Thermal response and QCD CEP

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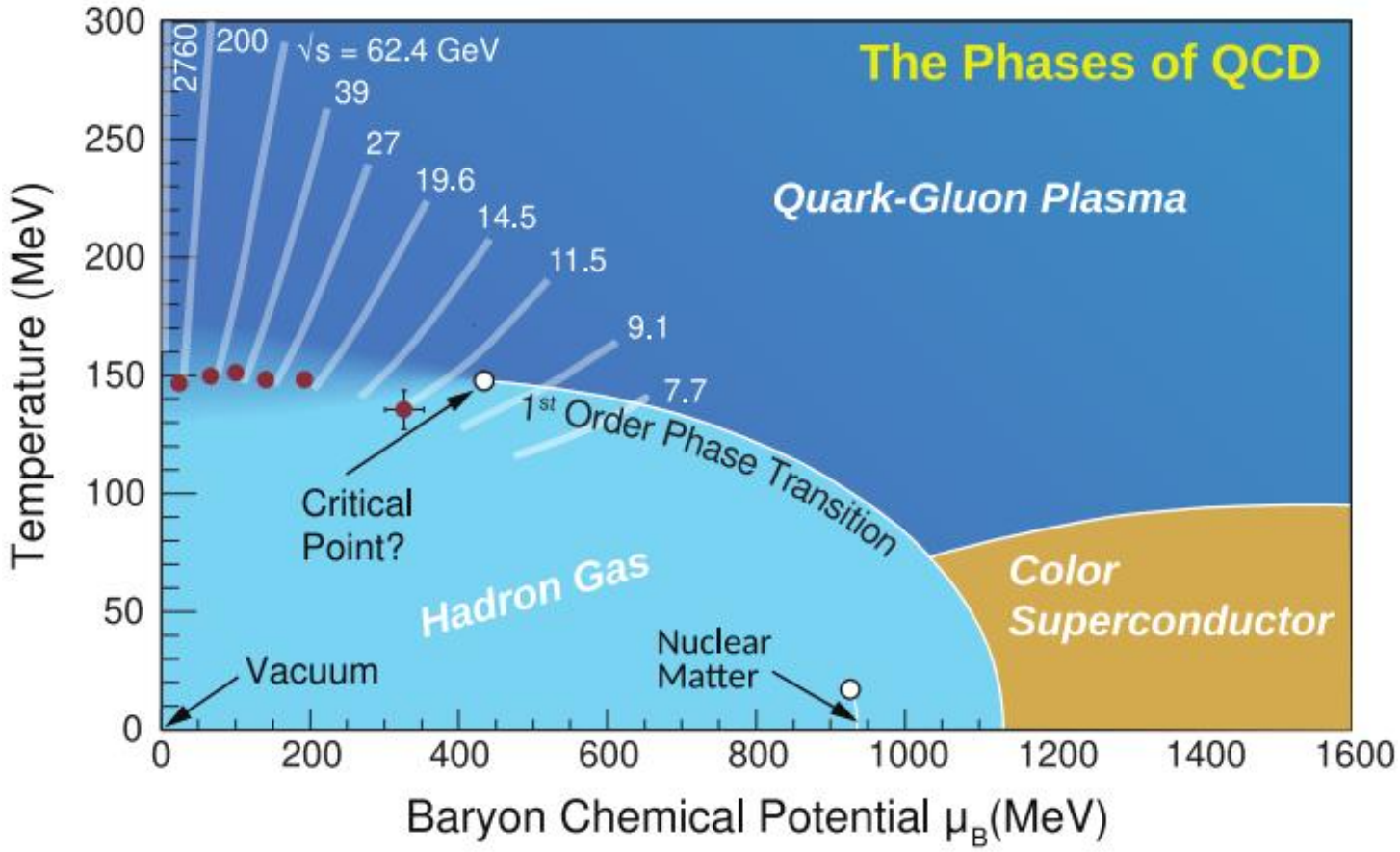
"QCD Critical End Point and Effective Field Theory Methods "
C3NT workshop, CCNU, Oct. 21, 2026



Based on Yu-Shan Mu, Li Yan and Xu-Guang Huang, arXiv:2609.14000

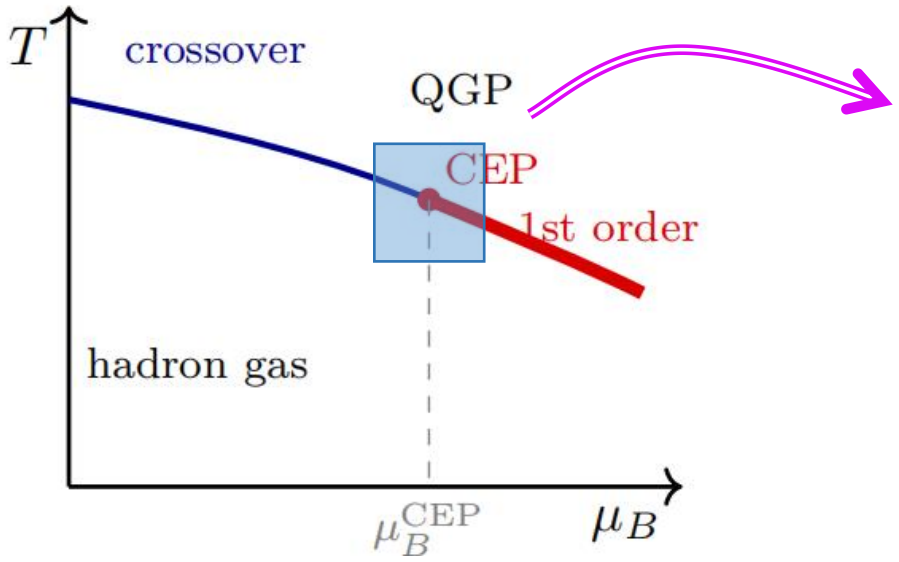
Scan QCD phases and CEP by heavy-ion collisions

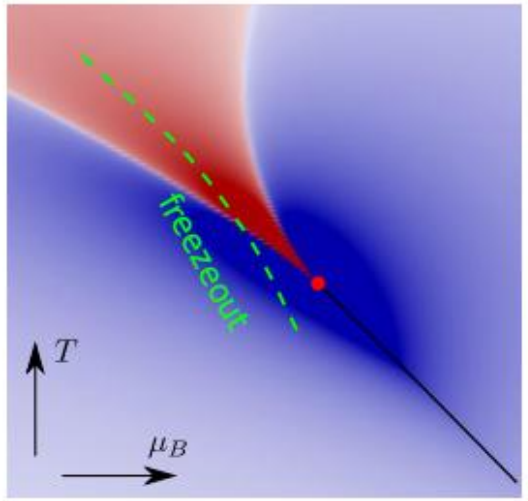
- How to detect QCD phases, and in particular, QCD CEP if it exists?

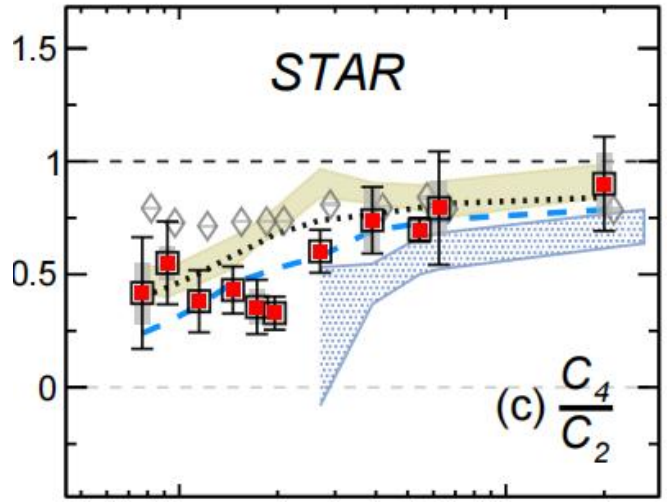


Current strategy: fluctuations of δn_B

- QCD phases and QCD CEP: [M. Stephanov 06', STAR 25']







- Freeze-out across QCD critical region,

$$\langle \delta n_B \delta n_B \rangle \sim \chi_B \sim \xi^{\gamma/\nu} \quad \ldots \quad \langle (\delta n_B)^m \rangle \sim \xi^x$$

Fluctuation–dissipation relation: two sides of the same coin

- Near equilibrium, fluctuations are related to thermal response by FDT

$$\begin{pmatrix} \Delta s \\ \Delta n_B \end{pmatrix} = X \begin{pmatrix} \Delta T \\ \Delta \mu_B \end{pmatrix}, \quad X = \begin{pmatrix} \chi_s & \chi_{TB} \\ \chi_{TB} & \chi_B \end{pmatrix}$$

Cumulants of n_B,

$$\langle \delta n_B \delta n_B \rangle \sim \chi_B$$

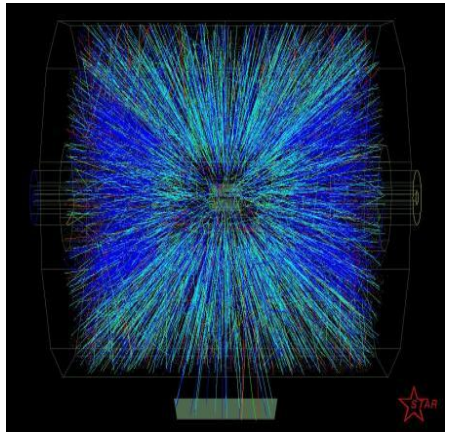
Variations of (T, μB) in response to variations of (s, n_B),

$$\chi^{-1} \sim \delta(T, \mu_B)/\delta(s, n_B))$$

Realizable in realistic central collisions

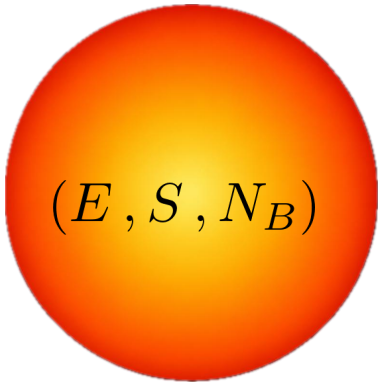
Particles from heavy-ion collisions = an effective QCD fireball

$(E\,,S\,,N_B)$

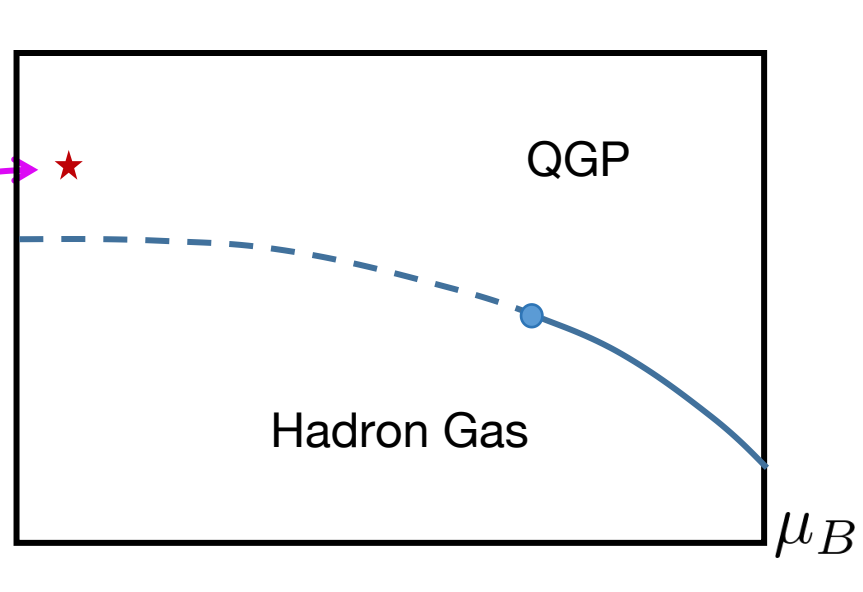


$([p_T]\,,N_{ch}\,,N_B)$

$\leftrightarrow$



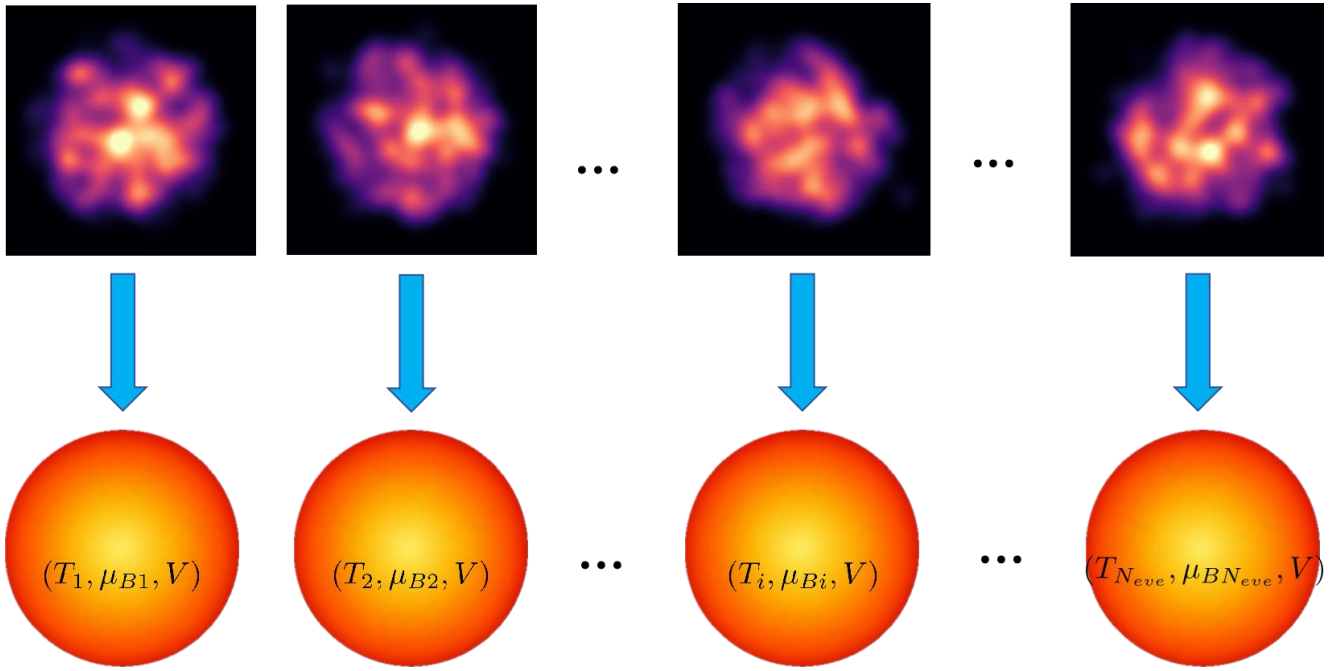
(T,μ_B,V)

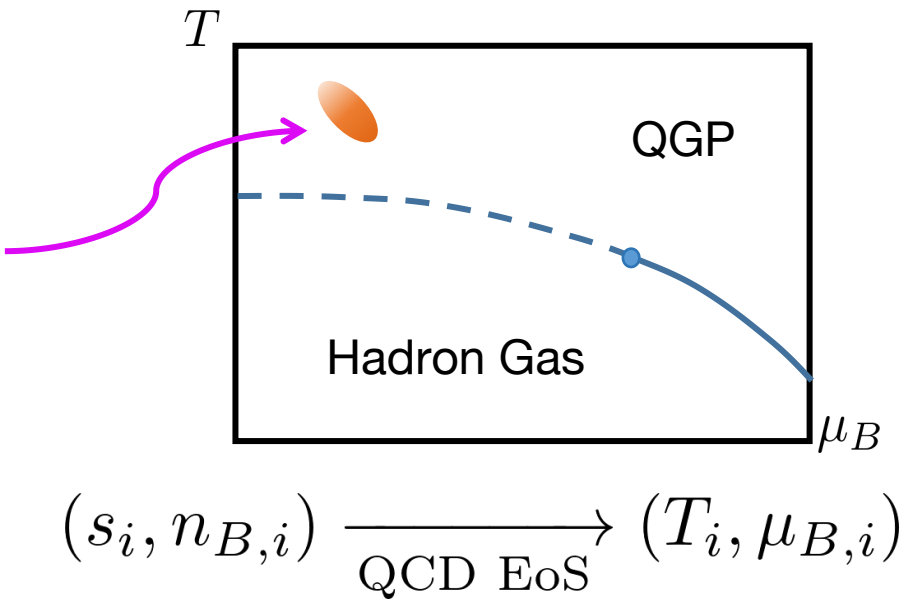


- Multi-Particle State (MPS) thermally equivalent to a smooth and uniform QCD fireball:

$$E^{\rm MPS} = E^{\rm eff} = \underbrace{e(T,\mu_B)}_{\text{QCD EoS}} V\,, \quad S^{\rm MPS} = S^{\rm eff} = \underbrace{s(T,\mu_B)}_{\text{QCD EoS}} V\,, \quad N_B^{\rm MPS} = N_B^{\rm eff} = \underbrace{n_B(T,\mu_B)}_{\text{QCD EoS}} V$$

Effective fireballs from event to event in one centrality class (UCC)





- Initial-state fluctuations provide natural variations of S and NB from event to event.
- Ideally, the central events would resemble the GCE of <T> and <μB>.

MPS in ultra-central collisions (UCC) detect c_s^2

speed of sound: ($\mu_B \approx 0$, LHC energies and top RHIC energies)

- Initial-state quantum fluct. introduce variations in entropy from event to event: ΔS

$$\text{const. V} \Rightarrow \quad \underbrace{\Delta T}_{\text{resp.}} = \underbrace{C_V^{-1}}_{\text{thermal resp. func.}} \underbrace{(T\Delta S)}_{\text{source}} = \frac{c_s^2}{S}(T\Delta S)\,, \qquad C_V \sim \chi_s$$

$$\Delta \log[p_T] = c_s^2 \Delta \log N_{ch}$$

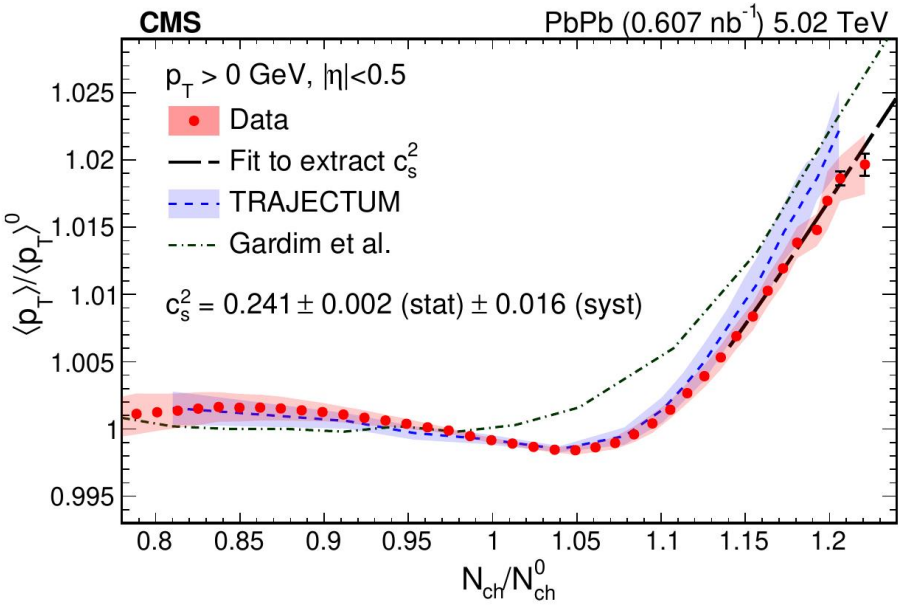
- Assumptions and conditions:

  1. Linear correlations: $[p_T] \sim T$, $N_{ch} \sim S$

  2. Saturated effective volume in UCC: $V = \text{const.}$

[F. Gardim et al, 19']

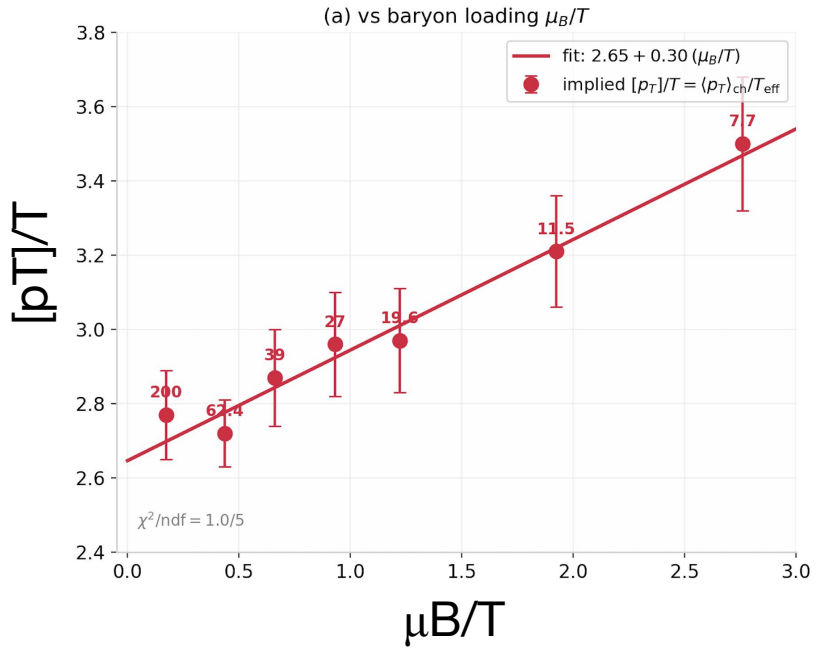
[CMS collaboration, 2401.06896]

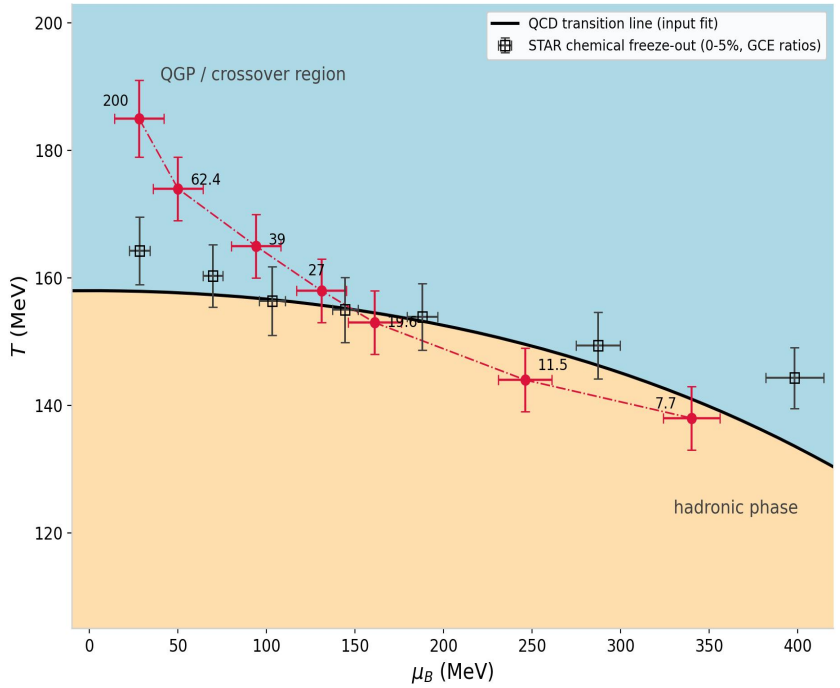


Effective (T, μ_B) in BES

- Location of effective T and μB can be estimated via exp. data:

$$\text{net baryons: } \frac{\langle \mu_{B,\text{eff}} \rangle}{\langle T_{\text{eff}} \rangle} \approx \frac{1}{2} \log \left(\frac{\langle N_p \rangle}{\langle N_{\bar{p}} \rangle} \right) \qquad \text{system energy: } \frac{\langle [p_T] \rangle}{\langle T_{\text{eff}} \rangle} = r(\langle \mu_{B,\text{eff}} \rangle / \langle T_{\text{eff}} \rangle)$$





[STAR 17']

The ratio <[pT]>/<T> = r (μB, T) is determined via hydro simulations.

Near-thermal condition of central collisions

* Note that variations of S and N_B may not be sufficiently small.

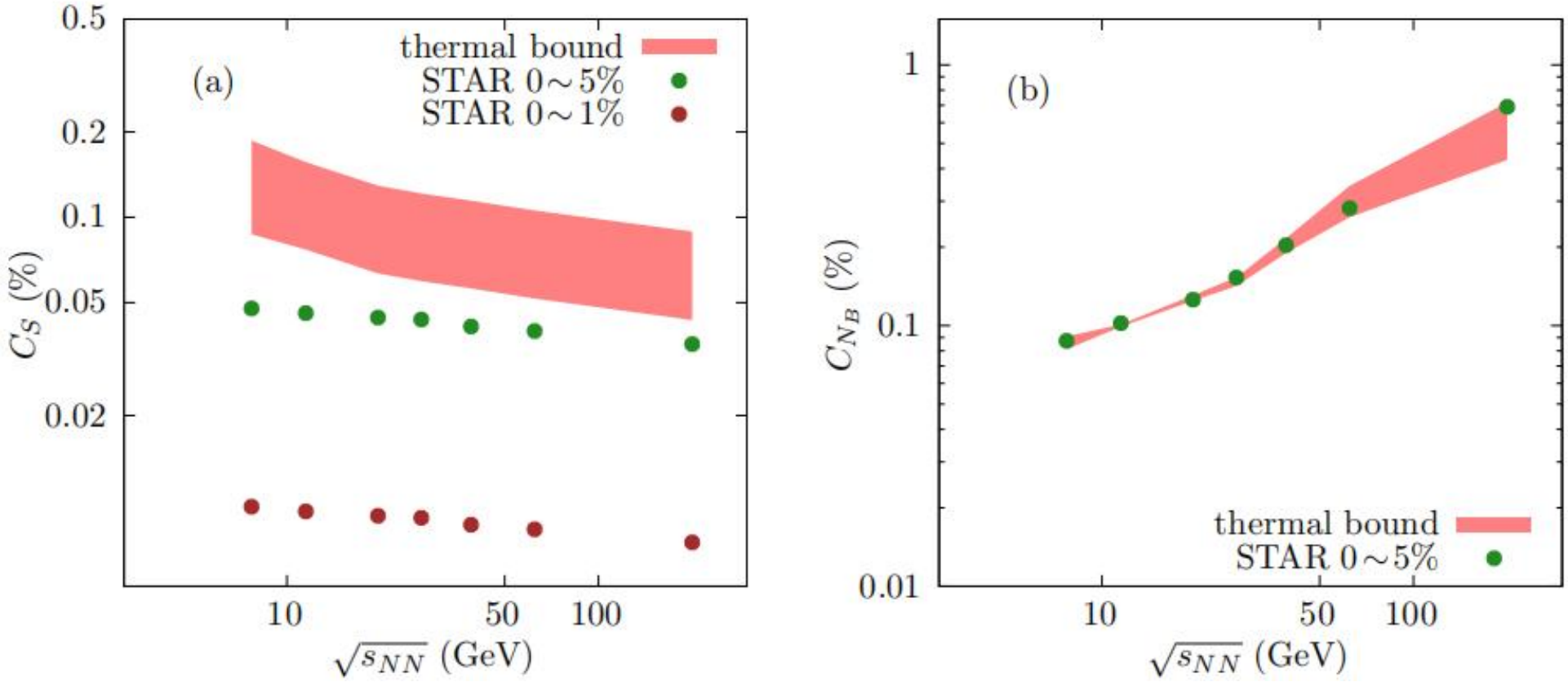
Thermal bounds on variations from thermal GCE probability,

$$P(S, N_B) \propto \exp\left[-\frac{V}{2T}\Delta Y^T X^{-1} \Delta Y\right], \qquad \Delta Y = (\Delta S, \Delta N_B)$$

$$\Rightarrow \quad C_s \equiv \frac{\sqrt{\langle(\Delta S)^2\rangle}}{\langle S\rangle} \leq C_s^{\rm th} = \frac{\sqrt{T\chi_s}}{s}, \quad C_{N_B} \equiv \frac{\sqrt{\langle(\Delta N_B)^2\rangle}}{\langle N_B\rangle} \leq C_{N_B}^{\rm th} = \frac{\sqrt{T\chi_B}}{n_B}$$

- Thermal bounds in realistic collisions are evaluated at the mean values of T and μB
- Cs ≈ $C_{\rm Nch}$ (via S ~ Nch), both Cs and $C_{\rm NB}$ are measurable.

Thermalization condition confronts STAR BES data: near-thermal state



measured fluctuations $\leq$ thermal bound: small deviations around a thermal state

[STAR 21', We have used MUSES EoS, 2402.0863]

Thermodynamic response at finite baryon chemical potential

Response of intensive variables to variations of extensive densities: inverse susceptibility matrix

$$\begin{pmatrix} \Delta T \\ \Delta \mu_B \end{pmatrix} = X^{-1} \begin{pmatrix} \Delta s \\ \Delta n_B \end{pmatrix} = \frac{1}{D} \begin{pmatrix} \chi_B & -\chi_{TB} \\ -\chi_{TB} & \chi_s \end{pmatrix} \begin{pmatrix} \Delta s \\ \Delta n_B \end{pmatrix}, \qquad D = \det X = \chi_s \chi_B - \chi_{TB}^2$$

In relative variations:

$$\frac{\Delta T}{\langle T \rangle} = R_s \frac{\Delta s}{\langle s \rangle} - R_{N_B} \frac{\Delta n_B}{\langle n_B \rangle}, \qquad \frac{\Delta \mu_B}{\langle \mu_B \rangle} = -\frac{\langle T \rangle}{\langle \mu_B \rangle} \left[\frac{\chi_{TB}}{\chi_B} R_s \frac{\Delta s}{\langle s \rangle} - \frac{\chi_s}{\chi_{TB}} R_{N_B} \frac{\Delta n_B}{\langle n_B \rangle} \right]$$

with the two response coefficients

$$\boxed{R_s = \frac{\langle s \rangle}{\langle T \rangle} \frac{\chi_B}{D}, \qquad R_{N_B} = \frac{\langle n_B \rangle}{\langle T \rangle} \frac{\chi_{TB}}{D}}$$

NB: The resp. form follows from GCE analysis, which could break down in the vicinity of CEP.

Physical content of the response coefficients

- $\mu_B \to 0$ limit: $R_s \to c_s^2$, recovering the UCC result at LHC

$$c_s^2 = \left(\frac{\partial P}{\partial e}\right)_\sigma = \frac{\Delta \log T}{\Delta \log S} = \frac{\Delta \log[p_T]}{\Delta \log N_{\rm ch}}$$

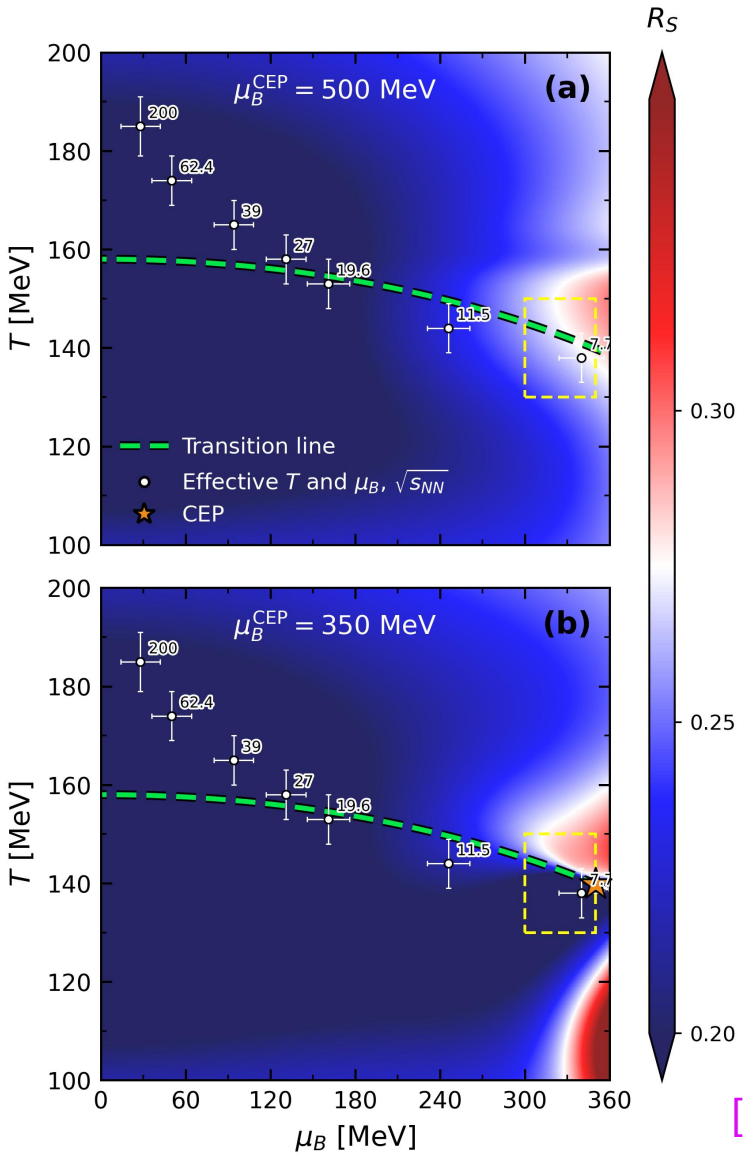
- General relation with speed of sound:

$$c_s^2 = \frac{n_B^2}{(e+P)D}\left(\sigma^2 \chi_B - 2\sigma \chi_{TB} + \chi_s\right) \ , \qquad \sigma \equiv s/n_B$$

- UCC with $\Delta\sigma \simeq 0$ selection: exact linear relation, $\dfrac{\Delta[p_T]}{\langle [p_T] \rangle} \simeq c_s^2 \, \dfrac{1+\mu_B/(\sigma T)}{1+\lambda/\sigma} \, \dfrac{\Delta N_{\rm ch}}{\langle N_{\rm ch} \rangle}$,

$$\lambda \equiv \left(\frac{\partial \mu_B}{\partial T}\right)_\sigma$$

Sensitivity to QCD CEP: Rs



Response coefficients inversely proportional to susceptibilities

$$R_s\,, R_{N_B} \sim \chi^{-1}$$

- sensitive to phase structures.
- Near QCD CEP, Rs and R_{NB} are expected to diverge.

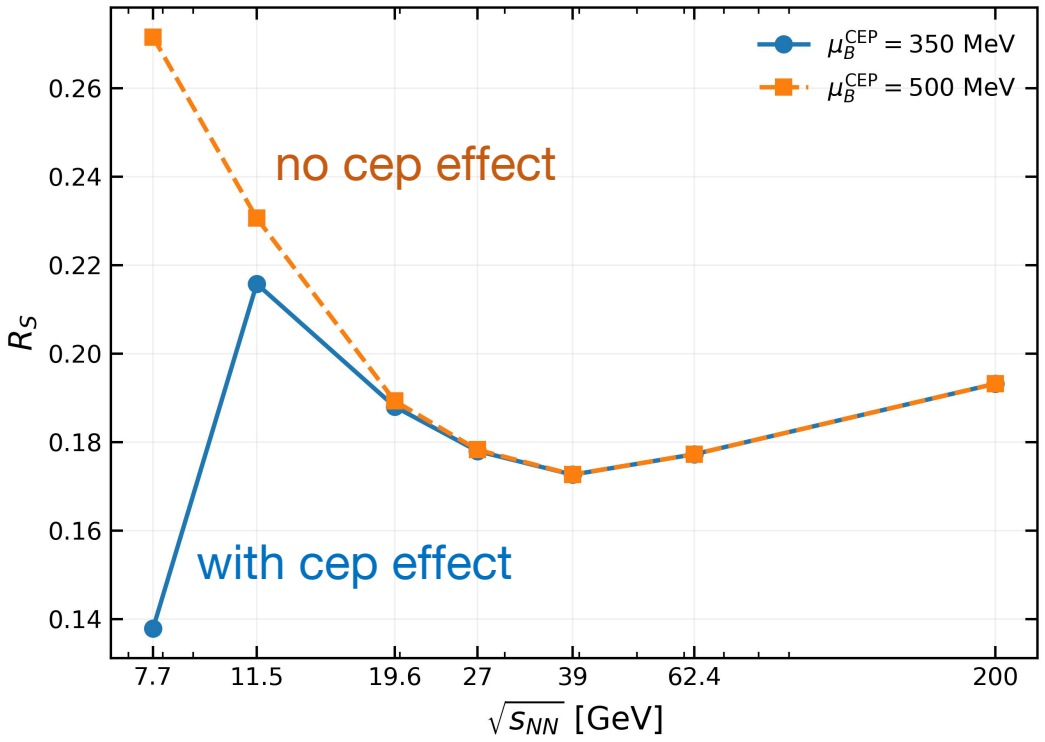
$$R = R^{\mathrm{reg}} + R^{\mathrm{sing}}$$

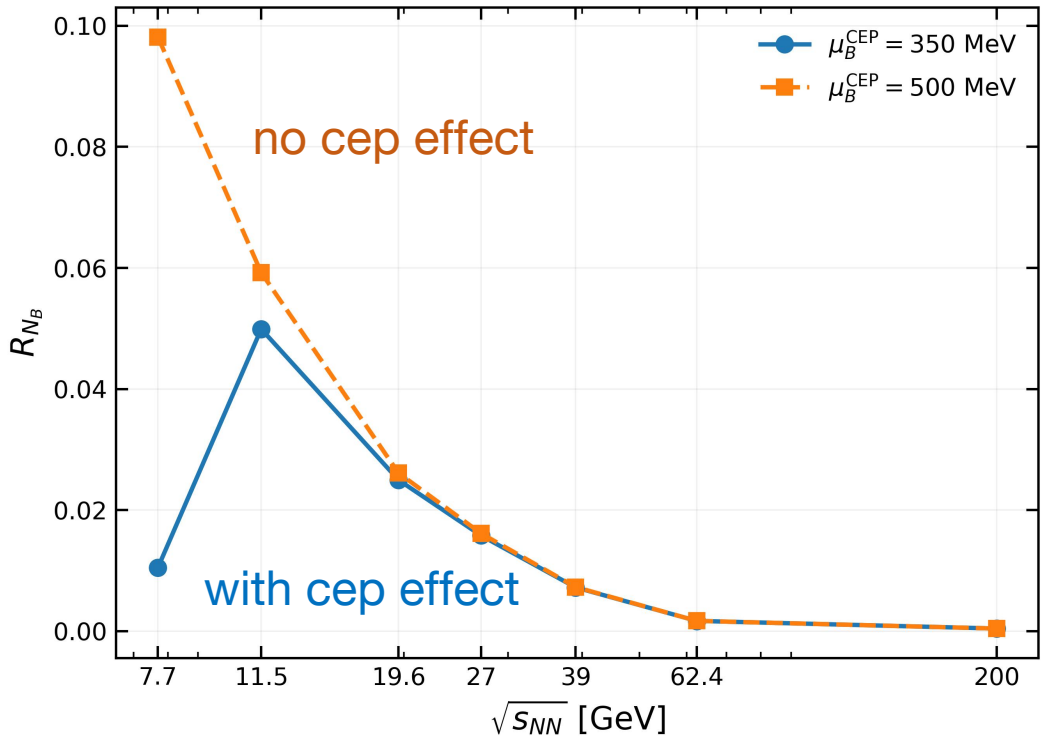
- A quantitative/realistic test, see the next talk by Yu-Shan

hydro + lattice-based EoS with CEP

[MUSES EoS, 2402.0863]

CEP effects across BES on thermal response





Thermal resp. coefficients are sensitive to CEP, as expected

Summary and conclusion

- We propose to study thermal resp. as probe of QCD CEP:

$$\chi's \text{ in the thermal response coefficents.}$$

- In central collisions (<5%), thermal resp. is well approached in experiments.
- Fluct. of T and muB in central collisions are determined by thermal resp. (hypothesis),

$$\frac{\Delta T}{\langle T\rangle} = R_s \frac{\Delta s}{\langle s\rangle} - R_{N_B} \frac{\Delta n_B}{\langle n_B\rangle} \qquad \frac{\Delta \mu_B}{\langle \mu_B\rangle} = -R'_s \frac{\Delta s}{\langle s\rangle} + R'_{N_B} \frac{\Delta n_B}{\langle n_B\rangle}$$

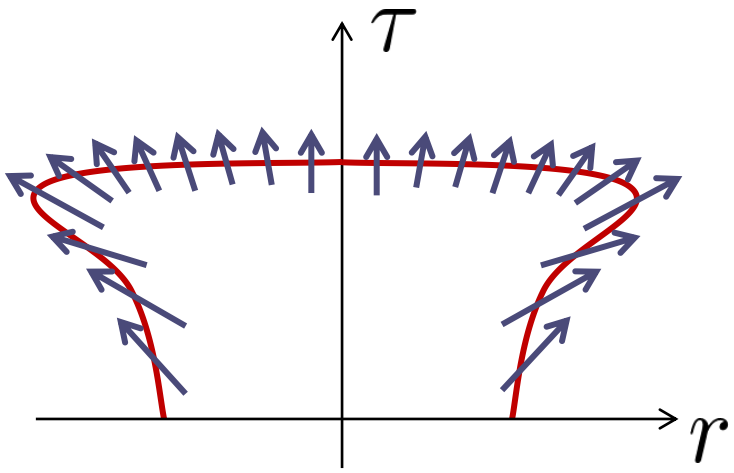
Determine T and μ_B with Hydro modeling (2+1D)

$T^{\mu\nu}$, J^{μ}_{B} — T_c, μ_{Bc}, w and ρ [MUSES EoS, 2402.0863]

TRENTo + MUSIC + MUSES EoS + iSS + UrQMD

sample *s* and n_B independently,

- Extract *T* and μ_B from the hydro freeze-out surface

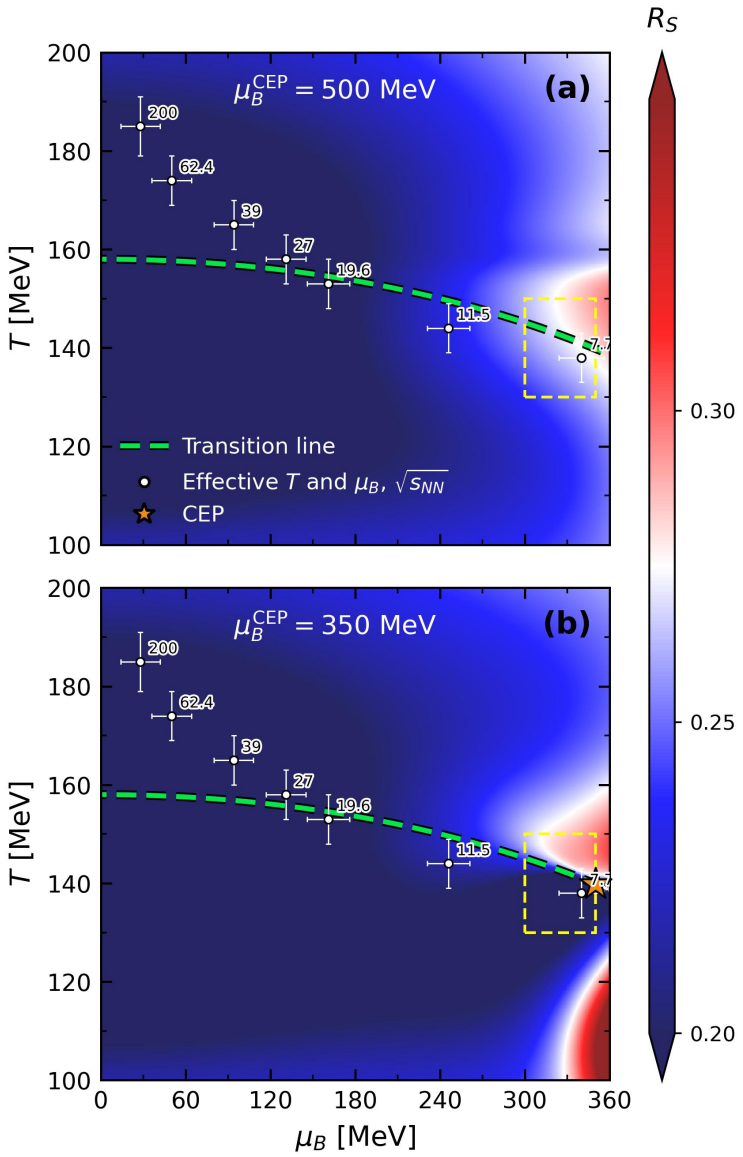


$$\begin{cases} E = \int_\Sigma d\sigma_\mu T^{0\mu} = e^{EOS}(T,\mu_B)V \\ S = \int_\Sigma d\sigma_\mu s^{\mu} = s^{EOS}(T,\mu_B)V \\ N_B = \int_\Sigma d\sigma_\mu n_B^{\mu} = n_B^{EOS}(T,\mu_B)V \end{cases}$$

* We consider (at least transient) thermalization of QCD system above 3.5 GeV in BES.

[(NCQ scaling) STAR collaboration, 2504.02531]

Sensitivity to QCD CEP -- R_{NB}



Response coefficients inversely proportional to susceptibilities

$$R_s\,, R_{N_B} \sim \chi^{-1}$$

- sensitive to phase structures.
- Near QCD CEP, Rs and R_{NB} are expected to diverge.

$$R = R^{\mathrm{reg}} + R^{\mathrm{sing}}$$

- A quantitative/realistic test, see the next talk by Yu-Shan

hydro + lattice-based EoS with CEP

Volume saturation and correction

- In the UCC, as volume saturates, the resp. can be extracted with specified variations, Cs=C_{Nch}, C_{NB}, e.g.,

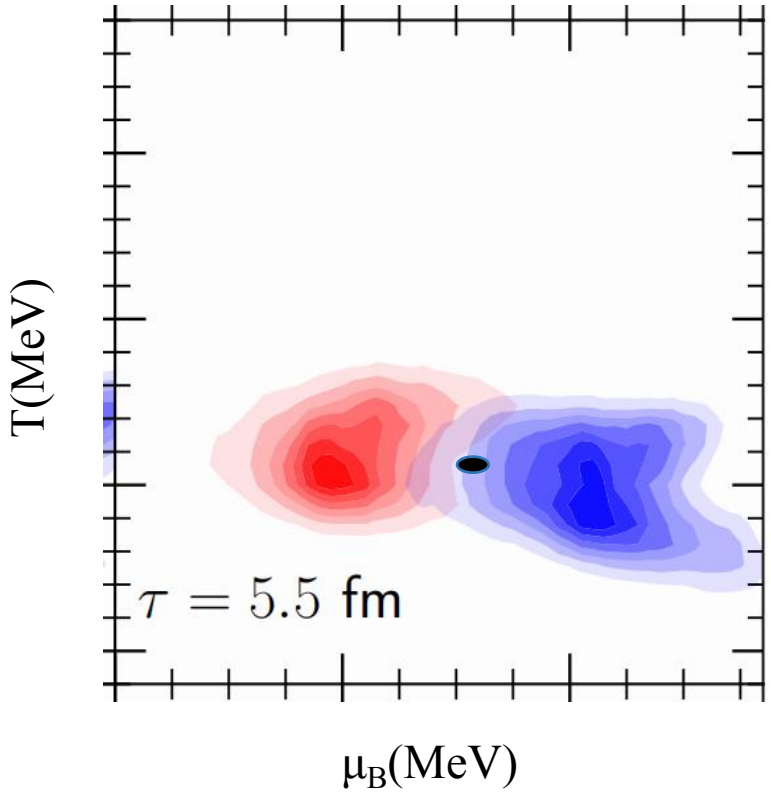
$$\frac{\Delta T}{\langle T\rangle} = R_s \frac{\Delta s}{\langle s\rangle} - R_{N_B} \frac{\Delta n_B}{\langle n_B\rangle} \quad \rightarrow \quad \frac{\Delta T}{\langle T\rangle} = R_s \frac{\Delta S}{\langle S\rangle} - R_{N_B} \frac{\Delta N_B}{\langle N_B\rangle}$$

- Without volume saturation,

$$\frac{\Delta T}{\langle T\rangle} = R_s \frac{\Delta s}{\langle s\rangle} - R_{N_B} \frac{\Delta n_B}{\langle n_B\rangle} = R_s \left(\frac{\Delta S}{\langle S\rangle} - \frac{\Delta V}{\langle V\rangle}\right) - R_{N_B} \left(\frac{\Delta N_B}{\langle N_B\rangle} - \frac{\Delta V}{\langle V\rangle}\right)$$

Why thermal fluctuations are sub-leading

1 event



VS

1k events

