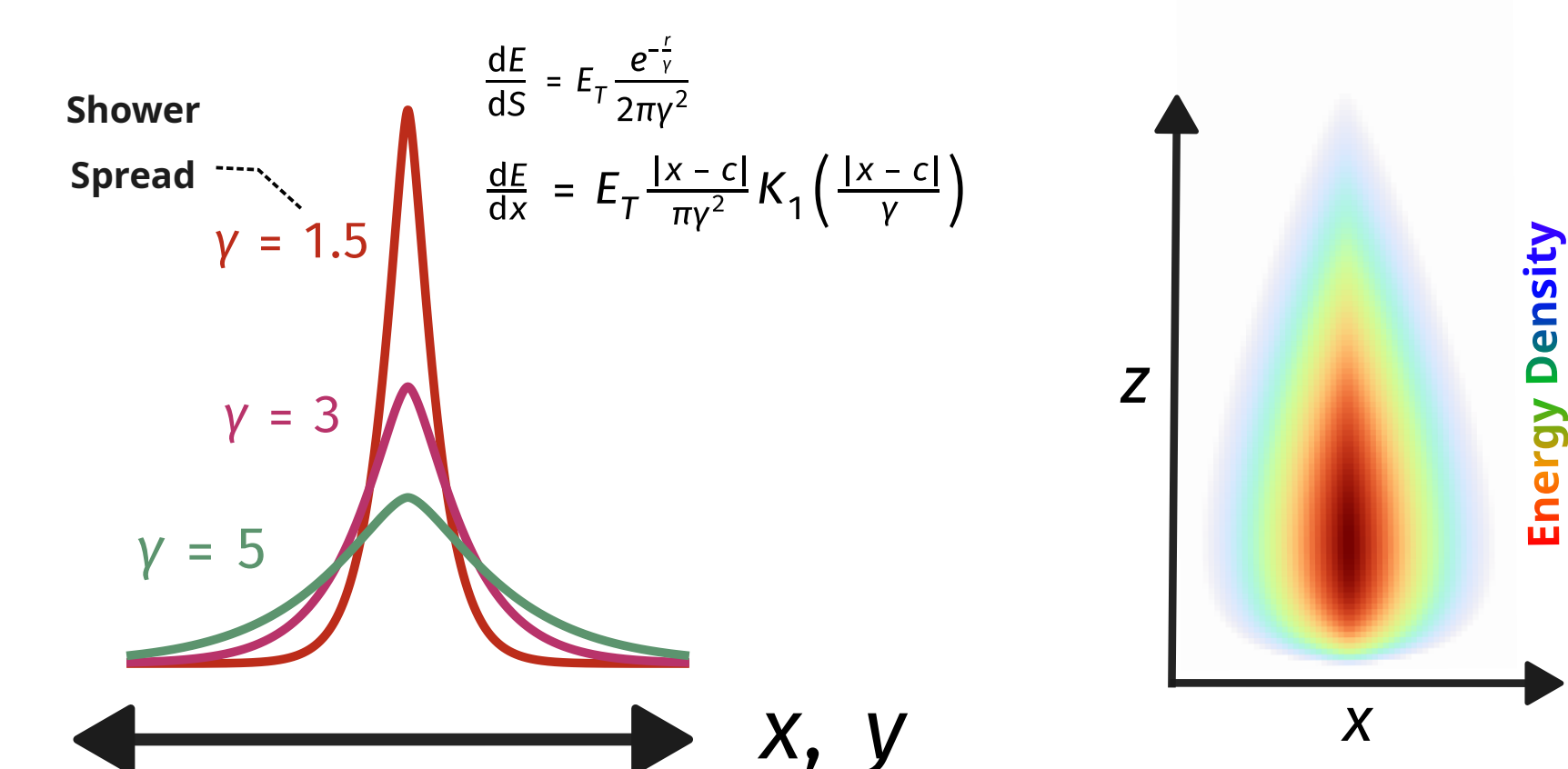


Fitted Shower Shape Models

► Longitudinal Distribution: Gamma

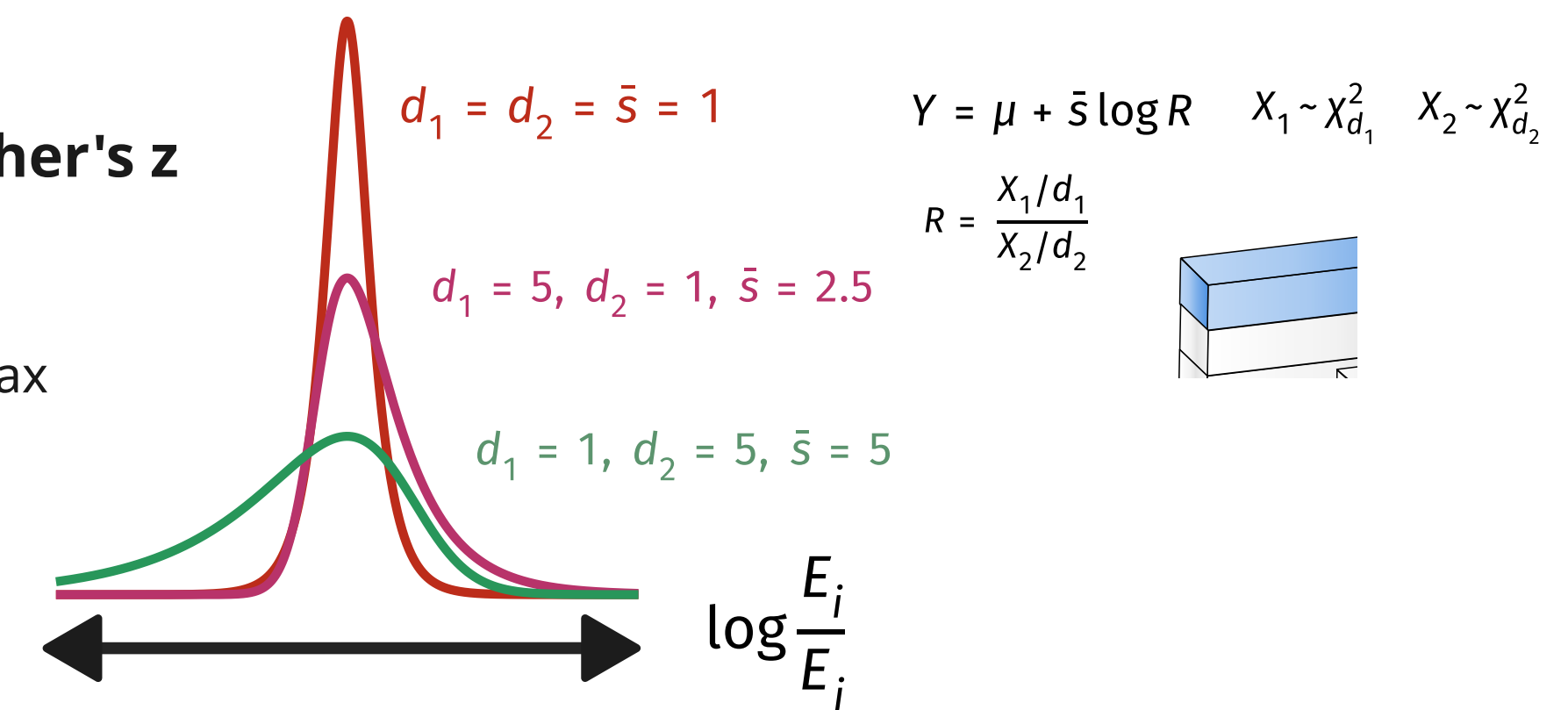


► Lateral Distribution: Bessel-K



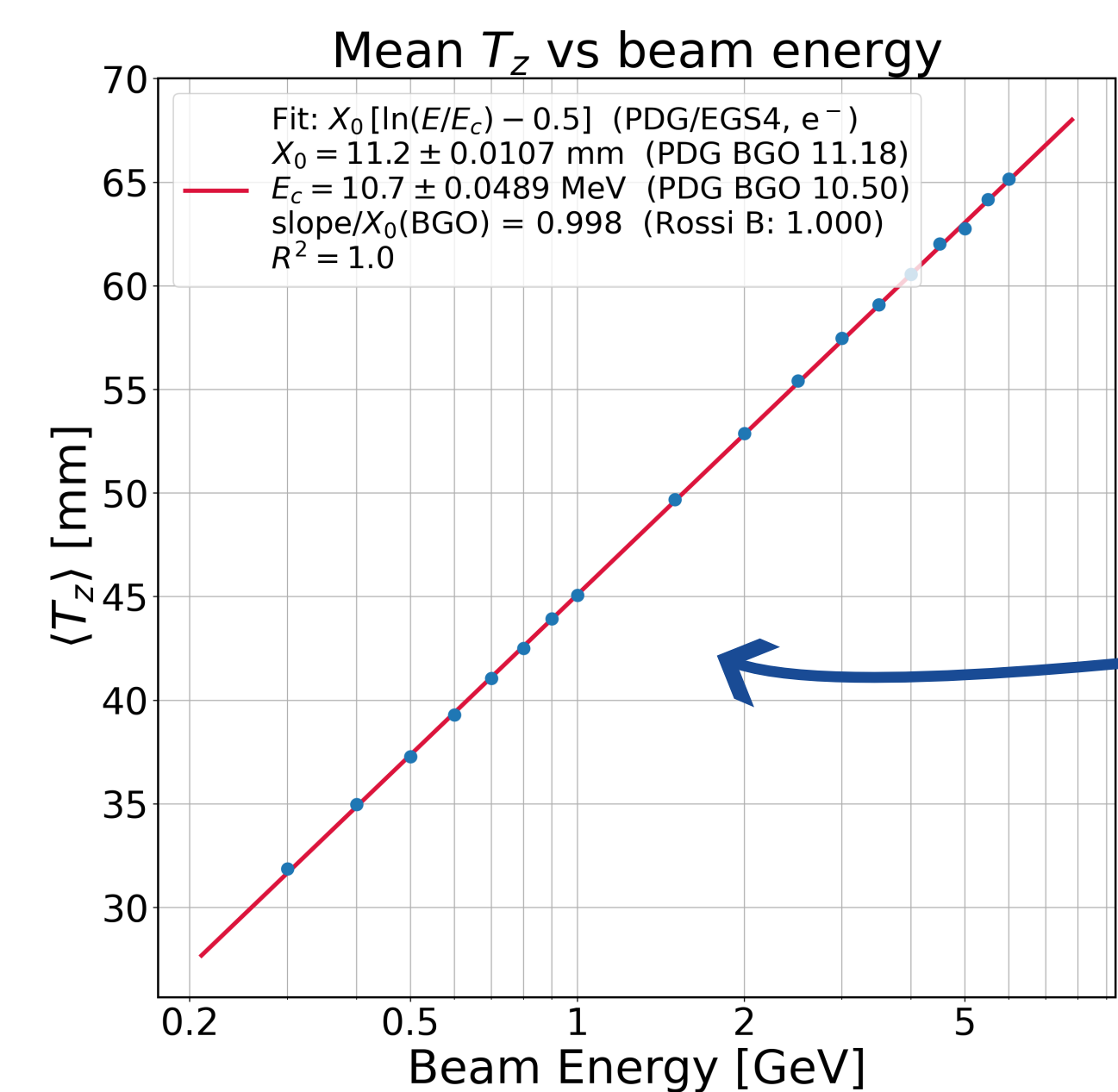
► Log Energy Fraction Distribution: Fisher's z (Difference of Log Gamma Variates)

Log ratios related to energy fractions by Softmax
 $[f_0, f_1, \dots, f_D] = \text{Softmax}\left(\left[\log \frac{E_0}{E_D}, \log \frac{E_1}{E_D}, \dots, \log \frac{E_{D-1}}{E_D}, 0\right]\right)$



Key Results

Physics Measurements of BGO and energy fractions

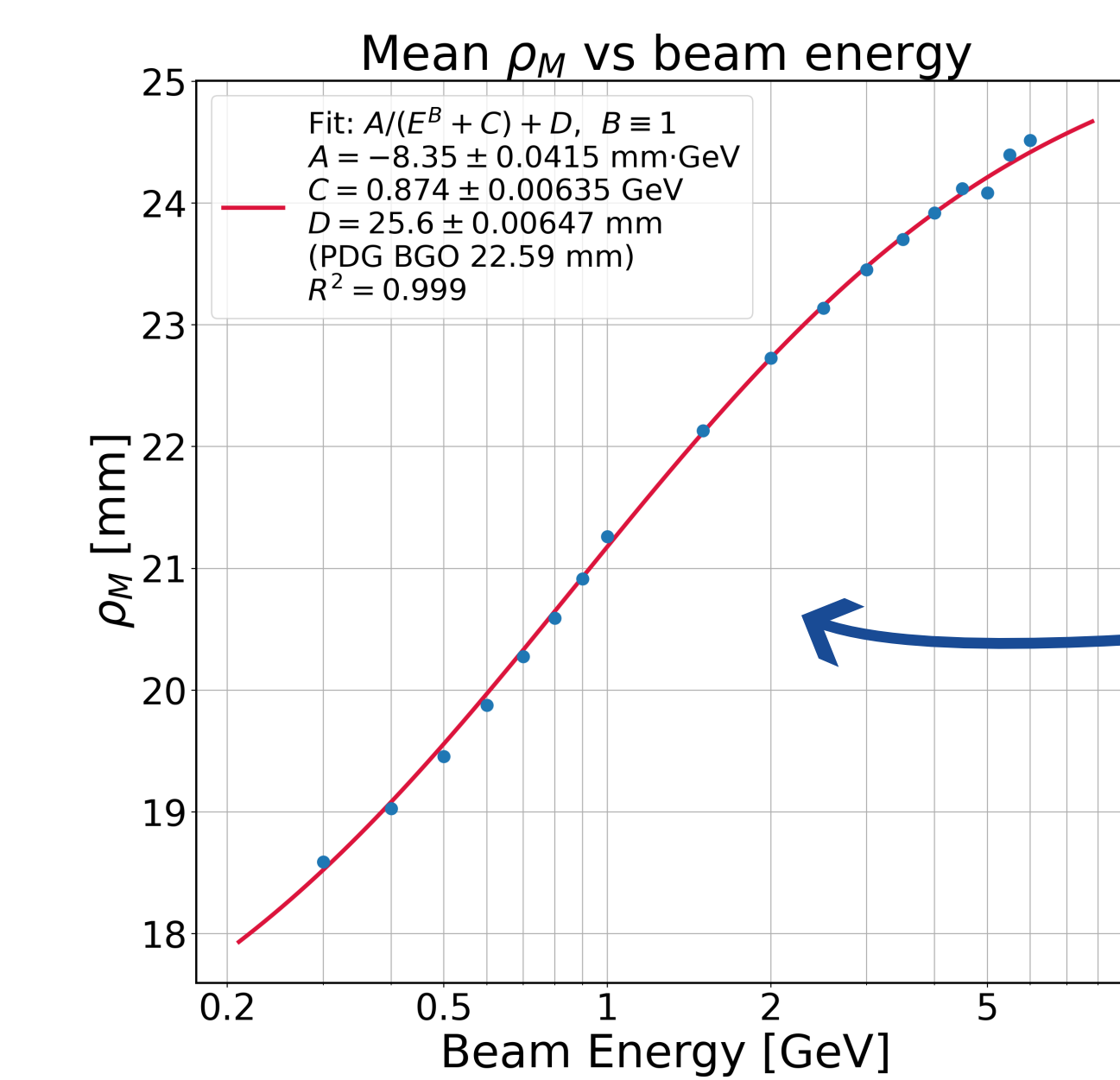


Key Result:
 $\langle T_z \rangle = X_0 \left(\log \left(\frac{E}{E_c} \right) - 0.5 \right)$
► **Radiation Length (X_0):**
• Distance after which electron loses $\sim 64\%$ of its total energy;

PDG, BGO	Measurement
11.18 mm	$11.2 \pm 0.01 \text{ mm}$

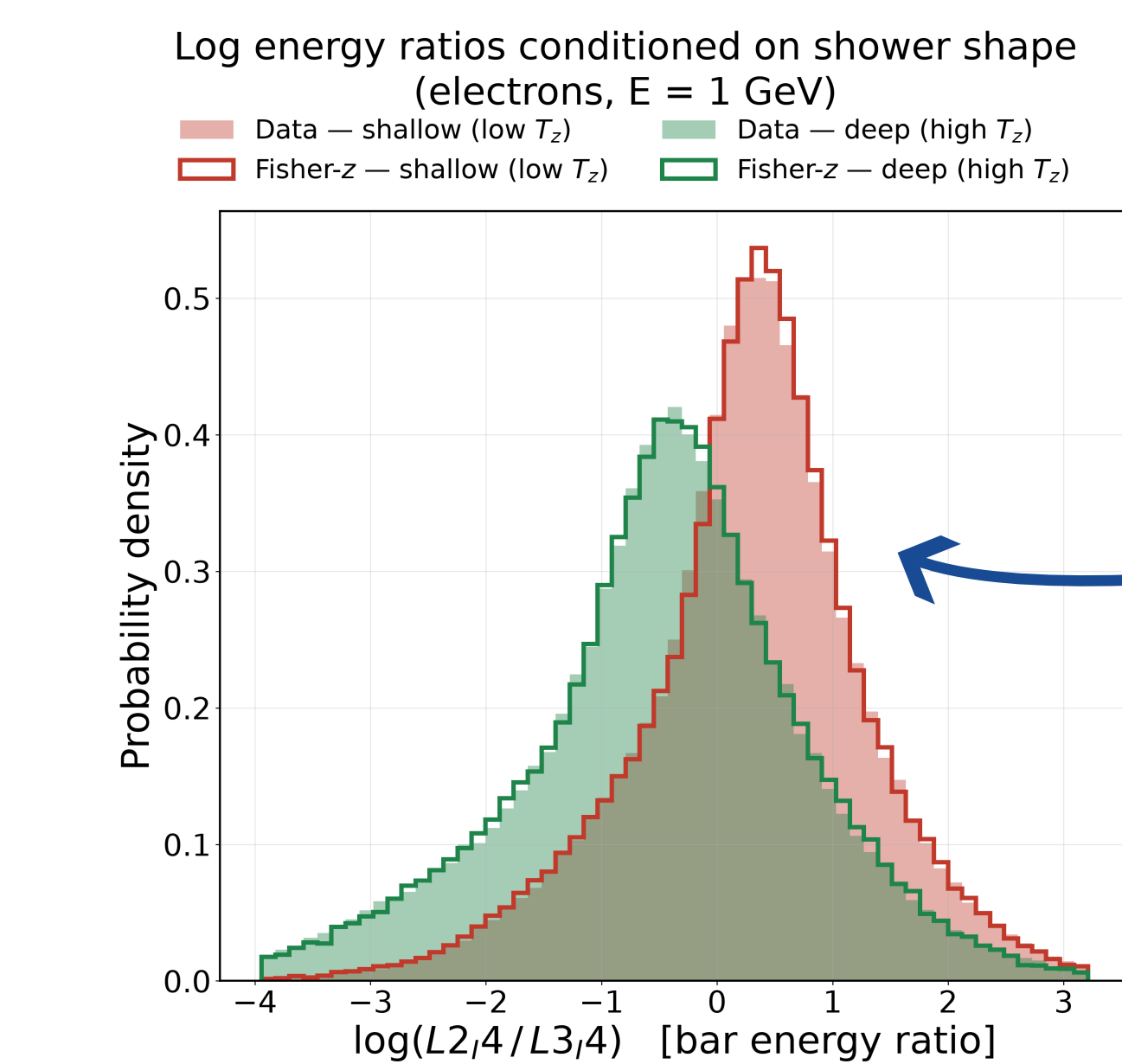
► **Critical Energy (E_c):**
• Below critical energy, EM shower extinction more probable than multiplication

PDG, BGO	Measurement
10.5 MeV	$10.7 \pm 0.05 \text{ MeV}$



Key Result:
 $\langle \rho_M \rangle = \frac{A}{(E^B + C)} + D$
► **Effective Moliere Radius (D):**
• Radius containing 90% of energy;
• Larger than theory due to granularity/energy threshold

PDG, BGO	Measurement
22.59 mm	$25.6 \pm 0.06 \text{ mm}$



Key Result:
► Log fraction distribution fitted by Fisher's z distribution;
► Log fraction fully conditional on shower shape
► Scale indicates correlation between cells;

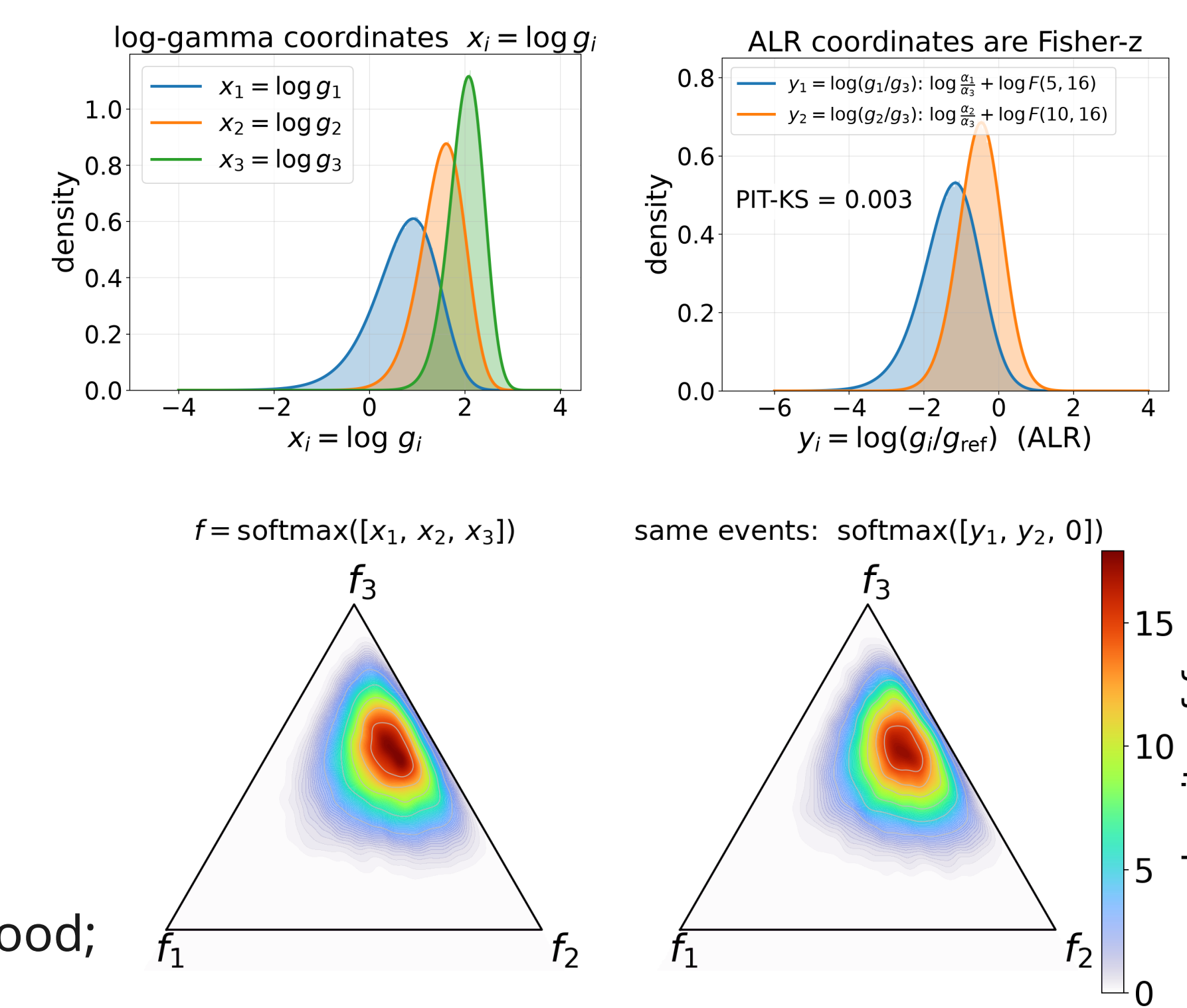
Generative Model using energy fraction distribution

- A fixed shower shape means log energy ratios drawn from Fisher's z distribution;
- Softmax maps log ratios to energy fractions;
- Equivalent to Softmax of log-Gamma variates;

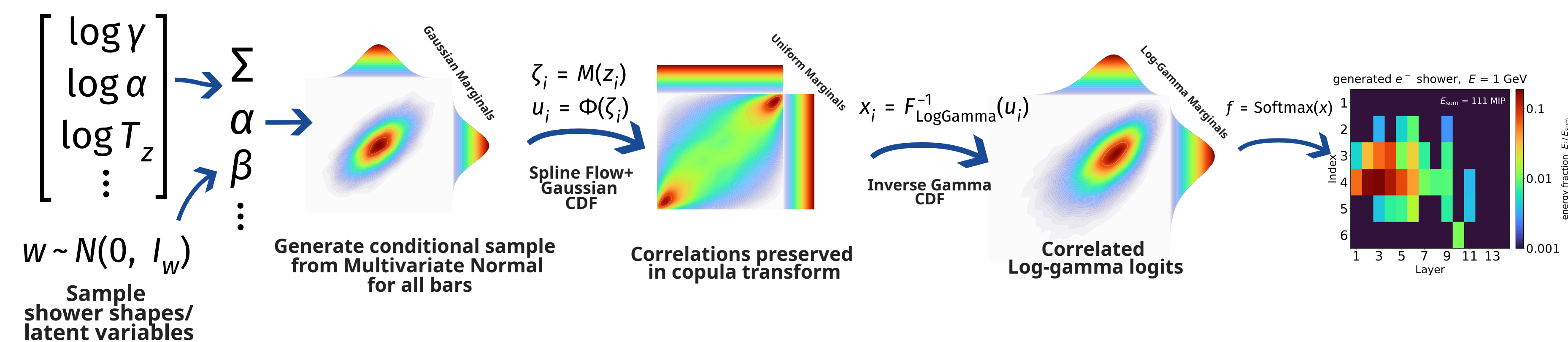
► Fast shower simulation method:

- Generate correlated log-Gamma variates;**
- Apply conditional per-bar spline operation;**
- Apply Softmax operation;**
- Obtain energy fractions for each bar**

- Correlations from Gaussian Copula method;
- Fit of marginal/correlation matrix with log-likelihood;

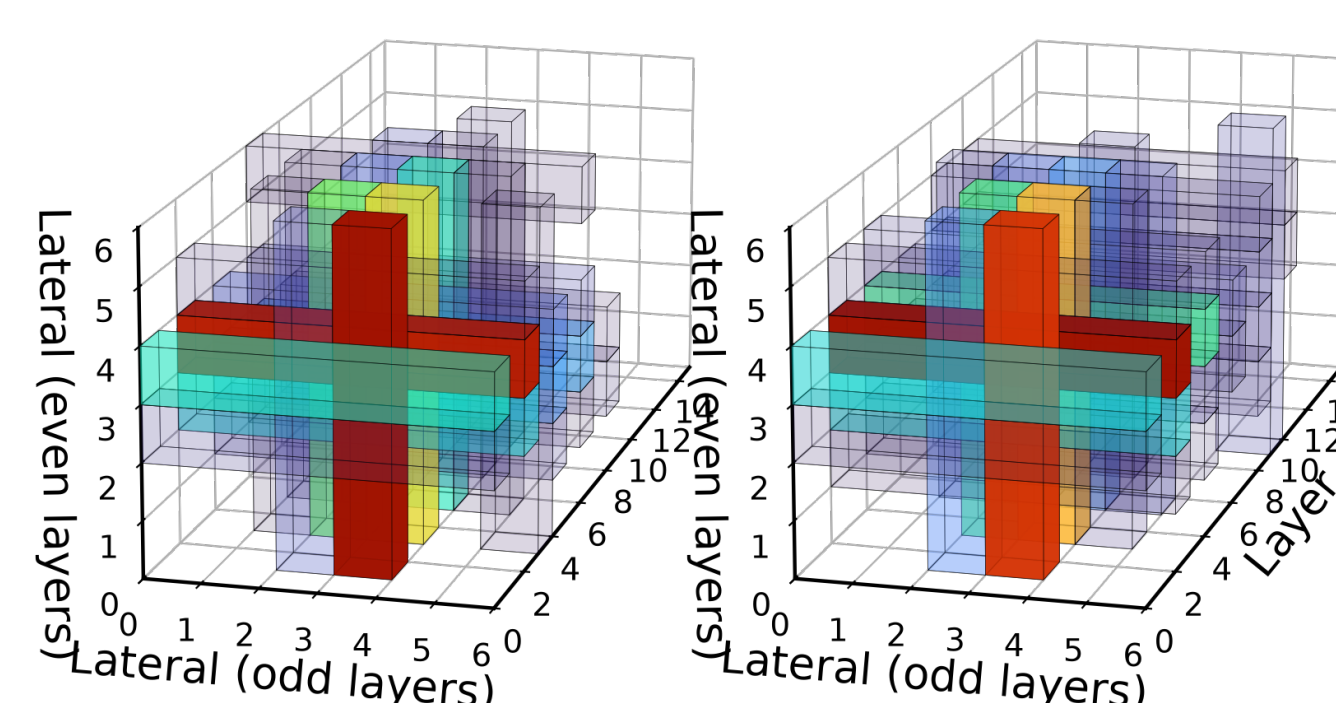


Copula Generative Model To Sample Full EM Shower Fractions



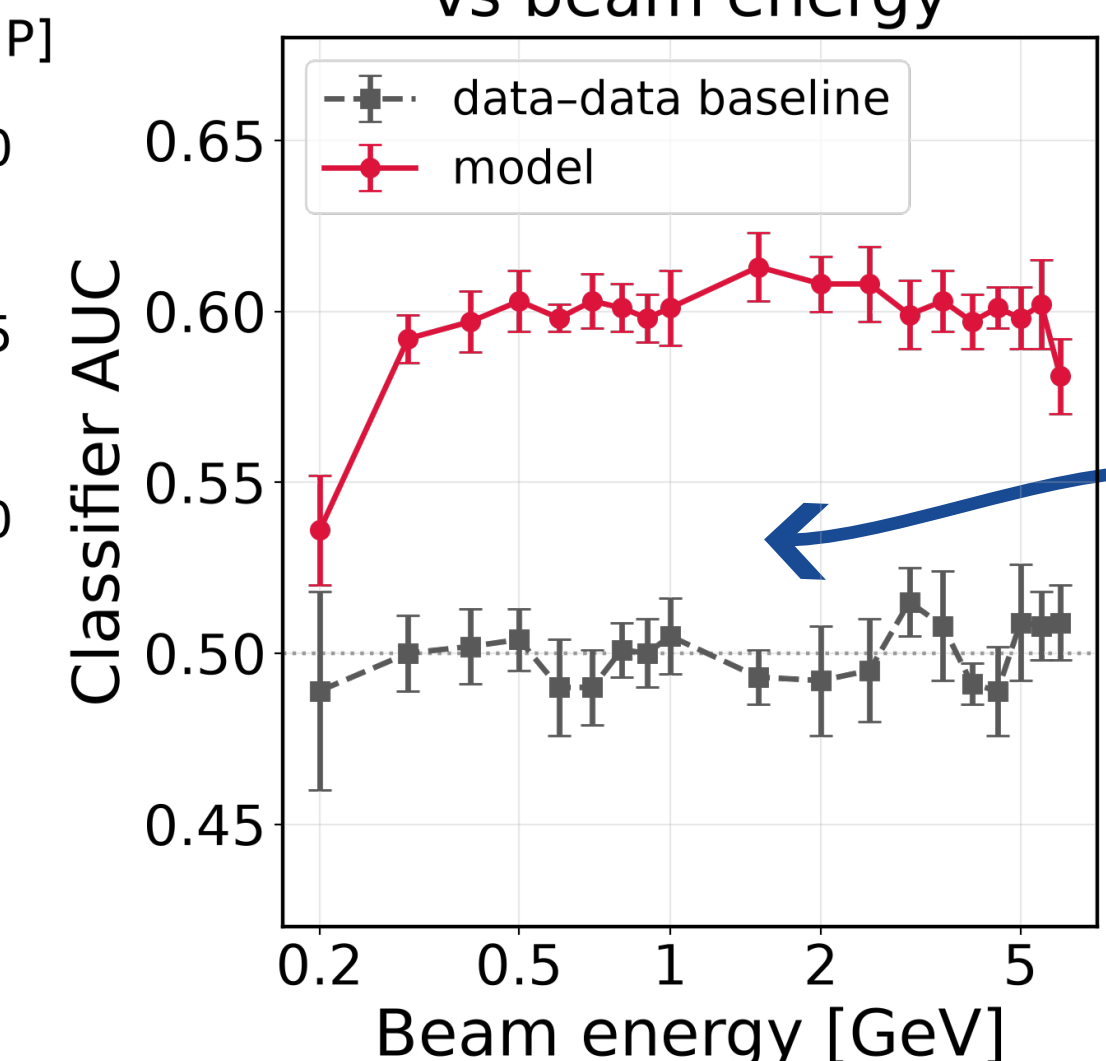
Data

Model



$E = 1 \text{ GeV}$, $E_{\text{sum}} = 111 \text{ MIP}$

Data-generated classifier AUC vs beam energy



Key Result:

- DNN Classifier can only distinguish generated vs. real up to AUC=0.6;
- Global shower variables reconstructed with excellent agreement;
- Average generation time on GPU: 33 μs