JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Critical Pion Dynamics near the QCD Chiral Transition

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Strong-interaction matter under extreme conditions

Based on:
YRC, J. V. Roth, and L. von Smekal, in preparation
Y.-y. Tan, YRC, W.-j. Fu and W.-j. Li, Nat. Commun. 16, 2916 (2025)

UNTERSTÜTZT VON / SUPPORTED BY
Alexander von HUMBOLDT STIFTUNG

Outline

1. Introduction
2. Model G within the real-time fRG approach
3. Numerical results
   - Static critical behavior and scaling functions
   - Dynamic critical behavior and scaling functions
   - Damping and propagation of pseudo-Goldstone modes
4. Summary and Outlook

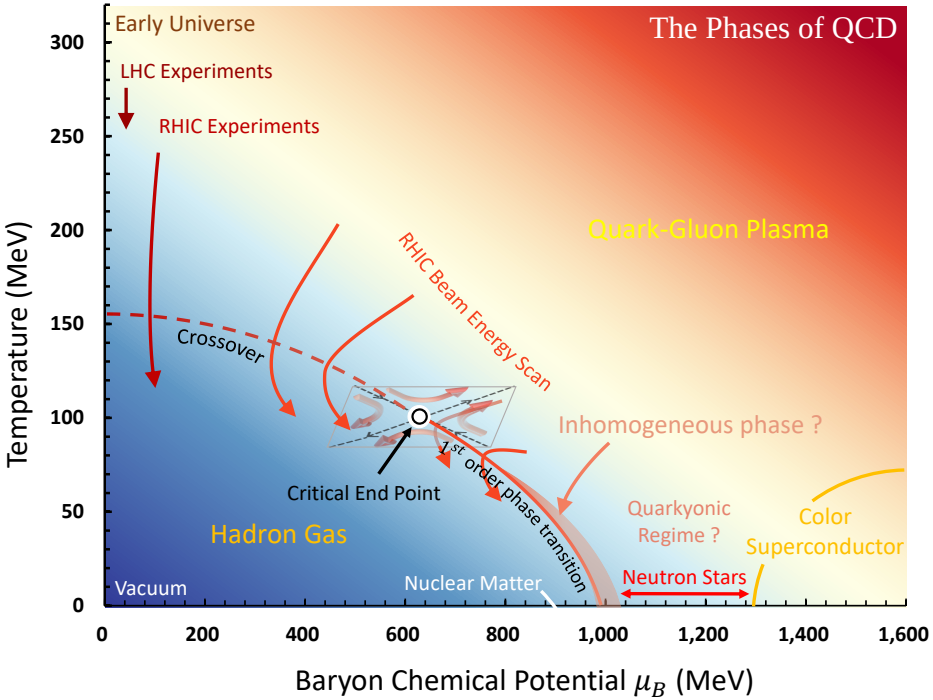
1 Introduction

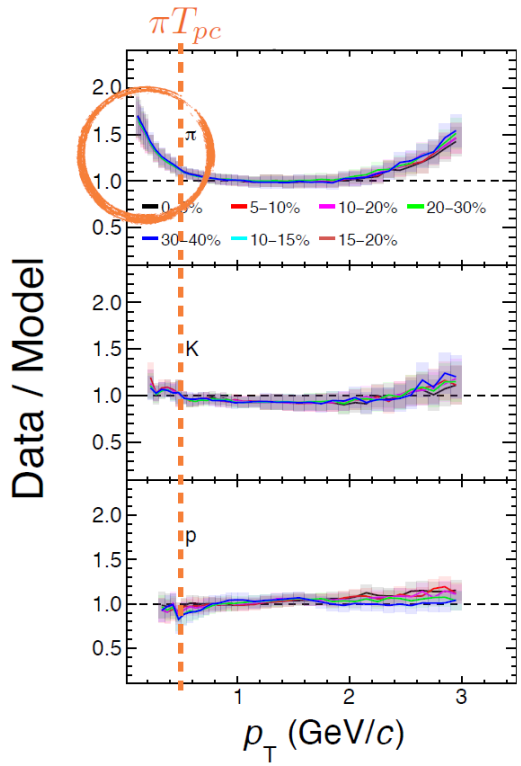
2 Model G within the real-time fRG approach

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Introduction





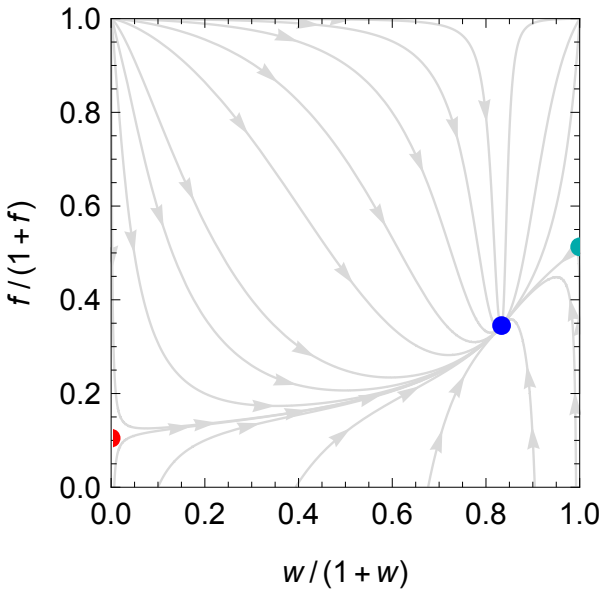
- QCD chiral phase transition ($N_f = 2$): statics: $O(4)$ universality class, dynamics: Model G
- Imprint of chiral crossover: **low-p_T pion enhancement** – soft pion dynamics near T_{pc}
- Understand the non-equilibrium effects and critical dynamics

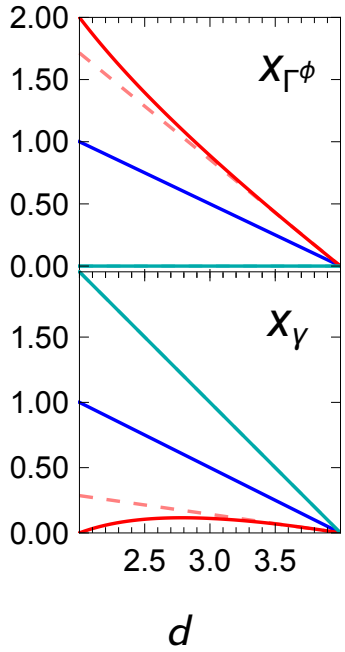
Pisarski, Wilczek, PRD 29, 338 (1984); Rajagopal, Wilczek, NPB 399, 395 (1993); Lu *et al.*, Nucl. Sci. Tech. 36, 142 (2025)

Introduction

Real-time fRG

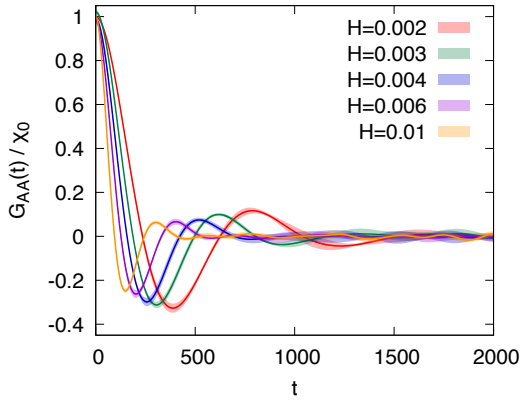
$f \propto g^2 T\, k^{d-4}/(\Gamma_{\phi,k}\gamma_{n,k}), \quad w \propto \Gamma_{\phi,k}/\gamma_{n,k}$

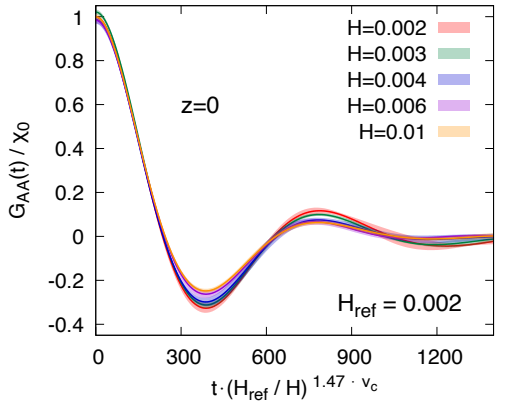




- weak scaling: $w^* = 0$ or ∞, $z_\phi \neq z_n$
- strong scaling: $0 < w^* < \infty$, $z_\phi = z_n$
  IR stable $\Rightarrow$ governs the critical dynamics

Langevin simulations





$z = 1.47 \pm 0.01\,(\text{stat.}) \quad \simeq d/2$

FRG: Roth, Ye, Schlichting, von Smekal, JHEP 01, 118 (2025); PRD 111, L111901 (2025)

simulations: Florio, Grossi, Soloviev, Teaney, PRD 105, 054512 (2022); Florio, Grossi, Teaney, PRD 109, 054037 (2024)

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Dynamic universality model: Model G

Order parameter ϕ_a + conserved $O(N)$ charge densities $n_{ab} = -n_{ba}$

Hohenberg, Halperin, RMP 49, 435 (1977); Sasvári, Schwabl, Szépfalusy (1975); Rajagopal, Wilczek, NPB 399, 395 (1993)

Langevin equation

$$\partial_t \phi_a = -\Gamma_\phi \frac{\delta F}{\delta \phi_a} + \frac{g}{2}\{\phi_a, n_{bc}\}\frac{\delta F}{\delta n_{bc}} + \theta_a \,,$$

$$\partial_t n_{ab} = \gamma_n \nabla^2 \frac{\delta F}{\delta n_{ab}} + g\{n_{ab}, \phi_c\}\frac{\delta F}{\delta \phi_c} + \frac{g}{2}\{n_{ab}, n_{cd}\}\frac{\delta F}{\delta n_{cd}} + \nabla \cdot \boldsymbol{\zeta}_{ab}$$

Landau-Ginzburg-Wilson free energy

$$F = \int_{\boldsymbol{x}} \left\{ \tfrac{1}{2}(\partial_i \phi_a)^2 + \tfrac{m^2}{2}\phi_a\phi_a + \tfrac{\lambda}{4!N}(\phi_a\phi_a)^2 + \tfrac{1}{4\chi} n_{ab} n_{ab} \right\}$$

- **Irreversible part**
  - dissipative and diffusive terms with the kinetic coefficients Γ_ϕ and γ_n
  - thermal noise terms θ_a, $\nabla \cdot \boldsymbol{\zeta}_{ab}$, fixed by the FDT
  - equilibrium distribution $P_{\rm eq} \propto e^{-F/T}$
- **Reversible part**: mode couplings through the Poisson brackets

  $\{\phi_a, n_{bc}\} = \delta_{ac}\phi_b - \delta_{ab}\phi_c \,,$ $\quad\quad$ $\{n_{ab}, n_{cd}\} = \delta_{ac} n_{bd} + \delta_{bd} n_{ac} - \delta_{ad} n_{bc} - \delta_{bc} n_{ad}$

Effective action of Model G

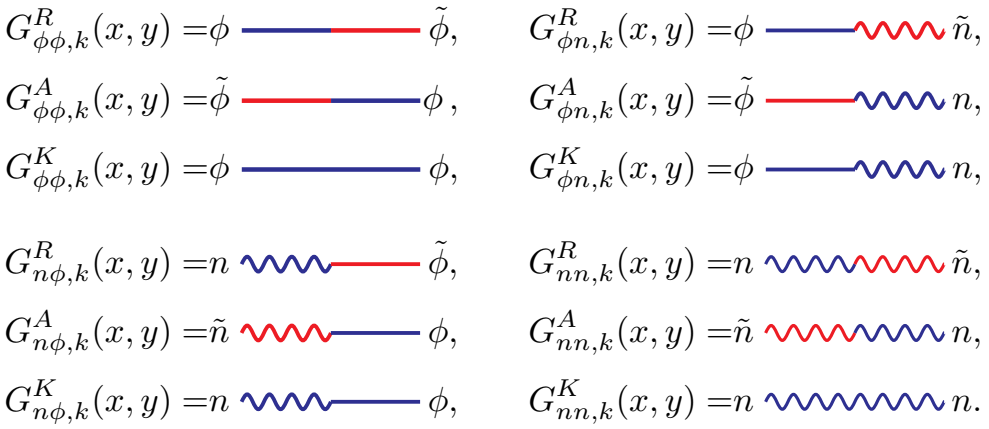
MSR effective action

$$\Gamma_k = \int_x \Big[-\tilde{\phi}_a \Big(\partial_t \phi_a + \Gamma_{\phi,k} \frac{\delta F_k}{\delta \phi_a} - \frac{g}{2} \{\phi_a, n_{bc}\} \frac{\delta F_k}{\delta n_{bc}} \Big) - \frac{1}{2} \tilde{n}_{ab} \Big(\partial_t n_{ab} - \gamma_{n,k} \nabla^2 \frac{\delta F_k}{\delta n_{ab}} - g \{n_{ab}, \phi_c\} \frac{\delta F_k}{\delta \phi_c} - \frac{g}{2} \{n_{ab}, n_{cd}\} \frac{\delta F_k}{\delta n_{cd}} \Big) + iT \Gamma_{\phi,k} \tilde{\phi}_a^2 - \frac{i}{2} T \tilde{n}_{ab} \gamma_{n,k} \nabla^2 \tilde{n}_{ab} \Big]$$

- dynamic anomalous dimensions: $x_{\Gamma_\phi} = -\partial_t \ln \Gamma_{\phi,k}$, $x_{\gamma_n} = -\partial_t \ln \gamma_{n,k}$
- static anomalous dimension: LPA: $Z_{\phi,k} = 1$, $\eta_\phi = 0$; LPA′: $\eta_\phi = -\partial_t \ln Z_{\phi,k}$ ($Z_{n,k} = 1/\chi_n$ not renormalized)
- dynamic critical exponents: $z_\phi = 2 - \eta_\phi - x_{\Gamma_\phi}$, $z_n = 2 - x_{\gamma_n}$
- g not renormalized: Ward identity of the time-dependent $O(N)$ symmetry

Martin, Siggia, Rose, PRA 8, 423 (1973)

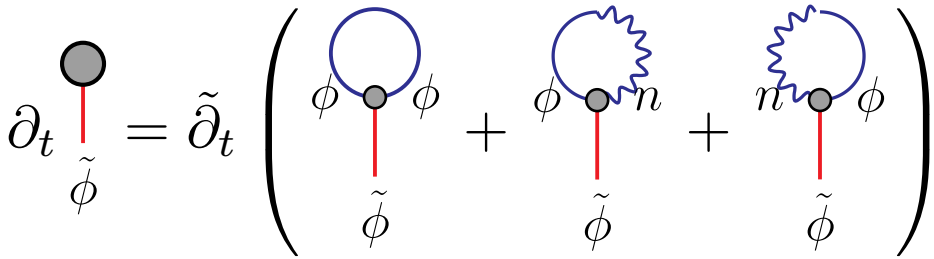
Propagators and flow of the effective potential



- straight: ϕ, wavy: n
- blue: physical fields (ϕ, n), red: response fields $(\tilde{\phi}, \tilde{n})$
- regulator only for ϕ; none for n: static charge fluctuations are non-critical

- symmetric phase: $\Gamma_k^{(2)}$ diagonal in (ϕ, n) $\Rightarrow$ $G_{\phi n} = G_{n\phi} = 0$
- broken phase ($\sigma_0 \neq 0$): mix the pions π_i with the isoaxial-vector charges n_{0i} $\Rightarrow$ off-diagonal $G_{\phi n}$, $G_{n\phi} \neq 0$

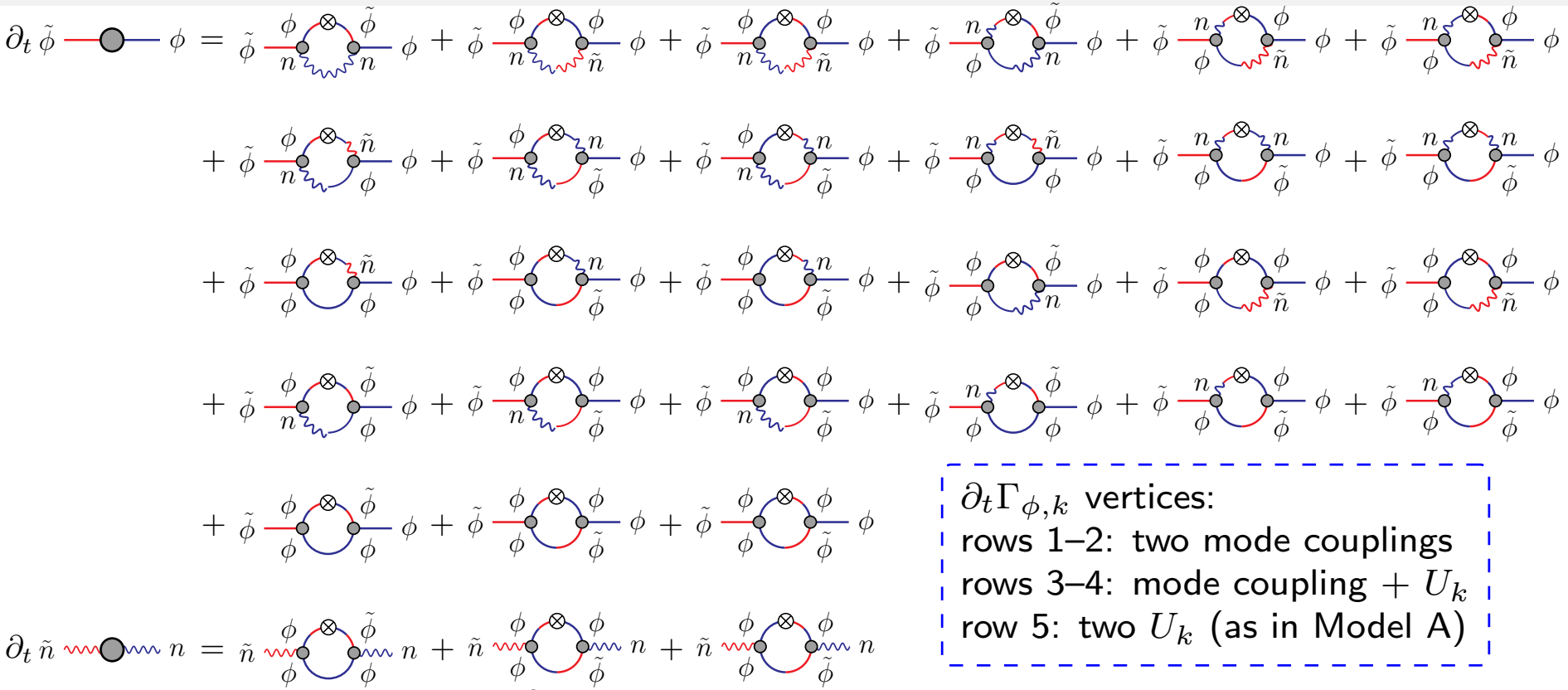
Flow of the effective potential



- the two mixed diagrams (present only for $\sigma_0 \neq 0$) cancel each other

$\Rightarrow$ static: Euclidean $O(N)$ model, independent of $\Gamma_{\phi,k}$, $\gamma_{n,k}$, g

Flow of the kinetic coefficients

∂_t  ϕ

∂_t

$\partial_t \Gamma_{\phi,k}$ vertices:
rows 1–2: two mode couplings
rows 3–4: mode coupling + U_k
row 5: two U_k (as in Model A)

$$\partial_t \Gamma_{\phi,k} = \frac{1}{N-1} \lim_{p\to 0} (-i) \frac{\partial}{\partial \boldsymbol{p}^2} P_T \, \partial_t \Gamma^{(2)}_{\tilde{\phi}\phi,k}(p) - \Gamma_{\phi,k} \partial_t Z_{\phi,k} \,, \qquad P_T : \text{ pions } \pi_i$$

$$\partial_t \gamma_{n,k} = \frac{1}{N-1} \lim_{p\to 0} (-i) \frac{\partial}{\partial \boldsymbol{p}^2} P_A \, \partial_t \Gamma^{(2)}_{\tilde{n}n,k}(p) \,, \qquad P_A : \text{ axial charges } n_{0i}$$

- assess contributions from the effective-potential vertex
- symmetric phase: only the first three diagrams contribute $\partial_t \Gamma_{\phi,k}$
- $\partial_t \gamma_{n,k}$: two ϕ propagators, both phases

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Static critical exponents

Scaling form of the free energy density

τ: reduced temperature
h: reduced external field

$$f_s(\tau, h) = \ell^{-d} f_s\left(\tau\, \ell^{y_\tau}, h\, \ell^{y_h}\right)$$

critical behavior of the physical observables:

$$M(\tau, h=0) = (-\tau)^{\beta}\,, \quad M(\tau=0, h) = h^{1/\delta}\,, \quad \chi \sim |\tau|^{-\gamma}\,, \quad \xi \sim |\tau|^{-\nu}$$

	LPA			LPA′			
	β	δ	ν	β	δ	η_ϕ	ν
$O(2)$	0.3526	4.9936	0.7052	0.3573	4.7452	0.0438	0.6846
$O(3)$	0.3799	4.9933	0.7598	0.3805	4.7609	0.0409	0.7311
$O(4)$	0.4009	4.9932	0.8018	0.3996	4.7801	0.0373	0.7705

- consistent with earlier fRG results in the same truncation

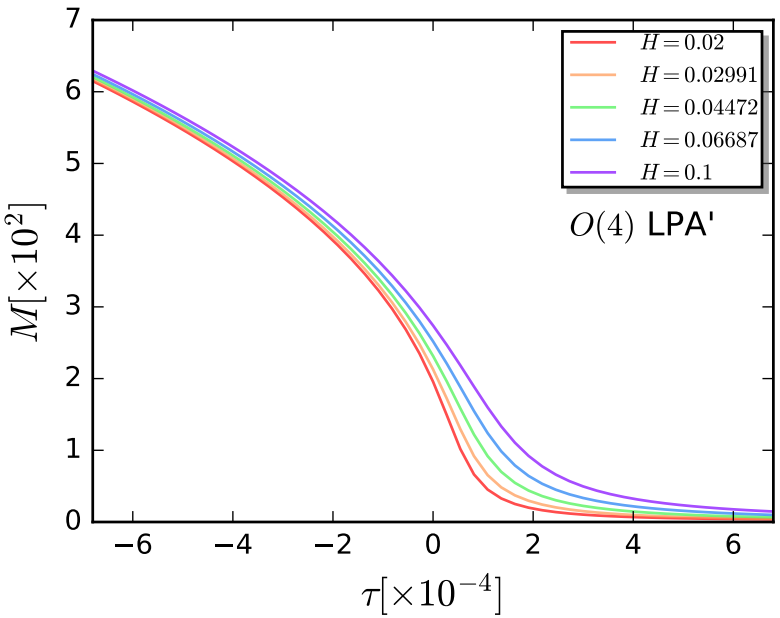
Scaling function of the order parameter

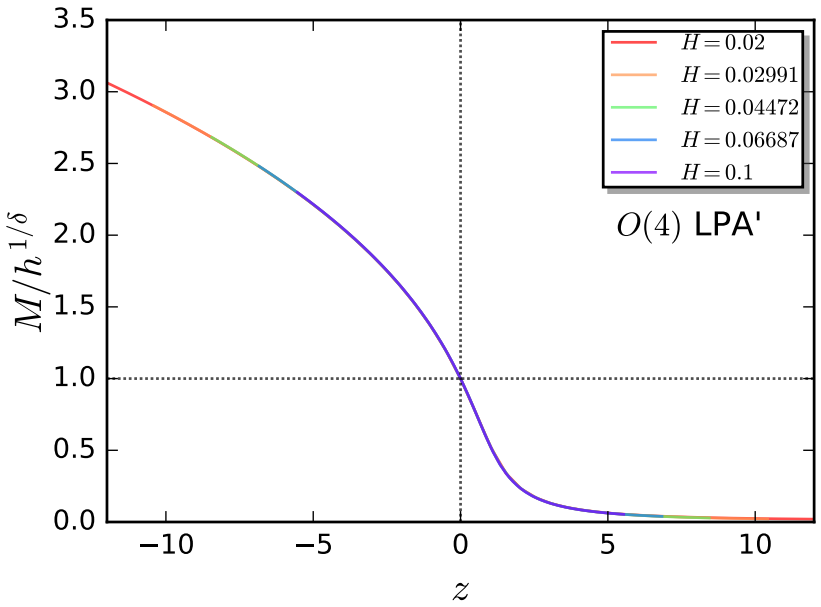
Magnetic equation of state with scaling variable $z \equiv \tau/h^{1/(\beta\delta)}$

$$M = h^{1/\delta} f_G(z)$$

normalization: $f_G(0) = 1$

asymptotic behavior: $f_G \xrightarrow{z\to-\infty} (-z)^\beta, \quad f_G \xrightarrow{z\to\infty} z^{-\gamma}$





- left: order parameter M as a function of τ for several different values of c
- right: rescaled order parameter $M/h^{1/\delta}$ as a function of the scaling variable z

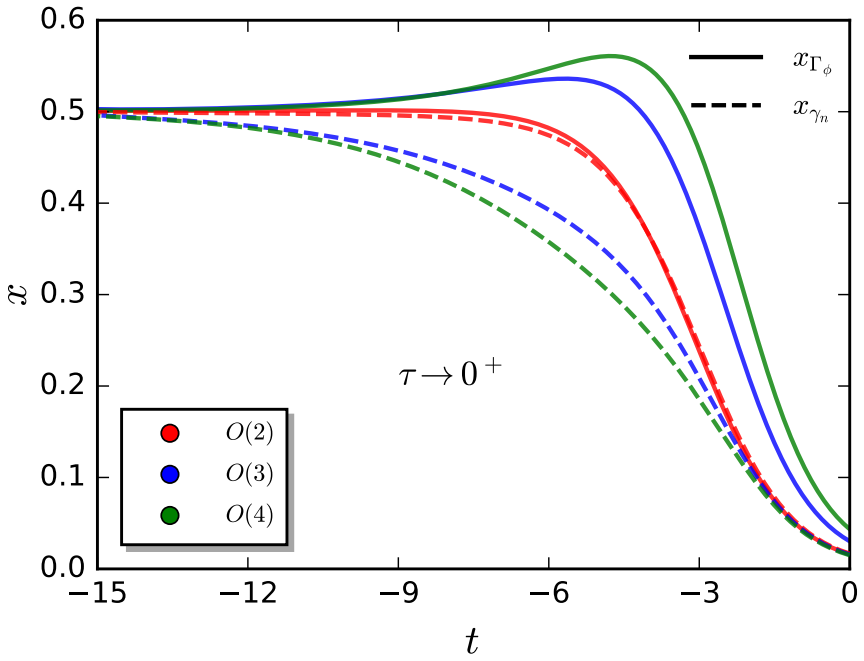
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Symmetric phase: dynamic anomalous dimensions

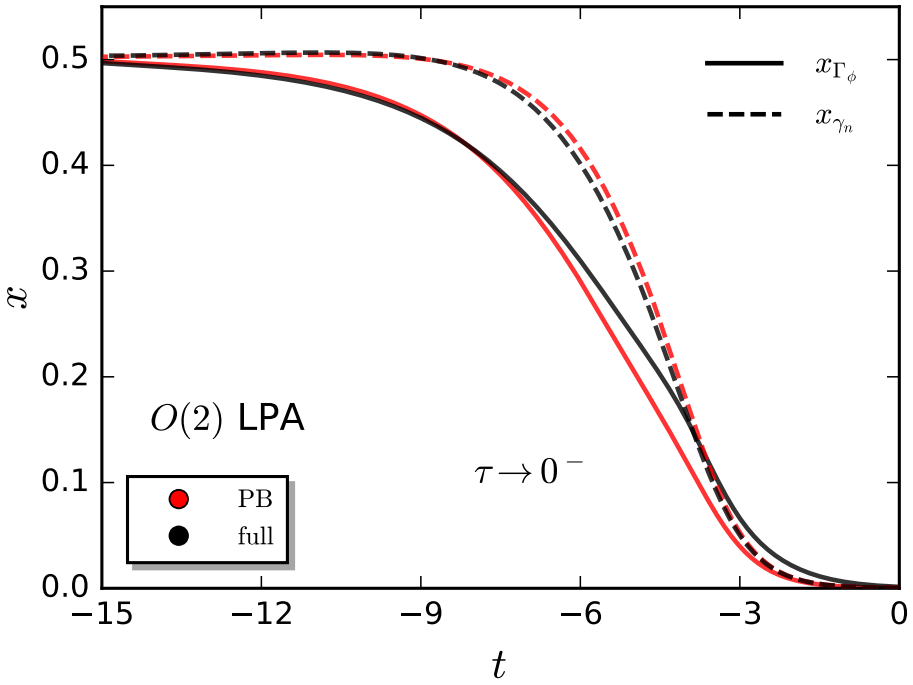


- $\tau \to 0^+$, $\sigma_0 = 0$: no ϕ-n mixing
  $\Rightarrow$ driven by the mode couplings only
- $\eta_\phi = 0$ (expansion at $\rho_0 = 0$)
- $N = 2$: $x_{\Gamma_\phi} \approx x_{\gamma_n}$ along the whole flow
- $N = 3, 4$: x_{Γ_ϕ} overshoots and approaches the fixed point from above, x_{γ_n} from below

IR ($t = -15$): $|x_{\Gamma_\phi} - \frac{1}{2}|,\ |x_{\gamma_n} - \frac{1}{2}| < 0.005$ for $N = 2, 3, 4$

✓ strong scaling, $z_\phi = z_n = \frac{3}{2}$

Broken phase: Poisson brackets vs. full flow

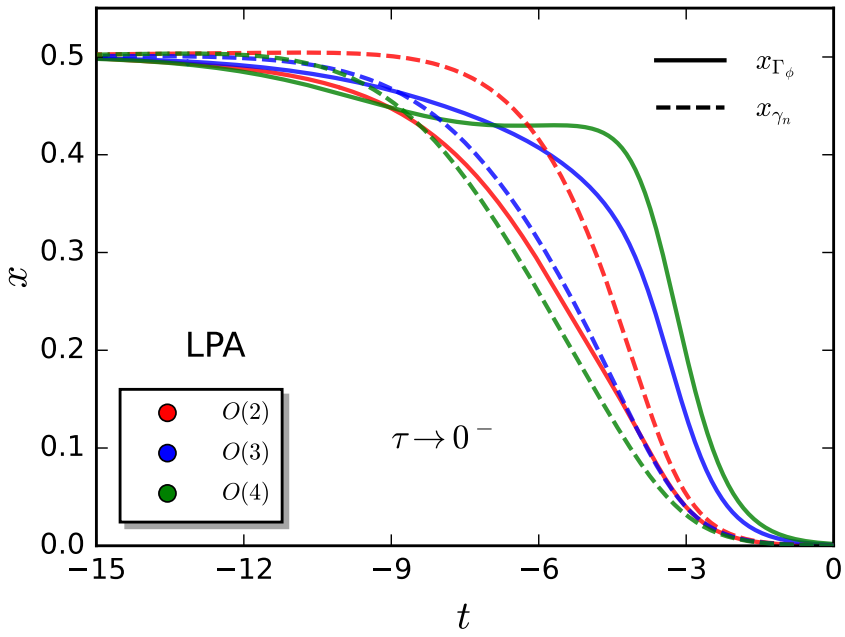


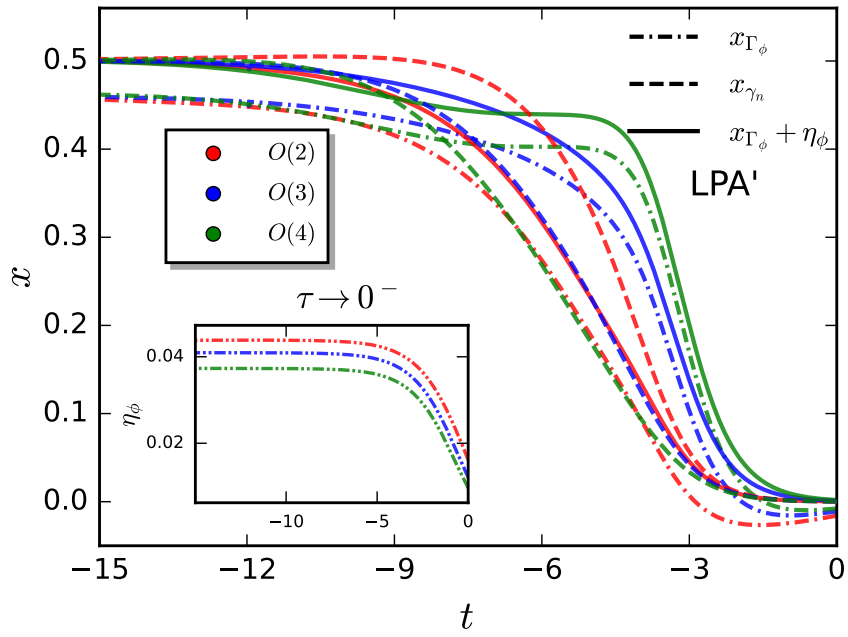
mode-coupling vertices only
Poisson Brackets(PB) vs. all diagrams (full)

- U_k vertices modify the flow of $\Gamma_{\phi,k}$ only at intermediate scales
- $\gamma_{n,k}$ (dashed) is affected only slightly: U_k enters indirectly through $\Gamma_{\phi,k}$ in the propagators
- IR agreement, PB (full):
  $x_{\Gamma_\phi} \simeq 0.4986\ (0.4967)$,
  $x_{\gamma_n} \simeq 0.5027\ (0.5037)$

★ the flows of the kinetic coefficients are dominated by the Poisson brackets(PB)

Broken phase: dynamic anomalous dimensions





Broken phase ($\tau \to 0^-$, $\sigma_0 \neq 0$): LPA (left panel), LPA′ (right panel)

- LPA: both x_{Γ_ϕ} and x_{γ_n} approach $\frac{1}{2}$ in the IR
- LPA′: $\eta_\phi \neq 0$ (inset) $\Rightarrow$ x_{Γ_ϕ} is reduced below $\frac{1}{2}$, while $x_{\gamma_n} \to \frac{1}{2}$ as before
- strong scaling still holds: $x_{\Gamma_\phi} + \eta_\phi \to \frac{1}{2}$ although $x_{\Gamma_\phi} \neq x_{\gamma_n}$ (nontrivial check of strong scaling)

Summary of the dynamic anomalous dimensions

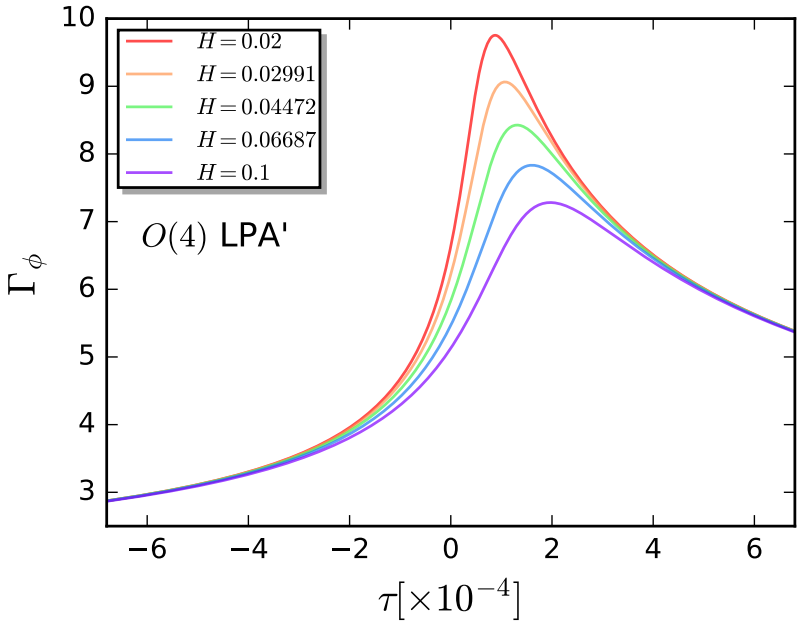
Infrared values ($t = -15$) of the dynamic anomalous dimensions

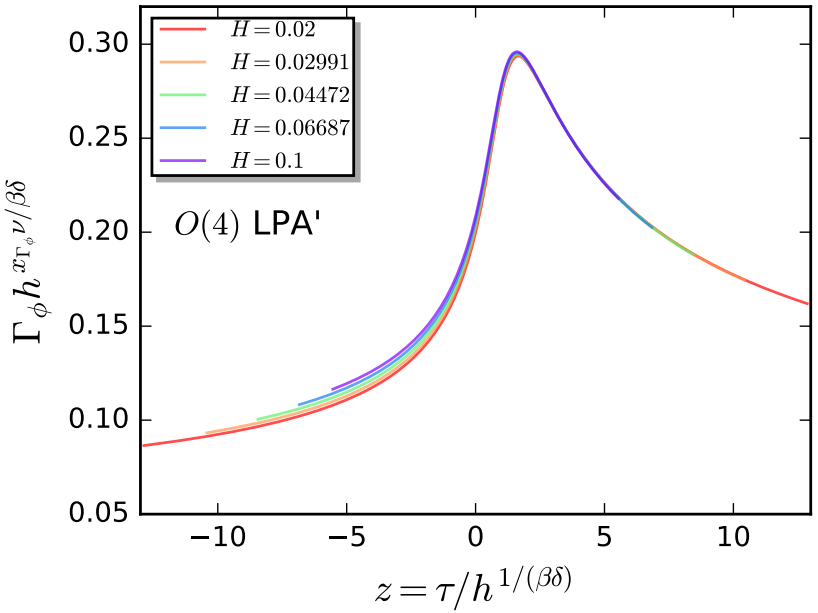
		$O(2)$	$O(3)$	$O(4)$
symmetric	x_{Γ_ϕ}	0.5009	0.5028	0.5014
	x_{γ_n}	0.4996	0.4961	0.4952
broken, LPA	x_{Γ_ϕ}	0.4986	0.4986	0.4981
	x_{γ_n}	0.5027	0.5010	0.5024
broken, LPA′	η_ϕ	0.0438	0.0409	0.0373
	x_{Γ_ϕ}	0.4563	0.4592	0.4618
	$x_{\Gamma_\phi} + \eta_\phi$	0.5001	0.5001	0.4991
	x_{γ_n}	0.5022	0.4999	0.5013

★ $z_\phi = z_n = d/2$ in both phases and both truncations

Scaling function of the kinetic coefficient Γ_ϕ

$$\Gamma_\phi = h^{-x_{\Gamma_\phi}\nu/(\beta\delta)} f_\Gamma(z)$$

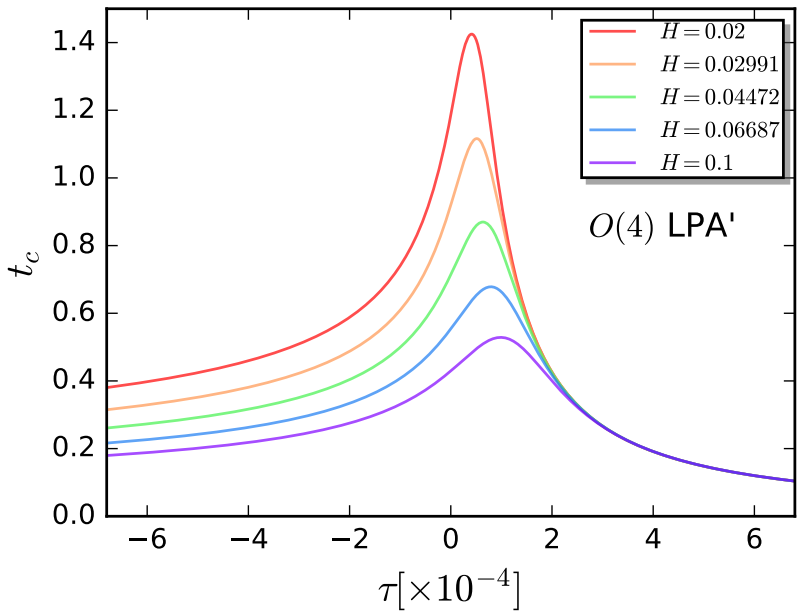


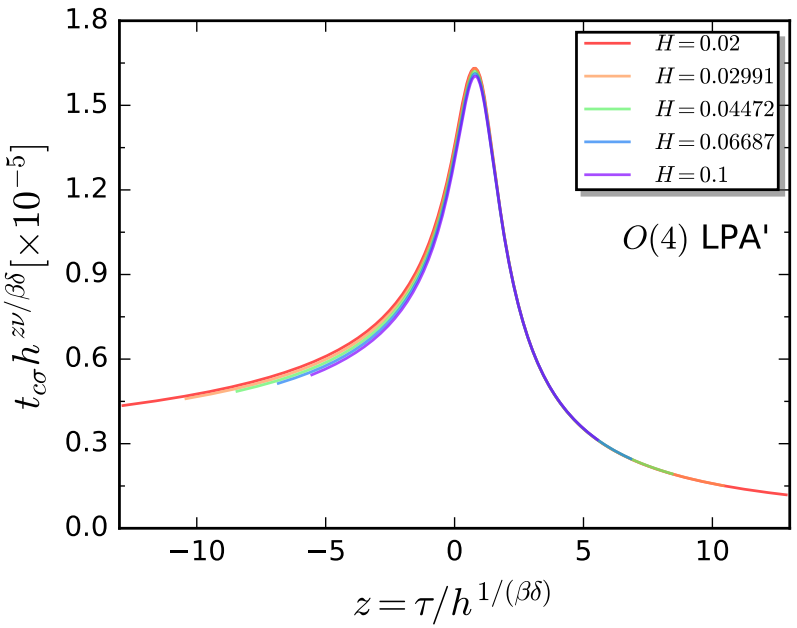


- Γ_ϕ exhibits a pronounced maximum at $\tau > 0$ close to T_c
- With decreasing c, the maximum becomes higher and moves towards $\tau = 0$
- right: all c collapse onto a single curve $\Rightarrow$ scaling function $f_\Gamma(z)$

Scaling function of the relaxation time

$$t_c = 1/(\Gamma_\phi Z_\phi m_\sigma^2)\,, \qquad t_c = h^{-z\nu/(\beta\delta)} f_t(z)$$





- relaxation time of the σ mode: diverges in the chiral limit (critical slowing down)
- collapse with the strong-scaling value $\zeta = 3/2$; Model A: $\zeta \simeq 2.02 \Rightarrow$ slowing down weaker in Model G

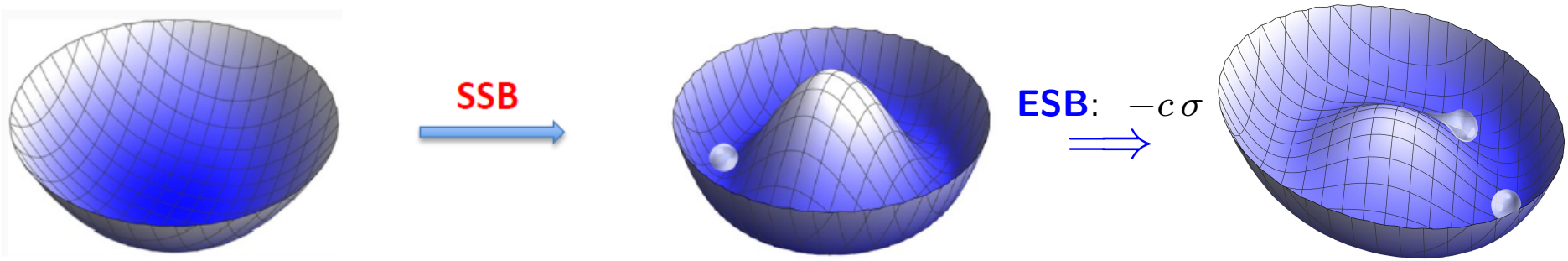
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From Goldstone to pseudo-Goldstone modes



$O(N) \to O(N-1)$: massless Goldstones π_i gapped: $m_\pi \neq 0$

$O(N)$ model with explicit breaking

- exact symmetry: SSB $\Rightarrow$ $N-1$ massless Goldstones π_i
- ESB: Goldstones acquire a small mass gap $\Rightarrow$ pseudo-Goldstone modes
- GMOR relation: $m_\pi^2 f_\pi^2 = m_q \bar{\sigma}$

Why pseudo-Goldstones matter

- lightest modes below T_c $\Rightarrow$ they control the large-distance, long-time dynamics: transport, soft-pion spectra
- same structure in many systems: pions (chirality), pinned CDWs (translations) $\Rightarrow$ universality

Gell-Mann, Oakes, Renner, PR 175, 2195 (1968); Burgess, Phys. Rept. 330, 193 (2000); Amoretti *et al.*, PRL 123, 211602 (2019)

pseudo-Goldstone damping in Model A

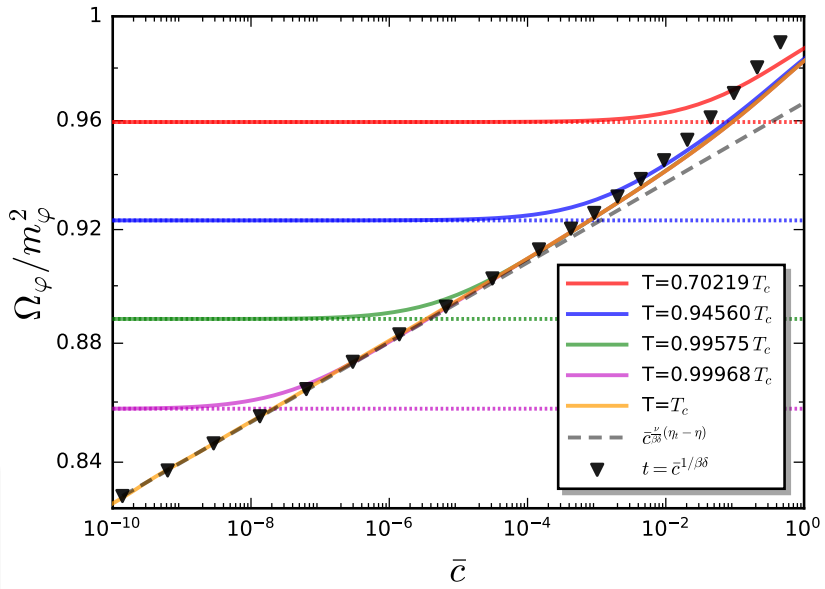
Dispersion relation of the pions
(pseudo-Goldstone modes π) below T_c

$$\omega(\boldsymbol{p}) \simeq \pm v_s \sqrt{m_\pi^2 + \boldsymbol{p}^2} - \frac{i}{2}\left[\Omega_\pi + (D_\pi + D_n)\boldsymbol{p}^2\right]$$

v_s: pion velocity, m_π: screening mass,
Ω_π: damping rate, D_π, D_n: diffusivities

Damping rate of pion

$$\Omega_\pi / m_\pi^2 \simeq D_\pi + \mathcal{O}\left(m_\pi^2/T^2\right)$$



Tan, YRC, Fu, Li, Nat. Commun. 16, 2916 (2025)

- Model A (no conserved charges): $\omega(q) = -i\, D_\pi\left(m_\pi^2 + q^2\right)$ $\Rightarrow$ only damped
- $T < T_c$: $\Omega_\pi/m_\pi^2 \to D_\pi(T)$ for $\bar{c} \to 0$ (dotted lines)
- $T = T_c$: $\Omega_\pi/m_\pi^2 \propto m_\pi^{\Delta_\eta}$, $\Delta_\eta = -x_{\Gamma_\phi} - \eta_\phi$ $\Rightarrow$ controlled by critical fluctuations

Questions: (i) does $\Omega_\pi \simeq D_\pi m_\pi^2$ survive at T_c? (ii) role of the conserved charges?

Son, Stephanov, PRL 88, 202302 (2002); Grossi *et al.*, PRD 102, 014042 (2020); Delacrétaz *et al.*, PRL 128, 141601 (2022)

pseudo-Goldstone modes in Model G

Linearize the Langevin equations around σ_0

$$\partial_t \pi_i = -\Gamma_\phi Z_\phi \big(-\nabla^2 + m_\pi^2 \big) \pi_i + \frac{g\sigma_0}{\chi_n} n_{0i} + \theta_i$$

$$\partial_t n_{0i} = \frac{\gamma_n}{\chi_n} \nabla^2 n_{0i} - g\sigma_0 Z_\phi \big(-\nabla^2 + m_\pi^2 \big) \pi_i + \nabla \cdot \boldsymbol{\zeta}_{0i}$$

- pions mix with the axial charges n_{0i}; σ and vector charges n_{ij} decouple at linear order
- $\boldsymbol{p} = 0$: $\partial_t n_{0i} = -g\, c\, \pi_i \;\rightarrow$ axial charges only approximately conserved (PCAC)

Dispersion relation

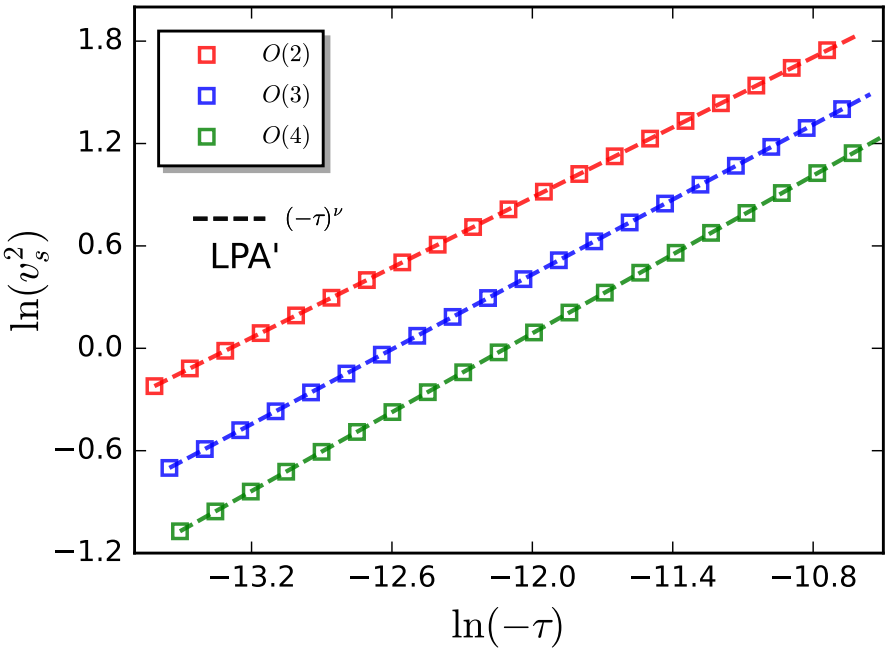
$$\omega_\pm(\boldsymbol{p}) = -\tfrac{i}{2}\big[D_\pi(\boldsymbol{p}^2 + m_\pi^2) + D_n \boldsymbol{p}^2\big] \pm \tfrac{1}{2}\sqrt{4v_s^2(\boldsymbol{p}^2 + m_\pi^2) - \big[D_\pi(\boldsymbol{p}^2 + m_\pi^2) - D_n \boldsymbol{p}^2\big]^2}$$

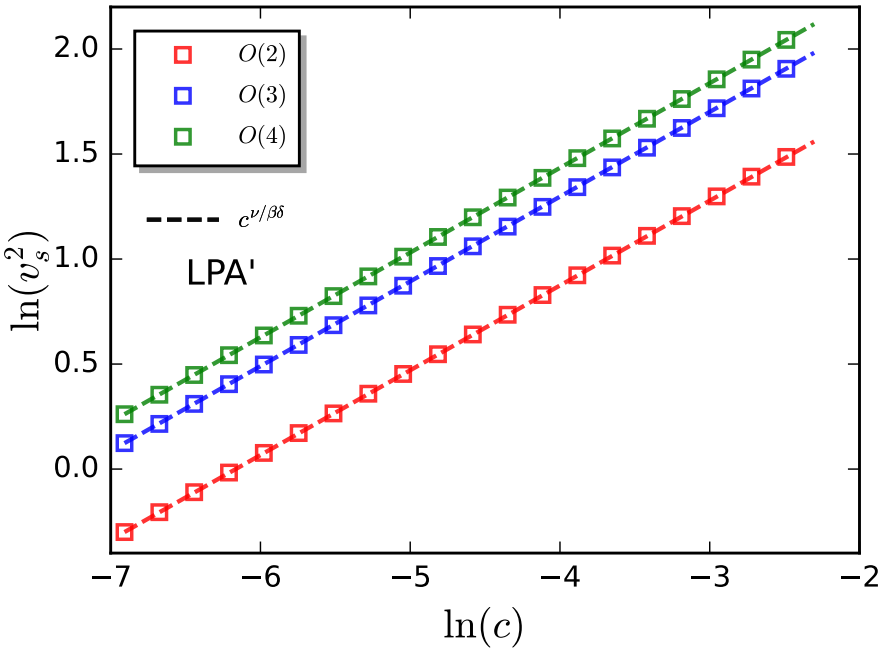
$$D_\pi = \Gamma_\phi Z_\phi, \quad D_n = \gamma_n/\chi_n, \quad v_s^2 = g^2 Z_\phi \sigma_0^2/\chi_n = g^2 \rho_s/\chi_n, \quad m_\pi^2 = c/(\sigma_0 Z_\phi)$$

- the mixing leads to a propagation and damping mode of pion (v_s: propagation, $\Omega_\pi = D_\pi m_\pi^2$: damping)

Critical behavior of the pion velocity

$$v_s^2 = g^2 \rho_s / \chi, \quad \text{stiffness } \rho_s = Z_\phi \sigma_0^2$$



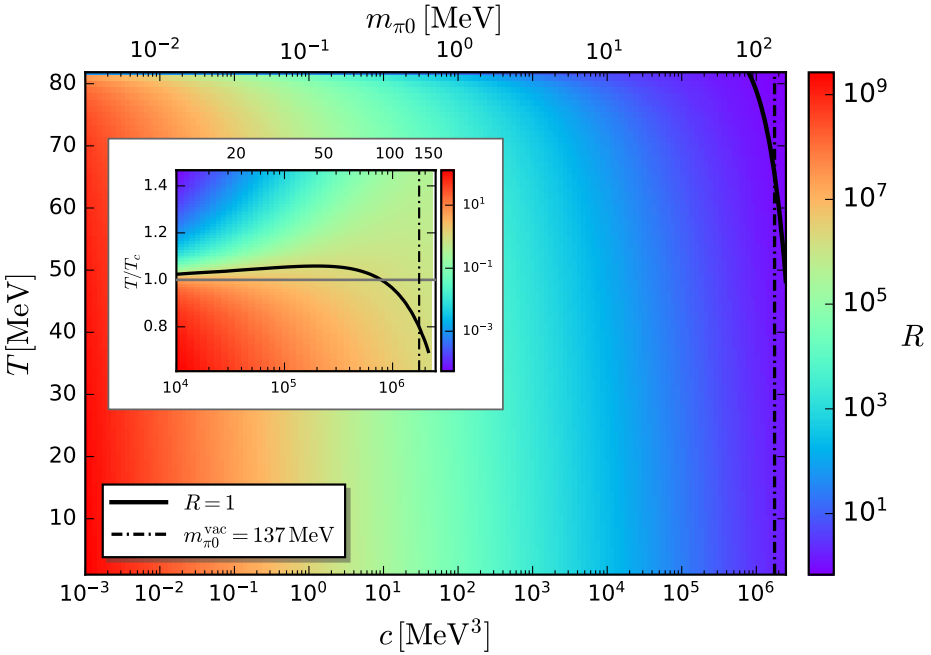


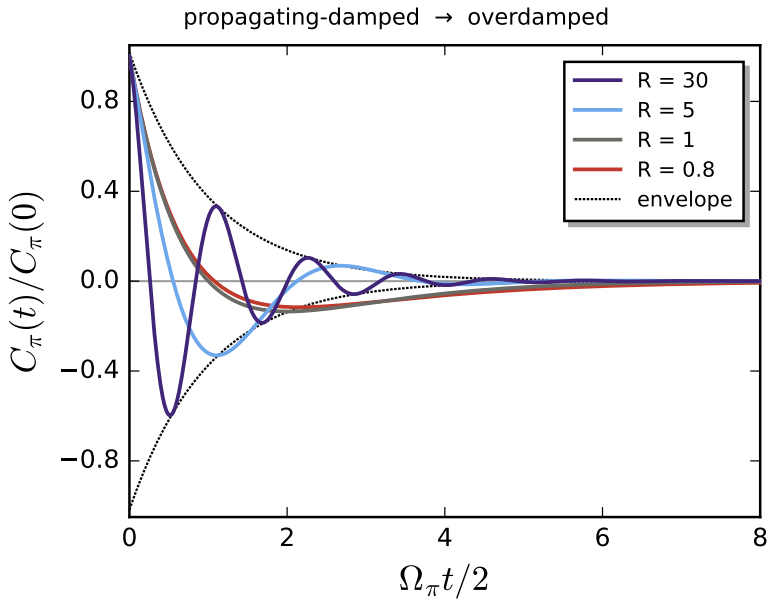
- Josephson's relation: $\rho_s \propto \xi^{-(d-2)}$
- left — chiral limit ($c = 0$): $v_s^2 \propto (-\tau)^\nu$; right — $T = T_c$, finite c: $v_s^2 \propto c^{\nu/(\beta\delta)}$

$v_s^2 \propto \rho_s/\chi \to 0$ at T_c: the pion velocity vanishes at the critical point, in agreement with Son & Stephanov $(v_s^2 = f_s^2/f_t^2,\ f_s^2 = \rho_s,\ f_t^2 = \chi)$

Son, Stephanov, PRL 88, 202302 (2002); PRD 66, 076011 (2002)

Propagating vs. over-damped pions





Pion dispersion relation at $\boldsymbol{p}=0$

$$R=\frac{4v_s^2}{D_\pi^2 m_\pi^2}$$

$$\omega_\pm(\boldsymbol{p}=0)=-\frac{i}{2}\Omega_\pi\pm\frac{1}{2}\Omega_\pi\left(R-1\right)^{1/2}$$

- $R\gg 1$ in almost the whole phase: propagating, weakly damped pions
- at $T\approx T_c$: propagating $\to$ overdamped $(R<1)$ with increasing c (upper right)

1 Introduction

2 Model G within the real-time fRG approach

3 Numerical results

4 Summary and Outlook

Summary and Outlook

- Model G within the real-time fRG: symmetric *and* broken phase, explicit breaking c
- Statics unaffected by the mode couplings, scaling functions $f_G(z)$, $f_Z(z)$. Scaling functions of the kinetic coefficients Γ_ϕ, γ_n and of the relaxation time
- Strong scaling $x_{\Gamma_\phi} + \eta_\phi = x_{\gamma_n} = 1/2 \Rightarrow z = d/2$ in both phases and both truncations
- Pseudo-Goldstone modes: overdamped in Model A $\rightarrow$ propagating and damping in Model G

Outlook

* momentum and frequency dependence of the kinetic coefficients and spectral functions
* assess the size of the dynamic critical regime
* link to the low-p_T pion enhancement in heavy-ion collisions

Model G
$z = d/2$

Thanks for your attention!

Backup

Retarded two-point functions

Retarded two-point correlation functions in space of $(\omega, \boldsymbol{p})$

$$\Gamma^{(2)}_{\tilde{\phi}_a \phi_b, k}(\omega, \boldsymbol{p}) = \Big(i\omega - \Gamma_{\phi,k}\big[Z_{\phi,k}\boldsymbol{p}^2 + U_k'(\rho)\big]\Big)\delta_{ab} - \Gamma_{\phi,k} U_k''(\rho)\,\phi_a\phi_b$$

$$\Gamma^{(2)}_{\tilde{\phi}_a n_{ef}, k}(\omega, \boldsymbol{p}) = g\, Z_{n,k}\big(\delta_{af}\phi_e - \delta_{ae}\phi_f\big)$$

$$\Gamma^{(2)}_{\tilde{n}_{bc} \phi_d, k}(\omega, \boldsymbol{p}) = g\big(Z_{\phi,k}\boldsymbol{p}^2 + U_k'(\rho)\big)\big(\delta_{bd}\phi_c - \delta_{cd}\phi_b\big)$$

$$\Gamma^{(2)}_{\tilde{n}_{bc} n_{ef}, k}(\omega, \boldsymbol{p}) = \big(i\omega - Z_{n,k}\gamma_{n,k}\boldsymbol{p}^2\big)\big(\delta_{be}\delta_{cf} - \delta_{bf}\delta_{ce}\big)$$

- retarded propagator $G^R = \big(\Gamma^{(2)}_{\tilde{\psi}\psi}\big)^{-1}$, $\psi = (\phi_a, n_{ab})$
- diagonal entries: relaxation of ϕ_a $(\Gamma_{\phi,k})$ and diffusion of n_{ab} $(D_n = \gamma_{n,k}\boldsymbol{p}^2)$
- off-diagonal entries $\propto$ Poisson brackets: π_i–n_{0i} mixing only in the broken phase

Three-point vertices

Feynman rules for the three-point vertices

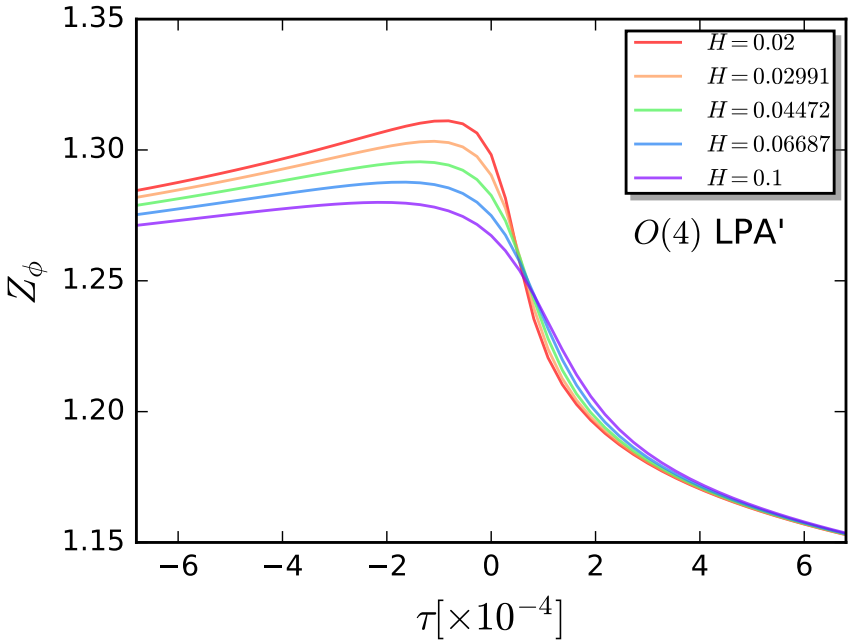
$$\Gamma^{(3)}_{\tilde{\phi}_a \phi_b n_{cd},k}(p,q,r) = g\, Z_{n,k} \left(\delta_{ad}\delta_{bc} - \delta_{ac}\delta_{bd}\right)$$
$$\Gamma^{(3)}_{\tilde{n}_{ab} \phi_c \phi_d,k}(p,q,r) = g\, Z_{\phi,k} \left(\boldsymbol{r}^2 - \boldsymbol{q}^2\right)\left(\delta_{ad}\delta_{bc} - \delta_{ac}\delta_{bd}\right)$$
$$\Gamma^{(3)}_{\tilde{\phi}_a \phi_b \phi_c,k}(p,q,r) = -\Gamma_{\phi,k}\left[U_k''(\rho)\left(\delta_{ab}\phi_c + \delta_{bc}\phi_a + \delta_{ca}\phi_b\right) + U_k^{(3)}(\rho)\,\phi_a\phi_b\phi_c\right]$$

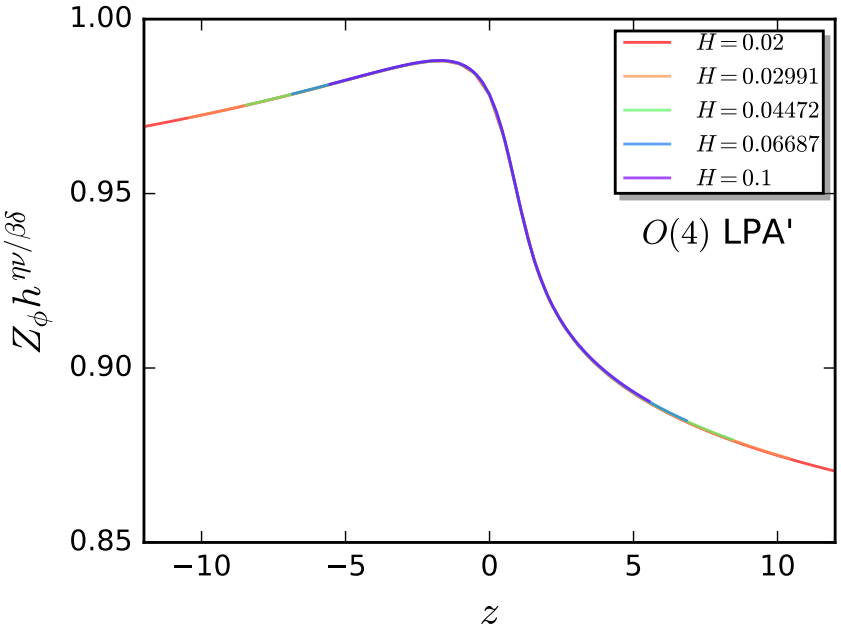
- two vertices from mode couplings, one from the potential U_k

Scaling function of Z_ϕ

Wave function renormalization (LPA′)

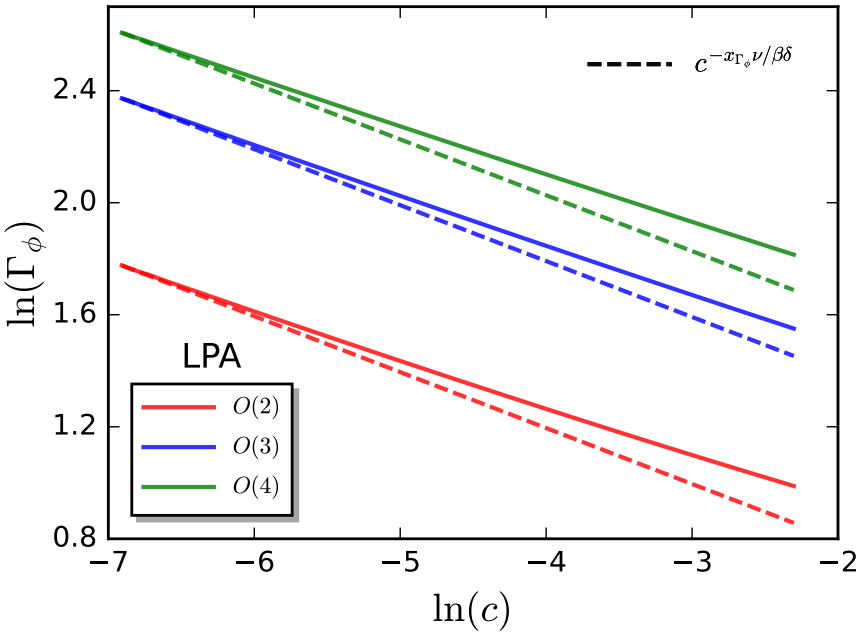
$$Z_\phi = h^{-\eta\nu/(\beta\delta)} f_Z(z)$$

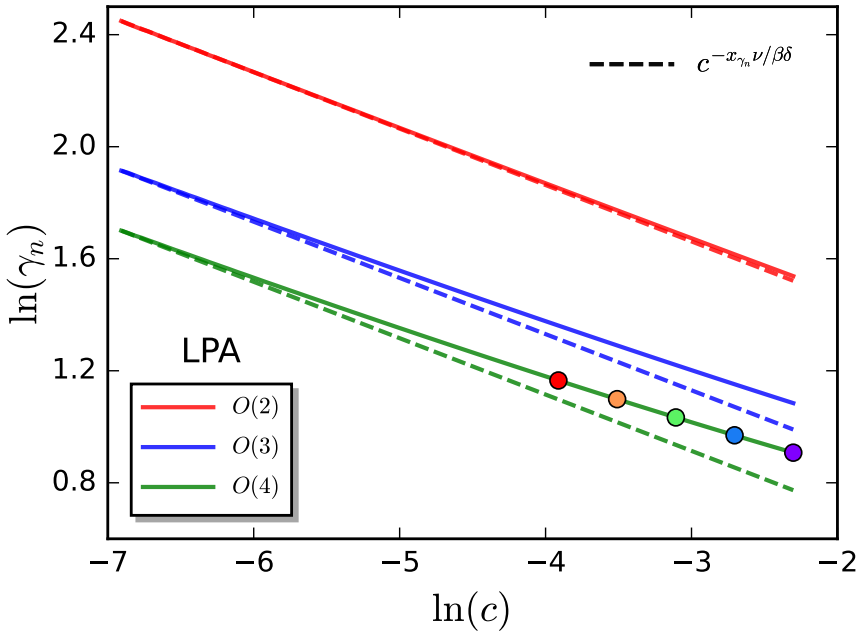




- critical behavior: $Z_\phi \propto |\tau|^{-\eta\nu}$ (c=0), $Z_\phi \propto c^{-\eta\nu/(\beta\delta)}$ at $T = T_c$
- a maximum slightly below T_c, followed by a rapid decrease across the crossover region.
- with decreasing c, the maximum becomes higher and moves towards $\tau = 0$

Kinetic coefficients at T_c: critical behavior in c

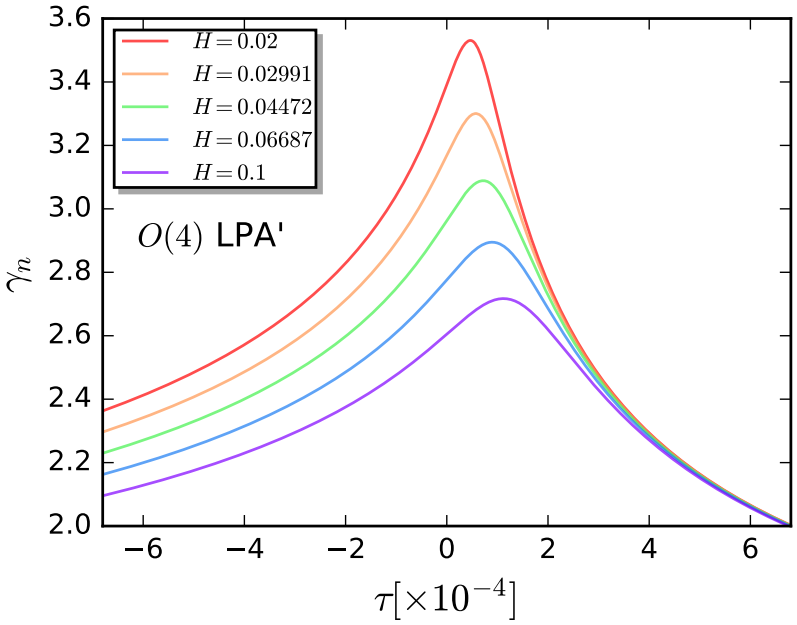


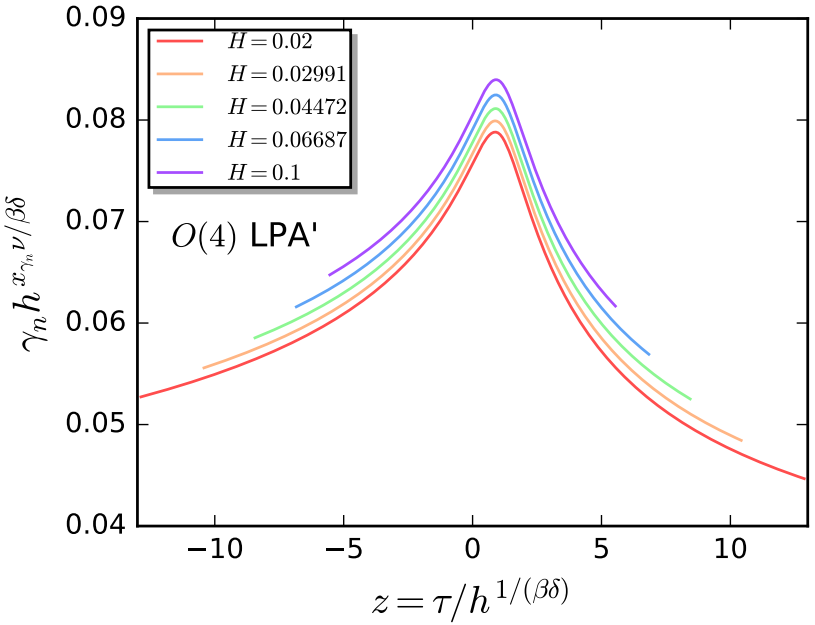


- at T_c $(z = 0)$: $\Gamma_\phi \propto c^{-x_{\Gamma_\phi}\nu/(\beta\delta)}$, $\gamma_n \propto c^{-x_{\gamma_n}\nu/(\beta\delta)}$ (slopes $\simeq 0.20$)
- dashed: asymptotic power laws; bending at larger $c \Rightarrow$ corrections to scaling
- dots: the five c values of the scaling plots $\Rightarrow$ incomplete collapse of $f_\gamma(z)$

Scaling function of the kinetic coefficient γ_n

$$\gamma_n = h^{-x_{\gamma_n}\nu/(\beta\delta)} f_\gamma(z)$$





- γ_n also shows a peak close to T_c
- curves for different c merge only in the symmetric phase; in the broken phase γ_n depends on c for all τ
- after rescaling: same shape, but no complete collapse