# New insights on freeze out in heavy-ion collisions via functional QCD

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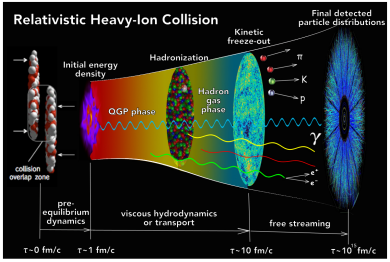


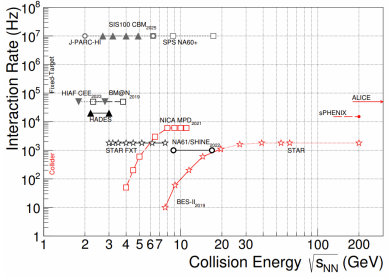

fqcd-collaboration.github.io

Experimental research on QCD phase diagram at high baryon density is entering a new era - higher precision, luminosity and statistics: CBM (FAIR), CEE+ (HIAF), ...

Measurements are done after **freeze out**, leading to many key questions, e.g.:

- how does the freeze-out state indicates the property of strong interaction matter?
- importantly, if there is a critical end point, would it be reflected in the observables at freeze out?




(Plot: Nucl.Phys.A 982 (2019) 163-169)

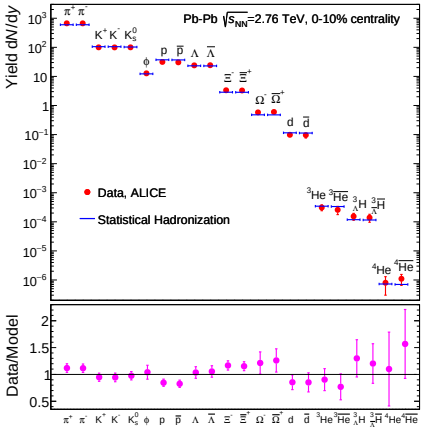
# Freeze out: where we take measurements in HIC

- Particles yields & ratios between different (hadron) species are stabilized - “no more in-elastic collisions”: **chemical** freeze out (focus of this talk).
- Particle (momentum) spectra are stabilized - “no more collisions”: **kinetic**.

Hydrodynamic simulation requires explicitly the definition of freeze-out hypersurface for extracting observables.

Pheno.: thermal statistical models are remarkably successful for explaining the chemical freeze out.
see e.g. [Andronic et al. 1710.09425]

This talk:
• equilibrium QCD analysis at high baryon density (**functional**).



# QCD phase structure and thermodynamics

YL, F. Gao, Y.-x. Liu, J. M. Pawlowski: Phys. Rev. D 113, 054019

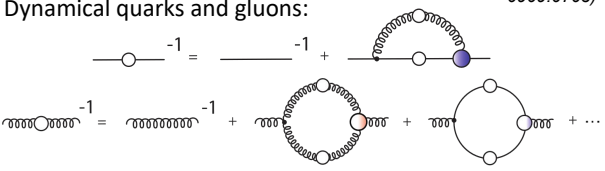
# Continuum functional QCD

Recent reviews: *Fischer and Pawlowski, 2606.03703, 2603.11135*

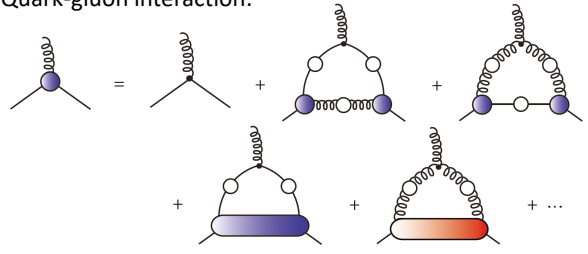
## Dyson Schwinger equations [DSE]

*(Plots: Eichmann, 0909.0703)*

Dynamical quarks and gluons:

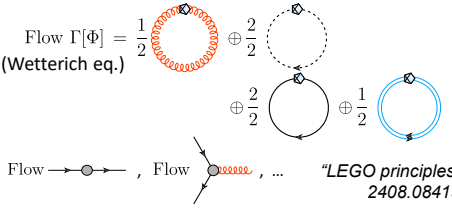


Quark-gluon interaction:



*Alkofer & von Smekal, 0007355  Roberts & Schmidt, 0005064*
*Eichmann at al, 1606.09602  Fischer, 1810.12938  …*

## functional renormalization group [fRG]

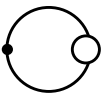



*"LEGO principles" 2408.08413*

*Pawlowski, 0512261  Gies, 0611146*
*Dupuis at al., 2006.04853  Fu, 2205.00468  …*

$\langle \mathbf{vac.}\,|\bar{q}q|\mathbf{vac.}\rangle$ vs $\sum_{\psi \in \text{asymptotic states:}\ \pi, K, p, n, \ldots} \langle \text{vac.}\,|\psi\rangle\langle\psi|\text{vac.}\rangle$

$$\langle \bar{q}(x)\Gamma q(x)\rangle = -\mathrm{Tr}\,\langle \Gamma q\bar{q}\rangle = -\Gamma$$



Capturing chiral dynamics.

The DSE point of view:

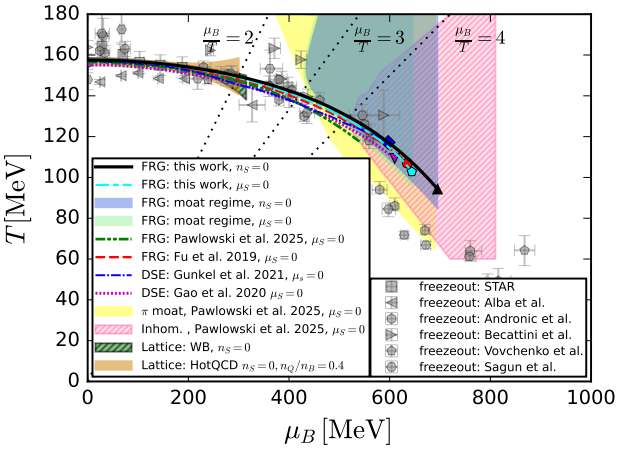
- Feedbacks of dynamical quarks & gluons [Fischer 1405.4762] [Gao 2102.13053], and resonances [Gunkel 2106.08356]
- Symmetries in the quark-gluon interaction:

$$\Gamma_\mu = \lambda_1 \gamma_\mu + \lambda_4 \sigma_{\mu\nu} k^\nu + ...$$

$\lambda_1, \lambda_4, ...$ driven by Slavnov-Taylor identities. [Gao & Pawlowski 2010.13705] [**YL** et al. 2310.18383]

Further indication on CEP and new phases: moats, inhomogeneity ...

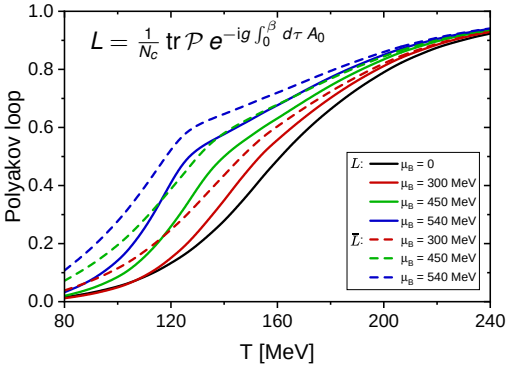
Plot taken from: [2606.03703]

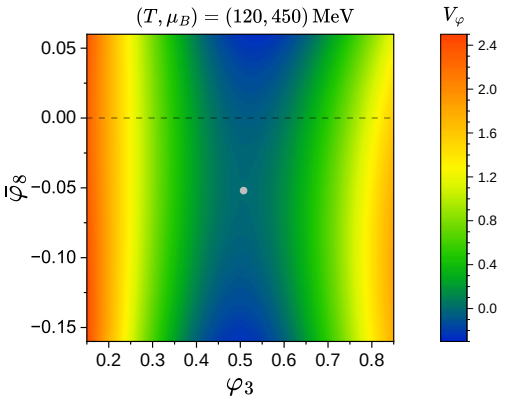


Dynamics in the gauge background ($A_0$): [YL 2504.05099] [Fischer 1306.6022] [Fister 1301.4163]

$$\frac{\delta(\Gamma - S)}{\delta A_0} = \frac{1}{2} - - - \frac{1}{6} +$$

$$A_0 = 2\pi \frac{T}{g}\left(\varphi_3 \frac{\lambda_3}{2} + \varphi_8 \frac{\lambda_8}{2}\right).$$

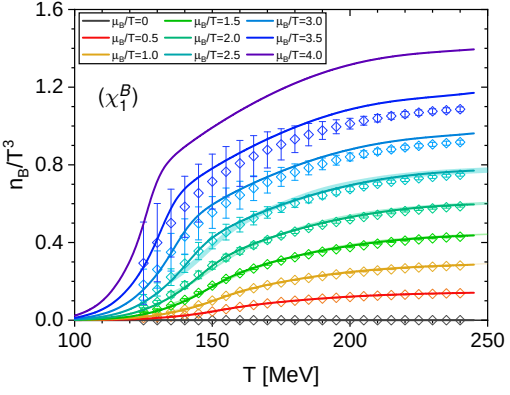


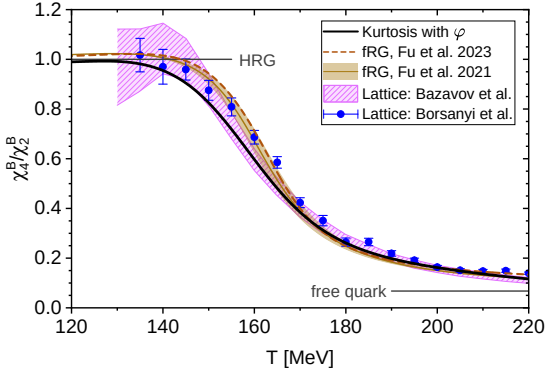



Chiral & confining dynamics "explain" the finite-density sector: [YL et al. 2504.05099]

$$\chi_k^B = \frac{\partial^k (P/T^4)}{\partial(\mu_B/T)^k} \propto \partial_{\mu_B}^{(k-1)} n_B = -\frac{1}{3} \sum_{q=u,d,s} \mathrm{Tr}\,[\gamma_0\, \partial_{\mu_B}^{(k-1)} \boldsymbol{G_{qq}}]\,;$$

$$G_{qq}(p) = \mathrm{i}\gamma_0(\omega_n + \mathrm{i}\mu + g\boldsymbol{A_0})\, \boldsymbol{Z_q^E}(p) + \mathrm{i}\vec{\gamma}\cdot\vec{p}\, \boldsymbol{Z_q^M}(p) + \boldsymbol{M_q}(p).$$



# Linking freeze out with the thermodynamics

YL, C. Fischer, F. Gao, Y.-x. Liu, J. M. Pawlowski: 2603.09336

To distinguish $T_f$ and $\mu_{B,f}$ at different $\sqrt{s_{\rm NN}}$ in HIC, we consider:

$$\chi_1^B/\chi_2^B \ \& \ \chi_3^B/\chi_1^B, \quad \text{and map them to experimental data:}$$

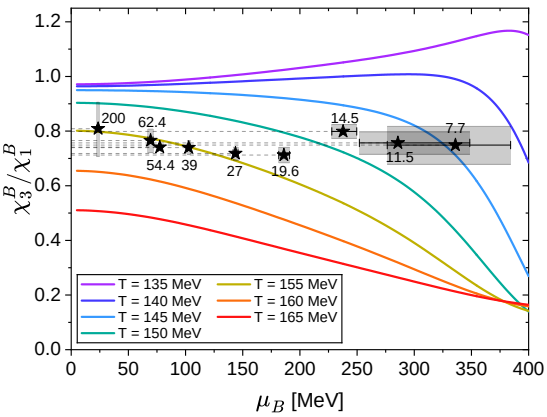
**(I)** At small $\mu_B$:

$$\chi_3^B/\chi_1^B(T,\mu_B) = \chi_4^B/\chi_2^B(T,0)+O(\mu_B^2);$$

The temperature dependence of kurtosis at vanishing $\mu_B$ shows a transition from hadronic DOFs (= 1) to quark-gluon DOFs ($\approx 0.1$):

thermometer ($T$ detector) !

see [Bazavov et al. 1208.1220] [Borsanyi et al. 1305.5161] [Fu and Pawlowski 1508.06504] [Fu et al. 2308.15508]



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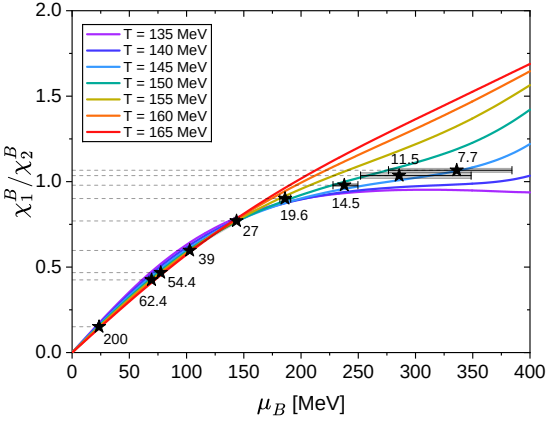
**(II)** $\chi_1^B/\chi_2^B$ is insensitive within a wide range of temperature ($T_c \pm 15\,\text{MeV}$):

$\mu_B$ detector !

A Taylor-expansion point of view:

$\chi_1^B/\chi_2^B(T,\mu_B) = \mu_B/T + O(\mu_B^3)$.

The $\mu_B$ slope $\approx T_c^{-1} \approx (155\,\text{MeV})^{-1}$ around chiral crossover.

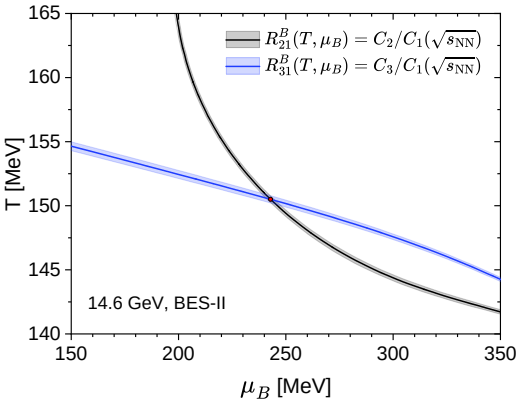


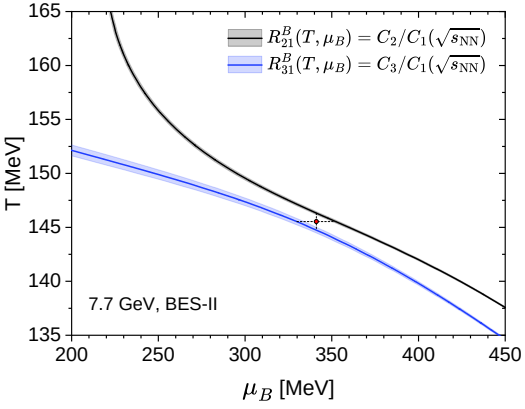
# Systematic approach to extract freeze out

- More complicated at large $\mu_B$: both $T$ and $\mu_B$ aspects play a role: **trajectory analysis**.

The intersection systematically measures $(T_f, \mu_{B,f})$: mean & error bars at different $\sqrt{s_{\rm NN}}$ !

- At low $\sqrt{s_{\rm NN}}$ ($\approx 7.7$ GeV), there could be no intersection point between equilibrium QCD and experiment... indication of new dynamics?

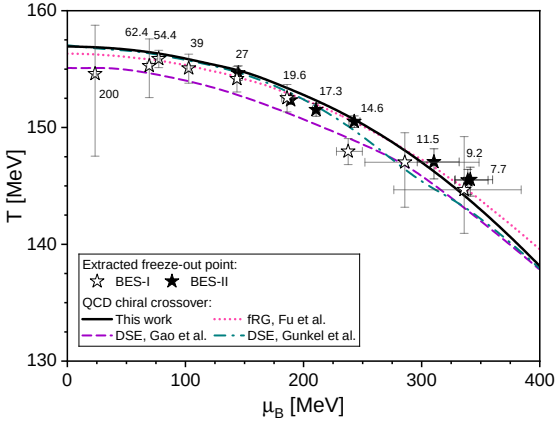


# Freeze out versus QCD phase structure

Strong overlap between freeze out and chiral crossover (up to 7.7 GeV).

For chiral crossover line: $T_c(0) \approx 155\,\text{MeV}$, $\kappa_2^\chi \approx 0.015$. [Fischer & Pawlowski, 2603.11135]




Parameterising a freeze out line:

$$T_f = T_0[1-\kappa_2^f(\mu_{B,f}/T_0)^2-\kappa_4^f(\mu_{B,f}/T_0)^4+\cdots].$$

- Direct fit over the data points implies:

$$T_0 \approx T_c(0) \quad \text{and} \quad \kappa_2^f \lesssim \kappa_2^\chi;$$

- Freeze out happens after a stabilized hadronization (hence after crossover) $\rightarrow$

$$\kappa_2^f \geq \kappa_2^\chi;$$

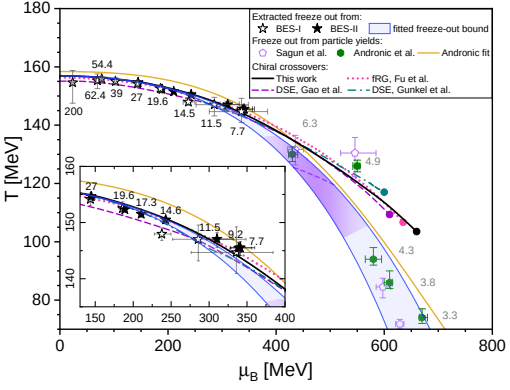
- Data + physics constraints implies:

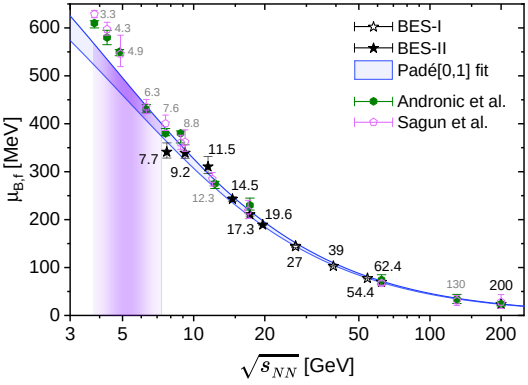
$\kappa_2^f = \kappa_2^\chi$, i.e. overlap with crossover.

# Freeze out in theoretical and experimental landscape

Extrapolation of freeze out towards low energy: $T_f(\mu_{B,f})$ & $\mu_{B,f} = a/(1 + b\sqrt{s_{\rm NN}})$.

- ○ Main uncertainties of $T_f(\mu_{B,f})$ is in $\kappa_4^f$: fixed by the closest data point (SPS 6.3 GeV).
- • Agrees with the thermal statistical model freeze-out data for relatively large $\sqrt{s_{\rm NN}}$;
- • Apparent discrepancy is shown near and below 5 GeV.



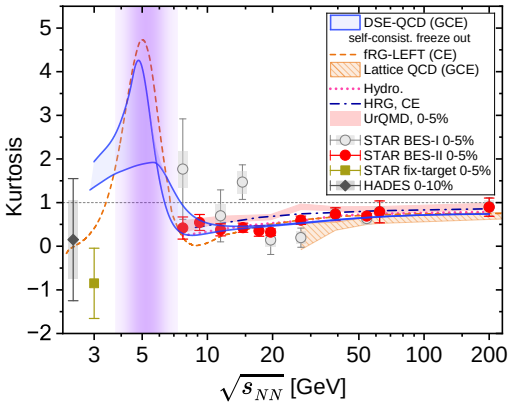



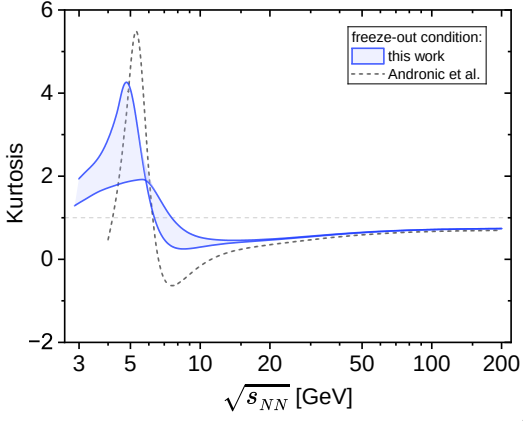
# Applications: kurtosis along freeze out

- Kurtosis peak height is sensitive on the freeze out condition; while the peak position is relatively stable (estimated by the purple region).

→ Different aspects point to similar collision energy (5 GeV) for potentially new physics.

- The prediction of kurtosis would strongly depend on the freeze out condition.



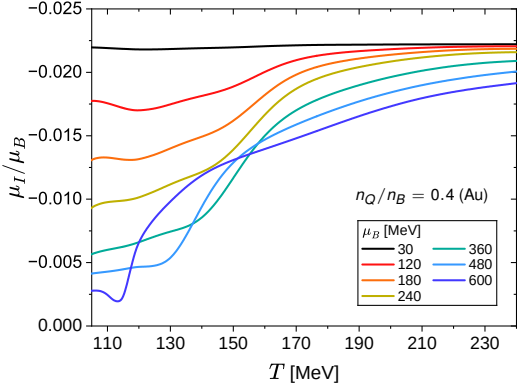



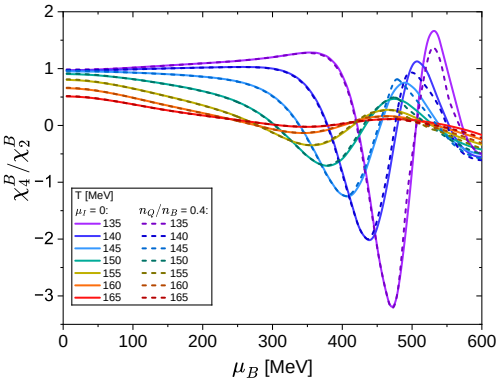
The charge number (initial proton vs. neutron) of collided nuclei is known:

$$n_Q/n_B = Z/A, \qquad \text{e.g. Au+Au: } Z/A \approx 0.4. \qquad \text{also: } n_S = 0$$

Symmetric case ($n_u = n_d$): $n_Q/n_B \equiv 0.5 \rightarrow$ always a finite isospin asymmetry in HIC.

- For Au, we found $|\mu_I/\mu_B| \sim 2/9\,|Z/A - 0.5| \sim 2\%$ ; $(\mu_u = \mu_B/3 + \mu_I, \quad \mu_d = \mu_B/3 - \mu_I)$
- Kurtosis does not change qualitatively, while the peak height may be affected sizably.



- Systematic determination of freeze out: trajectory analysis on $\chi_1/\chi_2$ & $\chi_3/\chi_1$.
- Freeze out coincides precisely with QCD chiral crossover for $\sqrt{s_{\rm NN}} \geq 7.7\,\text{GeV}$.
- New dynamics at $\sqrt{s_{\rm NN}} \lesssim 5\,\text{GeV}$ is indicated from multiple perspectives.
- Baryon number kurtosis baseline in equilibrium QCD is obtained:
  - an important step towards finalizing theoretical predictions for experiment.

Further steps to go:

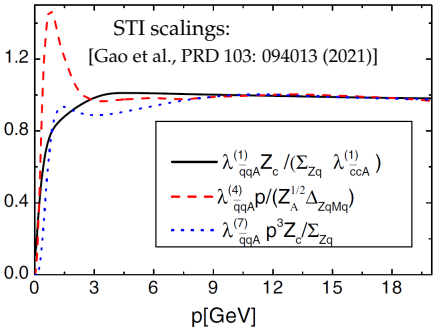
- Strangeness neutrality + charge-to-mass ratio constraint ($n_Q/n_B$).
- Theoretical access to distinguish proton and neutron cumulants.
- Non-equilibrium dynamics, canonical corrections and other realistic effects.
- ...

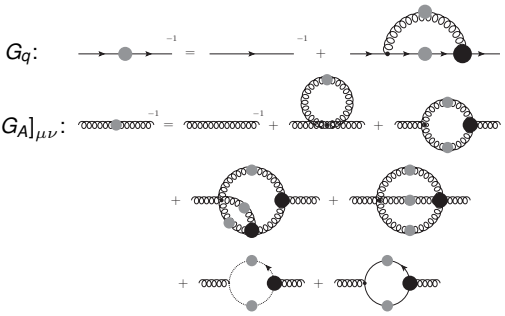
**Thanks for your attention!**

fqcd-collaboration.github.io

# Back-up

# Symmetries in the quark-gluon vertex




$$[\Gamma_{q\bar{q}A}]_\mu = \lambda^{(1)}_{q\bar{q}A}\,\gamma_\mu + \lambda^{(4)}_{q\bar{q}A}\,\sigma_{\mu\nu}k^\nu + \lambda^{(7)}_{q\bar{q}A}\,\frac{\mathrm{i}}{3}\Big\{\sigma_{\alpha\beta}\gamma_\mu + \sigma_{\beta\mu}\gamma_\alpha + \sigma_{\mu\alpha}\gamma_\beta\Big\} l^\alpha k^\beta + \ldots$$

$$G_q^{-1}(p) = Z_q(p^2)\left[\mathrm{i}\gamma\cdot p + M_q(p^2)\right],$$

$$[G_A]_{\mu\nu}(p) = [Z_A(p^2)p^2]^{-1}.$$

Physics in the Pauli term: [Cyrol et al. PRD (2018)], [Qin et al. PLB (2013)] (anomalous chromo-magnetic moment)

## Polyakov loop potential via DSE

In one-loop truncation, the $A_0$ DSE can be explicitly formulated with the propagators:

$$V(\varphi) = V_{\text{glue}}(\varphi) + V_q(\varphi) = \frac{1}{2} V_{\text{Weiss}}(\varphi) + V_a(\varphi) + V_c(\varphi) + V_q(\varphi).$$

Explicitly, in SU($N_c = 2$): $\quad A_0 = \frac{2\pi T}{g}\varphi\lambda_3, \quad \frac{\delta V[A_0]}{\delta A_0} \to V'(\varphi) \equiv \partial_\varphi V(\varphi);$

$$V_a'(\varphi) = \sum\!\!\!\!\!\!\int_k (\omega_k + 2\pi T\varphi)[G_a^E(\vec{k}, \omega_k + 2\pi T\varphi) + 2G_a^M(\vec{k}, \omega_k + 2\pi T\varphi)],$$

$$V_c'(\varphi) = -2\sum\!\!\!\!\!\!\int_k (\omega_k + 2\pi T\varphi) G_c(\vec{k}, \omega_k + 2\pi T\varphi),$$

$$V_q'(\varphi) = -\frac{1}{2}\sum\!\!\!\!\!\!\int_p \text{tr}_{C,D}\left[\lambda^3\gamma_0\, G_q(\vec{p}, \omega_p + \pi T\varphi\lambda^3)\right].$$

For SU($N_c = 3$), one can refer to the eigenvalue decomposition:

adjoint: $\left\{\pm\varphi_3, \pm(\varphi_3 + \sqrt{3}\varphi_8)/2, \pm(\varphi_3 - \sqrt{3}\varphi_8)/2, 0, 0\right\}$, $\quad$ fund.: $\left\{(\pm\varphi_3 + \varphi_8/\sqrt{3})/2, -\varphi_8/\sqrt{3}\right\}$.

[Fischer et al., Phys. Lett. B 732: 273 (2014)] [Fister and Pawlowski, Phys. Rev. D 88: 045010 (2013)]
[Abbott, Nucl. Phys. B 185 (1981) 189-203]

**Quark number density** also enables thermodynamically consistent determination (Maxwell relations) of EoS: pressure, entropy density, energy density ...

$$P(T,\mu_B) = P(T,0) + \int_0^{\mu_B} \boldsymbol{n_B(T,\mu)}\, d\mu; \qquad s(T,\mu_B) = s(T,0) + \int_0^{\mu_B} \frac{\partial \boldsymbol{n_B(T,\mu)}}{\partial T}\, d\mu.$$

EoS at zero chemical potential are well accessible by lattice QCD.

