1. The Standard Model of particle physics

1.1 Introduction

aka Glashow–Weinberg–Salam model

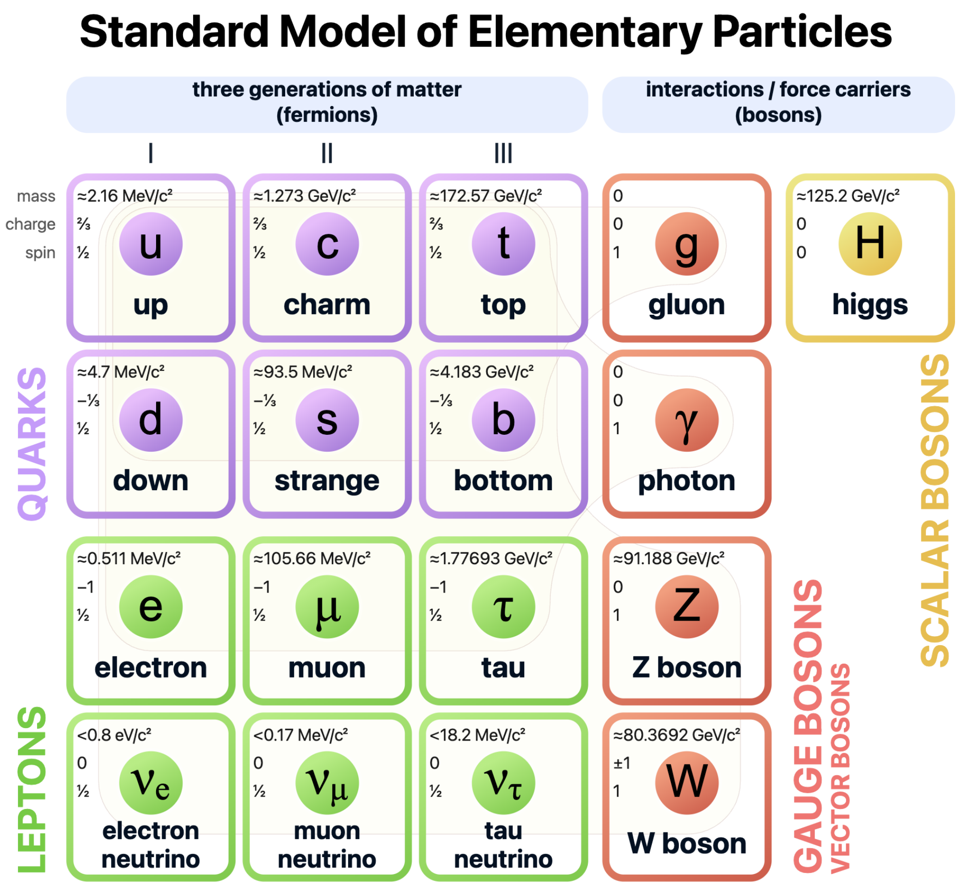
unified model for strong force, weak force and electromagnetism

GR is usually part of SM, but easy to add, though not relevant to colliders

most fundamental level phyiscs of almost all known matter in universe

often very accurate, eg, in QED, in presence of small couplings

poster child for reductionist science



SM Lagrangian

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{lepton}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{yukawa}} + \mathcal{L}_{\text{higgs}} \tag{1.1}$$

$$\mathcal{L}_{\text{quark}} = iq_I^{\dagger\alpha i}\bar{\sigma}^\mu \left(D_\mu q_I\right)_{\alpha i} + i\bar{u}_{I\alpha}^\dagger\bar{\sigma}^\mu \left(D_\mu \bar{u}_I\right)^\alpha + i\bar{d}_{I\alpha}^\dagger\bar{\sigma}^\mu \left(D_\mu \bar{d}_I\right)^\alpha \tag{1.2}$$

$$\mathcal{L}_{\text{lepton}} = i\ell_I^{\dagger i}\bar{\sigma}^\mu \left(D_\mu\right)_i^j \ell_{Ij} + i\bar{e}_I^\dagger\bar{\sigma}^\mu D_\mu \bar{e}_I, \tag{1.3}$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^{a\mu\nu}W^a_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \tag{1.4}$$

$$\mathcal{L}_{\text{yukawa}} = -\varepsilon^{ij}\varphi_i q_{I\alpha j} y'_{IJ}\bar{d}^\alpha_J - \varphi^{\dagger i} q_{I\alpha i} y''_{IJ}\bar{u}^\alpha_J - \varepsilon^{ij}\varphi_i \ell_{Ij} y_{IJ}\bar{e}_J + h.c. \tag{1.5}$$

$$\mathcal{L}_{\text{higgs}} = -\left(D^\mu\varphi\right)^\dagger D_\mu\varphi - \frac{1}{4}\lambda\left(\varphi^\dagger\varphi - \frac{1}{2}v^2\right)^2 \tag{1.6}$$

- renormalizable `most important in IR`
- spontaneous symmetry breaking / higgs mechanism
- chiral gauge theory / parity vaiolation (actually CP violation)

  `LH is different from RH. parity is maximally violated in weak interactions`
- gauge anomaly cancellation

  `gauge anomalies must cancel to keep gauge symmetry/redundancy`

  `vectorlike gauge theory is anomaly free; nontrivial for chiral gauge theory`
- flavor structure purely from Yukawa couplings

  `fermion masses (HUGE differences), mixing angles, CP violation`
- gauge interactions are flavor universal
- accidental symmetries: $U(1)_B \times U(1)_L$
- Wilsonian UV completeness & asymptotic freedom

SM is not final answer in physics: dark energy, dark matter, neutrino mass, baryogenesis, ...

`potential problem: higgs potential not bounded from below`

`fine-tuning problem / hierarchy problem` $\Longrightarrow$ `SUSY`

thus Beyond SM is needed

1.1.1 Spacetime symmetries

Minkowski space $\eta_{\mu\nu} = (-1, 1, 1, 1)$

Lorentz invariance

$$x'^\mu = \Lambda^\mu{}_\nu x^\nu, \quad \eta_{\mu\nu}\Lambda^\mu{}_\rho\Lambda^\nu{}_\sigma = \eta_{\rho\sigma} \tag{1.7}$$

`Lorentz group: 4 disconnected subspaces; connected by discrete P and T`

`Lorentz symmetry above = orthochronous` $(\Lambda^0{}_0 > 0)$ `proper` $(\det(\Lambda) = +1)$ `part`

Action should be Lorentz scalar `indices contracted`

Building blocks: $\phi,\ \psi^\alpha/\chi^a,\ A^\mu,\ \partial_\mu, \eta_{\mu\nu}, \epsilon_{\mu\nu\rho\sigma}, \gamma^\mu/\bar{\sigma}^\mu, \varepsilon^{ab}...$

1.1.2 Field contents and internal symmetries

- gauge bosons according to $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
- LH Weyl fermions in 3 copies of $\left(1,2,-\frac{1}{2}\right) \oplus (1,1,+1) \oplus \left(3,2,+\frac{1}{6}\right) \oplus \left(\overline{3},1,-\frac{2}{3}\right) \oplus \left(\overline{3},1,+\frac{1}{3}\right)$
- complex scalar in rep $\left(1,2,-\frac{1}{2}\right)$

If use Dirac fields, we have

$$\psi = \psi_L + \psi_R, \quad \psi_{L,R} = P_{L,R}\psi, \quad P_L = \frac{1-\gamma^5}{2}, \quad P_R = \frac{1+\gamma^5}{2} \tag{1.8}$$

$\left(0,\frac{1}{2}\right) \cong$ `conjugate of` $\left(\frac{1}{2},0\right)$

For example, electron Dirac field

$$e = \begin{pmatrix} e_L \\ e_R \end{pmatrix} = \begin{pmatrix} e \\ \bar{e}^\dagger \end{pmatrix} \tag{1.9}$$

1.2 Spontaneous symmetry breaking

Mechanics

$$L = \frac{1}{2}\dot{x}^2 - V(x), \quad V(x) = \frac{1}{2}\omega^2 x^2 + \frac{1}{24}\lambda x^4 \tag{1.10}$$

If $\omega^2 > 0$, minimum $x = 0$

If $\omega^2 < 0$, minima $x = v = \pm(-6\omega^2/\lambda)^{1/2}$ **draw figure**

Lagrangian: $x \to -x$, Z_2 symmetry

Vacuum: say, $x = (-6\omega^2/\lambda)^{1/2}$, Z_2 is spontaneously broken

`quantum mechanically, no spontaneous symmetry breaking: quantum tunnelling`

$$\begin{pmatrix} \langle 0+|\,H\,|0+\rangle & \langle 0+|\,H\,|0-\rangle \\ \langle 0-|\,H\,|0+\rangle & \langle 0-|\,H\,|0-\rangle \end{pmatrix} = \begin{pmatrix} E_0 & -\Delta \\ -\Delta & E_0 \end{pmatrix} \implies \begin{vmatrix} E_0 - E & -\Delta \\ -\Delta & E_0 - E \end{vmatrix} = 0 \implies E = E_0 \mp \Delta$$

$|0+\rangle + |0-\rangle$ `is true vacuum and` $|0+\rangle - |0-\rangle$ `has slightly higher energy`

Scalar field

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{1}{24}\lambda\varphi^4 \tag{1.11}$$

If $m^2 > 0$, minimum: $\varphi(x) = 0$

If $m^2 < 0$, minima: $\varphi(x) = +v$ and $\varphi(x) = -v$

$m^2 < 0 \implies \varphi$ `is tachyon; for our case, vacuum is not correctly identified`

Lagrangian: $\varphi(x) \to -\varphi(x)$

Vacuum:, say, $\varphi(x) = +v$, Z_2 is spontaneously broken

`quantum vacua are similar; obtained by computing quantum effective potential`

`field theory has infinite set of oscillators, coupled by` $(\nabla\varphi)^2$

`quantum tunneling suppressed by infinite volume` $\implies \langle 0+|0-\rangle = 0$

`spontaneous symmetry breaking happens even in the quantum theory`

True perturbative field $\rho(x) = \varphi(x) - v$

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\rho\partial_\mu\rho - \frac{1}{6}\lambda v^2\rho^2 - \frac{1}{6}\lambda v\rho^3 - \frac{1}{24}\lambda\rho^4 \tag{1.12}$$

mass $\frac{1}{2}m_\rho^2 = \frac{1}{6}\lambda v^2 = |m^2|$

Scalar U(1) gauge theory

$$\mathcal{L} = -\left(D^\mu\varphi\right)^\dagger D_\mu\varphi - V(\varphi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad V(\varphi) = m^2\varphi^\dagger\varphi + \frac{1}{4}\lambda\left(\varphi^\dagger\varphi\right)^2, \quad m^2 < 0 \tag{1.13}$$

Gauge covariant derivative $D_\mu = \partial_\mu - igA_\mu$

minima $|\varphi| = v/\sqrt{2}$, $v = \left(4\left|m^2\right|/\lambda\right)^{1/2}$ **draw figure**

Lagrangian: $\varphi \to e^{-i\alpha}\varphi$, $U(1)$ symmetry

Vacuum/VEV: say, $\varphi = v/\sqrt{2}$

`made global U(1) transformation to set phase of VEV to 0`

True perturbative field $\varphi(x) = \frac{1}{\sqrt{2}}(v + \rho(x))e^{-i\chi(x)/v}$

$\rho(x)$ `and` $\chi(x)$ `are real scalars`

$$V(\varphi) = \frac{1}{4}\lambda v^2\rho^2 + \frac{1}{4}\lambda v\rho^3 + \frac{1}{16}\lambda\rho^4 \tag{1.14}$$

χ `not in potential, massless` $\implies$ `Goldstone boson for broken U(1);` **see figure**

Unitary gauge `equivalent to setting` $v = 0$

$$\text{gauge transformation:} \quad \varphi \to \varphi e^{+i\chi/v} \implies \varphi = \frac{1}{\sqrt{2}}(v + \rho) \tag{1.15}$$

$$\begin{aligned} -\left(D^\mu\varphi\right)^\dagger D_\mu\varphi &= -\frac{1}{2}\left(\partial^\mu\rho + ig(v+\rho)A^\mu\right)\left(\partial_\mu\rho - ig(v+\rho)A_\mu\right) \\ &= -\frac{1}{2}\partial^\mu\rho\partial_\mu\rho - \frac{1}{2}g^2(v+\rho)^2A^\mu A_\mu \implies m_A = gv \end{aligned} \tag{1.16}$$

gauge field is now massive

this is called Higgs mechanism; φ is called Higgs field

important bit is not Goldstone boson being eaten, but spontaneously breaking

quantum level, unitarity gauge is problematic; use R_ξ gauge: $\mathcal{L}_{\rm gf} = -\xi^{-1}G^2$

sometimes it suffice to look at Goldstone boson sector in decoupling limit

Scalar nonabelian gauge theory

complex scalar field φ in rep R of gauge group

$$\mathcal{L} \supset -\left(D^\mu\varphi\right)^\dagger D_\mu\varphi \tag{1.17}$$

Covariant derivative $(D_\mu\varphi)_i = \partial_\mu\varphi_i - igA^a_\mu\left(T^a_{\rm R}\right)_i{}^j\varphi_j$

Vacuum $\varphi_i = v_i/\sqrt{2}$ v_i is up to global gauge transformation

Vector mass term

$$\mathcal{L}_{\rm mass} = -\frac{1}{2}\left(M^2\right)^{ab}A^{a\mu}A^b_\mu, \quad \left(M^2\right)^{ab} = \frac{1}{2}g^2 v_i^*\left\{T^a_{\rm R}, T^b_{\rm R}\right\}_{ij} v_j \tag{1.18}$$

$A^{a\mu}A^b_\mu$ is symmetric $\implies$ replaced $T^a_{\rm R}T^b_{\rm R}$ with $\frac{1}{2}\left\{T^a_{\rm R}, T^b_{\rm R}\right\}$.

generator T^a is said to be spontaneously broken if $\left(T^a_{\rm R}\right)_{ij} v_j \neq 0$

each broken generator $\implies$ a nonzero mass for gauge fields

unbroken generators (if any) form gauge group with massless gauge fields

Goldstone's theorem

Each broken generator results in a massless Goldstone boson.

Consider real scalars ϕ_i with potential $V(\varphi)$ infinitesimal parameter θ

$$V\left(\left(1 - i\theta^a T^a\right)\phi\right) = V(\phi) \implies \frac{\partial V}{\partial\phi_j}T^a_{jk}\phi_k = 0 \tag{1.19}$$

invariance of potential under global gauge transformation

Differentiate wrt $\phi_i \implies$

$$\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}T^a_{jk}\phi_k + \frac{\partial V}{\partial\phi_j}T^a_{ji} = 0 \quad \xrightarrow{\phi_k = v_k} \quad m^2_{ij}(T^a v)_j = 0, \quad m^2_{ij} = \left.\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right|_{\phi=v}$$

$(T^a v) \neq 0 \implies$ a zero eigenvalue in mass-squared matrix

a zero eigenvalue for every linearly independent broken generator

Example: SU(N) group

`assume complex scalar field` φ `in fundamental representation`

global SU(N) transformation $\Longrightarrow \varphi = (0, ...0, v)^T$ with real v

${(T^a)_i}^j$ `with 0 entries in last column are unbroken`

`they form unbroken SU(N-1)`

Broken generators:

1) $N-1$ of them: ${(T^a)_i}^N = \frac{1}{2}$ for $i \neq N \Longrightarrow$ SU(N-1) fundamental rep $M = \frac{1}{2}gv$;

2) $N-1$ of them: ${(T^a)_i}^N = -\frac{i}{2}$ for $i \neq N \Longrightarrow$ SU(N-1) fundamental rep $M = \frac{1}{2}gv$;

3) T^{N^2-1}=$[2N(N-1)]^{-\frac{1}{2}}\,\mathrm{diag}(1, ..., 1, -(N-1)) \Longrightarrow$ SU(N-1) singlet $M = [(N-1)/2N]^{\frac{1}{2}}gv$

1.3 Standard Model Lagrangian: EW bosons

Electroweak part of gauge group: SU(2)$_L \times$U(1)$_Y$ $\rightarrow$ U(1)$_{\mathrm{EM}}$

Higgs $\varphi = \frac{1}{\sqrt{2}}\binom{\phi_1+i\phi_2}{\phi_3+i\phi_4}$: $(2, -\frac{1}{2})$

`Higgs get a VEV` $\Longrightarrow$ `spontaneously breaks SU(2)`$_L$`×U(1)`$_Y$ `to U(1)`$_{\mathrm{EM}}$

$$(D_\mu\varphi)_i = \partial_\mu\varphi_i - i\left[g_2 W_\mu^a T^a + g_1 B_\mu Y\right]_i^{\ j}\varphi_j, \quad T^a = \frac{1}{2}\sigma^a, \quad Y = -\frac{1}{2}I \tag{1.20}$$

Y `is hypercharge; not EM charge` Q

$$g_2 W_\mu^a T^a + g_1 B_\mu Y = \frac{1}{2}\begin{pmatrix} g_2 W_\mu^3 - g_1 B_\mu & g_2\left(W_\mu^1 - iW_\mu^2\right) \\ g_2\left(W_\mu^1 + iW_\mu^2\right) & -g_2 W_\mu^3 - g_1 B_\mu \end{pmatrix} \tag{1.21}$$

Potential

$$V(\varphi) = \frac{1}{4}\lambda\left(\varphi^\dagger\varphi - \frac{1}{2}v^2\right)^2 \tag{1.22}$$

VEV/Vacuum v `global gauge transformation` $\Longrightarrow$ `VEV only in 1st component`

$$\varphi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix} \tag{1.23}$$

$\mathcal{L} \supset -\left(D^\mu\varphi\right)^\dagger D_\mu\varphi \Longrightarrow$ (replace φ by its VEV) $\sigma^{a\dagger} = \sigma^a$

$$\mathcal{L}_{\mathrm{mass}} = -\frac{1}{8}v^2(1,\ 0)\begin{pmatrix} g_2 W_\mu^3 - g_1 B_\mu & g_2\left(W_\mu^1 - iW_\mu^2\right) \\ g_2\left(W_\mu^1 + iW_\mu^2\right) & -g_2 W_\mu^3 - g_1 B_\mu \end{pmatrix}^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.24}$$

Define weak mixing angle `aka Weinberg angle`

$$\theta_{\mathrm{w}} \equiv \tan^{-1}\left(g_1/g_2\right) \tag{1.25}$$

Define $(s_{\rm w} \equiv \sin\theta_{\rm w}, c_{\rm w} \equiv \cos\theta_{\rm w})$

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right), \quad Z_\mu \equiv c_{\rm w}W_\mu^3 - s_{\rm w}B_\mu, \quad A_\mu \equiv s_{\rm w}W_\mu^3 + c_{\rm w}B_\mu \tag{1.26}$$

Then

$$\begin{aligned}\mathcal{L}_{\rm mass} &= -\frac{1}{8}g_2^2v^2(1,0)\begin{pmatrix} \frac{1}{c_{\rm w}}Z_\mu & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & \cdots \end{pmatrix}^2\begin{pmatrix}1\\0\end{pmatrix}. \\ &= -\left(g_2v/2\right)^2 W^{+\mu}W_\mu^- - \frac{1}{2}\left(g_2v/2c_{\rm w}\right)^2 Z^\mu Z_\mu \equiv -M_{\rm w}^2W^{+\mu}W_\mu^- - \frac{1}{2}M_{\rm z}^2Z^\mu Z_\mu\end{aligned} \tag{1.27}$$

$$M_{\rm w} = \frac{g_2v}{2} \simeq 80.4GeV, \quad M_{\rm z} = \frac{M_{\rm w}}{\cos\theta_{\rm w}} \simeq 91.2GeV, \quad \sin^2\theta_{\rm w} = 0.231$$

$\theta_{\rm w}$ depends on renormormalization scheme; in $\overline{\rm MS}$, $\sin^2\theta_{\rm w} = 0.231$ at $\mu = M_{\rm z}$

in on-shell scheme: define it as $\cos\theta_{\rm W} = M_{\rm W}/M_{\rm Z} = 0.223$

N.B.: A^μ is massless (unbroken $U(1)_{\rm EM}$), which is the EM field

Higgs in unitary gauge

3 of 4 Higgs fields eaten by $W^\pm$and Z^0

$$\varphi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}v + H(x)\\0\end{pmatrix} \tag{1.28}$$

H is the narrow sense Higgs

Kinetic term $\mathcal{L} \supset -\frac{1}{2}\partial^\mu H\partial_\mu H$

Higgs potential

$$V(\varphi) = \frac{1}{4}\lambda v^2H^2 + \frac{1}{4}\lambda vH^3 + \frac{1}{16}\lambda H^4 \tag{1.29}$$

Higgs mass $m_{\rm H} = (\frac{1}{2}\lambda v^2)^{1/2} \simeq 125$GeV

H-gauge boson interactions: $\mathcal{L}_{\rm mass} \to \mathcal{L}_{\rm mass}\big|_{v\to v+H}$

Gauge kinetic terms

$$\mathcal{L} = -\frac{1}{4}W^{a\mu\nu}W_{\mu\nu}^a - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \tag{1.30}$$

$W_{\mu\nu} \equiv \partial_\mu W_\nu - \partial_\nu W_\mu - ig\left[W_\mu, W_\nu\right] = W_{\mu\nu}^aT^a$

$$\begin{aligned}W_{\mu\nu}^a &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2\epsilon^{abc}W_\mu^bW_\nu^c \\ B_{\mu\nu} &\equiv \partial_\mu B_\nu - \partial_\nu B_\mu\end{aligned} \tag{1.31}$$

Define $D_\mu \equiv \partial_\mu - ig_2W_\mu^3 = \partial_\mu - ig_2\left(s_{\rm w}A_\mu + c_{\rm w}Z_\mu\right)$ on $W_\mu^+ \Longrightarrow$

$$\begin{aligned}\frac{1}{\sqrt{2}}\left(W_{\mu\nu}^1 - iW_{\mu\nu}^2\right) &= D_\mu W_\nu^+ - D_\nu W_\mu^+, \\ \frac{1}{\sqrt{2}}\left(W_{\mu\nu}^1 + iW_{\mu\nu}^2\right) &= D_\mu^\dagger W_\nu^- - D_\nu^\dagger W_\mu^-,\end{aligned} \tag{1.32}$$

W_μ^+ has EM charge $Q = +1 \Longrightarrow e = g_2 \sin\theta_\mathrm{W}$ `electron charge = ` $-e$

Define EM $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $Z_{\mu\nu} \equiv \partial_\mu Z_\nu - \partial_\nu Z_\mu$

$$\begin{aligned} W^3_{\mu\nu} &= \partial_\mu W^3_\nu - \partial_\nu W^3_\mu - i g_2 \left(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu\right) \\ &= s_\mathrm{w} F_{\mu\nu} + c_\mathrm{w} Z_{\mu\nu} - i g_2 \left(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu\right), \\ B_{\mu\nu} &= c_\mathrm{w} F_{\mu\nu} - s_\mathrm{w} Z_{\mu\nu} \end{aligned} \tag{1.33}$$

Collecting everything together, Lagrangian in unitary gauge

$$\begin{aligned} \mathcal{L} = &- \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} Z^{\mu\nu} Z_{\mu\nu} - D^{\dagger\mu} W^{-\nu} D_\mu W^+_\nu + D^{\dagger\mu} W^{-\nu} D_\nu W^+_\mu \\ &+ ie \left(F^{\mu\nu} + \cot\theta_\mathrm{w} Z^{\mu\nu}\right) W^+_\mu W^-_\nu \\ &- \frac{1}{2} \left(e^2/\sin^2\theta_\mathrm{w}\right) \left(W^{+\mu} W^-_\mu W^{+\nu} W^-_\nu - W^{+\mu} W^+_\mu W^{-\nu} W^-_\nu\right) \\ &- \left(M^2_\mathrm{w} W^{+\mu} W^-_\mu + \frac{1}{2} M^2_\mathrm{z} Z^\mu Z_\mu\right) \left(1 + v^{-1} H\right)^2 \\ &- \frac{1}{2} \partial^\mu H \partial_\mu H - \frac{1}{2} m^2_\mathrm{H} H^2 - \frac{1}{2} m^2_\mathrm{H} v^{-1} H^3 - \frac{1}{8} m^2_\mathrm{H} v^{-2} H^4, \end{aligned} \tag{1.34}$$

`remember` $D_\mu = \partial_\mu - ie\left(A_\mu + \cot\theta_\mathrm{w} Z_\mu\right)$

1.4 SM model Lagrangian: leptons

`grouped into three families or generations:` e `and` ν_e, μ `and` ν_μ, τ `and` ν_τ

Name	Symbol	Mass (MeV)	Q
electron	e	0.511	-1
electron neutrino	ν_e	0	0
muon	μ	105.7	-1
muon neutrino	ν_μ	0	0
tau	τ	1777	-1
tau neutrino	ν_τ	0	0

(1.35)

`consider one generation first`

Left-handed Weyl fields `group into Dirac fields later`

$\ell = \binom{\nu}{e}$: $(2, -\frac{1}{2})$, doublet

$\bar{e}$: $(1, +1)$, singlet `bar over` e `is part of name of field`

e is left-handed electron, $\bar{e}$ is left-handed position

antiparticle of LH electron (positron) = RH positron (electron)

makes gauge and chiral structure manifest; simplifies anomaly calculations

often: $\ell_L = (\nu_L, e_L)^T$, $e_R = \binom{0}{\bar{e}^\dagger}$ for 4-component Dirac fields with projectors

Kinetic terms $\bar{\sigma}^\mu = (I, -\sigma^a)$

$$\mathcal{L}_{\text{kin}} = i\ell^{\dagger i}\bar{\sigma}^\mu (D_\mu \ell)_i + i\bar{e}^\dagger \bar{\sigma}^\mu D_\mu \bar{e} \tag{1.36}$$

$$\begin{aligned} (D_\mu \ell)_i &= \partial_\mu \ell_i - ig_2 W_\mu^a (T^a)_i{}^j \ell_j - ig_1 \left(-\frac{1}{2}\right) B_\mu \ell_i \\ D_\mu \bar{e} &= \partial_\mu \bar{e} - ig_1(+1) B_\mu \bar{e} \end{aligned} \tag{1.37}$$

chiral gauge theory $\Longrightarrow$ parity violating

if only Dirac fields without chiral projection $\Longrightarrow$ vectorlike gauge theory

QED and QCD are vectorlike, but EW sector is chiral

No mass term for ℓ and/or $\bar{e}$

no singlet from $\left(2, -\frac{1}{2}\right) \otimes \left(2, -\frac{1}{2}\right)$, $\left(2, -\frac{1}{2}\right) \otimes (1, +1)$, $(1, +1) \otimes (1, +1)$

Yukawa interaction

$$\mathcal{L}_{\mathsf{Yuk}} = -y\varepsilon^{ij}\varphi_i \ell_j \bar{e} + h.c. \tag{1.38}$$

group theoretically: $\left(2, -\frac{1}{2}\right) \otimes \left(2, -\frac{1}{2}\right) \otimes (1, +1) = (1, 0) \oplus (3, 0)$

In unitary gauge $\varphi = \binom{\varphi_1}{\varphi_2} \to \binom{(v+H)/\sqrt{2}}{0}$ remember $\ell = \binom{\nu}{e}$

$$\mathcal{L}_{\text{Yuk}} = -\frac{1}{\sqrt{2}} y(v+H)\left(\ell_2 \bar{e} + h.c.\right) = -\frac{1}{\sqrt{2}} y(v+H)\left(e\bar{e} + \bar{e}^\dagger e^\dagger\right) \tag{1.39}$$

Define Dirac field for electron (and positron)

$$\mathcal{E} \equiv \begin{pmatrix} e \\ \bar{e}^\dagger \end{pmatrix} \tag{1.40}$$

$$\mathcal{L}_{\text{Yuk}} = -\frac{1}{\sqrt{2}} y(v+H)\overline{\mathcal{E}}\mathcal{E} \implies m_e = \frac{yv}{\sqrt{2}} \tag{1.41}$$

neutrino remains massless

Define Dirac field for neutrino (with L projection)

$$\mathcal{N}_{\text{L}} \equiv P_{\text{L}} \mathcal{N} = \begin{pmatrix} \nu \\ 0 \end{pmatrix}, \quad P_{\text{L}} = \frac{1}{2}(1 - \gamma_5) \tag{1.42}$$

For example

$$i\nu^\dagger \bar{\sigma}^\mu \partial_\mu \nu = i\overline{\mathcal{N}}_{\text{L}} \,\partial\!\!\!/ \mathcal{N}_{\text{L}} \tag{1.43}$$

`alternatively define Majorana field for neutrino` $\mathcal{N} \equiv \binom{\nu}{\nu^\dagger}$

`Majorana field is like real scalar, Dirac field is like complex scalar`

Then

$$g_2 W^1_\mu T^1 + g_2 W^2_\mu T^2 = \frac{g_2}{\sqrt{2}} \begin{pmatrix} 0 & W^+_\mu \\ W^-_\mu & 0 \end{pmatrix} \tag{1.44}$$

$$g_2 W^3_\mu T^3 + g_1 B_\mu Y = \frac{e}{s_\mathrm{w}} \left(s_\mathrm{w} A_\mu + c_\mathrm{w} Z_\mu \right) T^3 + \frac{e}{c_\mathrm{w}} \left(c_\mathrm{w} A_\mu - s_\mathrm{w} Z_\mu \right) Y \tag{1.45}$$

$$= e \left(A_\mu + \cot\theta_\mathrm{w} Z_\mu \right) T^3 + e \left(A_\mu - \tan\theta_\mathrm{w} Z_\mu \right) Y \tag{1.46}$$

$$= e \left(T^3 + Y \right) A_\mu + e \left(\cot\theta_\mathrm{w} T^3 - \tan\theta_\mathrm{w} Y \right) Z_\mu \tag{1.47}$$

A_μ is EM field $\Longrightarrow$ electric charge Q

$$Q = T^3 + Y \tag{1.48}$$

Since

$$T^3 \nu = +\frac{1}{2}\nu, \quad T^3 e = -\frac{1}{2} e, \quad T^3 \bar{e} = 0 \tag{1.49}$$

$$Y \nu = -\frac{1}{2}\nu, \quad Y e = -\frac{1}{2} e, \quad Y \bar{e} = +\bar{e} \tag{1.50}$$

we have, as expected,

$$Q\nu = 0, \quad Qe = -e, \quad Q\bar{e} = +\bar{e}. \tag{1.51}$$

Replace Y with Q

$$\begin{aligned} g_2 W^3_\mu T^3 + g_1 B_\mu Y &= eQA_\mu + e \left[\left(\cot\theta_\mathrm{w} + \tan\theta_\mathrm{w} \right) T^3 - \tan\theta_\mathrm{w} Q \right] Z_\mu \\ &= eQA_\mu + \frac{e}{s_\mathrm{w} c_\mathrm{w}} \left(T^3 - s_\mathrm{w}^2 Q \right) Z_\mu \end{aligned} \tag{1.52}$$

In terms of Dirac fields

$$\begin{aligned} \left(g_2 W^3_\mu T^3 + g_1 B_\mu Y \right) \mathcal{E} &= \left[-eA_\mu + \frac{e}{s_\mathrm{w} c_\mathrm{w}} \left(-\frac{1}{2} P_\mathrm{L} + s_\mathrm{w}^2 \right) Z_\mu \right] \mathcal{E} \\ \left(g_2 W^3_\mu T^3 + g_1 B_\mu Y \right) \mathcal{N}_\mathrm{L} &= \frac{e}{s_\mathrm{w} c_\mathrm{w}} \left(+\frac{1}{2} \right) Z_\mu \mathcal{N}_\mathrm{L} \end{aligned} \tag{1.53}$$

Collect things together

$$\mathcal{L}_\mathrm{int} = \frac{1}{\sqrt{2}} g_2 W^+_\mu J^{-\mu} + \frac{1}{\sqrt{2}} g_2 W^-_\mu J^{+\mu} + \frac{e}{s_\mathrm{w} c_\mathrm{w}} Z_\mu J^\mu_\mathrm{z} + e A_\mu J^\mu_\mathrm{EM}, \tag{1.54}$$

with currents

$$\begin{aligned} J^{+\mu} &\equiv \overline{\mathcal{E}}_{\mathrm{L}}\gamma^\mu\mathcal{N}_{\mathrm{L}}, \quad J^{-\mu} \equiv \overline{\mathcal{N}}_{\mathrm{L}}\gamma^\mu\mathcal{E}_{\mathrm{L}}, \\ J_3^\mu &\equiv \frac{1}{2}\overline{\mathcal{N}}_{\mathrm{L}}\gamma^\mu\mathcal{N}_{\mathrm{L}} - \frac{1}{2}\overline{\mathcal{E}}_{\mathrm{L}}\gamma^\mu\mathcal{E}_{\mathrm{L}}, \quad J_{\mathrm{EM}}^\mu \equiv -\overline{\mathcal{E}}^\mu\mathcal{E} \\ J_{\mathrm{Z}}^\mu &\equiv J_3^\mu - s_{\mathrm{W}}^2 J_{\mathrm{EM}}^\mu \end{aligned} \tag{1.55}$$

For all 3 generations, simply add generation index I

$$\mathcal{L}_{\text{kin}} = i\ell_I^{\dagger i}\bar{\sigma}^\mu\left(D_\mu\right)_i^j\ell_{jI} + i\bar{e}_I^\dagger\bar{\sigma}^\mu D_\mu\bar{e}_I \tag{1.56}$$

$$\mathcal{L}_{\text{Yuk}} = -\varepsilon^{ij}\varphi_i\ell_{jI}y_{IJ}\bar{e}_J + h.c. \tag{1.57}$$

y_{IJ} is complex 3×3 Yukawa matrix

Use 2 unitary matrices L and $\bar{E}$

$$\ell_I \to L_{IJ}\ell_J, \quad \bar{e}_I \to \bar{E}_{IJ}\bar{e}_J \implies y \to L^{\mathrm{T}}y\bar{E} = \mathrm{diag}(y_1, y_2, y_3) \tag{1.58}$$

kinetic terms unchanged

simply add I to each field + sum over in Lagrangian

eg, leptons $\mathcal{E}_I$: $m_{e_I} = y_I v/\sqrt{2}$

1.5 Standard Model Lagrangian: quarks

Quarks are spin-$\frac{1}{2}$ and triplets of color/SU(3) group

again 3 generations: u, d, c, s, t, b

Start with 1 generation

$q = \binom{u}{d}$: $\left(3, 2, +\frac{1}{6}\right)$, $\bar{u}$: $\left(\overline{3}, 1, -\frac{2}{3}\right)$, $\bar{d}$: $\left(\overline{3}, 1, -\frac{2}{3}\right)$

Covariant derivative

$$\begin{aligned} \left(D_\mu q\right)_{\alpha i} &= \partial_\mu q_{\alpha i} - ig_3 G_\mu^a \left(T_3^a\right)_\alpha{}^\beta q_{\beta i} - ig_2 W_\mu^a \left(T_2^a\right)_i{}^j q_{\beta j} \\ &\qquad - ig_1\left(+\frac{1}{6}\right)B_\mu q_{\alpha i} \\ \left(D_\mu\bar{u}\right)^\alpha &= \partial_\mu\bar{u}^\alpha - ig_3 G_\mu^a \left(T_3^a\right)^\alpha{}_\beta\bar{u}^\beta - ig_1\left(-\frac{2}{3}\right)B_\mu\bar{u}^\alpha \\ \left(D_\mu\bar{d}\right)^\alpha &= \partial_\mu\bar{d}^\alpha - ig_3 G_\mu^a \left(T_3^a\right)^\alpha{}_\beta\bar{d}^\beta - ig_1\left(+\frac{1}{3}\right)B_\mu\bar{d}^\alpha \end{aligned} \tag{1.59}$$

α is color index

Kinetic terms

$$\mathcal{L}_{\mathsf{kin}} = iq^{\dagger\alpha i}\bar{\sigma}^{\mu}\left(D_{\mu}q\right)_{\alpha i} + i\bar{u}^{\dagger}_{\alpha}\bar{\sigma}^{\mu}\left(D_{\mu}\bar{u}\right)^{\alpha} + i\bar{d}^{\dagger}_{\alpha}\bar{\sigma}^{\mu}\left(D_{\mu}\bar{d}\right)^{\alpha} \tag{1.60}$$

$\left(3, 2, +\frac{1}{6}\right) \oplus \left(\overline{3}, 1, -\frac{2}{3}\right) \oplus \left(\overline{3}, 1, +\frac{1}{3}\right)$ `for LH Weyl fields is complex`

`so it is chiral gauge theory, thus parity violating`

`no group singlet from tensor product` $\Longrightarrow$ `no mass term for` $q, \bar{u}, \bar{d}$

Yukawa Lagrangian

$$\mathcal{L}_{\mathrm{Yuk}} = -y'\varepsilon^{ij}\varphi_i q_{\alpha j}\bar{d}^{\alpha} - y''\varphi^{\dagger i}q_{\alpha i}\bar{u}^{\alpha} + h.c. \tag{1.61}$$

`1st term due to` $\left(1, 2, -\frac{1}{2}\right) \otimes \left(3, 2, +\frac{1}{6}\right) \otimes \left(\overline{3}, 1, +\frac{1}{3}\right) = (1, 1, 0) \oplus \ldots$

`2nd term due to` $\left(1, 2, +\frac{1}{2}\right) \otimes \left(3, 2, +\frac{1}{6}\right) \otimes \left(\overline{3}, 1, -\frac{2}{3}\right) = (1, 1, 0) \oplus \ldots$

`no other renormalizable terms`

Unitary gauge $\varphi_1 \to \frac{1}{\sqrt{2}}(v+H)$, $\varphi_2 \to 0$

$$\mathcal{L}_{\mathrm{Yuk}} = -\frac{1}{\sqrt{2}}y'(v+H)q_{\alpha 2}\bar{d}^{\alpha} - \frac{1}{\sqrt{2}}y''(v+H)q_{\alpha 1}\bar{u}^{\alpha} + h.c. \tag{1.62}$$

$$= -\frac{1}{\sqrt{2}}y'(v+H)\left(d_{\alpha}\bar{d}^{\alpha} + \bar{d}^{\dagger}_{\alpha}d^{\dagger\alpha}\right) - \frac{1}{\sqrt{2}}y''(v+H)\left(u_{\alpha}\bar{u}^{\alpha} + \bar{u}^{\dagger}_{\alpha}u^{\dagger\alpha}\right) \tag{1.63}$$

Define Dirac fields

$$\mathcal{D}_{\alpha} \equiv \begin{pmatrix} d_{\alpha} \\ \bar{d}^{\dagger}_{\alpha} \end{pmatrix}, \quad \mathcal{U}_{\alpha} \equiv \begin{pmatrix} u_{\alpha} \\ \bar{u}^{\dagger}_{\alpha} \end{pmatrix}. \tag{1.64}$$

$$\mathcal{L}_{\mathrm{Yuk}} = -\frac{1}{\sqrt{2}}y'(v+H)\overline{\mathcal{D}}^{\alpha}\mathcal{D}_{\alpha} - \frac{1}{\sqrt{2}}y''(v+H)\overline{\mathcal{U}}^{\alpha}\mathcal{U}_{\alpha} \tag{1.65}$$

Quark masses

$$m_d = \frac{y'v}{\sqrt{2}}, \quad m_u = \frac{y''v}{\sqrt{2}} \tag{1.66}$$

Previously found

$$\begin{gathered} g_2 W^1_{\mu}T^1 + g_2 W^2_{\mu}T^2 = \frac{g_2}{\sqrt{2}}\begin{pmatrix} 0 & W^+_{\mu} \\ W^-_{\mu} & 0 \end{pmatrix} \\ g_2 W^3_{\mu}T^3 + g_1 B_{\mu}Y = eQA_{\mu} + \frac{e}{s_{\mathrm{w}}c_{\mathrm{w}}}\left(T^3 - s^2_{\mathrm{w}}Q\right)Z_{\mu} \end{gathered} \tag{1.67}$$

with $Q = T^3 + Y$

Since

$$T^3 u = +\frac{1}{2}u, \quad T^3 d = -\frac{1}{2}d, \quad T^3\bar{u} = 0, \quad T^3\bar{d} = 0 \tag{1.68}$$

$$Yu = +\frac{1}{6}u, \quad Yd = +\frac{1}{6}d, \quad Y\bar{u} = -\frac{2}{3}\bar{u}, \quad Y\bar{d} = +\frac{1}{3}\bar{d} \tag{1.69}$$

we have, as expected

$$Qu = +\frac{2}{3}u, \quad Qd = -\frac{1}{3}d, \quad Q\bar{u} = -\frac{2}{3}\bar{u}, \quad Q\bar{d} = +\frac{1}{3}\bar{d} \tag{1.70}$$

$$\begin{aligned}\left(g_2 W_\mu^3 T^3 + g_1 B_\mu Y\right)\mathcal{U} &= \left[+\frac{2}{3}eA_\mu + \frac{e}{s_\mathrm{w}c_\mathrm{w}}\left(+\frac{1}{2}P_\mathrm{L} - \frac{2}{3}s_\mathrm{w}^2\right)Z_\mu\right]\mathcal{U} \\ \left(g_2 W_\mu^3 T^3 + g_1 B_\mu Y\right)\mathcal{D} &= \left[-\frac{1}{3}eA_\mu + \frac{e}{s_\mathrm{w}c_\mathrm{w}}\left(-\frac{1}{2}P_\mathrm{L} + \frac{1}{3}s_\mathrm{w}^2\right)Z_\mu\right]\mathcal{D}\end{aligned} \tag{1.71}$$

Collecting everything together

$$\mathcal{L}_\mathrm{int} = \frac{1}{\sqrt{2}}g_2 W_\mu^+ J^{-\mu} + \frac{1}{\sqrt{2}}g_2 W_\mu^- J^{+\mu} + \frac{e}{s_\mathrm{w}c_\mathrm{w}}Z_\mu J_\mathrm{z}^\mu + eA_\mu J_\mathrm{EM}^\mu, \tag{1.72}$$

$$\begin{aligned}&J^{+\mu} \equiv \overline{\mathcal{D}}_\mathrm{L}\gamma^\mu\mathcal{U}_\mathrm{L}, \qquad J^{-\mu} \equiv \overline{\mathcal{U}}_\mathrm{L}\gamma^\mu\mathcal{D}_\mathrm{L}, \\ &J_3^\mu \equiv \frac{1}{2}\overline{\mathcal{U}}_\mathrm{L}\gamma^\mu\mathcal{U}_\mathrm{L} - \frac{1}{2}\overline{\mathcal{D}}_\mathrm{L}\gamma^\mu\mathcal{D}_\mathrm{L}, \qquad J_\mathrm{EM}^\mu \equiv +\frac{2}{3}\overline{\mathcal{U}}\gamma^\mu\mathcal{U} - \frac{1}{3}\overline{\mathcal{D}}\gamma^\mu\mathcal{D} \\ &J_\mathrm{z}^\mu \equiv J_3^\mu - s_\mathrm{w}^2 J_\mathrm{EM}^\mu\end{aligned} \tag{1.73}$$

Consider all 3 generations

$$\mathcal{L}_{kin} = iq^{\dagger I\alpha i}\bar{\sigma}^\mu\left(D_\mu\right)_{\alpha i}{}^{\beta j}q_{I\beta j} + i\bar{u}_{I\alpha}^\dagger\bar{\sigma}^\mu\left(D_\mu\right)^\alpha{}_\beta\bar{u}_I^\beta + i\bar{d}_{I\alpha}^\dagger\bar{\sigma}^\mu\left(D_\mu\right)^\alpha{}_\beta\bar{d}_I^\beta \tag{1.74}$$

$$\mathcal{L}_\mathrm{Yuk} = -\varepsilon^{ij}\varphi_i q_{I\alpha j}y'_{IJ}\bar{d}_J^\alpha - \varphi^{\dagger i}q_{I\alpha i}y''_{IJ}\bar{u}_J^\alpha + h.c. \tag{1.75}$$

In unitary gauge

$$\mathcal{L}_\mathrm{Yuk} = -\frac{1}{\sqrt{2}}(v+H)d_{\alpha I}y'_{IJ}\bar{d}_J^\alpha - \frac{1}{\sqrt{2}}(v+H)u_{\alpha I}y''_{IJ}\bar{u}_J^\alpha + h.c. \tag{1.76}$$

Unitary matrices $U, D, \bar{U}, \bar{D}$

$$d_I \to D_{IJ}d_J, \ \bar{d}_I \to \bar{D}_{IJ}\bar{d}_J, \ u_I \to U_{IJ}u_J, \ \bar{u}_I \to \bar{U}_{IJ}\bar{u}_J \tag{1.77}$$

`kinetic terms are changed; except for coupings to` $W^\pm$

$$y' \to D^\mathrm{T}y'\bar{D} = \mathrm{diag}(y'_1, y'_2, y'_3), \qquad y'' \to U^\mathrm{T}y''\bar{U} = \mathrm{diag}(y''_1, y''_2, y''_3) \tag{1.78}$$

$\Longrightarrow$ down/$\mathcal{D}_I$ quark masses: $m_{d_I} = y'_I v/\sqrt{2}$, up/$\mathcal{U}_I$ masses $m_{u_I} = y''_I v/\sqrt{2}$

For neutral currents, simply add generation indices

For charged currents, they changed to

$$J^{+\mu} = \overline{\mathcal{D}}_{\mathrm{L}I}V_{IJ}^\dagger\gamma^\mu\mathcal{U}_{\mathrm{L}J}, \quad J^{-\mu} = \overline{\mathcal{U}}_{\mathrm{L}I}V_{IJ}\gamma^\mu\mathcal{D}_{\mathrm{L}K} \tag{1.79}$$

$V \equiv U^\dagger D$ is the Cabibbo–Kobayasi–Maskawa (CKM) matrix

`no this complication in lepton sector because there was 1 Yukawa term`

3×3 unitary matrix has 9 real parameters, but we can phase rotate

$$\mathcal{D}_I \to e^{i\alpha_I}\mathcal{D}_I, \qquad \mathcal{U}_I \to e^{i\beta_I}\mathcal{U}_I, \tag{1.80}$$

`it leaves kinetic and Yukawa terms invariant`

`make 1st row and column of` V_{IJ} `real, eliminating 5/9 parameters`

Left with θ_1 (Cabibbo angle), θ_2, θ_3, and δ (CP violating phase)

$$V = \begin{pmatrix} c_1 & +s_1c_3 & +s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ -s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} \tag{1.81}$$

$c_i = \cos\theta_i,\ \ s_i = \sin\theta_i$, $s_1 \simeq 0.224, s_2 \simeq 0.041, s_3 \simeq 0.016, \delta \simeq 40^\circ$

$e^{i\delta}$ `only in some elements +` $T^{-1}iT = -i \implies$ `EW time reversal violation`

`at high energies,` g_3 `is weak` $\implies$ `can use quarks`

`eg,` $W^\pm, Z^0$ `decay into quarks,` $\alpha_3(M_z) \equiv g_3^2(M_z)/4\pi = 0.12$`, QCD loops are a few %`

`at low energies, must use hadron; eg, neutron deay`

DoF/Fields counting:

$$\begin{aligned} &2\times(8[g]+1[\gamma]) + 3\times 3[W^\pm + Z^0] + 2\times 2[\text{helicity}]\times 3[e+\mu+\tau] + 2\times 3[\nu+\nu_\mu+\nu_\tau] \\ &+ 3[\text{color}]\times 2\times 2[\text{spin}]\times 6[u+d+c+s+t+b] + 1[\text{higgs}] = 118 \end{aligned} \tag{1.82}$$

Theory parameter counting:

$$\begin{aligned} &6[\text{quark masses } m_q] + 3[\text{lepton masses } m_l] + 1[\text{QCD coupling } g_3] + 1[\text{electric charge } e] + \\ &+ 1[\text{Weinberg angle } \theta_\mathrm{W}] + 1[\text{higgs mass } m_H] + 1[W \text{ mass } m_W] + 4[\text{CKM } \theta_1,\theta_2,\theta_3,\delta] = 18 \end{aligned}$$

1.6 EFT and BSM

At low energies, below the masses of $W^\pm, Z^0$

`leading order in double expansion of gauge couplings and` $O(1/M_\mathrm{w}, 1/M_\mathrm{z})$

$$\mathcal{L}_\mathrm{mass} = -M_\mathrm{w}^2 W^{+\mu}W^-_\mu - \frac{1}{2}M_\mathrm{z}^2 Z^\mu Z_\mu \tag{1.83}$$

$$\mathcal{L}_\mathrm{int} = \frac{1}{\sqrt{2}}g_2 W^+_\mu J^{-\mu} + \frac{1}{\sqrt{2}}g_2 W^-_\mu J^{+\mu} + \frac{e}{s_\mathrm{w}c_\mathrm{w}}Z_\mu J^\mu_\mathrm{z} + eA_\mu J^\mu_\mathrm{EM} \tag{1.84}$$

`ignore kinetic terms and other interactions of` $W^{\pm}, Z^0$

Solve EoM and substitute into Lagrangian

`ie, tree-level Feynman diagrams with single` $W^{\pm}$ `or` Z^0 `propagator` $g^{\mu\nu}/M^2_{\mathrm{w,z}}$

$$\begin{aligned}\mathcal{L}_{\mathrm{eff}} &= \frac{g_2^2}{2M_{\mathrm{w}}^2} J^{+\mu} J^-_{\mu} + \frac{e^2}{2s_{\mathrm{w}}^2 c_{\mathrm{w}}^2 M_{\mathrm{z}}^2} J^{\mu}_{\mathrm{z}} J_{\mathrm{z}\mu} \\ &= \frac{e^2}{2s_{\mathrm{w}}^2 M_{\mathrm{w}}^2} \left(J^{+\mu} J^-_{\mu} + J^{\mu}_{\mathrm{z}} J_{\mathrm{z}\mu}\right) = 2\sqrt{2} G_{\mathrm{F}} \left(J^{+\mu} J^-_{\mu} + J^{\mu}_{\mathrm{z}} J_{\mathrm{z}\mu}\right)\end{aligned} \tag{1.85}$$

Fermi constant $G_{\mathrm{F}} \equiv \frac{e^2}{4\sqrt{2}\sin^2\theta_{\mathrm{w}} M_{\mathrm{w}}^2} \simeq 1.166 \times 10^{-5} \mathrm{GeV}^{-2}$

G_F `has negative mass dimensions`

$\mathcal{L}_{\mathrm{eff}}$ is not renormalizable/irrelevant terms, or dim-6 operator

Example: amplitude squared for $\nu_e e^- \to \nu_e e^-$ `define` $C_{\mathrm{V}} = \frac{1}{2} + s^2_{\mathrm{W}}$ `and` $C_{\mathrm{A}} = \frac{1}{2}$

$$\left\langle |\mathcal{T}|^2 \right\rangle = 2G_{\mathrm{F}}^2 \left[\left(C_{\mathrm{V}}^2 + C_{\mathrm{A}}^2\right) \left(s^2 + u^2 - 4m_e^2(s+u) + 6m_e^4\right) + 2C_{\mathrm{V}} C_{\mathrm{A}} \left(s^2 - u^2 - 2m_e^2(s-u)\right)\right]$$

Example: muon decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu$,

electron $\mathcal{E}$, muon $\mathcal{M}$, electron neutrino $\mathcal{N}_e$, muon neutrino $\mathcal{N}_m$

`only charged currents` $J^{\pm}_{\mu}$ `contribute; ignore` π `terms`

$$\begin{aligned} J^{+\mu} &= \overline{\mathcal{E}}_{\mathrm{L}} \gamma^{\mu} \mathcal{N}_{e\mathrm{L}} + \overline{\mathcal{M}}_{\mathrm{L}} \gamma^{\mu} \mathcal{N}_{m\mathrm{L}}, \\ J^{-\mu} &= \overline{\mathcal{N}}_{e\mathrm{L}} \gamma^{\mu} \mathcal{E}_{\mathrm{L}} + \overline{\mathcal{N}}_{m\mathrm{L}} \gamma^{\mu} \mathcal{M}_{\mathrm{L}} \end{aligned} \tag{1.86}$$

use 4-fermion effector operator

$$\mathcal{L}_{\text{eff}} = 2\sqrt{2} G_{\mathrm{F}} \left(\overline{\mathcal{E}}_{\mathrm{L}} \gamma^{\mu} \mathcal{N}_{e\mathrm{L}}\right) \left(\overline{\mathcal{N}}_{m\mathrm{L}} \gamma_{\mu} \mathcal{M}_{\mathrm{L}}\right) = -4\sqrt{2} G_{\mathrm{F}} \left(\overline{\mathcal{M}}^{\mathrm{c}} P_{\mathrm{L}} \mathcal{N}_e\right) \left(\overline{\mathcal{E}} P_{\mathrm{R}} \mathcal{N}_m^{\mathrm{C}}\right) \tag{1.87}$$

`used Fierz identity`

muon decay rate ($m_e \ll m_\mu$)

$$\Gamma = \frac{G_{\mathrm{F}}^2 m_\mu^5}{192\pi^3} \tag{1.88}$$

`no need to know about W and Z and higgs and so on`

Standard model EFT

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \mathrm{EFTcorrections} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{dim-6}} + \mathcal{L}_{\mathrm{dim-8}} + \dots \tag{1.89}$$

`no dim-5 and dim-7 if imposing B and L conservation`

EFT corrections encodes BSM physics