

Searching for the QCD critical end point through fluctuations of intensive quantities

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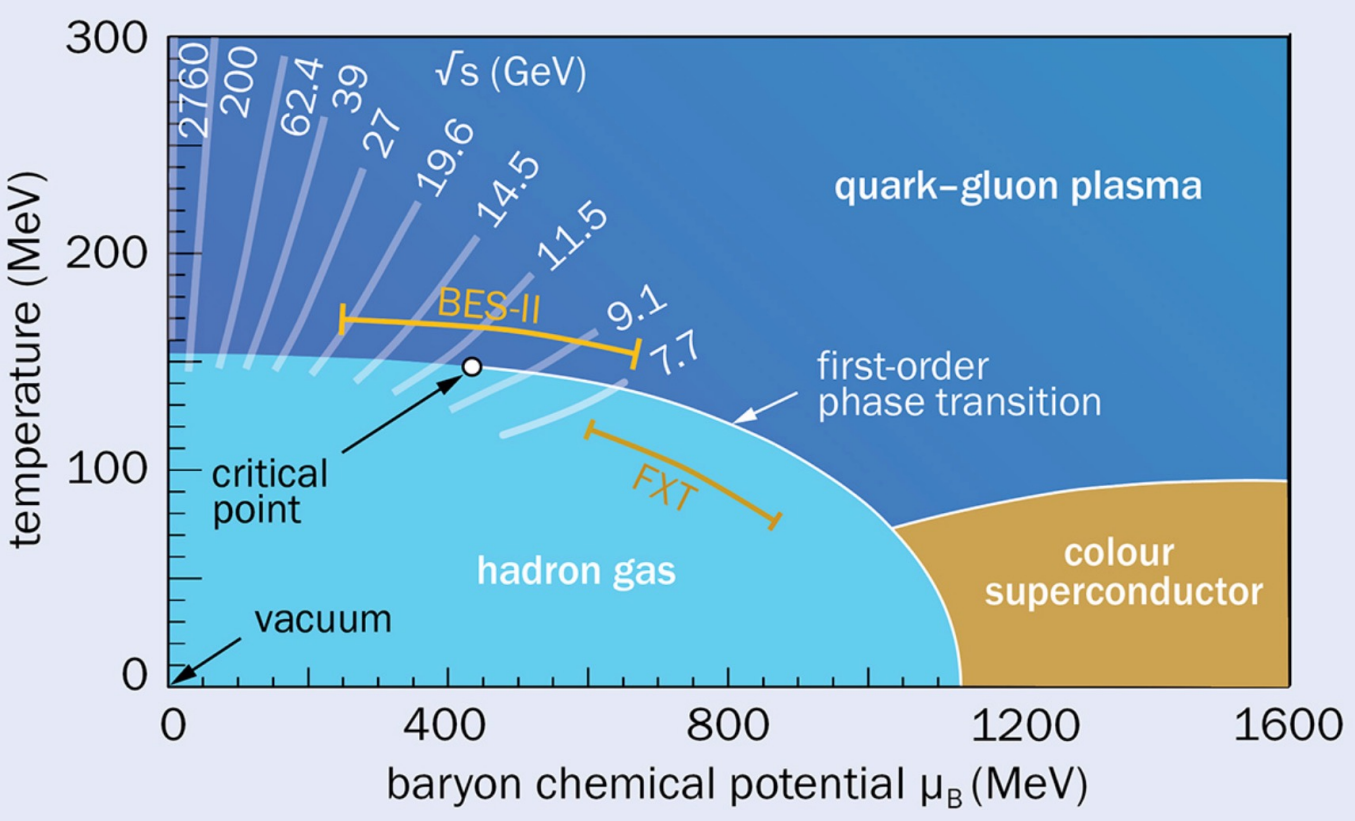
Based on 2609.14000 and work in progress

Workshop on QCD Critical Point and Effective Field Theory methods

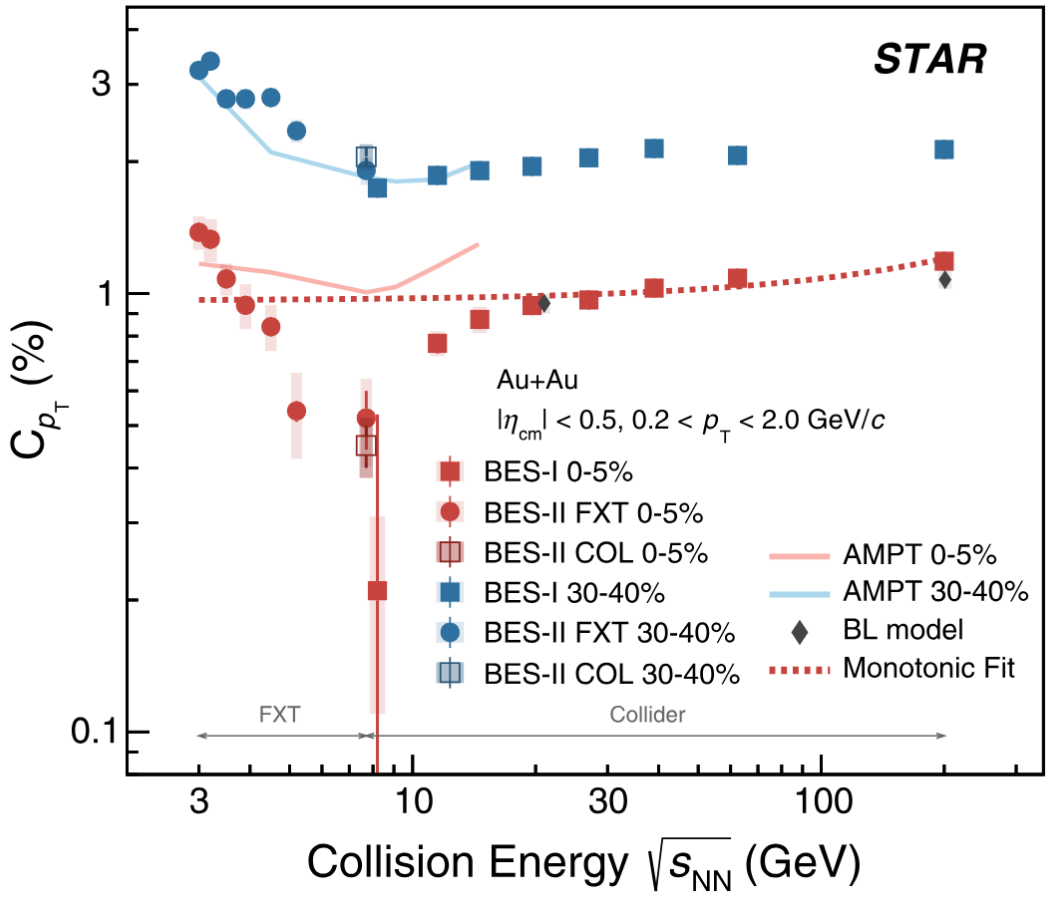
October 21, 2026

With Li Yan and Xu-Guang Huang

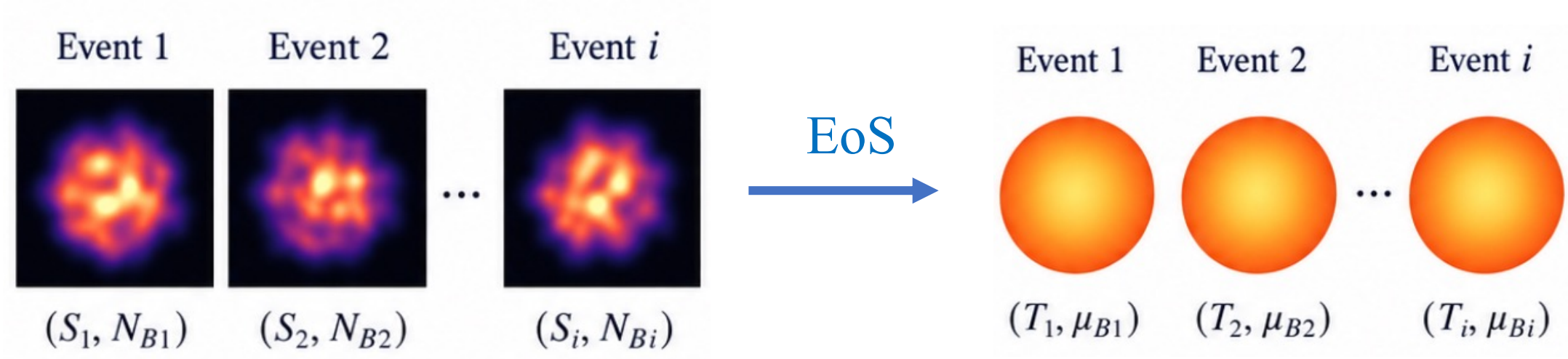
Motivation and QCD phase diagram



[Phys. Rev. Lett. 137, 132301 (2026)]



Fluctuations of intensive quantities: $[p_T]$ ~ T, μ_B



$$\begin{pmatrix} \Delta T \\ \Delta \mu_B \end{pmatrix} = R(T, \mu_B) \begin{pmatrix} \Delta s \\ \Delta n_B \end{pmatrix} \rightarrow \Delta[p_T]$$

EoS determines how density fluctuations are converted into fluctuations of temperature and chemical potential.

Hydrodynamic modeling at finite chemical potential

We carry out event-by-event hydro to test the thermal response hypothesis

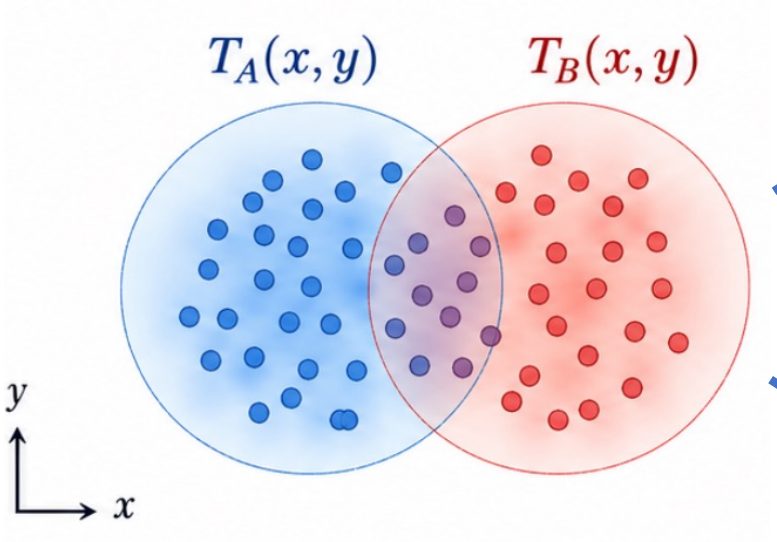
$$\text{hydro modeling} = \text{I.C.} + \underbrace{\text{hydro EoM}}_{\partial_\mu T^{\mu\nu}=0 \;\; \partial_\mu n_B^\mu = 0} + \underbrace{\text{hydro fluct.}}_{\text{not included}} + \text{EoS with CEP} + \text{iSS} + \text{UrQMD}$$

Initial s and nB

Embed with MUSES EoS

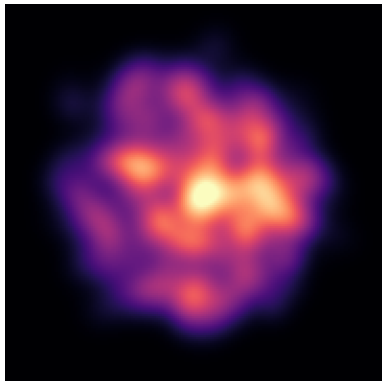
[MUSES collaboration, 2402.0863]

Fluctuating initial conditions

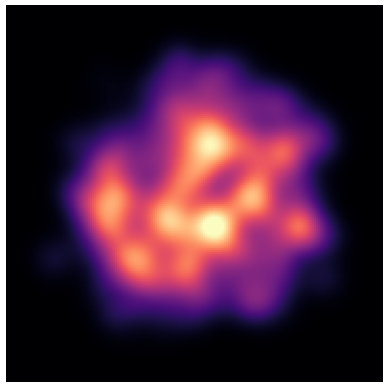


[Phys.Rev.C 92 (2015) 1, 011901]

s (x, y)



n_B(x, y)



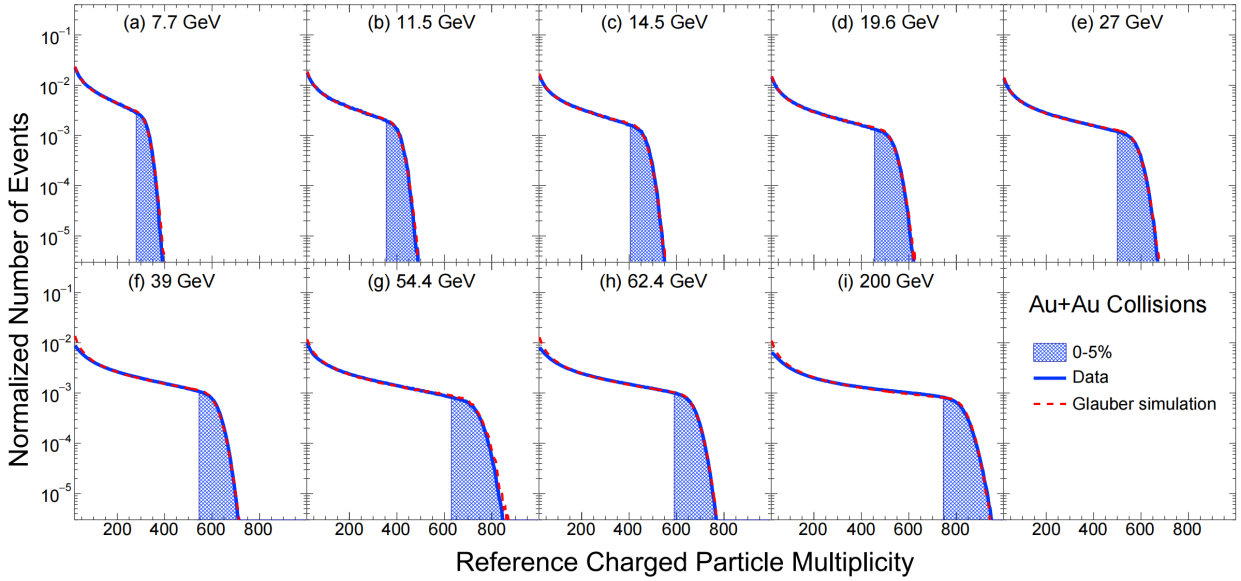
- $s(x,y) \propto \left[\frac{T_A(x,y)^p + T_B(x,y)^p}{2}\right]^{1/p}$,
- $w_i^{(s)} \sim \Gamma(k_s, \frac{1}{k_s})$

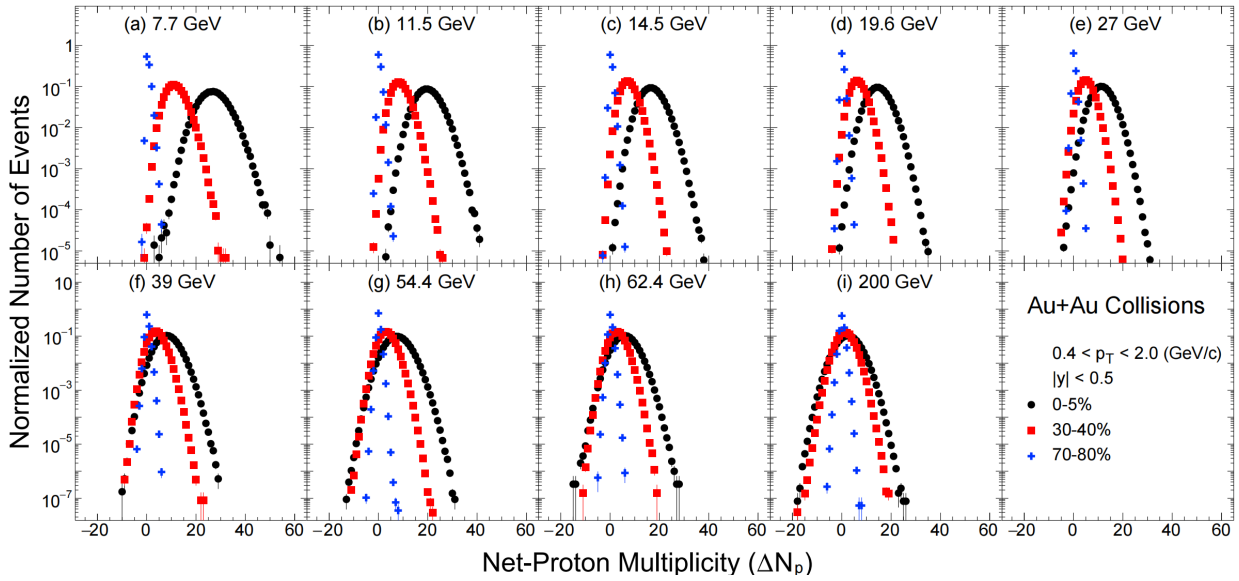
- $n_B(x,y) \propto \left[\frac{T_A(x,y)^p + T_B(x,y)^p}{2}\right]^{1/p}$,
- $w_i^{(B)} \sim \Gamma(k_B, \frac{1}{k_B})$

Fluctuating initial conditions fixed by STAR BES

ΔS $w_i^{(s)} \sim \Gamma(k_s, \frac{1}{k_s})$

$\Delta \mathrm{N}_B$ $w_i^{(B)} \sim \Gamma(k_B, \frac{1}{k_B})$

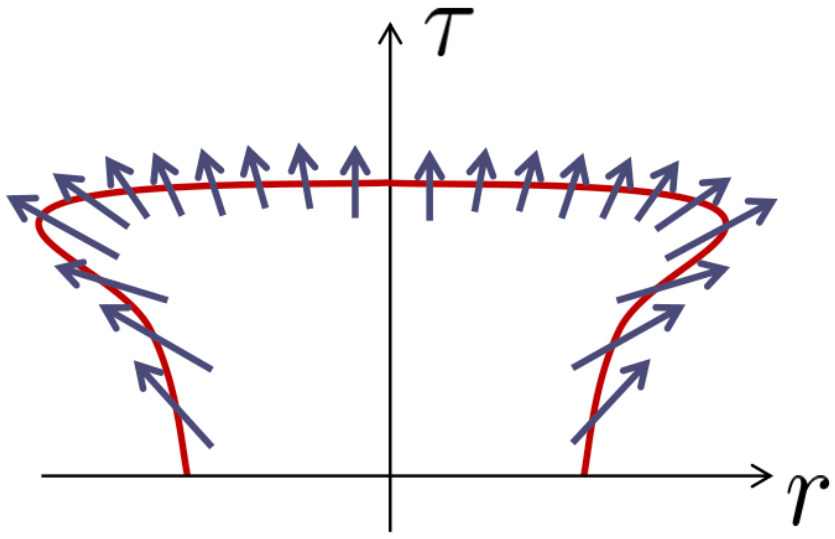




k_s and k_B are tuned to reproduce the measured variance.

Effective thermodynamic quantities from hydrodynamics

- Extract T_{eff} and $\mu_{B\mathrm{eff}}$ from 4D freeze out hypersurface



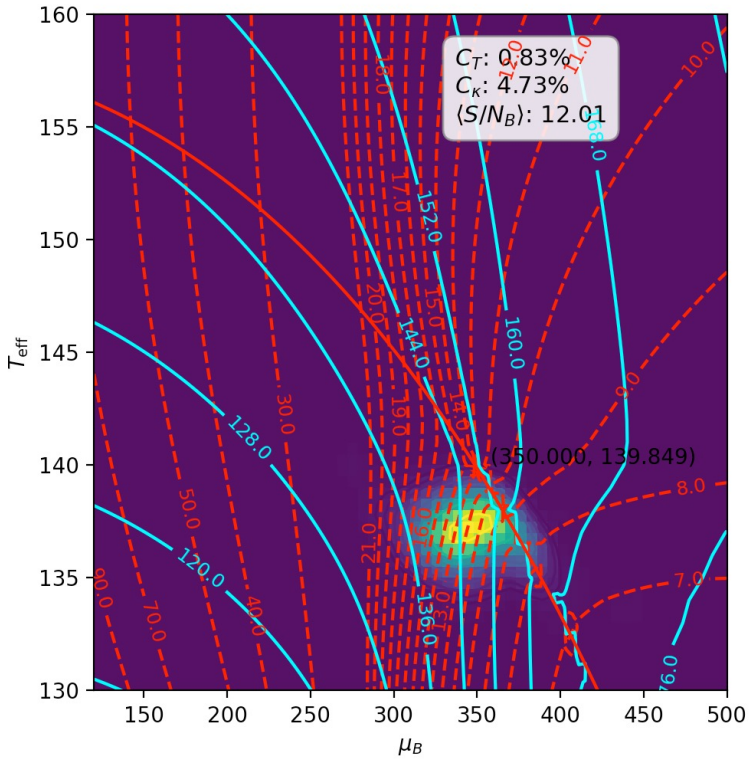
$$E = \int d\sigma_\mu T^{0\mu} = e\left(T_{eff}, \mu_{B,eff}\right)\ V_{eff}$$

$$\mathrm{S} = \int d\sigma_\mu\, s u^\mu = s\left(T_{eff}, \mu_{B,eff}\right)\ V_{eff}$$

$$N_B = \int d\sigma_\mu\, n_B u^\mu = s\left(T_{eff}, \mu_{B,eff}\right)\ V_{eff}$$

- Each event $\rightarrow (T_{\mathrm{eff}},\ \mu_{B\mathrm{eff}})_i$

T and muB from event-by-event in central class



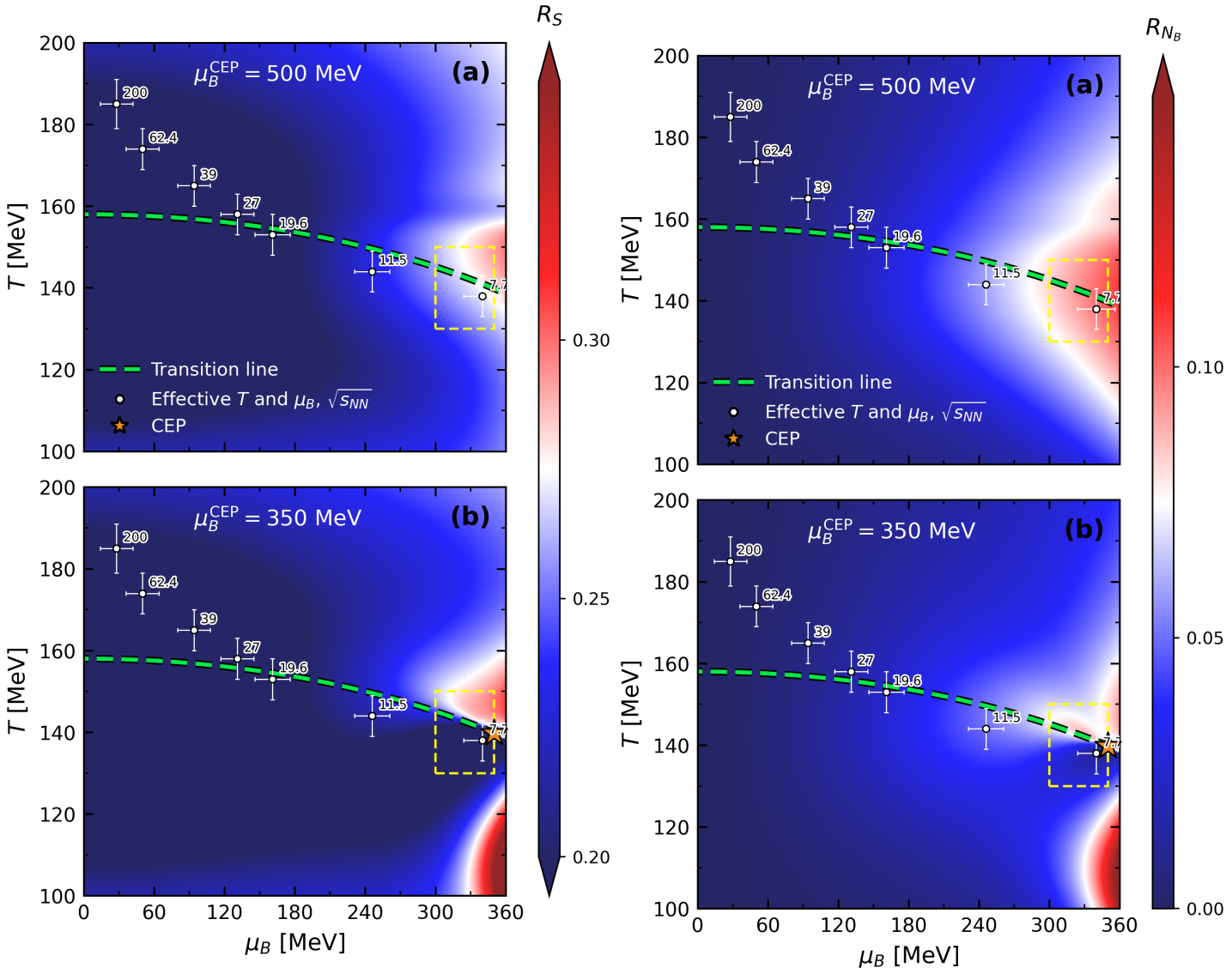
$$\begin{pmatrix} \Delta T \\ \Delta \mu_B \end{pmatrix} = R(T, \mu_B) \begin{pmatrix} \Delta s \\ \Delta n_B \end{pmatrix}$$

- $\frac{\Delta T}{\langle T \rangle} = R_S \frac{\Delta s}{\langle s \rangle} - R_{N_B} \frac{\Delta n_B}{\langle n_B \rangle}$
- $\frac{\Delta \mu_B}{\langle \mu_B \rangle} = -\frac{\langle T \rangle}{\langle \mu_B \rangle} \left[\frac{\chi_{TB}}{\chi_B} R_S \frac{\Delta s}{\langle s \rangle} - \frac{\chi_s}{\chi_{TB}} R_{N_B} \frac{\Delta n_B}{\langle n_B \rangle} \right]$

with $R_S \equiv \frac{\langle s \rangle}{\langle T \rangle} \frac{\chi_B}{\chi_s \chi_B - \chi_{TB}^2}$, $R_{N_B} \equiv \frac{\langle n_B \rangle}{\langle T \rangle} \frac{\chi_{TB}}{\chi_s \chi_B - \chi_{TB}^2}$

Thermal response: ebe variations of T and μ_B are determined by R_S, R_{N_B}, wrt Δs,Δn_B

Sensitivity of thermodynamic response to critical point

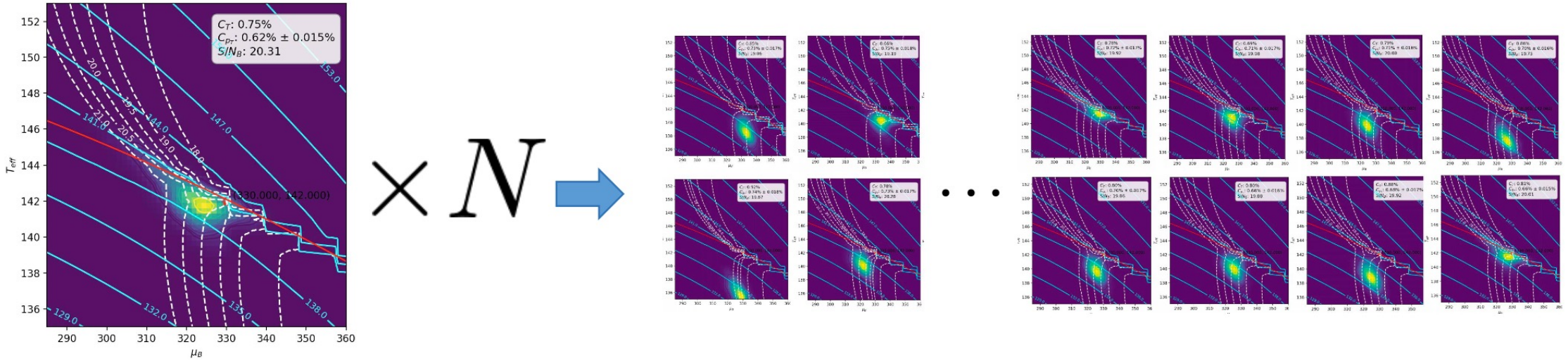


$$R_S \equiv \frac{\langle s \rangle}{\langle T \rangle} \frac{\chi_B}{\chi_S \chi_B - \chi_{TB}^2}$$

$$R_{N_B} \equiv \frac{\langle n_B \rangle}{\langle T \rangle} \frac{\chi_{TB}}{\chi_S \chi_B - \chi_{TB}^2}$$

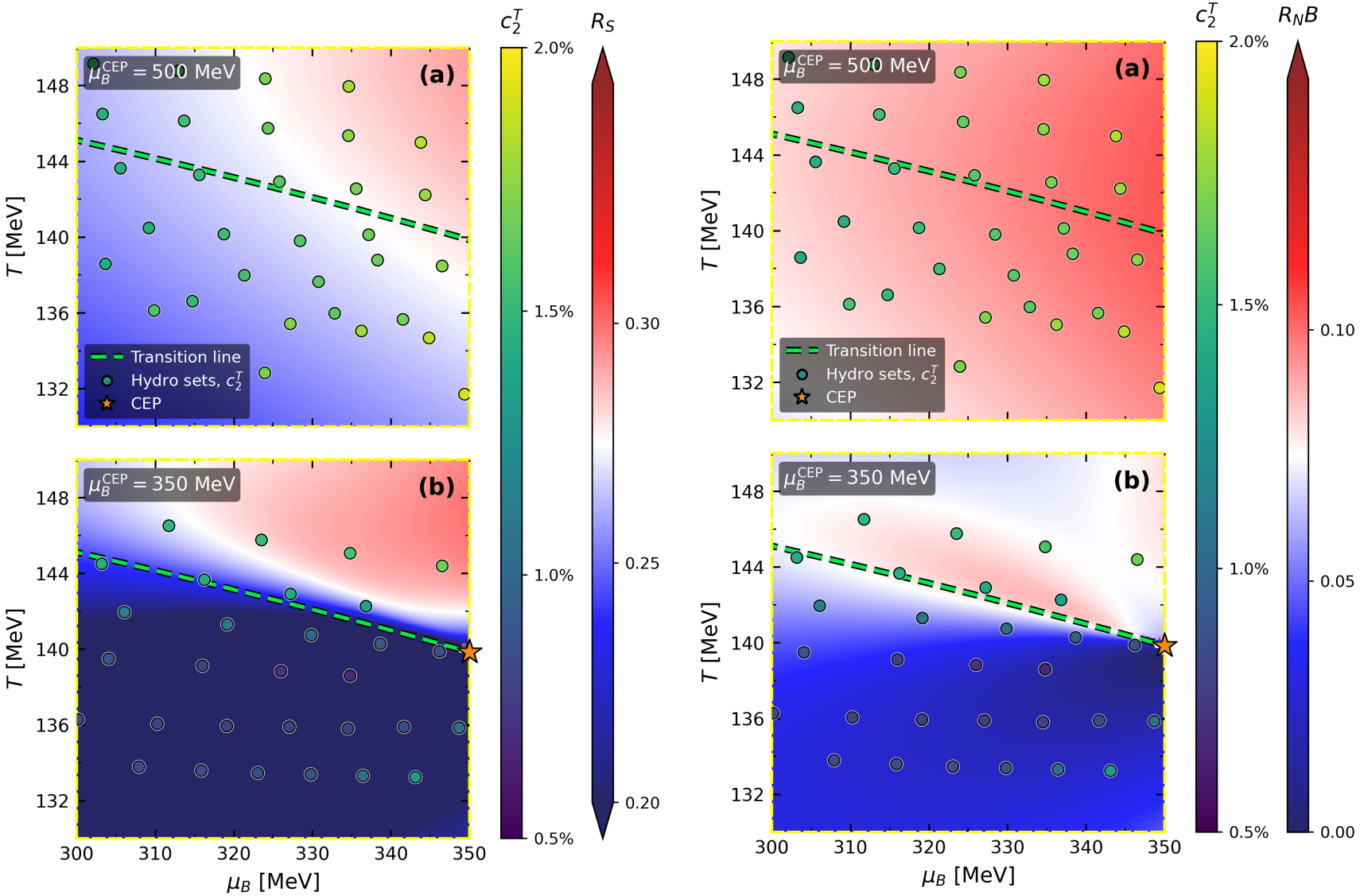
- Sensitive to phase structure, in particular sensitive to CEP
- We assume muBC=350 around 7.7

Scan around 7.7 GeV

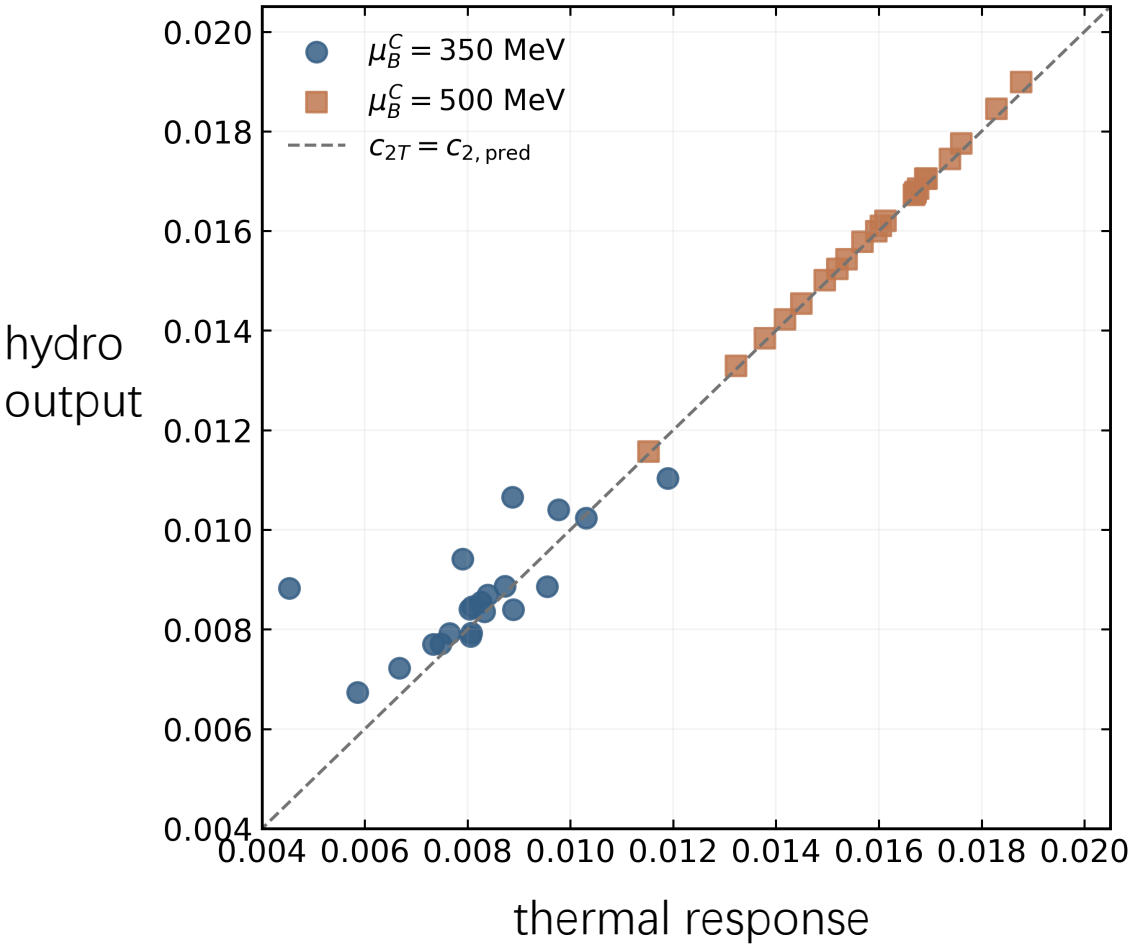


- Au-Au @ 7.7 GeV, 0-5 %, with fixed initial S and N_B fluctuations.
- Scan roughly 30 sets each for μ_{BC} = 350 MeV and μ_{BC} = 500 MeV.
- Each set with 1k events, wrt one set of mean value of T and μ_B .
- We define and calculate $c_T \equiv \frac{\sqrt{(\Delta T)^2}}{\langle T \rangle}$, $c_{pT} \equiv \frac{\sqrt{(\Delta [p_T])^2}}{\langle [p_T] \rangle}$

Suppression of temperature fluctuations



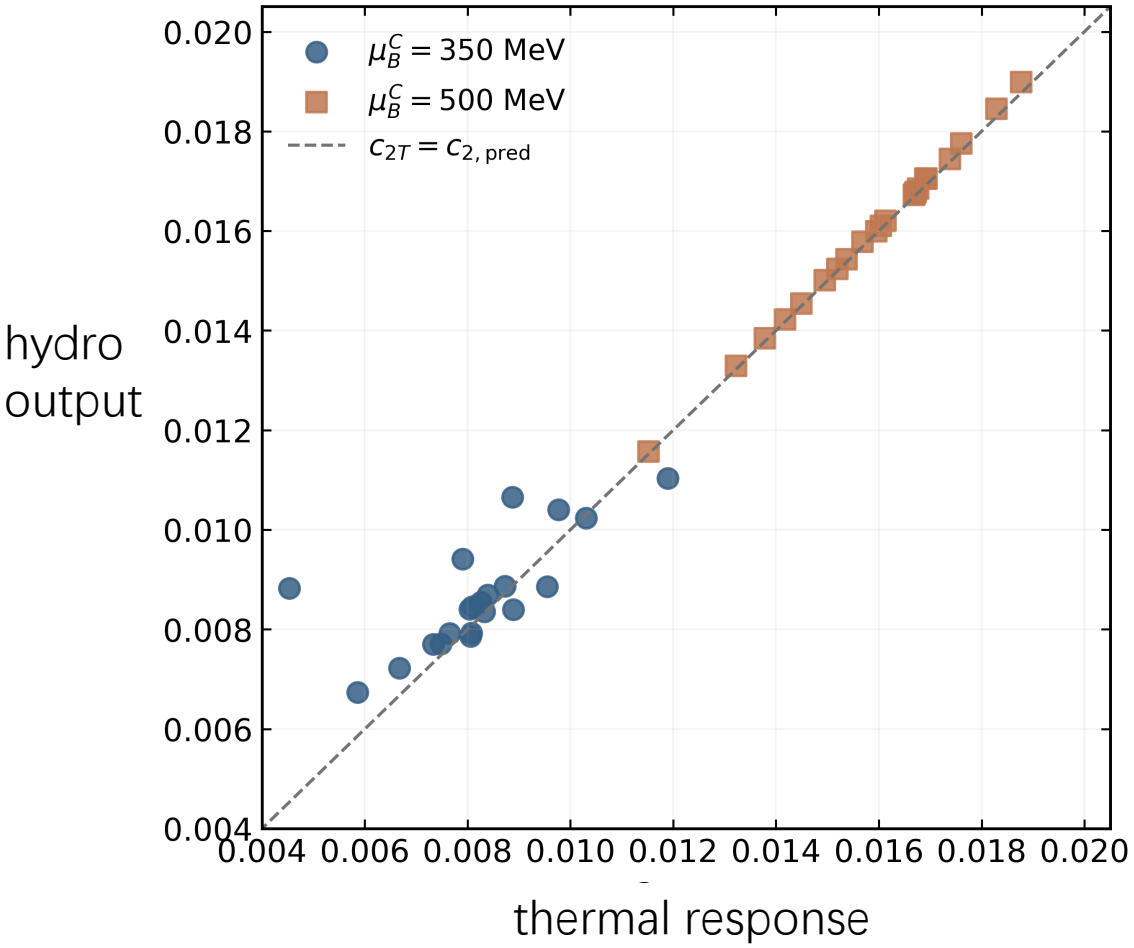
Numerical test of the thermal response in HIC: c_T



For μ_{BC} = 500 MeV (with no CEP effect):

- Amazingly well-captured by thermal resp, with r ~ 1.
- Note that these are event-by-event collisions in 0-5 %, NOT ideal thermal state!

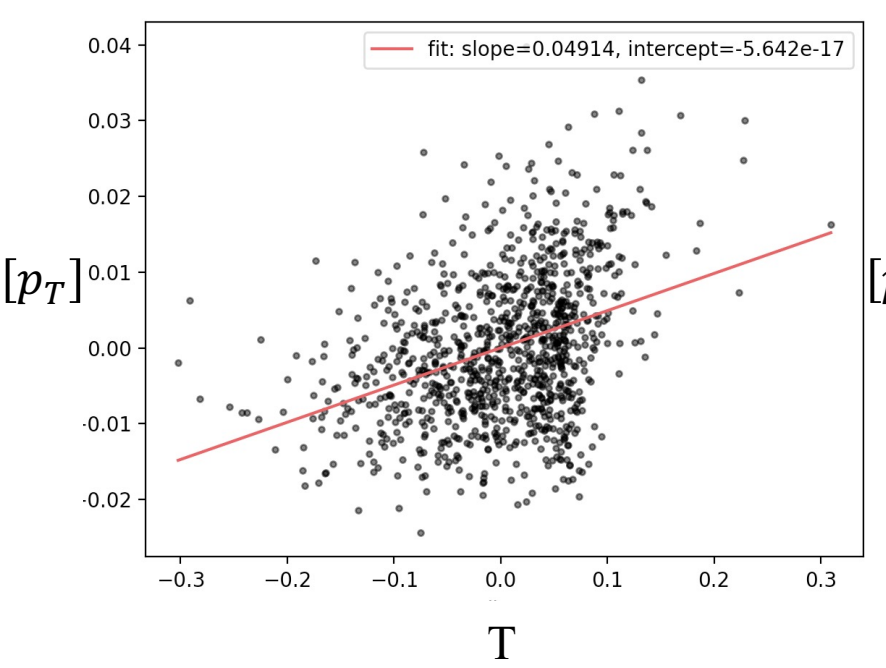
Numerical test of the thermal response in HIC: c_T

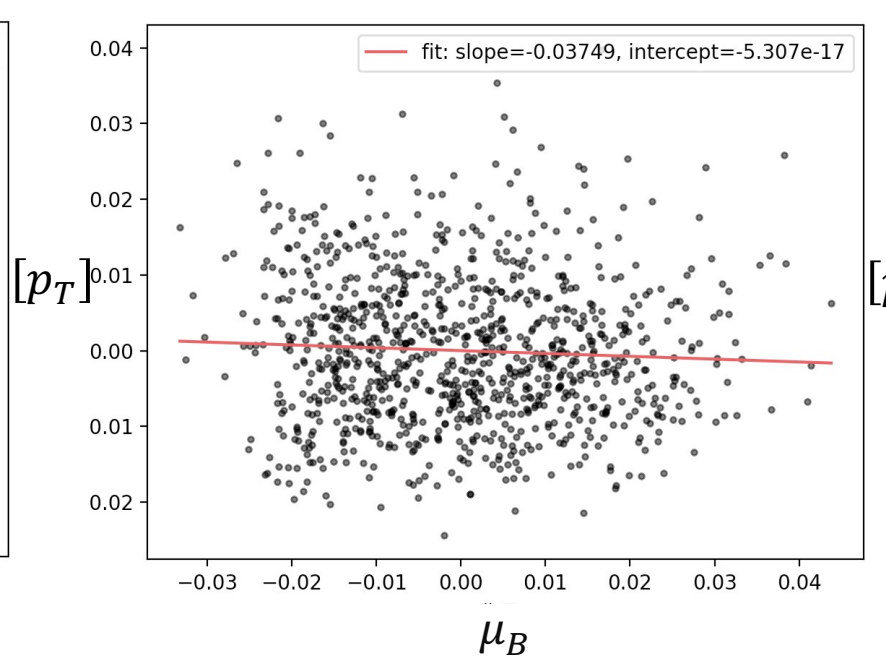


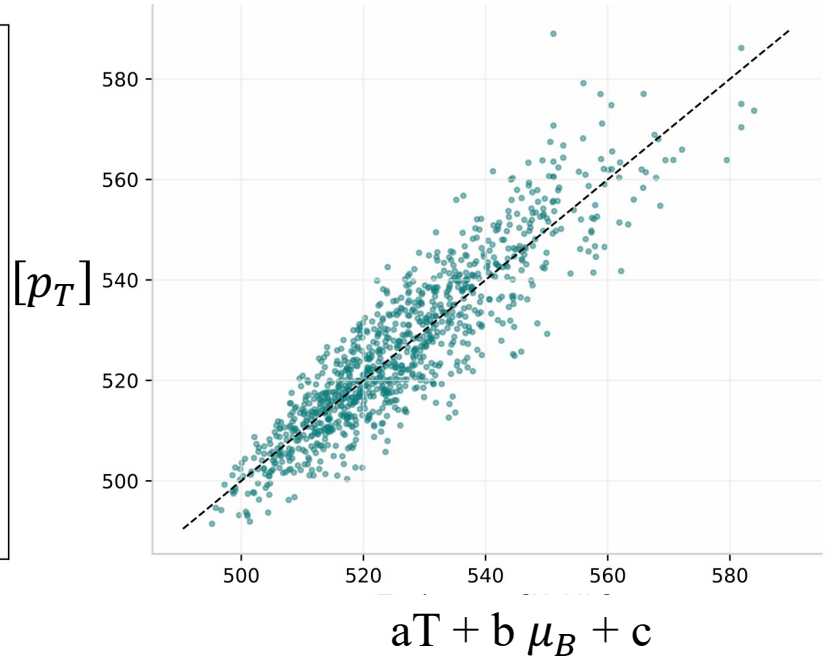
For $\mu_{BC} = 350$ MeV (with CEP effect):

- Overall good with r ~ 0.9;
- There seems to be a trend that thermal response breaks down when approaching CEP with non-linear order response, etc.

event-by-event $[p_T]$ vs (T, μ)







- Mean transverse momentum at finite μ_B: $[p_T]$ = aT + b μ_B + c

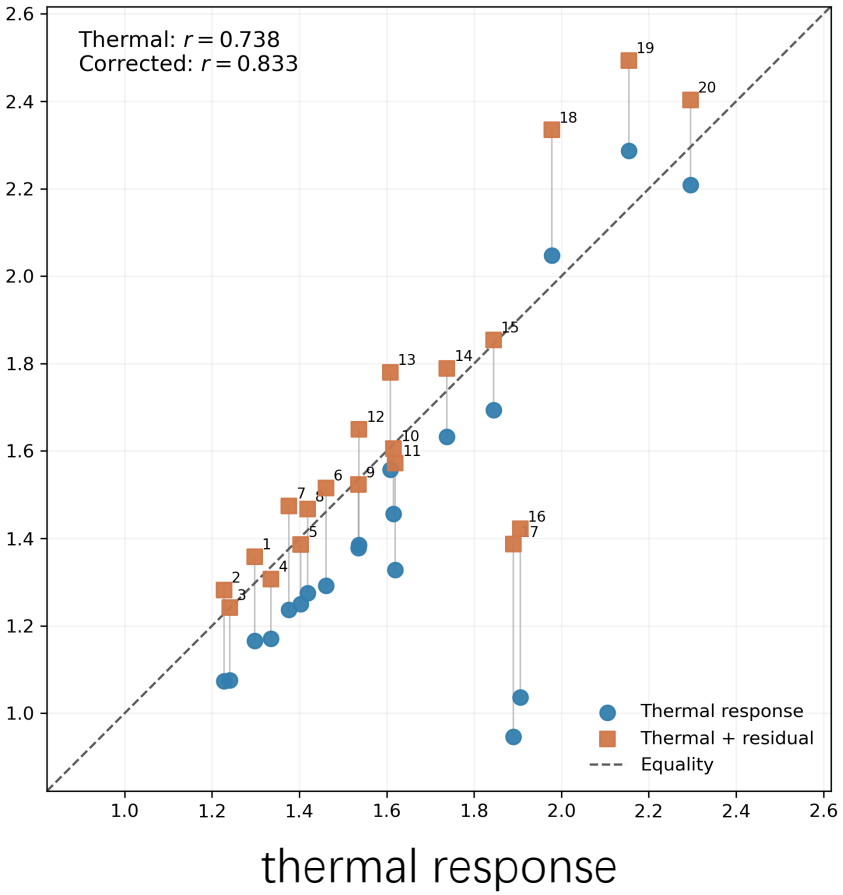
  kinetic contribution: $[p_T] \sim$ T (T_{local} + fireball expansion)

  chemical contribution: $[p_T] \sim \mu_B$ ($[p_T]$ of baryon > meson)

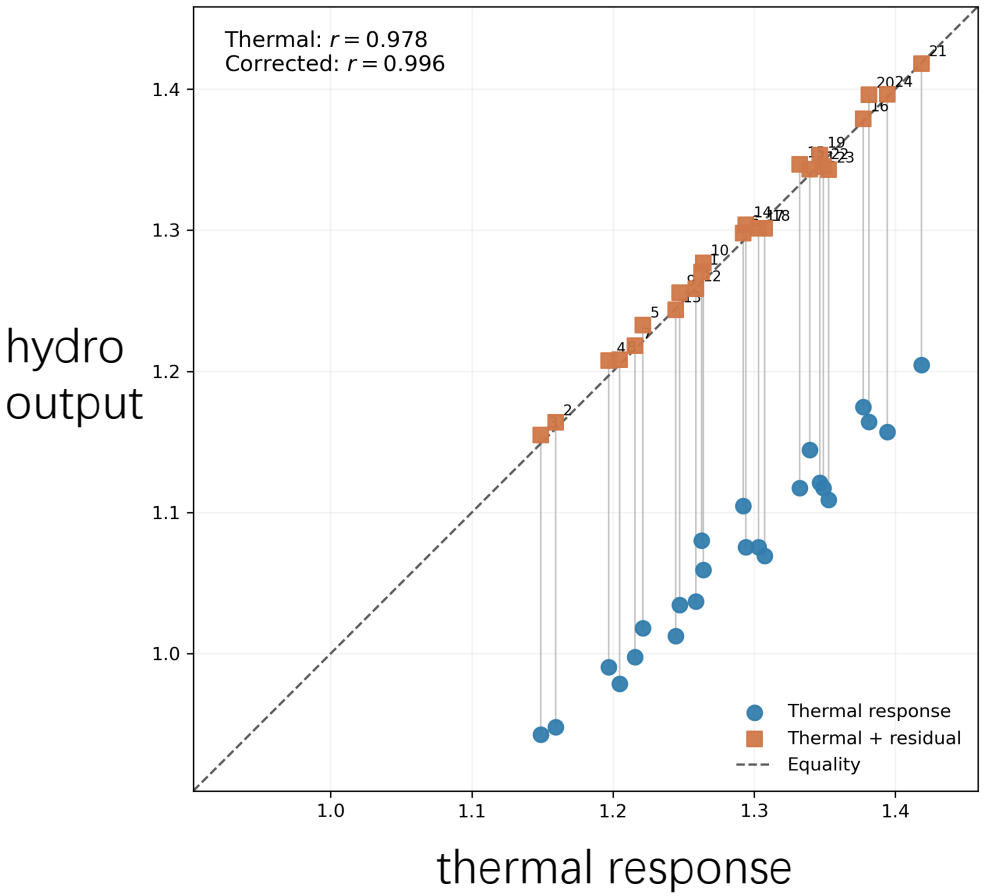
- Suppose a, b, c as smooth functions of T and μ_B

Numerical test of the thermal response in HIC: c_{p_T}

$\mu_{BC} = 350$ MeV



$\mu_{BC} = 500$ MeV



Fluctuations of $[p_T]$ is a sensitive probe of QCD EoS

Summary and conclusion

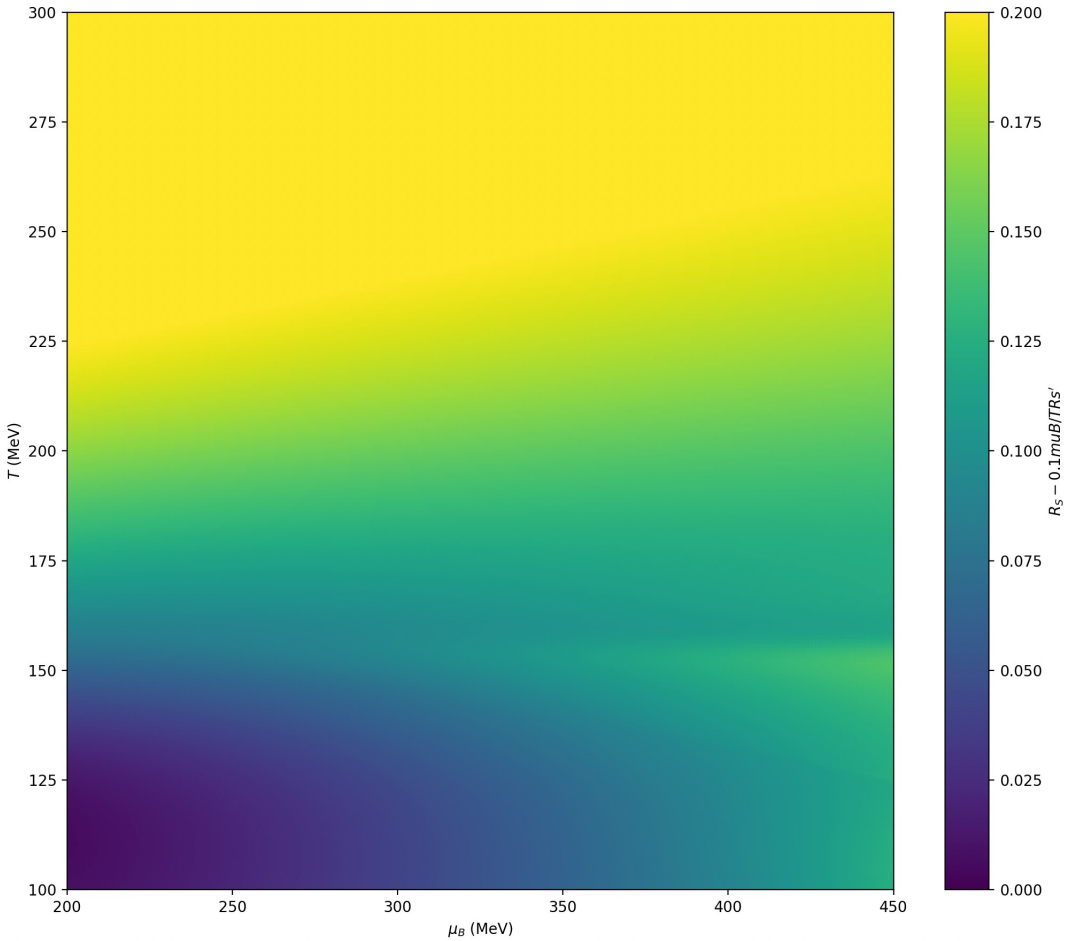
- We carry out event-by-event hydro simulations at finite muB with EoS embedded with CEP.
- Around 7.7 GeV, we tested and justified thermal response hypothesis.
- Fluctuations of intensive quantities probe sensitively QCD phase structure, in particular,

fluctuations of $[p_T]$ are potentially a smoking gun for the search of QCD CEP

Backup

Thermal response coefficient for $[p_T]$

$\mu_{BC} = 500$ MeV



$\mu_{BC} = 350$ MeV

