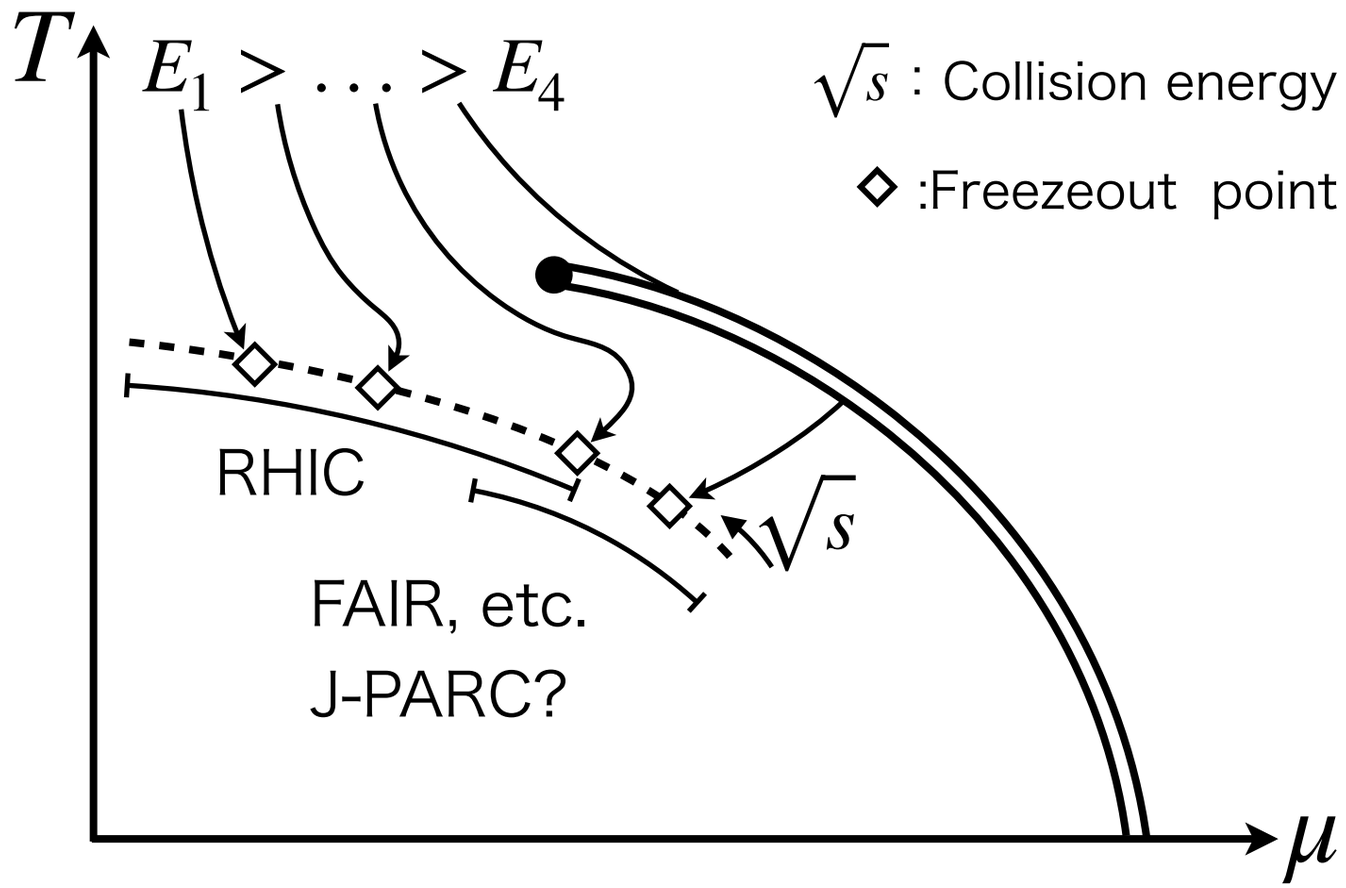
Theoretical Challenges in the Search for the QCD Critical Point

Noriyuki Sogabe (The University of Osaka)

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Certral China Center for Nuclear Theory

QCD critical point searech



- Relativistic Heavy Ion Collider (RHIC)、Beam-Energy-Scan (BES)
  (BES-I: 2010-2017, BES-II: 2019-2021)
- Future low-energy collisions: FAIR, HIAF, NICA, J-PARC

Flcutuations

Pandav (STAR), Critical Point and Onset of Deconfinement (CPOD) 2024

A review by Stephanov (2024) arXiv:2410.0286

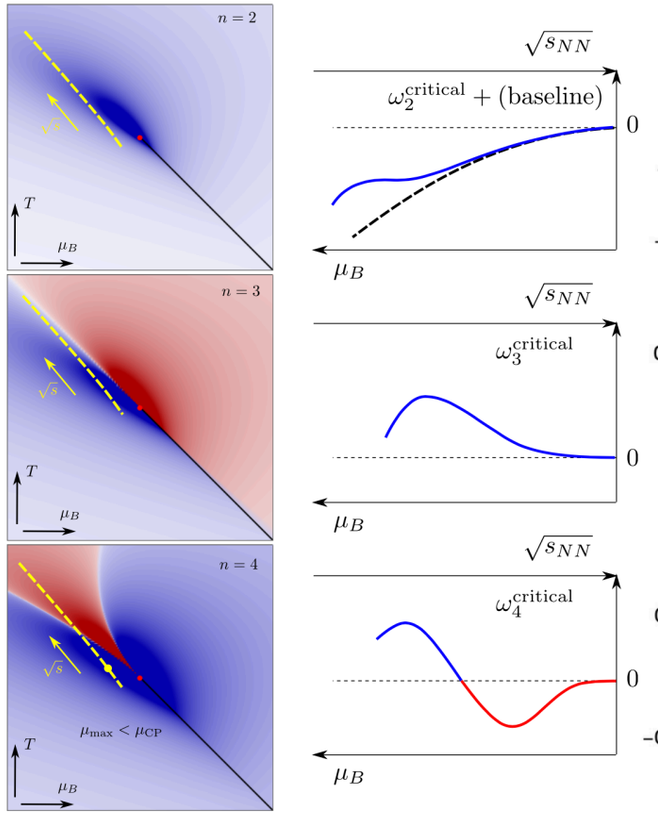
Baryon number fluctuation

$$\frac{\partial n(\mu, T)}{\partial \mu} \sim \langle \delta n \delta n \rangle_{\text{cum.}}$$

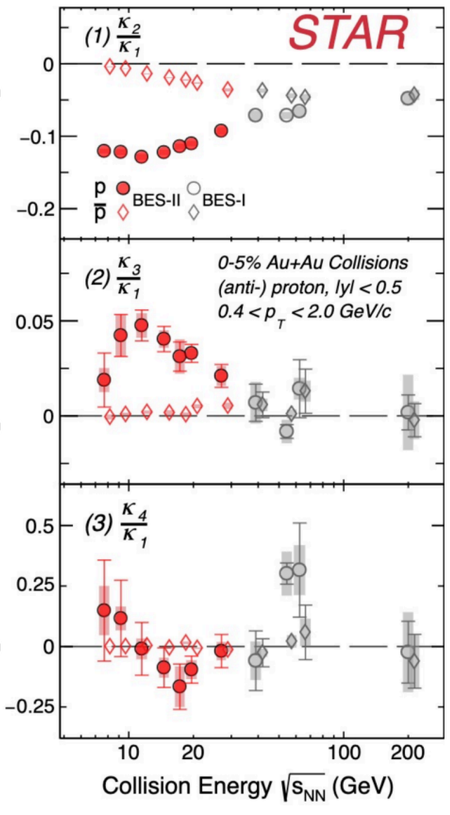
$$\frac{\partial^2 n}{\partial \mu^2} \sim \left\langle (\delta n)^3 \right\rangle_{\text{cum}}$$

$$\frac{\partial^3 n}{\partial \mu^3} \sim \left\langle (\delta n)^4 \right\rangle_{\text{cum}}$$

Theory



Experiment



Consistent with equilibrium expectation

Toward quantitative comparison with experiment:

Nonequilibrium effects on fluctuations

Theoretical challenges and Outline

1. Non-equilibrium dymamics of fluctuations

An EFT approach

NS and Yi Yin, JHEP (2022)

2. Fireball evolution near the critical point

Nonmonotonic specific entropy

Pradeep, NS, Stephanov, and Yee, PRC (2024)

3. Fluctuation dynamics along isentropes in a QCD phase diagram with a critical point

NS (in progress)

Baryon fluctuation out of equilibrium

Evolution of the n-th cumulant:

$$W_n(t; \boldsymbol{q}_1, \ldots, \boldsymbol{q}_{n-1}) \equiv \langle \delta n(t, \boldsymbol{q}_1) \ldots \delta n(t, \boldsymbol{q}_n) \rangle_{\text{cum}}$$

Under background hydrodynamic evolution

- Diffusive (Langevin) equation approach

  An, Basar, Stephanov, and Yee (2021)

- Nonequilibrium effective field theory (EFT) on the Schwinger-Keldysh path Crossley, Glorioso, and Liu (2017), Haehl, Loganayagam, and Rangamani (2016) Jensen, Pinzani-Fokeeva, and Yarom (2018)

$$\langle \delta n(t, \boldsymbol{q}_1) \ldots \delta n(t, \boldsymbol{q}_n) \rangle_{\text{connected}} \propto \int \mathscr{D}n \mathscr{D}\theta_{\text{a}} \, \mathrm{e}^{\mathrm{i} I_{\text{eff}}[n, \theta_{\text{a}}]} \delta n(t, \boldsymbol{q}_1) \ldots \delta n(t, \boldsymbol{q}_n)$$

Effective Lagrangian

NS and Yin (2021)

- Baryon number: $n = J^t_{\rm r} = \dfrac{\delta I_{\rm eff}}{\delta A^t_{\rm a}}, \quad \mu \equiv \mathscr{A}_{{\rm r}t},$

$$\mathscr{L}_{\rm eff} = n\mathscr{A}^{\rm a}_0 - \lambda(n)\boldsymbol{\mathscr{A}}^{\rm a}\cdot(\partial_t\boldsymbol{\mathscr{A}}^{\rm r} - {\rm i}T\boldsymbol{\mathscr{A}}^{\rm a}) + \text{NLO} + \cdots \qquad (\mathscr{A}^{\rm r/a}_\mu = A^{\rm r/a}_\mu + \partial_\mu\theta^{\rm r/a})$$

with U(1), KMS symmetry, etc.

See lecture notes by Glorioso and Liu (2018)

- Equation of motion :

$$\frac{\delta I_{\rm eff}}{\delta\theta_{\rm a}} = 0 \;\Leftrightarrow\; \partial_t n + \nabla\cdot\boldsymbol{J} = 0; \quad \boldsymbol{J} = -\lambda(n)\nabla\mu + 2{\rm i}\lambda(n)\nabla\theta_{\rm a} + \cdots$$

- NLO = “Non-universal terms” Jain and Kovtun (2020)

  HO in $\partial \to$ relevant for W_4 dynamics in the critical regime: $|\boldsymbol{q}|\xi \gg 1$

Pertuvative expansion from background NS and Yin (2021)

- Scale separation between the background evolution and the fluctuation dynamics
- Deviation from the background value: $\delta n = n - n_0$, etc.

$$\mu = \frac{1}{\chi}\delta n + \xi^2 \nabla^2 \delta n + \cdots, \quad \lambda(n) = \lambda + \lambda' \delta n + \cdots, \qquad \chi = \frac{\partial n_0(\mu, T)}{\partial \mu}$$

Correlation length: $\xi(\mu, T)$, Diffusion rate: $\lambda(\mu, T)$

$$\mathscr{L}_{\text{eff}} = (\mathscr{L})_2 + (\mathscr{L})_3 + (\mathscr{L})_4 + \cdots$$

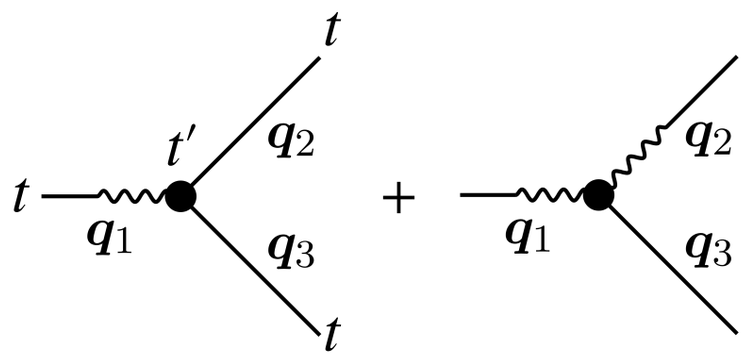
Diffusion eq. + Noise Nonlinear couplings/noise

Nonequilibrium fluctuation dynamics

NS and Yin (2021)

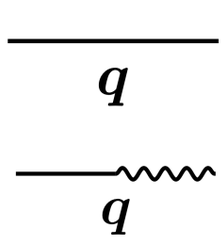
- Diagram calculation

$W_3(t;\boldsymbol{q}_1,\ldots,\boldsymbol{q}_3) =$



$W_4(t,\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3)$ as well

- Feynman rules:



$= G_{\rm rr}(t,t',\boldsymbol{q})$

$= G_{\rm ra}(t,t',\boldsymbol{q}) = G_{\rm ar}(t',t,\boldsymbol{q})$

$= G_{\rm ar}(t,t',\boldsymbol{q})$

+ vertices

- Schwinger-Dyson equations :

$$-\partial_t G_{\rm ar}(t,t',\boldsymbol{q}) + \Gamma(t,\boldsymbol{q}) G_{\rm ar}(t,t',\boldsymbol{q}) = \delta(t-t')$$

$$\partial_t G_{\rm rr}(t,t',\boldsymbol{q}) + \Gamma(t,\boldsymbol{q}) G_{\rm rr}(t,t',\boldsymbol{q}) = \lambda(t)\boldsymbol{q}^2 G_{\rm ar}(t,t',\boldsymbol{q})$$

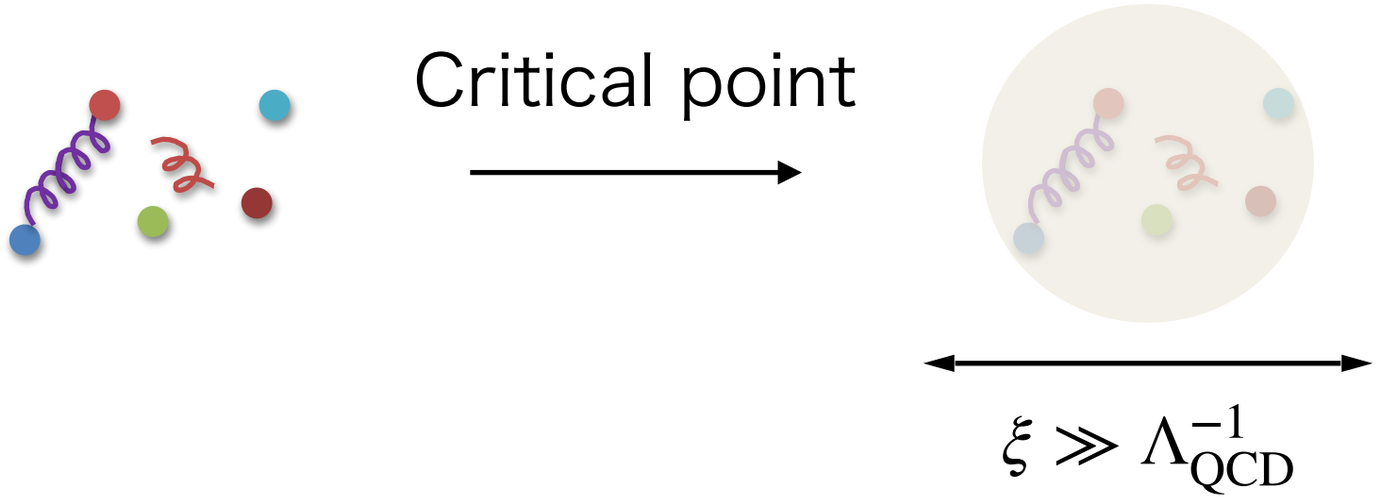
$$G_{\rm ar}(t'-0,t',\boldsymbol{q}) = 1\,, \quad G_{\rm rr}(t_{\rm I},t_{\rm I},\boldsymbol{q}) = \bar{W}_2(t_{\rm I},\boldsymbol{q})$$

$$\Gamma(t,\boldsymbol{q}) \equiv \lambda(t)\boldsymbol{q}^2 \bar{W}_2^{-1}(t,\boldsymbol{q})\,, \quad \bar{W}_2^{-1}(t_{\rm I},\boldsymbol{q}) = \chi^{-1}(t_{\rm I}) + \xi(t_{\rm I})^2\boldsymbol{q}^2$$

Dynamical background evolution: $\chi = \chi(\mu(t),T(t))$, $\xi = \xi(\mu(t),T(t))$, etc.

Ideal hydrodynamics

- Entropy and baryon number conservation $\longrightarrow \hat{s} = \dfrac{s}{n} = \text{const}.$
- The universality of critical phenomena:

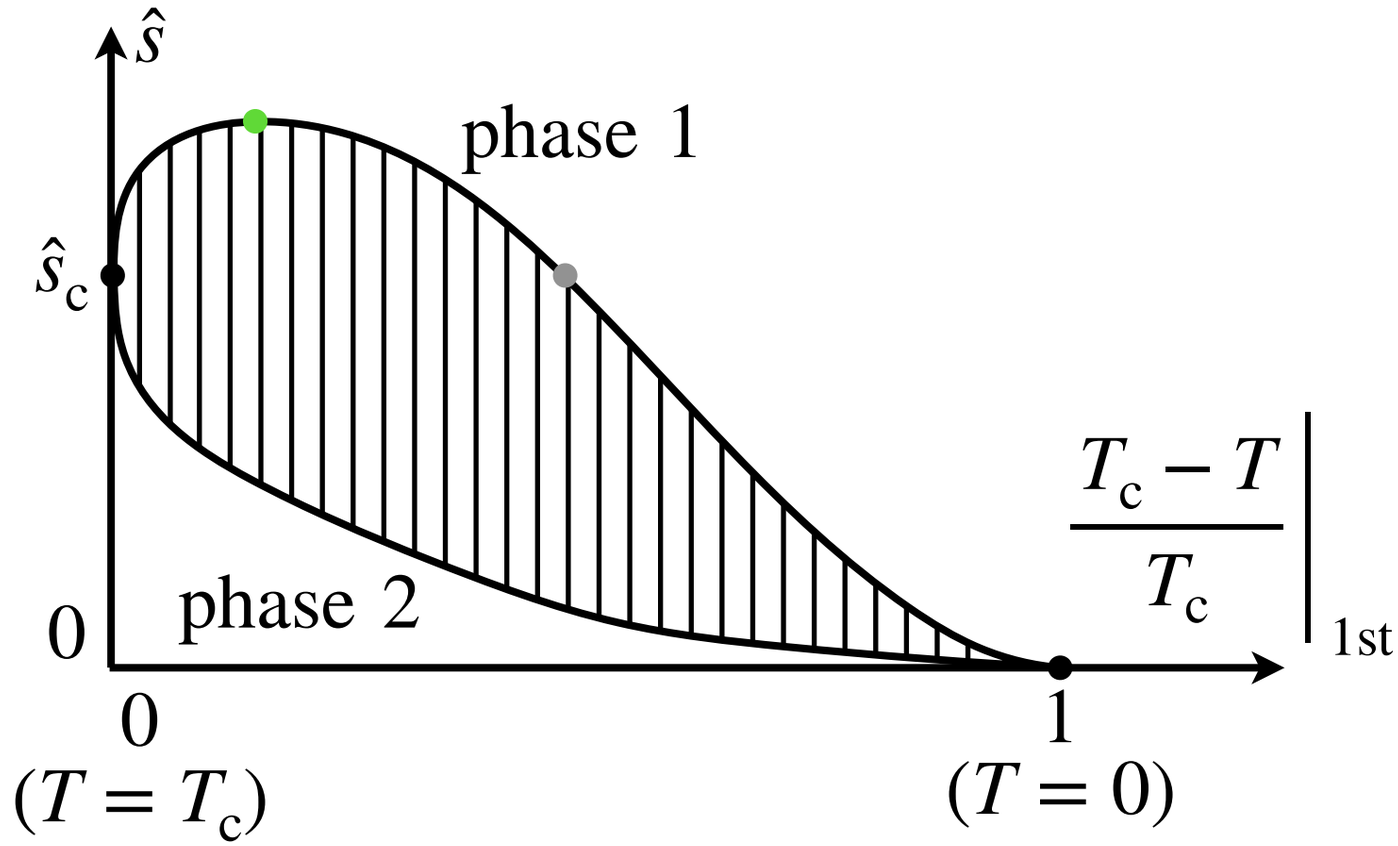


- QCD thermodynamics ≃ 3D Ising model

  $\longrightarrow$ Universal behaviors of isentropes

Non-monotonic specific entropy

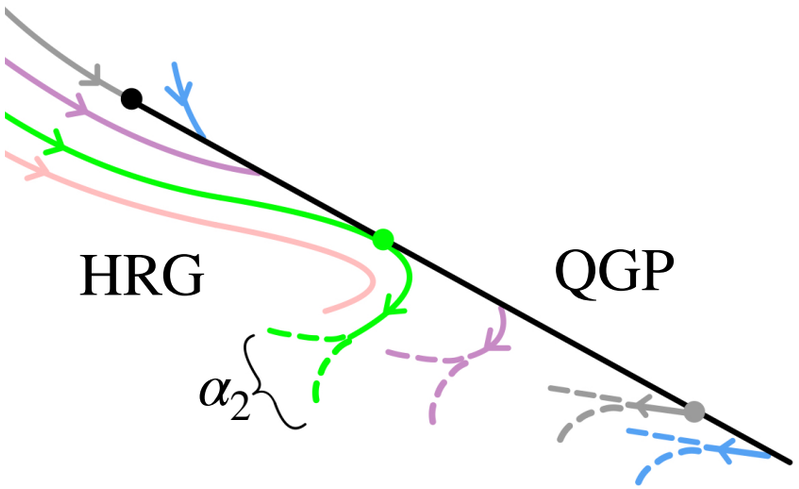
Pradeep, NS, Stephanov, and Yee (2024)



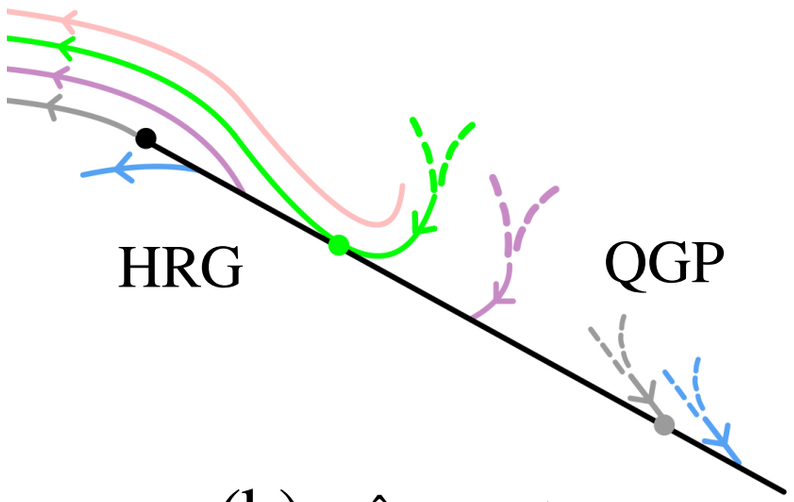
- Near T_c : $\hat{s} \sim (\text{order parameter}) \sim \pm (T - T_c)^{\beta}$ $(\beta = 0.326)$
- The third law of thermodynamics: $\hat{s}(T = 0) = 0$

Classification of isentropes

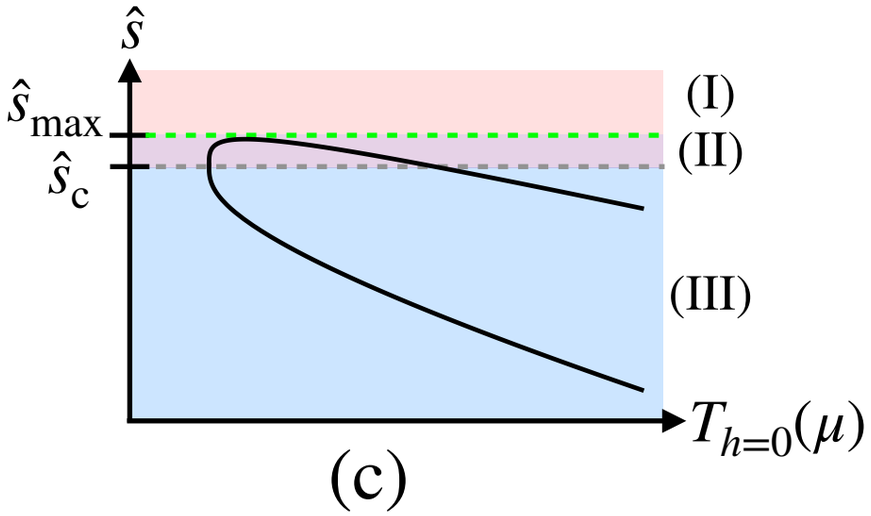
Pradeep, NS, Stephanov, and Yee (2024)



(a) $\hat{s}_c > \cot\alpha_1$



(b) $\hat{s}_c < \cot\alpha_1$



(c)

	(a)	(b)
(I)	Crossover	
(II)	HRG → HRG	QGP → QGP
(III)	QGP → HRG	

(d)

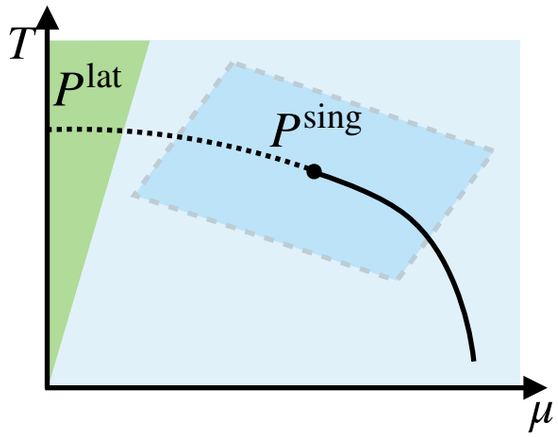
Fluctuation dynamics along isentropes with a QCD Critical Point

- 3D Ising equation of state with lattice QCD (T'-expansion)

  Kahangirwe et. al. (2024)

- Background evolution:
  - Ideal, boost-invariant Bjorken expansion
  - Freeze-out curve, $T_{\mathrm{fz}}(\mu)$

    Andronic Braun-Munzinger, K. Redlich, J. Stachel, (2018)

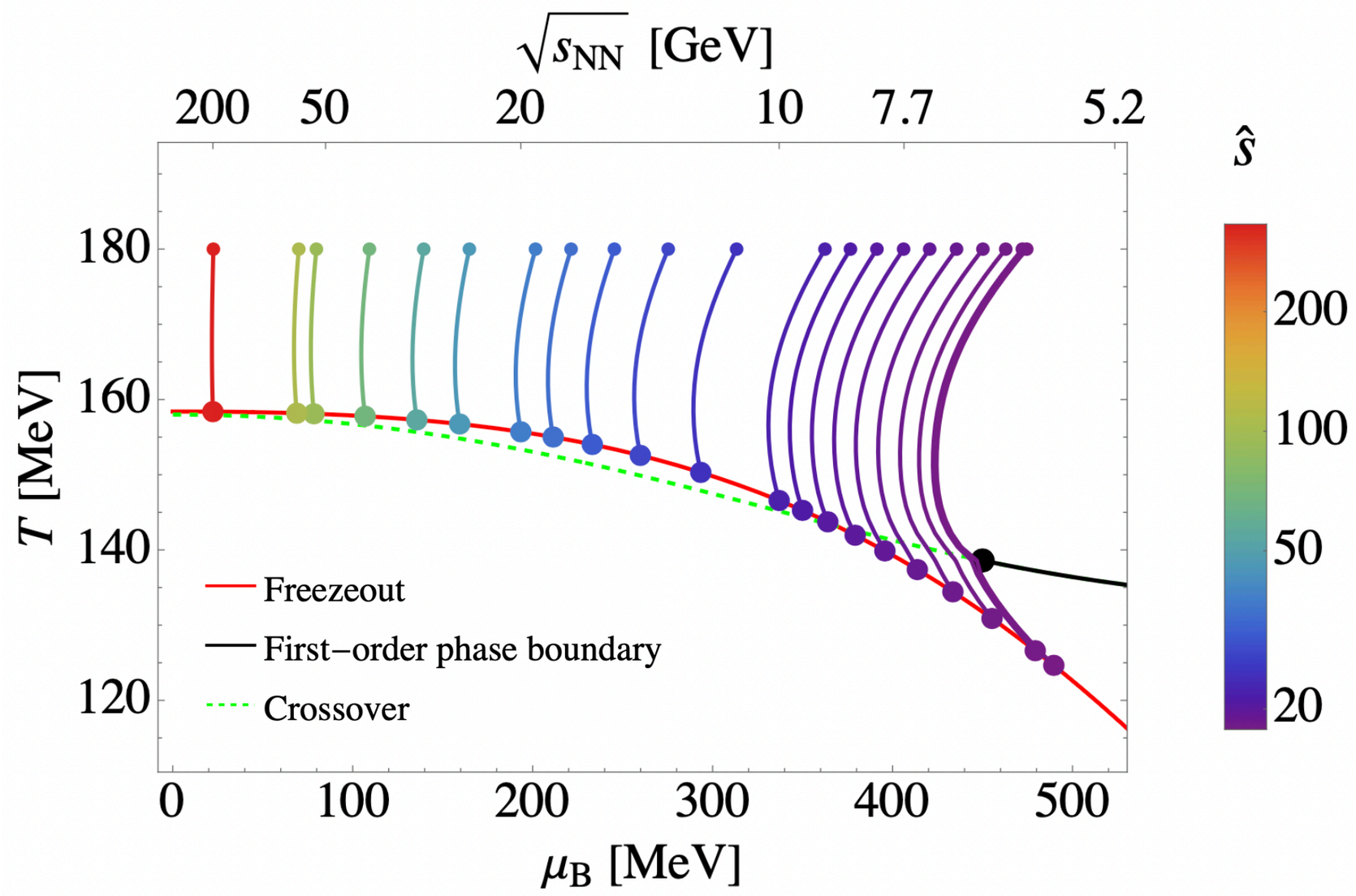


- Compute $W_2(t, \boldsymbol{q}), W_3(t, \boldsymbol{q}_1, \boldsymbol{q}_2), \ldots$

- [in progress] space-time integral (experimental acceptance), convert to proton portion

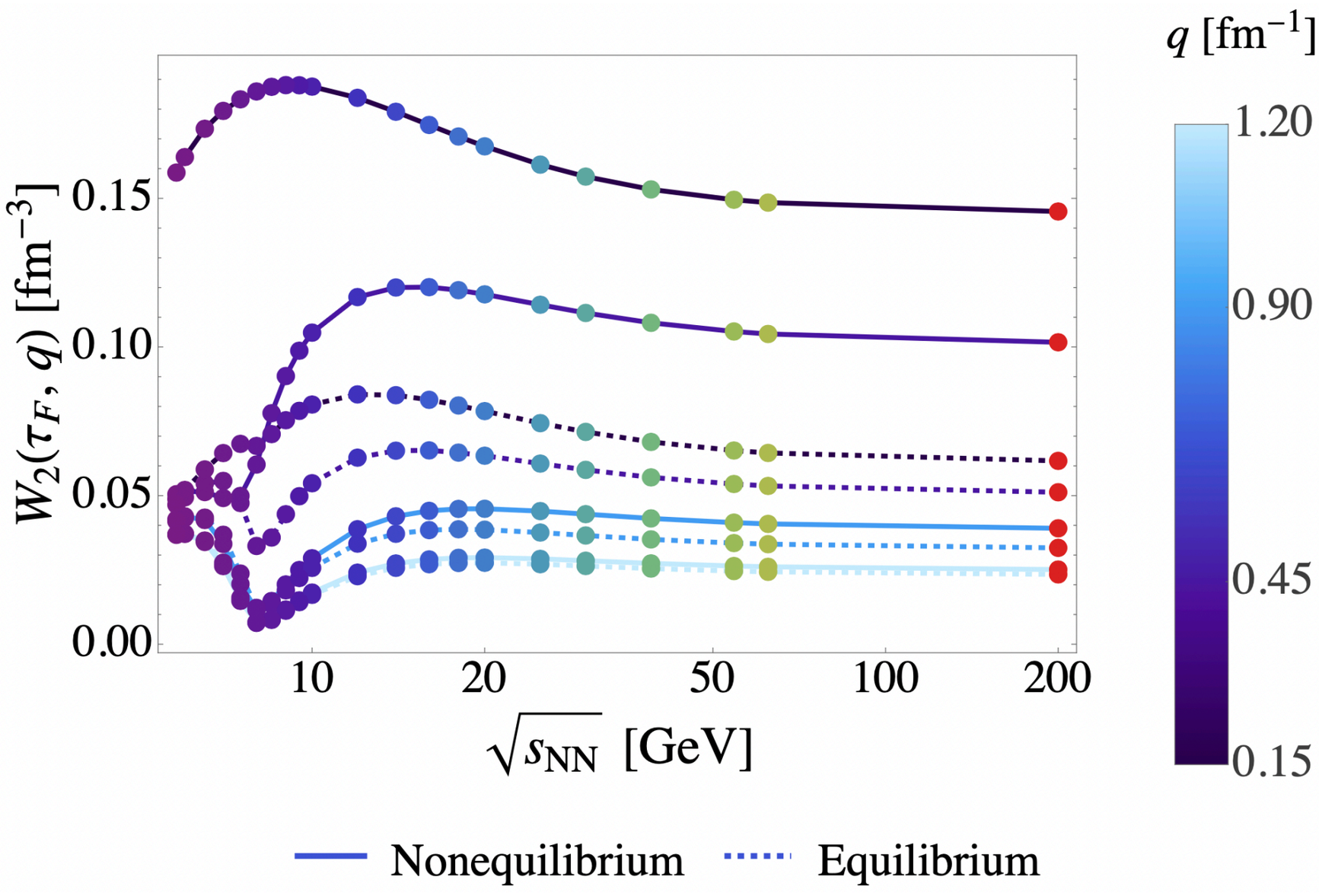
Fluctuation dynamics along isentropes with a QCD Critical Point

NS (in progress)



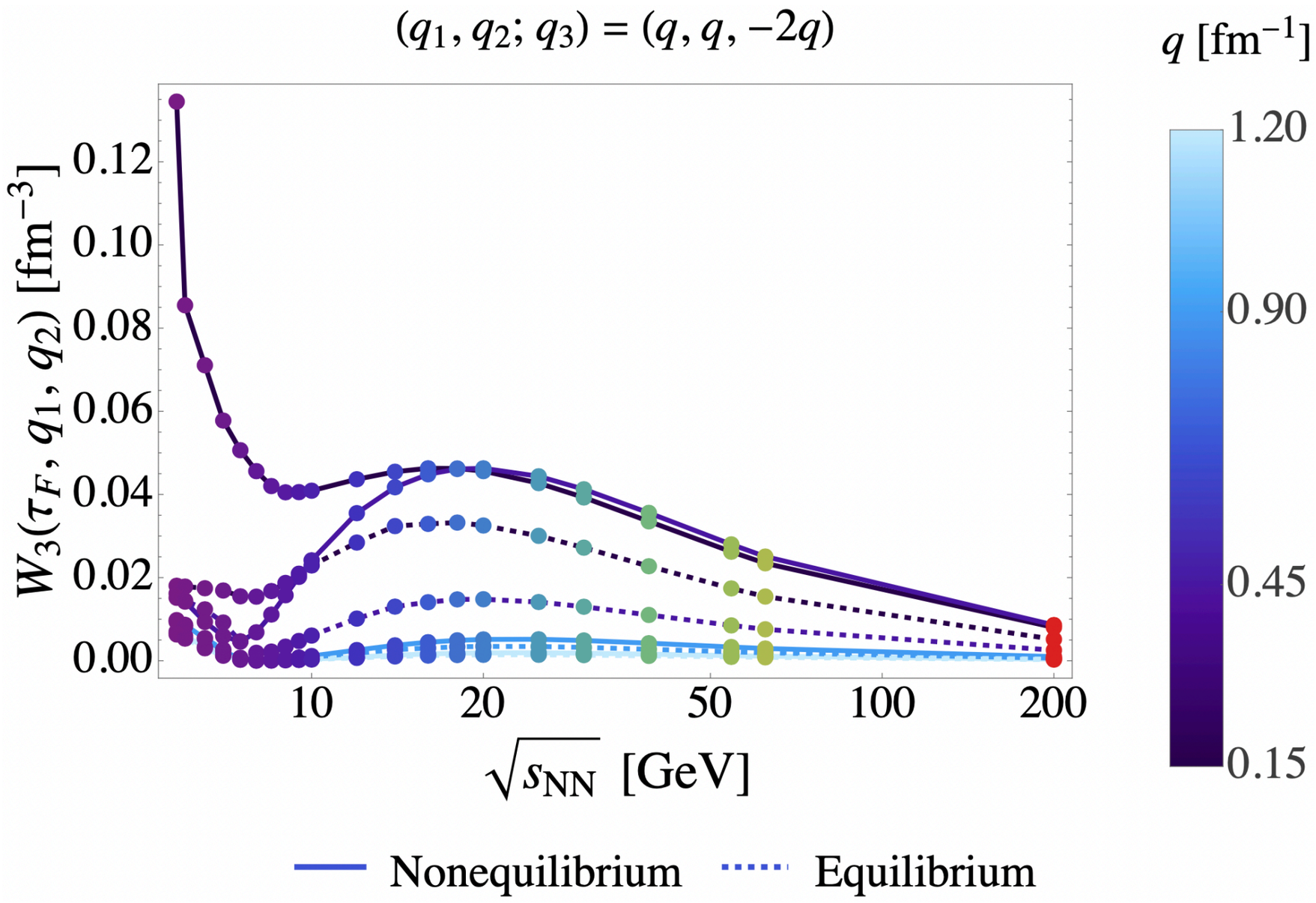
Two-point correlator at τ_{F}

NS (in progress)



Three-point correlator at τ_{F}

NS (in progress)



Other configuration as well

Summary and outlook

- Toward the QCD critical point search: Understanding fluctuation dynamics in an expanding fireball

  NS and Yi Yin, JHEP 2022, 124 (2022) arXiv:2111.14667

  Pradeep, NS, Stephanov, and Yee, PRC (2024) arXiv:2402.09519

- A simplified setup: Evolution of Non-Gaussian Fluctuations along isentropes with a QCD Critical Point NS (in progress)

- Outlook:

Fourie

$$W_2(\tau_{\rm F}, \boldsymbol{q}),\ W_3(\tau_{\rm F}, \boldsymbol{q}_1, \boldsymbol{q}_2), \cdots \longrightarrow G_2(\tau_{\rm F},\ \Delta\eta), G_3(\tau_{\rm F}, \Delta\eta_1, \Delta\eta_2), \cdots$$

Integration over experimental acceptance

$$\longrightarrow I_2(\tau_{\rm F}),\ I_3(\tau_{\rm F}), \cdots$$

Conversion to proton-number fluctuations + HRG subtraction

Maximum-entropy freezeout (Paradep and Stephanov, 2022)

$$\longrightarrow \kappa_2/\kappa_1,\ \kappa_3/\kappa_1, \cdots$$